Dispersive representation of the $K\pi$ and $\pi\pi$ form factors: application to hadronic τ decays

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Quark confinement and the hadron spectrum XI
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Outline

1. Introduction and Motivation

2. Description of the hadronic form factors

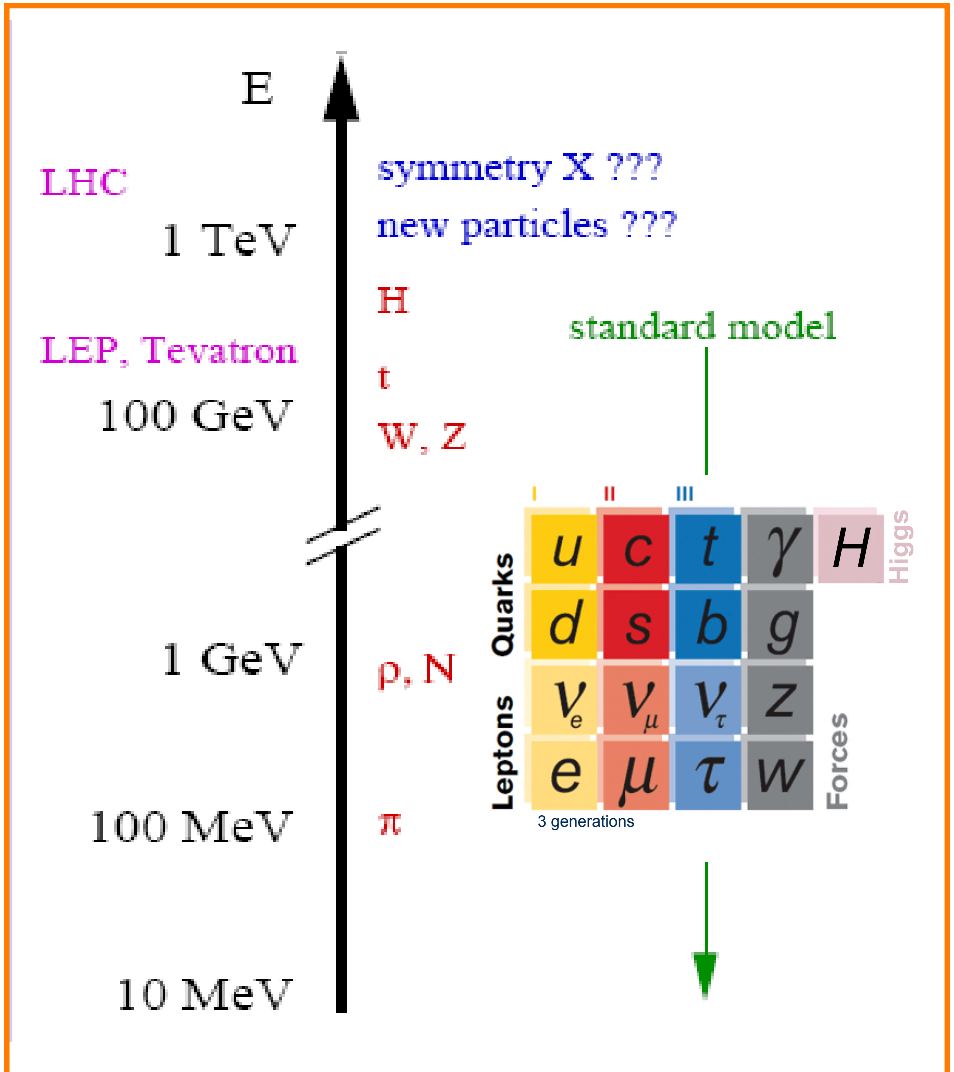
3. Probing lepton flavour violation in the Higgs sector with hadronic τ decays

4. Determination of V_{us} from $\tau \rightarrow K\pi\nu_{\tau}$ decays

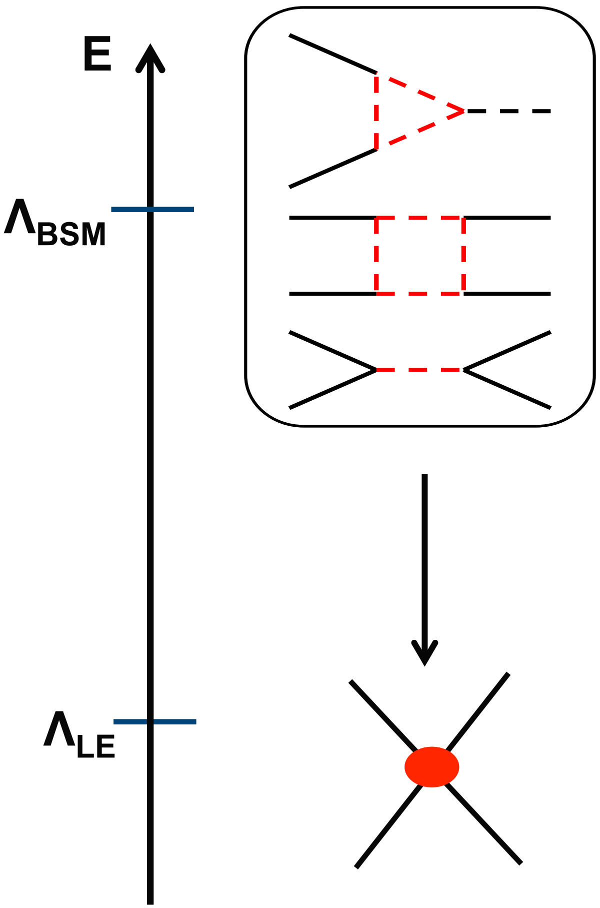
5. Conclusion and Outlook

1. Introduction and Motivation

1.1 Searching for physics beyond the SM



1.1 Searching for NP: Low energy & Colliders



High energy: → if Λ_{BSM} ~ TeV ⇨ sensitive to new resonances, direct discovery

→ if Λ_{BSM} >> TeV ⇨ EFT approach

$$pp \to R \qquad R = Z', h, \tilde{\nu}, l\ldots$$

Low energy: if Λ_{LE} << Λ_{BSM} ⇨ EFT approach sensitive to scale + flavour structure of couplings

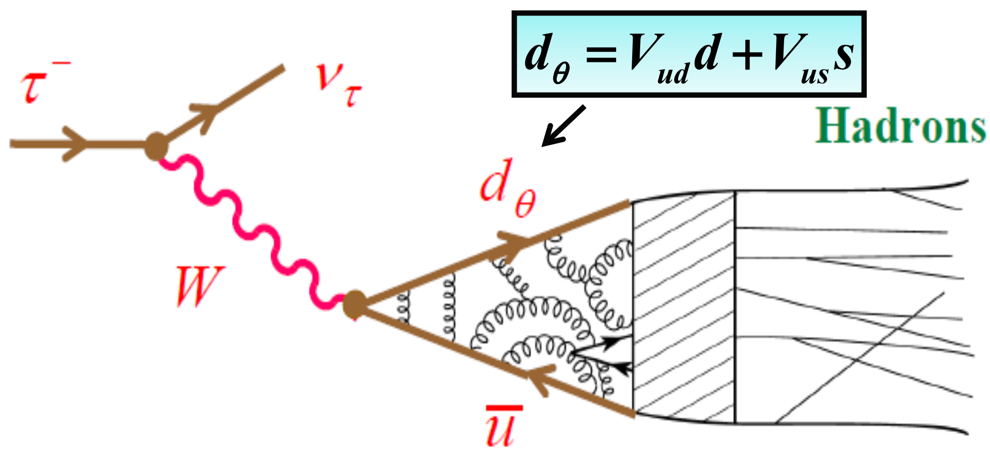
$$L = L_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

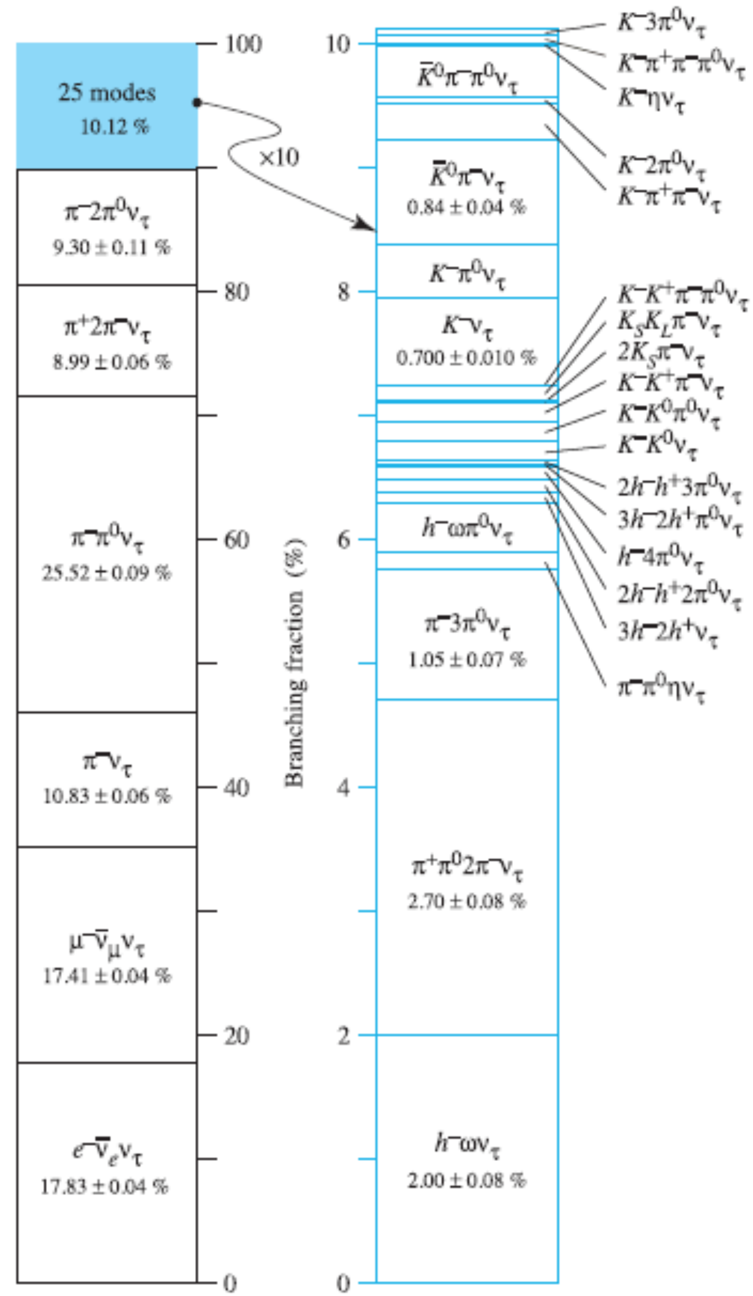
⇨ Reconstruct the underlying dynamics

PDG'14

1.2 Hadronic τ-decays

- τ: The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !





PDG'14

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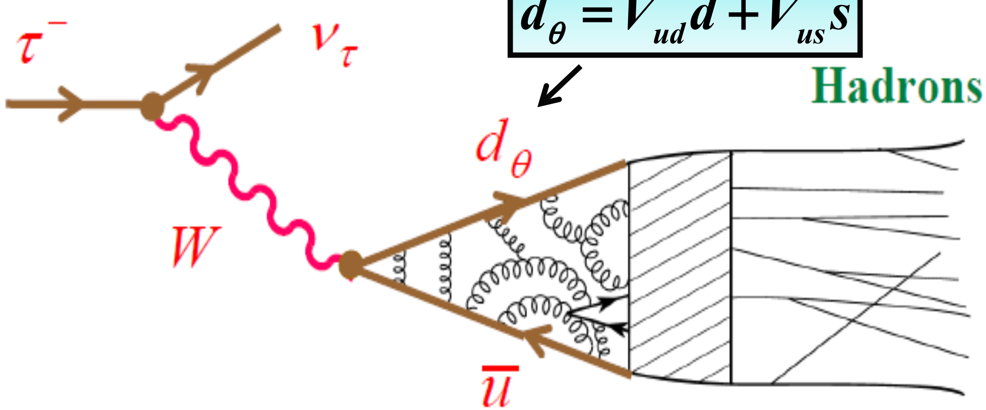
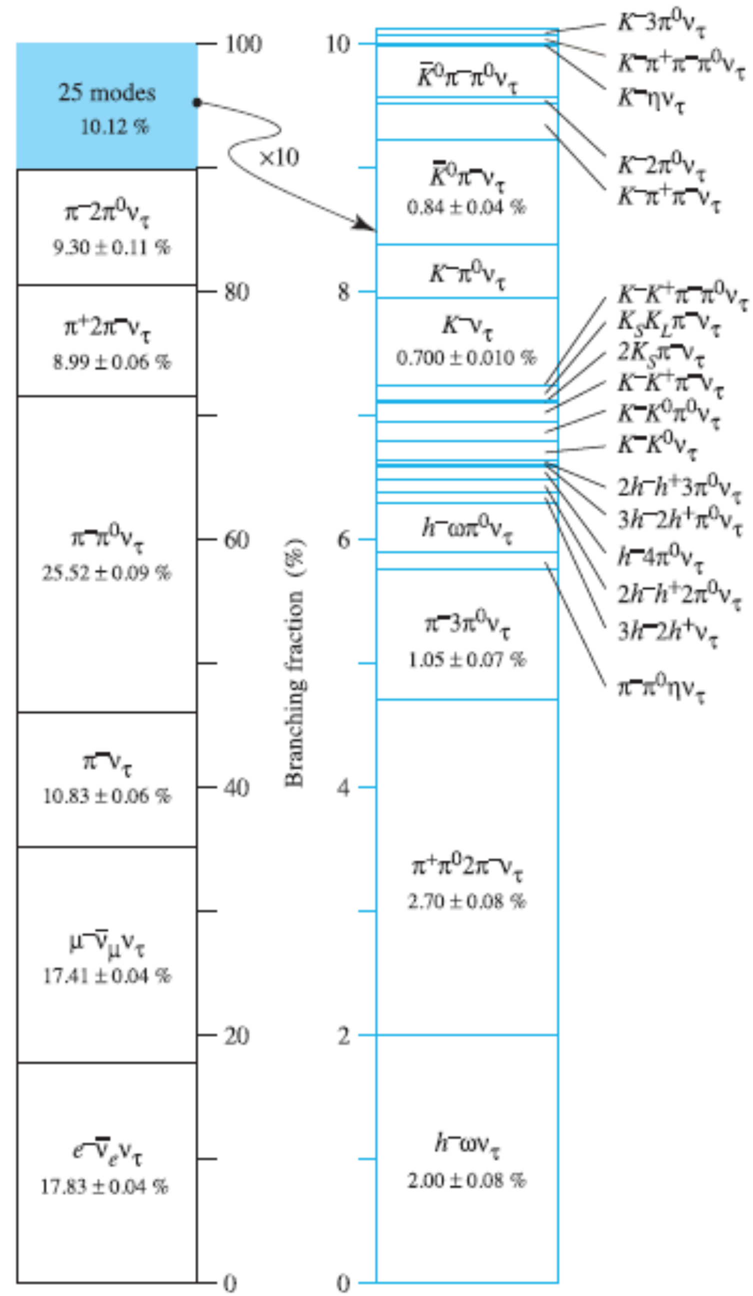
$$d_\theta = V_{ud}\, d + V_{us}\, s$$



Very rich phenomenology

Test of QCD and EW interactions

- For the tests:
  - Precise measurements needed
  - Hadronic uncertainties under control



1.2 Exclusive hadronic process $\tau \to PP\nu_\tau$

- Experimental measurement : decay rate

$$\boxed{d\Gamma\left(\tau \to H\nu_\tau\right) = \frac{G_F^2}{4m_\tau}\left|V_{CKM}\right|^2 L_{\mu\nu} H^{\mu\nu} dP_S}$$

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- The amplitude:

$$\boxed{M\left(\tau \to H\nu_\tau\right) = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}_{\nu_\tau} \gamma^\mu \left(1-\gamma^5\right) u_\tau H_\mu}$$

$$\boxed{H_\mu = \left\langle PP\right| \left(V_\mu - A_\mu\right) e^{iL_{QCD}} \left|0\right\rangle = \left(Lorentz\ \text{struct.}\right)_\mu^i F_i\left(q^2\right)}$$

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$$q^2 = s = \left(p_P + p_P\right)^2$$

- Challenge : determination of the form factors to extract SM parameters or NP

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parametrization of the ffs

SM param., NP

ChPT + Analiticity + Unitarity
Dispersion Relations
Models

Experimental Data
TAUOLA etc

FFs: masses, widths,couplings

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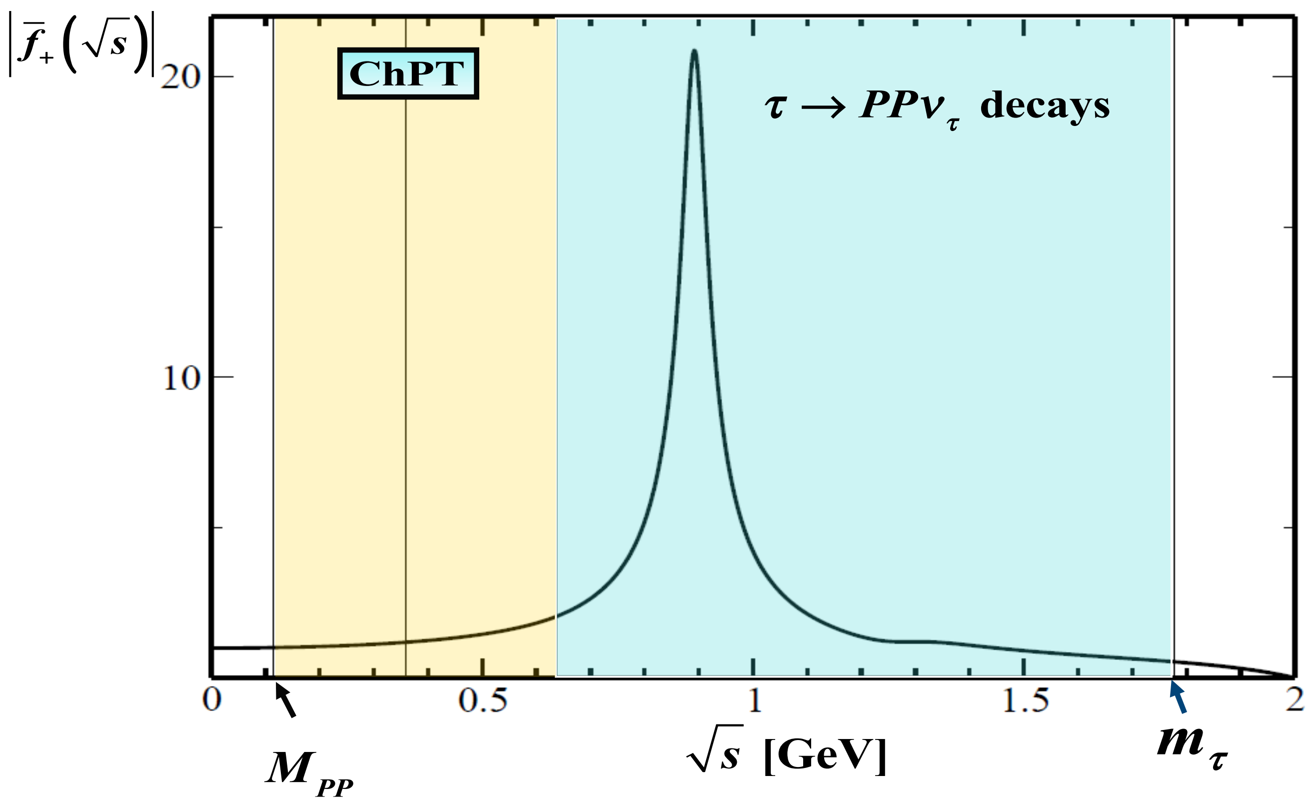
2. Description of the hadronic form factors

2.1 The challenge

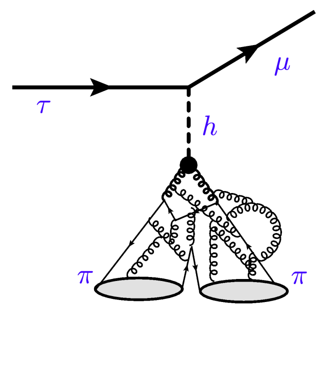
- $\boxed{M_{PP} < E < m_\tau \sim 1.77 \text{ GeV}}$ ⟹ need a description beyond E = 1GeV

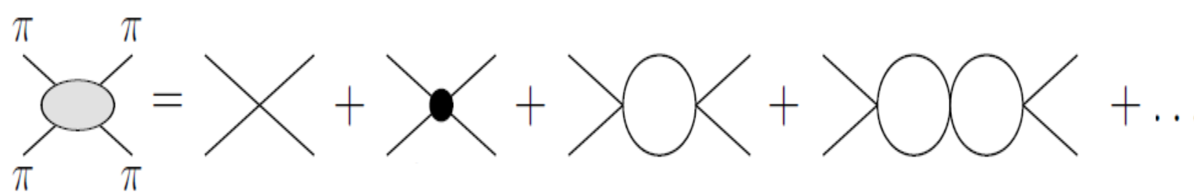
- ChPT is not valid on the full range: describe dynamics of π, K, η not resonances



2.1 The challenge

- *Large final state interactions for* PP:





➡ need to use *dispersion relations* to make the extrapolation from ChPT to higher energy

2.2 Dispersion relations: Method

- Unitarity ⇒ the discontinuity of the form factor is known

$$\boxed{\frac{1}{2i}\, disc\ F_{PP}(s) = \mathrm{Im}\, F_{PP}(s) = \sum_n F_{PP\to n}\left(\mathrm{T}_{n\to PP}\right)^*}$$

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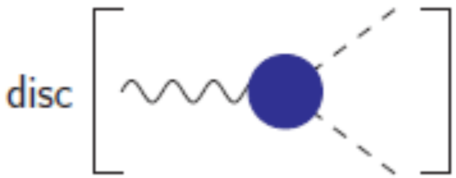

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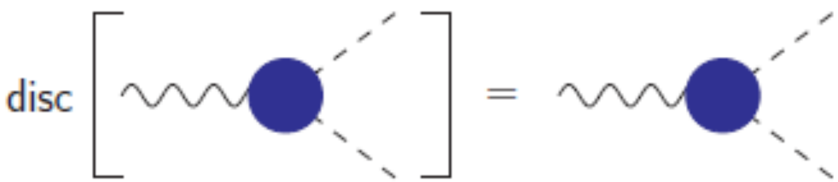

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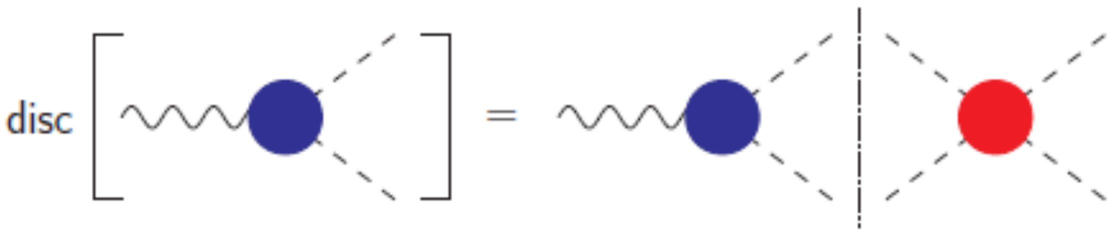

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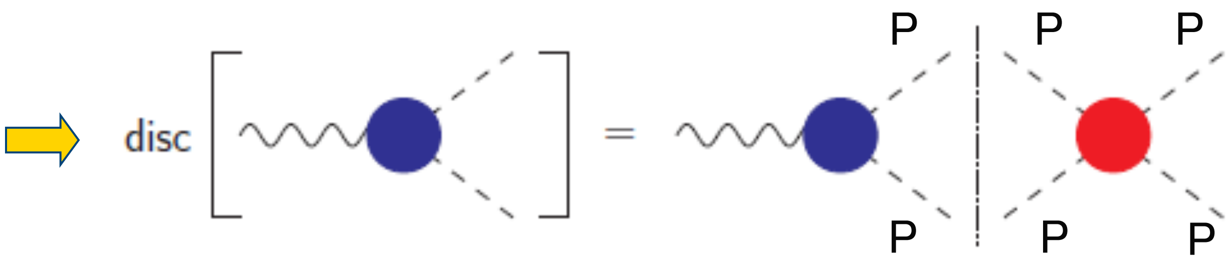
- Only one channel n = PP (elastic region)



$$\boxed{\frac{1}{2i}\, disc\; F_I(s) = \mathrm{Im}\, F_I(s) = F_I(s)\;\; \sin\delta_I(s) e^{-i\delta_I(s)}}$$

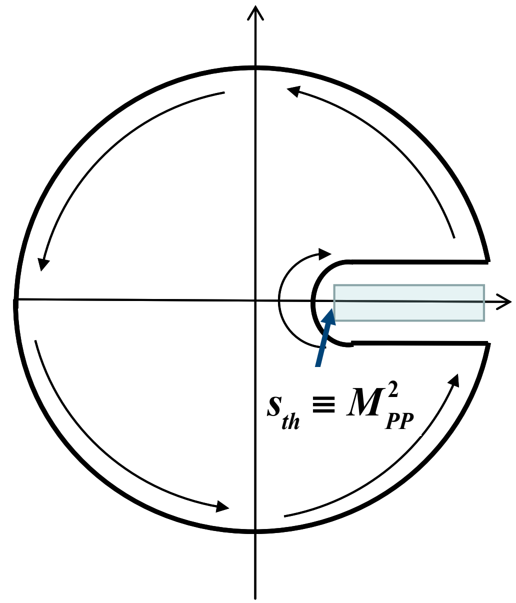
PP scattering phase
known from experiment

Watson's theorem

2.2 Dispersion relations: Method

- Knowing the discontinuity of F ⇨ write a dispersion relation for it
- Cauchy Theorem and Schwarz reflection principle

$$F(s)=\frac{1}{\pi}\oint\frac{F(s')}{s'-s}ds' \;\Rightarrow\; \frac{1}{2i\pi}\int_{M_{PP}^2}^{\infty}\frac{disc\left[F(s')\right]}{s'-s-i\varepsilon}ds'$$



$$\boxed{F(s)=\frac{1}{\pi}\int_{M_{PP}^2}^{\infty}\frac{\mathrm{Im}\left[F(s')\right]}{s'-s-i\varepsilon}ds'}$$

F can be reconstructed everywhere from the knowledge of $\mathrm{Im}\left[F(s)\right]$

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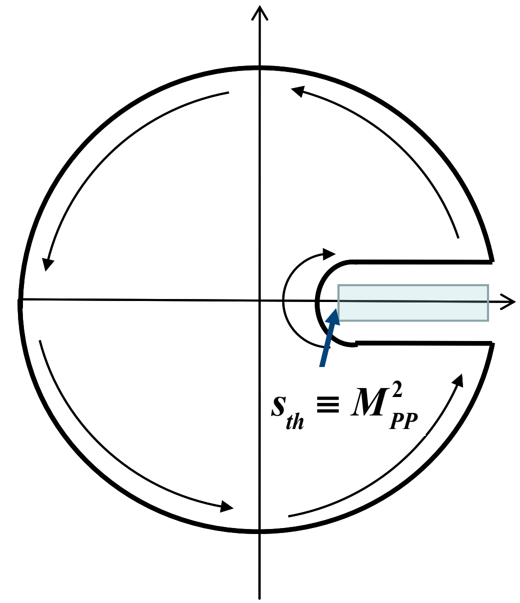


- If F does not drop off fast enough for $|s| \to \infty$
  ⇨ subtract the DR

$$\boxed{F(s) = P_{n-1}(s) + \frac{s^n}{\pi}\int_{M_{PP}^2}^{\infty} \frac{ds'}{s'^n}\frac{\mathrm{Im}\left[F(s')\right]}{\left(s'-s-i\varepsilon\right)}}$$

$P_{n-1}(s)$ polynomial

2.2 Dispersion relations: Method

- Solution: Use analyticity to reconstruct the form factor in the entire space

⇨ Omnès representation : $F_I(s) = P_I(s)\,\Omega_I(s)$

polynomial   Omnès function

- Omnès function : $$\boxed{\Omega_I(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta_I(s')}{s'-s-i\varepsilon}\right]}$$

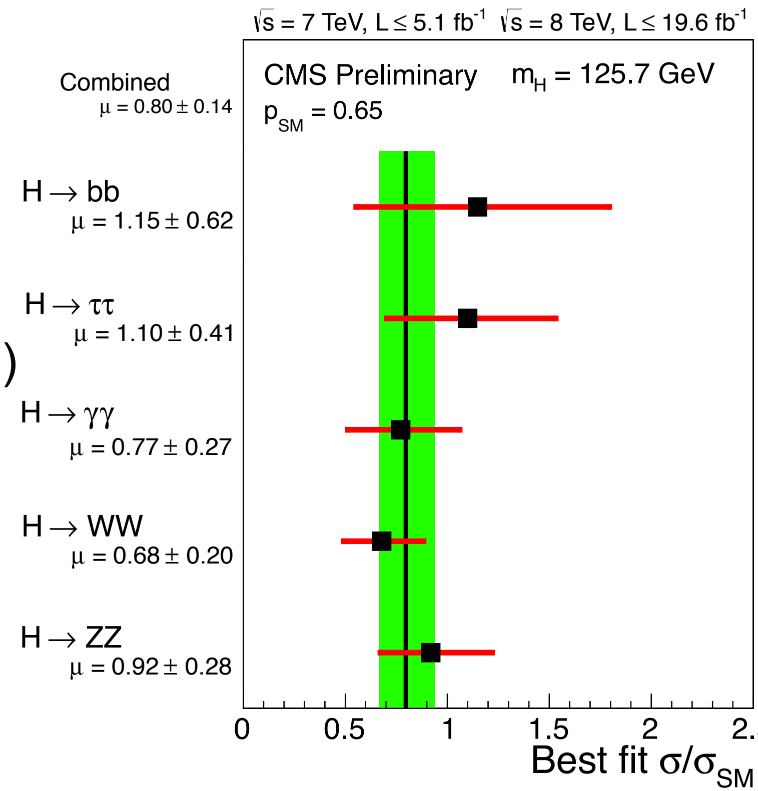
- Polynomial: $P_I(s)$ not known but determined from a matching to experiment or to ChPT at low energy

3. Probing non-standard Higgs couplings with τ decays

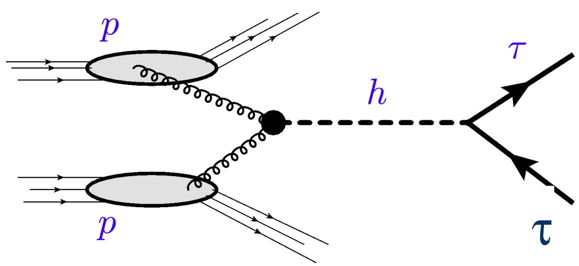
Celis, Cirigliano, E.P.'14

3.1 Introduction

- Discovery of a 125 GeV scalar particle : Standard Higgs? ⇨ Need to study its properties (couplings, spin, interactions, etc.)



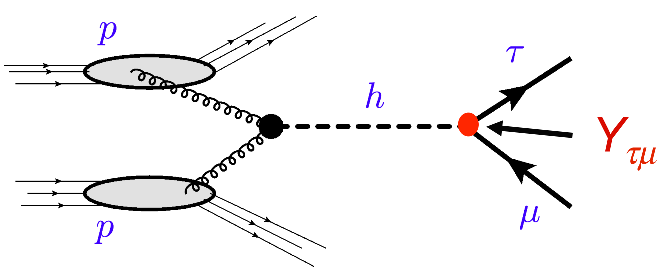
- LHC:



- Consider the possibility of non-standard LFV couplings of the Higgs ⇨ arise in several models

Goudelis, Lebedev,Park'11
Davidson, Grenier'10

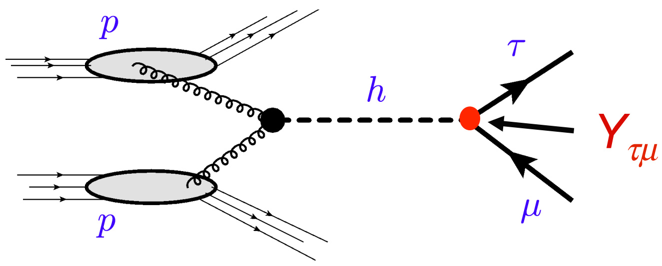
⇨



3.2 Testing LFV couplings of the Higgs

- How can it be tested?
  - High energy: LHC

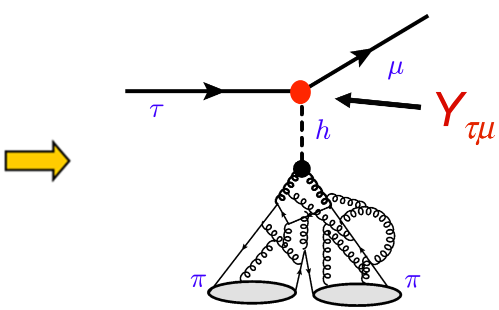
$$L_Y = -m_k \bar{f}_L^k f_R^k - Y_{ij}\left(\bar{f}_L^i f_R^j\right) h + h.c. + \ldots$$



In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

Hadronic part treated with perturbative QCD

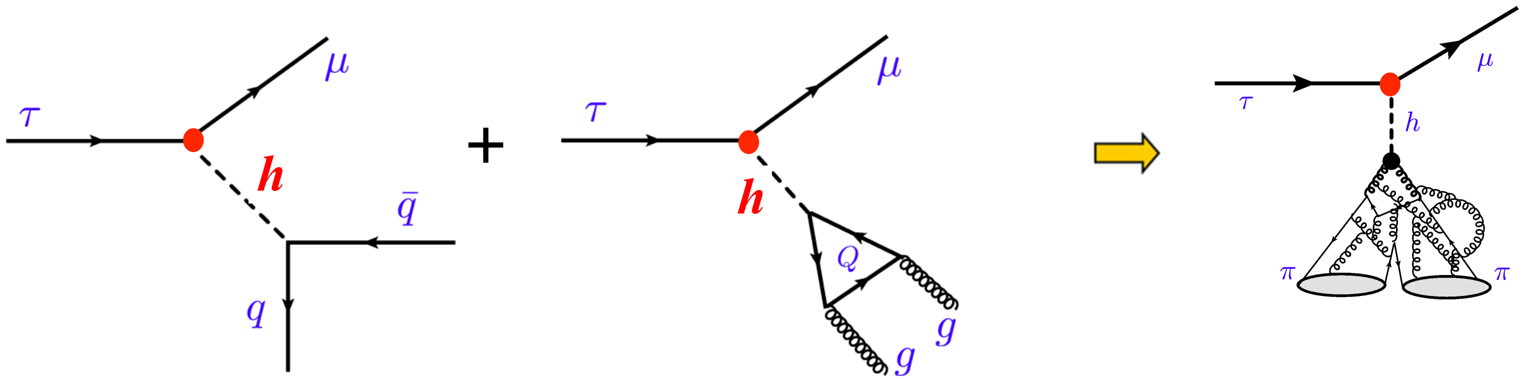
  - Low energy: Reverse the process



Hadronic part treated with non-perturbative QCD

3.3 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



- Problem : Have the hadronic part under control, ChPT not valid at these energies!

Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

Dreiner, Hanart, Kubis, Meissner'13

3.3 Constraints from $\tau \to \mu\pi\pi$

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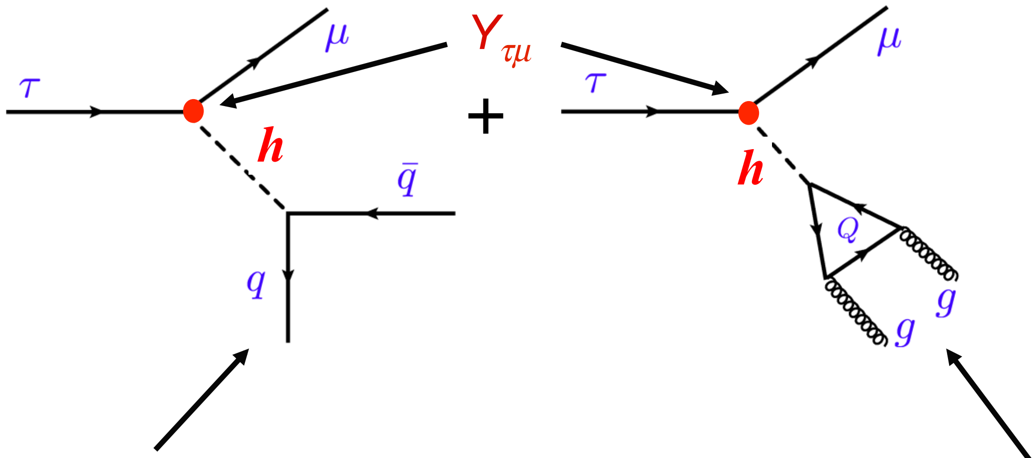
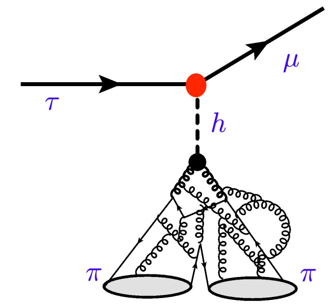




$$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s) \qquad \langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$$

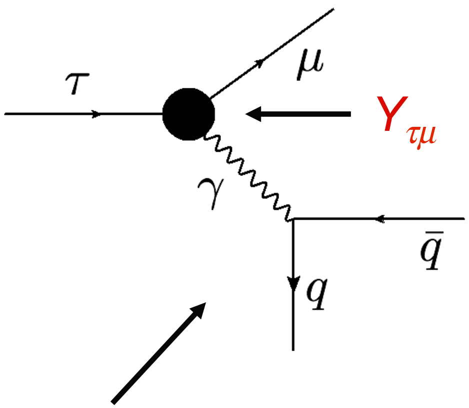
$$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\boxed{\Gamma_{\tau\to\mu\pi\pi} \propto \int \left| \Gamma_\pi(s) + \Delta_\pi(s) + \theta_\pi(s) \right|^2 Y^2_{\tau\mu}} \quad \text{with} \quad s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

3.3 Constraints from $\tau \rightarrow \mu\pi\pi$

- Contribution from dipole diagrams



$$\Gamma_{\tau \rightarrow \mu\pi^+\pi^-} \propto \int \left| F_V(s) \right|^2 Y_{\tau\mu}^2$$

$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \big| \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \big| 0 \rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Diagram only there in the case of $\tau^- \rightarrow \mu^-\pi^+\pi^-$ absent for $\tau^- \rightarrow \mu^-\pi^0\pi^0$

  ⇨ neutral mode more model independent

3.4 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

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  - Theoretically: Dispersive parametrization for $F_V(s)$

Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_V'' - \lambda_V'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{\left(s' - s - i\varepsilon\right)}\right]}$$

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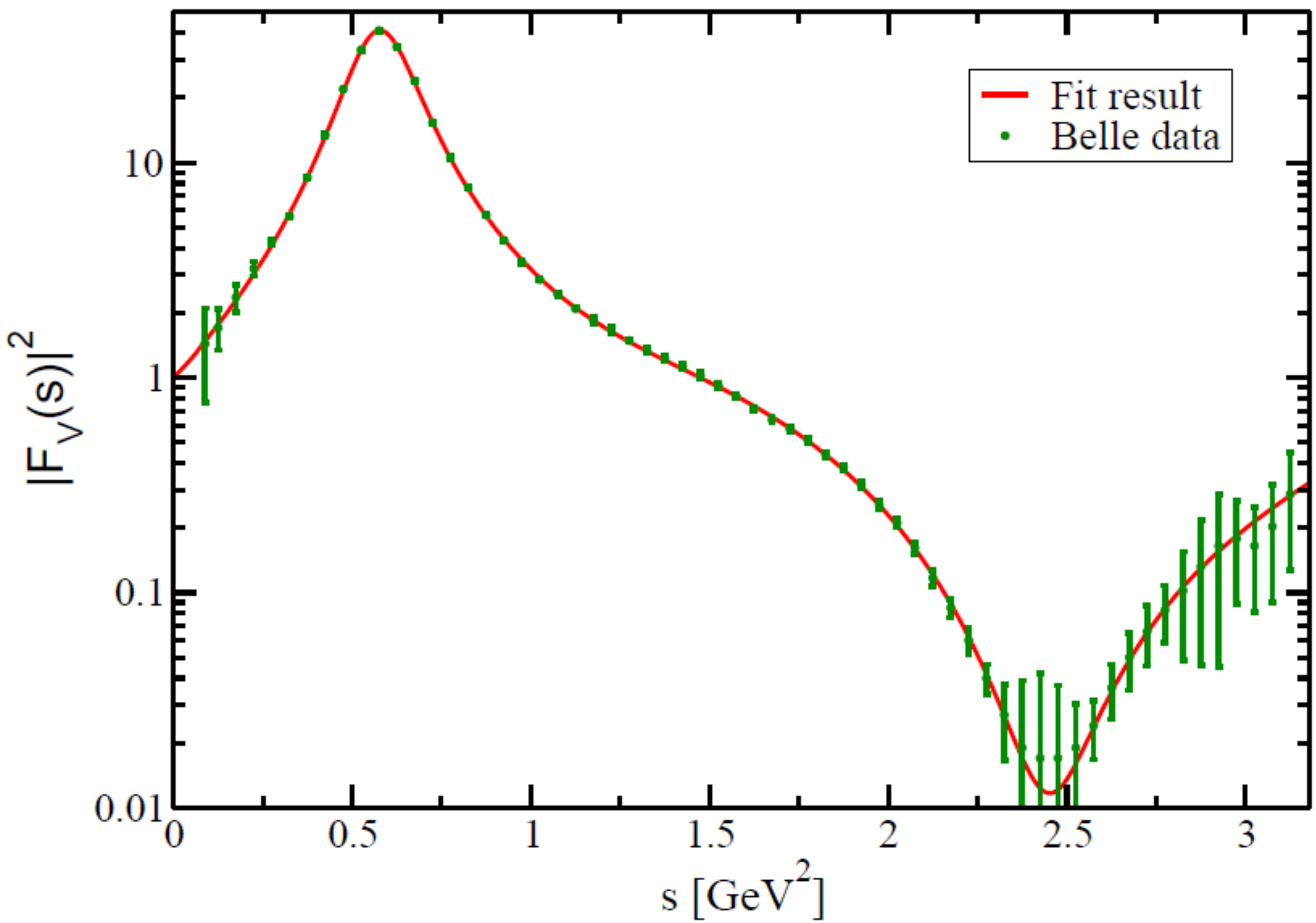
Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

3.4 Determination of the form factors : $F_V(s)$

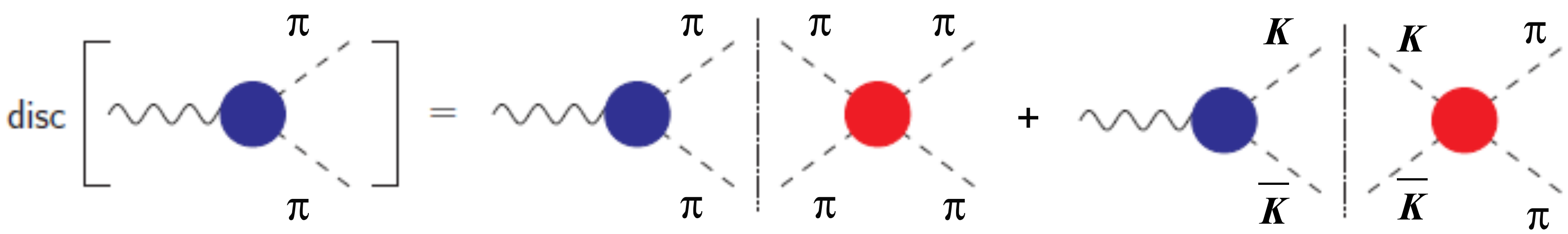


Determination of $F_V(s)$ thanks to very precise measurements of Belle!

3.5 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- Here no experimental data to determine the polynomial

- $\boxed{4m_\pi^2 < s < \left(m_\tau - m_\mu\right)^2 \sim \left(1.77\ \mathrm{GeV}\right)^2}$ two channels contribute $\pi\pi$ and $K\bar{K}$



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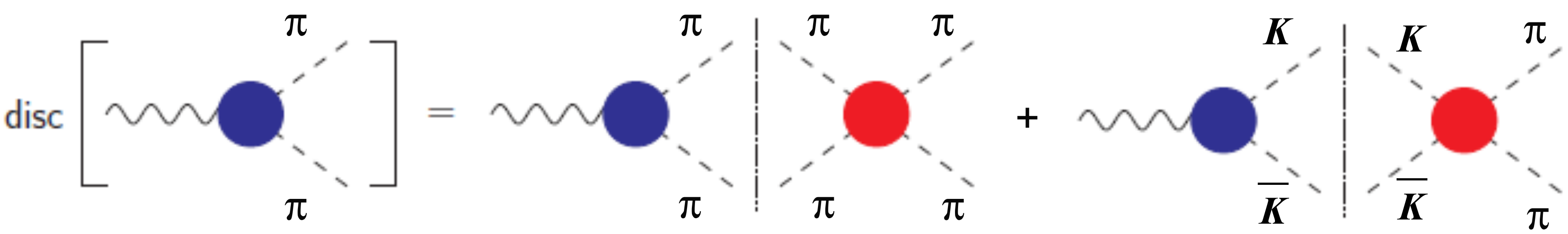

- Generalization of previous method ⇨ *coupled channel analysis*

Donoghue, Gasser, Leutwyler'90

Moussallam'99

Scattering matrix $\pi\pi \to \pi\pi,\ \pi\pi \to K\bar{K}$

$$K\bar{K} \to \pi\pi,\ K\bar{K} \to K\bar{K}$$

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Donoghue, Gasser, Leutwyler'90
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Unitarity $\Rightarrow$ $$\Gamma^\star_m(s) = \sum_n \{\delta_{mn} + 2\, i\, T_{mn}(s)\, \sigma_n(s)\}^\star\, \Gamma_n(s)$$

Scattering matrix $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$
$K\bar{K} \to \pi\pi$, $K\bar{K} \to K\bar{K}$

$$\boxed{\sigma_{1,2}(s) = \sqrt{1 - \frac{4M^2_{\pi,K}}{s}}\,\theta\left(s - 4M^2_{\pi,K}\right)}$$

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Scattering matrix $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$
$K\bar{K} \to \pi\pi$, $K\bar{K} \to K\bar{K}$

- Solve the dispersive integral equations iteratively starting with Omnès functions

$$\boxed{\mathrm{Im}X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{T^*_{nm}\sigma_m(s)X_m^{(N)}\right\}} \longrightarrow \boxed{\mathrm{Re}X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s}\mathrm{Im}X_n^{(N+1)}}$$

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Moussallam'99

Unitarity $\Rightarrow$ $$\Gamma^\star_m(s) = \sum_n \{\delta_{mn} + 2\,i\,T_{mn}(s)\,\sigma_n(s)\}^\star\,\Gamma_n(s)$$

Scattering matrix $\pi\pi \rightarrow \pi\pi$, $\pi\pi \rightarrow K\bar{K}$
$K\bar{K} \rightarrow \pi\pi$, $K\bar{K} \rightarrow K\bar{K}$

- Solve the dispersive integral equations iteratively starting with Omnès functions

$$\boxed{\mathrm{Im}X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\,\{T^*_{nm}\sigma_m(s)X_m^{(N)}\}} \longrightarrow \boxed{\mathrm{Re}X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'-s}\mathrm{Im}X_n^{(N+1)}}$$

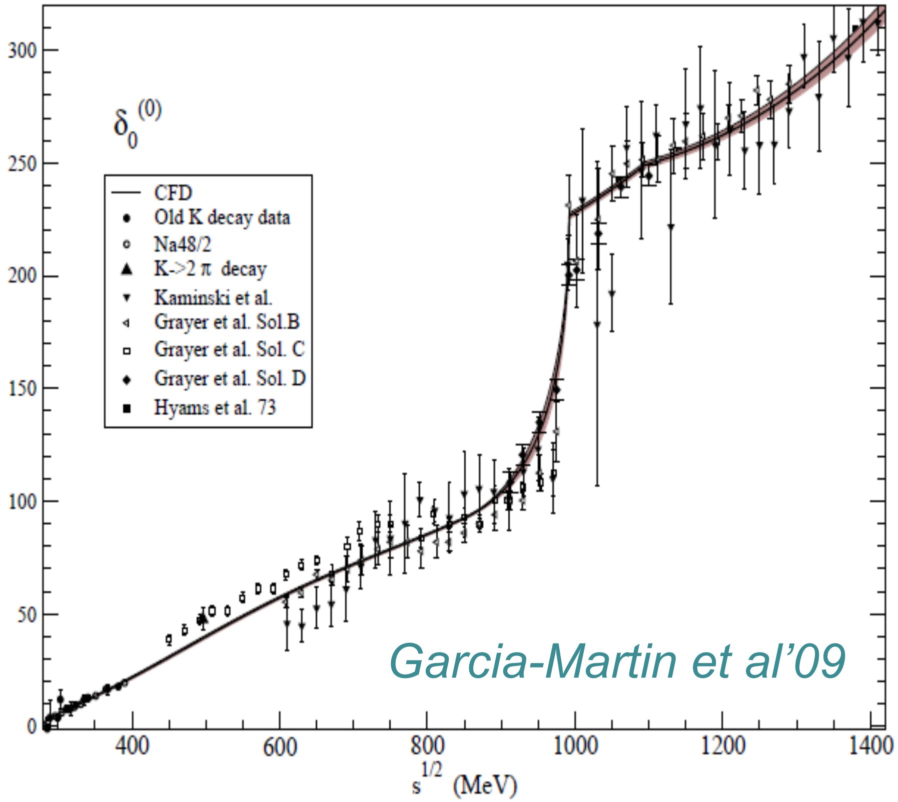
- According to *Muskhelishvili*, 2 sets of solutions $\{C_1(s), D_1(s)\}$, $\{C_2(s), D_2(s)\}$

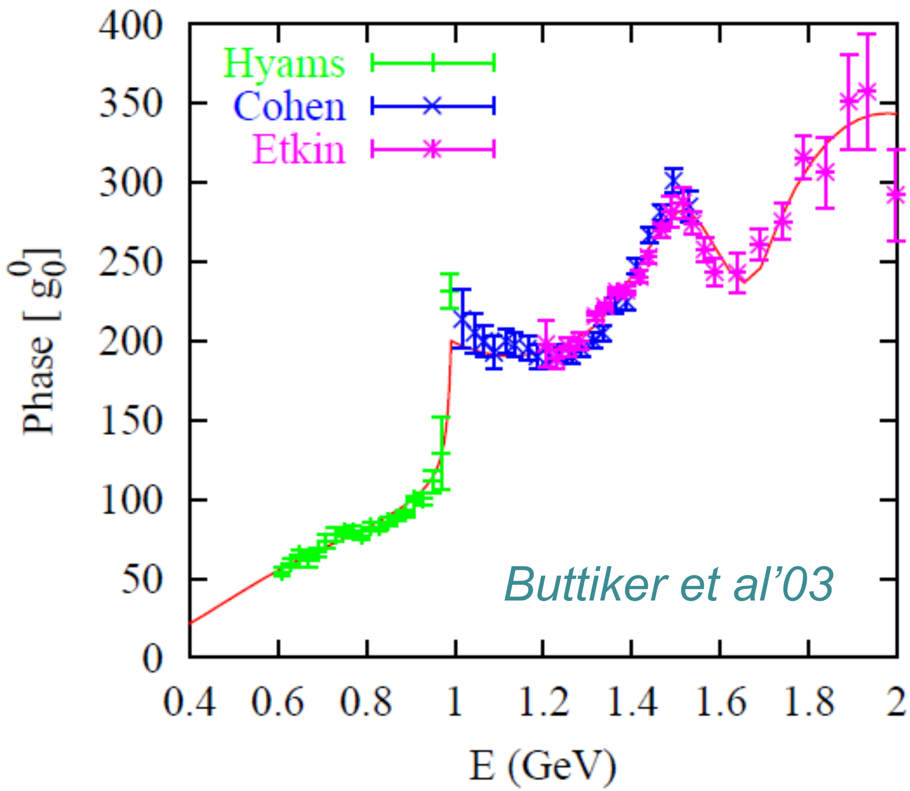
FFs linear combinations : $$\boxed{\Gamma_n(s) = P_\Gamma(s)C_n(s) + Q_\Gamma(s)D_n(s)}$$

Determined from a matching to ChPT + lattice

3.5 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- Inputs : $\pi\pi \to \pi\pi, K\bar{K}$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \to 1/s$ (*Brodsky & Lepage*) + ChPT:

Feynman-Hellmann theorem: ⇨

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$

⇨

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}}\Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}}\Delta_K(0) = \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{2}M_\pi^2\right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!

  ⇒ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\; M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05}\,(M_K^2 - 1/2 M_\pi^2)$$

Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- General solution

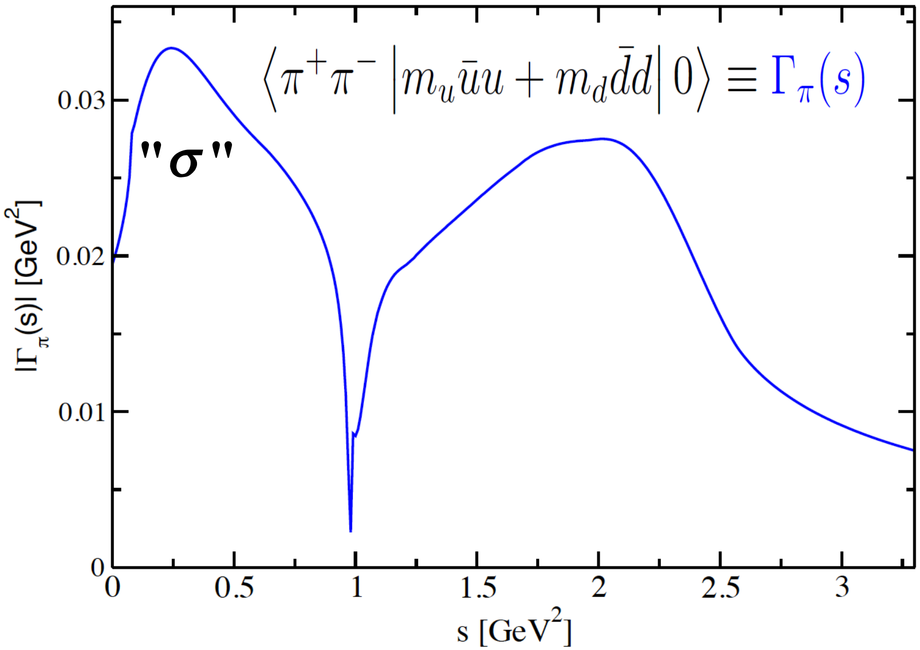
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

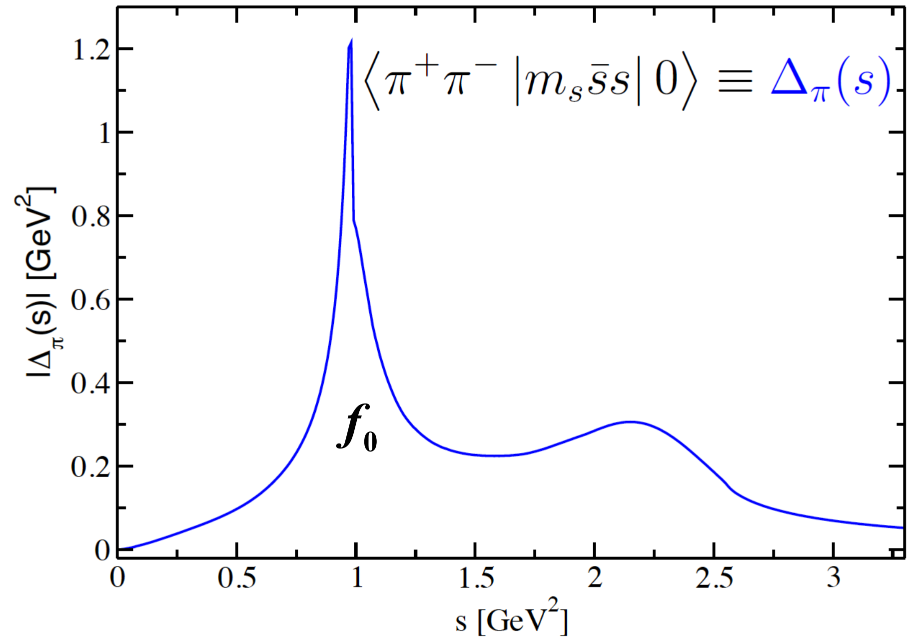
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

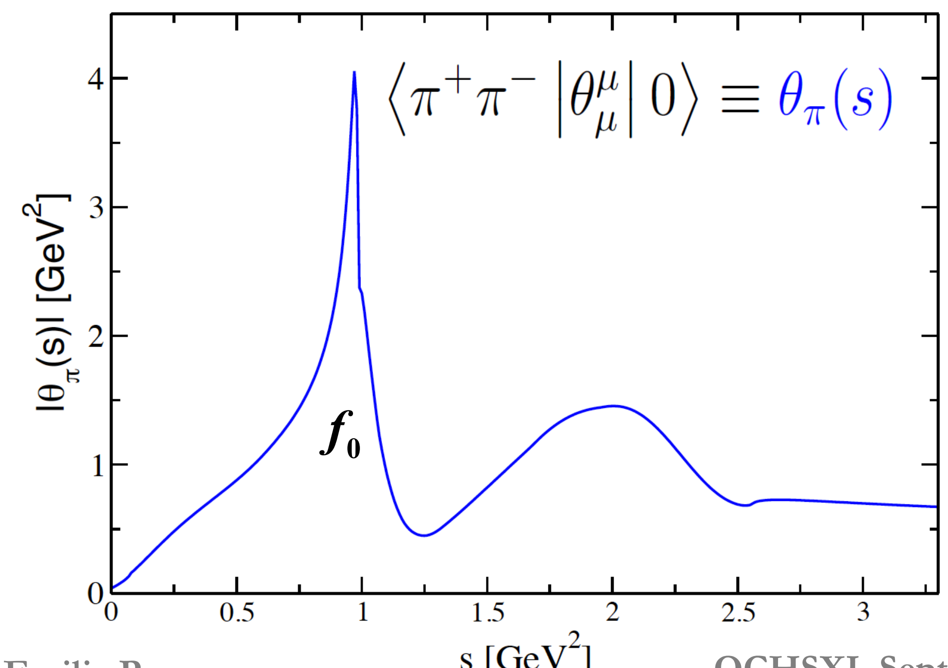
⇨ Relax the constraints and match to ChPT

$$\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s \end{aligned}$$

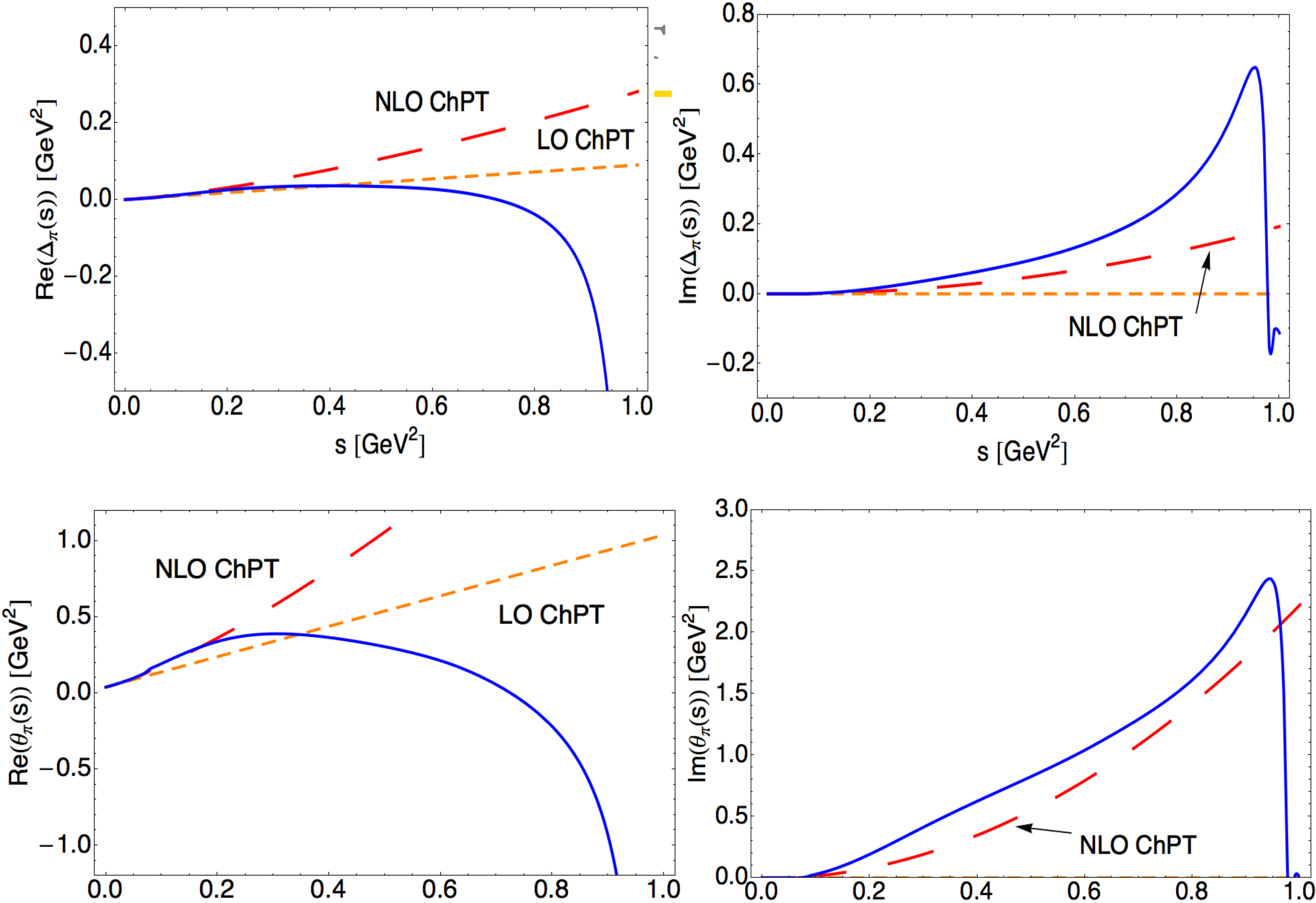
$\boxed{\dot{\theta}_{\pi,K} = \left.\frac{d\theta}{ds}\right|_{s=0} = 1}$ at LO in ChPT ⇨ SU(3) corrections: $\boxed{\dot{\theta}_K = 1.15 \pm 0.1}$



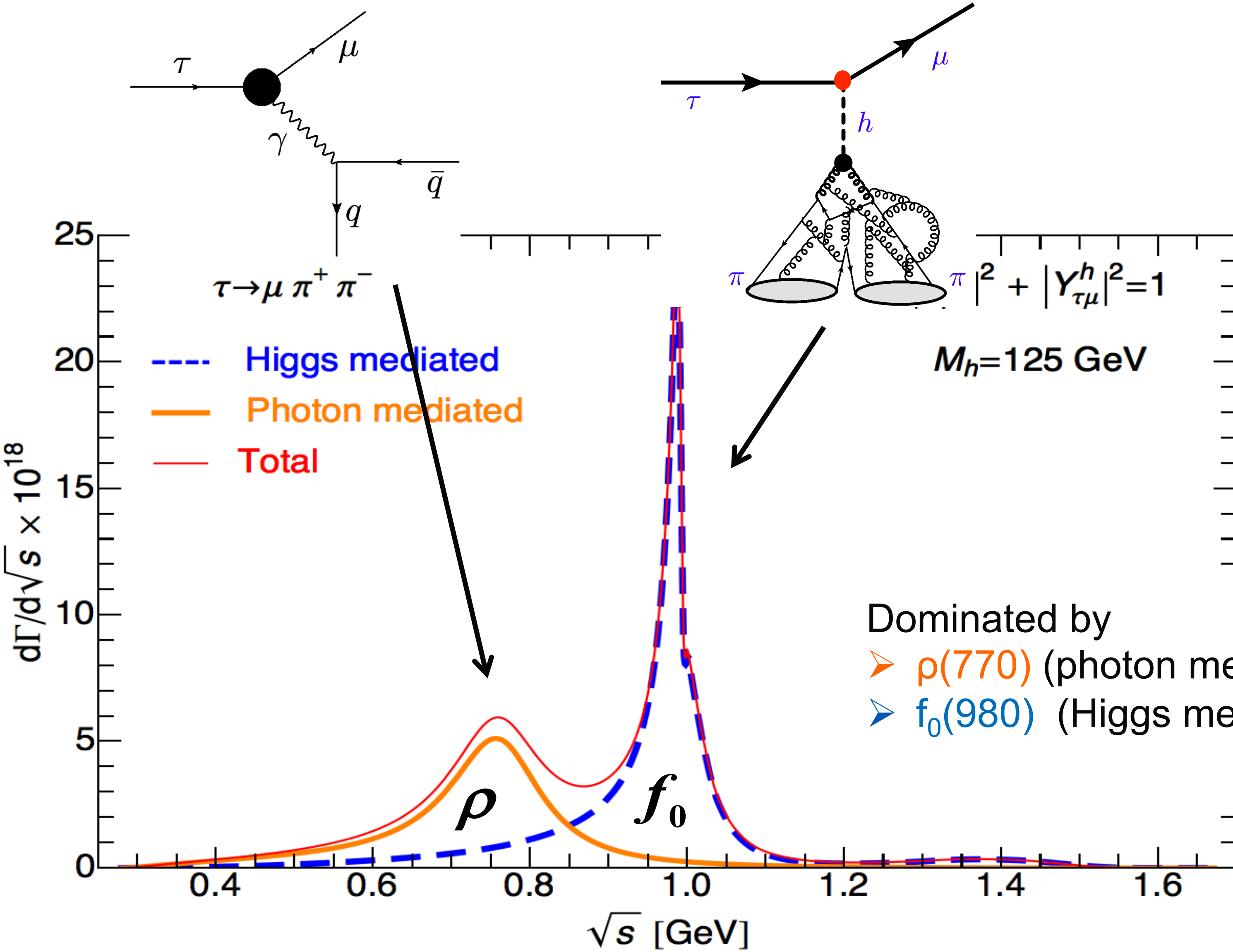




Dispersion relations:
Model-independent method,
based on first principles
that extrapolates ChPT
based on data


NLO ChPT
LO ChPT
Re(Δ_π(s)) [GeV^2]
s [GeV^2]
Im(Δ_π(s)) [GeV^2]
NLO ChPT
s [GeV^2]
NLO ChPT
LO ChPT
Re(θ_π(s)) [GeV^2]
s [GeV^2]
Im(θ_π(s)) [GeV^2]
NLO ChPT
s [GeV^2]

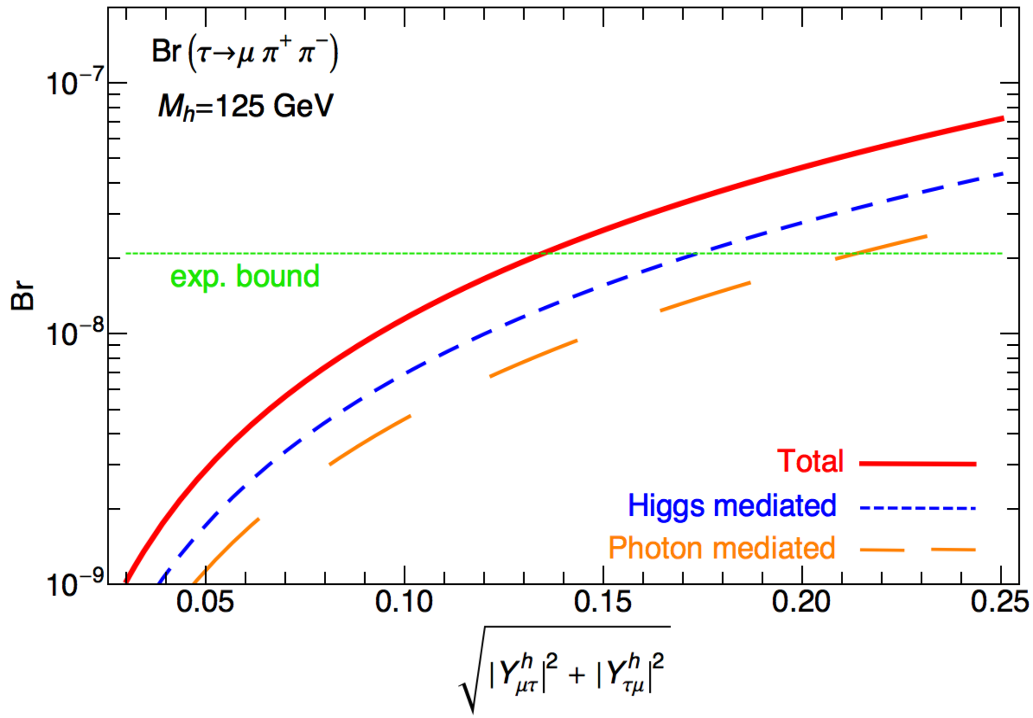
3.6 Results



Dominated by

- ρ(770) (photon mediated)
- f_0(980) (Higgs mediated)

3.6 Results



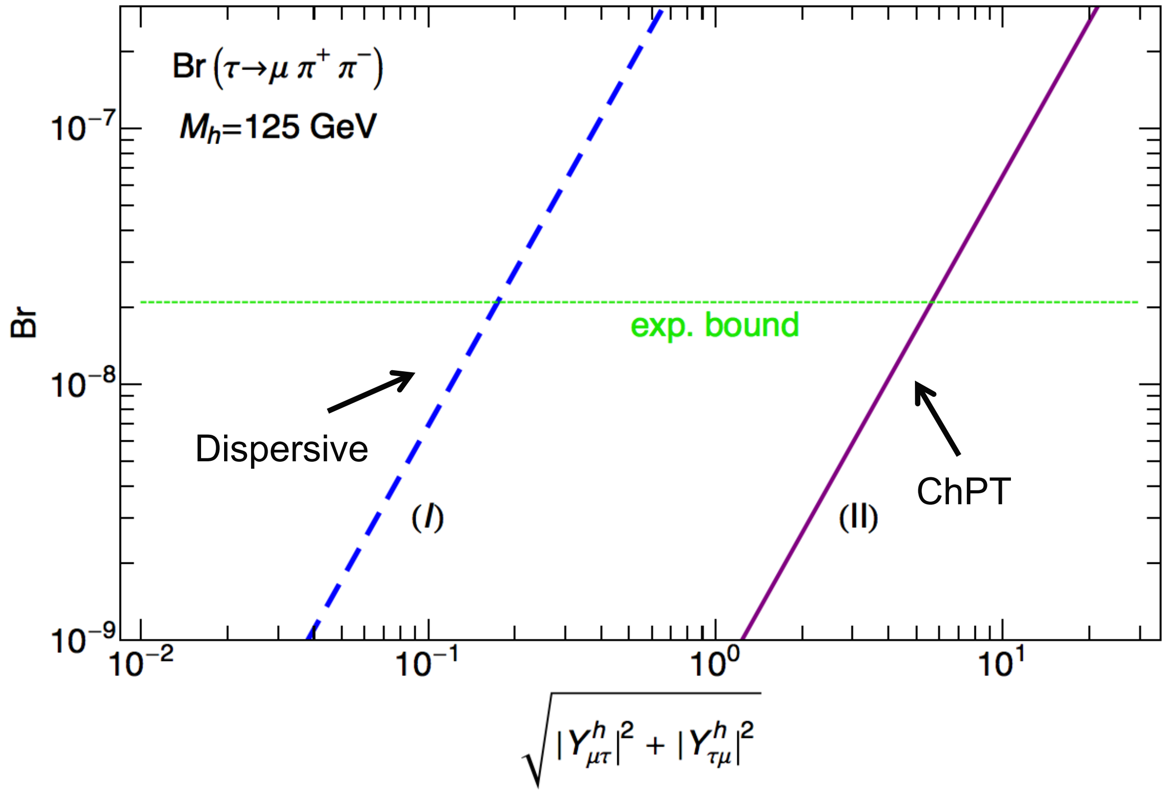
Process	$(\mathrm{BR} \times 10^8)$ 90% CL	$\sqrt{\|Y^h_{\mu\tau}\|^2 + \|Y^h_{\tau\mu}\|^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

Less stringent but more robust handle on LFV Higgs couplings

?

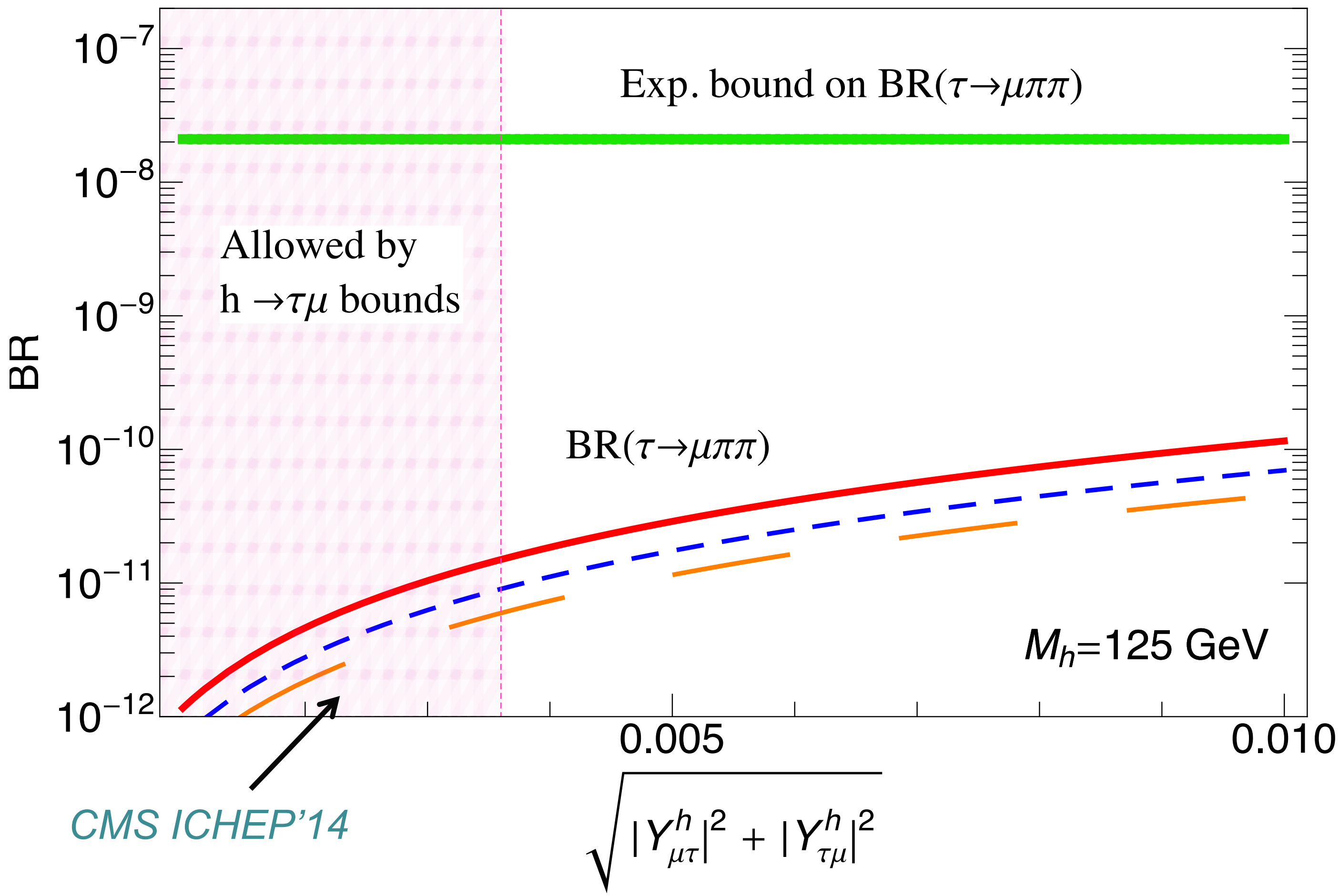
Belle'08'11'12 except last from *CLEO'97*

3.7 Impact of our work



- Rigorous treatment of hadronic part ⇨ bound reduced by one order of magnitude! ⇨ Very *robust bounds*!
- ChPT, EFT only valid at low energy for $\boxed{\mathbf{p} \ll \Lambda = 4\pi f_\pi \sim 1\ \mathrm{GeV}}$
  ⇨ *not valid up to* $E = \left(m_\tau - m_\mu\right)$!

3.8 Comparison with LHC result



- The LHC result gives stringent bounds on tau LFV!

4. Determination of V_{us} from $\tau \to K\pi\nu_\tau$ decays

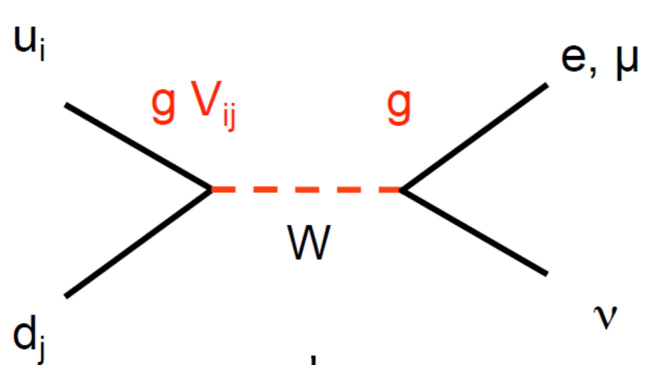
4.1 Test of New Physics : V_{us}

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element *V_{us}*
  - Fundamental parameter of the Standard Model
    Check unitarity of the first row of the CKM matrix:
    ⇨ *Cabibbo Universality*

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |\underbrace{V_{ub}}_{\substack{\text{Negligible} \\ \text{(B decays)}}}|^2 \stackrel{?}{=} 1}$$

  - Input in UT analysis

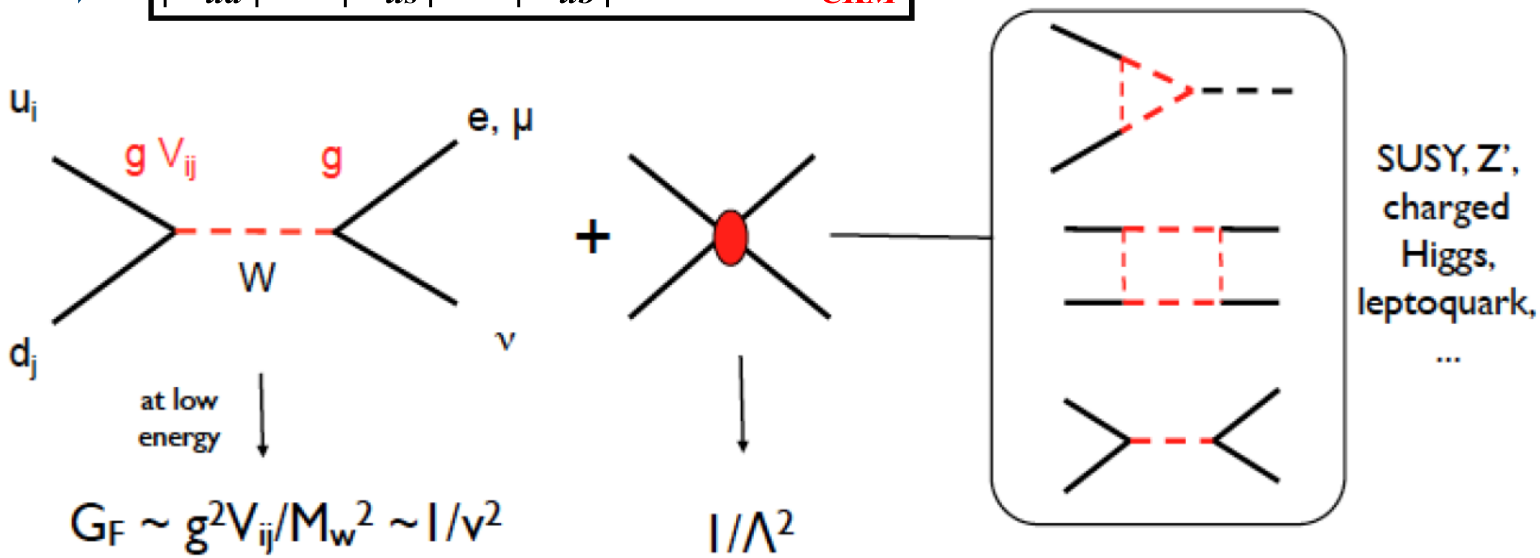
- Look for *new physics*
  - In the Standard Model : W exchange ⇨ only V-A structure



4.1 Test of New Physics : V_{us}

- BSM: sensitive to tree-level and loop effects of a large class of models

$$\left|V_{ud}\right|^2+\left|V_{us}\right|^2+\left|V_{ub}\right|^2=1+\Delta_{CKM}$$



BSM effects : $\boxed{\Delta_{CKM} \sim (v/\Lambda)^2}$

- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed K_{l3}, hadronic τ decays

4.2 $\tau \to K\pi\nu_\tau$ decays

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \bar{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\text{EM}}^{K\tau} + \tilde{\delta}_{\text{SU(2)}}^{K\pi}\right)^2}$$

4.3 Phase space integrals

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau |V_{us}|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

$$\boxed{I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)}$$

Hadronic matrix element: Crossed channel from $K \to \pi l \nu_l$

$$\boxed{\langle \mathrm{K}\pi | \, \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \, | 0 \rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s} (p_K + p_\pi)_\mu \right] \underbrace{f_+(s)}_{\text{vector}} - \frac{\Delta_{K\pi}}{s} (p_K + p_\pi)_\mu \underbrace{f_0(s)}_{\text{scalar}}}$$

with $s = q^2 = (p_K + p_\pi)^2$, $\overline{f}_{0,+}(t) = \dfrac{f_{0,+}(t)}{f_+(0)}$

➡ Use a *dispersive parametrization* to combine with K_{l3} analysis

Determination of the Kπ FFs: Dispersive representation

Bernard, Boito, E.P.'11

- $\bar{f}_0(s)$: dispersion relation with 3 subtractions: 2 in s=0 and 1 in s = $(m_K+m_\pi)^2$

Callan-Treiman

$$\bar{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}}\left(\ln C + (s-\Delta_{K\pi})\left(\frac{\ln C}{\Delta_{K\pi}} - \frac{\lambda_0'}{m_\pi^2}\right) + \frac{\Delta_{K\pi}s(s-\Delta_{K\pi})}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^2}\frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)}\right)\right]$$

- $\bar{f}_+(s)$: dispersion relation with 3 subtractions in s=0 *Boito, Escribano, Jamin'09,'10*

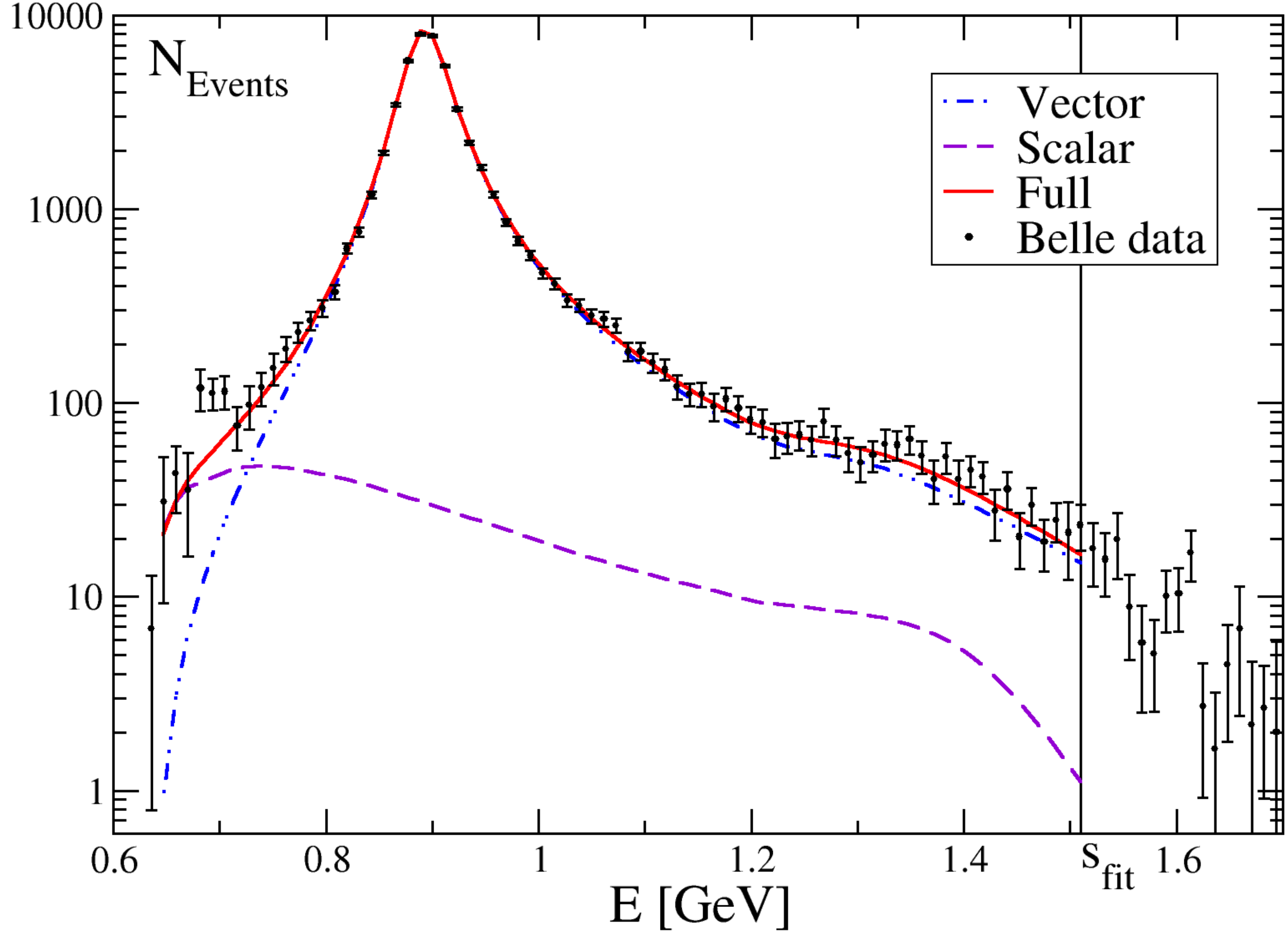
$$\bar{f}_+(s) = \exp\left[\lambda_+'\frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_+'' - \lambda_+'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_+(s')}{(s'-s-i\varepsilon)}\right]$$

Extracted from a model including
2 resonances K*(892) and K*(1414)

Jamin, Pich, Portolés'08

Fit to the $\tau \to K\pi\nu_\tau$ decay data + K_{l3} constraints

Bernard, Boito, E.P.'11



4.4 Extraction of V_{us}

- Decay rate master formula

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{EM}^{K\tau} + \cancel{\tilde{\delta}_{SU(2)}^{K\pi}}\right)^2$$

$$BR\left(\tau \to \overline{K^0}\pi^-\nu_\tau\right) = (0.416 \pm 0.008)\%$$

Belle'14

$$S_{ew} = 1.0201$$

Marciano & Sirlin'88, Braaten & Li'90, Erler'04

$$\delta_{EM}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%$$

$$I_{K^0}^\tau = 0.50432 \pm 0.01721$$

$$f_+(0) = 0.9661(32)$$

$$\Rightarrow \quad f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\exp}$$

$$\Rightarrow \quad \left|V_{us}\right| = 0.2216 \pm 0.0027$$

4.4 Extraction of V_{us}

- Decay rate master formula

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\boxed{\Gamma\left(\tau \to \bar{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \cancel{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}}\right)^2}$$

$$\boxed{f_+(0) = 0.9661(32)}$$

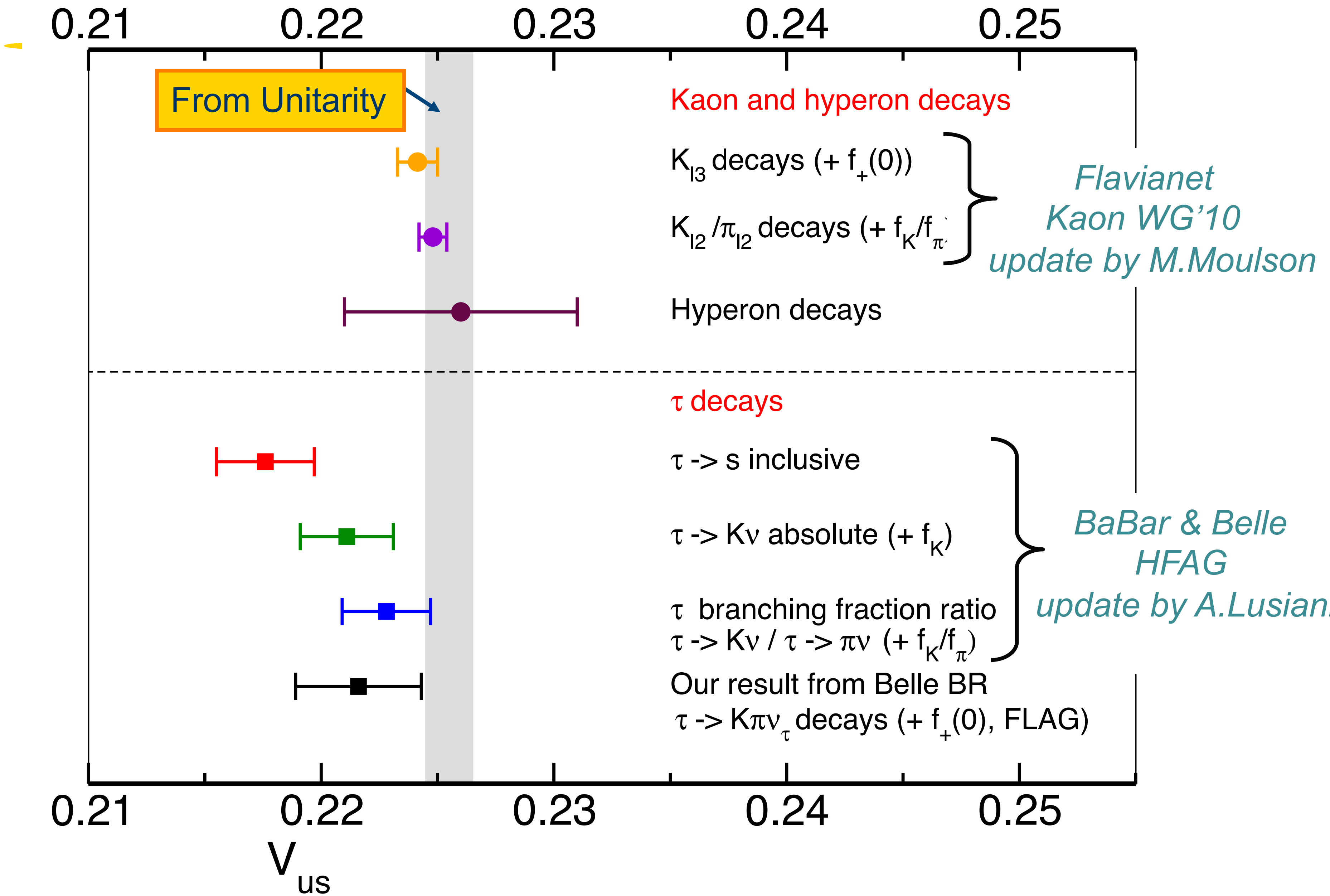
FLAG'13

$$\Rightarrow \boxed{f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\exp}} \Rightarrow \boxed{\left|V_{us}\right| = 0.2216 \pm 0.0027}$$

- Result of fit to K_{l3} + $\tau \longrightarrow K\pi\nu_\tau$ and $K\pi$ scattering data including inelasticities in the dispersive FFs

$$\Rightarrow \boxed{f_+(0)\left|V_{us}\right| = 0.2163 \pm 0.0014}$$

Bernard'14



0.21
0.22
0.23
0.24
0.25
From Unitarity
Kaon and hyperon decays
K_{l3} decays (+ $f_+(0)$)
K_{l2}/π_{l2} decays (+ f_K/f_π)
Flavianet
Kaon WG'10
update by M.Moulson
Hyperon decays
τ decays
τ -> s inclusive
τ -> Kν absolute (+ f_K)
τ branching fraction ratio
τ -> Kν / τ -> πν (+ f_K/f_π)
BaBar & Belle
HFAG
update by A.Lusiani
Our result from Belle BR
τ -> $K\pi\nu_\tau$ decays (+ $f_+(0)$, FLAG)
V_{us}

4.5 New determination of V_{us} from predicting τ strange BRs

Antonelli, Cirigliano, Lusiani, E.P. '13

- A sizeable fraction of the strange branching ratio is due to the decay $\tau \to K\nu_\tau$ and $\tau \to K\pi\nu_\tau$, which can be predicted theoretically from
  - kaon physics measurement
  - FF information ⇒ $\boxed{I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)}$

- $\tau \to K\nu_\tau$:

$$\mathrm{BR}(\tau \to K\nu_\tau) = \frac{m_\tau^3}{2m_K m_\mu^2}\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}\left(\frac{1-m_K^2/m_\tau^2}{1-m_\mu^2/m_K^2}\right)^2 \frac{\tau_\tau}{\tau_K}\,\delta_{\mathrm{EM}}^{\tau/K}\,\mathrm{BR}(K_{\ell 2})$$

computed *measured*

- $\tau \to K\pi\nu_\tau$

$$\mathrm{BR}(\tau \to \bar{K}\pi\nu_\tau) = \frac{2m_\tau^5}{m_K^5}\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}\frac{I_K^\tau}{I_K^\ell}\frac{\left(1+\delta_{\mathrm{EM}}^{K\tau}+\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}{\left(1+\delta_{\mathrm{EM}}^{K\ell}+\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}\frac{\tau_\tau}{\tau_K}\,\mathrm{BR}(K \to \pi e\bar{\nu}_e)$$

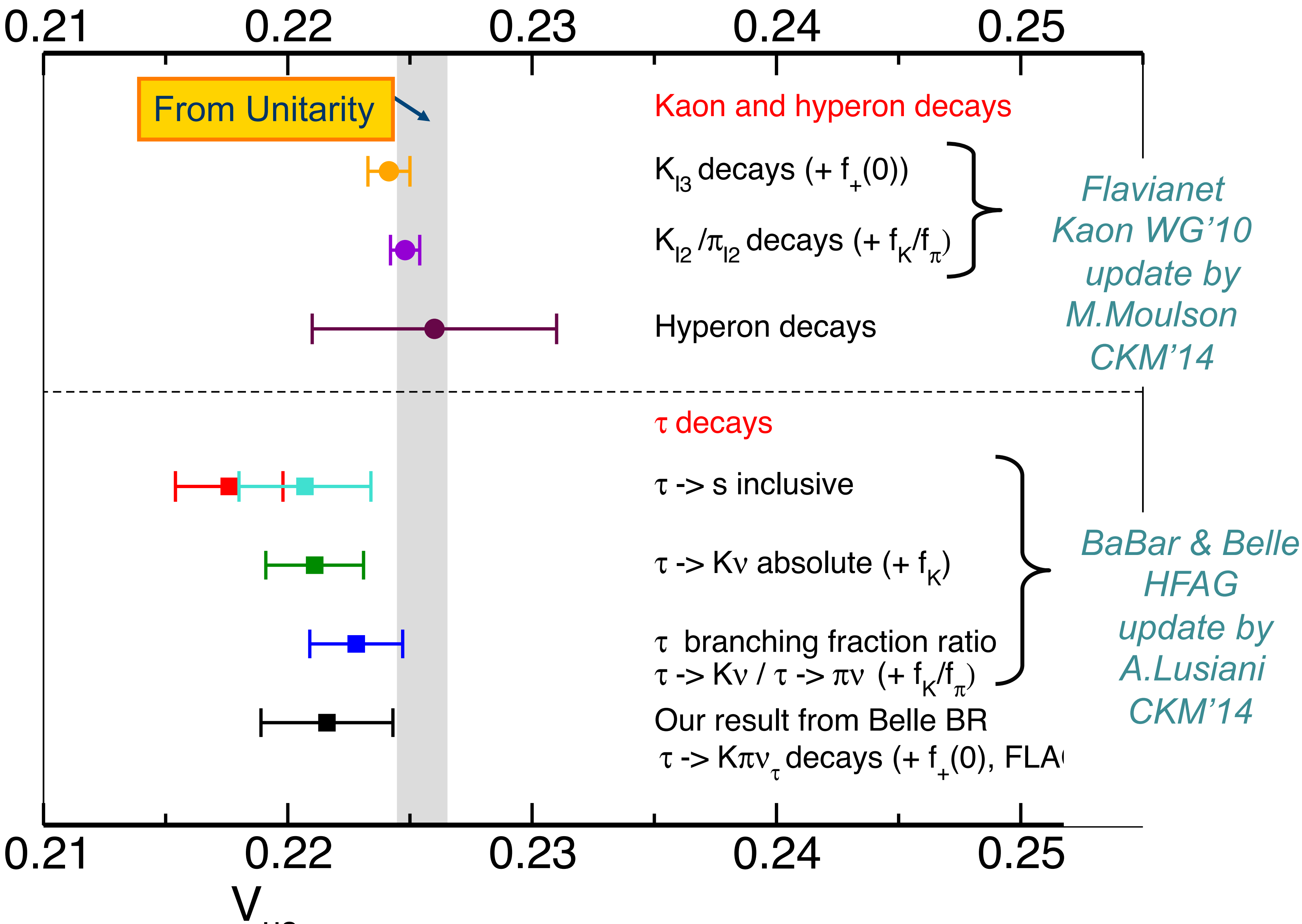
4.5 New determination of V_{us} from predicting τ strange BRs

Antonelli, Cirigliano, Lusiani, E.P. '13

- A sizeable fraction of the strange branching ratio is due to the decay $\tau \to K\nu_\tau$ and $\tau \to K\pi\nu_\tau$, which can be predicted theoretically from
  - kaon physics measurement
  - FF information $\Rightarrow$ $\boxed{I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)}$

Branching fraction	HFAG Winter 2012 fit	Prediction
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	$(0.713 \pm 0.003) \cdot 10^{-2}$
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	$(0.4707 \pm 0.0181) \cdot 10^{-2}$
$\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	$(0.8566 \pm 0.0299) \cdot 10^{-2}$
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	$(2.9648 \pm 0.0656) \cdot 10^{-2}$

$$\boxed{|V_{us}| = 0.2176 \pm 0.0021} \Rightarrow \boxed{|V_{us}| = 0.2207 \pm 0.0027}$$



0.21
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From Unitarity
Kaon and hyperon decays
K_{l3} decays (+ $f_+(0)$)
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Hyperon decays
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τ decays
τ -> s inclusive
τ -> Kν absolute (+ f_K)
τ branching fraction ratio
τ -> Kν / τ -> πν (+ f_K/f_π)
Our result from Belle BR
τ -> Kπν_τ decays (+ $f_+(0)$, FLA
BaBar & Belle HFAG update by A.Lusiani CKM'14
V_{us}

5. Conclusion and Outlook

5.1 Conclusion

- Low energy experiments a powerful probe of the Standard Model and New Physics

- Most of the case, need to control the hadronic uncertainties: need to know hadronic matrix elements, form factors etc.

- In this talk 2 examples involving hadronic τ decays:
  - looking for non-standard lepton flavour violating couplings of the Higgs in $\tau \to \mu\pi\pi$
  - extract V_{us} from $\tau \to K\pi\nu_\tau$

- We need to know the $\pi\pi$ and $K\pi$ form factors

  ⇨ Use dispersion relations

- Dispersion relations rely on analyticity, unitarity and crossing symmetry

  ⇨ Rigorous treatment of two and three hadronic final state

5.2 Outlook

- For reaching a high level of precision, theoretical challenges : in the dispersion relation
  - ➢ include inelasticities
  - ➢ Take isospin breaking and electromagnetic corrections into account

⇨ Work in this direction at JLab, e.g., use Regge phenomenology

Talk by V. Mathieu

Bern-Bonn collaboration: combine NREFT and dispersion relations

- Apply dispersion relations to other processes:
  - ➢ 3 body τ decays: $\tau \to \pi\pi\pi\nu_\tau, \tau \to K\pi\pi\nu_\tau$, etc
  - ➢ heavy mesons: D, B decays
  - ➢ baryons: nucleons, etc

6. Back-up

5.1 Conclusion

- Low energy experiments a powerful probe of the Standard Model and New Physics

- Most of the case, need to control the hadronic uncertainties: need to know hadronic matrix elements, form factors etc.

- In this talk 2 examples involving hadronic τ decays:
  - looking for non-standard lepton flavour violating couplings of the Higgs in $\tau \to \mu\pi\pi$
  - extract V_{us} from $\tau \to K\pi\nu_\tau$

- We need to know the $\pi\pi$ and $K\pi$ form factors

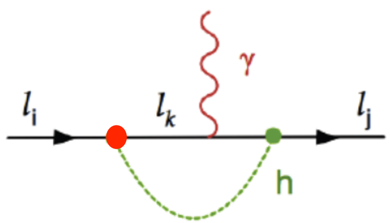
  ⇨ Use dispersion relations

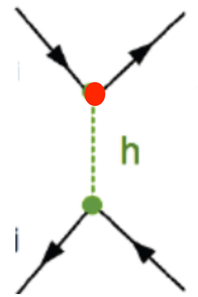
- Very exciting example of how low-energy probes can be effective for looking for new physics

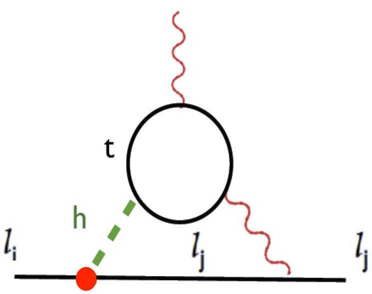
3.3 The role of $\tau \to \mu\pi\pi$

- Other processes to constrain LFV considered at low energy in flavour physics ⇨ leptonic sector

Harnick, Koop, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12







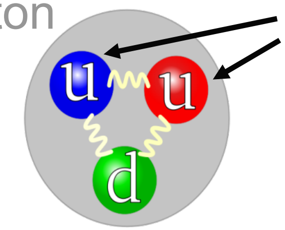
- But decays depending on the high energy completion of the theory unknown!

- $\tau \to \mu\pi\pi$ is a tree level process ⇨ less sensitive to the model of NP and establish a direct connection with the LHC!
  Taken into account for the first time for the Higgs in *Celis, Cirigliano & E.P.'14*

1.2 Hadronic physics

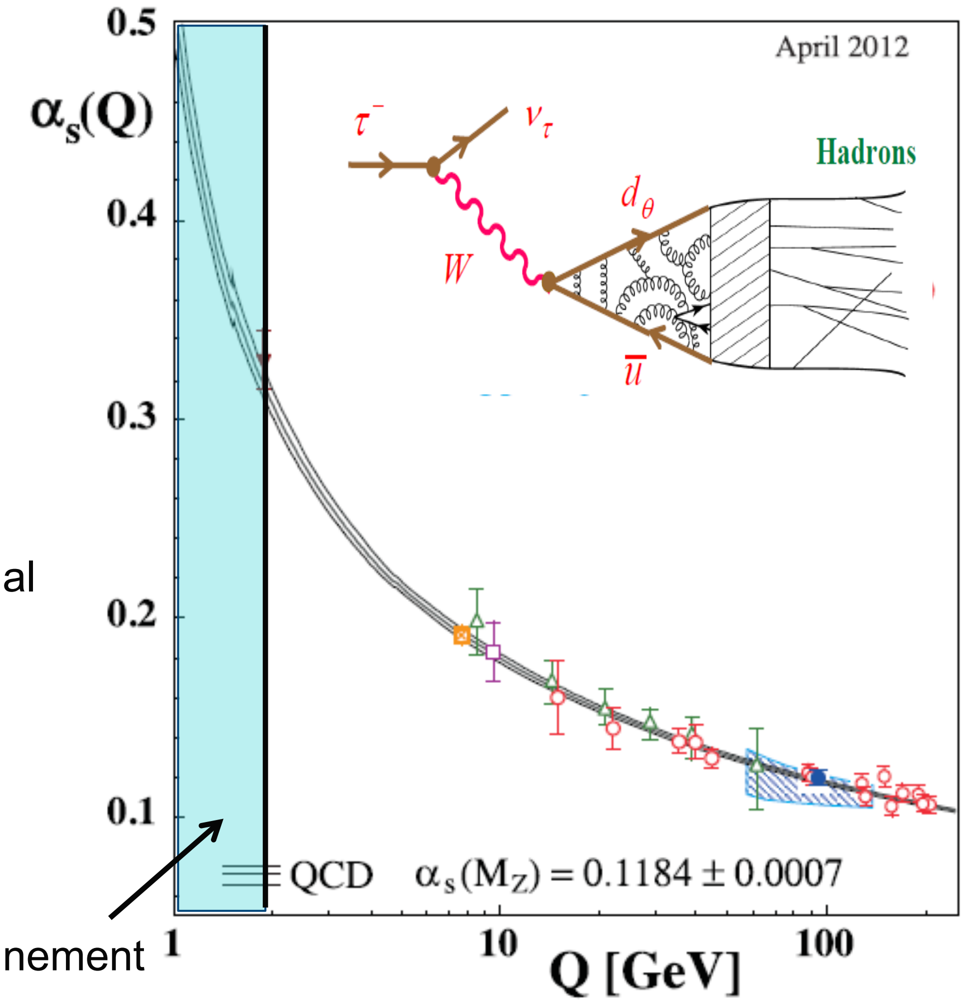
- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement

Proton



Quarks

PDG'12



Confinement

➢ Low energy (Q <~1 GeV), long distance: *α_S becomes large !*

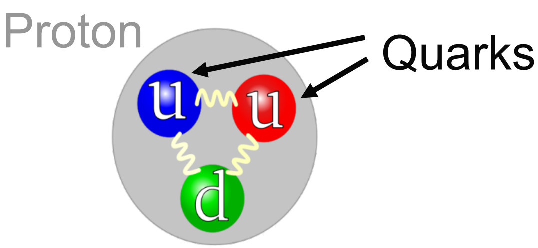
⇨ *Non-perturbative QCD*

A perturbative expansion in the usual sense fails

⇨ Use of alternative approaches, expansions…

1.2 Hadronic physics

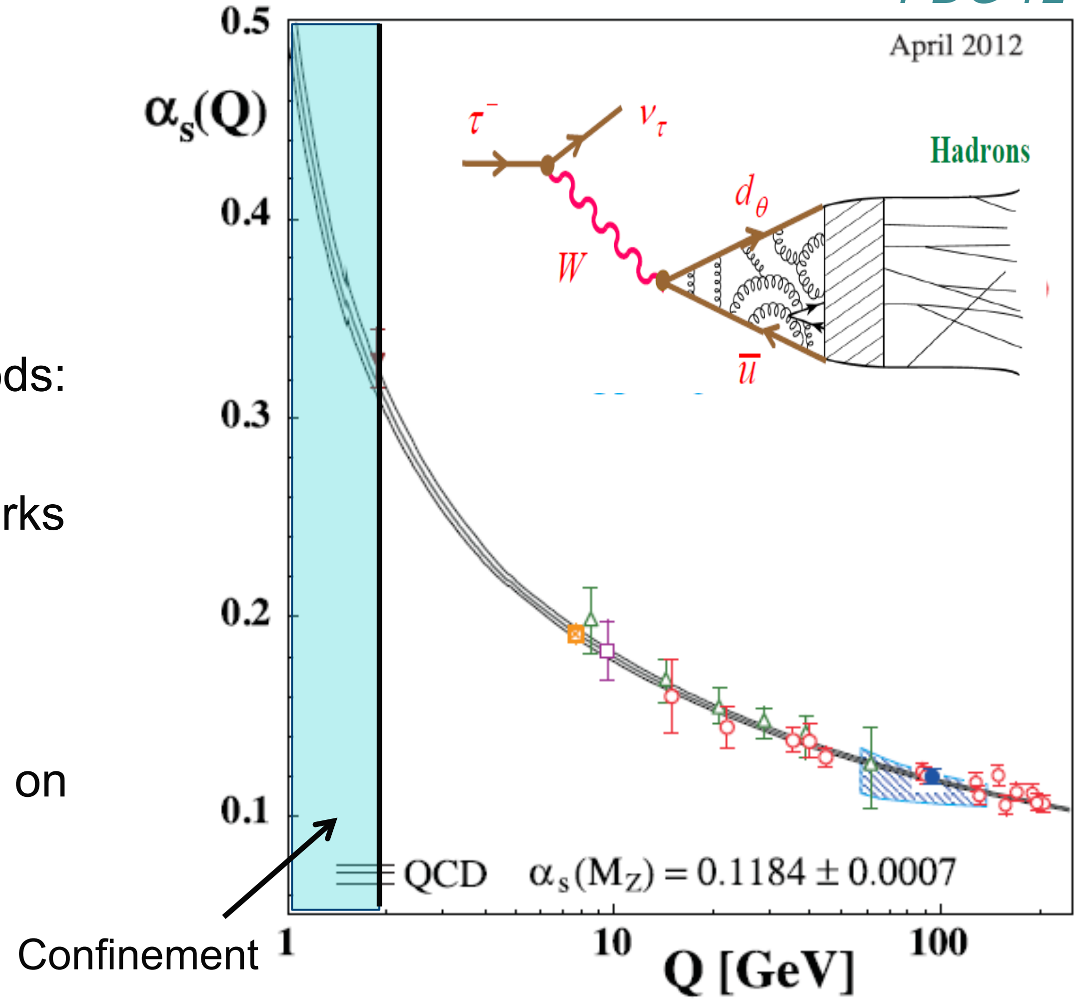
- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement



➢ Model independent methods:
  - Effective field theory

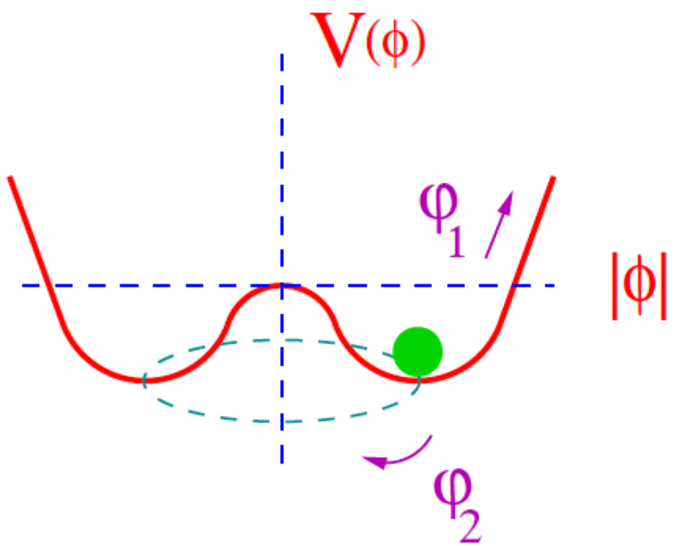
    Ex: ChPT for light quarks
  - Dispersion relations
  - Numerical simulations on the lattice

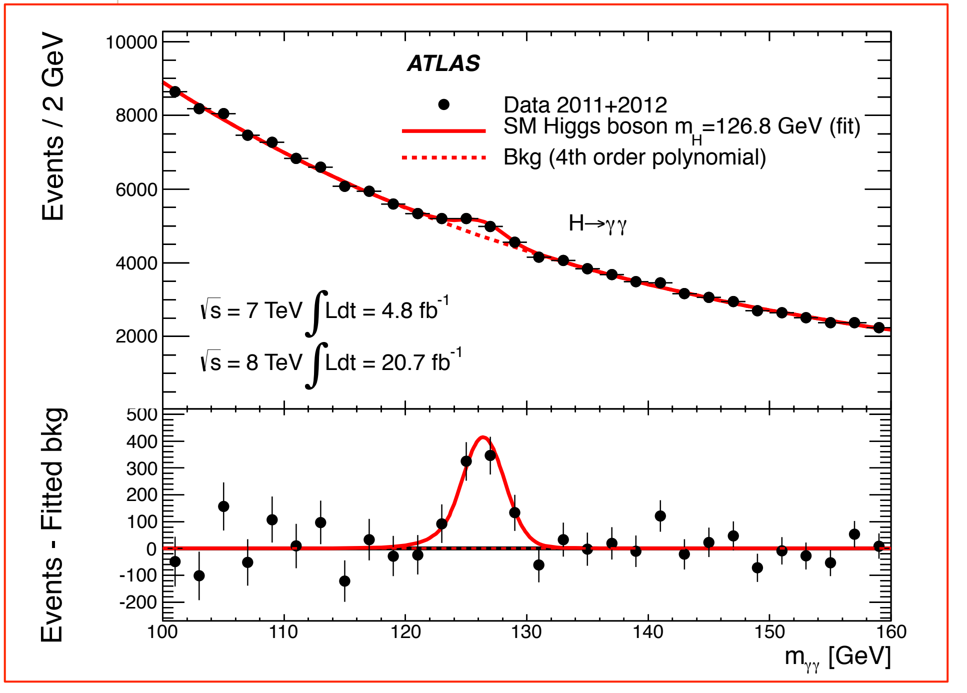
PDG'12



3.1 Introduction

- Discovery of a 125 GeV scalar particle: Missing piece of the Standard Model





- When the Higgs developps a v.e.v, choose a particular direction
  ⇨ breaks the electroweak symmetry

4.2 $\tau \to K\pi\nu_\tau$ decays

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau\left[\gamma\right]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Experimental inputs from HFAG *Banerjee et al.'12*

4.2.1 Electroweak corrections

- Master formula for $\tau \rightarrow K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \rightarrow \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

$$\boxed{S_{\mathrm{ew}} = 1.0201}$$

Marciano & Sirlin'88, Braaten & Li'90, Erler'04

4.2.2 Electromagnetic corrections

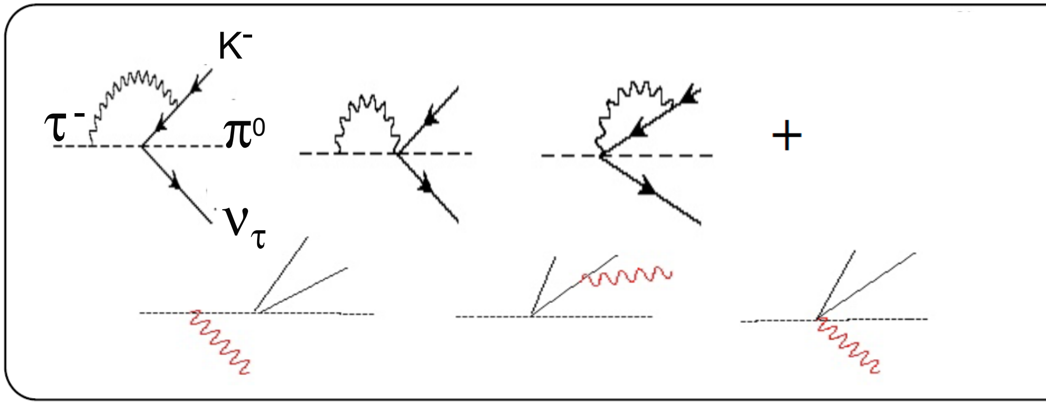
- Master formula for $\tau \rightarrow K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \rightarrow \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

→ ChPT to O(p²e²)

→ Counter-terms neglected

based on $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$



Cirigliano, Neufeld, Ecker'02

Antonelli, Cirigliano, Lusiani, E.P.'13

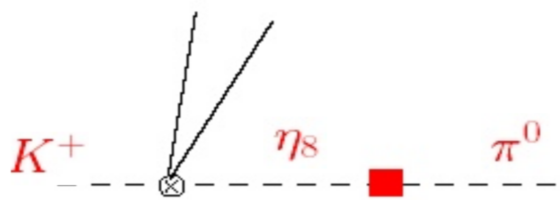
⇒ $\boxed{\delta_{\mathrm{EM}}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%}$ and $\boxed{\delta_{\mathrm{EM}}^{K^-\tau} = (-0.2 \pm 0.2)\%}$

4.2.3 Isospin breaking corrections

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

$$\boxed{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1}$$



$$\Rightarrow \quad \boxed{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi} = \left(2.9 \pm 0.4_{\mathrm{mixing}} \pm 0.5\right)\%}$$

+ IB in the K^{*-} to $K\pi$ coupling

Antonelli, Cirigliano, Lusiani, E.P.'13

3.1.6 Extraction of V_{us}

- Result for $\tau \rightarrow K\pi\nu_\tau$: $f_+(0)|V_{us}| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\exp}$

$\Rightarrow$ $|V_{us}| = 0.2216 \pm 0.0027$ with $f_+(0) = 0.9661(32)$ *FLAG'13*

- To be compared to results for K_{l3}: *FLAVIAnet Kaon WG, talk by M. Moulson*

FLAG'13

$f_+(0)|V_{us}| = 0.2165 \pm 0.0004$ $\Rightarrow$ $|V_{us}| = 0.2241 \pm 0.0004_{\exp} \pm 0.0008_{\text{theo}}$

$|V_{us}| = 0.2241 \pm 0.0009$

- Not competitive yet but interesting cross check of V_{us} determination from K_{l3} and inclusive τ result

$f_+(0)|V_{us}| = 0.2163 \pm 0.0014$

3.1.6 Extraction of V_{us}

- Result for $\tau \rightarrow K\pi\nu_\tau$: $\boxed{f_+(0)|V_{us}| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{exp}}$

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FLAG'13

$\boxed{f_+(0)|V_{us}| = 0.2165 \pm 0.0004}$ $\Rightarrow$ $\boxed{|V_{us}| = 0.2241 \pm 0.0004_{exp} \pm 0.0008_{theo}}$

$\boxed{|V_{us}| = 0.2241 \pm 0.0009}$

- Not competitive yet but interesting cross check of V_{us} determination from K_{l3} and inclusive τ result

Bernard'14

- Result of fit to K_{l3} + $\tau \rightarrow K\pi\nu_\tau$ and $K\pi$ scattering data including inelasticities in the dispersive FFs $\boxed{f_+(0)|V_{us}| = 0.2163 \pm 0.0014}$

3.2 V_{us} from $\tau \to K\nu_\tau$ / $\tau \to \pi\nu_\tau$

- $$\boxed{\frac{\Gamma(\tau \to K\nu[\gamma])}{\Gamma(\tau \to \pi\nu[\gamma])} = \frac{\left(1-m_{K^\pm}^2/m_\tau^2\right)}{\left(1-m_{\pi^\pm}^2/m_\tau^2\right)} \frac{f_K^2}{f_\pi^2} \frac{|V_{us}|^2}{|V_{ud}|^2} (1+\delta_{\mathrm{LD}})}$$

  - δ_{LD} : Long-distance radiative corrections

    $\Rightarrow$ $\boxed{\delta_{\mathrm{LD}} = 1.0003 \pm 0.0044}$

  - Brs from *HFAG'12 with update by A.Lusiani*
  - F_K/ F_π from lattice average: $\boxed{\frac{f_K}{f_\pi} = 1.1940 \pm 0.0050}$ *FLAG'13*
  - V_{ud} : $|V_{ud}| = 0.97425(22)$ *Towner & Hardy'08*

$\Rightarrow$ $\boxed{|V_{us}| = 0.2228 \pm 0.0019}$ 1.2σ away from unitarity

3.3 V_{us} from $\tau \to K\nu_\tau$

- $$\boxed{BR(\tau \to K\nu[\gamma]) = \frac{G_F^2 m_\tau^3 S_{EW} \tau_\tau}{16\pi h}\left(1 - \frac{m_{K^\pm}^2}{m_\tau^2}\right) f_K^2 |V_{us}|^2}$$

In principle less precise than ratios

➢ Inputs from *HFAG'12 with update by A.Lusiani*

➢ F_K from lattice average $\boxed{f_K = (156.3 \pm 0.9)\ \text{MeV}}$ *FLAG'13*

⇨ $\boxed{|V_{us}| = 0.2211 \pm 0.0020}$ 1.9σ away from unitarity

4. New determination of V_{us} from predicting τ strange BRs

Antonelli, Cirigliano, Lusiani, E.P. '13

4.1 Introduction

- Modes measured in the strange channel for $\tau \to s$:

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \overline{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \overline{K}^0 \pi^0 \nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \overline{K}^0 \pi^0 \pi^0 \nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

HFAG'12

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$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
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$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

HFAG'12

~70% of the decay modes crossed channels from Kaons!

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$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

HFAG'12

~70% of the decay modes crossed channels from Kaons!

Up to ~90% Including the 2p modes

4.2 Prediction of the strange BR $\tau \to K\nu_\tau$

- The BRs of these 3 modes can be predicted using Kaon BRs very precisely measured + form factor information

  - $\tau \to K\nu_\tau$:

$$\mathrm{BR}(\tau \to K\nu_\tau) = \frac{m_\tau^3}{2m_K m_\mu^2}\boxed{\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}}\left(\frac{1-m_K^2/m_\tau^2}{1-m_\mu^2/m_K^2}\right)^2\boxed{\frac{\tau_\tau}{\tau_K}}\boxed{\delta_{\mathrm{EM}}^{\tau/K}}\boxed{\mathrm{BR}(K_{\ell 2})}$$

  - Inputs needed:
    → Experimental : BR(K_{l2}), lifetimes

    → Theoretical : Short distance EW corrections
    Long distance EM corrections

⇒ $$BR(\tau^- \to K^-\nu_\tau) = (0.713 \pm 0.003)\%$$

4.3 Prediction of the strange BR $\tau \to K\pi\nu_\tau$

- The BRs of these 3 modes can be predicted using Kaon BRs very precisely measured + form factor information

  - $\tau \to K\pi\nu_\tau$:

$$\mathrm{BR}(\tau \to \bar{K}\pi\nu_\tau) = \frac{2m_\tau^5}{m_K^5}\boxed{\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}}\boxed{\frac{I_K^\tau}{I_K^\ell}}\frac{\left(1+\boxed{\delta_{\mathrm{EM}}^{K\tau}}+\boxed{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}}\right)^2}{\left(1+\boxed{\delta_{\mathrm{EM}}^{K\ell}}+\boxed{\delta_{\mathrm{SU(2)}}^{K\pi}}\right)^2}\boxed{\frac{\tau_\tau}{\tau_K}}\boxed{\mathrm{BR}(K \to \pi e\bar{\nu}_e)}$$

  - Inputs needed :
    - The K_{e3} branching ratios, lifetimes
    - Phase space integrals ⇨ use the dispersive parametrization for the form factors
    - The electromagnetic and isospin-breaking corrections

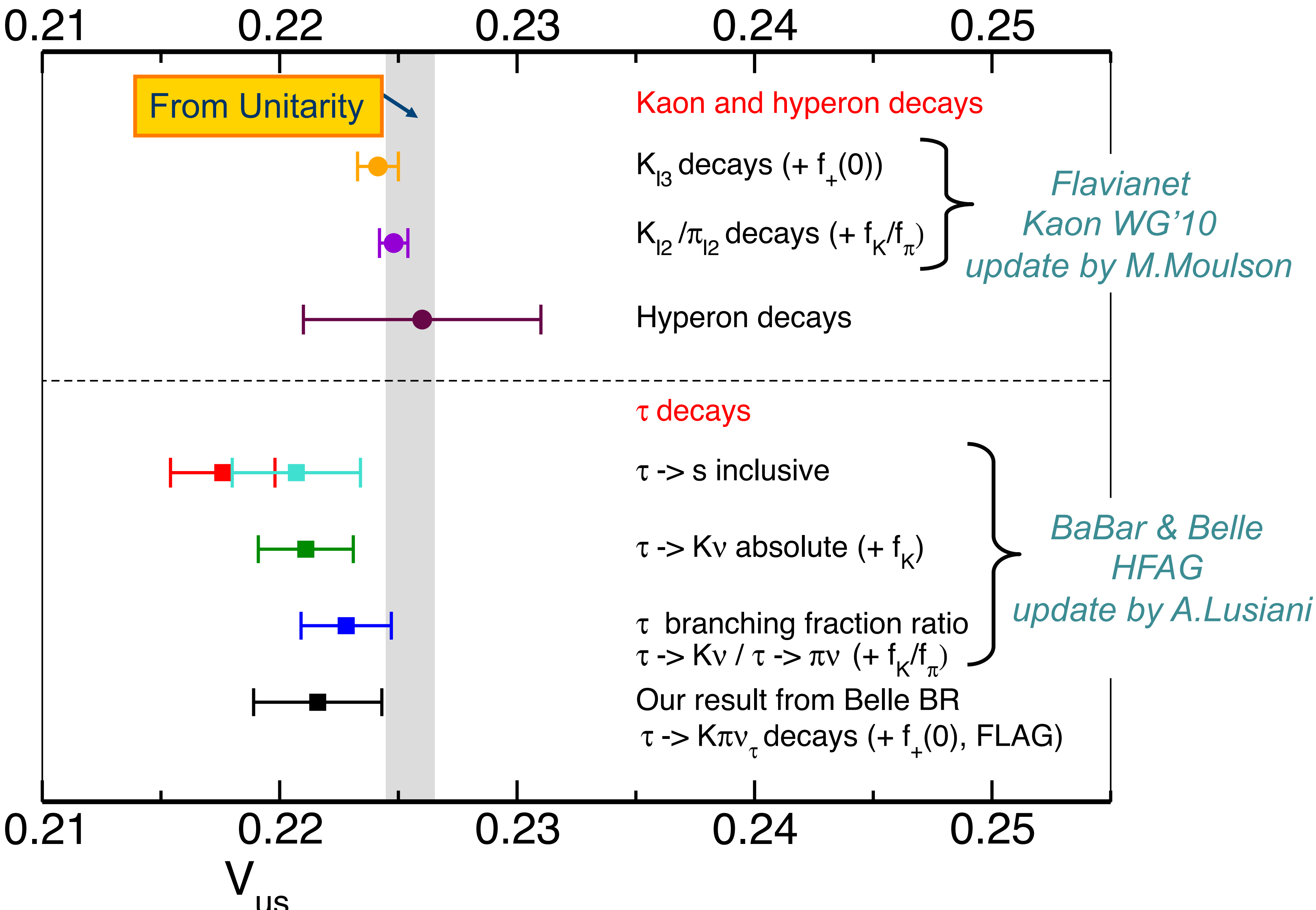
⇨ $\boxed{BR\left(\tau \to \overline{K^0}\pi^-\nu_\tau\right) = (0.857 \pm 0.030)\%}$ and $\boxed{BR\left(\tau \to K^-\pi^0\nu_\tau\right) = (0.471 \pm 0.018)\%}$

4.4 Extraction of V_{us}

Mode	BR	% err	BR(K_{e3})	τ_K	τ_τ	I_K^τ / I_K^e	Δ_{EM}	$\Delta_{\mathrm{SU(2)}}$
$\tau^- \to \bar{K}^0 \pi^- \nu_\tau$	0.8566 ± 0.0299	3.49	0.22	0.42	0.36	3.41	0.47	0
$\tau^- \to K^- \pi^0 \nu_\tau$	0.4707 ± 0.0181	3.84	0.06	0.12	0.34	3.65	0.48	1.01

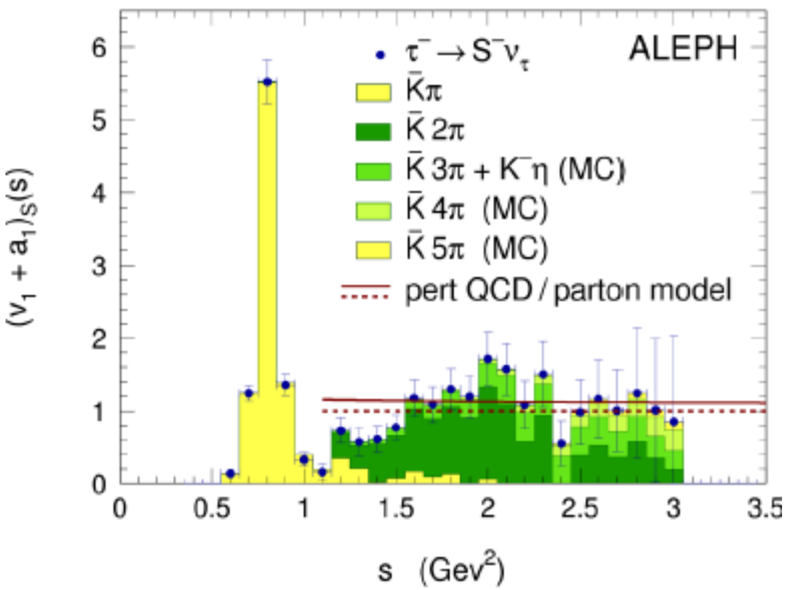
Branching fraction	HFAG Winter 2012 fit	Prediction
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	$(0.713 \pm 0.003) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	$(0.4707 \pm 0.0181) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	$(0.8566 \pm 0.0299) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	$(2.9648 \pm 0.0656) \cdot 10^{-2}$

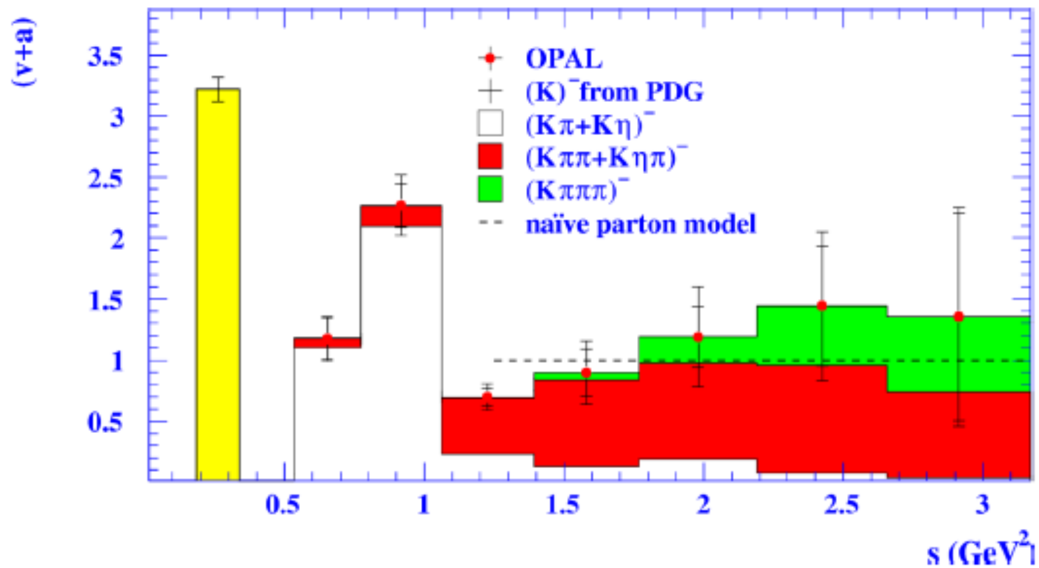
$$\boxed{|V_{us}| = 0.2176 \pm 0.0021} \Rightarrow \boxed{|V_{us}| = 0.2207 \pm 0.0027}$$


0.21
0.22
0.23
0.24
0.25
From Unitarity
Kaon and hyperon decays
K l3 decays (+ f +(0))
K l2 /π l2 decays (+ f K/f π)
Flavianet
Kaon WG'10
update by M.Moulson
Hyperon decays
τ decays
τ -> s inclusive
τ -> Kν absolute (+ f K)
τ branching fraction ratio
τ -> Kν / τ -> πν (+ f K/f π)
BaBar & Belle
HFAG
update by A.Lusiani
Our result from Belle BR
τ -> Kπν τ decays (+ f +(0), FLAG)
V us

4.5 Prospects : τ strange Brs

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

Smaller $\tau \to$ K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller V_{us}

$$R_\tau^S\Big|_{\text{old}} = 0.1686(47) \quad \Rightarrow \quad R_\tau^S\Big|_{\text{new}} = 0.1615(28)$$

$$|V_{us}|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}} \quad \Rightarrow \quad |V_{us}|_{\text{new}} = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}$$

5.1 Conclusion

- Studying τ physics ⇨ very interesting tests of the Standard Model e.g. V_{us}
- Inclusive τ decays : ⇨ $\boxed{|V_{us}| = 0.2176 \pm 0.0019_{exp} \pm 0.0010_{th}}$

  Error dominated by experiment ⇨ Potentially the more precise extraction of V_{us}

Antonelli, Cirigliano, Lusiani, E.P. '13

- Simulated *New flavour factory* data from *Belle* data :
  Same central values but uncertainties rescaled assuming 40 ab^{-1} luminosity

Mode	BR	% err	BR(K_{e3})	τ_K	τ_τ	I_K^τ / I_K^e	$\Delta_{\rm EM}$	$\Delta_{\rm SU(2)}$
$\tau^- \to \bar{K}^0 \pi^- \nu_\tau$	0.8427 ± 0.0122	1.45	0.22	0.41	0.34	1.24	0.46	0
$\tau^- \to K^- \pi^0 \nu_\tau$	0.4631 ± 0.0079	1.71	0.06	0.12	0.34	1.25	0.47	1.00

$\boxed{|V_{us}| = 0.2176 \pm 0.0019_{exp} \pm 0.0010_{th}}$ ⇨ $\boxed{|V_{us}| = 0.2211 \pm 0.0006_{exp} \pm 0.0010_{th}}$

- *Promising!* Competitive with kaon physics!

⇨ $\boxed{|V_{us}| = 0.2255 \pm 0.0005_{exp} \pm 0.0008_{th}}$

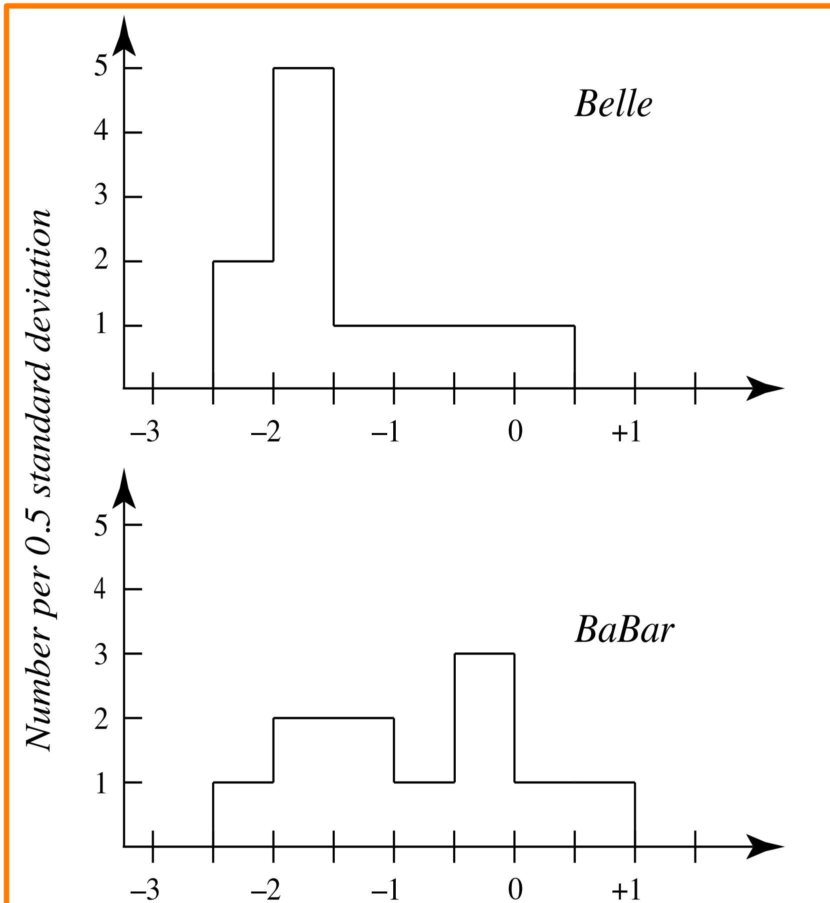
5.2 Outlook : Experimental challenges : strange τ Brs

- *PDG 2014*: « Nineteen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.08 » (-1.41 for the 11 Belle measurements and -0.75 for the 11 BaBar measurements)

  ⇒ Supported by predictions from kaon X channel

- Measured modes by the 2 B factories:

Mode	BaBar – Belle Normalized Difference ($\#\sigma$)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	$+1.4$
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	-2.9
$K^-K^+\pi^-\nu_\tau$	-2.9
$K^-K^+K^-\nu_\tau$	-5.4
$\eta\ K^-\nu_\tau$	-1.0
$\phi\ K^-\nu_\tau$	-1.3



5.2 Outlook

- Experimental challenges :

strange τ BRs:
PDG 2014: « Nineteen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.08 »

⇨ Supported by predictions from kaon X channel measurements

⇨ More *precise measurements*

- Theoretical challenges :
  - ➢ Having the hadronic uncertainties under control: OPE vs. Lattice QCD or ChPT
  - ➢ Isospin breaking
  - ➢ Electromagnetic corrections

Details on the parametrization of the phase

- Model for the phase: ➡ $\boxed{\tan\phi_V = \dfrac{\mathrm{Im}\,\tilde{F}_V(s)}{\mathrm{Re}\,\tilde{F}_V(s)}}$

Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\tilde{F}_V(s) = \frac{\tilde{M}_\rho^2 + \left(\alpha' e^{i\phi'} + \alpha'' e^{i\phi''}\right) s}{\tilde{M}_\rho^2 - s + \kappa_\rho \,\mathrm{Re}\left[A_\pi(s) + \frac{1}{2}A_K(s)\right] - i\tilde{M}_\rho\tilde{\Gamma}_\rho(s)} - \frac{\alpha' e^{i\phi'} s}{D(\tilde{M}_{\rho'},\tilde{\Gamma}_{\rho'})} - \frac{\alpha'' e^{i\phi''} s}{D(\tilde{M}_{\rho''},\tilde{\Gamma}_{\rho''})}$$

with $D(\tilde{M}_R,\tilde{\Gamma}_R) = \tilde{M}_R - s + \kappa_R \mathrm{Re} A_\pi(s) - i\tilde{M}_R\tilde{\Gamma}_R(s)$

and $\tilde{\Gamma}_R(s) = \tilde{\Gamma}_R \dfrac{s}{\tilde{M}_R^2} \dfrac{\left(\sigma_\pi^3(s) + 1/2\ \sigma_K^3(s)\right)}{\left(\sigma_\pi^3(\tilde{M}_R^2) + 1/2\ \sigma_K^3(\tilde{M}_R^2)\right)}$

$$\kappa_R(s) = \frac{\tilde{\Gamma}_R}{\tilde{M}_R} \frac{s}{\pi\left(\sigma_\pi^3(\tilde{M}_R^2) + 1/2\ \sigma_K^3(\tilde{M}_R^2)\right)}$$

Details on the fit

- The minimized quantity:

$$\chi^2 = \sum_{i=1}^{62} \left(\frac{(|F_V(s)|^2)_i^{\rm theo} - (|F_V(s)|^2)_i^{\rm exp}}{\sigma_{(|F_V(s)|^2)_i^{\rm exp}}} \right)^2 + \left(\frac{\lambda'_V - \lambda'^{\,\rm sr}_V}{\sigma_{\lambda'^{\,\rm sr}_V}} \right)^2 + \left(\frac{\alpha_{2v} - \alpha_{2v}^{\rm sr}}{\sigma_{\alpha_{2v}^{\rm sr}}} \right)^2$$

- 2 sum-rules are added such that $F_V(s) \to 1/s$ *Brodsky & Lepage*

$$\lambda'^{\,\rm sr}_V = \frac{m_\pi^2}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\phi_V(s')}{s'^2}$$

$$(\lambda''_V - \lambda'^2_V)^{\rm sr} = \frac{2m_\pi^4}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\phi_V(s')}{s'^3} \equiv \alpha_{2v}^{\rm sr}$$

Results for the $\pi\pi$ vector form factor

$$\tilde{F}_V(s) = \frac{\tilde{M}_\rho^2 + \left(\alpha' e^{i\phi'} + \alpha'' e^{i\phi''}\right) s}{\tilde{M}_\rho^2 - s + \kappa_\rho \,\mathrm{Re}\left[A_\pi(s) + \frac{1}{2} A_K(s)\right] - i\tilde{M}_\rho \tilde{\Gamma}_\rho(s)} - \frac{\alpha' e^{i\phi'} s}{D(\tilde{M}_{\rho'}, \tilde{\Gamma}_{\rho'})} - \frac{\alpha'' e^{i\phi''} s}{D(\tilde{M}_{\rho''}, \tilde{\Gamma}_{\rho''})}$$

$\lambda'_V \times 10^3$	36.7 ± 0.2
$\lambda''_V \times 10^3$	3.12 ± 0.04
$\tilde{M}_\rho$[MeV]	833.9 ± 0.6
$\tilde{\Gamma}_\rho$[MeV]	198 ± 1
$\tilde{M}_{\rho'}$[MeV]	1497 ± 7
$\tilde{\Gamma}_{\rho'}$[MeV]	785 ± 51
$\tilde{M}_{\rho''}$[MeV]	1685 ± 30
$\tilde{\Gamma}_{\rho''}$[MeV]	800 ± 31
α'	0.173 ± 0.009
ϕ'	-0.98 ± 0.11
α''	0.23 ± 0.01
ϕ''	2.20 ± 0.05
$\chi^2/d.o.f$	$38/52$