Hamiltonian approach to QCD in Coulomb gauge: from the vacuum to finite temperatures

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in collaboration with
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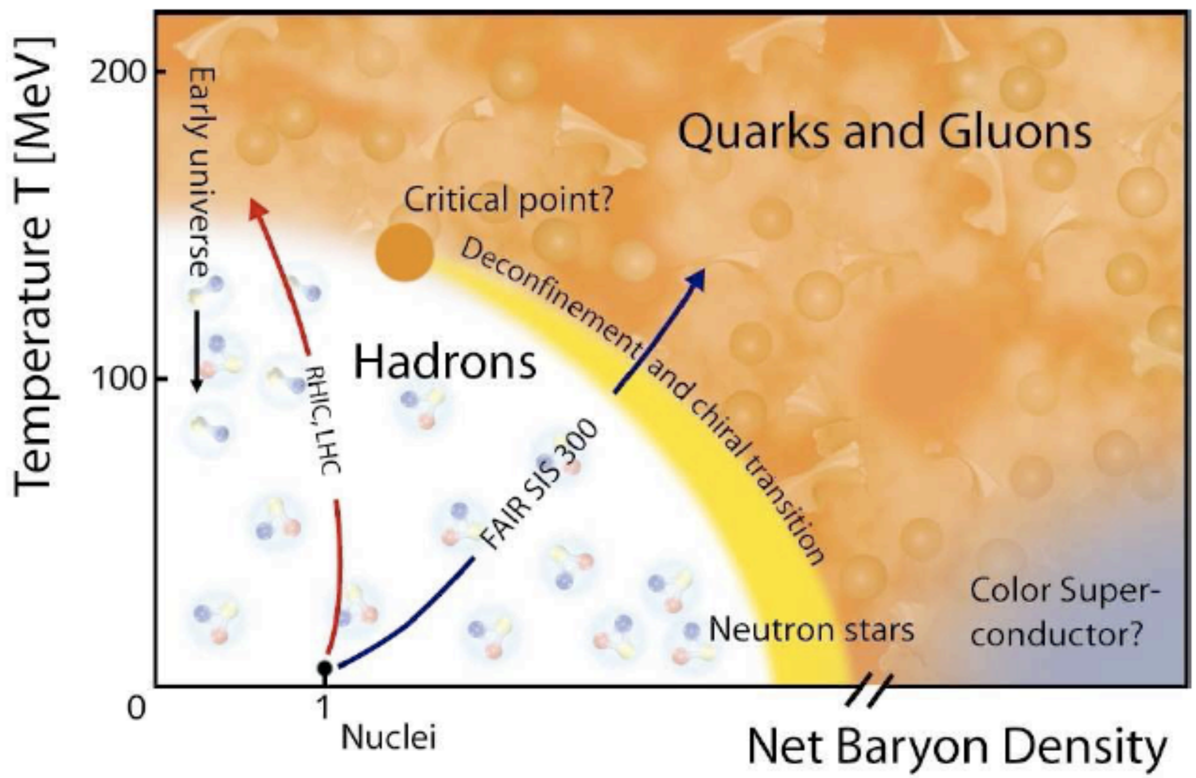
Phase diagram of QCD



lattice: SU(3) at finite chemical potential
- complex quark determinant
- continuum approaches required

Outline

- introduction:
    - Hamiltonian approach to QCD in Coulomb gauge
- novel Hamiltonian approach to finite tempature QFT: compactification of a spatial dimension
- YMT at finite temperature in Coulomb gauge
- effective potential for the Polyakov loop
- conclusions

Hamiltonian approach to YMT in Coulomb gauge

$$\partial A = 0$$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2) + H_C$$

$$\Pi^{\perp} = \delta / i\delta A^{\perp}$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D = \partial + gA$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density: $\rho = -A^{\perp}\Pi^{\perp} + \rho_m$

$$\langle \Phi | \ldots | \Psi \rangle = \int_{\Lambda} DAJ(A)\Phi^*(A)\ldots\Psi(A)$$

$$H\Psi[A] = E\Psi[A]$$

Variational approach

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\,\omega(x,y)\,A(y)\right]$$

gluon propagator $\langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

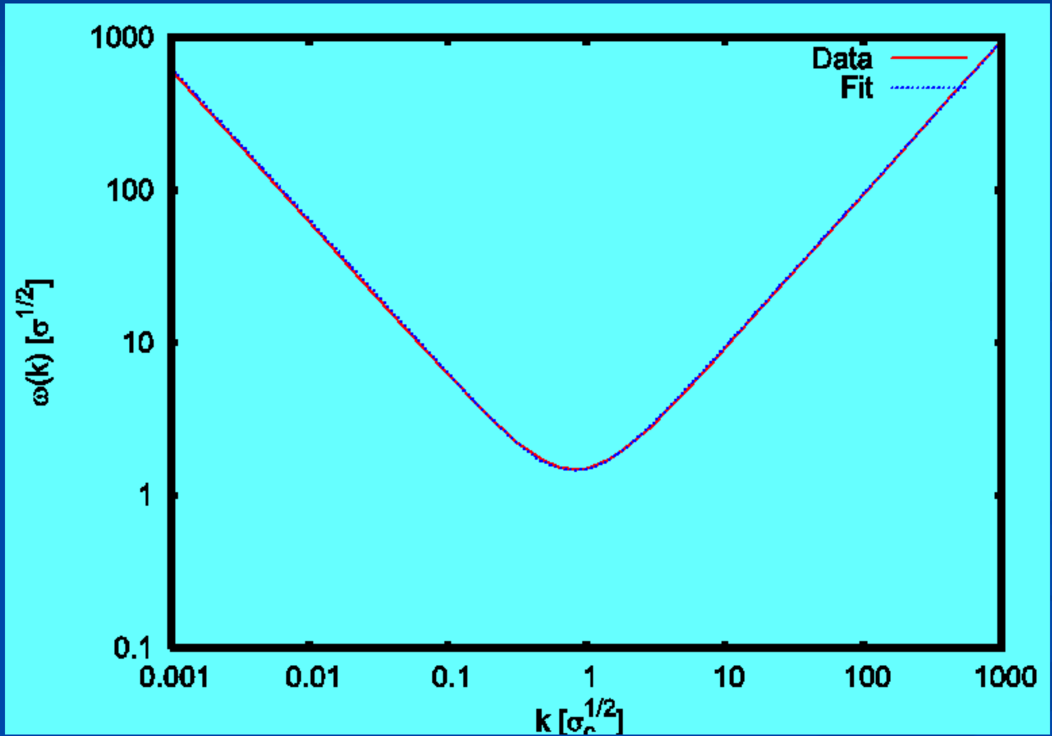
variational kernel $\omega(x,x')$ determined from

$$\langle\Psi|H|\Psi\rangle \to \min$$

Numerical results

gluon energy

D. Epple, H. R., W.Schleifenbaum, PRD 75 (2007)



$$IR: \quad \omega(k) \sim 1/k \qquad\qquad UV: \quad \omega(k) \sim k$$

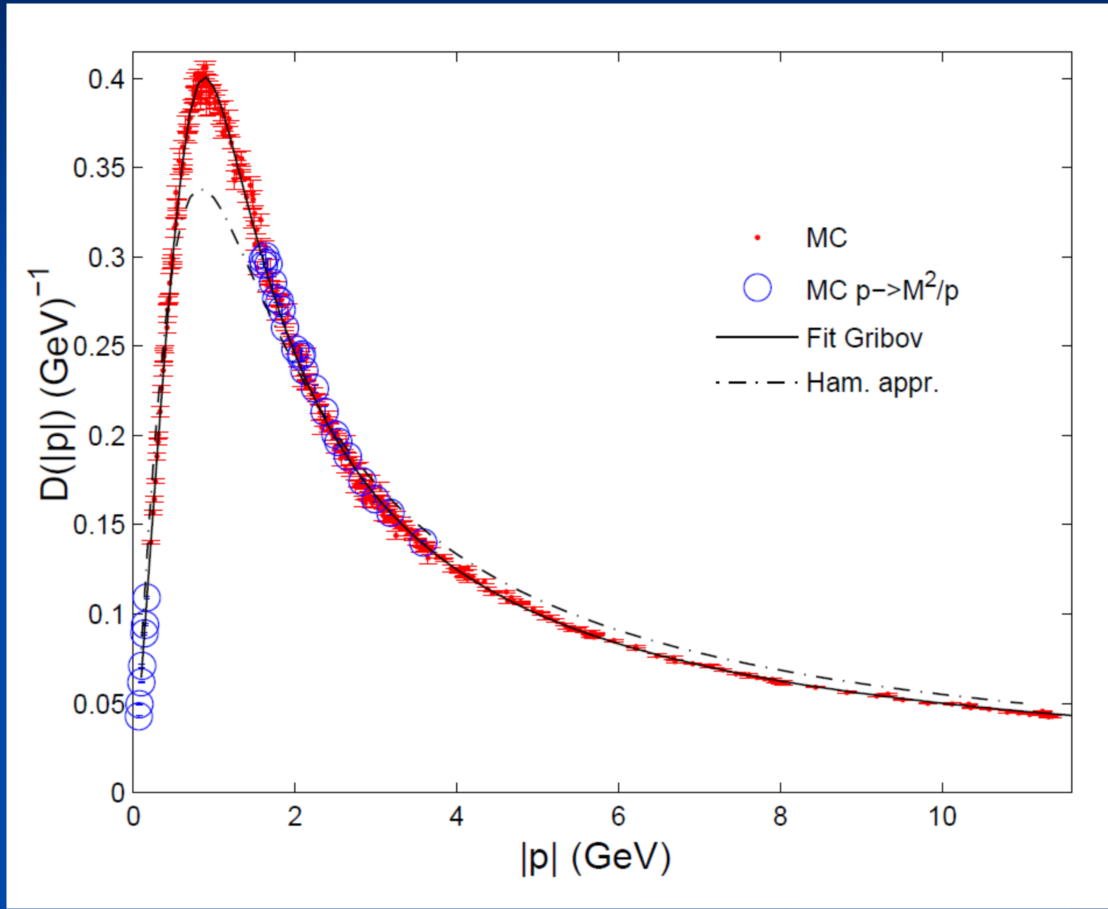
Static gluon propagator in D=3+1

$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 GeV$$



missing strength in
mid momentum regime:
missing gluon loop

G. Burgio, M.Quandt , H.R., **PRL102(2009)**

Hamiltonian approach to YMT at finite T

Reinhardt, Campagnari, Szczepaniak, PRD84(2011)
Heffner, Reinhardt, Campagnari, Phys. Rev D85(2012)

- Grand canonical ensemble with $\mu = 0$
  - quasi-particle varaional ansatz for the density matrix

$$D = \exp(-\tilde{H} / T)$$

  - minimization of the free energy

$$F = \langle H \rangle_T - TS \to \min \qquad \langle \dots \rangle_T = \frac{Tr(D \dots)}{TrD}$$

  - entropy

$$S = -Tr(D \ln D)$$

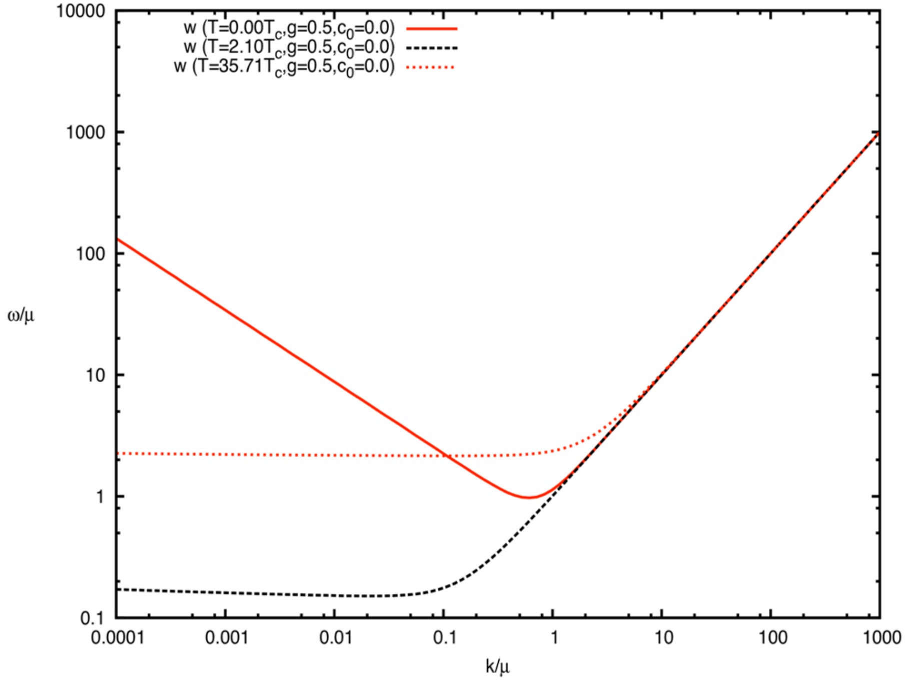
$$\langle AA \rangle_T = (1 + 2n)/(2\omega)$$

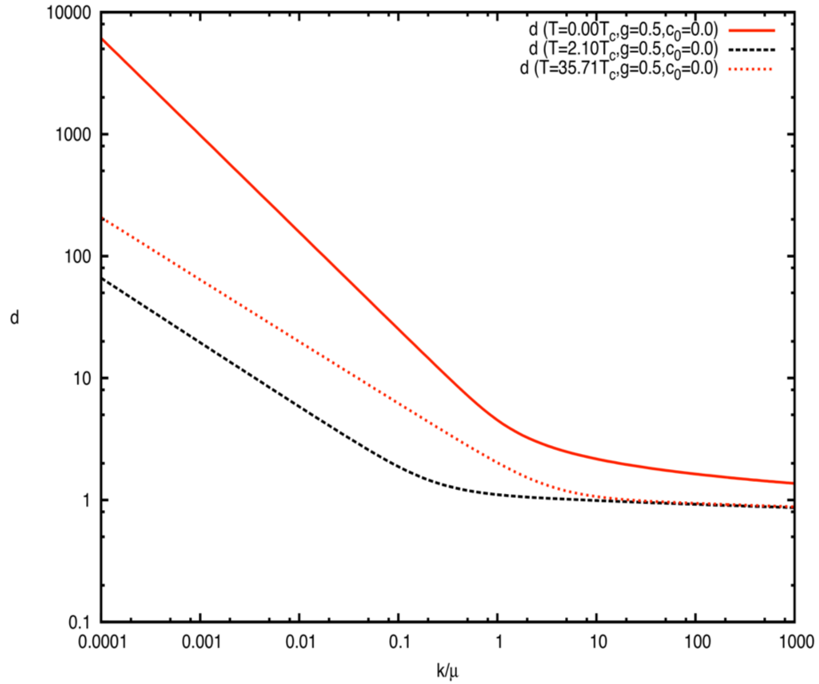
$$\langle (-D\partial)^{-1} \rangle = d/(-\Delta)$$

$$n(k) = \left[\exp(\omega(k)/T) - 1\right]^{-1}$$

gluon energy

ghost form factor





IR-exponent of ghost

Heffner, H.R., Campagnari
Phys. Rev D85(2012)

$$\left\langle (-D\partial)^{-1} \right\rangle = d/(-\Delta)$$

$$d(p) \sim p^{-\beta} \qquad p \to 0$$

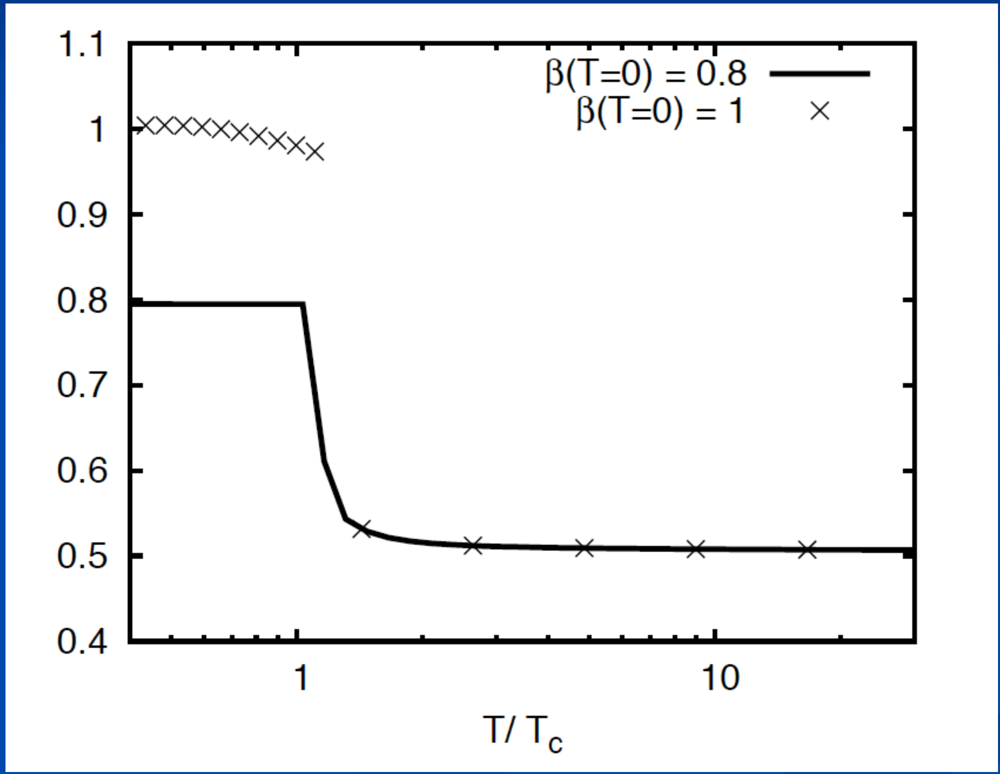
input : $SU(2)-lattice:$

$$\omega(k) = \sqrt{k^2 + M^4/k^2}$$

Gribov mass $M = 860....880\,MeV$

$\Rightarrow \; T_C = 275......290\,MeV$

lattice : $T_C = 295\,MeV$



Alternative Hamiltonian approach to finite temperature QFT

- no ansatz for the density matrix required

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013) and to be published

- motivation: Polyakov loop

$$P[A_0](\vec{x}) = \frac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

  - <P[A_0]> order parameter of confinement

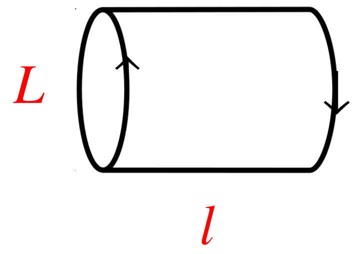
- Hamiltonian approach
  - Weyl gauge $A_0=0$

- How to calculate the Polkov loop in the Hamiltonian approach?

Finite temperature QFT

- compactification of (Euclidean) time
- bc: $A(x^0 = L/2) = \mathrm{A}(x^0 = -L/2)$ Bose fields

  $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields



$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\quad l \to \infty$

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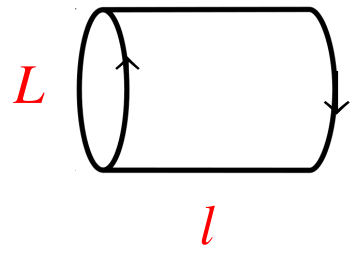

$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\qquad l \to \infty$
- exploit the O(4)-invariance of the Euclidean Lagrangian
  - O(4)-rotation $x^0 \to x^3 \qquad A^0 \to A^3 \qquad \gamma^0 \to \gamma^3$
  - one compactified spatial dimension
  - bc: $A(x^3 == L/2) = A(x^3 = -L/2)$ Bose fields
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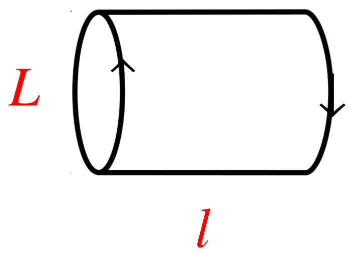
- spatial manifold: $\mathbb{R}^2 \times S^1(L)$



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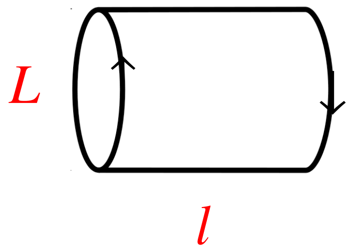

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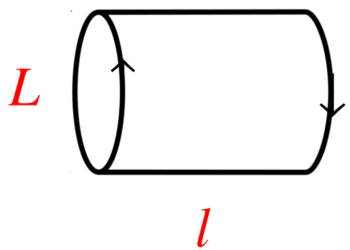


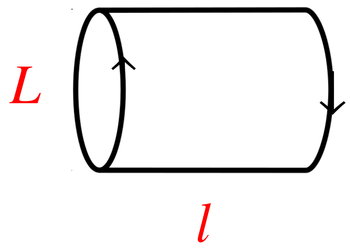
- *temperature is now encoded in one „spatial“ dimension while „time“ has infinite extension independent of the temperature*

Finite temperature QFT

- partition function

$$Z(L) = \lim_{l\to\infty} Tr \exp(-lH(L)) = \lim_{l\to\infty} \sum_n \exp(-lE_n(L)) = \lim_{l\to\infty} \exp(-lE_0(L))$$

- ground state energy $E_0(L) = l^2 L e(L)$
  - on the spatial manifold: $\mathbb{R}^2 \times S^1(L)$



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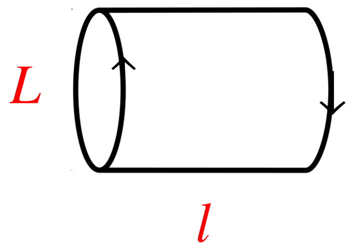
- ground state energy $E_0(L) = l^2 Le(L)$
  - on the spatial manifold: $\mathbb{R}^2 \times S^1(L)$



- pressure: $p = -e(L)$
- energy density: $\varepsilon = \partial[Le(L)]/\partial L$
- Dirac fermions with finite chemical potential

$$h = \vec{\alpha}\cdot\vec{p} + \beta m \rightarrow h + i\mu\alpha^3$$

Relativistic Bose gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \, \frac{p^2}{\omega(p)} n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1} \qquad \omega(p) = \sqrt{p^2 + m^2}$$

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- energy density on $\mathbb{R}^2 \times S^1(L)$

$$e(L) = \frac{1}{2}\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty} \sqrt{m^2 + p_\perp^2 + \omega_n^2} \qquad \omega_n = \frac{2\pi n}{L}$$

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  - Poisson resummation

$$P = -e(L) = -\frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\left(\frac{m}{n\beta}\right)^2 K_{-2}(n\beta m)$$

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- massles bosons: m=0

$Stephan - Boltzmann - law$

$$P = \frac{\zeta(4)}{\pi^2}T^4 = \frac{\pi^2}{90}T^4$$

Relativistic Fermi gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)}(n_+(p) + n_-(p)) \qquad n_\pm(p) = \frac{1}{e^{\beta(\omega(p)\mp\mu)}+1} \qquad \omega(p) = \sqrt{p^2 + m^2}$$

Relativistic Fermi gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)}(n_+(p) + n_-(p)) \qquad n_\pm(p) = \frac{1}{e^{\beta(\omega(p)\mp\mu)}+1} \qquad \omega(p) = \sqrt{p^2+m^2}$$

- energy density on $\mathbb{R}^2 \times S^1(L)$

$$e(L) = -2\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2 + p_\perp^2 + (\omega_n + i\mu)^2} \qquad \omega_n = \frac{2n+1}{L}\pi$$

  - Poisson resummation

$$P = -e(L) = -\frac{2}{\pi^2}\sum_{n=-\infty}^{\infty}\cos\left[n\beta(\tfrac{\pi}{\beta} - i\mu)\right]\left(\frac{m}{n\beta}\right)^2 K_{-2}(n\beta m)$$

- massles Dirac fermions: m=0

$$P = \frac{1}{12\pi^2}\left[\frac{7}{15}\pi^4T^4 + 2\pi^2T^2\mu^2 + \mu^4\right]$$

Hamiltonian approach to YMT in Coulomb gauge on $\mathbb{R}^2 \times S^1(L)$

- trial ansatz $$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\omega(x,y)A(y)\right]$$

  transversal projector in $\mathbb{R}^3$: $\quad t_{kl}(x) = \delta_{kl} - \frac{\partial_k^x \partial_l^x}{\partial_x^2}$

  $$t_{kl}(x) = t_{kl}^{\perp}(x) + t_{kl}^{\parallel}(x),$$

  transversal projector in $\mathbb{R}^2$

  $$t_{kl}^{\perp}(x) = (1-\delta_{k3})\left(\delta_{kl} - \frac{\partial_k^x \partial_l^x}{\Delta^{\perp}}\right)(1-\delta_{k3})$$

- varational kernel $$\omega_{kl}(x,y) = t_{kl}^{\perp}(x)\omega_{\perp}(x,y) + t_{kl}^{\parallel}(x)\omega_{\parallel}(x,y)$$

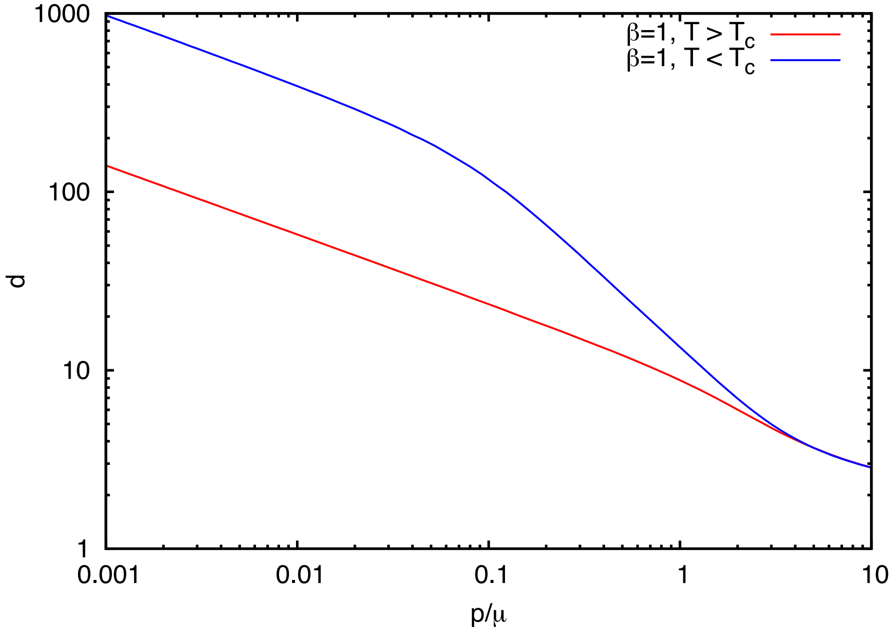
- gluon propagator $$\langle A(x)A(y)\rangle = t_{kl}^{\perp}(x)(2\omega_{\perp}(x,y))^{-1} + t_{kl}^{\parallel}(x)(2\omega_{\parallel}(x,y))^{-1}$$

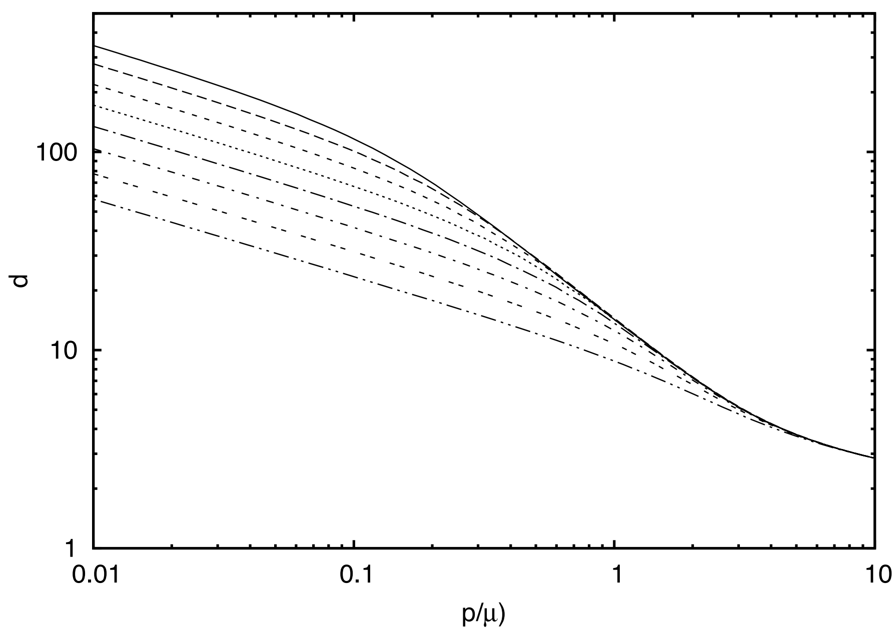
- $T \to 0$ $\qquad \omega_{\perp}(p) = \omega_{\parallel}(p) \qquad \chi_{\perp}(p) = \chi_{\parallel}(p) \qquad p_3 = \frac{2n\pi}{L}$

- $T \to \infty$ $\qquad \omega_{\perp}(p) = \omega(p)_{d=2} \qquad \chi_{\parallel}(p) = 0 \qquad \omega_{\parallel}(p) = p$

The ghost form factor on $\mathbb{R}^2 \times S^1(\beta)$

$$d(p_\perp, n=0)$$

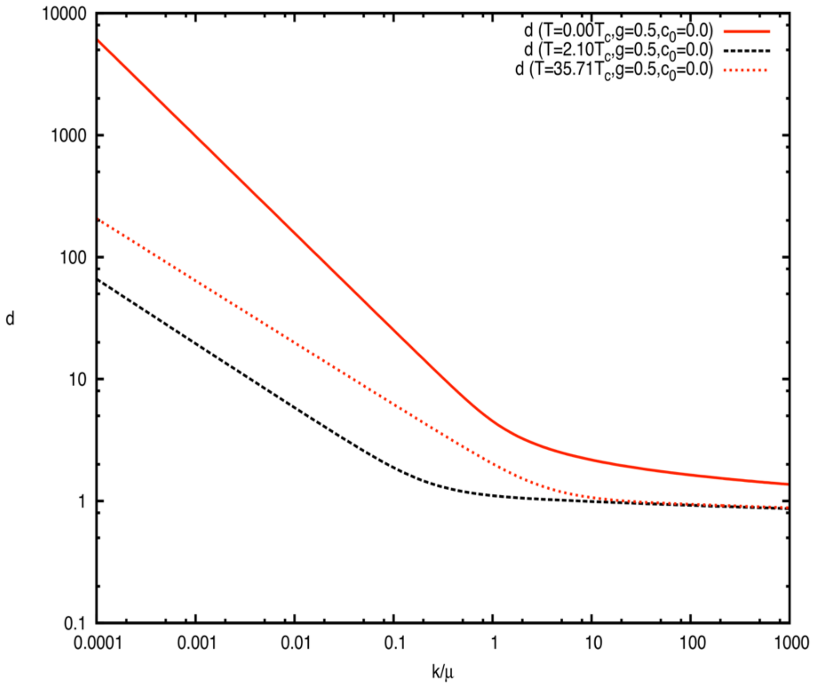




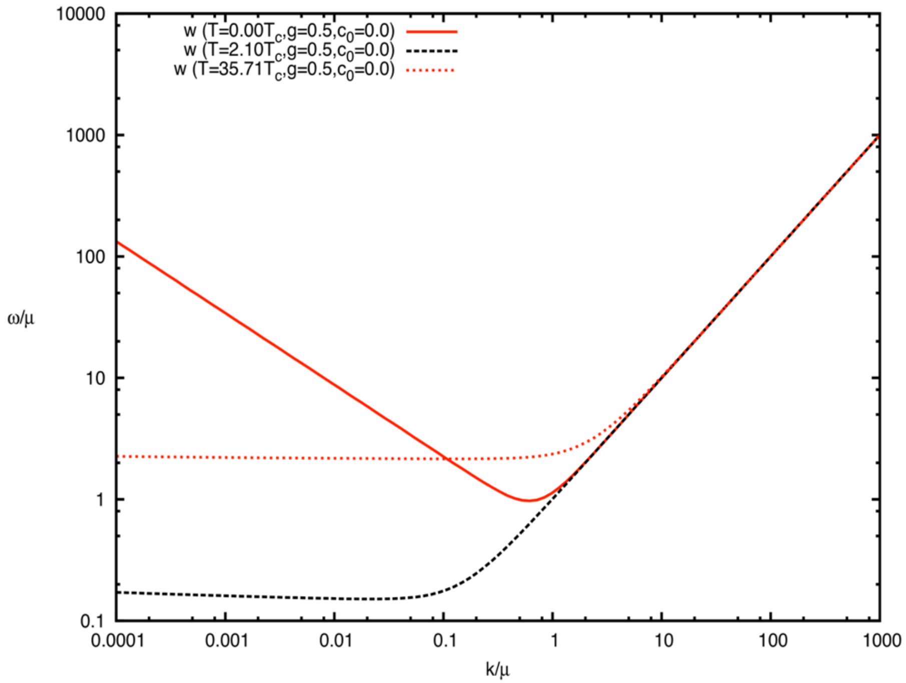
$T \to \infty$: dimensional reduction

Grand canonical ensemble

ghost form factor

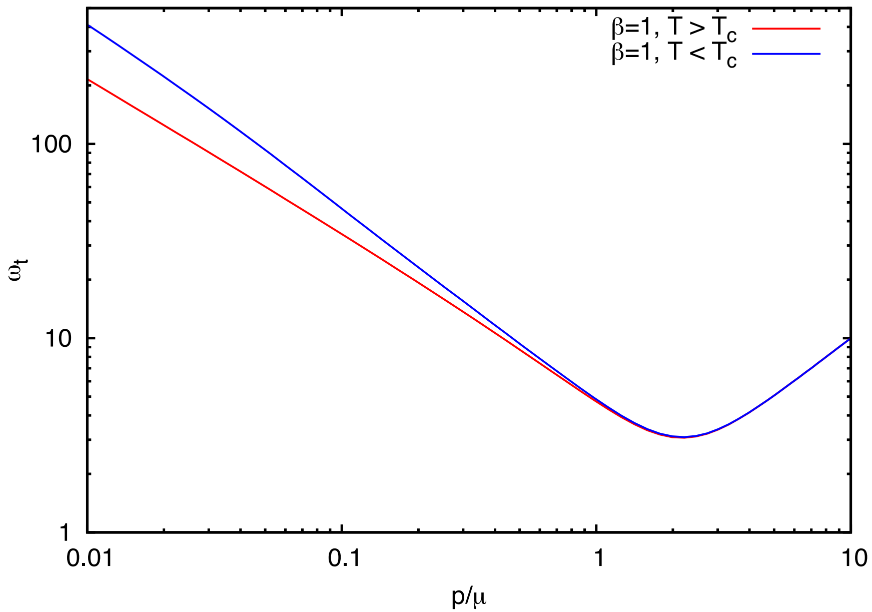


gluon energy

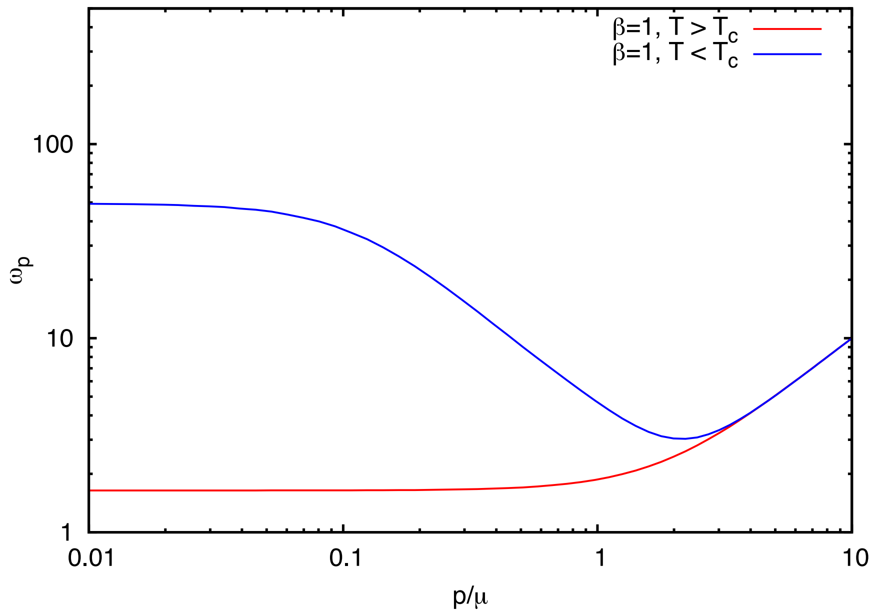


The gluon energy on $\mathbb{R}^2 \times S^1(\beta)$

transverse $\omega_\perp(p_\perp, n=0)$



parallel $\omega_\parallel(p_\perp, n=0)$

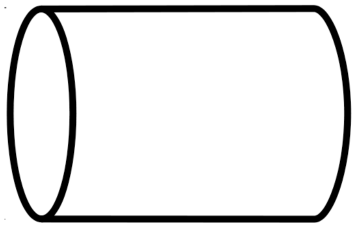


Polyakov loop

- YMT at finite temperature T :compact Euclidean time

$$P[A_0](\vec{x}) = \frac{1}{d_r} trP \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$$T^{-1} = L$$



- order parameter for confinement: $\langle P[A_0](\vec{x}) \rangle \sim \exp\left[-F_\infty(\vec{x}) L \right]$

  - conf. phase: center symmetry $\langle P[A_0](\vec{x}) \rangle = 0$
  - deconf. phase: center symmetry-broken $\langle P[A_0](\vec{x}) \rangle \neq 0$

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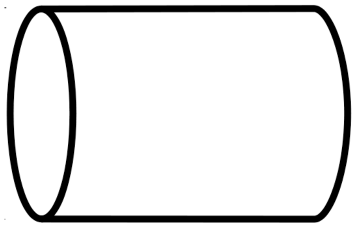

- order parameter for confinement: $\langle P[A_0](\vec{x}) \rangle \sim \exp\left[-F_\infty(\vec{x})L \right]$
  - conf. phase: center symmetry $\langle P[A_0](\vec{x}) \rangle = 0$
  - deconf. phase: center symmetry-broken $\langle P[A_0](\vec{x}) \rangle \neq 0$
- Polyakov gauge $\partial_0 A_0 = 0, \quad A_0 = diagonal$ $\quad SU(2): \; P[A_0](\vec{x}) = \cos(\frac{A_0(\vec{x})L}{2})$
  - fundamental modular region $0 < A_0 L/2 < \pi$ $\quad P[A_0] - unique\ function\ of\ A_0$
  - Jensen´s inequality: $\langle P[A_0](\vec{x}) \rangle \;\leq\; P[\langle A_0(\vec{x}) \rangle]$
  - alternative order parameters: $\langle P[A_0](\vec{x}) \rangle \quad P[\langle A_0(\vec{x}) \rangle] \quad \langle A_0(\vec{x}) \rangle$

- *F.Marhauser and J. M. Pawlowski, arXiv:0812.11144*
- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*
- *J. Braun,T.K. Herbst,arXiv1205.0779*

Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle$ − const, diagonal (Polyakov gauge)
- effective potential $e[a_0] \rightarrow \min \qquad \Rightarrow a_0 = \bar{a}_0$
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$

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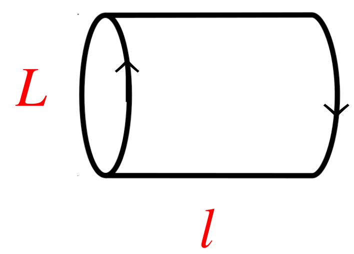
- ordinary Hamiltonian approach assumes Weyl gauge $A_0 = 0$
  - O(4)-invariance

- compactify (instead of time) one spatial axis to a circle of circumference L and interprete L^{-1} as temperature

- Hamiltonian approach on $\mathbb{R}^2 \times S^1(L)$



- compactify x_3 − axis $\vec{a} = a\vec{e}_3$
- calculate the effective potential $e[a]$

The effective potential in the Hamiltonian approach

- effective potential $e(\vec{a})$ of a spatial background field $\vec{a}$

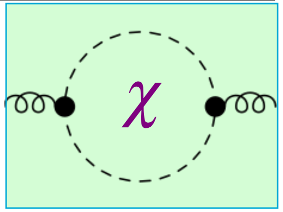
$$\langle H \rangle_{\vec{a}} = \min \langle H \rangle \qquad \langle \vec{A} \rangle = \vec{a}$$

$$\langle H \rangle_{\vec{a}} = (spatial\ volume) \times e(\vec{a})$$

$$e(\vec{a}) - effective\ potential$$

The gluon effective potential

- energy density $$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\chi(p^\sigma))$$



- background field $\vec{p}^\sigma=\vec{p}_\perp+(p_n-\sigma a)\vec{e}_3 \qquad p_n=2\pi n/L \qquad \sigma-roots$
- roots

$SU(2)$: $H_1=T_3 \qquad \sigma_1=0,\ \pm 1 \qquad$ *positive roots*

$SU(3)$: $H_1=T_3 \qquad H_2=T_8 \qquad \sigma=(1,0),\ \ (\tfrac{1}{2},\tfrac{1}{2}\sqrt{3}),\ \ (\tfrac{1}{2},-\tfrac{1}{2}\sqrt{3})$

- periodicity $e(a,L)=e(a+\mu_k/L,L) \qquad \exp\left(i\mu_k\right)=z_k\in Z(N)$

$\mu_k-coweights$

- input: $\omega(p),\chi(p)$ from the variational calculation in Coulomb gauge at T=0

C. Feuchter & H. Reinhardt,Phys. Rev.D71(2005)
D. Epple, H. Reinhardt, W. Schleifenbaum, Phys. Rev.D75(2007)

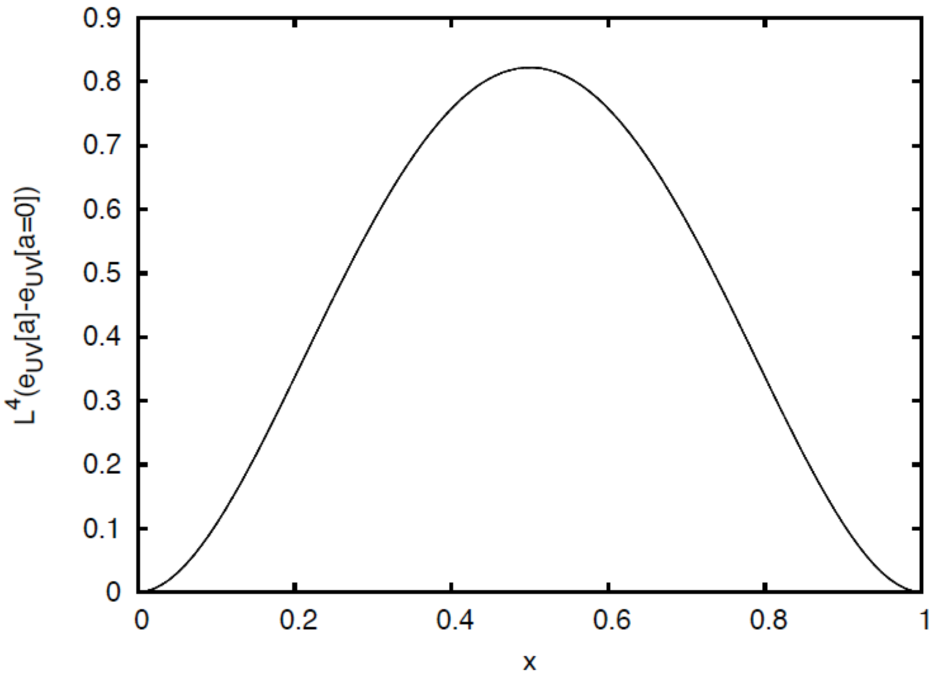
The gluon UV-effective potential

$$\chi(p)=0 \qquad \omega(p)=p \qquad e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\chi(p^\sigma))$$

$$e(a,L)=\frac{8}{\pi^2L^4}\sum_{n=1}^{\infty}\frac{\sin^2(naL/2)}{n^4}$$

$$=\frac{4\pi^2}{3L^4}\left(\underbrace{\frac{aL}{2\pi}}_{x}\right)^2\left[\frac{aL}{2\pi}-1\right]^2$$

N.Weiss 1-loop PT

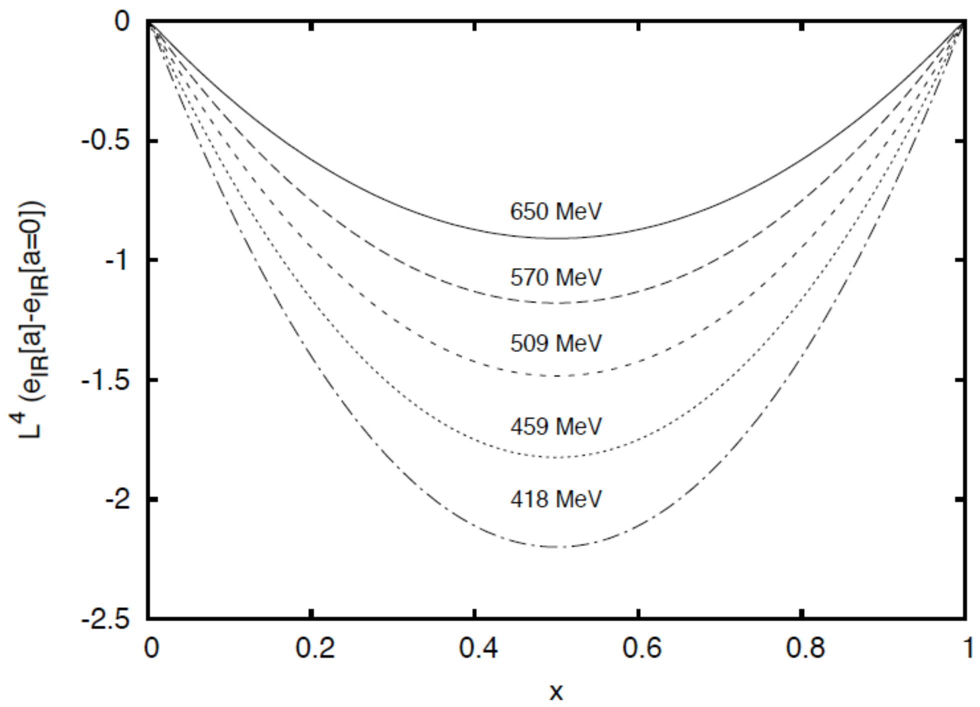


$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=0]=1 \qquad deconfining \quad phase$$

The gluon IR-effective potential

$$\chi(p)=0 \qquad \omega(p)=M^2/p \qquad e(a,L)=\sum_\sigma \frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\chi(p^\sigma))$$

$$e_{IR}(a,L)=-\frac{4M^2}{\pi^2L^2}\sum_{n=1}^{\infty}\frac{\sin^2(naL/2)}{n^2}=\frac{2M^2}{L^2}\underbrace{\left(\frac{aL}{2\pi}\right)}_{x}\left[\frac{aL}{2\pi}-1\right]$$



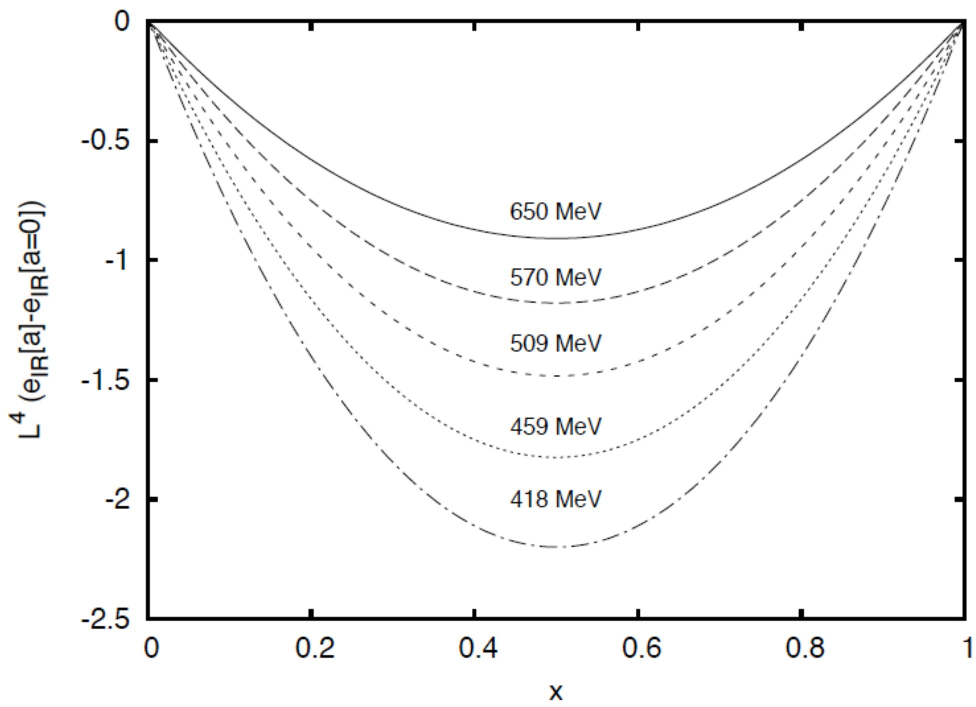
$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=\pi/L]=0 \qquad confining \quad phase$$

The gluon IR-effective potential

$\chi(p) = 0$ $\quad$ $\omega(p) = M^2 / p$ $\quad$ $e(a,L) = \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \chi(p^\sigma))$

$$e_{IR}(a,L) = -\frac{4M^2}{\pi^2 L^2} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^2}$$

$$= \frac{2M^2}{L^2} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x} \left[\frac{aL}{2\pi} - 1\right]$$



$Polyakov - loop \quad \langle P \rangle \simeq P[a_{min} = \pi / L] = 0 \quad confining \;\; phase$

deconfinement phase transition results from the interplay between the confining IR-potential and deconfining UV-potential

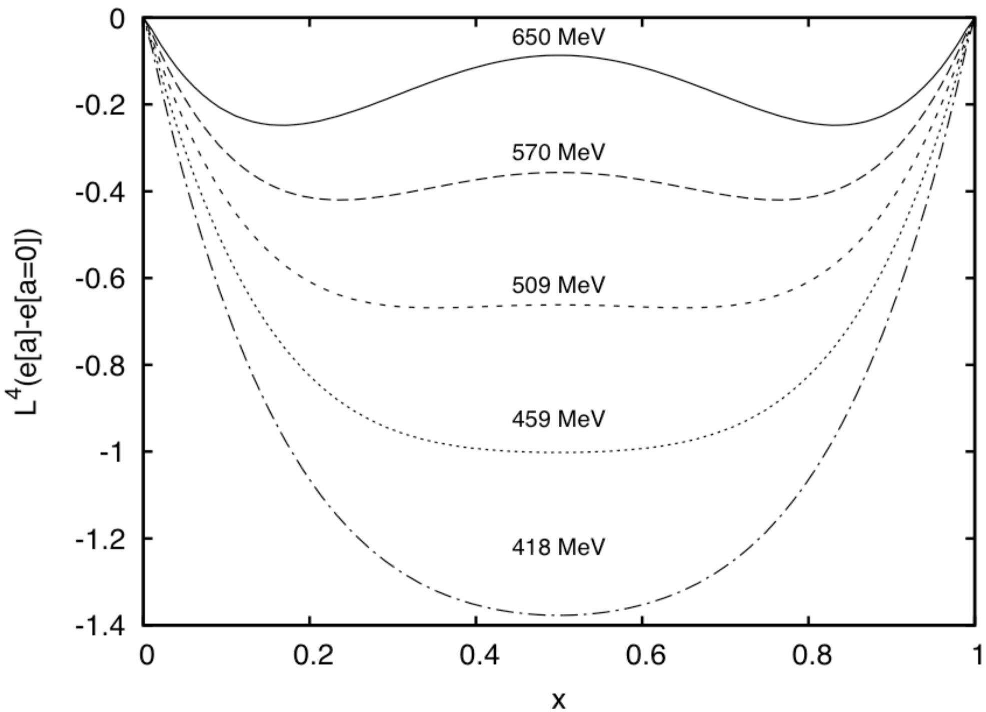
The gluon IR+UV effective potential:

$\chi(p) = 0$ $\omega(p) = p + M^2 / p$ $e(a,L) = e_{UV}(a,L) + e_{IR}(a,L)$

phase transition

critical temperature:

$T_C = \sqrt{3} M / \pi$

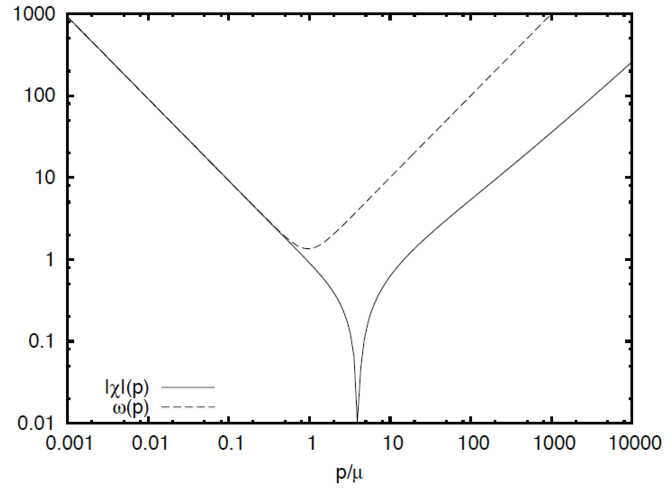


$lattice : M \simeq 880 MeV \quad \Rightarrow \quad T_C \simeq 485 MeV$

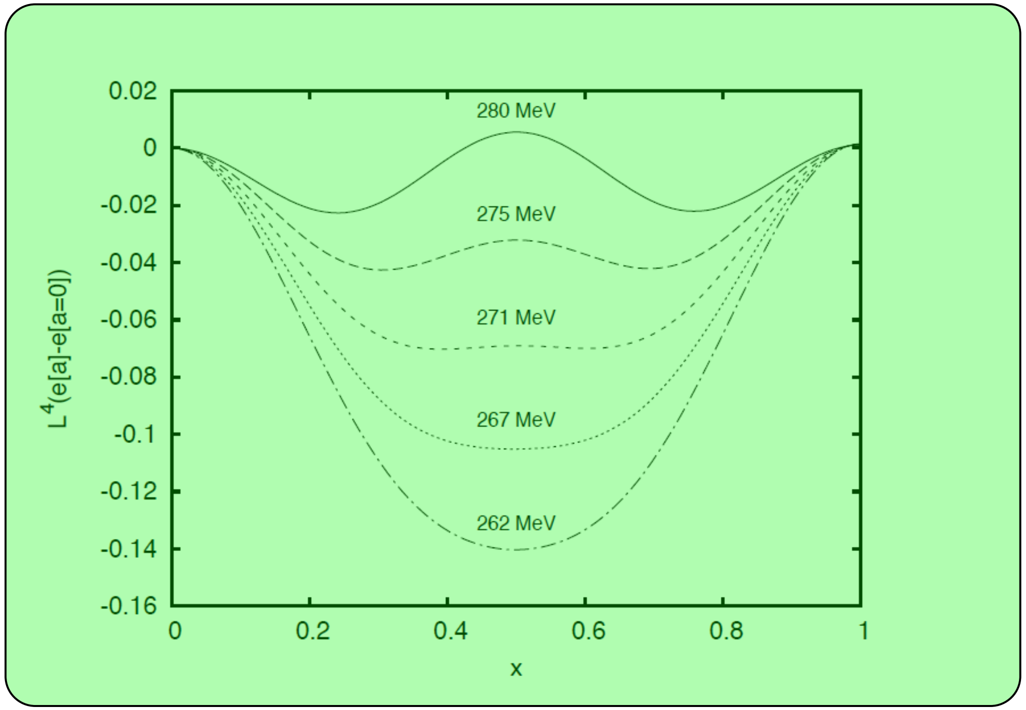
$\chi(p) = 0$ $\omega(p) = \sqrt{p^2 + M^4 / p^2}$ $T_C \simeq 432 MeV$

The full gluon effective potential

$$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}(\omega(p^{\sigma})-\chi(p^{\sigma}))$$



variational calculation in Coulomb gauge



SU(2)

critical temperature:

$$T_C \simeq 270\,MeV$$

The effective potential for SU(3)

SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive rooots

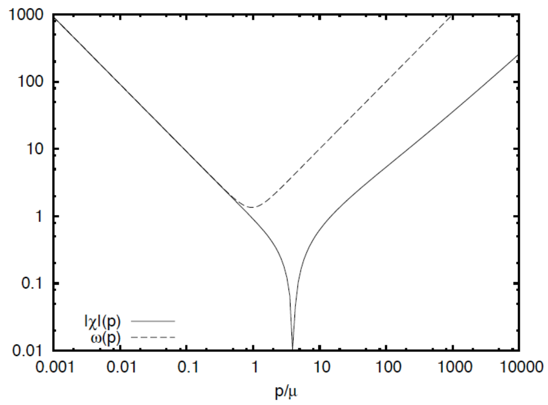
$$\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

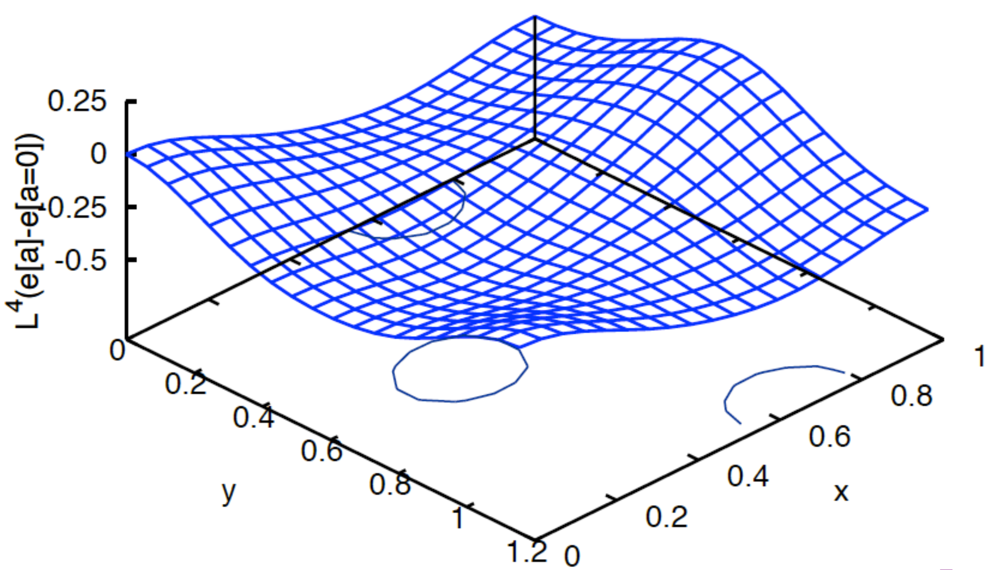
The full effective potential for SU(3)

$$e({\color{red}a},L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}\left(\omega(p^{\sigma})-\chi(p^{\sigma})\right)$$

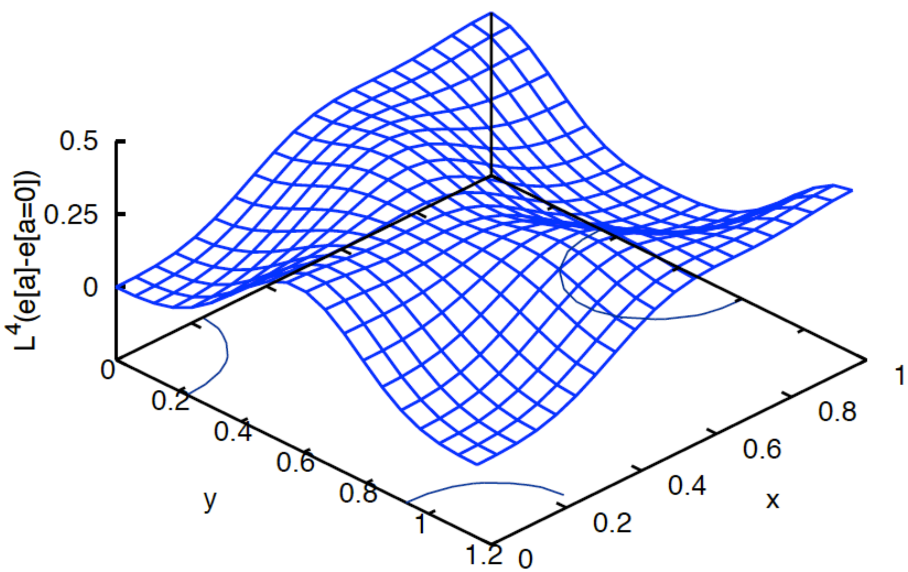
variational calculation in Coulomb gauge



$T < T_C$

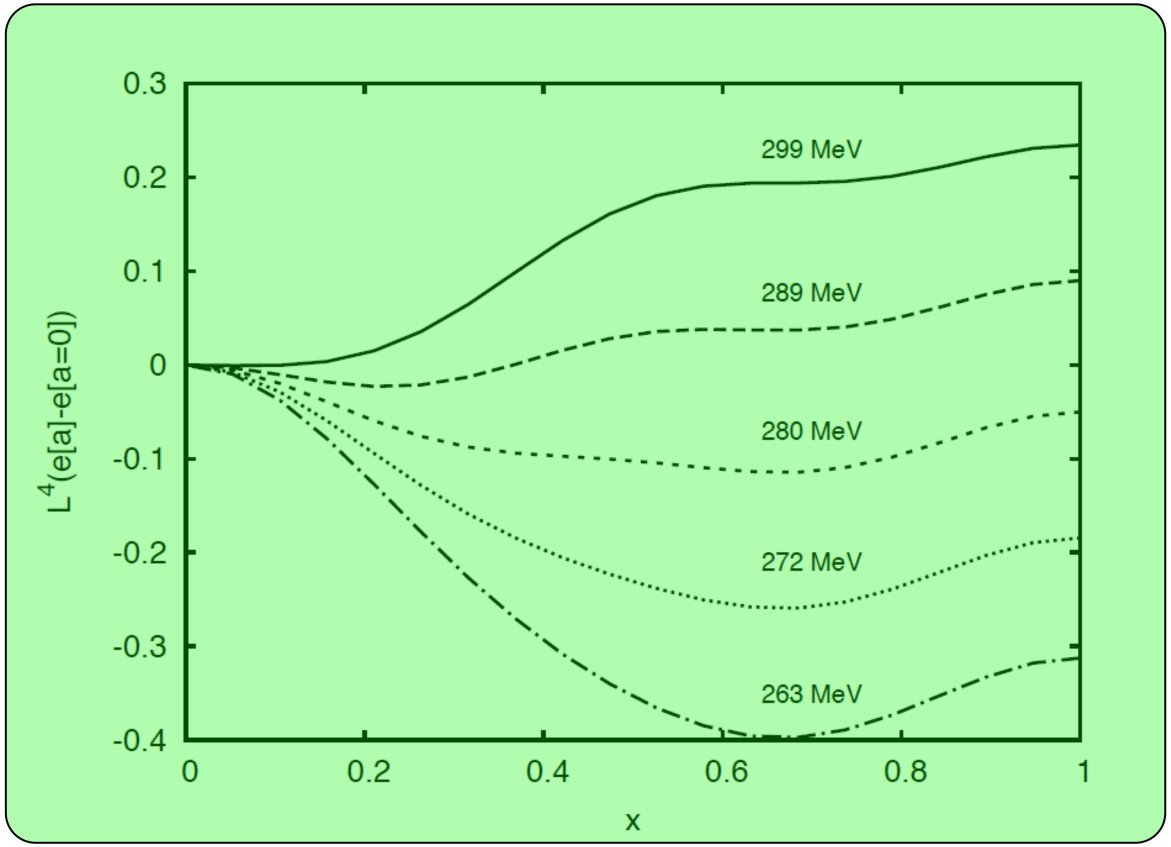


$T > T_C$



$x=\frac{a_3L}{2\pi}$, $\quad y=\frac{a_8L}{2\pi}$

Polyakov loop potential for SU(3)



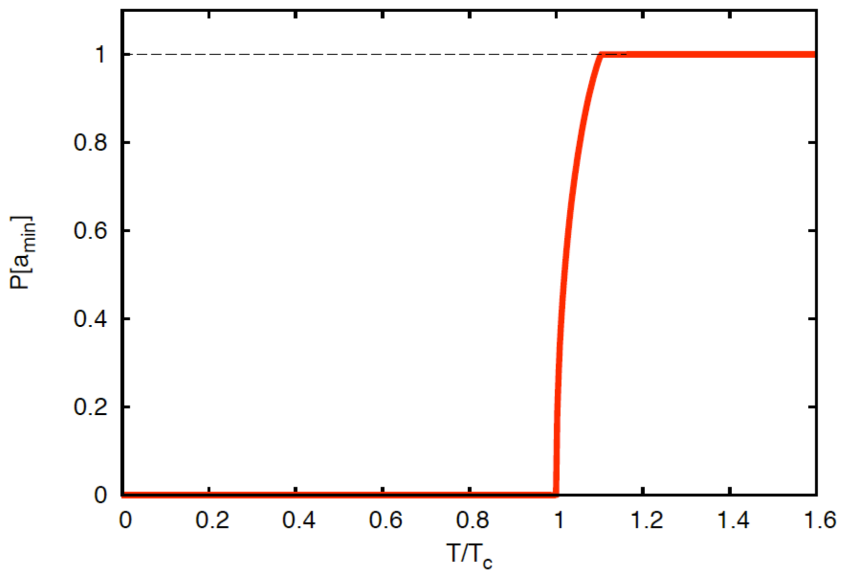
$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi} = 0$$

$$input: \; SU(2) - data:$$

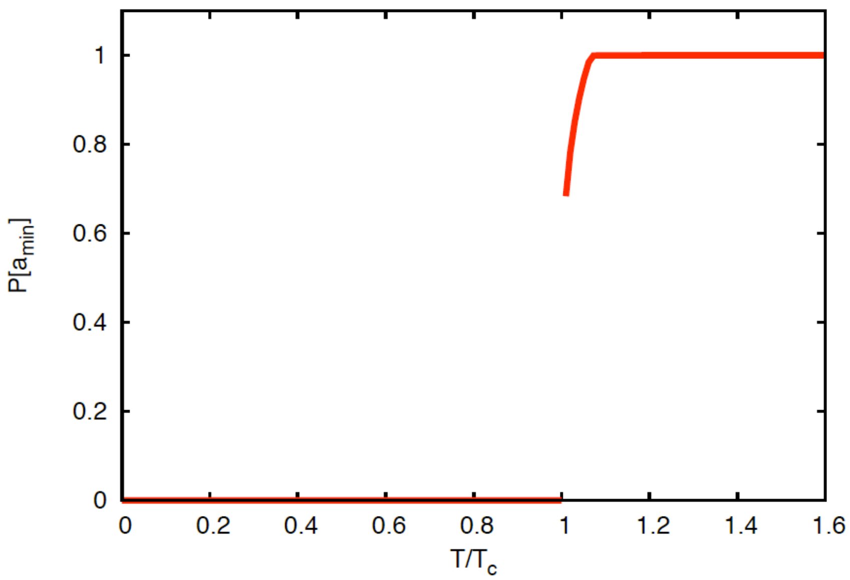
$$M = 880 MeV$$

$$T_C = 283 MeV$$

The Polyakov loop



SU(2)



SU(3)

critical temperature

$lattice:$ $T_C^{SU(2)} = 295\,MeV$ $T_C^{SU(3)} = 270\,MeV$

$this\ work:$ $T_C^{SU(2)} = 267\,MeV$ $T_C^{SU(3)} = 277\,MeV$

$FRG(Fister\ \&\ Pawlowski):$ $T_C^{SU(2)} = 230\,MeV$ $T_C^{SU(3)} = 275\,MeV$

Hamiltonian approach to QCD in Coulomb gauge

M. Pak & H. R., Phys.Lett.B707 (2012)
Phys. Rev. D88(2013)

quark wave functional

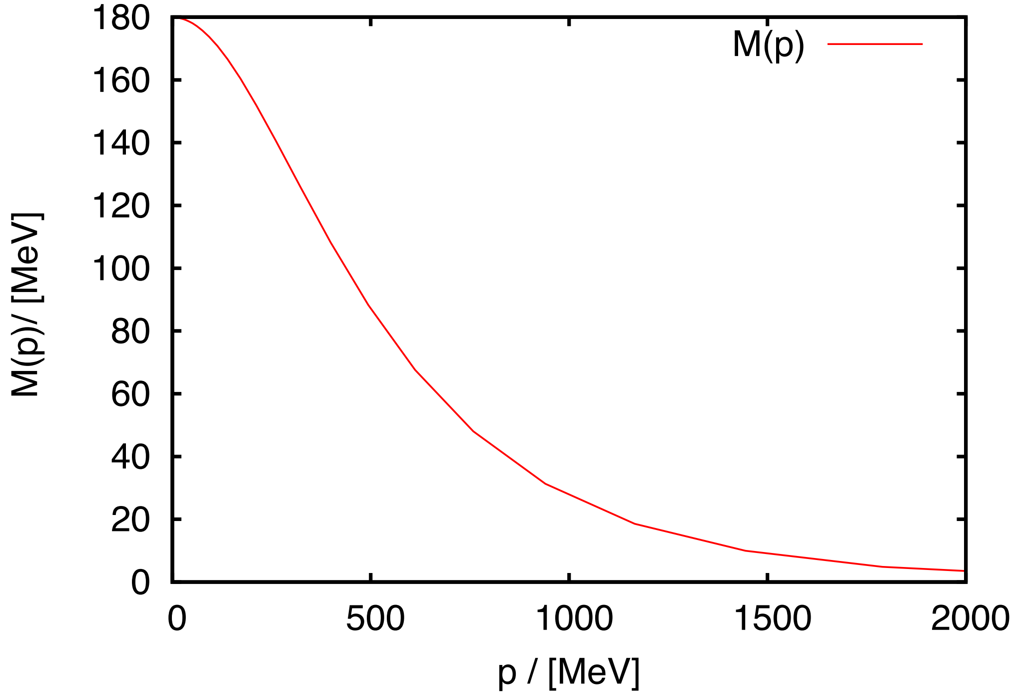
$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A})\Psi\right]|0\rangle$$

$\mathrm{v}=0:\quad BCS-wave\ function\ (Adler\ \&\ Davis)$

$\mathrm{v}\neq 0:\quad quark-gluon\ coupling$

The quark quasi-particles

effective quark energy $\varepsilon(p) = \sqrt{M^2(p) + p^2}$



-in the quak sector the Coulomb string tension was adjusted to produce a quark condensate of $\langle \bar{q}q \rangle = (-230\, MeV)^2$

The quark effective potential

- energy density

$$e(a,L) = -N_f \sum_{\sigma} \tfrac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp \varepsilon(p^\sigma)$$

- quasi quark energy $\varepsilon(p) = \sqrt{M^2(p) + p^2}$

- background field $\vec{p}^\sigma = \vec{p}_\perp + (p_n - \sigma a + i\mu)\vec{e}_3$ $\quad p_n = (2n+1)\pi / L \quad \sigma - weights$

$SU(2)$: $\quad H_1 = T_3 \quad\quad \sigma_1 = \pm\tfrac{1}{2}$

$SU(3)$: $\quad H_1 = T_3 \quad H_2 = T_8 \quad\quad \sigma = (\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}), \quad (-\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}), \quad (0, -\tfrac{1}{\sqrt{3}})$

- periodicity $\quad e(a,L) = e(a + 2\mu_k / L, L) \quad \exp(i\mu_k) = z_k \in Z(N)$

The quark UV-effective potential

$$\varepsilon(p) = p \qquad e(a,L) = -N_f \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp \varepsilon(p^\sigma)$$

$$e(a,L) = \frac{N_f}{24\pi^2 L^4} \sum_\sigma \left[\frac{7}{15}\pi^4 + 2\pi^2 L^2 (\mu + i\sigma \cdot a)^2 + L^4 (\mu + i\sigma \cdot a)^4 \right]$$

$SU(2): \quad \sigma \cdot a = \pm \frac{1}{2} \Rightarrow$ *real potential* **complex for SU(N>2)**

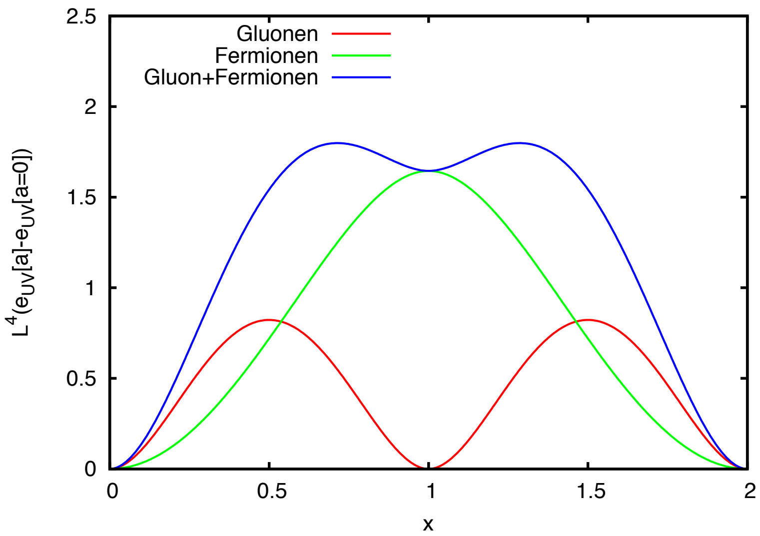
$$e(a=0,L) = \frac{N_f}{24\pi^2 L^4} \left[\frac{7}{15}\pi^4 + 2\pi^2 L^2 \mu^2 + L^4 \mu^4 \right] \quad \textit{pressure of massless fermions}$$

$$\bar{e}(a,L) = \frac{N_f \pi^2}{6L^4} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x}^2 \left[1 - \frac{1}{2}\left(\frac{aL}{2\pi}\right)^2 + 12\left(\frac{\mu L}{2\pi}\right)^2 \right]$$

$\mu = 0$ **N.Weiss 1-loop PT**

$Polyakov - loop \quad \langle P \rangle \simeq P[a_{\min} = 0] = 1$

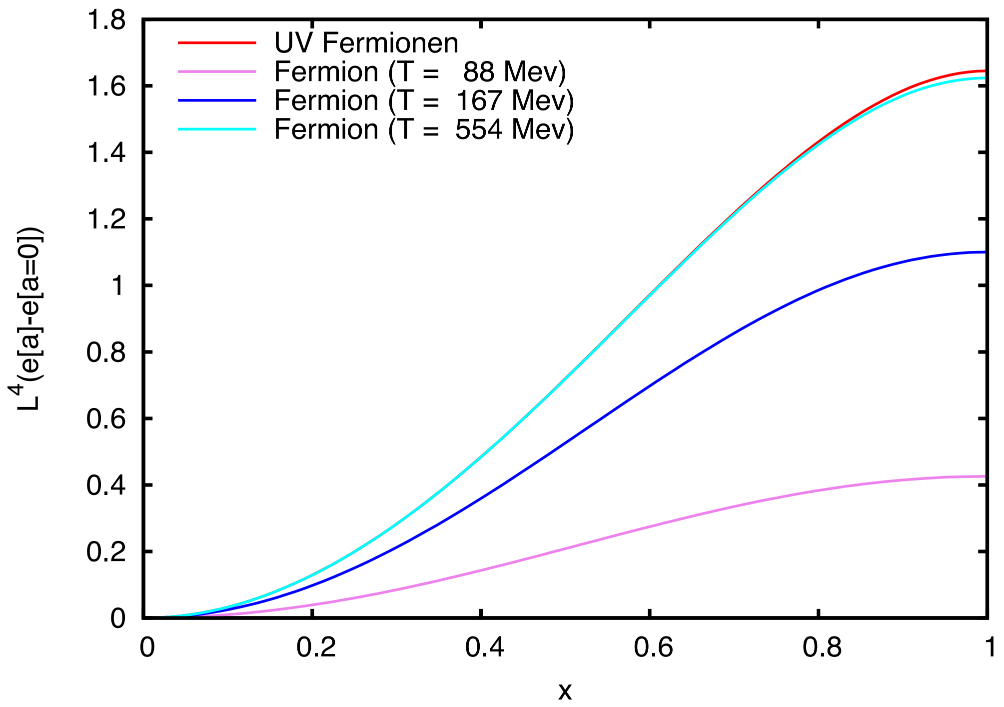
deconfining phase



The SU(2) quark Polyakov loop potential

$$e(a,L) = -N_f \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp \varepsilon(p^\sigma) \qquad \varepsilon(p) = \sqrt{M^2(p) + p^2}$$

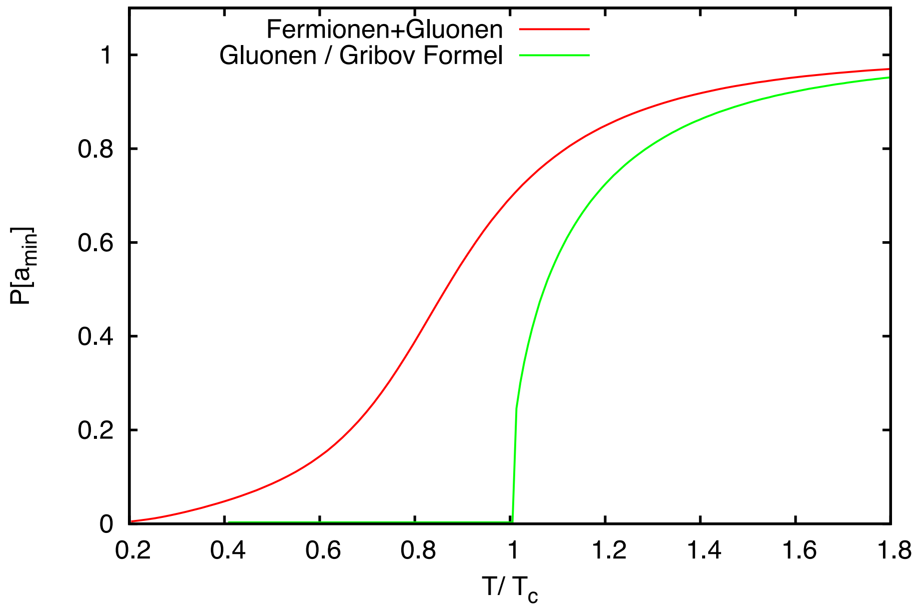
$$\vec{p}^{\sigma} = \vec{p}_\perp + (p_n - \sigma a + i\mu)\vec{e}_3 \qquad p_n = (2n+1)\pi / L \qquad \sigma - weights$$



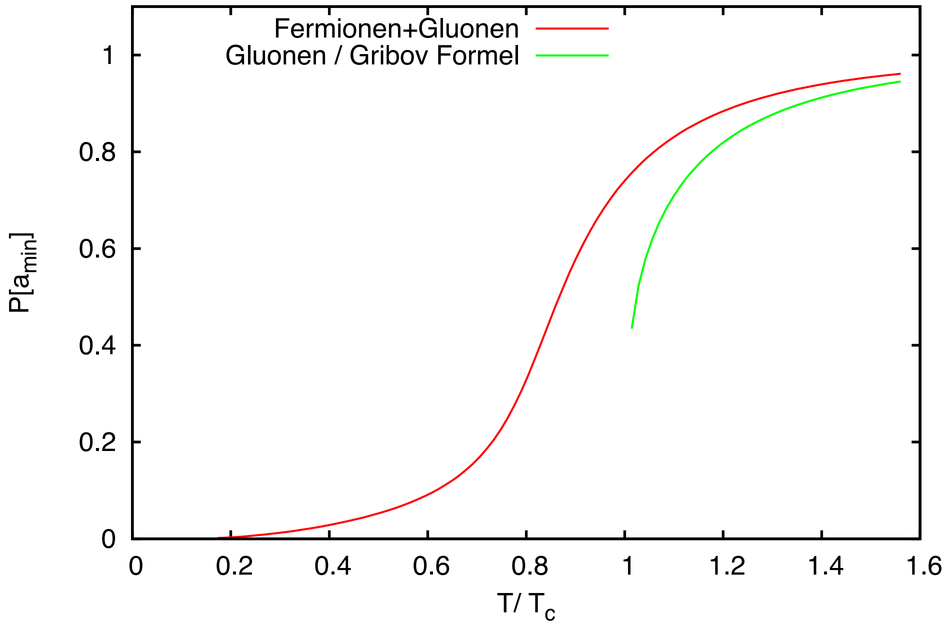
$\mu = 0$

- quarks are „deconfining“, i.e. lower the critical temperature

The Polyakov loop



SU(2)



SU(3)

Conclusions

- novel Hamiltonian approach to finite temperature QFT
- compactification of a spatial dimension
- requires calculation of the ground on the spatial manifold $R^2 \times S^1$
- reproduces results of the grand canonical ensemble

- effective potential of the Polyakov loop:
- input: static vacuum propagators obtained in the variational calculation in Coulomb gauge

- gluon potential:
  - deconfinement phase transition
    - SU(2): 2.order
    - SU(3): 1.order

$T_C \simeq 270\,MeV$

- inclusion of quarks:
  - lower the transition temperature
  - at small chemical potential:
    - the deconfinement phase transition is turned into a crossover

Thanks for your attention