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QCD with axial chemical potential: possible manifestations

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Outline

Collaboration with V. Andrianov, D. Espriu & X. Planells

Phenomenology of Local Parity breaking (LPB):

- Axial baryon charge $T_5 \Leftrightarrow$ axial chemical potential μ_5 (in finite volume of fireball), listen also D. Espriu's talk (Parallel IV: D4, Tuesday).
- Effective meson theory in a medium with LPB
  - Vector Meson Dominance (VMD) approach to LPB

    A.Andrianov, V.Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710:230 (2012)
  - Effective scalar/pseudoscalar meson theory with μ_5

    A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C , 73:2294 (2013)
  - Phase structure of the NJL quark model with μ_5

    A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C , 74:2776 (2014)
- Manifestation of LPB in heavy ion collisions (HIC): dilepton polarization asymmetry et al.

  A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Rev. D 90, 034024 (2014)

  Listen D. Espriu's talk (Parallel IV: D4, Tuesday).

Phenomenology of local parity breaking

Parity: reasons to believe it may be broken in a finite volume?!

- **Fluctuations of topological charge density in a finite volume**: quasi-instanton vacuum [D.Diakonov, V.Petrov, E.Shuryak, . . .] Color Glass Condensate [E.Iancu, A.Leonidov, L.McLerran, R.Venugopalan,. . .].
- **quantum fluctuations of θ parameter** (P-odd bubbles [T. D. Lee and G. C. Wick . . .]: their manifestation in Chiral Magnetic Effect (CME))[D. E. Kharzeev, A.Zhitnitsky, L. McLerran, K.Fukushima, H. J. Warringa (an earlier proposal: A.Vilenkin, 1980)]
- **New QCD phase** characterized by a spontaneous parity breaking due to formation of neutral pion-like background [A.A.Anselm . . . . . . A. A., V. A. Andrianov & D. Espriu]
- **High energy production** of pseudoscalar gluelumps ⇔ parity-odd bunches of gluon jets ⇒ **then a PB background remains inside a hot dense nuclear fireball in HIC !?**

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Phenomenology of local parity breaking

Local large fluctuations in the topological charge presumably exist in a hot environment.

- For *peripheral* heavy ion collisions they lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \Rightarrow$ large $\vec{E} \Rightarrow$ charge separation.

  D. Kharzeev, R. D. Pisarski & M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998)

  K. Buckley, T. Fugleberg, & A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814

  D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008)

- For *central* collisions (and light quarks) they correspond to an ephemeral phase with axial chemical potential $\mu_5 \neq 0$ located in "fluctons" of few-Fermi size.

  A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.

Axial baryon charge and axial chemical potential

QCD has a non-trivial vacuum structure with different topological sectors.

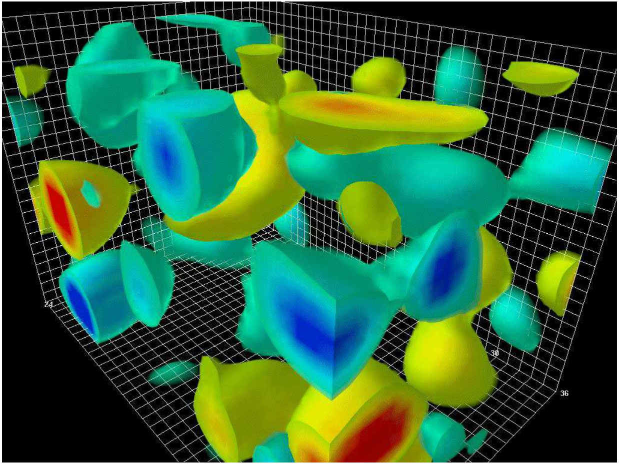
Topological charge T_5 may arise in a finite volume due to quantum fluctuations in a hot medium due to sphaleron transitions [Manton, McLerran, Rubakov, Shaposhnikov]

$$T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \mathrm{Tr} \left(G^j \partial^k G^l - i\frac{2}{3} G^j G^k G^l \right)$$

and survive for a sizeable lifetime in a heavy-ion fireball

$$\langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}} \simeq 5 \div 10 \text{ fm.}$$

Axial baryon charge and axial chemical potential



Lattice simulation of local fluctuations of the topological charge in the QCD vacuum [Leinweber].

Axial baryon charge and axial chemical potential

For the fireball lifetime one can control the value of $\langle \Delta T_5 \rangle$ introducing into the QCD Lagrangian a topological chemical potential μ_θ in a gauge invariant way via $\Delta\mathcal{L}_{top} = \mu_\theta \Delta T_5$, where

$$\Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2}\int_0^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right).$$

The partial conservation of axial current (broken by gluon anomaly)

$$\partial_\mu J_5^\mu - 2im_q J_5 = \frac{N_f}{8\pi^2}\text{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right)$$

predicts the induced axial charge (in the chiral limit $m_q \simeq 0$)

$$\frac{d}{dt}\left(Q_5^q - 2N_f T_5\right) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q}\gamma_0\gamma_5 q = \langle N_L - N_R \rangle$$

to be conserved during $\tau_{fireball}$.

Axial baryon charge and axial chemical potential

The characteristic left-right oscillation time is governed by inverse quark masses.

- For u, d quarks $1/m_q \sim 1/5\ \text{MeV}^{-1} \sim 40$ fm $\gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.
- For s quark $1/m_s \sim 1/150\ \text{MeV}^{-1} \sim 1$ fm $\ll \tau_{\text{fireball}}$ and $\langle Q_5^s \rangle \simeq 0$ due to left-right oscillations.

For u, d quarks QCD with a topological charge $\langle \Delta T_5 \rangle \neq 0$ can be equally described at the Lagrangian level by topological chemical potential μ_θ or by axial chemical potential μ_5

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta,$$

$$\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q$$

Axial baryon charge and axial chemical potential

LPB is investigated in e.m. interactions of leptons and photons with hot/dense nuclear matter via heavy ion collisions.

- e.m. interaction implies

$$Q_5^q \to \tilde{Q}_5 = Q_5^q - T_5^{\mathrm{em}}, \quad T_5^{\mathrm{em}} = \frac{N_c}{8\pi^2} \int_{\mathrm{vol.}} d^3x \, \varepsilon_{jkl} \, \mathrm{Tr} \left(\hat{A}^j \partial^k \hat{A}^l \right) .$$

- μ_5 is conjugated to (nearly) conserved $\tilde{Q}_5$
- Bosonization of $\tilde{Q}_5$ with chiral Lagrangian, VMD...

$$\boxed{\boxed{\langle \Delta T_5 \rangle \quad \Longleftrightarrow \quad \mu_5}}$$

The $\Longleftarrow$ sense means that if one is able to fit μ_5 from phenomenology, $\langle \Delta T_5 \rangle$ can be found using known techniques on the lattice.

How does $\tilde{Q}_5$ affect the hadronic phenomenology?

Effective meson theory in a medium with LPB

- Vector mesons

Low energy QCD can be described with the help of Vector Meson Dominance

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega \omega_\mu \mathbb{I} + \frac{1}{2}g_\rho \rho^0_\mu \tau_3,$$

$$(V_{\mu,a}) \equiv (A_\mu,\, \omega_\mu,\, \rho^0_\mu)$$

where $Q = \frac{\tau_3}{2} + \frac{1}{6}$, $g_\omega \simeq g_\rho \equiv g \simeq 6$.
In this framework, the following term is generated in the effective lagrangian for vector mesons

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]$$

with $\hat{\zeta}_\mu = \hat{\zeta}\delta_{\mu 0}$ for a spatially homogeneous and isotropic background ($\hat{} \equiv$ isospin content) and $\zeta \propto \mu_5$.
Two different cases of isospin structure for μ_5:

- Isosinglet pseudoscalar background ($T \gg \mu$) [RHIC, LHC]
- Pion-like (isotriplet) background ($\mu \gg T$) [FAIR, NICA]

Effective meson theory in a medium with LPB

Massive MCS electrodynamics for vector mesons

$$\mathcal{L}_{MCS} = -\,\tfrac{1}{4}\,F^{\alpha\beta}(x)F_{\alpha\beta}(x) + \tfrac{1}{2}\,m^2\,A_\nu(x)A^\nu(x) + \tfrac{1}{2}\,\zeta_\mu A_\nu(x)\tilde{F}^{\,\mu\nu}(x) + \mathrm{g.f.}$$

In momentum space wave Eqs.

$$\begin{cases} \left[g^{\,\lambda\nu}\,(\,k^2 - m^2\,) - k^{\,\lambda}k^{\,\nu} + i\,\varepsilon^{\,\lambda\nu\alpha\beta}\,\zeta_\alpha\,k_\beta\right]\mathbf{a}_\lambda(k) = 0 \\ k^{\,\lambda}\,\mathbf{a}_\lambda(k) = 0 \end{cases}$$

Energy spectrum:

Transversal polarizations

$$\begin{aligned} K^{\mu}_{\nu}\,\varepsilon^{\nu}_{\pm}(k) &= \left(k^2 - m^2 \pm \sqrt{\mathrm{D}}\right)\,\varepsilon^{\mu}_{\pm}(k); \\ \omega_{\,\mathbf{k}\,,\,\pm} &= \sqrt{\mathbf{k}^2 + m^2 \pm \zeta_0|\,\mathbf{k}\,|}; \quad \zeta_\mu = (\zeta_0, 0, 0, 0) \end{aligned}$$

Longitudinal polarization

$$\omega_{\,\mathbf{k}\,,\,L} = \sqrt{\mathbf{k}^2 + m^2}$$

Vector Meson spectrum in PB medium

After diagonalization of mass matrix

$$m_{V,\epsilon}^2 = m_V^2 - \epsilon\zeta|\vec{k}| \implies |\zeta|,$$

where $\epsilon = 0, \pm 1$ is the meson polarization.
The photon itself happens to be unaffected by a **singlet** $\hat{\zeta}$.
The position of the poles for $\pm$ polarized mesons is changing with wave vector $|\vec{k}|$.
Massive vector mesons split into three polarizations with masses $m_{V,+}^2 < m_{V,L}^2 < m_{V,-}^2$.

This splitting unambiguously signifies LPB. Can it be measured?
→ Manifestation of LPB in heavy ion collisions:

abnormal dilepton production in HIC from the decays $\rho, \omega \to e^+e^-$, listen the discussion in D.Espriu's talk.

Effective scalar/pseudosalar meson theory with μ_5

A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C , 73:2294 (2013)

The scalar sector can be estimated by using the spurion technique in the chiral Lagrangian with an isosinglet μ_5

$$D_\nu \Longrightarrow D_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \cdot\} = D_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu}.$$

Two new processes are likely to appear inside the fireball: the decays $\eta, \eta' \to \pi\pi$ that are strictly forbidden in QCD on parity grounds.

In a medium where parity is broken: are these processes relevant within the fireball? Can these particles reach thermal equilibrium?!

Effective scalar/pseudosalar meson theory with μ_5

No $\mathcal{O}(p^2)$ terms in the chiral Lagrangian involving vertices $\eta\pi\pi$.

$\mathcal{O}(p^4)$ terms lead to couplings such as

$$\mathcal{L}_{\eta\pi\pi} \sim \frac{16\mu_5}{F_\eta f_\pi^2} L\,\partial\eta\,\partial\pi\partial\pi,$$

where $L \sim 10^{-3}$. A rough estimate of the partial width shows a strong dependence on μ_5 as $\Gamma_{\eta\to\pi\pi} \propto \mu_5^2$ and gives values similar or higher than $\Gamma_{\rho\to\pi\pi} = 150$ MeV.

An effective Lagrangian is needed to include the lightest isoscalar degrees of freedom such as $\sigma(600)$ and $a_0(980)$, which will be mixed with their pseudoscalar partners η, η' and π, respectively.

Effective scalar/pseudosalar meson theory with μ_5

Generalized Σ model

Effective Lagrangian:

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left(D_\mu H D^\mu H^\dagger\right) + \frac{b}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right] + \frac{M^2}{2}\mathrm{Tr}\left(HH^\dagger\right)$$
$$-\frac{\lambda_1}{2}\mathrm{Tr}\left[(HH^\dagger)^2\right] - \frac{\lambda_2}{4}\left[\mathrm{Tr}\left(HH^\dagger\right)\right]^2 + \frac{c}{2}(\det H + \det H^\dagger)$$
$$+\frac{d_1}{2}\mathrm{Tr}\left[M(HH^\dagger H + H^\dagger H H^\dagger)\right] + \frac{d_2}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right]\mathrm{Tr}\left(HH^\dagger\right)$$

where

$$H = \xi\Sigma\xi, \quad \xi = \exp\left(i\frac{\Phi}{2f}\right), \quad \Phi = \lambda^a\phi^a, \quad \Sigma = \lambda^b\sigma^b.$$

The v.e.v. of the neutral scalars are defined as $v_i = \langle\Sigma_{ii}\rangle$ where $i = u, d, s$, and satisfy the following gap equations:

$$M^2 v_i - 2\lambda_1 v_i^3 - \lambda_2 \vec{v}^2 v_i + c\frac{v_u v_d v_s}{v_i} = 0.$$

Effective scalar/pseudosalar meson theory with μ_5

Generalized Σ model

For further purposes we need the non-strange meson sector and η_s

$$\Phi = \begin{pmatrix} \eta_q + \pi^0 & \sqrt{2}\pi^+ & 0 \\ \sqrt{2}\pi^- & \eta_q - \pi^0 & 0 \\ 0 & 0 & \sqrt{2}\eta_s \end{pmatrix}, \Sigma = \begin{pmatrix} v_u + \sigma + a_0^0 & \sqrt{2}a_0^+ & 0 \\ \sqrt{2}a_0^- & v_d + \sigma - a_0^0 & 0 \\ 0 & 0 & v_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

For $\mu_5 = 0$, we assume $v_u = v_d = v_s = v_0 \equiv f_\pi \approx 92$ MeV. The coupling constants (in units of MeV) are fitted to phenomenology assuming isospin symmetry via χ^2 minimization (`MINUIT`):

$$b = -3510100/m,\ M^2 = 1255600,\ c = 1252.2,\ \lambda_1 = 67.007,$$

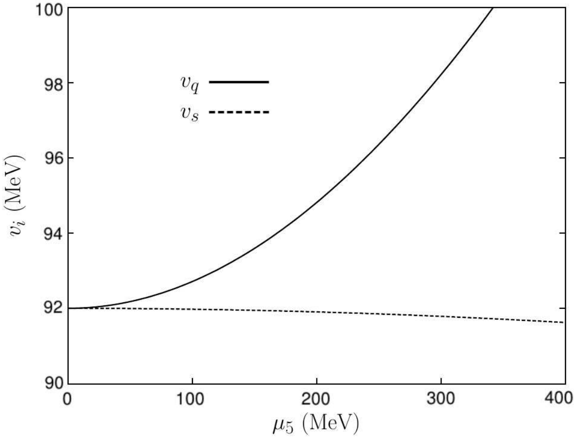
$$\lambda_2 = 9.3126,\ d_1 = -1051.7/m,\ d_2 = 523.21/m,$$

where $m \equiv m_q = (m_u + m_d)/2$ and $m/m_s \simeq 1/25$.

Effective scalar/pseudosalar meson theory with μ_5

Generalized Σ model

Vacuum: for non-vanishing isosinglet μ_5 we impose our solutions to be $v_u = v_d = v_q \neq v_s$.



Effective scalar/pseudosalar meson theory with μ_5

New eigenstates of strong interactions with LPB (isotriplet)

We present a simple case of mixing due to LPB in the isotriplet sector with π and a_0. The kinetic and mixing terms in the Lagrangian are given by

$$\mathcal{L} = \frac{1}{2}(\partial a_0)^2 + \frac{1}{2}(\partial \pi)^2 - \frac{1}{2}m_1^2 a_0^2 - \frac{1}{2}m_2^2 \pi^2 - 4\mu_5 a_0 \dot{\pi},$$

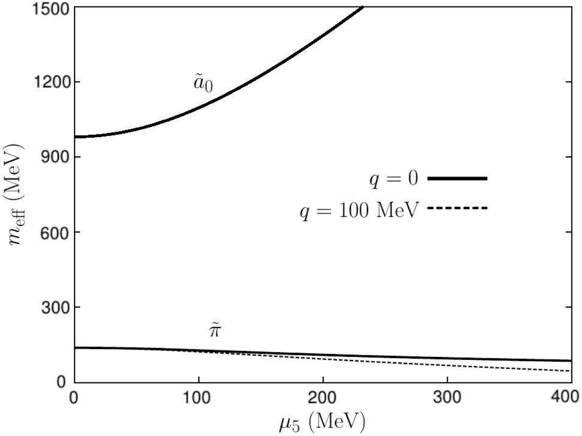
where

$$\begin{aligned} m_1^2 =& -2[M^2 - 2(3\lambda_1 + \lambda_2)v_q^2 - \lambda_2 v_s^2 \\ & - c v_s + 2(3d_1 + 2d_2) m v_q + 2d_2 m_s v_s + 2\mu_5^2] \\ m_2^2 =& \frac{2m}{v_q}\left[b + (d_1 + 2d_2)v_q^2 + d_2 v_s^2\right] \end{aligned}$$

Effective scalar/pseudosalar meson theory with μ_5

New eigenstates of strong interactions with LPB (isotriplet)

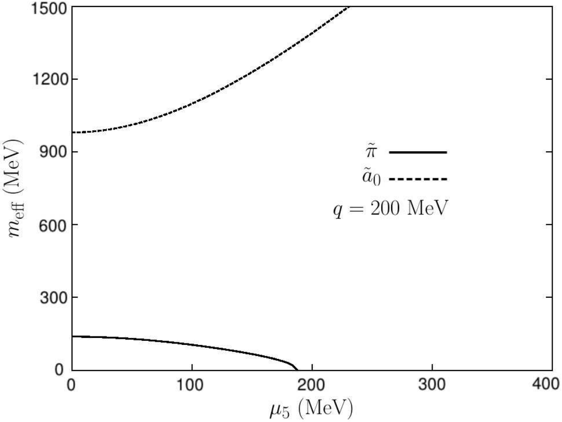
After diagonalization in the momentum representation, the new (momentum-dependent) eigenstates are defined $\tilde{\pi}$ and $\tilde{a}_0$.



Effective scalar/pseudosalar meson theory with μ_5

New eigenstates of strong interactions with LPB (isotriplet)

For high energies $k_0, |\vec{k}| > m_1 m_2/(4\mu_5) \equiv k^c_{\tilde{\pi}}$, in-medium $\tilde{\pi}$ goes tachyonic. Nevertheless, energies are always positive (no vacuum instabilities). $\tilde{a}_0$ mass shows an enhancement, but μ_5 has to be understood as a perturbatively small parameter. A better treatment of $\tilde{a}_0$ would require heavier degrees of freedom.



Effective scalar/pseudosalar meson theory with μ_5

New eigenstates of strong interactions with LPB (isosinglet)

In the isosinglet sector, we show the mixing of η, σ and η'. The kinetic and mixing terms in the Lagrangian are given by

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}[(\partial\sigma)^2 + (\partial\eta_q)^2 + (\partial\eta_s)^2] - \frac{1}{2}m_3^2\sigma^2 - \frac{1}{2}m_4^2\eta_q^2 - \frac{1}{2}m_5^2\eta_s^2 \\ & - 4\mu_5\sigma\dot{\eta}_q - 2\sqrt{2}cv_q\eta_q\eta_s,\end{aligned}$$

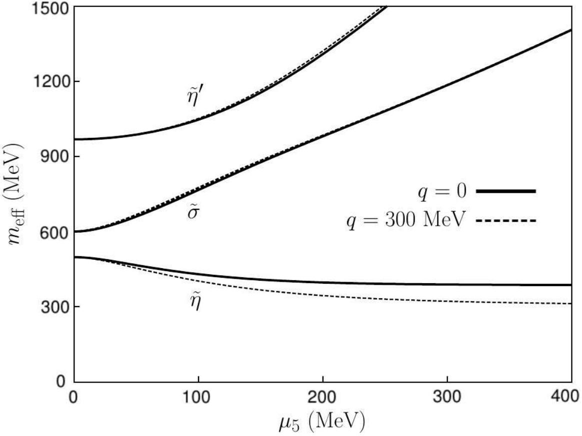
where

$$\begin{aligned}m_3^2 =& -2(M^2 - 6(\lambda_1+\lambda_2)v_q^2 - \lambda_2 v_s^2 + cv_s \\ & + 6(d_1+2d_2)mv_q + 2d_2 m_s v_s + 2\mu_5^2), \\ m_4^2 =& \frac{2m}{v_q}\left[b + (d_1+2d_2)v_q^2 + d_2 v_s^2\right] + 2cv_s, \\ m_5^2 =& \frac{2m_s}{v_s}[b + 2d_2 v_q^2 + (d_1+d_2)v_s^2] + \frac{cv_q^2}{v_s}.\end{aligned}$$

Effective scalar/pseudosalar meson theory with μ_5

New eigenstates of strong interactions with LPB (isosinglet)

After diagonalization, the new eigenstates are $\tilde{\sigma}$, $\tilde{\eta}$ and $\tilde{\eta}'$.



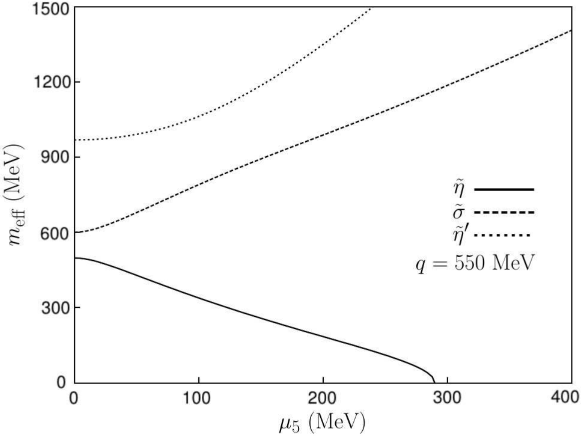
Effective scalar/pseudosalar meson theory with μ_5

New eigenstates of strong interactions with LPB (isosinglet)

Again, for high energies $k_0, |\vec{k}| > k^c_{\tilde{\eta}}$ with

$k^c_{\tilde{\eta}} \equiv \frac{m_3}{4\mu_5 m_5}\sqrt{m_4^2 m_5^2 - 8c^2 v_q^2}$, in-medium $\tilde{\eta}$ goes tachyonic.

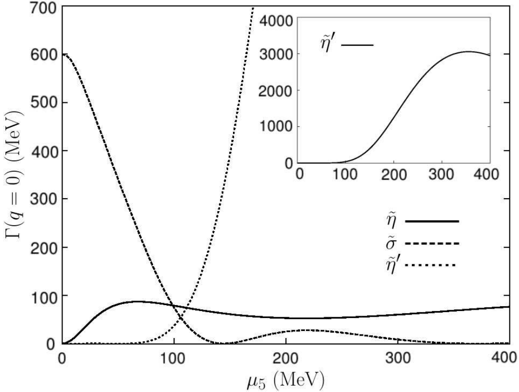
$\tilde{\eta}'$ mass also shows an enhancement and a better treatment would require the inclusion of heavier degrees of freedom.



Effective scalar/pseudosalar meson theory with μ_5

Decay widths (at rest)

The cubic couplings used to calculate the widths $\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}' \to \tilde{\pi}\tilde{\pi}$ from the Lagrangian. $\tilde{\eta}$ exhibits a smooth behaviour with $\langle \Gamma_{\tilde{\eta}} \rangle \sim 60$ MeV $\leftrightarrow$ mean free path ~ 3 fm $\lesssim L_{\text{fireball}} \sim 5 \div 10$ fm. Possible thermalization! Down to $\mu_5 \sim 100$ MeV, $\tilde{\sigma}$ width decreases and becomes stable.



Phase structure of the NJL quark model with μ_5

A. A. Andrianov, D. Espriu & X. Planells, Eur. Phys. J. C , 74:2776 (2014)

NJL Lagrangian with vector μ and axial μ_5 chemical potentials μ, N colors,

$$\mathcal{L} = \bar{\psi}(\not\partial + m - \mu\gamma_0 - \mu_5\gamma_0\gamma_5)\psi$$

$$-\frac{G_1}{N}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{G_2}{N}[(\bar{\psi}\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2],$$

Introduce two doublets of bosonic degrees of freedom $\{\sigma, \vec{\pi}\}$ and $\{\eta, \vec{a}\}$. Bosonic effective potential,

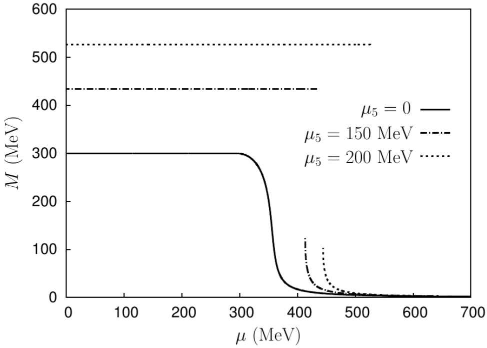
$$V_{\text{eff}} = \frac{N}{4G_1}(\sigma^2 + \vec{\pi}^2) + \frac{N}{4G_2}(\eta^2 + \vec{a}^2) - \text{Tr}\log\mathcal{M}(\mu, \mu_5),$$

The fermion operator

$$\mathcal{M}(\mu, \mu_5) = \not\partial + (M + \vec{\tau}\vec{a}) - \mu\gamma_0 - \mu_5\gamma_0\gamma_5 + i\gamma_5(\vec{\tau}\vec{\pi} + \eta),$$

with the introduction of a constituent quark mass $M \equiv m + \sigma$.

Phase structure of the NJL quark model with μ_5



Evolution of the constituent quark mass M depending on μ for different values of the axial chemical potential μ_5 setting $m = -5$ MeV, $G_1 = -40/\Lambda^2$ and $G_2 = -45/\Lambda^2$. The drawn lines correspond to locally stable phases. The transition to a chirally restored phase changes to a first order one as μ_5 increases. We observe the influence of μ_5 on the restoration of chiral symmetry

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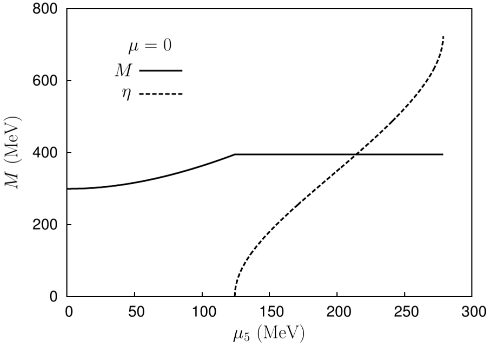

M and η dependence on μ_5 for $\mu < M_0$, $G_1 = -40/\Lambda^2$, $G_2 = -39.5/\Lambda^2$, $m = -5$ MeV and $\Lambda = 1$ GeV. This plot showing the evolution of M and η with respect to μ_5 for $\mu = 0$ (or any $\mu < M_0 \approx 300$ MeV). In the CSB phase M grows with μ_5 up to the critical value M^c, the point where this magnitude freezes out, and η acquires non-trivial values, also growing with the axial chemical potential. At $\mu_5 \simeq 0.28\Lambda$, this phase shows an endpoint and beyond, no stable solution exists.

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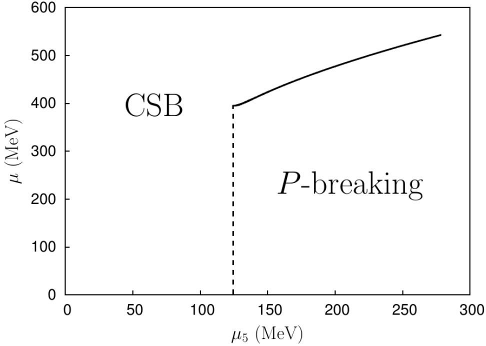

Transition line from the CSB to the P-breaking phase with $G_1 = -40/\Lambda^2$, $G_2 = -39.5/\Lambda^2$, $m = -5$ MeV and $\Lambda = 1$ GeV. The vertical dashed line is related to a 2nd order phase transition while the solid one corresponds to a 1st order one.

Conclusions

- LPB not forbidden by any physical principle in QCD at finite temperature/density.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential.
- LPB leads to unexpected modifications of the in-medium meson properties.
- Axial chemical potential triggers parity-odd condensation in first- vs. second-order phase transitions

Back up slides

Effective meson theory in a medium with LPB

Massive MCS electrodynamics (CFJ model)

$$\mathcal{L}_{MCS} = -\,\tfrac{1}{4}\, F^{\alpha\beta}(x)F_{\alpha\beta}(x) + \tfrac{1}{2}\, m^2\, A_\nu(x)A^\nu(x) + \tfrac{1}{2}\,\zeta_\mu A_\nu(x)\tilde{F}^{\,\mu\nu}(x) + \mathrm{g.f.}$$

In momentum space wave Eqs.

$$\left\{ \begin{array}{c} \left[g^{\,\lambda\nu}\left(k^2 - m^2\right) - k^{\,\lambda}k^{\,\nu} + i\,\varepsilon^{\,\lambda\nu\alpha\beta}\,\zeta_\alpha\, k_\beta \right] \mathbf{a}_\lambda(k) = 0 \\ k^{\,\lambda}\,\mathbf{a}_\lambda(k) = 0 \end{array} \right.$$

Projection onto different polarizations with the help of

$$S^\nu_\lambda = \delta^{\,\nu}_{\,\lambda}\,\mathtt{D} + k^\nu\, k_\lambda\,\zeta^2 + \zeta^{\,\nu}\,\zeta_\lambda\, k^2 - \zeta\cdot k\,(\zeta_\lambda\, k^\nu + \zeta^\nu\, k_{\,\lambda}); \quad \mathtt{D}\ \equiv\ (\zeta\cdot k)^2 - \zeta^2\, k^2$$

Transversal polarizations,

$$\boldsymbol{\pi}^{\,\mu\nu}_{\,\pm} \equiv \frac{S^{\,\mu\nu}}{2\,\mathtt{D}} \ \pm\ \frac{i}{2}\,\varepsilon^{\mu\nu\alpha\beta}\,\zeta_\alpha\, k_\beta\,\mathtt{D}^{\,-\frac{1}{2}}; \quad \varepsilon^{\,\mu}_{\,\pm}(k) = \boldsymbol{\pi}^{\,\mu\lambda}_{\,\pm}\,\epsilon^{(0)}_\lambda$$

Scalar and longitudinal polarizations,

$$\varepsilon^{\,\mu}_{\,S}(k)\ \equiv\ \frac{k^{\,\mu}}{\sqrt{k^2}}, \qquad \varepsilon^{\,\mu}_{\,L}(k)\ \equiv\ \left(\mathtt{D}\, k^2\right)^{-\frac{1}{2}}\left(k^2\,\zeta^{\,\mu} - k^{\,\mu}\,\zeta\cdot k\,\right)$$

Effective meson theory in a medium with LPB

Energy spectrum:

Transversal polarizations

$$\begin{aligned} K^{\mu}_{\nu}\,\varepsilon^{\nu}_{\pm}(k) &= \left(k^2 - m^2 \pm \sqrt{\mathtt{D}}\right)\varepsilon^{\mu}_{\pm}(k); \\ \omega_{\mathbf{k},\pm} &= \begin{cases} \sqrt{\mathbf{k}^2 + m^2 \pm \zeta_0 |\,\mathbf{k}\,|}; & \zeta_\mu = (\zeta_0, 0, 0, 0) \\ \sqrt{\mathbf{k}^2 + m^2 + \frac{1}{2}\zeta_x^2 \pm \zeta_x \sqrt{k_x^2 + m^2 + \frac{1}{4}\zeta_x^2}} & \zeta_\mu = (0, -\zeta_x, 0, 0) \end{cases} \end{aligned}$$

Effective scalar/pseudosalar meson theory with μ_5

Decay widths (moving $\tilde{\eta}$)

Small variations at low 3-momenta: 2 initial bumps slowly separate as one increases q. $\Gamma_{\tilde{\eta}}(\mu_5, q)$ exhibits a saddle point at $\mu_5^* \sim 240$ MeV and $q^* \sim 500$ MeV. For large 3-momenta, a third intermediate bump appears (creation of 2 tachyons) and grows fast as one increases q becoming the global maximum when $q \gtrsim 700$ MeV.