Chiral-symmetry breaking and confinement in Minkowski space

Elmar P. Biernat

Centro de Física Teórica de Partículas (CFTP), Instituto Superior Técnico (IST), Lisbon

September 12, 2014

Quark Confinement and the Hadron Spectrum XI, St. Petersburg

Collaboration

Franz Gross (Jefferson Lab)

Teresa Peña (CFTP/IST)

Emílio Ribeiro (CFIF/IST)

Alfred Stadler (U. Évora and CFTP/IST)

EB, F. Gross, T. Peña, A. Stadler. Phys. Rev. D 89, 016005 (2014); Phys. Rev. D 89, 016006 (2014)

EB, T. Peña, J. E. Ribeiro, A. Stadler, F. Gross. arXiv:1408.1625 [hep-ph]

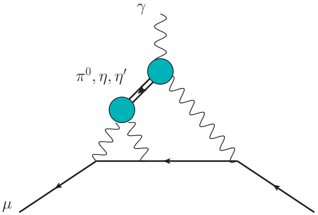
Meson phenomenology — background

- upcoming experiments at JLab (Hall A and D) and FAIR-GSI (Panda)
- theory: need better understanding of $q\bar{q}$ mesons

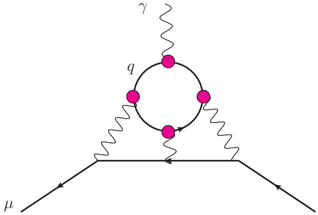
Meson phenomenology — background

- upcoming experiments at JLab (Hall A and D) and FAIR-GSI (Panda)
- theory: need better understanding of $q\bar{q}$ mesons
- QCD-based model descriptions of mesons using non-perturbative methods
- spectrum: learn about confining interaction
- structure: calculate form factors needed in various hadronic processes
  e.g. hadronic contribution to light-by-light scattering in prediction of muon g-2: search for new physics beyond the Standard Model (talks by Colangelo, Blum, Eichmann)

transition form factors



dressed quark current



Objectives and framework

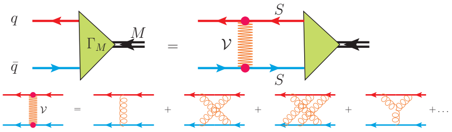
- aim: formulate dynamical model for all $q\bar{q}$ mesons

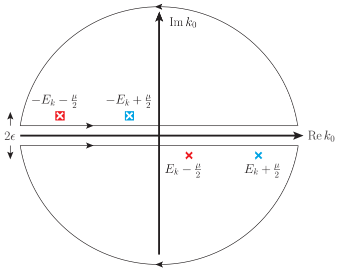
Objectives and framework

- aim: formulate dynamical model for all $q\bar{q}$ mesons
- Covariant Spectator Theory (CST):
  mini review: Stadler, Gross. Few Body Syst. (2011); Gross, Milana PRD (1991); (1992); (1994); Şavkli, Gross PRC (2001)
  - covariant and non-perturbative method
  - NJL-type mechanism for SχSB (similar to Dyson-Schwinger approach)
  - equations solved in Minkowski space (BUT: have to deal with singularities!)
  - confining interaction can also have scalar, pseudoscalar, etc... Lorentz structures
    Ikeda, Iida PoS Lattice (2010); Koike PLB (1989). Tiemeijer, Tjon PRC (1990); PLB (1992); PRC (1993)
  - beyond Bethe-Salpeter ladder approximation ($\exists$ Dirac limit!)

CST bound-state equation

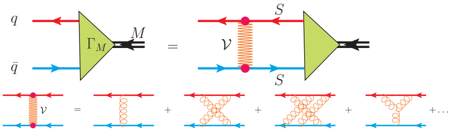
- Bethe-Salpeter equation (BSE)

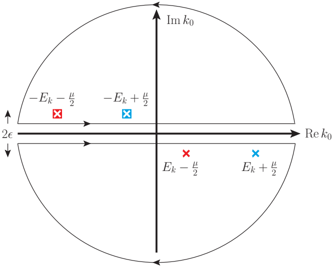




CST bound-state equation

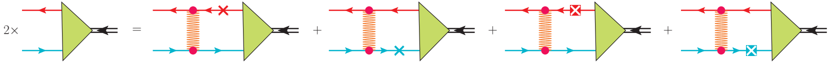
- Bethe-Salpeter equation (BSE)





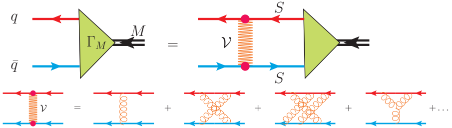
- idea of CST: in k_0-integration take only propagator pole contributions

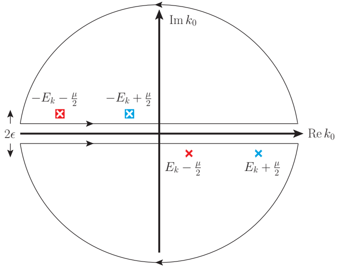
CST-BSE



CST bound-state equation

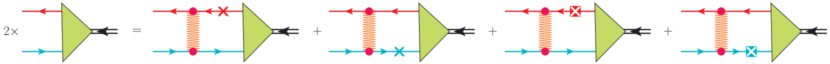
- Bethe-Salpeter equation (BSE)





- idea of CST: in k_0-integration take only propagator pole contributions

CST-BSE



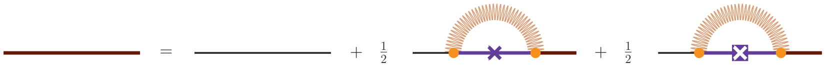
- efficient reorganization of BS series: kernel poles in higher-order irreducible diagrams
- 3-dimensional covariant loop integrals $\int d^3k \frac{m}{E_k} \equiv \int_k$

Dyson equation

- consistent dynamical model: dressed quark propagator S from solving the Dyson (mass gap) equation

CST-Dyson equation

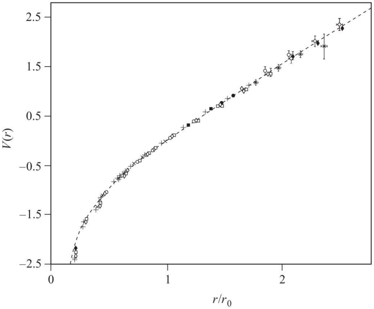
- consistent dynamical model: dressed quark propagator S from solving the Dyson (mass gap) equation
- CST-Dyson equation



- $S_0(p) = \frac{1}{m_0 - \not{p} - \mathrm{i}\epsilon} \longrightarrow S(p) = \frac{1}{m_0 + \Sigma(p^2) - \not{p} - \mathrm{i}\epsilon} \equiv \frac{Z(p^2)}{M(p^2) - \not{p} - \mathrm{i}\epsilon}$
  with mass function $M(p^2) = \frac{A(p^2)+m_0}{1-B(p^2)}$ generated dynamically (SχSB)
  m_0 bare quark mass
- constituent quark mass obtained from $m = M(p^2 = m^2)$

Confinement in CST

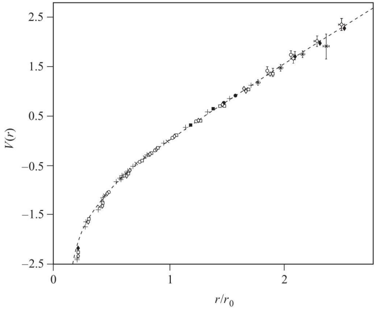
- linear confinement $V_L^{nr} = \sigma r$
  in momentum space:
  $\langle V_L^{nr}\psi\rangle(\vec{p}) = \sigma \int d^3k \frac{\psi(\vec{k})-\psi(\vec{p})}{(\vec{p}-\vec{k})^4}$



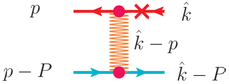
ALLTON *et al*, UKQCD Collab., PRD (2002)

Confinement in CST

- linear confinement $V_L^{nr} = \sigma r$
  in momentum space:
  $\langle V_L^{nr}\psi\rangle(\vec{p}) = \sigma \int d^3k \frac{\psi(\vec{k})-\psi(\vec{p})}{(\vec{p}-\vec{k})^4}$
- Covariant CST generalization:
  replace $\vec{q}^2 \to -q^2$
  $\langle V_L\psi\rangle(p) = \int_k V_L(p-\hat{k})\psi(\hat{k}) =$
  $\sigma \int_k \frac{\psi(\hat{k})-\psi(\hat{p}_R)}{(p-\hat{k})^4}$



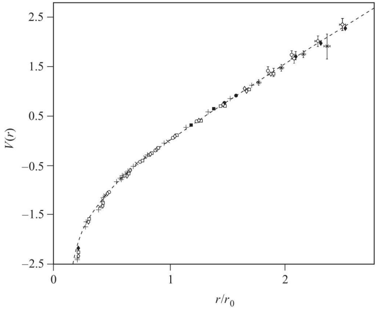
ALLTON *et al*, UKQCD Collab., PRD (2002)



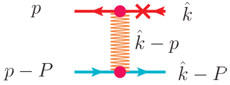
$\hat{k} = (E_k, \vec{k})$

Confinement in CST

- linear confinement $V_L^{nr} = \sigma r$
  in momentum space:
  $\langle V_L^{nr}\psi\rangle(\vec{p}) = \sigma \int d^3k \frac{\psi(\vec{k})-\psi(\vec{p})}{(\vec{p}-\vec{k})^4}$
- Covariant CST generalization:
  replace $\vec{q}^2 \to -q^2$
  $\langle V_L\psi\rangle(p) = \int_k V_L(p-\hat{k})\psi(\hat{k}) =$
  $\sigma \int_k \frac{\psi(\hat{k})-\psi(\hat{p}_R)}{(p-\hat{k})^4}$



ALLTON *et al*, UKQCD Collab., PRD (2002)



$\hat{k} = (E_k, \vec{k})$

- satisfies $\int_k V_L = \langle V_L\rangle = 0$ (covariant version of condition $V_L^{nr}(r=0)=0$)
- nonrelativistic limit $m \to \infty$: $V_L \to V_L^{nr}$ ✓

Chiral symmetry and AVWTI

- Consistency with chiral symmetry and its breaking: ensured by satisfying axial-vector Ward-Takahashi identity (AVWTI):

$$-\mathrm{i}(p_1 - p_2)_\mu \Gamma_5^\mu(p_1, p_2) + 2m_0 \Gamma_5(p_1, p_2) = S^{-1}(p_1)\gamma_5 + \gamma_5 S^{-1}(p_2)$$

Chiral symmetry and AVWTI

- Consistency with chiral symmetry and its breaking: ensured by satisfying axial-vector Ward-Takahashi identity (AVWTI):
  $-\mathrm{i}(p_1 - p_2)_\mu \Gamma_5^\mu(p_1, p_2) + 2m_0\Gamma_5(p_1, p_2) = S^{-1}(p_1)\gamma_5 + \gamma_5 S^{-1}(p_2)$
- constrains scalar, pseudoscalar and tensor structures of kernel
  $\mathcal{V}(p-k) = V_L(p-k)\Big[\lambda_S(\mathbf{1}\otimes\mathbf{1}) + \lambda_P(\gamma^5\otimes\gamma^5) + \lambda_V(\gamma^\mu\otimes\gamma_\mu) + \lambda_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu) + \frac{\lambda_T}{2}(\sigma^{\mu\nu}\otimes\sigma_{\mu\nu})\Big] + V_C(p-k)\Big[\kappa_V(\gamma^\mu\otimes\gamma_\mu) + \kappa_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu)\Big]$

Chiral symmetry and AVWTI

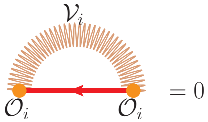
- Consistency with chiral symmetry and its breaking: ensured by satisfying axial-vector Ward-Takahashi identity (AVWTI):

$$-\mathrm{i}(p_1 - p_2)_\mu \Gamma_5^\mu(p_1, p_2) + 2m_0\Gamma_5(p_1, p_2) = S^{-1}(p_1)\gamma_5 + \gamma_5 S^{-1}(p_2)$$

- constrains scalar, pseudoscalar and tensor structures of kernel

$$\mathcal{V}(p-k) = V_L(p-k)\Big[\lambda_S(\mathbf{1}\otimes\mathbf{1})+\lambda_S(\gamma^5\otimes\gamma^5) + \lambda_V(\gamma^\mu\otimes\gamma_\mu) + \lambda_A(\gamma^5\gamma^\mu\otimes \gamma^5\gamma_\mu) + \tfrac{\lambda_T}{2}(\sigma^{\mu\nu}\otimes\sigma_{\mu\nu})\Big] + V_C(p-k)\Big[\kappa_V(\gamma^\mu\otimes\gamma_\mu) + \kappa_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu)\Big]$$

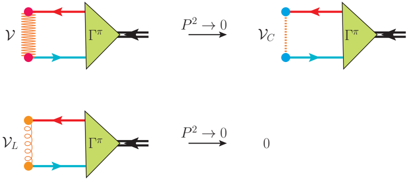
1. V_L does not contribute to scalar part A (dynamical quark mass) of CST-Dyson equation because of $\langle V_L\rangle = 0$
2. scalar and pseudoscalar structures cancel in B for $\lambda_S = \lambda_P$



$\Rightarrow$ V_L can have Lorentz scalar, pseudoscalar and tensor structures!

Chiral symmetry and AVWTI

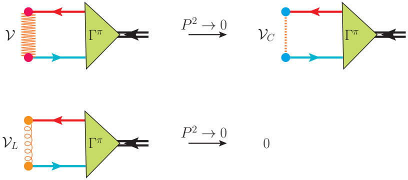
- Consistency with chiral symmetry and its breaking: ensured by satisfying axial-vector Ward-Takahashi identity (AVWTI):
  $-\mathrm{i}(p_1 - p_2)_\mu \Gamma_5^\mu(p_1, p_2) + 2m_0\Gamma_5(p_1, p_2) = S^{-1}(p_1)\gamma_5 + \gamma_5 S^{-1}(p_2)$
- constrains scalar, pseudoscalar and tensor structures of kernel
  $\mathcal{V}(p-k) = V_L(p-k)\Big[\lambda_S(\mathbf{1}\otimes\mathbf{1}) + \lambda_S(\gamma^5\otimes\gamma^5) + \lambda_V(\gamma^\mu\otimes\gamma_\mu) + \lambda_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu) + \frac{\lambda_T}{2}(\sigma^{\mu\nu}\otimes\sigma_{\mu\nu})\Big] + V_C(p-k)\Big[\kappa_V(\gamma^\mu\otimes\gamma_\mu) + \kappa_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu)\Big]$



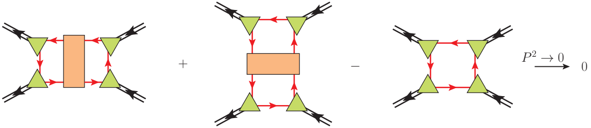
- V_L does not contribute to massless pion CST-BSE because of $\langle V_L \rangle = 0$
- χ limit: $\Gamma^\pi(p,p) \sim A(p^2)\gamma^5$

Chiral symmetry and AVWTI

- Consistency with chiral symmetry and its breaking: ensured by satisfying axial-vector Ward-Takahashi identity (AVWTI):
  $-\mathrm{i}(p_1 - p_2)_\mu \Gamma_5^\mu(p_1, p_2) + 2m_0\Gamma_5(p_1, p_2) = S^{-1}(p_1)\gamma_5 + \gamma_5 S^{-1}(p_2)$
- constrains scalar, pseudoscalar and tensor structures of kernel
  $\mathcal{V}(p-k) = V_L(p-k)\Big[\lambda_S(\mathbf{1}\otimes\mathbf{1}) + \lambda_S(\gamma^5\otimes\gamma^5) + \lambda_V(\gamma^\mu\otimes\gamma_\mu) + \lambda_A(\gamma^5\gamma^\mu\otimes \gamma^5\gamma_\mu) + \frac{\lambda_T}{2}(\sigma^{\mu\nu}\otimes\sigma_{\mu\nu})\Big] + V_C(p-k)\Big[\kappa_V(\gamma^\mu\otimes\gamma_\mu) + \kappa_A(\gamma^5\gamma^\mu\otimes\gamma^5\gamma_\mu)\Big]$



- V_L does not contribute to massless pion CST-BSE because of $\langle V_L\rangle = 0$
- χ limit: $\Gamma^\pi(p,p)\sim A(p^2)\gamma^5$
- Adler consistency-zero of π-π scattering (to all orders!) in χ-limit reproduced ✓

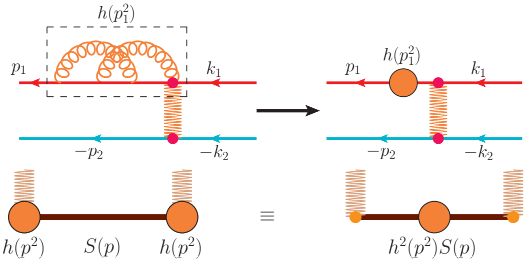


Simple exploratory model ('a proof of principle')

- Covariant CST generalization of nonrelativistic 'linear+constant' potential $V(r) = \sigma r - C \approx$ "Cornell" potential
  - $\mathcal{V}_L = [\mathbf{1} \otimes \mathbf{1} + \gamma_5 \otimes \gamma_5] V_L$
    does not contribute to self energy $\Rightarrow \Sigma_L = 0$!
  - $\mathcal{V}_C = [\gamma^\mu \otimes \gamma_\mu] h(p_1) h(p_2) h(\hat{k}_1) h(k_2) C 2 \frac{E_k}{m} \delta^3(p-k)$

Simple exploratory model ('a proof of principle')

- Covariant CST generalization of nonrelativistic 'linear+constant' potential $V(r) = \sigma r - C \approx$ "Cornell" potential
  - $\mathcal{V}_L = [\mathbf{1} \otimes \mathbf{1} + \gamma_5 \otimes \gamma_5] V_L$
    does not contribute to self energy $\Rightarrow \Sigma_L = 0$!
  - $\mathcal{V}_C = [\gamma^\mu \otimes \gamma_\mu] h(p_1) h(p_2) h(\hat{k}_1) h(k_2) C 2 \frac{E_k}{m} \delta^3(p - k)$
- phenomenological strong quark form factors for each off-shell quark line at interaction vertex Gross, Riska PRC (1987); Surya, Gross, PRC (1996)



h form factors absorbed in modified propagators: $\tilde{S}(p) = h^2(p^2)S(p) \Leftrightarrow$ reduced vertex functions $\Gamma_R(p_1, p_2) = h^{-1}(p_1^2)\Gamma(p_1, p_2)h^{-1}(p_2^2)$

- satisfy corresponding AVWTI $\Rightarrow$ Adler consistency zero reproduced ✓

Result for quark mass function

- $\mathcal{V}_C$ contributes only to A

  $\Rightarrow$ dressed quark mass function $M(p^2) = C\, h^2(m^2) h^2(p^2) + m_0$ with $C = m_\chi + c_1 m_0 + \mathcal{O}(m_0^2)$

  $\overset{m_0 \to 0}{\longrightarrow} M_\chi(p^2) = m_\chi h^2(p^2)$ with $h(p^2) = \left(\frac{\Lambda_\chi^2 - m_\chi^2}{\Lambda_\chi^2 - p^2}\right)^2$

Result for quark mass function

- $\mathcal{V}_C$ contributes only to A

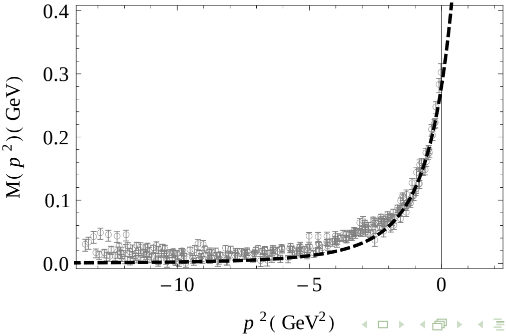
  $\Rightarrow$ dressed quark mass function $M(p^2) = C\,h^2(m^2)h^2(p^2) + m_0$ with $C = m_\chi + c_1 m_0 + \mathcal{O}(m_0^2)$

  $\overset{m_0\to 0}{\longrightarrow} M_\chi(p^2) = m_\chi h^2(p^2)$ with $h(p^2) = \left(\frac{\Lambda_\chi^2 - m_\chi^2}{\Lambda_\chi^2 - p^2}\right)^2$

- fix 2 parameters by fit to (Euclidean) LQCD data extrapolated to chiral limit (first 50 points up to $p^2 > -1.94 GeV^2$ fit)

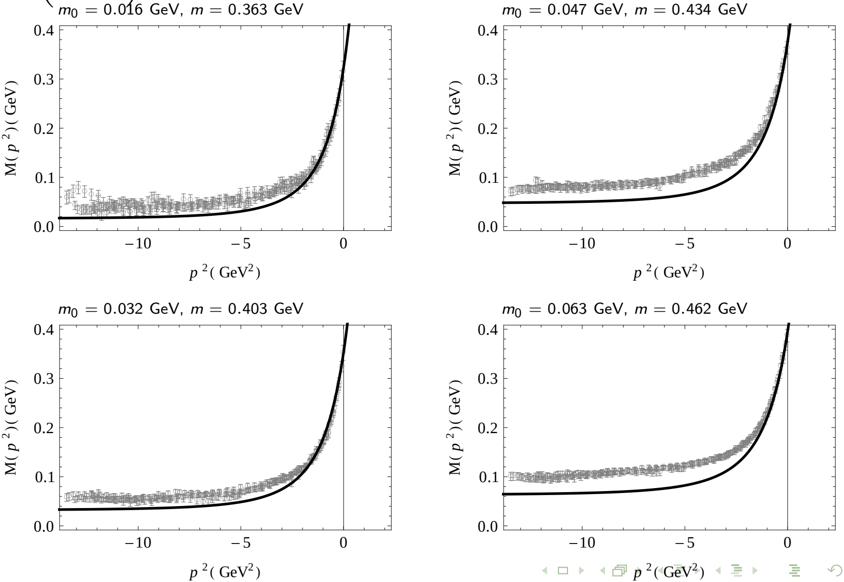
  $\Lambda_\chi = 2.04$ GeV and $m_\chi = 0.308$ GeV

Lattice QCD data from Bowman *et al* PRD (2005) extrapolated to chiral limit



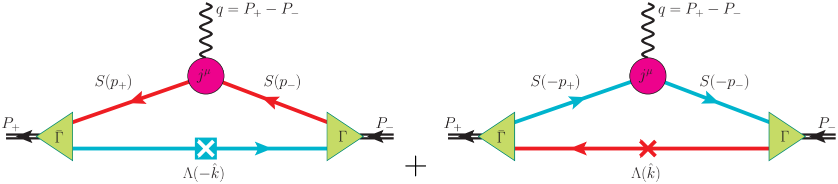
Results for quark functions with finite m_0

$h(p^2) = \left(\frac{\Lambda_\chi^2 - m_\chi^2}{\Lambda^2 - p^2}\right)^2$, $\Lambda = m + \Lambda_\chi - m_\chi$, solve $M(m^2) = m$ and global fit: $c_1 \approx 12$



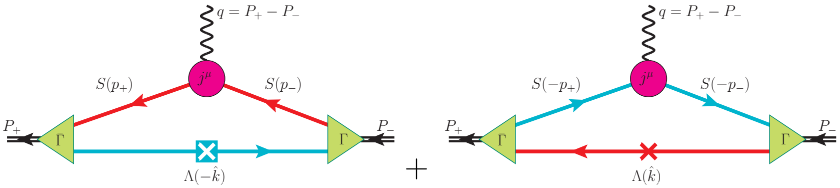
Pion electromagnetic form factor

- CST pion current: triangle diagram with 6 propagator poles
- relativistic impulse approximation (RIA): keep only spectator quark poles



Pion electromagnetic form factor

- CST pion current: triangle diagram with 6 propagator poles
- relativistic impulse approximation (RIA): keep only spectator quark poles

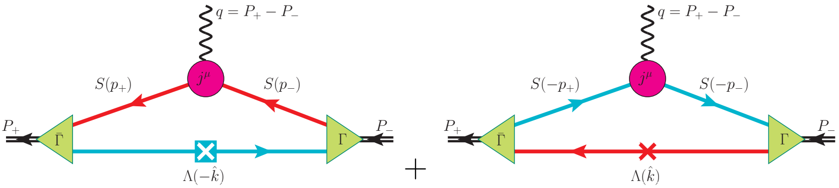


- analysis of pole structure ⇒ RIA works well for
  1. large Q^2 (any m_π) ✓
  2. large m_π (any Q^2) ✓

  for small Q^2 and small m_π: other poles contribute significantly
  ⇒ cannot be ignored!
  ⇒ need complete impulse approximation (to do!)

Pion electromagnetic form factor

- CST pion current: triangle diagram with 6 propagator poles
- relativistic impulse approximation (RIA): keep only spectator quark poles

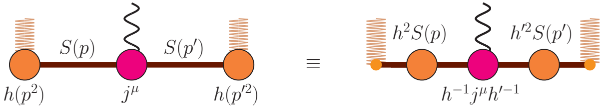


- analysis of pole structure ⇒ RIA works well for
  1. large Q^2 (any m_π) ✓
  2. large m_π (any Q^2) ✓

  for small Q^2 and small m_π: other poles contribute significantly
  ⇒ cannot be ignored!
  ⇒ need complete impulse approximation (to do!)
- ingredients:
  1. pion vertex function Γ_π: use simple CST off-shell extension near χ-limit
  2. dressed electromagnetic quark current j^μ

Quark-photon vertex

- derive dressed quark current using the prescription by Gross and Riska

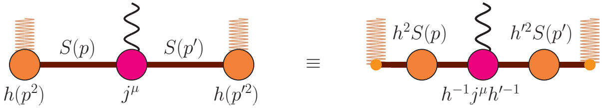


$\Rightarrow$ reduced off-shell quark current $j_R^\mu(p',p) = h^{-1}(p'^2) j^\mu(p',p) h^{-1}(p^2)$

- gauge invariance: impose vector Ward-Takahashi identity (VWTI) $(p'-p)_\mu j_R^\mu(p',p) = \tilde{S}^{-1}(p) - \tilde{S}^{-1}(p')$
- Lorentz structure $j_R^\mu = f(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu}q_\nu}{2m}) + \delta'\Lambda'\gamma^\mu + \delta\gamma^\mu\Lambda + g\Lambda'\gamma^\mu\Lambda$ with $\Lambda = \frac{M(p)-\not{p}}{2M(p)}$ and off-shell form factors f, δ, δ', g determined by VWTI in terms of $h(p^2)$

Quark-photon vertex

- derive dressed quark current using the prescription by Gross and Riska



$\Rightarrow$ reduced off-shell quark current $j_R^\mu(p', p) = h^{-1}(p'^2) j^\mu(p', p) h^{-1}(p^2)$

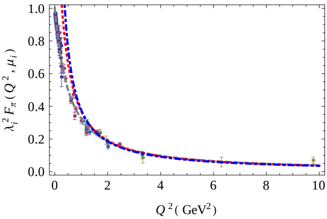
- gauge invariance: impose vector Ward-Takahashi identity (VWTI) $(p' - p)_\mu j_R^\mu(p', p) = \tilde{S}^{-1}(p) - \tilde{S}^{-1}(p')$
- Lorentz structure $j_R^\mu = f(\gamma^\mu + \kappa \frac{i\sigma^{\mu\nu} q_\nu}{2m}) + \delta' \Lambda' \gamma^\mu + \delta \gamma^\mu \Lambda + g \Lambda' \gamma^\mu \Lambda$ with $\Lambda = \frac{M(p) - \not{p}}{2M(p)}$ and off-shell form factors f, δ, δ', g determined by VWTI in terms of $h(p^2)$
- j_R^μ differs in chiral limit from Ball-Chiu current by transverse piece

Results for pion form factor

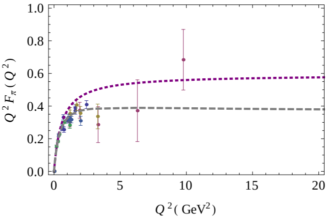
- Pion form factor calculated with different pion masses
- best fit to data with $m_\pi = 0.42$ GeV (allows to use RIA also at small Q^2)
  data: Amendolia *et al* 1986; Brown *et al* 1973; Bebek *et al* 1974; 1976; 1978; Huber *et al* 2008

$m_\pi = 0.42$ GeV, $m_\pi = 0.28$ GeV,

$m_\pi = 0.14$ GeV



simple ρ pole

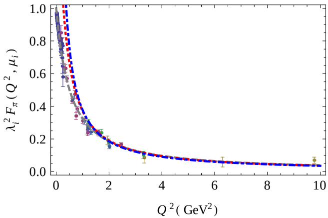


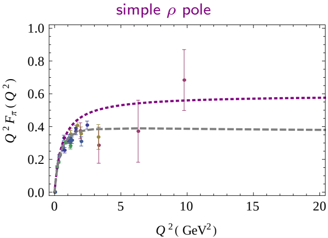
Results for pion form factor

- Pion form factor calculated with different pion masses
- best fit to data with $m_\pi = 0.42$ GeV (allows to use RIA also at small Q^2)
  data: Amendolia *et al* 1986; Brown *et al* 1973; Bebek *et al* 1974; 1976; 1978; Huber *et al* 2008

$m_\pi = 0.42$ GeV, $m_\pi = 0.28$ GeV,

$m_\pi = 0.14$ GeV





- independent of h form factors ✓
- scaling relation

$$F_\pi(Q^2, \lambda m_\pi) \overset{Q^2 \gg m_\pi^2}{\simeq} \lambda^2 F_\pi(Q^2, m_\pi)$$

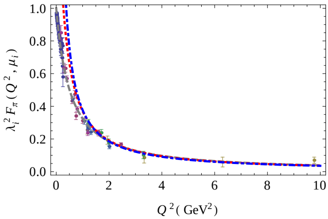
- RIA fails for small pion masses m_π and small Q^2 (understood! ✓)

Results for pion form factor

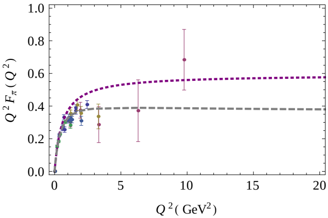
- Pion form factor calculated with different pion masses
- best fit to data with $m_\pi = 0.42$ GeV (allows to use RIA also at small Q^2)
  data: Amendolia *et al* 1986; Brown *et al* 1973; Bebek *et al* 1974; 1976; 1978; Huber *et al* 2008

$m_\pi = 0.42$ GeV, $m_\pi = 0.28$ GeV,

$m_\pi = 0.14$ GeV



simple ρ pole



- independent of h form factors ✓
- scaling relation

$$F_\pi(Q^2, \lambda m_\pi) \overset{Q^2 \gg m_\pi^2}{\simeq} \lambda^2 F_\pi(Q^2, m_\pi)$$

- RIA fails for small pion masses m_π and small Q^2 (understood! ✓)

- F_π has correct monopole fall-off

$$F_\pi(Q^2) \overset{Q^2 \gg \mu^2}{\sim} \frac{1}{Q^2 + \nu^2} \checkmark$$

with $\nu \simeq 0.63$ GeV

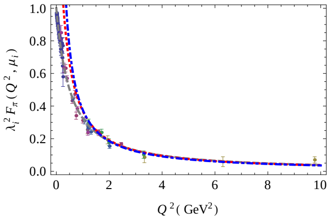
- Model works well without ρ pole contribution (VMD). ✓

Results for pion form factor

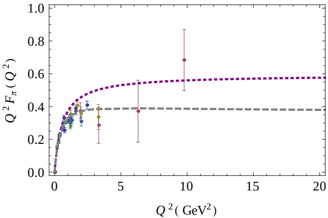
- Pion form factor calculated with different pion masses
- best fit to data with $m_\pi = 0.42$ GeV (allows to use RIA also at small Q^2)
  data: Amendolia *et al* 1986; Brown *et al* 1973; Bebek *et al* 1974; 1976; 1978; Huber *et al* 2008

$m_\pi = 0.42$ GeV, $m_\pi = 0.28$ GeV,

$m_\pi = 0.14$ GeV



simple ρ pole



- independent of h form factors ✓
- scaling relation

$$F_\pi(Q^2, \lambda m_\pi) \overset{Q^2 \gg m_\pi^2}{\simeq} \lambda^2 F_\pi(Q^2, m_\pi)$$

- RIA fails for small pion masses m_π and small Q^2 (understood! ✓)

- F_π has correct monopole fall-off

$$F_\pi(Q^2) \overset{Q^2 \gg \mu^2}{\sim} \frac{1}{Q^2+\nu^2}$$ ✓

with $\nu \simeq 0.63$ GeV

- Model works well without ρ pole contribution (VMD).
  Why?

Summary/Outlook

- Covariant Spectator Theory: dynamical model $q\bar{q}$ mesons
  - solved in Minkowski space
  - describes confinement and spontaneous chiral symmetry breaking
  - consistency between one-body Dyson and two-body Bethe-Salpeter equations
- first model calculations:
  - dressed quark mass function in Minkowski space with Euclidean LQCD data used to fix parameters
  - qualitative study of pion form factor in CST with simple pion vertex function and dressed quark current: reasonable results ✓

Outlook and work in progress:

1. calculate dressed quark current dynamically by solving inhomogeneous vector CST-BSE
2. refine kernel: include vector structures for confining part; add one-gluon exchange
3. solve CST bound-state equation and fit light meson spectrum
4. π-π scattering away from chiral limit: expect deviation from Weinberg result

   WEINBERG PRL (1966)

Acknowledgements/Support

CFTP

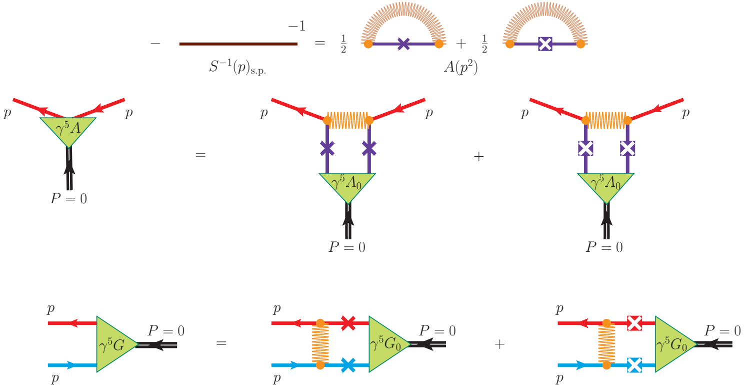
TÉCNICO LISBOA

FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Jefferson Lab

NJL-Mechanism for SχSB

- chiral limit ($m_0 = 0$): scalar part (s.p.) of CST-DE for A and CST-BSE for a massless pion become identical, $\Gamma_{\pi\chi}(p,p) \sim A(p^2)\gamma^5$



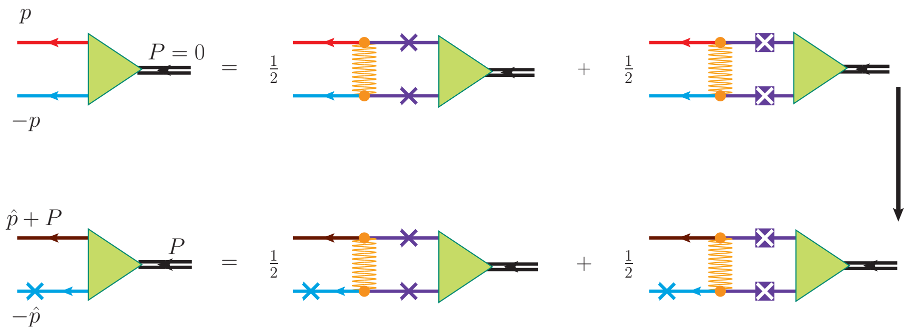
$\Rightarrow$ a massless pion state exists! Goldstone pion in chiral limit associated with SχSB ✓

- $m_0 > 0$: the equation for A ensures that there is no solution of the equation for a massless pion ✓ Gross, Milana. PRD (1991)

A simple pion vertex function

$\Gamma_\pi(p_1, p_2) =$
$G_1(p_1^2, p_2^2)\gamma^5 + G_+(p_1^2, p_2^2)(\not{p}_1\gamma^5 + \gamma^5\not{p}_2) + G_-(p_1^2, p_2^2)(\not{p}_1\gamma^5 - \gamma^5\not{p}_2) + G_3(p_1^2, p_2^2)\not{p}_1\gamma^5\not{p}_2$

$\xrightarrow{\text{chiral limit, rest frame}} \Gamma_\pi(p, p) = G(p^2)\gamma^5$ with $G(p^2) \propto A(p^2) = m_\chi h^2(p^2)$



real pion away from chiral limit: assume that γ^5 structure dominates
$\Rightarrow$ CST pion vertex function near chiral limit
$\Gamma(p_1, \hat{p}_2) = G_0 h(p_1^2)\gamma^5$ and $\Gamma(\hat{p}_1, p_2) = G_0 h(p_2^2)\gamma^5$