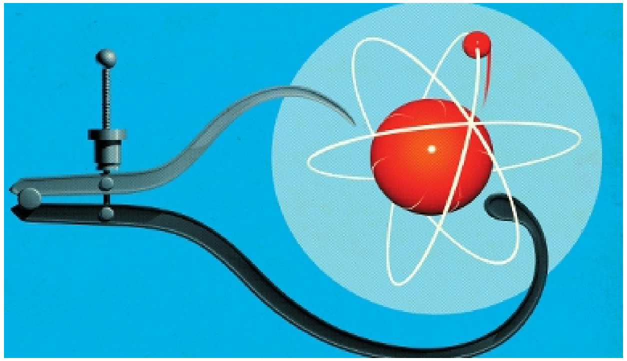
The hadronic corrections to muonic hydrogen Lamb shift from ChPT and the proton radius



arxiv: 1403.3408
arxiv: 1406.4524
work in collaboration with Antonio Pineda

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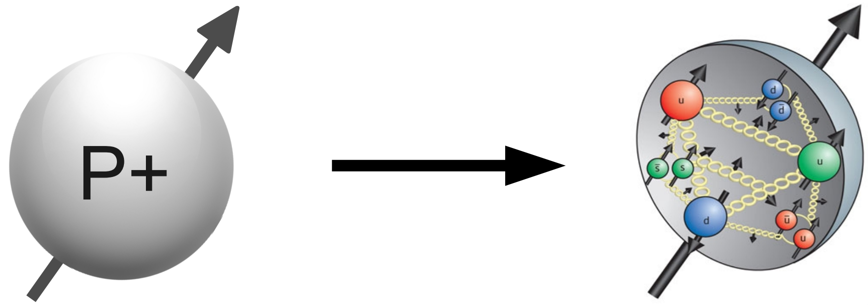
Clara Peset

Outline

- Model independent prediction of the proton radius
- Effecive field theory framework: pNRQED
- Contributions to the Lamb shift
  - QED- like contributions
  - hadronic contributions (TPE)
- Summary

Definition of the proton radius



Rutherford 1917-20

$\tau_p > 10^{34}$ *years*

QCD

$$r_p^2 = 6\frac{dG_{p,E}(q^2)}{dq^2}\Big|_{q^2=0}$$

IR divergent!

NRQCD Lagrangian:

$$\delta\mathcal{L} = -e\frac{c_D^{(p)}}{m_p^2} N_p^\dagger \nabla \cdot \mathbf{E} N_p$$

Caswell, Lepage '86

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2)$$

$$c_D = 1 + 2F_2 + 8F_1'$$

$$r^2_{p,\overline{MS}}(\nu) = \frac{3}{4m_p^2}\left(c_D^{\overline{MS}}(\nu) - 1\right)$$

General expression:

$$c_D^{(p)}(\nu) - c^{(p)}_{D,point-like}(\nu) \equiv \frac{4}{3}m_p^2 r_p^2$$

The proton radius puzzle

Value from μ-H: $r_p = 0.84087(39)$ fm *(Science '13)*

CODATA value (2012): $r_p = 0.8775(51)$ fm

7.1 σ away!

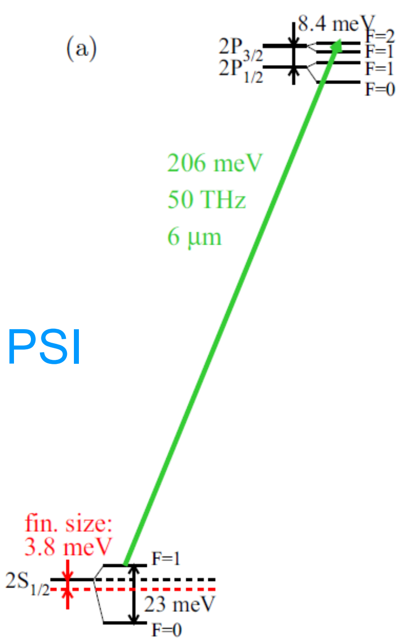
from Hydrogen spectroscopy and electron-proton scattering

Probability of the lepton being *within* the volume of the proton

$$\left(\frac{r_p}{a_B}\right)^3 = \left(\alpha\, m_r r_p\right)^3$$

$$m_\mu \approx 200\, m_e$$

8 million times larger for μH



PSI

But also larger UNKNOWN hadronic effects → Two Photon Exchange

LAMB SHIFT: $2S_{1/2} \rightarrow 2P_{3/2}$

displacement of nS energy levels

EFT for bound states (μp)

Scales in bound states — Muonic hydrogen (Coulomb interaction)

- Hard scale: $m_r \longrightarrow m_r$
- Soft scale: $|\boldsymbol{p}| \longrightarrow m_r\alpha$
- Ultrasoft scale: $E \longrightarrow m_r\alpha^2$

Well separated scales!

pNRQED

Scales in μH: $m_p \sim m_\rho, \quad m_\mu \sim m_\pi \sim m_r, \quad m_r\alpha \sim m_e$

Small expansion parameters: $\frac{m_\pi}{m_p} \sim \frac{m_\mu}{m_p} \approx \frac{1}{9}, \quad \frac{m_e}{m_r} \sim \frac{m_r\alpha}{m_r} \sim \alpha \approx \frac{1}{137}$

Energy levels: $$E(\mu p) = \frac{-m_r\alpha^2}{2n^2}\left(1 + c_1\alpha + c_2\alpha^2 + c_3\alpha^{3.}..\right)$$

$c_1 \sim c_1\left(\frac{m_\mu\alpha}{m_e}\right)$ pure QED, and $c_n \sim \sum_{j=0}^{\infty} c_n^{(j)}\left(\frac{m_\pi}{m_p}\right)^j; \quad c_n^{(j)} \sim c_n^{(j)}\left(\frac{m_r}{m_\mu}, \frac{m_\mu}{m_\pi}...\right)$

pNRQED

pNRQED is a theory of ultrasoft photons

HBET $\xrightarrow[m_\pi\,,\,\Delta]{}$ NRQED $\longrightarrow$ pNRQED

The pNRQED lagrangian:

$$\mathcal{L}_{pNRQED} = \int d^3\mathbf{x} d^3\mathbf{X} S^\dagger(\mathbf{x},\mathbf{X},t)\left\{ i\partial_0 - \frac{\mathbf{p}^2}{m_r} + \frac{\mathbf{p}^4}{m_\mu^3} + \frac{\mathbf{p}^4}{m_p^3} - \frac{\mathbf{P}^2}{2M} - V(\mathbf{x},\mathbf{p},\sigma_\mathbf{1},\sigma_\mathbf{2}) + e\mathbf{x}\cdot\mathbf{E}(\mathbf{X},t) \right\} S(\mathbf{x},\mathbf{X},t) - \int d^3\mathbf{x}\frac{d_1}{4}F_{\mu\nu}F^{\mu\nu}$$

The potential:

$$V = V^{(0)} + V^{(1)} + V^{(2)} + V^{(3)} + \ldots$$

$$V^{(i)} \propto \left(\frac{1}{m}\right)^i$$ + expansions in the small parameters

Matching coefficients

Hadronic contributions: d_2^{had} c_D^{had} c_3^{had}

EXAMPLE:

HBET: $$\delta\mathcal{L}_{(N,\Delta)l_\mu} = \frac{1}{m_p^2} c_{3,R}^{pl_\mu} \bar{N}_p \gamma^0 N_p \bar{l}_\mu \gamma_0 l_\mu$$

NRQED: $$\mathcal{L}_{Nl_\mu} = \frac{1}{m_p^2} c_{3,NR}^{pl_\mu} N_p^\dagger N_p l_\mu^\dagger l_\mu$$

pNRQED: $$V^{(2)}(r) = c_{3,NR}^{pl_\mu} \frac{\delta^{(3)}(\mathbf{r})}{m_p m_\mu}$$

$$c_{3,NR}^{pl_\mu} = c_{3,R}^{pl_\mu} + c_{3,point-like}^{pl_\mu} + c_3^{had}$$

Theoretical equation for the Lamb Shift:

$$\Delta E_L \;=\; 206.0243(30) - 5.2271(7)\frac{r_p^2}{\mathrm{fm}^2} + 0.0455(125) \;+\mathcal{O}(m_\mu\alpha^5\frac{m_\mu^3}{m_\rho^3}, m_\mu\alpha^6)\mathrm{meV}$$

CP, A. Pineda

“Pure” QED corrections

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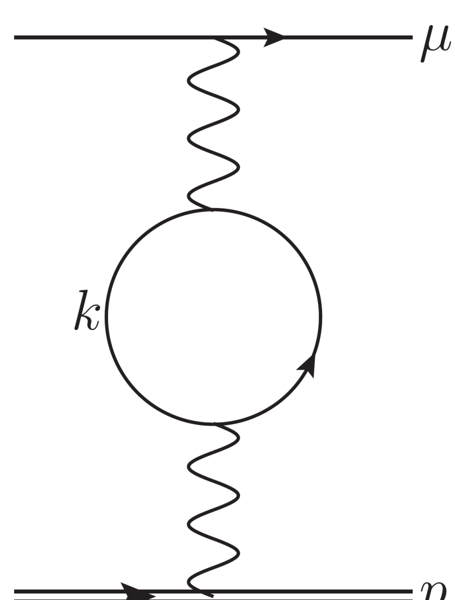
μH QED leading contribution: ELECTRON VACUUM POLARIZATION



$V_{\text{VP},\,\mu}$ $\quad \mathcal{O}(m_r\alpha^3)$

“Pure” QED corrections

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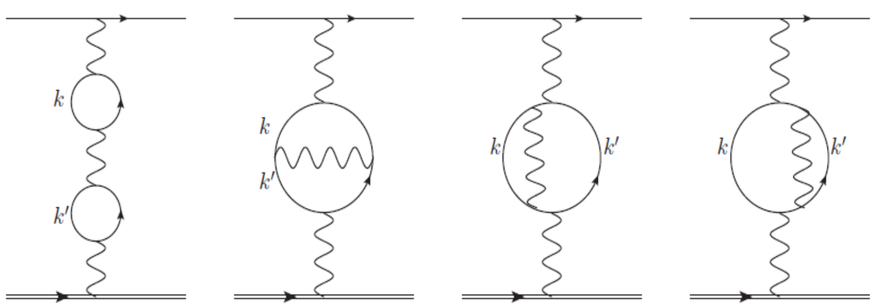
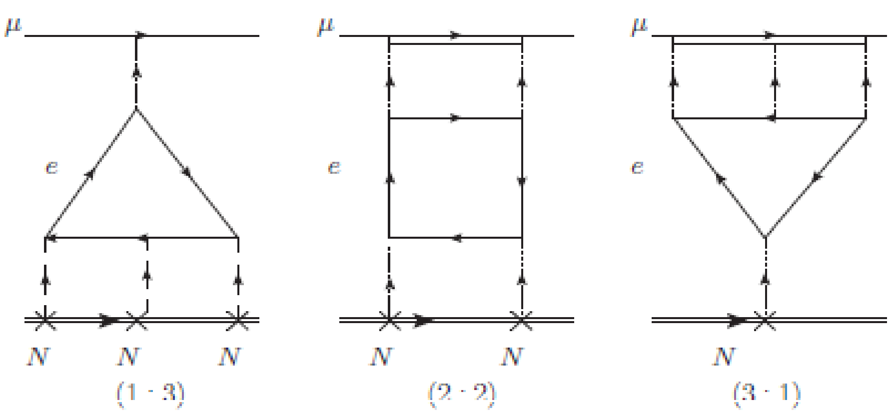
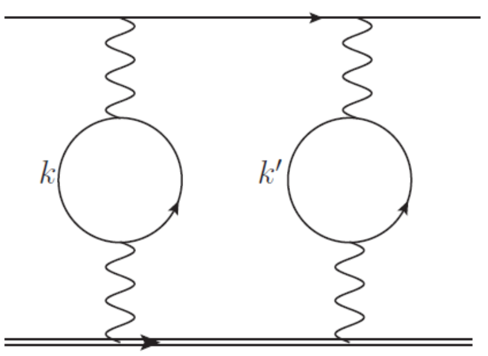
Other contributions up to $\mathcal{O}(m_r\alpha^5)$







“Pure” QED corrections

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) \; + \mathcal{O}\left(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6\right) \text{meV}$$

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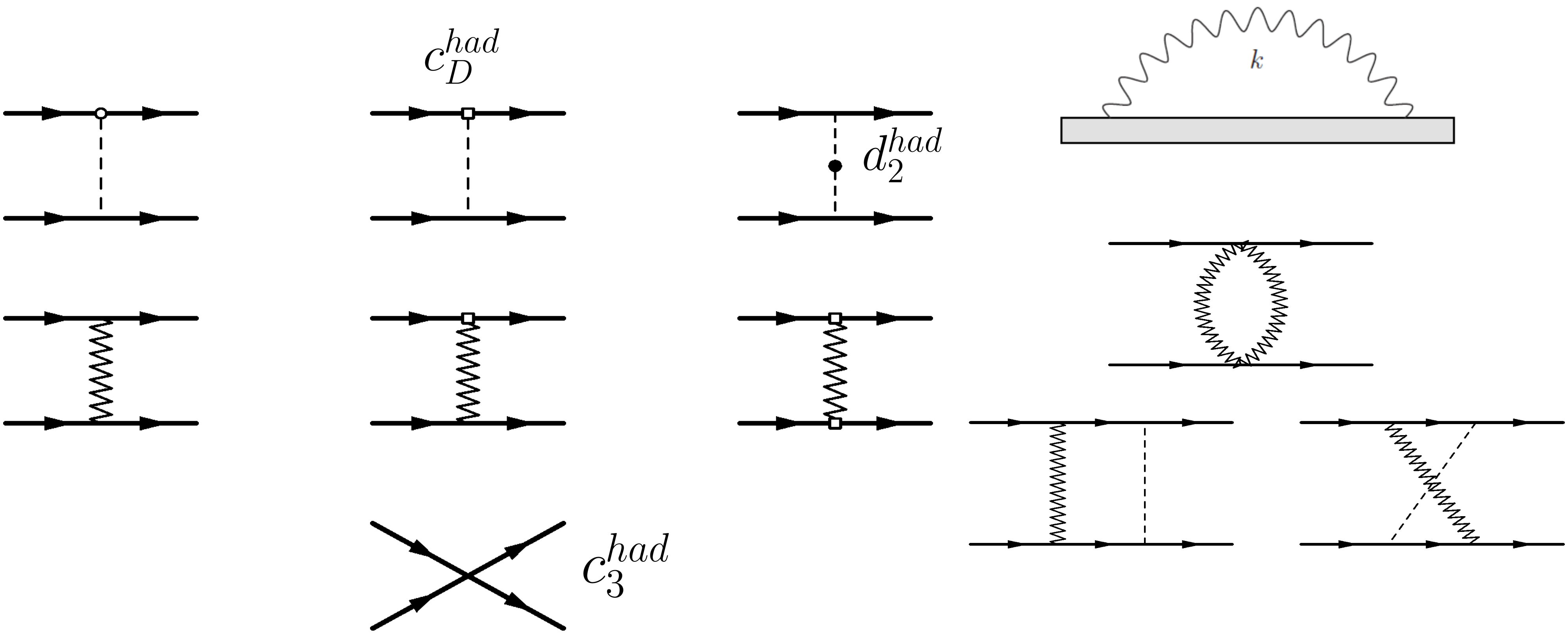
Relativistic contributions up to $\mathcal{O}(m_r\alpha^5)$



Summary and error estimates

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu\alpha^5\frac{m_\mu^3}{m_\rho^3}, m_\mu\alpha^6)\text{meV}$$

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$\mathcal{O}(m_r\alpha^3)$	$V_{\rm VP}^{(0)}$	205.00745
$\mathcal{O}(m_r\alpha^4)$	$V_{\rm VP}^{(0)}$	1.50795
$\mathcal{O}(m_r\alpha^4)$	$V_{\rm VP}^{(0)}$	0.15090
$\mathcal{O}(m_r\alpha^5)$	$V_{\rm VP}^{(0)}$	0.00752
$\mathcal{O}(m_r\alpha^5)$	$V_{LbL}^{(0)}$	-0.00089
$\mathcal{O}(m_r\alpha^4 \times \frac{m_\mu^2}{m_p^2})$	$V^{(2)} + V^{(3)}$	0.05747
$\mathcal{O}(m_r\alpha^5)$	$V_{\rm soft}^{(2)}$/ultrasoft	-0.71903
$\mathcal{O}(m_r\alpha^5)$	$V_{\rm VP}^{(2)}$	0.01876
$\mathcal{O}(m_\mu\alpha^6 \times \ln(\frac{m_\mu}{m_e}))$	$V^{(2)}; c_D^{(\mu)}$	-0.00127
$\mathcal{O}(m_\mu\alpha^6 \times \ln\alpha)$	$V_{\rm VP}^{(2)}; c_D^{(\mu)}$	-0.00454

Hadronic Effects

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) \quad +\mathcal{O}(m_\mu\alpha^5\frac{m_\mu^3}{m_\rho^3}, m_\mu\alpha^6)\text{meV}$$

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$\mathcal{O}(m_r\alpha^4 \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-5.1975\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r\alpha^5 \times m_r^2 r_p^2)$	$V_{\text{VP}}^{(2)}; c_D^{(p)}; r_p^2$	$-0.0283\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r\alpha^6 \ln\alpha \times m_r^2 r_p^2)$	$V^{(2)}; c_D^{(p)}; r_p^2$	$-0.0014\frac{r_p^2}{\text{fm}^2}$
$\mathcal{O}(m_r\alpha^5 \times \frac{m_r^2}{m_\rho^2})$	$V_{\text{VP}_{\text{had}}}^{(2)}; d_2^{\text{had}}$	$0.0111(2)$
$\mathcal{O}(m_r\alpha^5 \times \frac{m_r^2}{m_\rho^2}\frac{m_\mu}{m_\pi})$	$V^{(2)}; c_3^{\text{had}}$	$0.0344(125)$

$$\delta V^{had} = \frac{D_d^{had}}{m_p^2}\delta^{(3)}(\mathbf{r})$$

$$D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2}c_D^{had}$$

Definition of the proton radius

Hadronic Effects

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\text{fm}^2} + 0.0455(125) + \mathcal{O}(m_\mu\alpha^5\frac{m_\mu^3}{m_\rho^3}, m_\mu\alpha^6)\text{meV}$$

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F. Jegerlehner

$$\delta V^{had} = \frac{D_d^{had}}{m_p^2}\delta^{(3)}(\mathbf{r})$$

$$D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2}c_D^{had}$$

Hadronic Vacuum polarization:
Obtained from DR

Hadronic Effects

Theoretical equation for the Lamb Shift:

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$$\delta V^{had} = \frac{D_d^{had}}{m_p^2}\delta^{(3)}(\mathbf{r})$$

$$D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2}c_D^{had}$$

$$c_3^{\text{had}} = c_3^{\text{Born}} + c_3^{\text{pol}} \sim \alpha^2\frac{m_\mu}{m_\pi}\left(1 + \#\frac{m_\pi}{\Delta} + \ldots\right) + \mathcal{O}\left(\alpha^2\frac{m_\mu}{\Lambda_{\text{QCD}}}\right)$$

Large-Nc limit: we expect a large contribution from Δ(1232)

c_3^{had}: Born/Zemach contribution

$$c_{3,\mathrm{Born}}^{pl_i} = 4(4\pi\alpha)^2 M_p^2 m_{l_i} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{1}{\mathbf{q}^6} G_E^{(0)} G_E^{(2)}(-\mathbf{q}^2) = \frac{\pi}{3}\alpha^2 M_p m_\mu \langle r^3 \rangle_{(2)}$$

Zemach third momentum

Zemach momenta:

		$\langle r^3\rangle$	$\langle r^4\rangle$	$\langle r^5\rangle$	$\langle r^6\rangle$	$\langle r^7\rangle$	$\langle r^3\rangle_{(2)}$
EFT: CP, Pineda	π	0.4980	0.6877	1.619	5.203	20.92	0.9960
	$\pi\&\Delta$	0.4071	0.6228	1.522	4.978	20.22	0.8142
FITS:	Dipole	0.7706	1.083	1.775	3.325	7.006	2.023
	Kelly	0.9838	1.621	3.209	7.440	19.69	2.526
	Distler et al	1.16(4)	2.59(19)(04)	8.0(1.2)(1.0)	29.8(7.6)(12.6)	– – –	2.85(8)

large dependence on the fitted function & large difference with EFT

Born energy shift:

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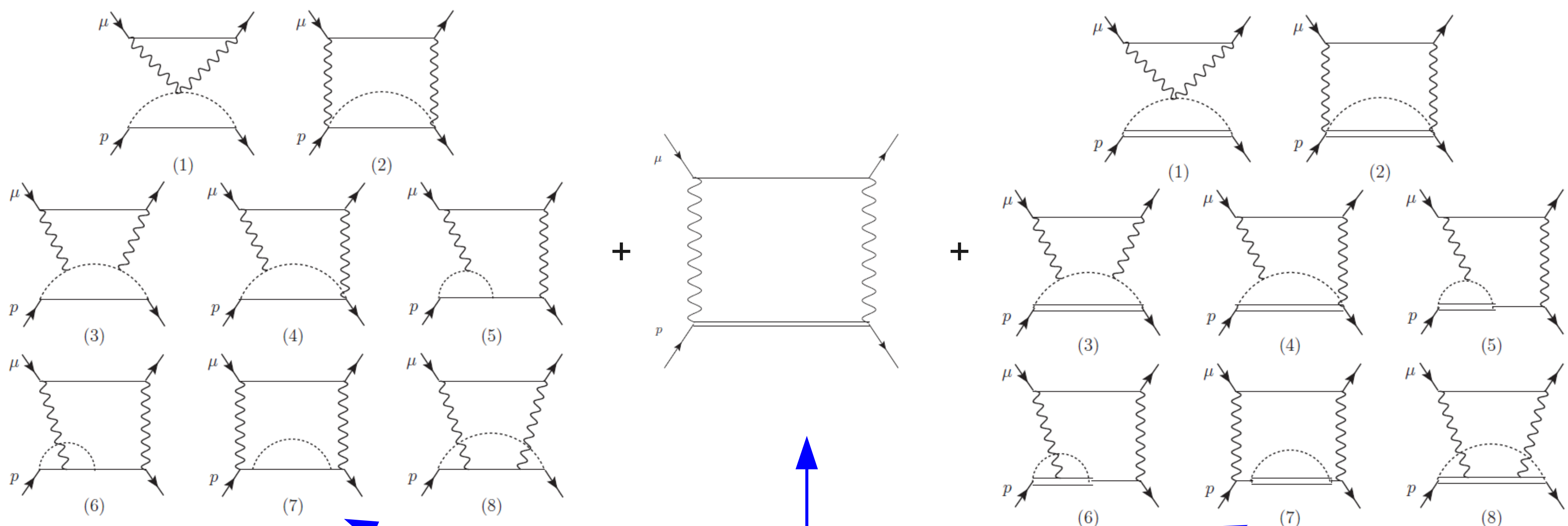
μeV	DR Pachucki	DR Carlson et al	HBET (π)	HBET ($\pi\&\Delta$)
ΔE_{Born}	23.2(1.0)	24.7(1.6)	10.1(5.1)	8.3(4.3)

we would expect less difference with the DR analysis

c_3^{had}: Polarizability

$$T_S^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) S_1(\rho, q^2) + \frac{1}{M_p^2}\left(p^\mu - \frac{M_p \rho}{q^2} q^\mu\right)\left(p^\nu - \frac{M_p \rho}{q^2} q^\nu\right) S_2(\rho, q^2)$$



$$\Delta E_{\text{pol}} = \frac{c_{3,\text{pol}}^{pl_\mu}}{M_p^2} \frac{1}{\pi} \left(\frac{m_r \alpha}{2}\right)^3 = 18.51(\pi\text{−loop}) - 1.58(\Delta\text{−tree}) + 9.25(\pi\Delta\text{−loop}) = 26.2(10.0)\mu\text{eV}$$

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$c_3^{\rm had}$: Polarizability

Different results for the polarizability contribution:

(μeV)	DR + Model				BχPT (π)	HBET (π)	(π&Δ)
$\Delta E_{\rm pol}$	12(2)	11.5	7.4(2.4)	15.3(5.6)	$8.2(^{+1.2}_{-2.5})$	18.5(9.3)	26.2(10.0)
	Pachucki	Martynenko	Carlson et al	Gorchtein et al	Alarcon et al		CP, Pineda

corrections to HBET are suppressed by $\frac{m_\mu}{m_\rho}$

The polarizability contribution from EFT is larger than the one computed using combinations of DR and different models

Total TPE energy shift:

$$\Delta E_{\rm TPE} = \Delta E_{\rm Born} + \Delta E_{\rm pol} = 28.59(\pi) + 5.86(\pi\&\Delta) = 34.4(12.5)\mu{\rm eV}$$

(LO) (NLO)

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The total contribution of TPE agrees with DR results better than when arbitrarily split into Born and polarizability pieces

Summary

We can theoretically predict the Lamb shift in a model independent way in an EFT framework

from μH: $r_p = 0.8412(15)$ fm

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CODATA value (from H): $r_p = 0.8775(51)$ fm

6.8σ away!

- The hadronic contributions are the main source of uncertainty, although it is not enough to account for the discrepancy with ep

- The main radius-independent hadronic contribution is the **Two Photon Exchange**

  - The EFT approach gives Born & polarizability contributions which are quite different from the ones obtained by DR plus different models

  - Both the agreements and the disagreements should be further understood.

- In conclusion, **the proton radius puzzle survives the EFT analysis**

THANK YOU!

Спасибо!

Hadronic corrections

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7)\frac{r_p^2}{\mathrm{fm}^2} + 0.0455(125) + \mathcal{O}\left(m_\mu \alpha^5 \frac{m_\mu^3}{m_\rho^3}, m_\mu \alpha^6\right)\mathrm{meV}$$

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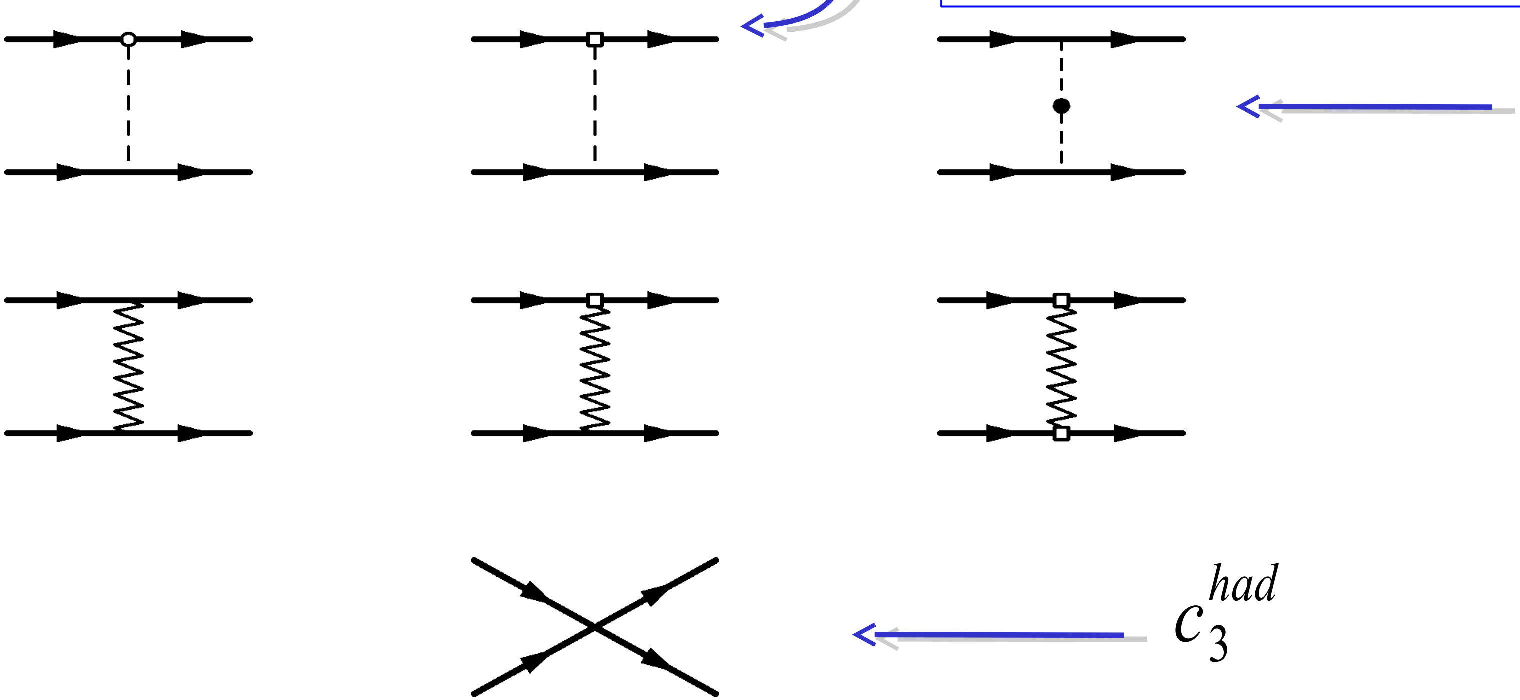
Scheme dependent $c_D^{(p)} \circ 1 + \frac{4}{3} r_p^2 m_p^2 + ¿\alpha + \ldots$

$$\delta\mathcal{L} = -e\frac{c_D^{(p)}}{m_p^2} N_p^\dagger \nabla \cdot \mathbf{E} N_p$$



d_2^{had}

c_3^{had}

Zemach momenta

Even momenta:

$$G_E(-\mathbf{k}^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \mathbf{k}^{2n} \int_0^{\infty} dr (4\pi) r^{2n} \rho_e(r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \mathbf{k}^{2n} \langle r^{2n} \rangle$$

Odd momenta:

$$\langle r^{2k+1} \rangle = \frac{\pi^{3/2}\Gamma[2+k]}{\Gamma[-1/2-k]} 2^{4+2k} \int \frac{d^3q}{(2\pi)^3} \frac{1}{\mathbf{q}^{2(2+k)}} \left[G_E(-\mathbf{q}^2) - \sum_{n=0}^{k} \frac{\mathbf{q}^{2n}}{n!} \left(\frac{d}{d\mathbf{q}^2} \right)^n G_E(-\mathbf{q}^2) \Big|_{\mathbf{q}^2=0} \right]$$

Polarizability energy shift

$$c_3^{pl_i} = -e^4 M_p m_{l_i} \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^4} \frac{1}{k_E^4 + 4 m_{l_i}^2 k_{0,E}^2} \left\{ (3k_{0,E}^2 + \mathbf{k}^2) S_1(ik_{0,E}, -k_E^2) - \mathbf{k}^2 S_2(ik_{0,E}, -k_E^2) \right\} + \mathcal{O}(\alpha^3)\,. \tag{4.1}$$