

Heavy Hybrids in pNRQCD

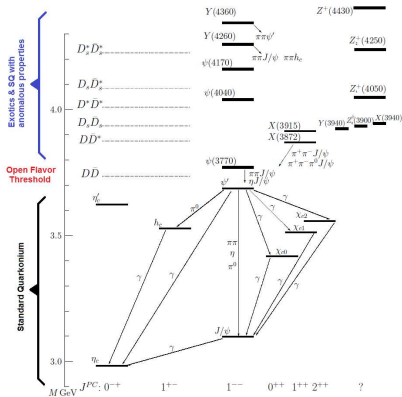
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XIth Quark Confinement and the Hadron Spectrum
Saint-Petersburg, September 8, 2014.

- ▶ In the last decade many new unexpected states have been found close or above threshold.



Voloshin

- ▶ The states that do not fit Quarkonium potential models are called Exotics and labeled Xs, Ys and Zs.
- ▶ These states are candidates for **non traditional hadronic states**, including four constituent quark or an excited gluon constituent.
- ▶ There is an ongoing experimental effort to study normal and Exotic quarkonium: new states, production mechanisms, decays and transitions, precision and high statistics data.
- ▶ BaBar, Belle2, BESIII, LHCb and Panda (under construction).

Quarkonium Hybrids

What are quarkonium Hybrids?

- ▶ A quarkonium hybrid consists of Q, \bar{Q} in a color octet configuration and a gluonic excitation g .

Born-Oppenheimer Hybrids:

The heavy quarks are nearly static, and the gluons adapt nearly instantaneously.

Born-Oppenheimer approximation Heavy Hybrids

- The gluonic static energies can be defined in NRQCD and computed on the lattice or, in the short range, using pNRQCD.
- The hybrid state energy levels are obtained solving the Schrödinger equation with $H_{kin} + E_g$.
- H_{kin} acts on the gluon wave functions. Additional approximations are needed, because the gluonic wave functions are not available.
- The mixing terms have to be taken into account because the static energies are degenerate at short distances.

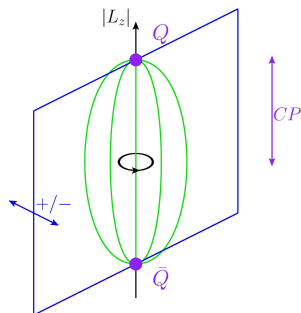
Pioneered by **Juge, Kuti, Morningstar 1999**

Symmetries of the static system

Static states classified by symmetry group $D_{\infty h}$

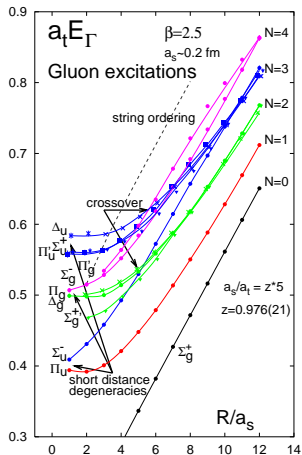
Representations labeled Λ_{η}^{σ}

- ▶ Λ rotational quantum number
 $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \dots$ corresponds to
 $\Lambda = \Sigma, \Pi, \Delta \dots$
- ▶ η eigenvalue of CP :
 $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- ▶ σ eigenvalue of reflections
- ▶ σ label only displayed on Σ states
(others are degenerate)



- The static energies correspond to the irreducible representations of $D_{\infty h}$.
- In general it can be more than one state for each irreducible representations of $D_{\infty h}$, usually denoted by primes, e.g. $\Pi_u, \Pi'_u, \Pi''_u \dots$

Lattice data on hybrid static energies



- ▶ Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- ▶ The rest of the static energies correspond to excited gluonic states that generate hybrids.
- ▶ The two lowest hybrid static energies are Π_u and Σ_u^- , they are nearly degenerate at short distances.
- ▶ The static energies have been computed in quenched lattice QCD, the most recent data by [Juge, Kuti, Morningstar, 2002](#) and [Bali and Pineda 2003](#).
- ▶ Quenched and unquenched calculations for Σ_g^+ and Π_u were compared in [Bali et al 2000](#) and good agreement was found below string breaking distance.

[Juge, Kuti, Morningstar 2002](#)

Radial wave functions and and Schrödinger equation

The **hybrid masses** are obtained by solving the Schrödinger equation with $H_{kin} + E_{n\Lambda\sigma}$

- ▶ Let us define Hybrid state as:

$$|N; j, m, s, l; n, \eta; \epsilon\rangle = \sum_{\Lambda} \int dr |j, m, s, l; n, \Lambda, \eta; \epsilon\rangle \frac{\psi_N^{\Lambda}(r)}{r}.$$

- ▶ $H_{kin} = -\frac{\partial_r^2}{2\mu} + \frac{L_{Q\bar{Q}}^2}{2\mu r^2}$ acts in principle on **both** the gluonic and heavy quark parts.
- ▶ The radial derivative acting on the gluonic state can be ignored.

Angular derivative

In spherical coordinates $\mathbf{K} = K_{\theta}\hat{\theta} + K_{\phi}\hat{\phi} + K_n\hat{n}$, and the rising and lowering operators $K_{\pm} = K_{\theta} \pm iK_{\phi}$.

$$\begin{aligned} L_{Q\bar{Q}}^2 &= (\mathbf{L} - \mathbf{K})^2 = \mathbf{L}^2 - 2\mathbf{L} \cdot \mathbf{K} + \mathbf{K}^2 \\ &= \mathbf{L}^2 - 2K_n^2 + \mathbf{K}^2 - L_- K_+ - L_+ K_- , \end{aligned}$$

- Acting on our J^{PC} eigenstates the first two terms give $l(l+1)$ and $-2\Lambda^2$.
- $\mathbf{K}^2 - L_- K_+ - L_+ K_-$ is not directly determined since we do not have the gluonic wave functions.
- $L_- K_+ + L_+ K_-$ mixes different channels.

To determine the last two terms we use:

1. $L_{Q\bar{Q}}^2/2\mu r^2$ is most important in the short range.
2. The static symmetry group $D_{\infty h}$ is extended to $O(3) \times C$ in the limit $r \rightarrow 0$.
 \Rightarrow approximate $\mathbf{K}^2 - L_- K_+ - L_+ K_-$ for its short range behavior.

\mathbf{K}^2

- $\langle \mathbf{K}^2 \rangle = k(k+1)$ with k from the K^{PC} multiplet corresponding to $n\Lambda_{\eta}^{\sigma}$ in the $r \rightarrow 0$ limit.
- For the three the lowest static energies, Σ_g^+ , Π_u and Σ_u^- , pNRQCD tells us these are 0^{++} , 1^{+-} and 1^{+-} respectively.

$L_- K_+ + L_+ K_-$

- If we ignore the mixing terms, for $\Lambda > 0$, there are two degenerate states with opposite parity. If we include them the degeneracy is lifted and we obtain an effect called Λ -doubling.
- The mixing involves gluonic static energies with λ and $\lambda \pm 1$.
- We will only consider mixing between static energies that are nearly degenerate.
- We have considered the mixing through **coupled Schrödinger equations**.

Gluonic static energies in pNRQCD

EFT of QCD for Quarkonium

- ▶ Quarkonium systems are non-relativistic bound states.
- ▶ **Multiscale system:** $m \gg \mathbf{p}_Q \gg E_b$, and Λ_{QCD} . m is the heavy-quark mass.
- ▶ We can exploit the **scale hierarchies** by building an **Effective Field Theory** (EFT).

pNRQCD for Hybrid static energies

- ▶ The short range $\mathbf{p}_Q \sim 1/r \gg \Lambda_{QCD}$ behavior of the static energies can be studied in weakly-coupled pNRQCD.
- ▶ In this region pNRQCD is obtained integrating out \mathbf{p}_Q (perturbative) and Λ_{QCD} nonperturbative.

- ▶ The Gluelumps are the adjoint sources in the presence of a gluonic field

$$H(\mathbf{R}, \mathbf{r}, t) = H^a(\mathbf{R}, t) \mathcal{O}^{a\dagger}(\mathbf{R}, \mathbf{r}, t),$$

Gluonic excitation operators up to dim 3

Λ_{η}^{σ}	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r} \cdot \mathbf{B}, \mathbf{r} \cdot (\mathbf{D} \times \mathbf{E})$
Π_u	1^{+-}	$\mathbf{r} \times \mathbf{B}, \mathbf{r} \times (\mathbf{D} \times \mathbf{E})$
$\Sigma_g^{+'}$	1^{--}	$\mathbf{r} \cdot \mathbf{E}, \mathbf{r} \cdot (\mathbf{D} \times \mathbf{B})$
Π_g	1^{--}	$\mathbf{r} \times \mathbf{E}, \mathbf{r} \times (\mathbf{D} \times \mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{B})$
Π_g'	2^{--}	$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{B} + \mathbf{D}(\mathbf{r} \cdot \mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r} \times \mathbf{D})^i (\mathbf{r} \times \mathbf{B})^j + (\mathbf{r} \times \mathbf{D})^j (\mathbf{r} \times \mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{E} + \mathbf{D}(\mathbf{r} \cdot \mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r} \times \mathbf{D})^i (\mathbf{r} \times \mathbf{E})^j + (\mathbf{r} \times \mathbf{D})^j (\mathbf{r} \times \mathbf{E})^i$

- ▶ We can see that in the short distance limit the $\Pi_u - \Sigma_u^-, \Pi_g - \Sigma_g^{+'}, \Delta_g - \Sigma_g^- - \Pi_g'$ and $\Delta_u - \Pi_u' - \Sigma_u^+$ multiplets must be degenerate.

Hybrid Static energies

- ▶ The hybrid static energy spectrum reads

$$E_H = 2m + V_H,$$

with

$$V_H = \lim_{T \rightarrow \infty} \frac{i}{T} \log \left\langle H^a(T/2) \mathcal{O}^a(T/2) H^b(-T/2) \mathcal{O}^b(-T/2) \right\rangle.$$

- ▶ Up to next-to-leading order in the **multipole expansion**.

$$V_H = V_o + \Lambda_H + b_H r^2,$$

- ▶ $V_o(r)$ is the octet potential, which can be computed in perturbation theory.
- ▶ Λ_H corresponds to the **gluelump mass**.

$$\Lambda_H = \lim_{T \rightarrow \infty} \frac{i}{T} \log \left\langle H^a(T/2) \phi_{ab}^{adj}(T/2, -T/2) H^b(-T/2) \right\rangle,$$

where $\phi^{adj}(T/2, -T/2)$ is a Wilson line.

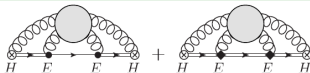
- ▶ We work in the **Renormalon Subtracted scheme** which improves the convergence of the octet potential.

Λ_H

- ▶ It is a non-perturbative quantity.
- ▶ It depends on the particular operator H^a , however it is the same for operators corresponding to different projections of the same gluonic operators.
- ▶ The gluelump masses have been determined in the lattice. *Foster et al 1999; Bali, Pineda 2004; Marsh Lewis 2014*
- ▶ At the subtraction scale $\nu_f = 1$ GeV: $\Lambda_{1+-}^{RS} = 0.87(15)$ GeV.

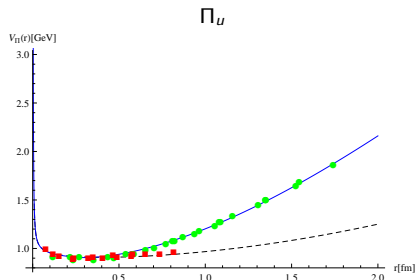
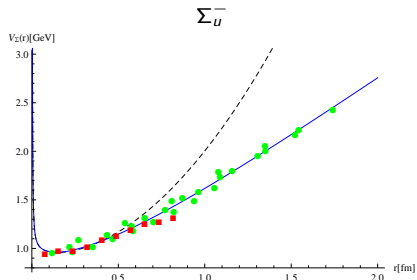
b_H

- ▶ It is a non-perturbative quantity.



- ▶ Proportional to r^2 due to rotational invariance and the multipole expansion.
- ▶ We are going to fix it through a fit to the static energies lattice data.
- ▶ Breaks the degeneracy of the potentials.

Static Hybrid potentials



Lattice data: Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.5)}$

Lattice data fitted for the $r = 0 - 0.5$ fm range, $b_{\Sigma}^{(0.5)} = 1.112 \text{ GeV}/\text{fm}^2$, $b_{\Pi}^{(0.5)} = 0.110 \text{ GeV}/\text{fm}^2$.

$$c_{\text{BP}} = 0.105 \text{ GeV}, \quad c_{\text{KJM}} = -0.471 \text{ GeV},$$

$V^{(0.25)}$

▶ $r \leq 0.25$ fm: pNRQCD potential, $b_{\Sigma}^{(0.25)} = 1.246 \text{ GeV}/\text{fm}^2$, $b_{\Pi}^{(0.25)} = 0.000 \text{ GeV}/\text{fm}^2$.

▶ $r > 0.25$ fm: phenomenological potential, $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.

Hybrid state masses from $V^{(0.5)}$

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.05	0.29	0.11	0.94	7.40	0.31	0.08	0.94	10.73	0.36	0.06	0.95
H'_1	4.23	0.27	0.20	0.91	7.54	0.30	0.16	0.91	10.83	0.36	0.11	0.92
H_2	4.09	0.21	0.13	1.00	7.43	0.23	0.10	1.00	10.75	0.27	0.07	1.00
H'_2	4.30	0.19	0.24	1.00	7.60	0.21	0.19	1.00	10.87	0.25	0.13	1.00
H_3	4.69	0.37	0.42	0.00	7.92	0.42	0.34	0.00	11.09	0.50	0.23	0.00
H_4	4.17	0.19	0.17	0.97	7.49	0.25	0.14	0.97	10.79	0.29	0.09	0.98
H_5	4.20	0.17	0.18	1.00	7.51	0.19	0.15	1.00	10.80	0.22	0.10	1.00

Consistency test:

- The potentials describe the lattice data well up to $r \lesssim 0.55 - 0.65$ fm which corresponds $\langle 1/r \rangle \gtrsim 0.36 - 0.30$ GeV.
- The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

Spin symmetry multiplets

H_1	$\{1^{--}, (0, 1, 2)^{-+}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0, 1, 2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1, 2, 3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1, 2, 3)^{-+}\}$	Π_u

Hybrid state masses from $V^{(0.25)}$

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H'_1	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H'_2	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Consistency test:

- The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

Conclusion:

- $V^{(0.25)}$ yields more consistent results.
- As expected the Born–Oppenheimer program works better in bottomonium than charmonium

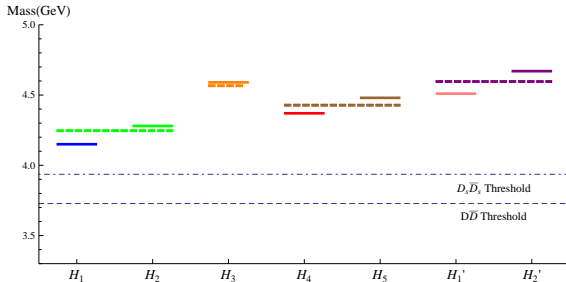
Spin symmetry multiplets

H_1	$\{1^{--}, (0, 1, 2)^{+-}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0, 1, 2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1, 2, 3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1, 2, 3)^{+-}\}$	Π_u
H_6	$\{3^{--}, (2, 3, 4)^{+-}\}$	Σ_u^-, Π_u
H_7	$\{3^{++}, (2, 3, 4)^{+-}\}$	Π_u

Λ -doubling effect

- ▶ In [Braaten et al 2014](#) a similar procedure was followed to obtain the hybrid masses.
- ▶ No Λ -doubling effect mixing terms were included, and phenomenological potentials fitting the lattice data.
- ▶ We can compare the results to estimate the size of the Λ -doubling effect.

Charmonium sector



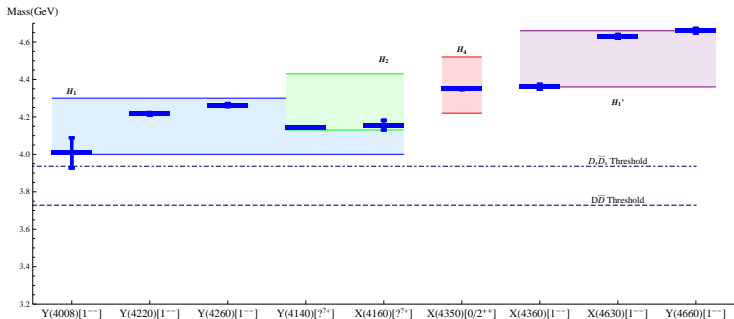
[Braaten et al 2014](#) results plotted in dashed lines.

- ▶ The mixing lowers the mass of the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$.

Identification with experimental states

Most of the candidates have 1^{--} or $0^{++}/2^{++}$ since the main observation channels are production by e^+e^- or $\gamma\gamma$ annihilation respectively.

- ▶ Charmonium states (Belle, CDF, BESIII, Babar):



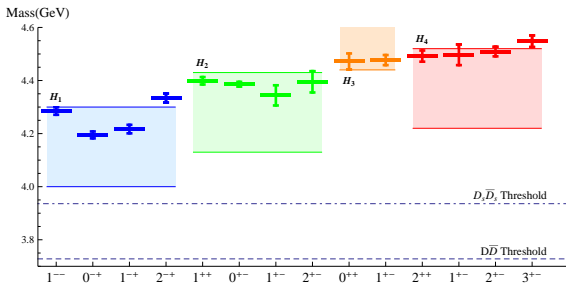
- ▶ Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.884 \pm 3.0$ (Belle). Possible H_1 candidate, $m_{H_1} = 10.79 \pm 0.15$.

However, except for $Y(4220)$, all other candidates observed decay modes violate Heavy Quark Spin Symmetry.

Comparison with direct lattice computations

Charmonium sector

- ▶ Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. *Liu et al 2012*
- ▶ They worked in the constituent gluon picture, which consider the multiplets H_2 , H_3 , H_4 as part of the same multiplet.
- ▶ Their results are given with the η_c mass subtracted.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- ▶ Our masses are 0.1 – 0.14 GeV lower except the for the H_3 multiplet, which is the only one dominated by Σ_u^- .
- ▶ Good agreement with the mass gaps between multiplets, in particular the Λ -doubling effect ($\delta m_{H_2-H_1}$).

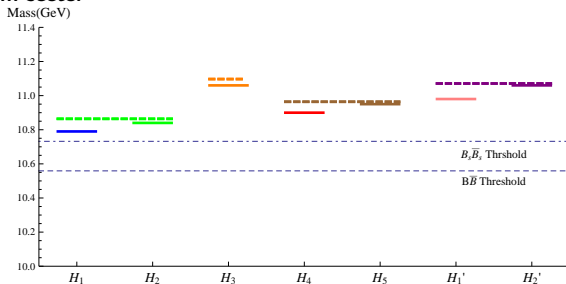
Conclusions

- ▶ We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the Λ -doubling terms by using coupled Schrödinger equations.
- ▶ The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.
- ▶ A large set of masses for spin symmetry multiplets for $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ has been obtained.
- ▶ Λ -doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.
- ▶ Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.
- ▶ Several experimental candidates for Charmonium hybrids belonging to the H_1 , H_2 , H_4 and H_1' multiplets.
- ▶ One experimental candidate to the bottomonium H_1 multiplet.

Thank you for your attention

Λ -doubling effect

Bottomonium sector



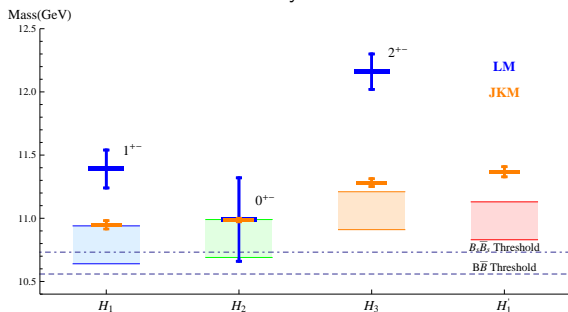
Braaten *et al* 2014 results plotted in dashed lines.

- ▶ The mixing lowers the mass of the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$.

Comparison with direct lattice computations

Bottomonium sector

- ▶ Calculations done by **Juge, Kuti, Morningstar 1999** and **Liao, Manke 2002** using quenched lattice QCD.
- ▶ **Juge, Kuti, Morningstar 1999** included no spin or relativistic effects.
- ▶ **Liao, Manke 2002** calculations are fully relativistic.



Error bands take into account the uncertainty on the glueball mass ± 0.15 GeV

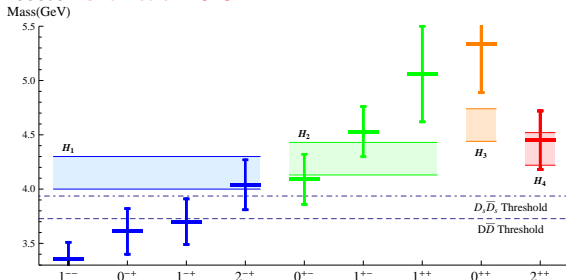
Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- ▶ Our masses are 0.15 – 0.25 GeV lower except the for the H_1' multiplet, which is larger by 0.36 GeV.
- ▶ Good agreement with the mass gaps between multiplets, in particular the Λ -doubling effect ($\delta m_{H_2-H_1}$).

Comparison with QCD sum rules

- ▶ A recent analysis of QCD sum rules for hybrid operators has been performed by [Chen et al 2013, 2014](#) for $b\bar{b}$ and $c\bar{c}$ hybrids, and $b\bar{c}$ hybrids respectively.
- ▶ Correlation functions and spectral functions were computed up to dimension six condensates which stabilized the mass predictions compared to previous calculations which only included up to dimension 4 condensates.

Charmonium sector [Chen et al 2013](#)

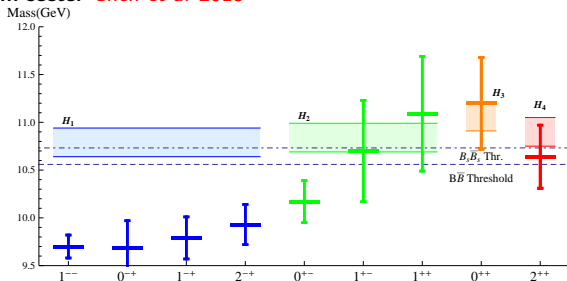


Error bands take into account the uncertainty on the gluon mass ± 0.15 GeV

- ▶ The spin average of the H_1 multiplet is 0.4 GeV lower than our mass.
- ▶ H_2 , H_3 and H_4 multiplets are incomplete.
- ▶ Large uncertainties compared to direct lattice calculations.

Comparison with QCD sum rules

Bottomonium sector *Chen et al 2013*



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- ▶ The spin average of the H_1 multiplet is 0.98 GeV lower than our mass.
- ▶ H_2 , H_3 and H_4 multiplets are incomplete.
- ▶ Large uncertainties compared to direct lattice calculations.