The reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$: developement of the analysis methods and selected results

D. Ryabchikov

Institute for High Energy Physics, Protvino

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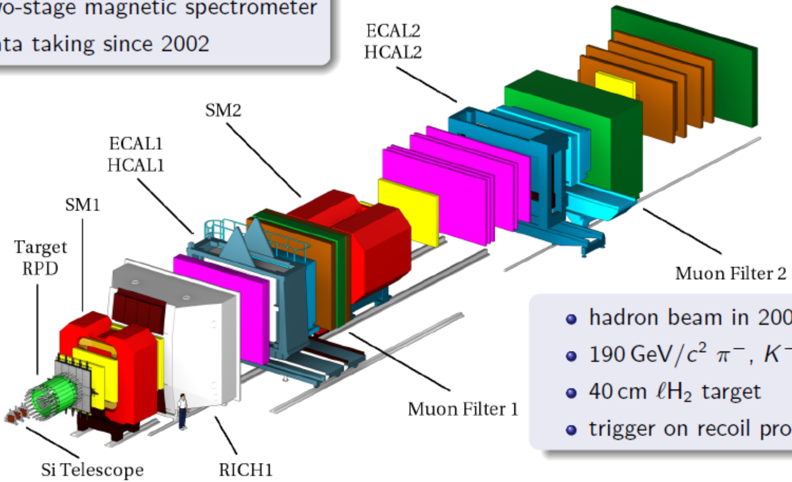
Plan of the talk

The reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$ at $p_{\pi^-} = 190$ GeV

- Mass-independent Partial-Wave Analysis (PWA)
- Mass-dependent analysis
- Model-independent extraction of $(\pi^+ \pi^-)$ isobars

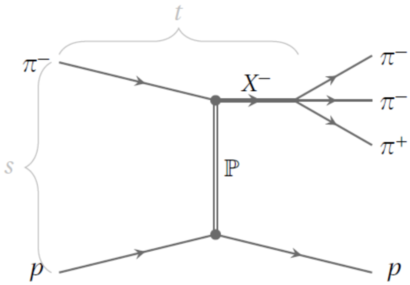
Apparatus

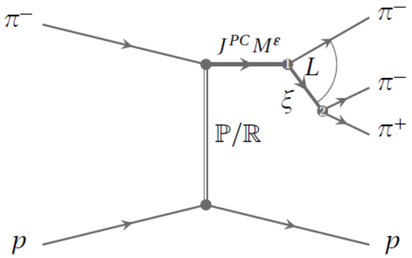
- fixed target experiment
- located at CERN's SPS
- two-stage magnetic spectrometer
- data taking since 2002



- hadron beam in 2008
- 190 GeV/c^2 π^-, K^-, $\bar{p}$
- 40 cm ℓH_2 target
- trigger on recoil proton

PWA basics





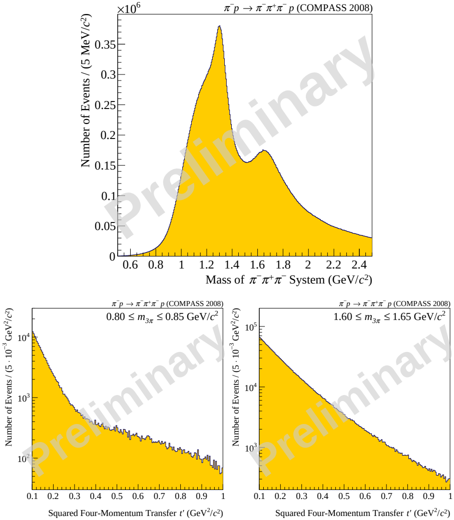
$$\|J^P M^\varepsilon\rangle \equiv \Theta(M)\left[|J^P M\rangle - \varepsilon P(-)^{J-M}\, |J^P -M\rangle\right] \quad \text{with} \quad \Theta(M) = \begin{cases} 1/\sqrt{2} & \text{for} M > 0 \\ 1/2 & \text{for} M = 0 \\ 0 & \text{for} M < 0 \end{cases}$$

$$\mathcal{I}(m_X, t', \tau) = \sum_r^{N_r} \sum_{\varepsilon=\pm 1} \left| \sum_\alpha T_\alpha^{r\varepsilon}(m_X, t')\, \psi_\alpha^\varepsilon(m_X, \tau) \right|^2 \quad \text{with} \quad \varrho^\varepsilon_{\alpha\beta}(m_X, t') = \sum_r^{N_r} T_\alpha^{r\varepsilon}\, T_\beta^{r\varepsilon *}$$

Mass-independent PWA

- Spin formalism using D-functions is applied for $\pi^-\pi^-\pi^+$ decay amplitudes (X-checked vs. Zemach tensors)
- 5 standart $\pi^+\pi^-$ isobars: $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$, $(\pi\pi)_S$ (AMP with $f_0(980)$ withdrawn) and $f_0(980)$ (FLATTE)
- rank=1 is used (rather narrow m(3π) and t' bins; helicity non-flip nature of Pomeron)
- 80 waves with $\epsilon = +1$, 7 waves with $\epsilon = -1$ and incoherent FLAT wave

The invariant mass and t' spectrums



50 million of high-quality $\pi^- \pi^- \pi^+$ events

Now let's compare the mass-independent intensity:

$\mathcal{I}(m_x, t', \tau) = \sum_\epsilon \sum_r \left|\sum_i T^\epsilon_{ir}(m_x, t')\bar{\psi}^\epsilon_i(\tau, m_X)\right|^2$ (1)

The intensity as a function of all phase-space variables:

$\mathcal{I}(m, t, \tau) =$

$\sum_\epsilon \sum_r \left|\sum_i \sum_k C^\epsilon_{ikr} BW_{ik}(m_X, \zeta, t)\sqrt{\int |\psi^\varepsilon_i(\tau', m)|^2 \, d\rho(\tau')}\bar{\psi}^\epsilon_i(\tau, m)\right|^2$ (2)

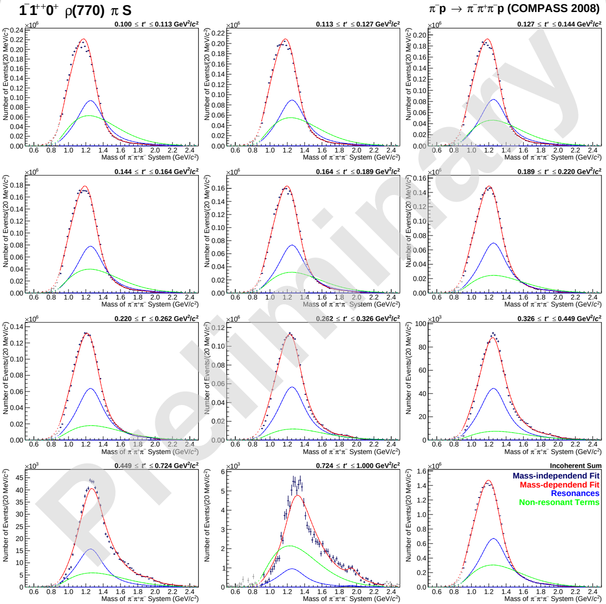
Parameters like T^ϵ_{ir} or C^ϵ_{ikr} are not defined in unique way what is measured is:

$\rho_{i,j} = \sum_r T^\epsilon_{ir} T^{\epsilon *}_{jr}$

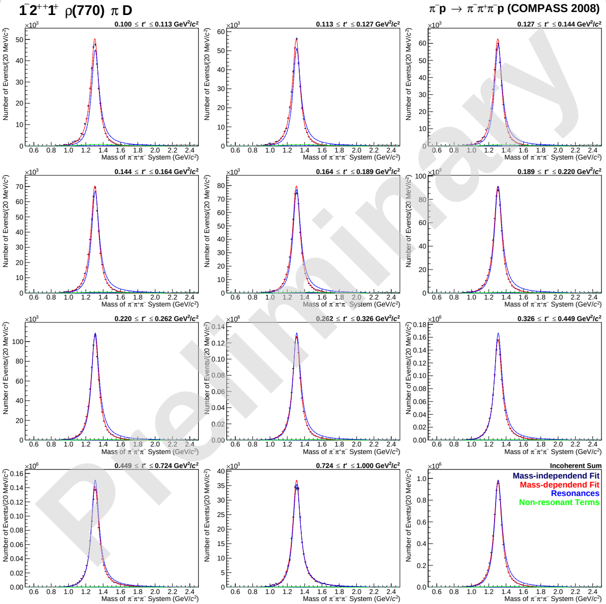
compare (1) and (2) $\rightarrow \rho_{i,j}(m, t) =$

$\sum_r \sum_{k,l} C^\epsilon_{ikr} C^{\epsilon *}_{jlr} BW_{ik}(m, t, \zeta) BW^*_{jl}(m, t, \zeta)\sqrt{\int |\psi^\varepsilon_i(\tau')|^2 \, d\rho(\tau')}\sqrt{\int \left|\psi^\varepsilon_j(\tau')\right|^2 d\rho(\tau')}$

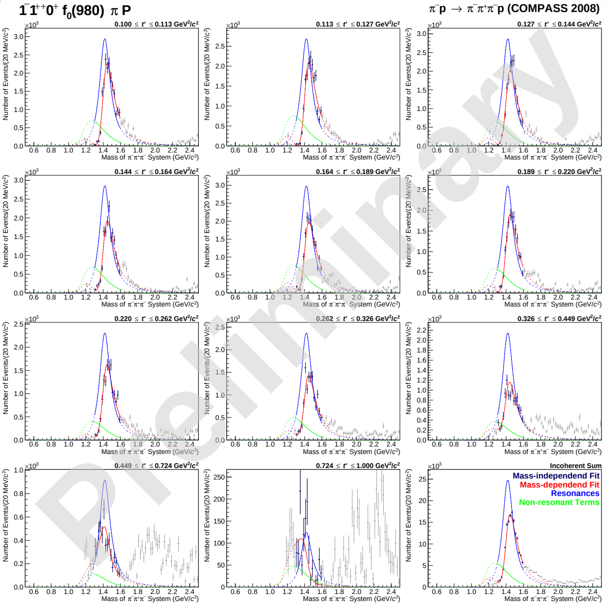
Mass-indep vs. mass-dep of $1^{++}0^{+}\rho(770)\pi^{-}$ in 11 t' bins



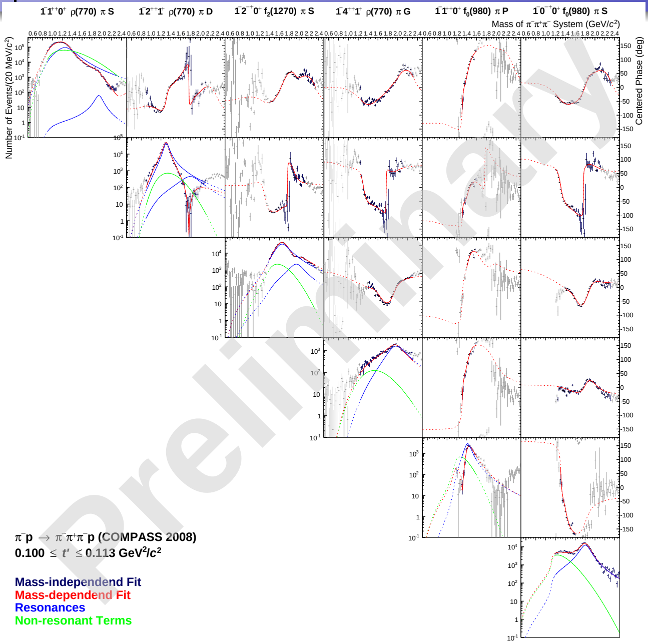
Mass-indep vs. mass-dep of $2^{++}1^{+}\rho(770)\pi^{-}D$ in 11 t' bins



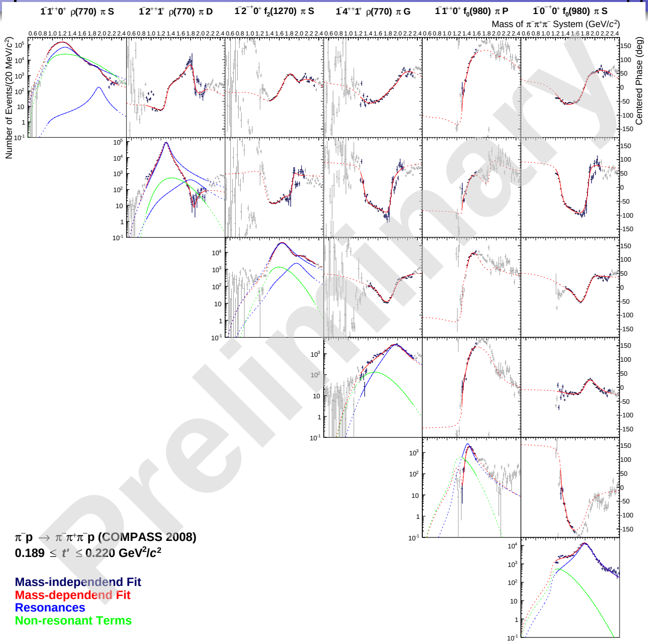
Mass-indep vs. mass-dep of $1^{++}0^{+} f_0(980)\pi^{-}$ in 11 t' bins



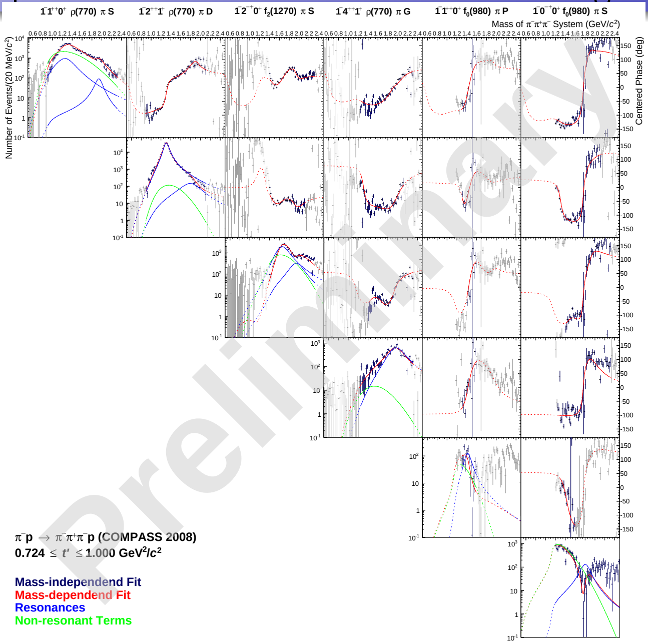
Mass-dependent fit in $\pi^-\pi^-\pi^+$ - "sub-matrix"lowest t'



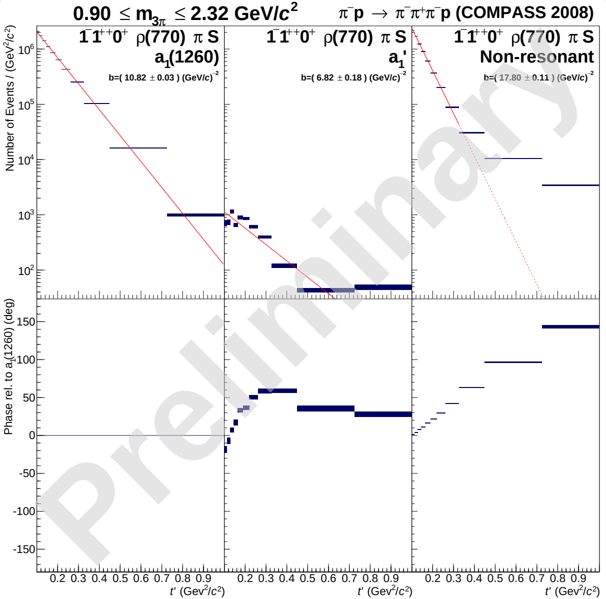
Mass-dependent fit in $\pi^-\pi^-\pi^+$ - "sub-matrix"higher t'



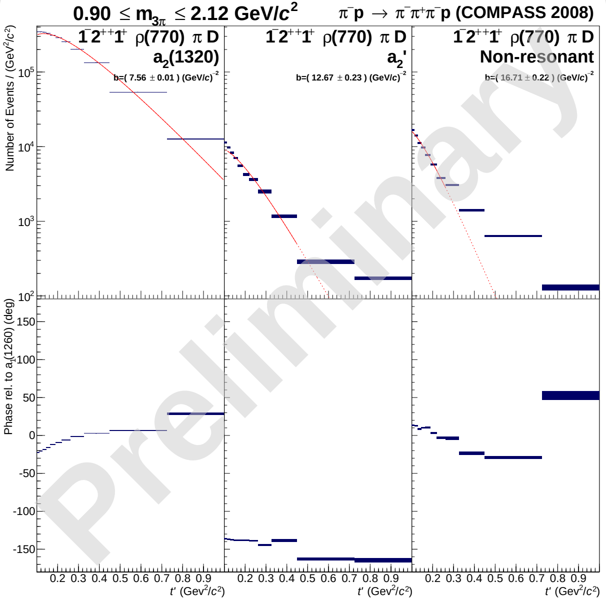
Mass-dep. fit in $\pi^-\pi^-\pi^+$ - "sub-matrix"the highest t'



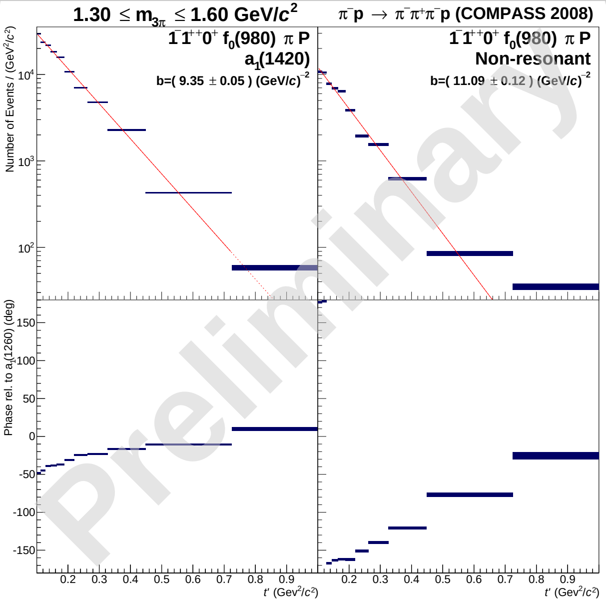
t'-dependence of mass-dependent fit components



t'-dependence of mass-dependent fit components



t'-dependence of mass-dependent fit components



Mass-dependent fit in $\pi^-\pi^-\pi^+$ resonance parameters table

Particle	J^{PC}	Mass Range [MeV/c^2]	Width Range [MeV/c^2]	PDG Values m [MeV/c^2]	PDG Values Γ [MeV/c^2]
			"Established" states		
$a_1(1260)$	1^{++}	1260–1290	360–420	1230 ± 40	250–600
$a_2(1320)$	2^{++}	1312–1315	108–115	$1318.3^{+0.5}_{-0.6}$	107 ± 5
$a_4(2040)$	4^{++}	1928–1959	360–400	1996^{+10}_{-9}	255^{+28}_{-24}
$\pi_2(1670)$	2^{-+}	1635–1663	265–305	1672.2 ± 3.0	260 ± 9
$\pi(1800)$	0^{-+}	1768–1807	212–280	1812 ± 12	208 ± 12
$\pi_2(1880)$	2^{-+}	1900–1990	210–390	1895 ± 16	235 ± 34
			States not in PDG summary table		
$a_1(1420)$	1^{++}	1412–1422	130–150	—	—
$a_1(1930)$	1^{++}	1920–2000	155–255	1930^{+30}_{-70}	155 ± 45
$a_2(1950)$	2^{++}	1740–1890	300–555	1950^{+30}_{-70}	180^{+30}_{-70}

The Method of extraction of 2π amplitudes

The mass-independent PWA intensity reads:
$\mathcal{I}(m,t,\tau) = \sum_\epsilon \sum_r \left| \sum_i T^\epsilon_{ir}(m,t) \bar{\psi}^\epsilon_i(\tau,m) \right|^2$
The decay amplitudes $\psi^\epsilon_i(\tau,m)$ contain multiplicative complex amplitudes of intermediate isobars.
Same multipicative functions (but depending on different kinematical variables) are contained in each linear term in case of bose- or isospin- symmetrisation – in arbitrary N-particle phase-space.

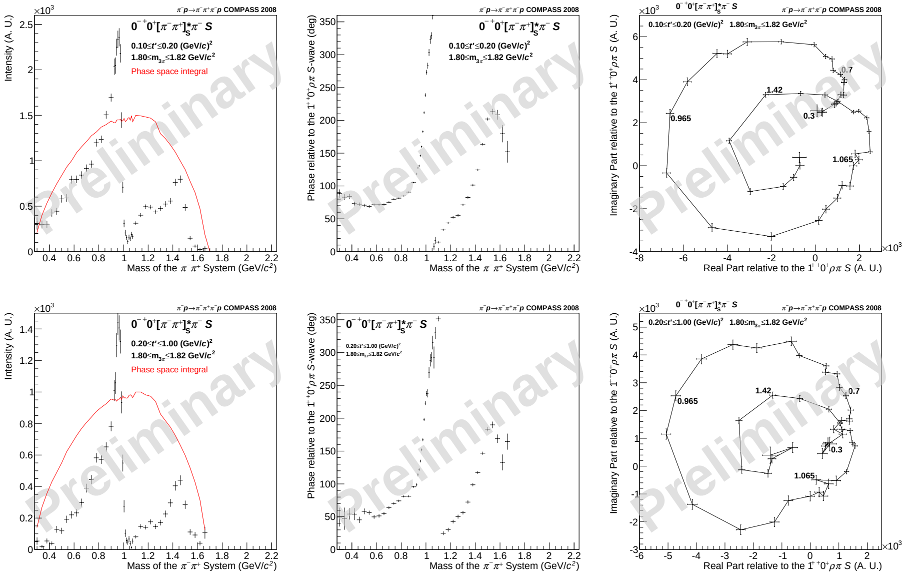
Example: $\Psi(\tau) = BW(m_{13})A(\Omega_{13},\Omega_{1(13)}) + BW(m_{23})A(\Omega_{23},\Omega_{1(23)})$

Let's express $BW(m) = \sum_k C_k \Theta_k$ Here Θk is set of functions $=1$ in each (non-equidistant) bin (m_k, m_{k+1}).
Then $\psi(\tau) = \sum_k C_k \psi_{\Theta_k}(\tau)$ where ψ_{Θ_k} has $\Theta(m_k, m_{k+1})$ instead of $BW(m)$ respectively.
Several selected amplitudes are decomposed to "theta-like" amplitudes. Using rank=1 fit will provide measurement of model-independent isobaric amplitudes in a given sub-systems, different for each decay quantum numbers.
Example: $0^{-+}, 1^{++}, 2^{-+} \to (\pi\pi)_S \pi^-$ in S,P and D waves (described below).

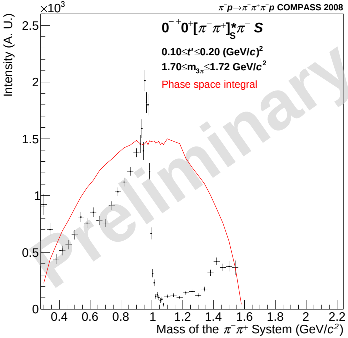
Mass-independent fit with step-like isobars

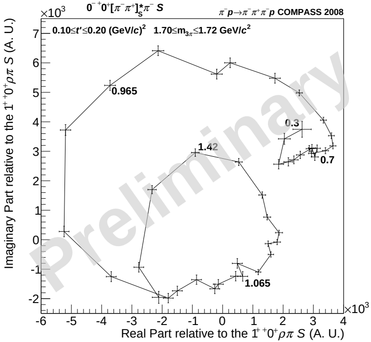
- 20 MeV bins in $0.5 < m(3\pi) < 2.5 GeV$ and 2 broad t' regions: $0.1 < t < 0.2 GeV^2$ and $0.2 < t < 1.0 GeV^2$
- $(\pi\pi)_S$ is represented by 62 complex steps Θ_k in 3 (S,P and D) waves $\rightarrow$ i.e. 372 fit parameters. The remaining 41 natural-parity exchange with rank=1 $\rightarrow$ 82 par and finally 7 unnatural-parity exchange with rank=2 $\rightarrow$ 26 par. In total 480 parameter.
- Fast fitting routine with analytical 1-st and 2-nd derivatives of log-likelihood.
- Fit continuality achieved by random 20 attempts for 50 bins: Nx1000 hours in parallel computing
- Measured 2D amplitudes $T_j(m(3\pi), m(2\pi))$ contain info on 3-body and 2-body states as $\Sigma_{k,l} C_{jkl} BW_{jk}(m(3\pi)) BW_{kl}(m(2\pi))$ (developement of 1D mass-dependent description)
- what we present, do not contain phase-space factors (they are explicitly shown on 2-body intensities)

$0^{-+} \to \pi(\pi\pi)_S$ wave at $\pi(1800)$ low and high t'

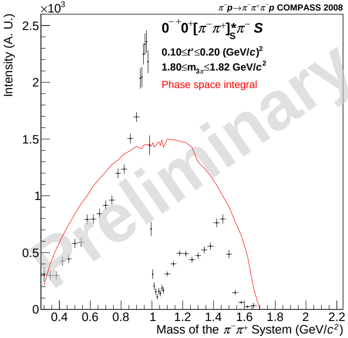


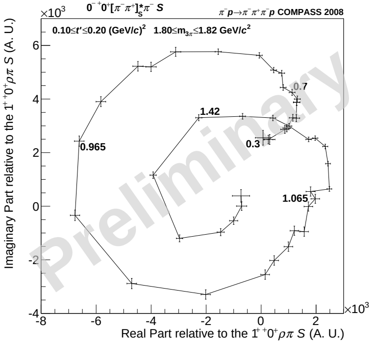
$0^{-+} \to \pi(\pi\pi)_S$ wave below $\pi(1800)$



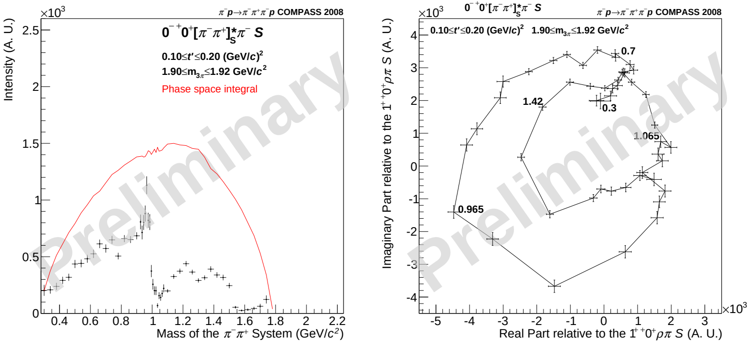


$0^{-+} \to \pi(\pi\pi)_S$ wave at $\pi(1800)$

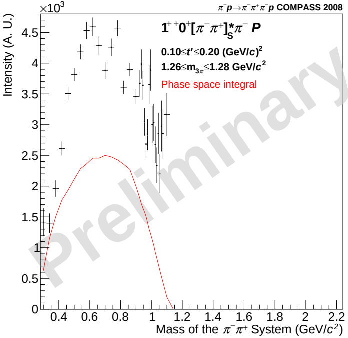


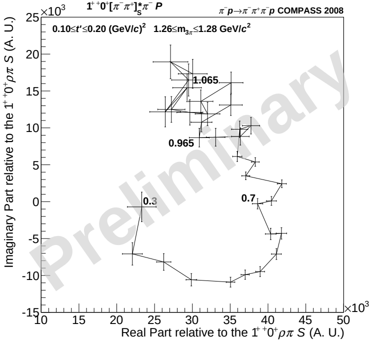


$0^{-+} \to \pi(\pi\pi)_S$ wave above $\pi(1800)$

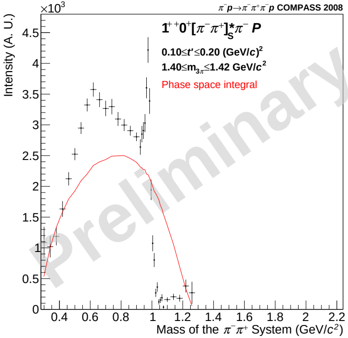


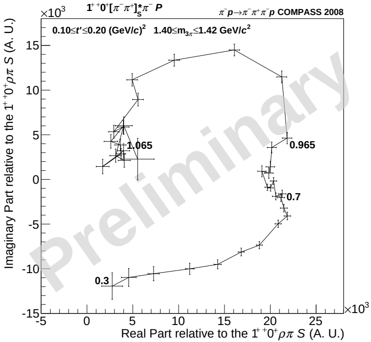
$1^{++} \to \pi(\pi\pi)_S$ wave below $a_1(1420)$



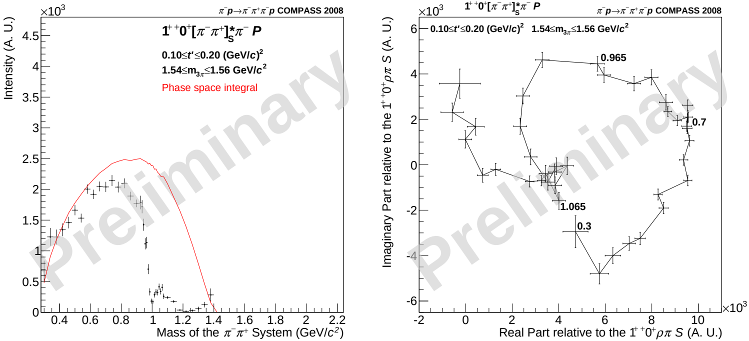


$1^{++} \to \pi(\pi\pi)_S$ wave at $a_1(1420)$

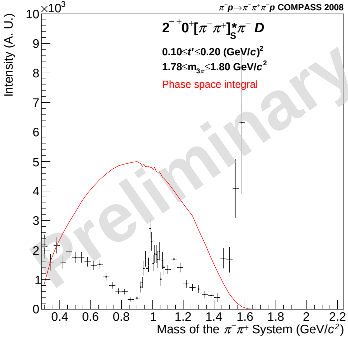


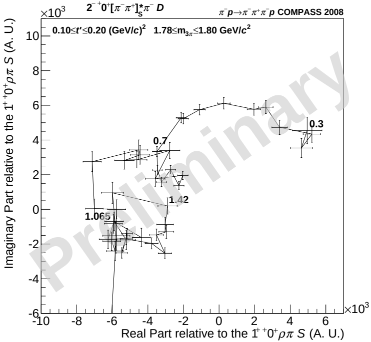


$1^{++} \to \pi(\pi\pi)_S$ wave above $a_1(1420)$

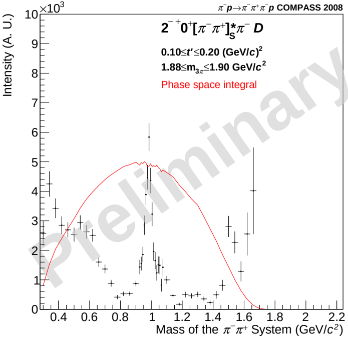


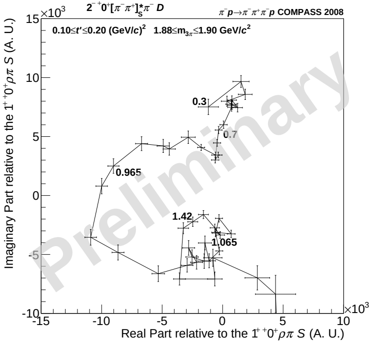
$2^{-+} \to \pi(\pi\pi)_S$ wave below $\pi_2(1880)$



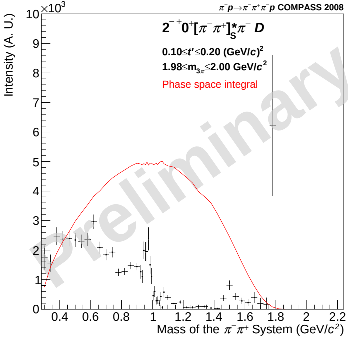


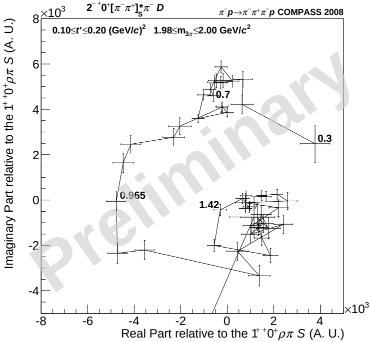
$2^{-+} \to \pi(\pi\pi)_S$ wave at $\pi_2(1880)$



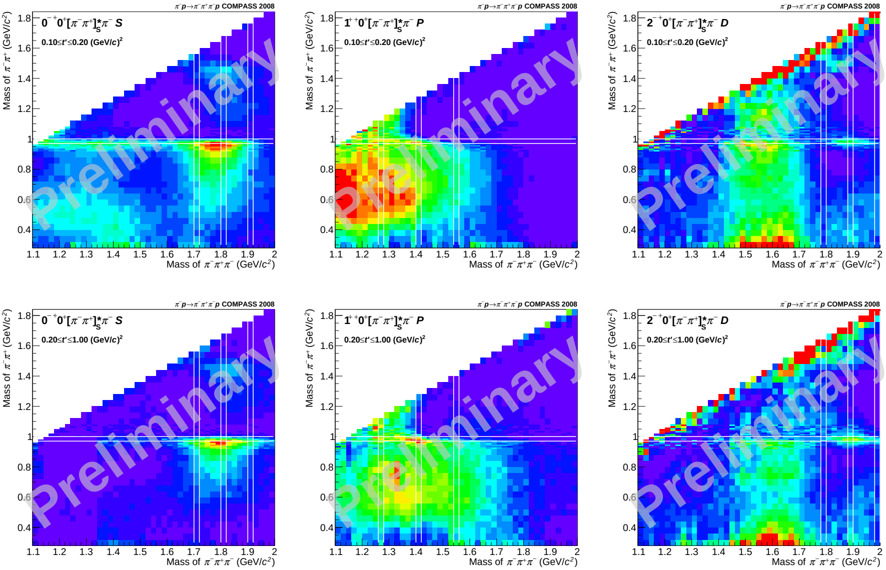


$2^{-+} \to \pi(\pi\pi)_S$ wave above $\pi_2(1880)$





$m(2\pi)$ vs $m(3\pi)$ for $(pipi)_S$ in $0^{-+}, 1^{++}$ and 2^{-+}



Summary

- mass-independent PWA in 20- MeV $m(3\pi)$ and 11 different increasing t' bins
  - various partial wave intensities have different peak behavior over t'
  - most relative phases, in contrast, do not change with t'
- mass-dependent fits using sub-density matrix in number of $m(3\pi)$ and t' bins
  - using 1-2 resonant terms and coherent background
  - complex couplings for each resonant or background term are free parameters in each t'-slice ("t'-independent analysis")
  - provide rather stable parameters of dominating resonance in each partial wave
  - need Breit-Wigner amplitude for new state $a_1(1420)$ having M = 1412-1422 MeV/c^2, Γ=130-150 MeV/c^2
- model-independent parametrization of selected 2π isobars
  - 3 independent $0^{-+}, 1^{++}0^{+}$ and $2^{-+}0^{+} \to (\pi\pi)_S\pi$ amplitudes
  - illustrates in at most model independent way decay modes $\pi(1800)$, $a_1(1420)$ and $\pi_2(1880)$ to $f_0(980)\pi$

Outlook

- mass-independent PWA
  - further study effects of modification of isobaric states
  - try some new isobars (example: $2^{-+} \to f_2(1565)\pi$)
- mass-dependent analysis
  - using different sets of amplitudes, increasing their number – in progress
  - formulate more advanced χ^2 taking into account covariance matrices of sub-density matrix elements - in progress
  - study of local minima of χ^2 and way of rejection of unphysical solutions - in progress
  - model both m(3pi) and t' dependence by smooth functions (true 2d analysis $\to$ reduction of number of parameters)
- model-independent parametrization of selected 2π isobars
  - try $(\pi\pi)_P$ and $(\pi\pi)_D$ as free isobars - in progress
  - feed back models for isobaric amplitudes into mass-independent PWA
  - for previous item - de-isobar further amplitudes ("bootstraping")