Light tetraquark bound-states

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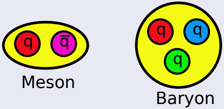
QCD boundstates

Guiding principles

- Strong coupling
    - Non-perturbative
- Confining theory
    - Single quarks cannot be observed
- Colorless observables
    - “Classical” objects

“Classical“ singlet states

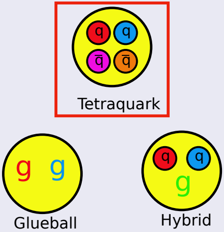


QCD boundstates

Guiding principles

- Strong coupling
  - Non-perturbative
- Confining theory
  - Single quarks cannot be observed
- Colorless observables
  - “Exotic” objects

”Exotic” singlet states



Tetraquarks

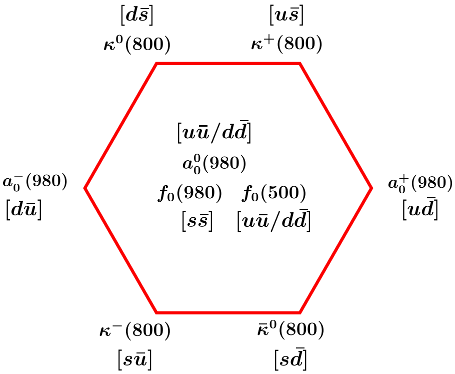
Reasons to investigate tetraquarks in general

- There is no reason from QCD why they should **not** exist.
- Increasing evidence from experiments eg. $Z_c(3900, 4020, \ldots)$, $X(3872)$ ....
- They are part of the spectrum in a variety of theoretical frameworks. lattice, sum rules, constituent models. . .

Reasons to investigate light scalar tetraquarks

- From a simple quark model point of view, the "better" candidate for the lightest scalar nonet. Jaffe(1965)
- The $1/N_c$ behavior in unitarized ChPT hints to a significant non-$q\bar{q}$ component for σ, κ, a_0. Pelaez (2004)
- Linear σ-model favors $qq\bar{q}\bar{q}$ for light scalars. Rischke et. al (2012)

Scalars and tetraquarks - a simple quark picture argument



$[d\bar{s}]$ $[u\bar{s}]$

$\kappa^0(800)$ $\kappa^+(800)$

$[u\bar{u}/d\bar{d}]$

$a_0^0(980)$

$a_0^-(980)$ $[d\bar{u}]$

$a_0^+(980)$ $[u\bar{d}]$

$f_0(980)$ $f_0(500)$

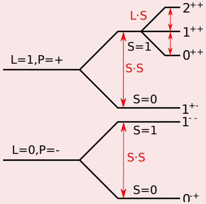
$[s\bar{s}]$ $[u\bar{u}/d\bar{d}]$

$\kappa^-(800)$ $\bar{\kappa}^0(800)$

$[s\bar{u}]$ $[s\bar{d}]$

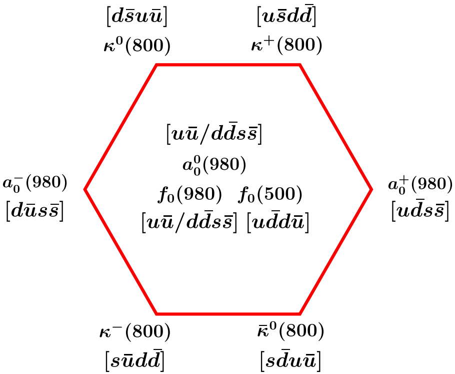
Meson nonet

- Wrong mass order in the nonet.
- 0^{++} : P-Wave.



- Decay channels.
- Width of f_0 vs. OZI-rule.

Scalars and tetraquarks - a simple quark picture argument



$[d\bar{s}u\bar{u}]$ $\kappa^0(800)$

$[u\bar{s}d\bar{d}]$ $\kappa^+(800)$

$[u\bar{u}/d\bar{d}s\bar{s}]$

$a_0^0(980)$

$f_0(980)$ $f_0(500)$

$[u\bar{u}/d\bar{d}s\bar{s}]$ $[u\bar{d}d\bar{u}]$

$a_0^-(980)$ $[d\bar{u}s\bar{s}]$

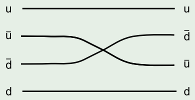
$a_0^+(980)$ $[u\bar{d}s\bar{s}]$

$\kappa^-(800)$ $[s\bar{u}d\bar{d}]$

$\bar{\kappa}^0(800)$ $[s\bar{d}u\bar{u}]$

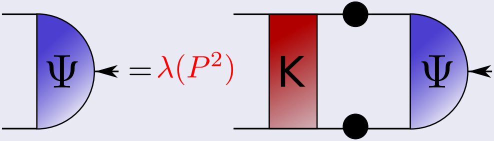
Tetraquark nonet

- Right mass ordering.
- 0^{++} : S-wave.
- Decay channels.
- Width of f_0 originates from the “gluon-less” decay.



Bound-state equations in QFT

Bethe-Salpeter equation

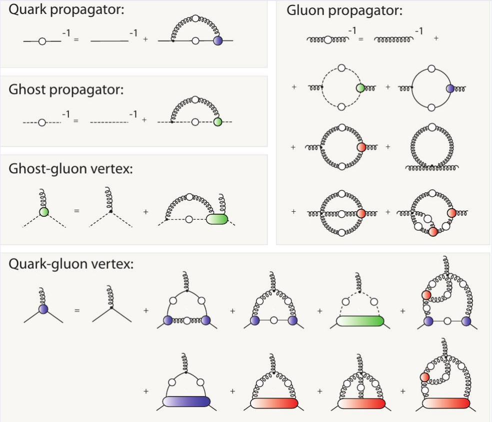


Features and ingredients

- Selfconsistent eigenvalue problem.
- Requires dressed propagators and suitable interaction.
- Determines mass **and** wavefunction.
- Fully covariant formulation.

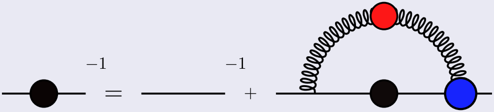
The model

DSE tower



The model

Full quark DSE



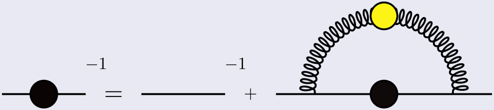
$$S(p) = \frac{1}{A(p^2)} \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

Kernel

$$K(p,q) = \frac{\delta\Sigma(p)}{\delta S(q)}$$

The model

Truncated DSE



Truncation scheme

- Effective gluon. Maris-Tandy(1997)
- Fixed to $f_\pi = 131$ MeV and $m_\pi = 138$ MeV.

Rainbow ladder - Motivation

Successfully applied to:

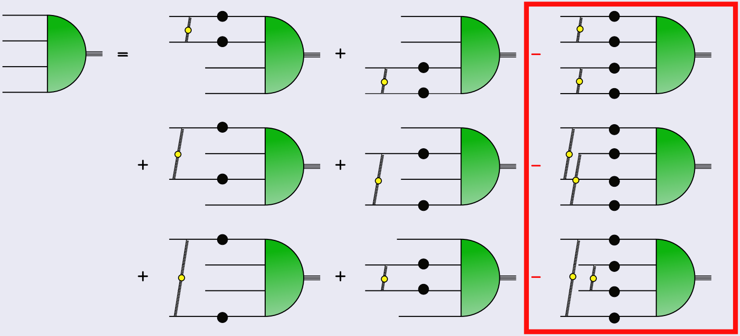
- Light meson spectrum. Fischer et. al (2014), . . .
- Charmonium and Bottomonium spectrum. Blank, Krassnigg (2011), . . .
- Baryon octet and decuplet masses. Sanchis-Alepuz, Fischer (2014), . . .
- EM formfactors of mesons and baryons.
- EM transition form factors. Maris, Tandy (2002)
- Hadronic LbL-scattering. Williams, Goecke, Fischer (2012)
- Hadronic decays. Mader et. al (2011)
- Nucleon compton scattering. Eichmann, Fischer (2013)
- . . .

Justifies

Investigate tetraquarks in the same framework.

Faddeev-Yakubowski equation

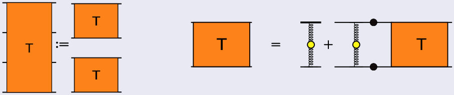
Four-body problem - Quark picture



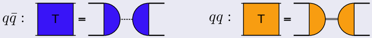
- Neglect (for now) three- and four-body interactions.
- Keep pair interaction. Treat overcounting properly.
- 512 wave functions, depend on 9 variables.

Reduction to a two-body equation

Seperable ansatz and scattering equation:



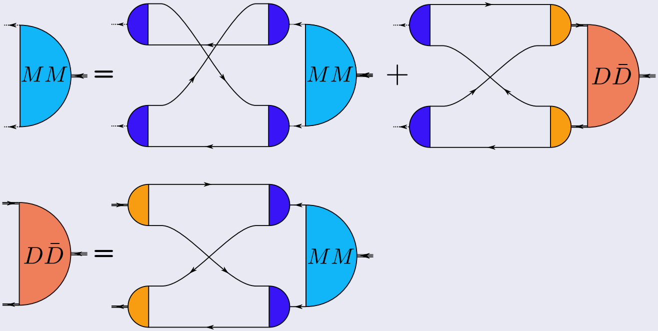
Ansatz for the two-body T-matrix:



- Use a seperable ansatz for the four-body T-Matrix.
- Utilize scattering equation to reformulate interaction via the T-matrix.
- Employ a pole dominance approximation for the T-matrices.
- Use an offshell ansatz for the factorized T-matrices.

Boundstate equation in the two-body approach

Two-body problem - Meson/Meson-Diquark/Antidiquark picture



- Interaction via quark-exchange.
- 2 amplitudes, depend on 2 variables.

Four-body equation vs. two-body equation

Structure of the tetraquark in the two-body approach:

- Structures of the meson-meson amplitude:

$$\{\pi \otimes \pi, \rho \otimes \rho, \dots\}$$

- Structures of the diquark-antidiquark amplitude:

$$\{D_s \otimes \bar{D}_s, D_{AV} \otimes \bar{D}_{AV}, \dots\}$$

Advantages

- Numerically easier to tackle.
- Physical interpretation of amplitudes.

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Disadvantages

- Technical/numerical restrictions.
- Not all color singlet structures possible.
- Depends on pole approximation and offshell ansatz for the boundstates.
- Difficult to include three/four-body interactions.

Four-body equation vs. two-body equation

Structure of the tetraquark in the four-body approach:

- Structures of the in meson-meson basis:

$$\{\gamma_5 \otimes \gamma_5,\ \mathbb{1} \otimes \mathbb{1},\ \gamma^\mu \otimes \gamma^\mu, \gamma_5\gamma^\mu \otimes \gamma_5\gamma^\mu \ldots\}\{1 \otimes 1_S, 8 \otimes 8_S\}$$

- Structures in diquark-antidiquark basis:

$$\{\mathcal{C}\gamma_5 \otimes \gamma_5\mathcal{C}^T, \mathcal{C}^T\mathbb{1} \otimes \mathbb{1}\mathcal{C}^T, \mathcal{C}\gamma^\mu \otimes \gamma^\mu\mathcal{C}^T, \ldots\}\{3 \otimes \bar{3}_S, 6 \otimes \bar{6}_S\}$$

Advantages

- Basis are complete. Different basis connected via Fierz transformations.
- Consistent equation within the used framework.
- Room for improvements (three/four-body interactions).

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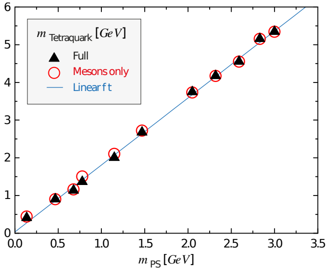
Disadvantages

- Numerically more demanding.
- Physical meaning of amplitudes not as clear as in the two-body approach.

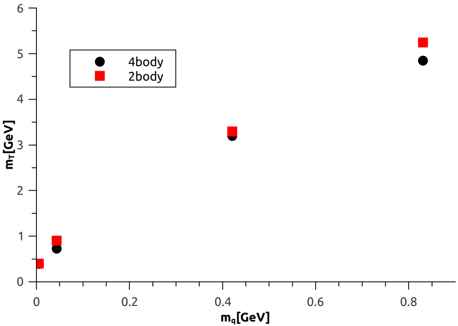
Results - Two-body equation



What can be learned from the two-body equation

- Bound 0^+ tetraquark state at $\approx$ 400 MeV.
- The Pion-Pion wavefunction is dominant.
- Tetraquark inherits quarkmass dependence of the pion.
- Possible narrow $cc\bar{c}\bar{c}$ state at 5.3 GeV??

Results - Four-body equation I



What can be learned from the four-body equation - Preliminary

- Bound 0^+ tetraquark state at $\approx$ 400 MeV.
- Overall agreement between both approaches.

Results - Four-body equation II

Particle	Mass [MeV]	Mass [MeV]
$f_0(500)$	400	$400 - 550$
κ	601	682 ± 29
$a_0/f_0(980)$	785	980 ± 20

What can be learned from the four-body equation - Preliminary

- Tetraquark states are in the right ballpark.
- Tendency to undershoot the mass. Missing three- and four-body interactions/mixing effects?

Conclusion and outlook

Conclusion

- The 0^+ tetraquark boundstate equation was derived and solved in the two-body and in the four-body approach.
- **Both** approaches give a mass of $\approx$ 400 MeV for the $f_0(500)$. The masses of a_0, κ agree qualitatively with values found in the literature.
- The $f_0(500)$ is dominated by the pion-pion contribution.

Conclusion and outlook

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- The $f_0(500)$ is dominated by the pion-pion contribution.

Outlook

- Solve for other quantum numbers.
- Solve the four-body equation with better numerics and the full structure.
- Include three-body interactions.

The end

Thank you for your attention!

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