Discriminating between two reformulations of SU(3) Yang-Mills theory on a lattice: Abelian monopole or non-Abelian monopole responsible for confinement

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Based on arXiv:1409.1599 and work in progress

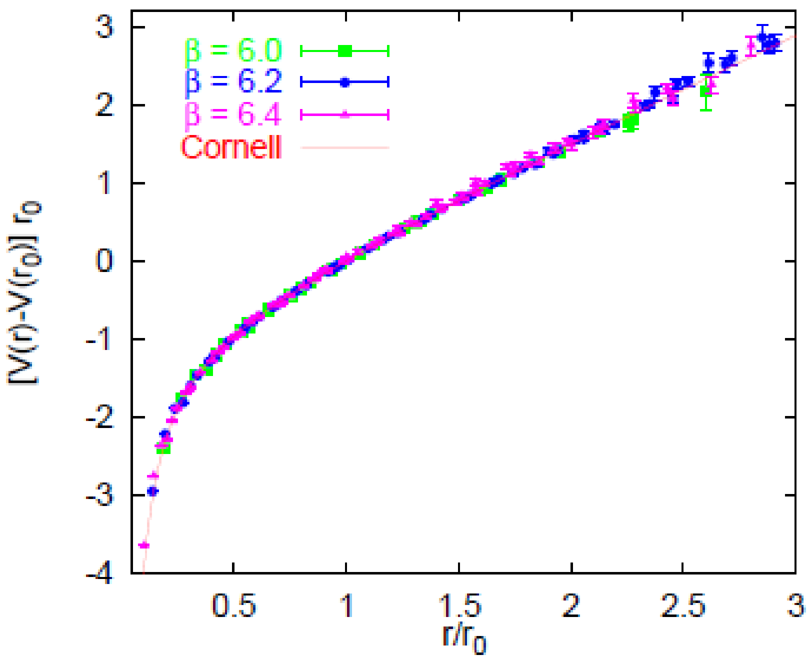
Introduction

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]
- What is the mechanism of the quark confinement?

➔dual super conductivity

Is the promising mechanism

Cf. center vortex in maximal center gauge. [Greensite]



G.S. Bali, [hep-ph/0001312], Phys. Rept. **343, 1–136 (2001)**

$$\text{Non-Abelian Wilson loop} \quad \left\langle \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]\right\rangle_{\mathrm{YM}}^{\text{no GF}} \sim e^{-\sigma_{NA}|S|},$$

dual superconductivity

- Dual superconductivity is a promising mechanism for quark confinement. [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]

superconductor

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

dual superconductor

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks

Evidences for the dual superconductivity::SU(2)

tring tension (Linear potential)

- ☐ Abelian dominance in the string tension [Suzuki & Yotsuyanagi, 1990]
- ☐ Abelian magnetic monopole dominance in the string tension [Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]

Chromo-flux tube (dual Meissner effect)

- ☐ Measurement of (Abelian) dual Meissner effect
- ◆ Observation of chromo-electric flux tubes and Magnetic current due to chromo-electric flux
- ◆ Type the super conductor is of order between Type I and Type II [Y.Matsubara, et.al. 1994]

- ✓ **only obtained in the case of special gauge such as MA gauge**
- ✓ **gauge fixing breaks the gauge symmetry as well as color symmetry**

Evidences for the dual superconductivity::SU(2)

Gauge decomposition method (a new lattice formulation)

- **Extracting the relevant mode *V* for quark confinement** by solving the defining equation in the gauge independent way (gauge-invariant way)

➢ For SU(2) case, the decomposition is a lattice compact representation of the Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.

➔ **we have showed in the series of lattice conferences that**

➢ V-field dominance, magnetic monopole dominance in string tension

➢ chromo-electric flux tube and dual Meissner effect.

Dual Superconductivity for SU(3) case

- **Abelian projection**
  - Abelian projection :: SU(3) ➔ U(1)xU(1) (Maximal torus)

  In the MA gauge [Suganuma et.al.]
  - The problem remains as the same as SU(2) case.
- Decomposition method (A new formulation of the lattice YM theory ➔ see the next slide)
  - Extension of SU(2) case

➔ **we have showed in the series of lattice conferences that**

- V-field dominance, magnetic monopole dominance in string tension,
- chromo-flux tube and dual Meissner effect.
- The first observation on quark confinement/deconfinement phase transition in terms of dual Meissner effect at finite temperature

A new formulation of Yang-Mills theory (on a lattice)

Decomposition of SU(N) gauge links

- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

□ SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$

□ SU(3) Yang-Mills link variables: **Two options**

maximal option : $U(1) \times U(1) \subset SU(3)$

✓ Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓ Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

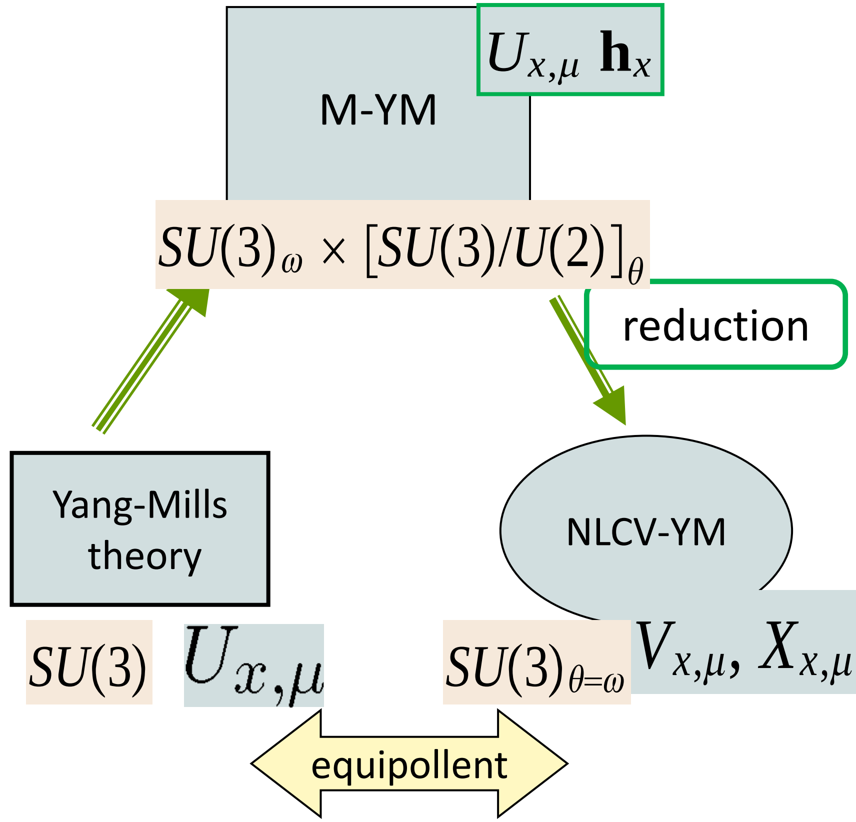
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_x$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Defining equation for the decomposition

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger(\det\hat{L}_{x,\mu})^{1/N} g_x^{-1} \quad V_{x,\mu} = X_{x,\mu}^\dagger U_{x,} = g_x\hat{L}_{x,\mu}U_{x,}(\det\hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L_{x,\mu}^\dagger}\right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}$$

continuum version by continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

The defining equation and the Wilson loop for the fundamental representation

By inserting the complete set of the coherent state $|\xi_x, \Lambda\rangle$ at every site on the Wislon loop C, $1 = \int |\xi_x, \Lambda\rangle d\mu(\xi_x) \langle \Lambda, \xi_x|$ we obtain

$$W_C[U] = \mathrm{tr}\left(\prod_{<x> \in C} U_{x,\mu} \right) = \prod_{<x,x+\mu> \in C} \int d\mu(\xi_x) \langle \Lambda, \xi_x | U_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle$$

$$= \prod_{<x,x+\mu> \in C} \int d\mu(\xi_x) \langle \Lambda, |(\xi_x^\dagger X_{x,\mu} \xi_x)(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu})|, \Lambda \rangle$$

where we have used $\xi_x \xi_x^\dagger = 1$.

For the stability group of $\tilde{H}$, the 1st defining equation

$$\xi V_{x,\mu} \xi^\dagger \in \tilde{H} \Leftrightarrow [\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}, \tilde{H}] \Leftrightarrow \mathbf{h}_x V_{x,\mu} - V_{x,\mu} \mathbf{h}_{x+\mu} = 0$$

implies that $|\Lambda\rangle$ is eigenstate of $\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}$:

$$(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}) |\Lambda\rangle = |\Lambda\rangle e^{i\phi}, \quad e^{i\phi} := \langle \Lambda | \xi_x^\dagger V_{x,\mu} \xi_{x+\mu} | \Lambda \rangle = \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle.$$

Then we have

$$W_C[U] = \int d\mu(\xi_x) \rho[X;\xi] \prod_{<x,x+\mu> \in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle$$

$$\rho[X;\xi] := \prod_{<x> \in C} \langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle$$

The defining equation and the Wilson loop for the fundamental representation (2)

By using the expansion of $X_{x,\mu}$: the 2nd defining equaiton, $\mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0$, derives

$$\langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = \mathrm{tr}(X_{x,\mu})/\mathrm{tr}(\mathbf{1}) + 2\mathrm{tr}(X_{x,\mu}\mathbf{h}_x)$$

$$= 1 + 2ig\epsilon tr(\mathcal{X}_\mu(x)\mathbf{h}(x)) + O(\epsilon^2).$$

Then we have $\rho[X;\xi] = 1 + O(\epsilon^2)$.

Therefore, we obtain

$$W_c[U] = \int d\mu(\xi_x) \prod_{<x,x+\mu>\in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = W_C[V]$$

Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ describing the original Yang-Mills theory and color fields.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory
- The configuration of the color fields $\mathbf{h}_x$ can be determined by the reduction condition such that the reduction functional is minimized for given $U_{x,\mu}$ to obtain equipollent theory to the original Yang-Mills theory $SU(3)_\omega \times [SU(3)/U(2)]_\theta \to SU(3)_{\omega=\theta}$

Determining $\mathbf{h}_x$ to minimize the reduction function for given $U_{x,\mu}$

$$F_{\mathrm{red}}[\mathbf{h}_x, U_{x,\mu}] = \sum_{x,\mu} \mathrm{tr}\left\{ (D_\mu^\epsilon[U_{x,\mu}]\mathbf{h}_x)^\dagger (D_\mu^\epsilon[U_{x,\mu}]\mathbf{h}_x) \right\}$$

- ■ This is invariant under the gauge transformation θ=ω
- ■ The extended gauge symmetry is reduced to the same symmetry as the original YM theory.
- ■ We choose a reduction condition of the same type as the SU(2) case

Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection

$$W_C[\mathcal{A}] = \int [d\mu(\xi)]_\Sigma \exp\left(-ig \int_{S:C=\partial\Sigma} dS^{\mu\nu} \sqrt{\frac{N-1}{2N}} \mathrm{tr}(2\mathbf{h}(x)\mathcal{F}_{\mu\nu}[\mathcal{V}](x)) \right)$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left(ig \sqrt{\frac{N-1}{2N}} (k, \Xi_\Sigma) + ig \sqrt{\frac{N-1}{2N}} (j, N_\Sigma) \right)$$

magnetic current $k := \delta^* F = {}^* dF$, $\quad \Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1}$

electric current $j := \delta F$, $\quad N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$

$\Delta = d\delta + \delta d$, $\quad \Theta_\Sigma := \int_\Sigma d^2 S^{\mu\nu}(\sigma(x)) \delta^D(x - x(\sigma))$

k and j are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

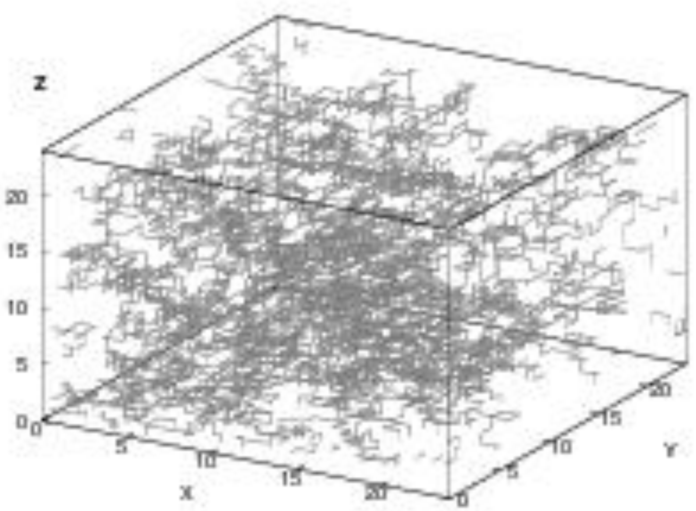
K.-I. Kondo PRD77 085929(2008)

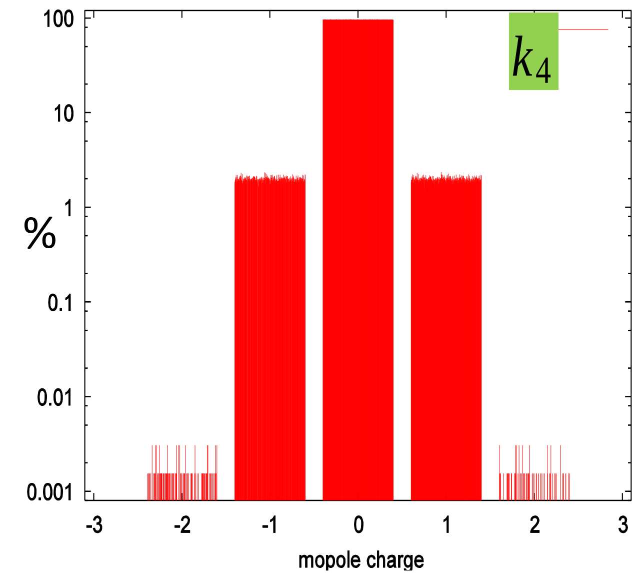
The lattice version is defined by using plaquette:

$$\Theta^8_{\mu\nu} := -\arg \mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x \right) V_{x,\mu} V_{x+\mu,\mu} V^\dagger_{x+\nu,\mu} V^\dagger_{x,\nu} \right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu \Theta^8_{\alpha\beta},$$

non-Abelian monopoles in vacuum





$$\Theta^8_{\mu\nu} := -\arg \operatorname{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x\right)V_{x,\mu}V_{x+\mu,\mu}V^\dagger_{x+\nu,\mu}V^\dagger_{x,\nu}\right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu\Theta^8_{\alpha\beta},$$

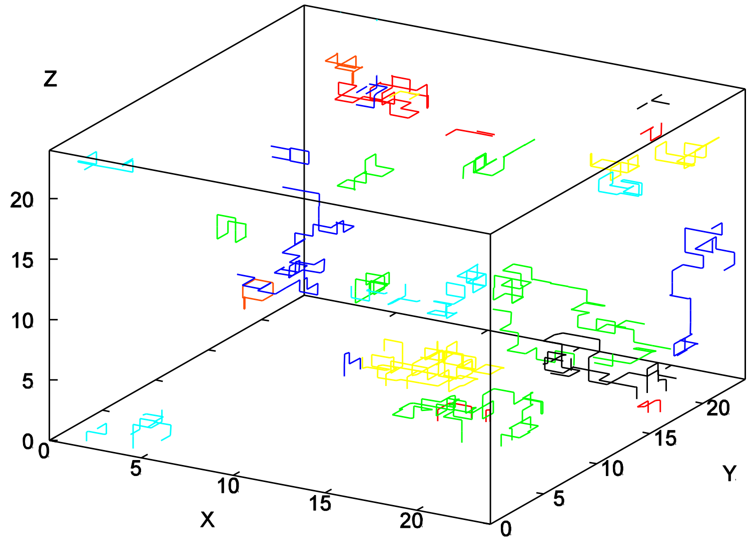
•The distribution of the monopole charges for 16^4 lattice β=5.7 400 configurations. The distribution of each configuration is shown by thin bar chart.

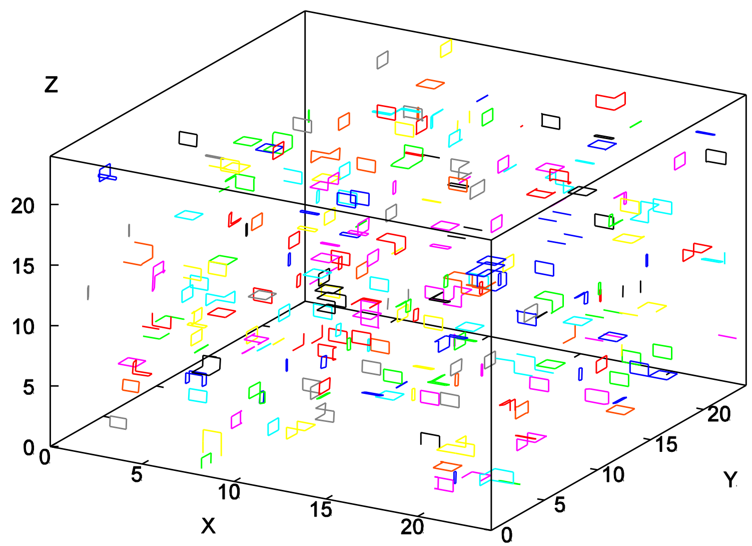
Non-Abelian magnetic monopole loops: 24^3 x8 lattice b=6.0 (T≠0)

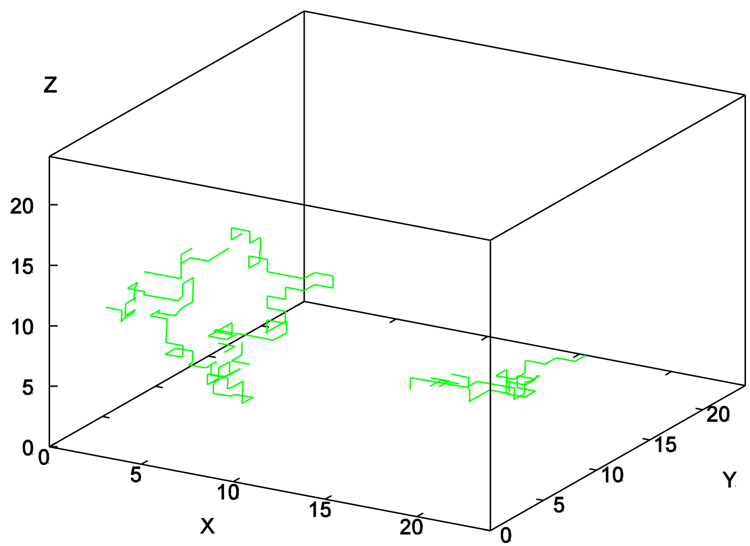
Projected view (x,y,z,t) ➔(x,y,z)

(left lower) loop length 1-10
(right upper) loop length 10 -- 100
(right lower) loop length 100 -- 1000

Monopole loop is winding to T direction.







■ SU(3) Yang-Mills theory

- In confinement of fundamental quarks, a restricted non-Abelian variable ***V*** **,** and the extracted non-Abelian magnetic monopoles play the dominant role (dominance in the string tension), in marked contrast to the Abelian projection.

gauge independent "Abelian" dominance

$$\frac{\sigma_V}{\sigma_U} = 0.92$$

$$\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82$$

Gauge independent non-Abalian monople dominance

$$\frac{\sigma_M}{\sigma_U} = 0.85$$

$$\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76$$

U* is from the table in R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. B517, 377 (1998).

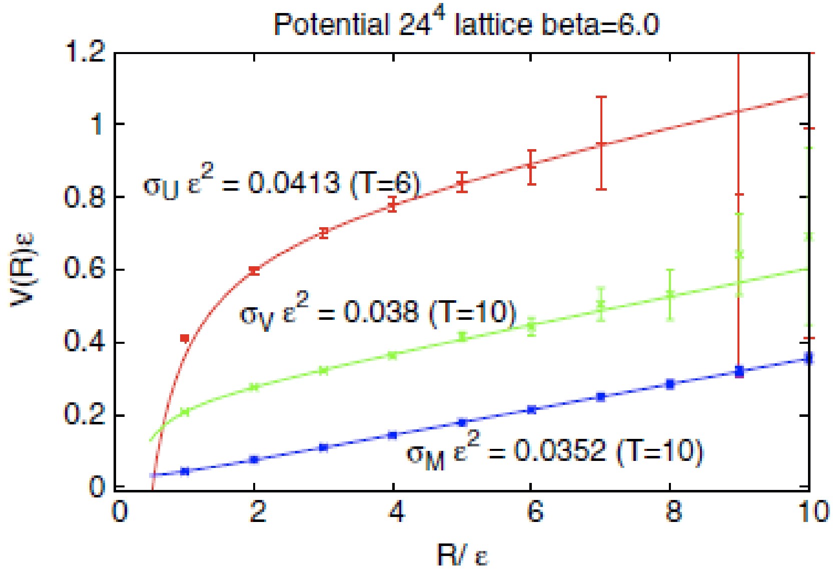


FIG. 1 (color online). $SU(3)$ quark-antiquark potentials as functions of the quark-antiquark distance R: (from tob to bottom) (i) full potential $V_f(R)$ (red curve), (ii) restricted part $V_r(R)$ (green curve), and (iii) ma;gnetic-monopole part $V_m(R)$ (blue curve), measured at $\beta = 6.0$ on 24^4 using 500 configurations where ϵ is the lattice spacing.

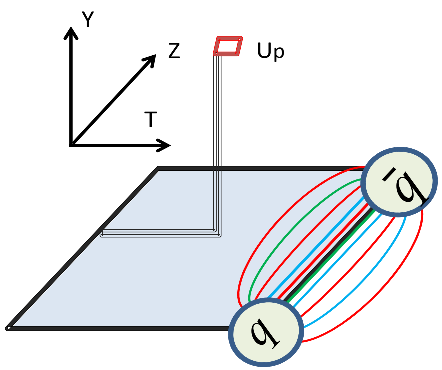
PRD 83, 114016 (2011)

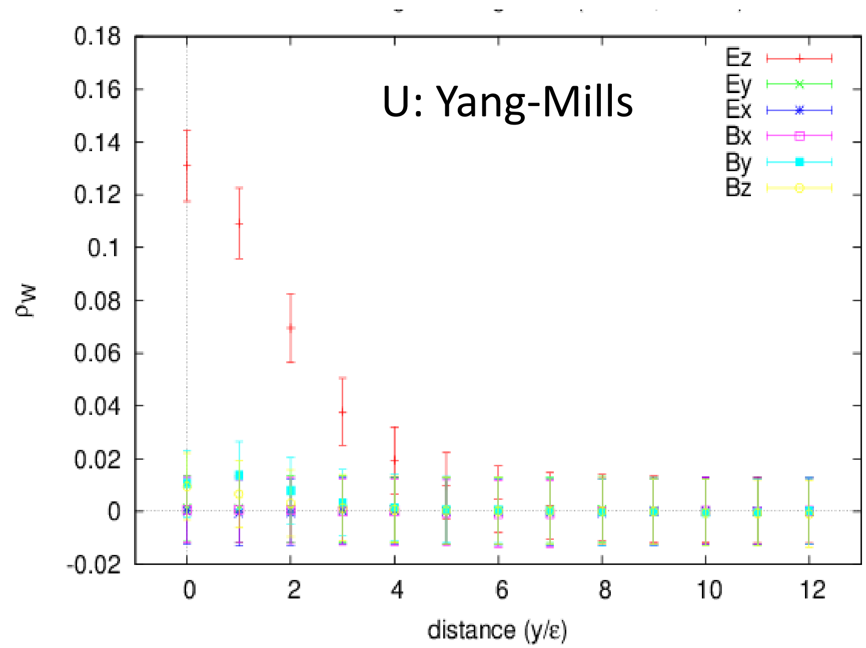
Chromo flux

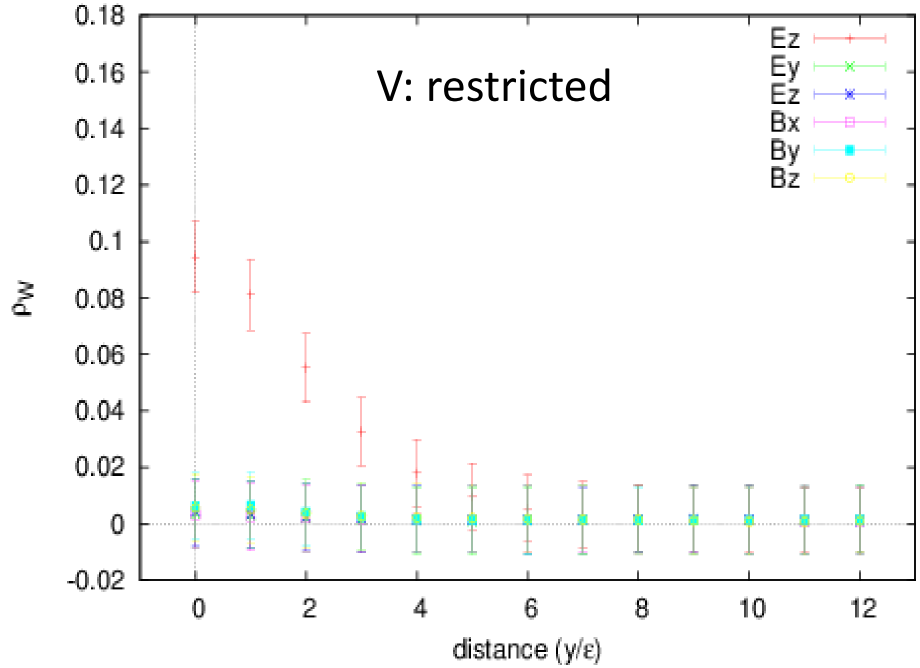
$$\rho_W = \frac{\langle \mathrm{tr}(WLU_pL^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

Gauge invariant correlation function: This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3) [Adriano Di Giacomo et.al. PLB236:199,1990 NPBB347:441-460,1990

$\mathrm{tr}(U_pLWL^\dagger)$

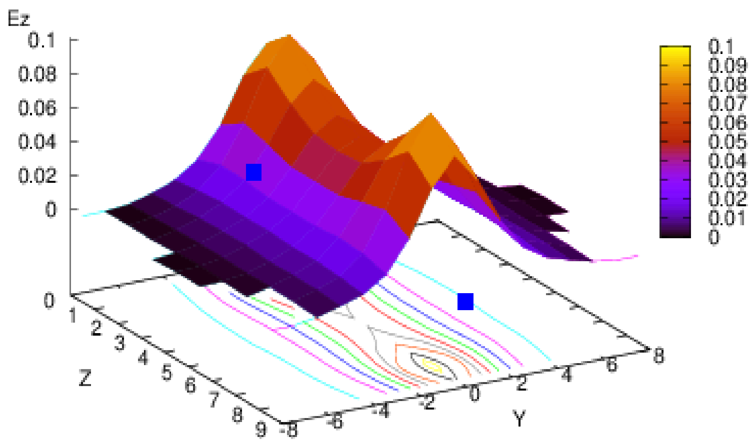




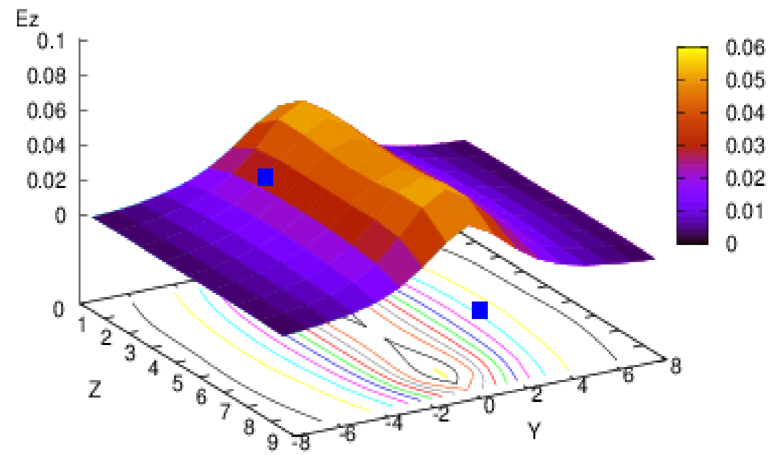


Chromo-electric (color flux) Flux Tube

Original YM filed



Restricted field



A pair of quark-antiquark is placed on z axis as the 9x9 Wilson loop in Z-T plane. Distribution of the chromo-electronic flux field created by a pair of quark-antiquark is measured in the Y-Z plane, and the magnitude is plotted both 3-dimensional and the contour in the Y-Z plane.

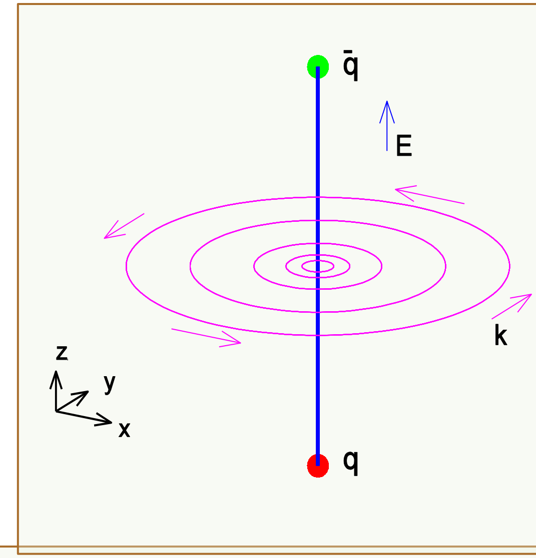
Flux tube is observed for V-field case. :: dual Meissner effect

Magnetic current induced by quark and antiquark pair

Yang–Mills equation (Maxell equation) fo rrestricted field V_μ, the magnetic current (monopole) can be calculated as

$$k = \delta^{*}F[V] = {}^{*}dF[V],$$

where $F[V]$ is the field strength of V, d exterior derivative, $*$ the Hodge dual and δ the coderivative $\delta := {}^{*}d^{*}$, respectively.



$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$, and $\mathbf{k} = {}^{*}dF[\mathbf{V}] = {}^{*}ddF[\mathbf{V}] = 0$
(Bianchi identity)

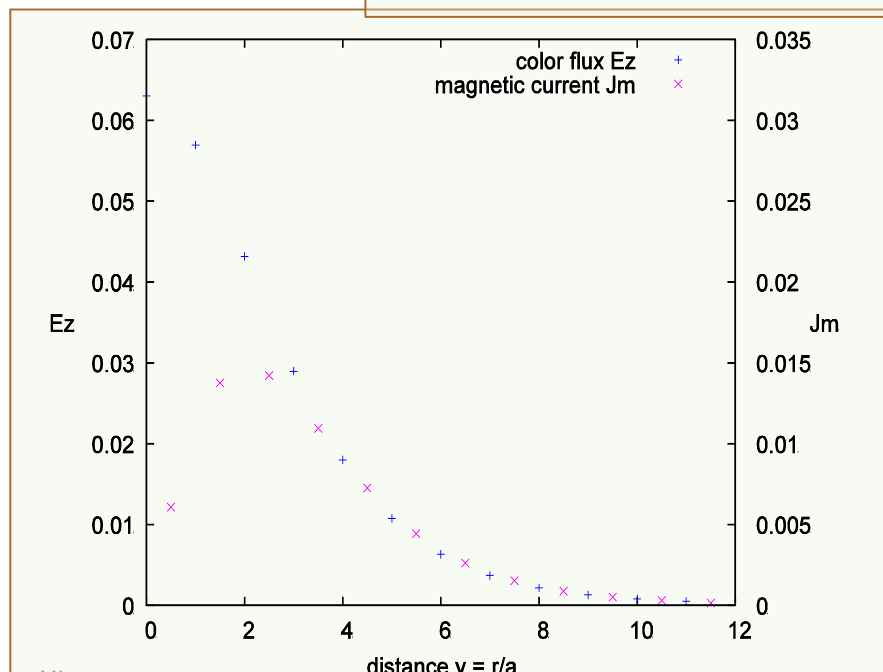


Figure: (upper) positional relationship of chromo-electric flux and magnetic current. (lower) combination plot of chromo-electric flux (left scale) and magnetic current(right scale).

COMPARISON WITH MAXIMAL OPTION

The decomposition of SU(3) link variable: maximal option

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

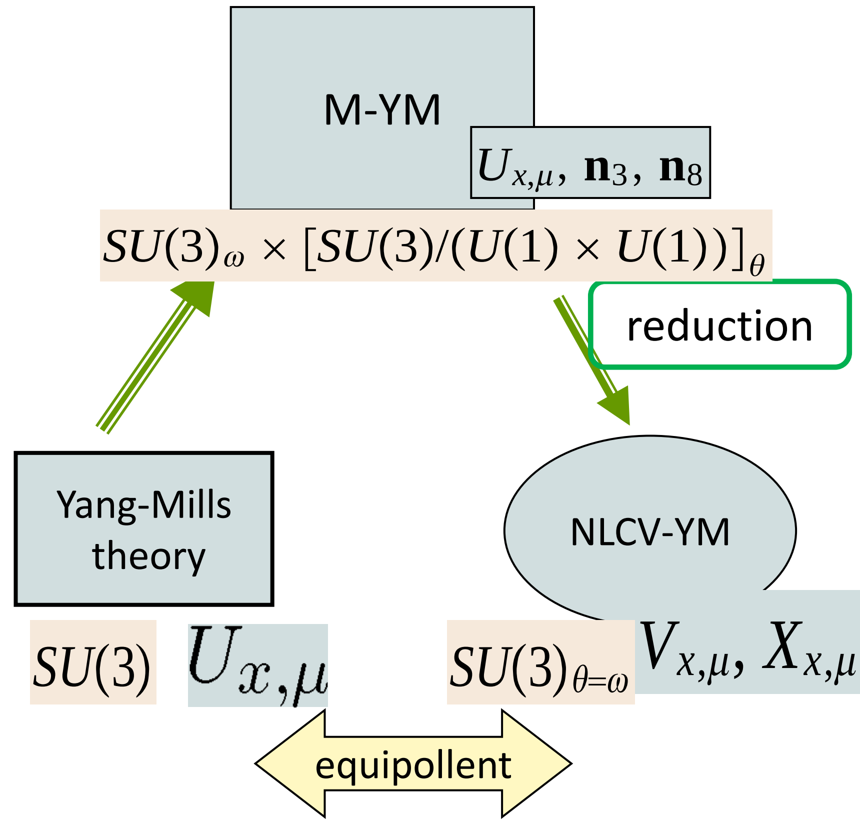
$$U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_{x}$$

$$\Omega_x \in G = SU(N)$$

$$\mathbf{n}^{(j)}_x \rightarrow \mathbf{n}^{(j)\prime}_x = \Theta_x \mathbf{n}^{(j)}_x \Theta^\dagger_x \quad j = 3, 8$$



Gauge invariant version of the Abelian projection to maximal torus group U(1) x U(1) in MA gauge.

Defining equation for the decomposition:: maximal option

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

By introducing the color field $\mathbf{n}_3 = \xi(\lambda^3/2)\xi^\dagger$, $\mathbf{n}_8 = \xi(\lambda^8/2)\xi^\dagger \in SU(3)_\omega \times [SU(3)/(U(1)\times U(1))]_\theta$ the set of the defineing equation for the decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$:

$$D^\epsilon_\mu[V] := \frac{1}{\epsilon}\left(U_{x,\mu}\mathbf{n}^{(j)}_{x+\mu} - \mathbf{n}^{(j)}_x U_{x,\mu}\right) = 0, \quad (j = 3, 8)$$

$$g_x := \exp(2\pi i n/N)\exp\left\{i\sum_{j=3,8}\alpha^{(j)}\mathbf{n}^{(j)}_x\right\} = \mathbf{1}$$

with corresponding to the continuum version of decomposition $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$

$$D_\mu[\mathcal{V}]\mathbf{n}^{(j)}_x = 0, \quad \mathrm{tr}(\mathbf{n}^{(j)}_x\mathcal{X}_\mu(x)) = 0, \quad j = 3, 8$$

$$X_{x,\mu} = \hat{K}^\dagger_{x,\mu}\det(K_{x,\mu})^{1/3}g_x^{-1}, \quad V_{x,\mu} = g_x\hat{K}_{x,\mu}\det(K_{x,\mu})^{-1/3}$$

where

$$\hat{K}_{x,\mu} := \left(\sqrt{K_{x,\mu}K^\dagger_{x,\mu}}\right)^{-1}K_{x,\mu}, \quad \hat{K}^\dagger_{x,\mu} := K^\dagger_{x,\mu}\left(\sqrt{K_{x,\mu}K^\dagger_{x,\mu}}\right)^{-1}$$

$$K_{x,\mu} = \mathbf{1} + 6(\mathbf{n}^{(3)}_x U_{x,\mu}\mathbf{n}^{(3)}_x + \mathbf{n}^{(8)}_x U_{x,\mu}\mathbf{n}^{(8)}_x)$$

Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ describing the original Yang-Mills theory and color fields.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory
- The configuration of the color fields $\mathbf{n}^{(j)}{}_x$ can be determined by the reduction condition such that the reduction functional is minimized for given $U_{x,\mu}$ $U_{x,\mu}$ to obtain equipollent theory to the original Yang-Mills theory

$$SU(3)_{\omega} \times [SU(3)/(U(1)\times U(1))]_{\theta} \rightarrow SU(3)_{\omega=\theta}$$

$$F_{\mathbf{red}}[\mathbf{n}_x^{(3)}, \mathbf{n}_x^{(8)}, V] = \sum_{x,\mu} \left(D_\mu[U]\mathbf{n}_x^{(3)}\right)^{\dagger} \left(D_\mu[U]\mathbf{n}_x^{(3)}\right) + \sum_{x,\mu} \left(D_\mu[U]\mathbf{n}_x^{(8)}\right)^{\dagger} \left(D_\mu[U]\mathbf{n}_x^{(8)}\right)$$

- This is invariant under the gauge transformation θ=ω
- The extended gauge symmetry is reduced to the same symmetry as the original YM theory.

By using the gauge transformation, Θ $(= \Omega)$,

$$\mathbf{n}_x^{(3)} \rightarrow T_3 = \boldsymbol{\lambda}^3/2, \quad \mathbf{n}_x^{(8)} \rightarrow \Theta_x T_8 \Theta_x^{\dagger} = T_8 = \boldsymbol{\lambda}^8/2$$

$$U_{x,\mu} \rightarrow U_{x,\mu}^{\Omega} := \Theta_x U_{x,\mu} \Theta_x^{\dagger}$$

we obtain the functional for MA gauge fixing.

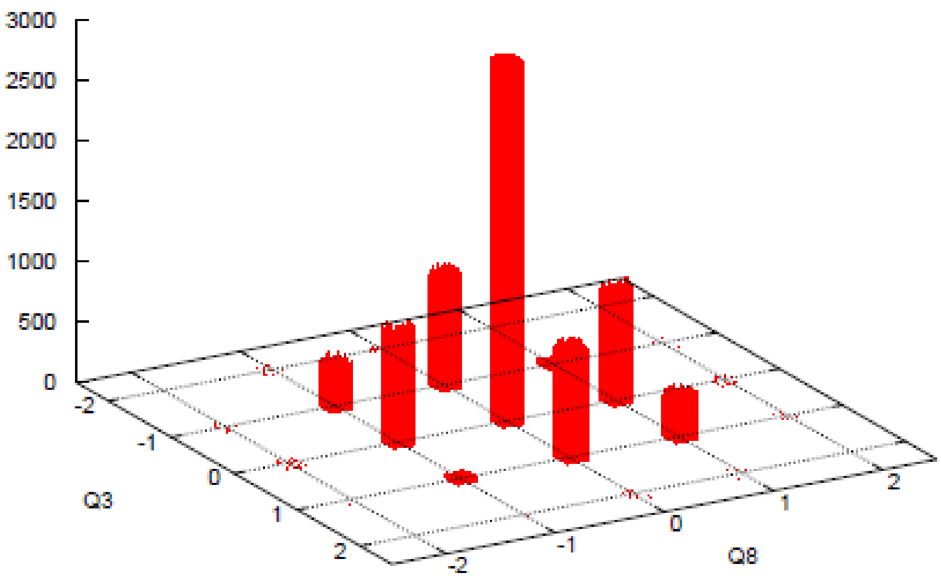
Magnetic monopole charge in the maximal option

$$Tr(V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\mu,\nu}V^{\dagger}_{x,\nu}) = \exp(i\mathcal{F}^{(3)}_{\mu\nu}n + i\mathcal{F}^{(8)}_{\mu\nu}m)$$

$$\Theta^{(3)}_{\mu\nu} = Tr\left(\left(\frac{1}{3} + n + \frac{1}{\sqrt{3}}m\right)V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\mu,\nu}V^{\dagger}_{x,\nu}\right)$$

$$\Theta^{(8)}_{\mu\nu} = Tr\left(\left(\frac{1}{3} - \frac{2}{\sqrt{3}}m\right)V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\mu,\nu}V^{\dagger}_{x,\nu}\right)$$

$$\kappa^{(k)}_{\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_{\nu}\Theta^{(k)}_{\alpha\beta}$$



What relation between the Wilson loop and the magnetic monopole in the maximal option?

Comparison of the Abelian dominance between minimal option and maximal option in a lattice data

- Wilson action with $\beta=6.0$ on the 24^4 lattice

1. 500 configurations by using pseudo heat bath.
2. Determine the color fields $\{\mathbf{h}(x)\}$ for the minimal option, $\{\mathbf{n}^{(3)}(x)\ \ \mathbf{n}^{(8)}(x)\}$ for maximal option by minimizing the reduction condition: $F[\mathbf{h}; U]$, $F[\mathbf{n}^{(3)}, \mathbf{n}^{(8)}; U]$, respectively.
3. The decomposition U=XV is obtained by using the formula by the defining equation for the decomposition.

Preliminary result

Static potential

- Wilson loop by the decomposed variable V

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \log\langle W_{(R,T)}[V]\rangle$$

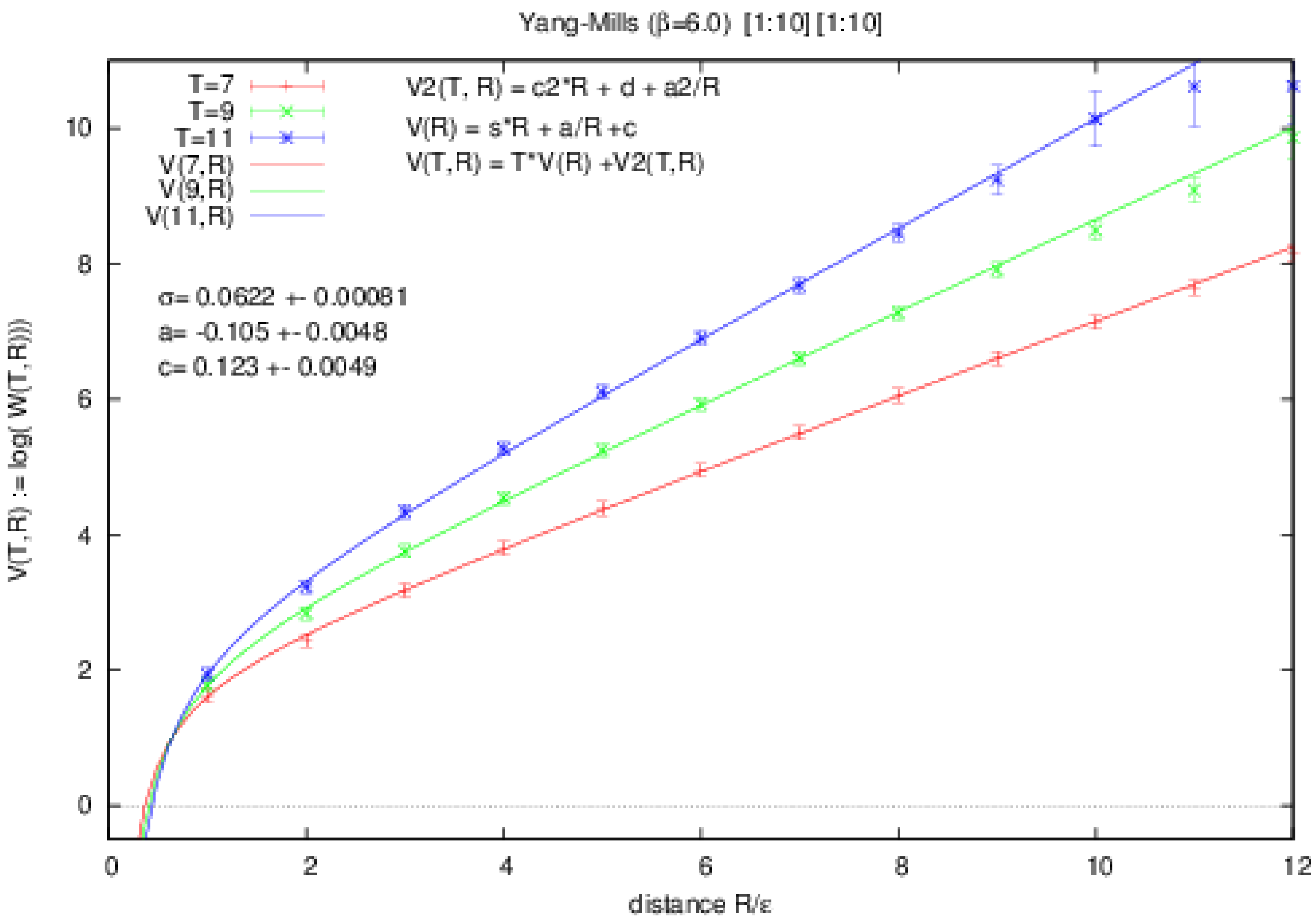
- To get the static potential

we fit the Wilson loop *$W_C[V]$* by the function *$V(R,T)$*

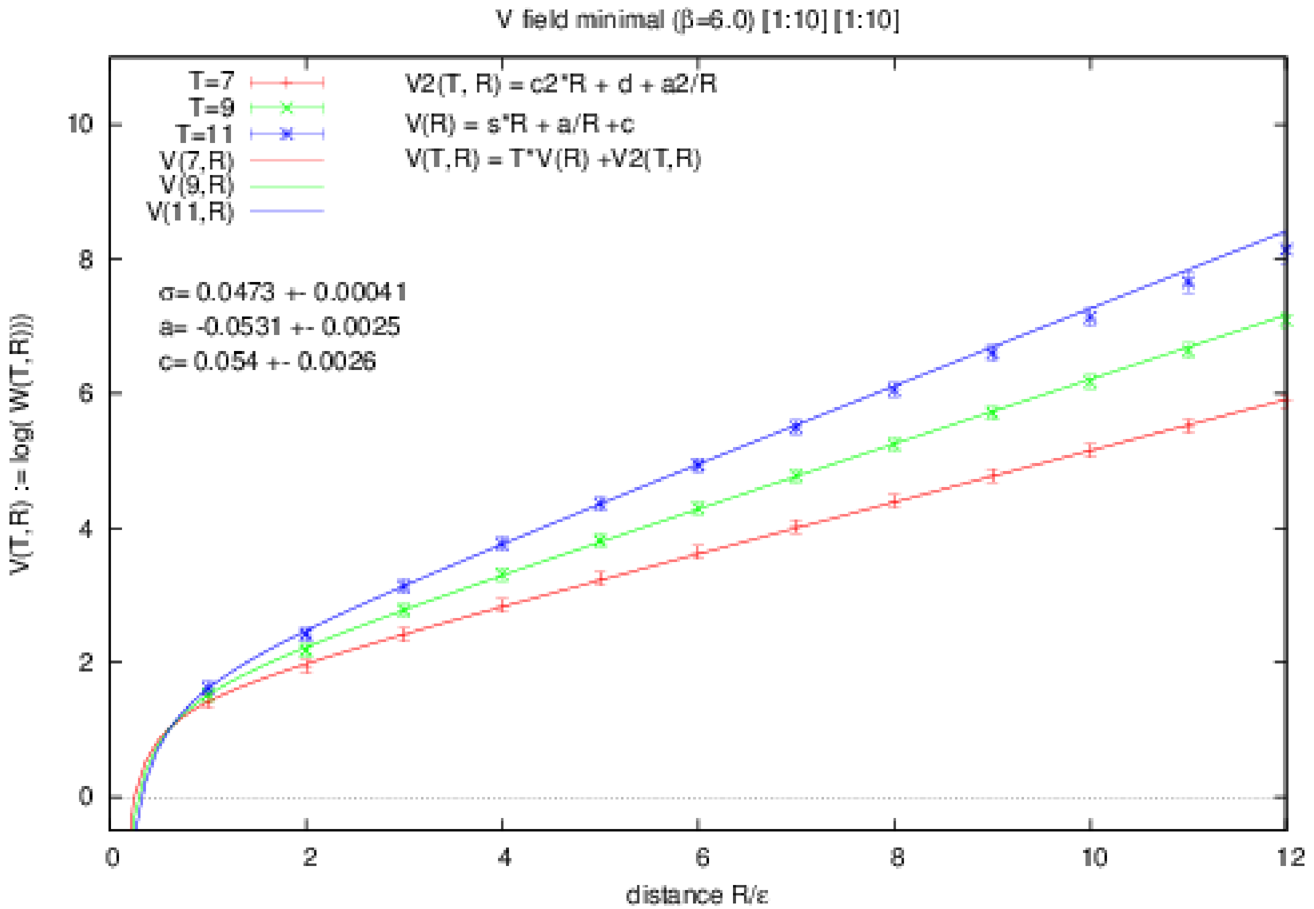
$$V(T,R) = -\log(\langle W[T,R]\rangle)$$

$$V(T,R) = sRT + cT + aT/R + c_2R + d + a_2R/T$$

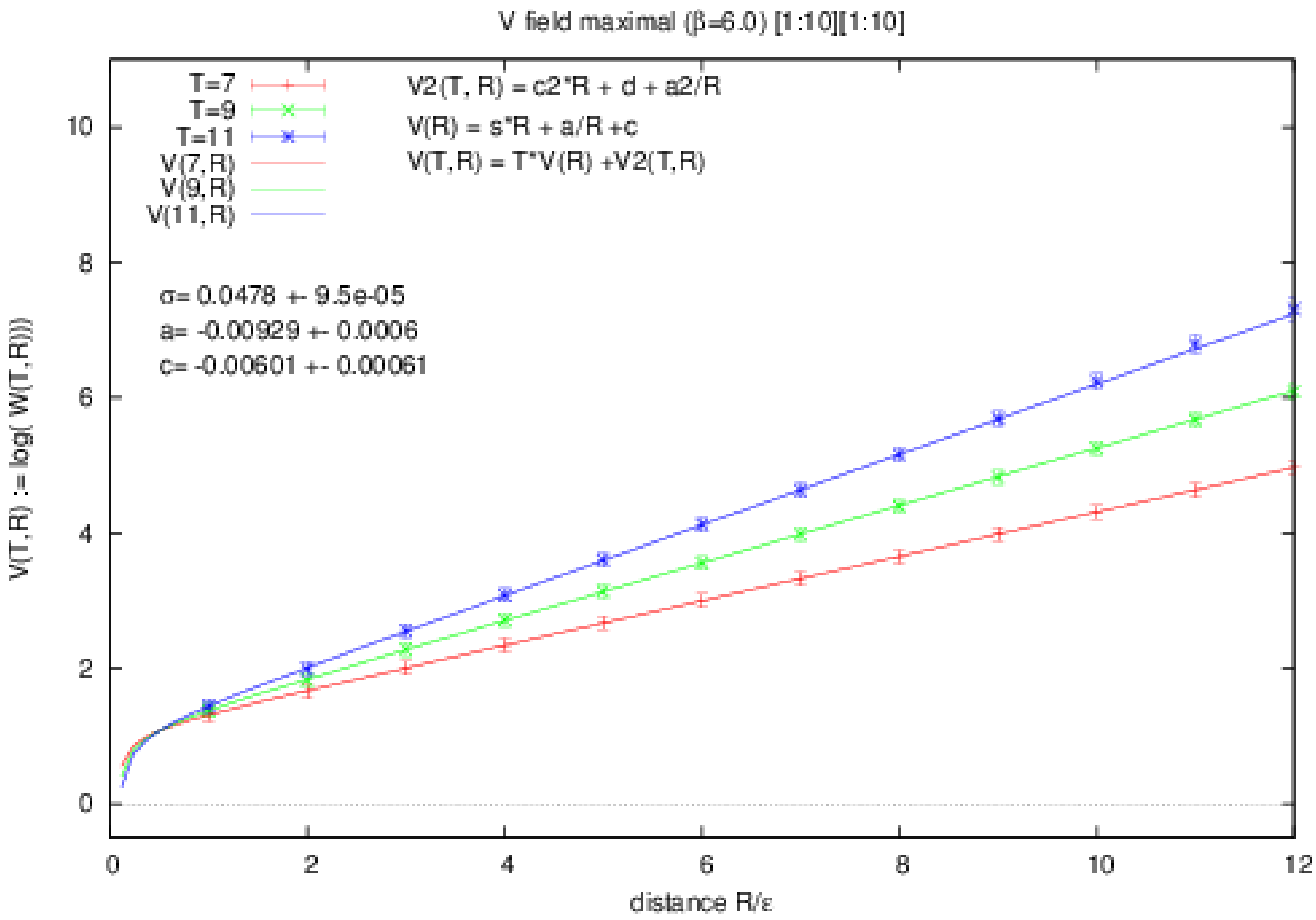
Yang-Mills field



Restricted field V: minimal option



Restricted field V :: maximal option

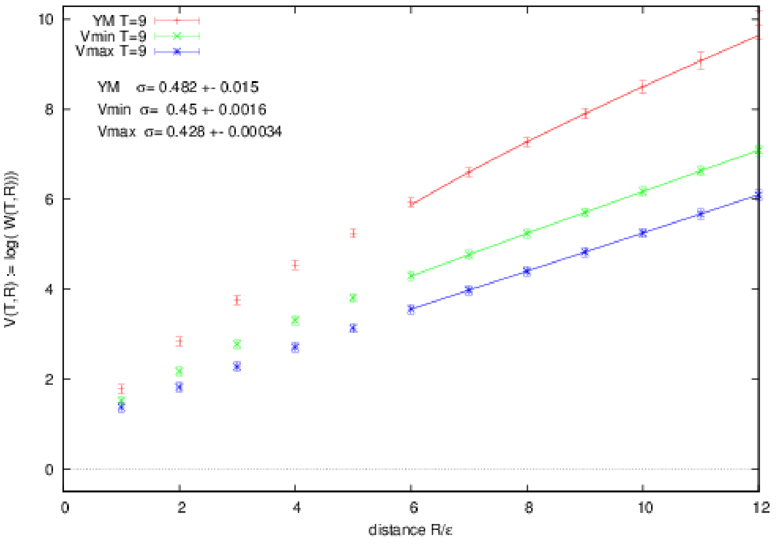


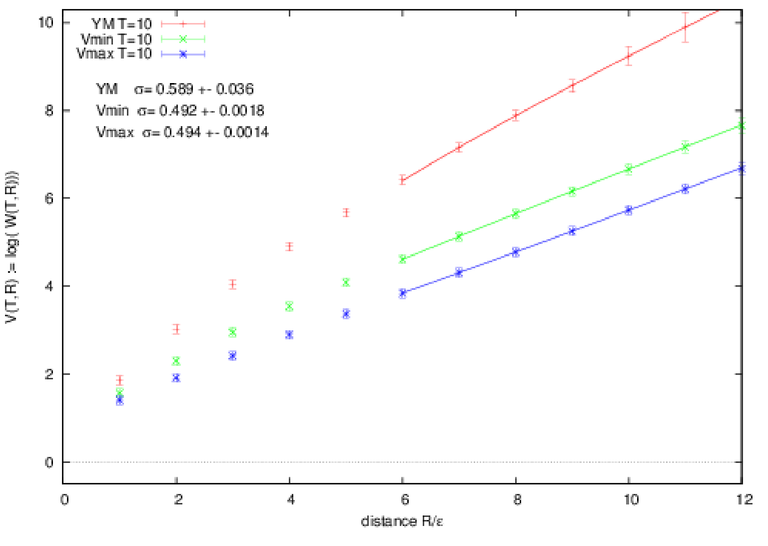
Minial vs Maximal in the Wilson loop

We obtain the **"Abelian dominance" in the string tension** for both the minimal option and the maximal option

- Comparison of Wilson loop values for T=9 and T=10 :
  -log <W[T,R]> vs R
- Each curve shows the result of fitting by using Chonel potential for the range R ~ [6,12]
- The Wilson loop average for the restricted field <Vmin> < <Vmax>
- However, the string tension is almost same for the both options.

➔ Further study by using higher statistics, off-axis measurement , etc.



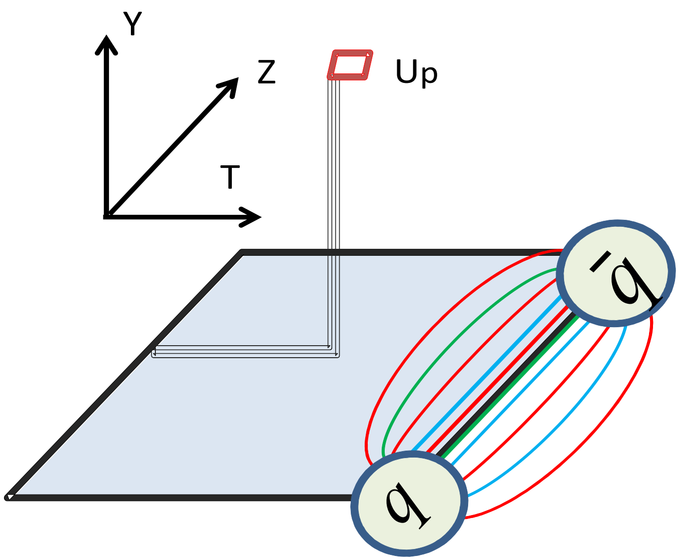


Measurement of color flux

$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger) \rangle}{\langle \mathrm{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{tr}(W) \mathrm{tr}(U_p) \rangle}{\langle \mathrm{tr}(W) \rangle}$$

Proposed by Adriano Di Giacomo et.al.
[Phys.Lett.B236:199,1990]
[Nucl.Phys.B347:441-460,1990]

$$\mathrm{tr}(U_p L W L^\dagger)$$



The field strength by quark and anti quark can be defined as

$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\, \rho_W(x)$$

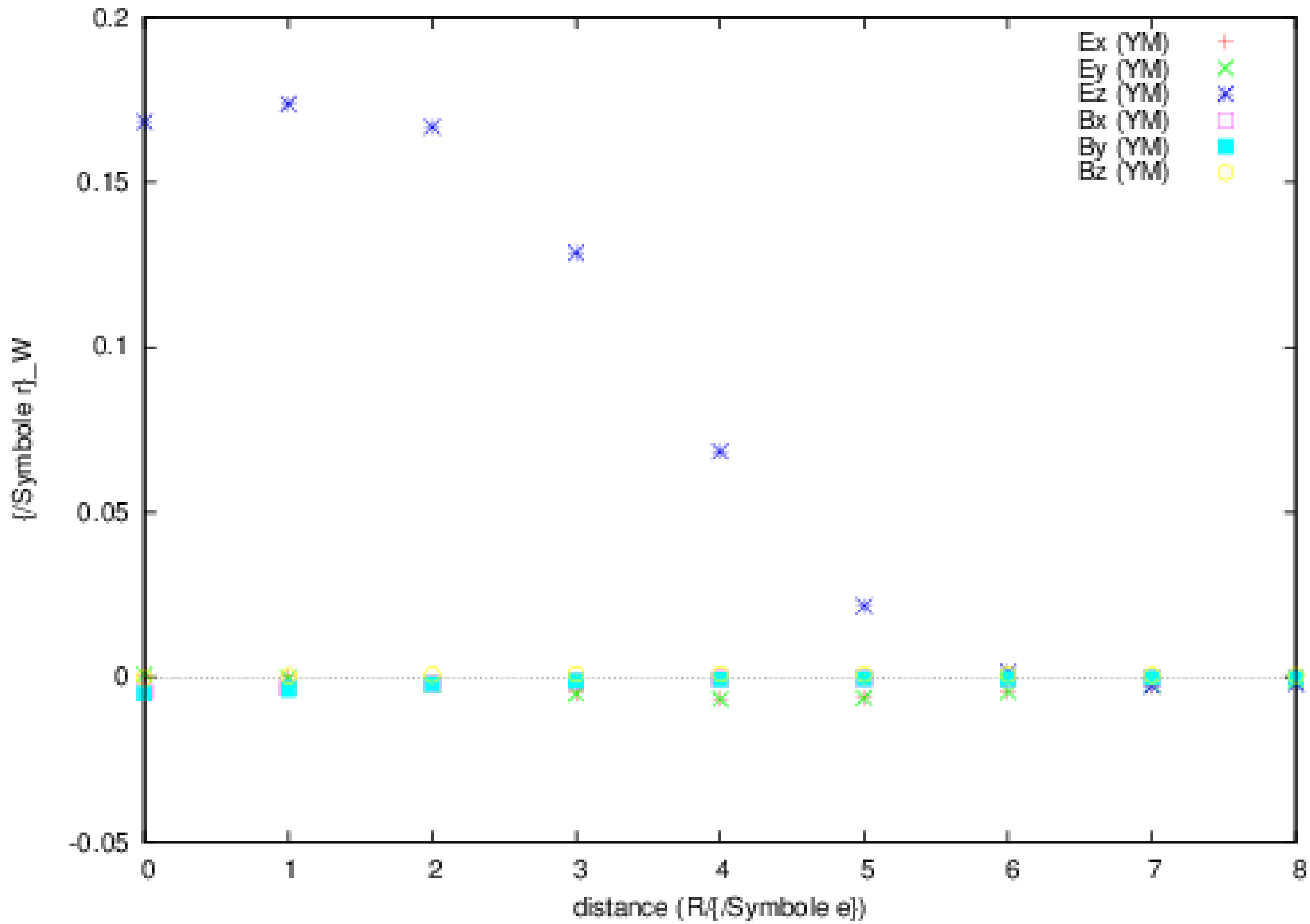
To know the difference between the decomposition, we measure the three types of probes and compare them.

$$O^{[YM]} = L[U] U_p L[U]^{-1} \quad :: \text{ original YM}$$

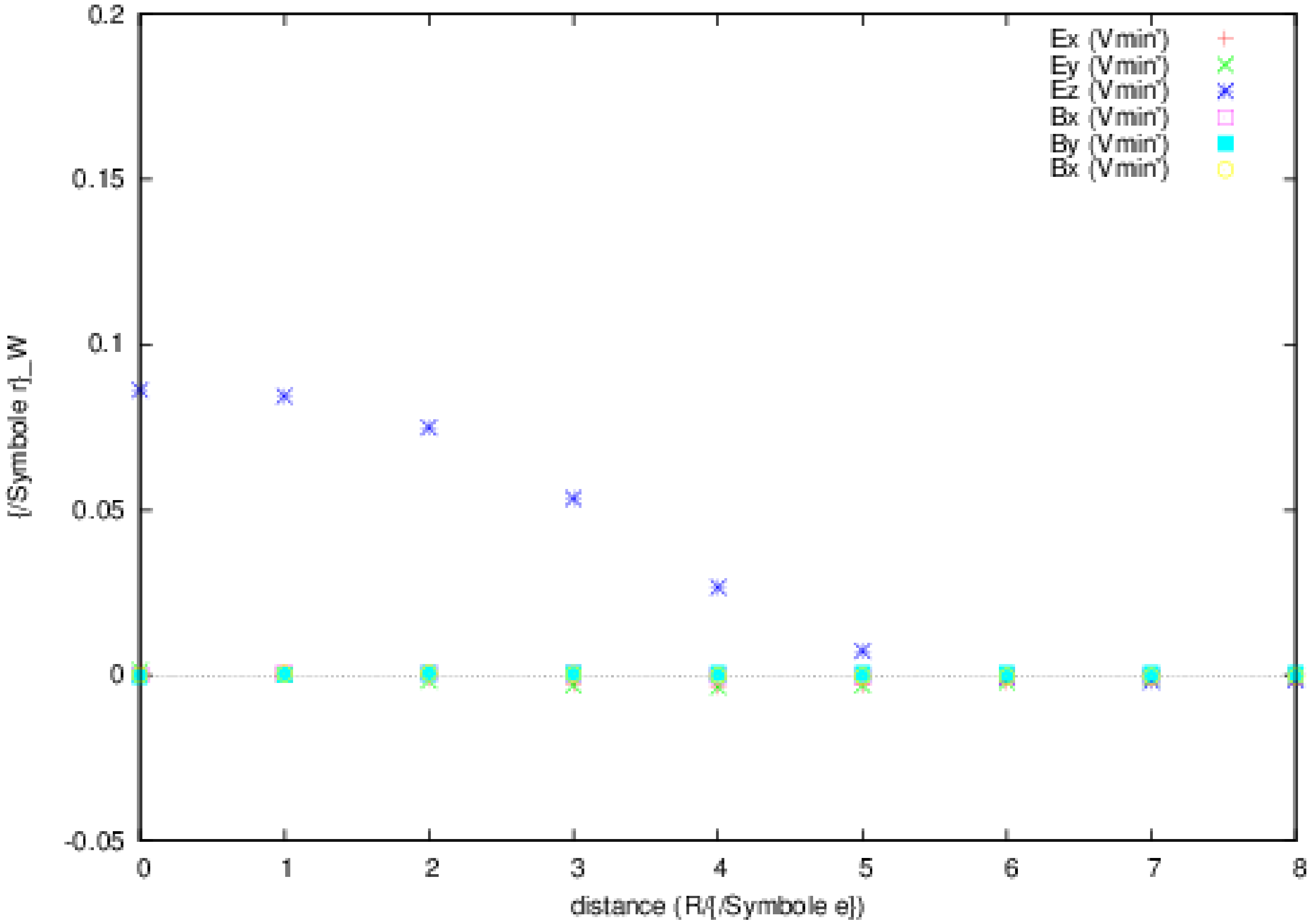
$$O^{[\mathrm{nin}]} = L[V^{[\mathrm{min}]}] V_p^{[\mathrm{min}]} L[V^{[\mathrm{min}]}]^{-1} \quad :: \text{ V field in minimal option}$$

$$O^{[\mathrm{max}]} = L[V^{[\mathrm{max}]}] V_p^{[\mathrm{max}]} L[V^{[\mathrm{max}]}]^{-1} \quad :: \text{ V field in maximal option}$$

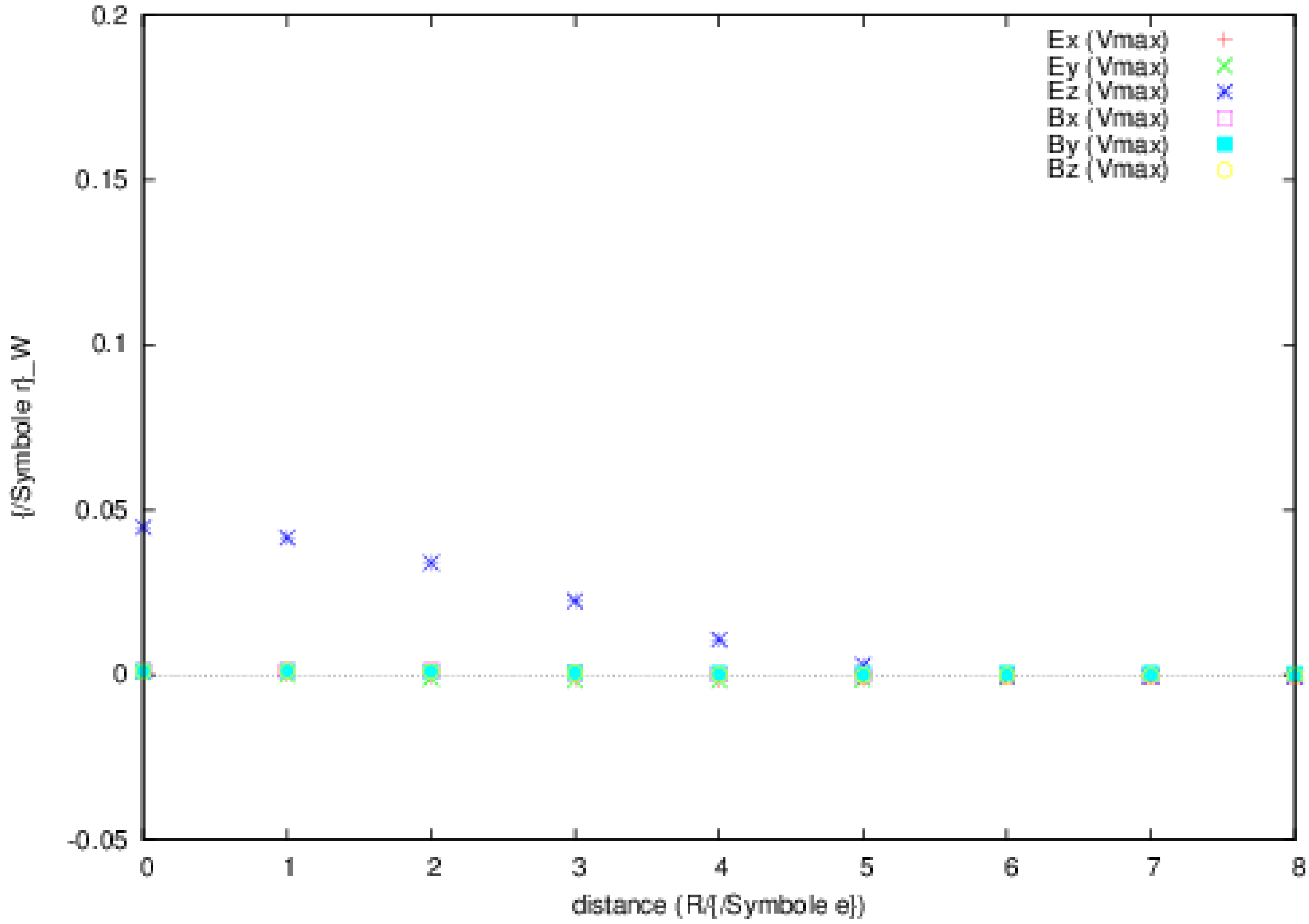
$$O^{[YM]} = L[U]U_p L[U]^{-1} \quad :: \text{original YM}$$



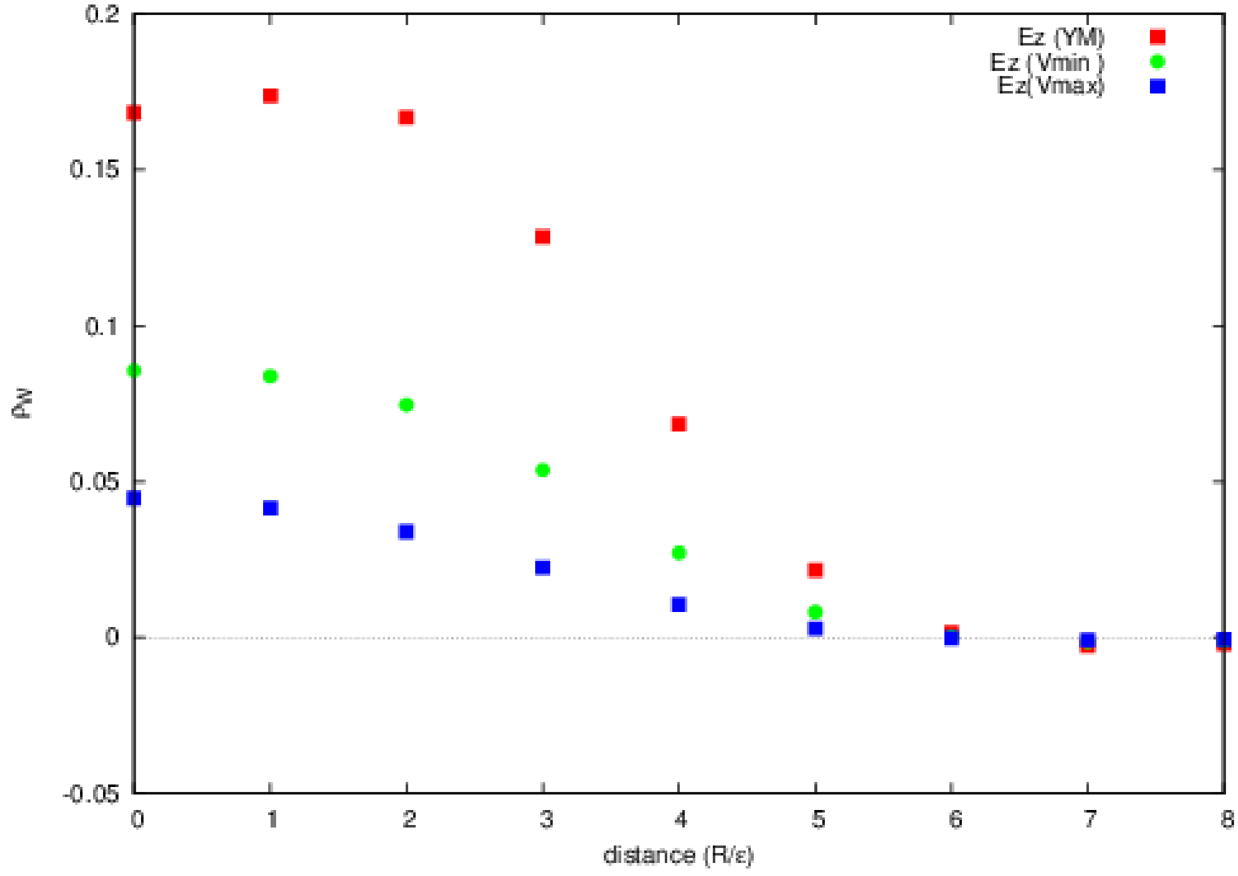
$$O^{[\mathrm{nin}]} = L[V^{[\mathrm{min}]}]V_p^{[\mathrm{min}]}L[V^{[\mathrm{min}]}]^{-1} \quad :: \text{V field in minimal option}$$



$$O^{[\max]} = L[V^{[\max]}]V_p^{[\max]}L[V^{[\max]}]^{-1} \quad :: \text{V field in maximal option}$$



Minimal v.s. Maxial



summary

- We have investigate dual superconductivity picture by using the new formulation of lattice Yang-Mills theory.
- We proposed the none-Abelian dual super conductivity, based on the minimal option of the new formulation, and have shown the numerical evidences:
- We have further investigated “Abelian” dual superconductivity by using the maximal option, in which we can study the picture conventional Abelian projection in MA gauge .(preliminary result)
  - We have found, “Abalian” dominance in the string tension as well as the in the minimal option.
  - We also have found the chromo-electric flux tube and dual Meissner effects.

  ➔ We do not find the qualitative differences between the maximal and minimal options.

Outlook

We need further investigate that

- The measurement with heigher statistics, and comparison with fine lattice with large physical volume.
- Distribution of chromo-flux and magnetic monopole (curents) in 2D (3D) space
- The measurement of the correlation between the magnetic monopole operator and Wilson loop

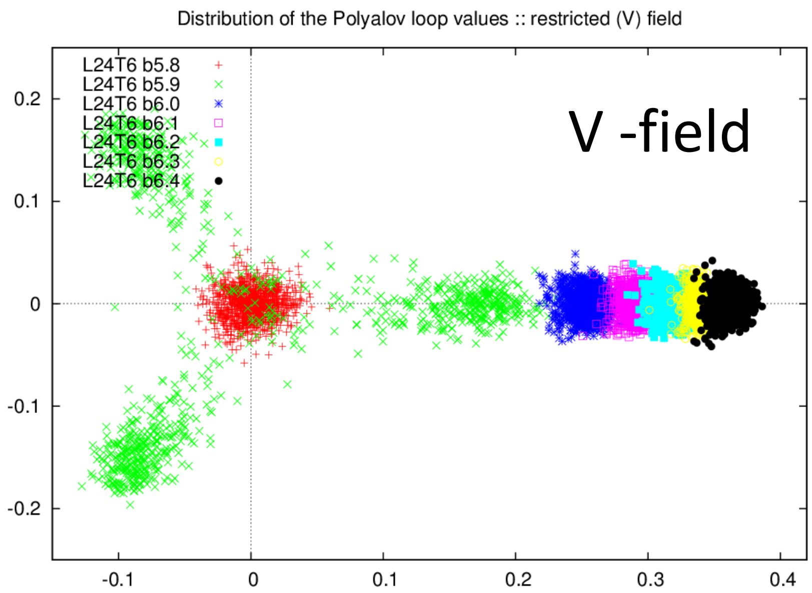
  ➔To investigate the what kind of the magnetic monopole play the dominant role in confinement.
- The measurement chromo flux tube and dual Meissner effect for the barion source.

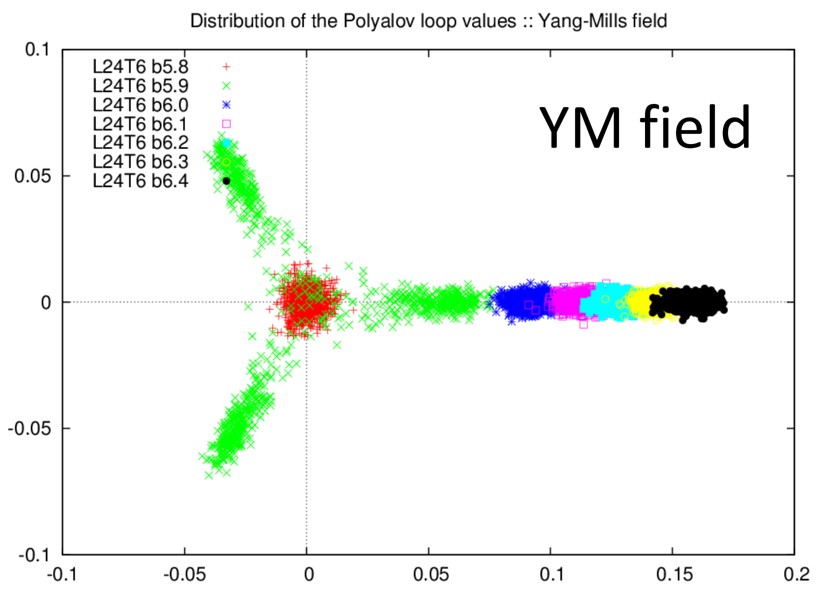
THANK YOU FOR YOUR ATTENTION

BACKUP

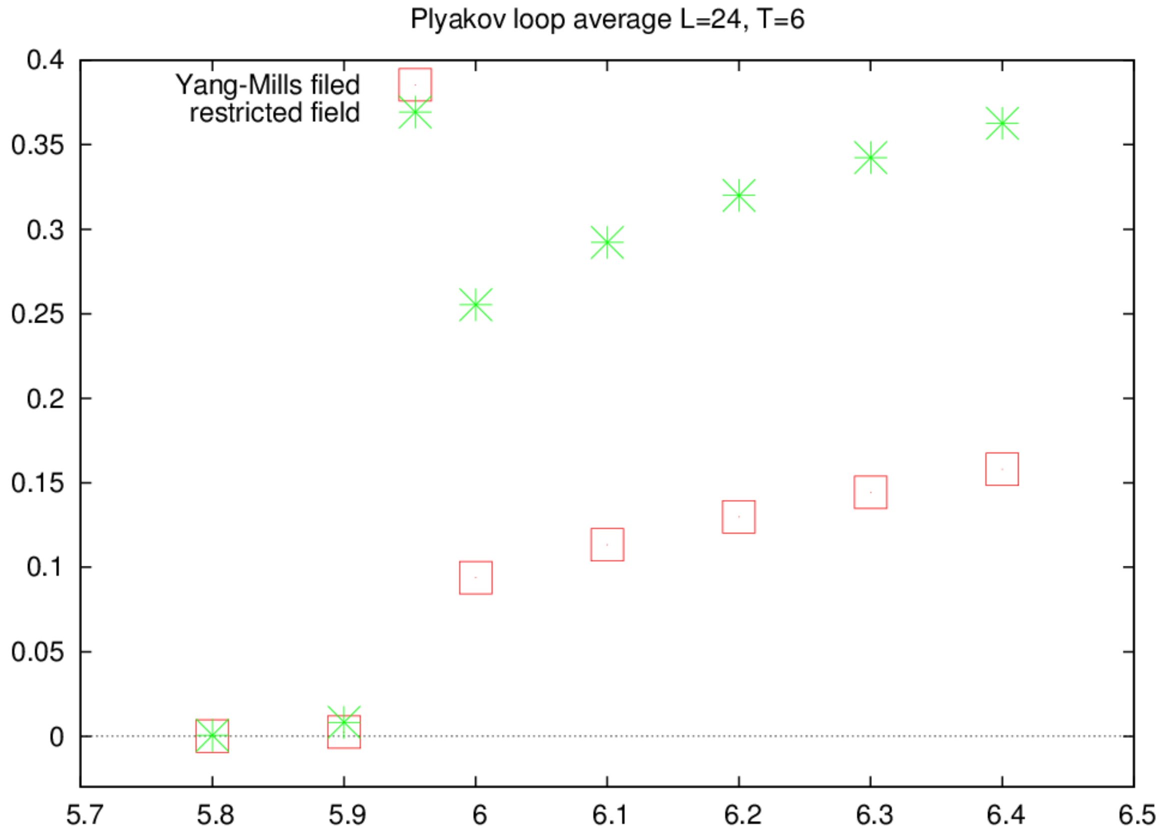
Distribution of Polyakov loop

$$P_U(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} U_{(x,t),4}\right) \text{ for original Yang–Mills filed}$$
$$P_V(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} V_{(x,t),4}\right) \text{ for restricted field}$$

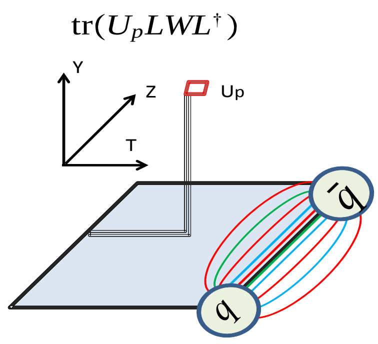


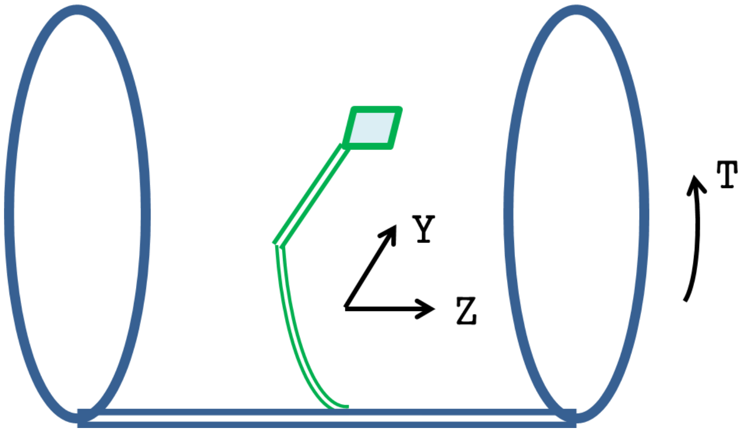


Polyakov loop average YM-field v.s. V - field



Chromo-electric flux at finite temperature

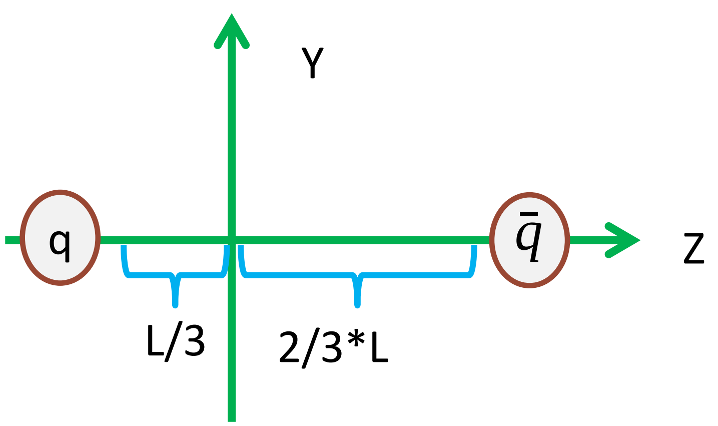




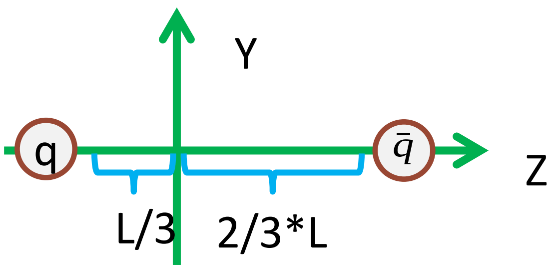
Size of Wilson loop T-direction = Nt
➔The quark and antiquark sources are given by **Plyakov loops**.

$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger) \rangle}{\langle \mathrm{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{tr}(W) \mathrm{tr}(U_p) \rangle}{\langle \mathrm{tr}(W) \rangle}$$

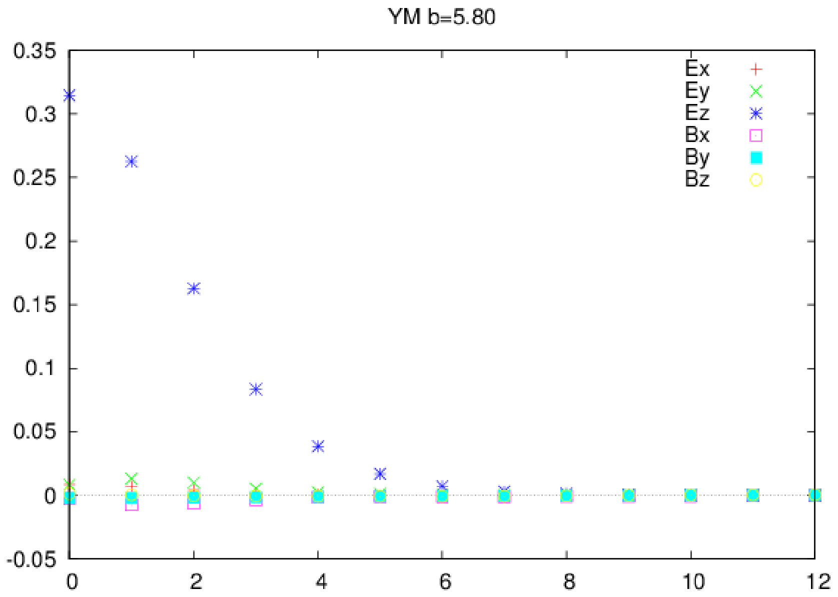
$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}} \rho_W(x)$$



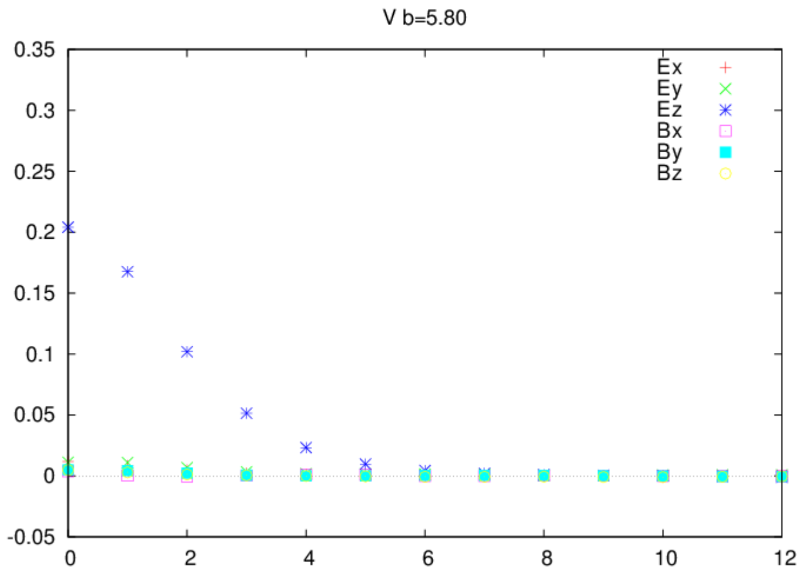
Chromo-flux β=5.8



YM field

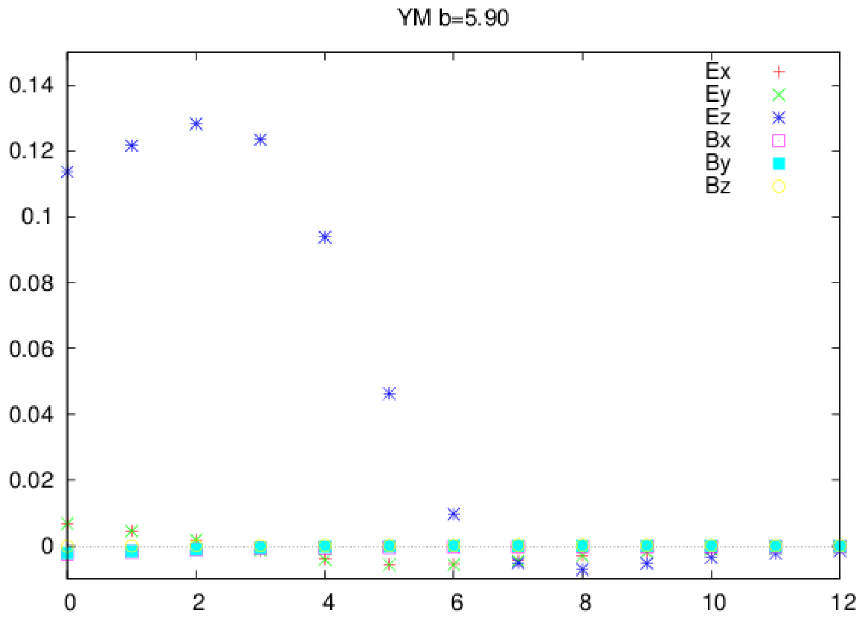


V field

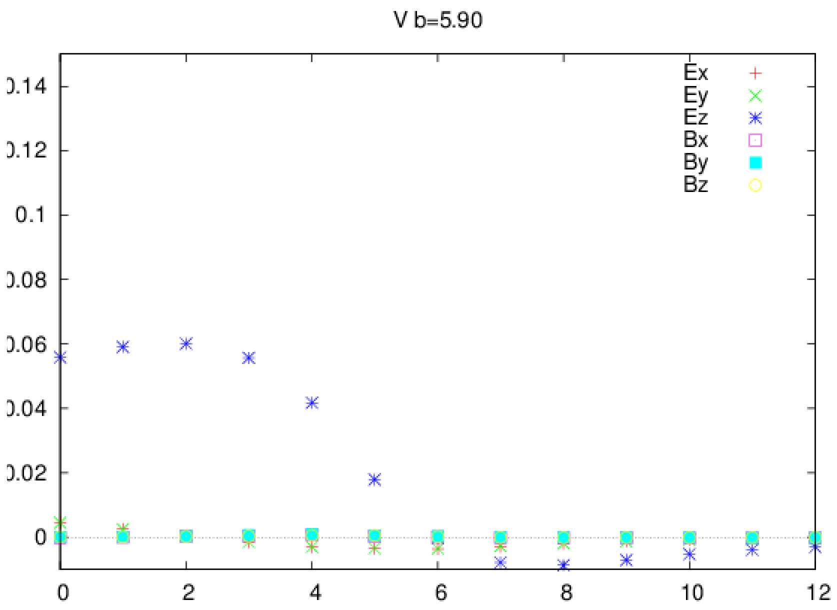


Chromo-flux β=5.9

YM field

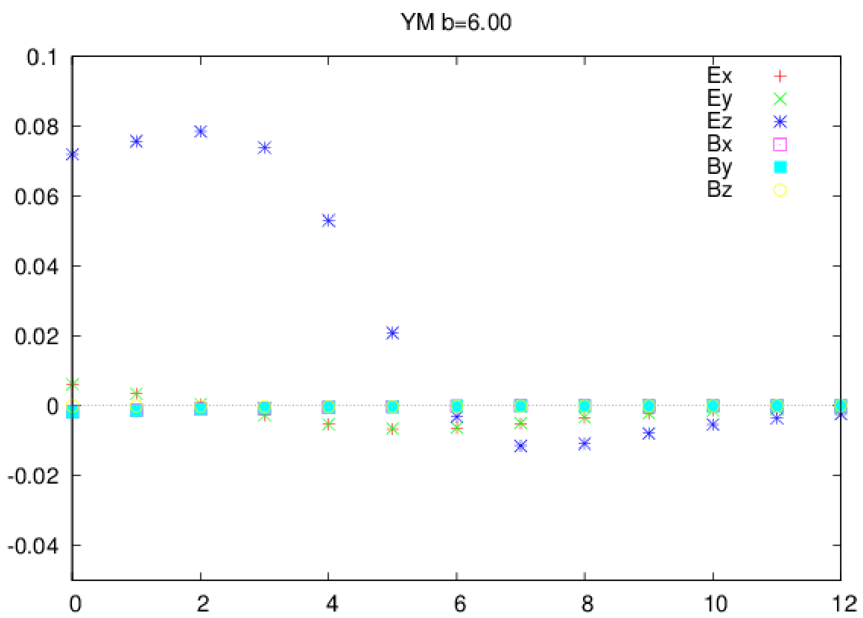


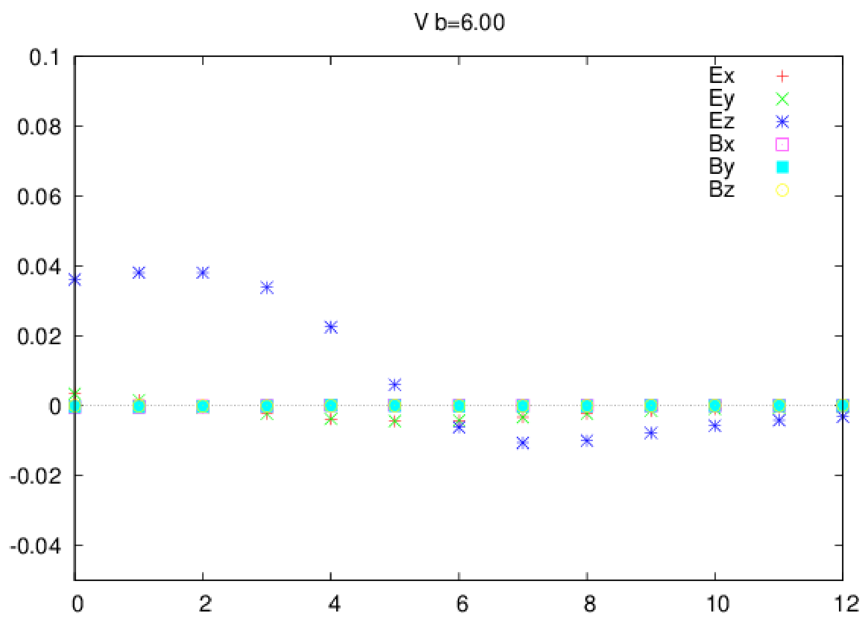
V field



Chromo-flux β=6.0

YM field | V field

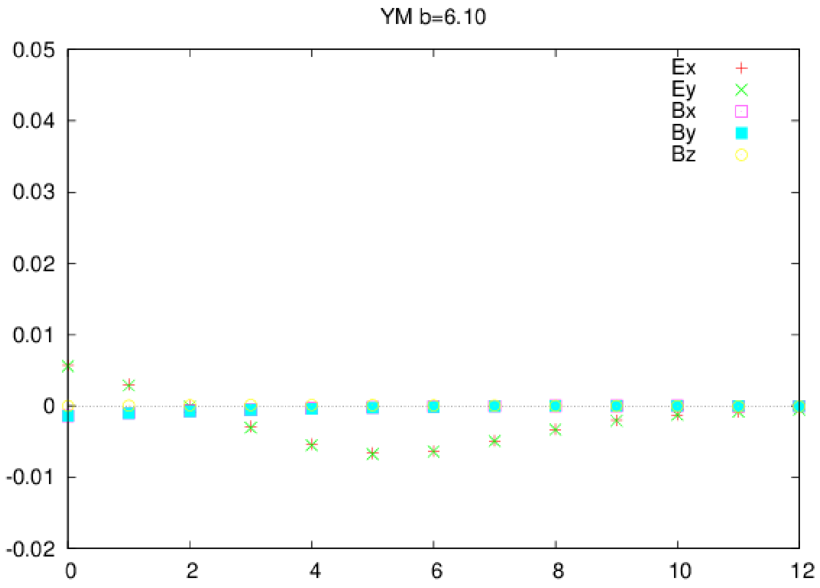


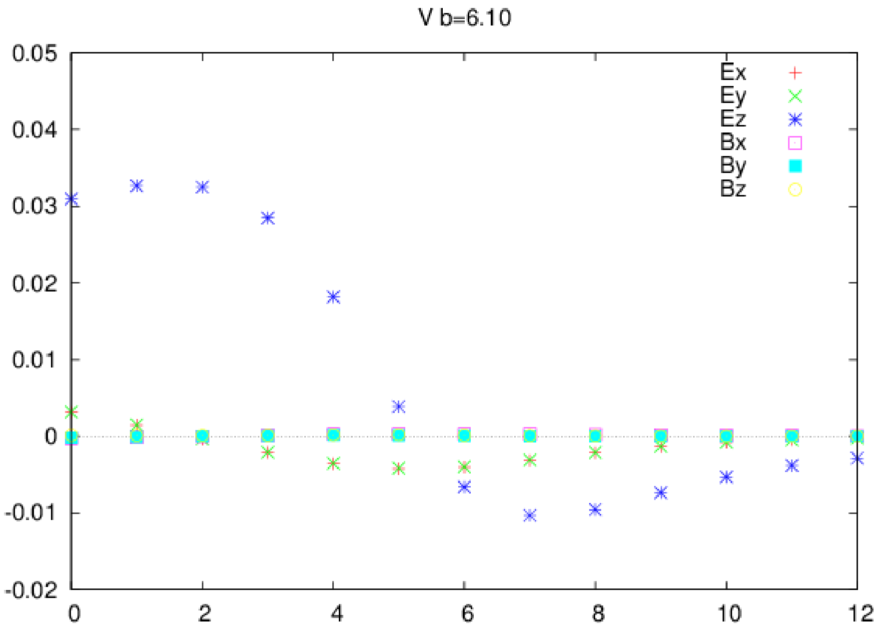


Chrono-flux β=6.1

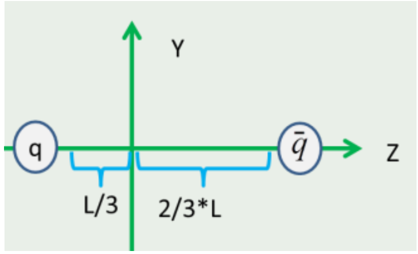
YM field

V field



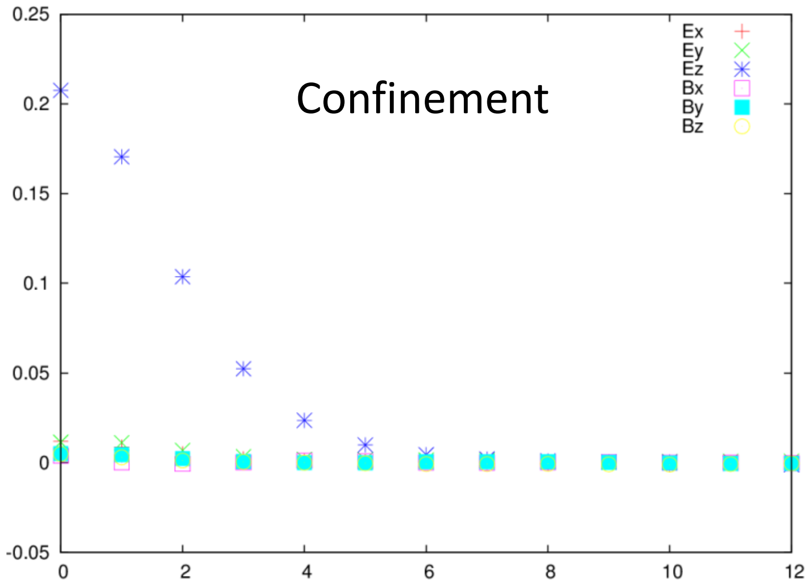


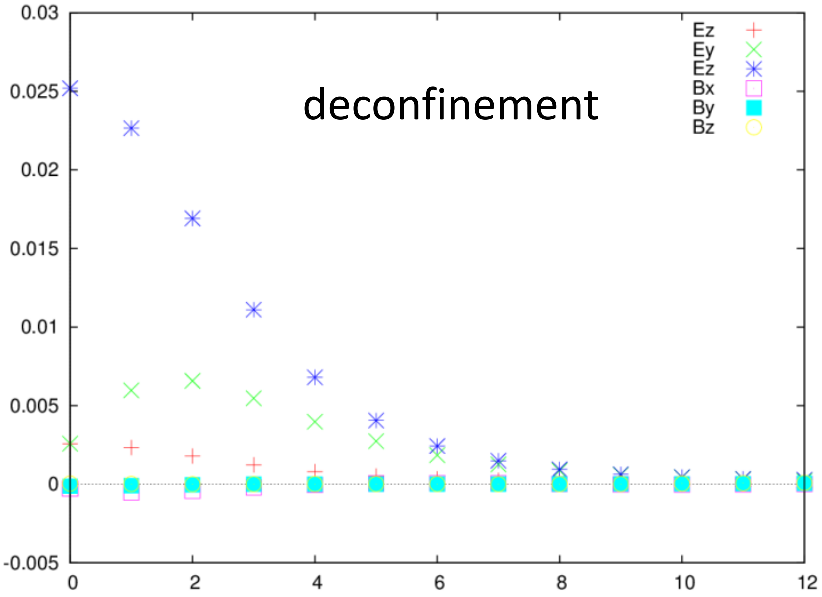
Chromo-electric flux in deconfinement phase



- $E_y \neq 0$ for deconfinemnte phase i.e., No sharp chromo-flux tube

➔ Disappearance of dual superconductivity.

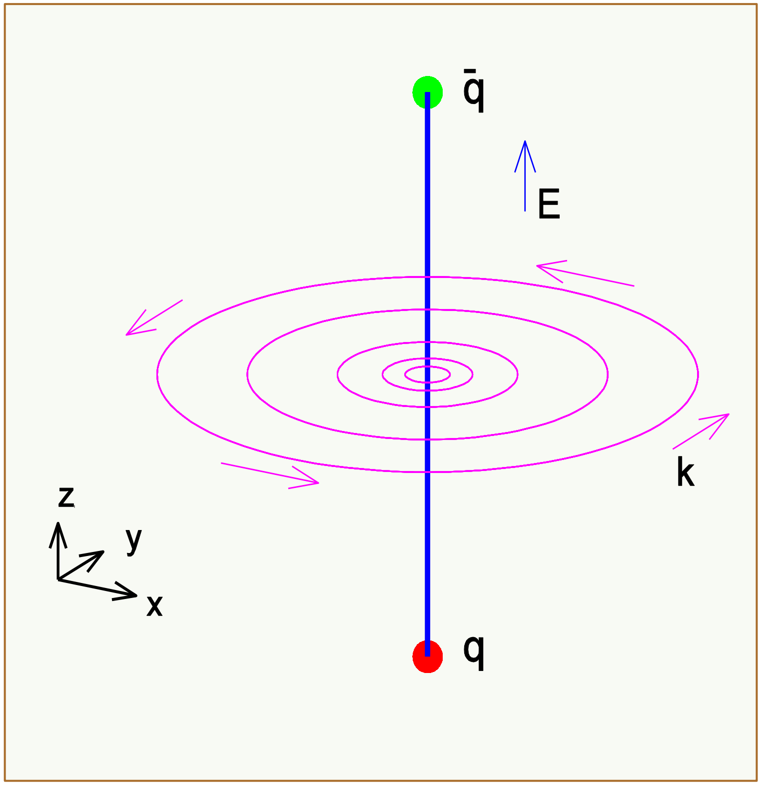




Chromo-magnetic current (monopole current)

- To know relation to the monopole condensation, we further need the measurement of magnetic current in Maxell equation for V field.

$$k = \delta^* F[V] = {}^* dF[V]$$



$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$,
and $\mathbf{k} = {}^* dF[\mathbf{V}] = {}^* ddF[\mathbf{V}] = 0$
(Bianchi identity)

Chromo-magnetic current k_x :: (conbied plot)