The conformal window in theories beyond QCD

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CP3 Origins

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Antipin, Gillioz, EM & Sannino; arXiv:1303.1525
Antipin, Di Chiara, Mojaza, EM & Sannino; arXiv:1205.6157

Quark Confinement and the Hadron Spectrum XI
Saint Petersburg

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  - Conformal bootstrap

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- Phenomenologically relevant
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Fixed points

Luty, Polchinski & Rattazzi; arXiv:1204.5221
Fortin, Grinstein & Stergiou; arXiv:1208.3674

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- When $\beta_g = 0$, the theory is at a fixed point.
- Quantum field theories at fixed points are scale invariant.
- Perturbatively, unitary, four dimensional, scale invariant quantum field theories are conformally invariant.

Banks-Zaks analysis

Banks & Zaks (1981)

$$\beta_g = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}C_2(G) - \frac{2}{3}T_2(F)\right) - \frac{g^5}{(4\pi)^4}\left(\frac{34}{3}C_2(G)^2 - \frac{10}{3}C_2(G)T_2(F) - 2T_2(F)C_2(F)\right) \quad (2)$$

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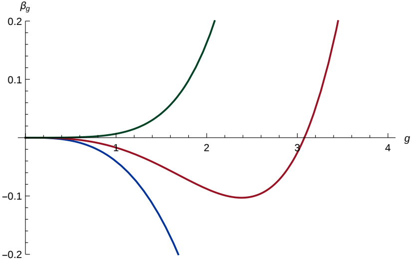

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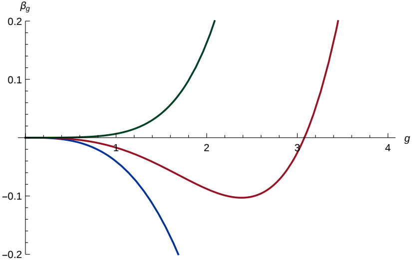
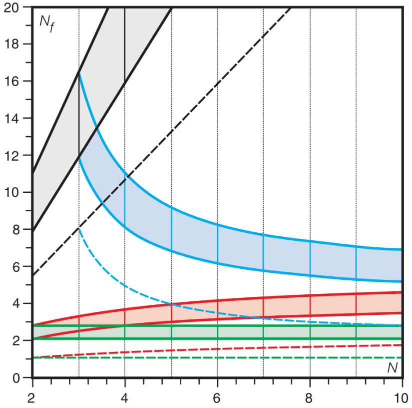


- Infrared free
- IR conformal
- Asymptotically free

Phase diagram of quantum field theories



Dietrich & Sannino;
arXiv:hep-ph/0611341

- Fundamental
- Two-index antisymmetric
- Two-index symmetric
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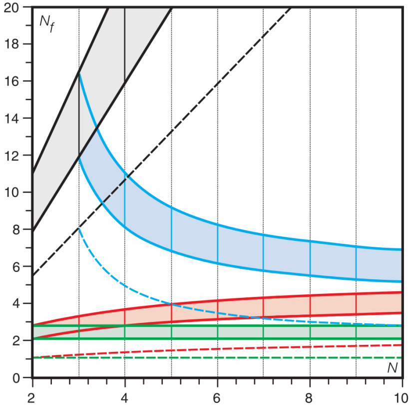

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- Fundamental
- Two-index antisymmetric
- Two-index symmetric
- Adjoint

- High N_f: Infrared free
- Medium N_f: IR conformal
- Low N_f: Asymptotically free

Theories with multiple couplings

$$\mathcal{L} = \mathcal{L}_{kin} - \frac{1}{2}\left(y_{JK;A}\Psi^J\Psi^K\Phi^A + h.c.\right) - \frac{1}{4!}\lambda_{ABCD}\Phi^A\Phi^B\Phi^C\Phi^D \qquad (3)$$

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and have

$$\mu\frac{\mathrm{d}g_i}{\mathrm{d}\mu} = \beta_{g_i}(g_{i'}, y_{JK;E}, \lambda_{ABCD}) \tag{4}$$

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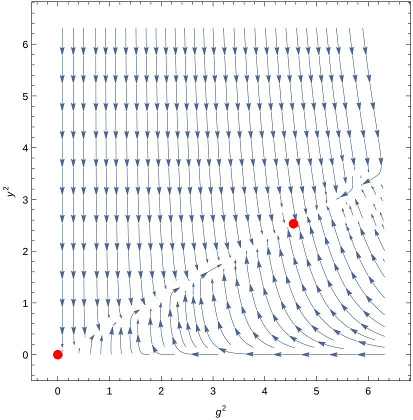
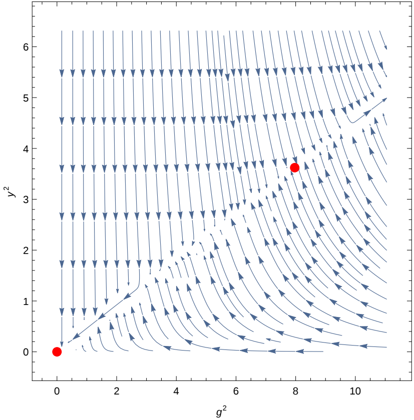
We will use rescaled couplings

$$a_g = \frac{g^2}{(4\pi)^2} \qquad a_y = \frac{y^2}{(4\pi)^2} \qquad a_\lambda = \frac{\lambda}{(4\pi)^2} \tag{7}$$

Renormalization group flows





The Weyl consistency conditions

World's shortest review

Jack & Osborn (1990), Osborn (1991)

The function $\tilde{a}$ is defined in analogy with the Zamolodchikov c function.

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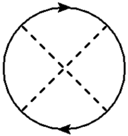
- $\mathcal{O}(a_g^{-2})$ for gauge couplings
- $\mathcal{O}(a_y^{-1})$ for Yukawa couplings
- $\mathcal{O}(1)$ for quartic couplings

The Weyl consistency conditions II

Four loop diagram with quartic and Yukawa couplings from the *a* function



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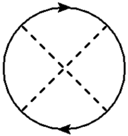
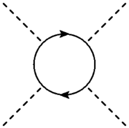


One loop diagram from β_λ



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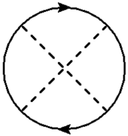

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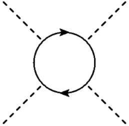
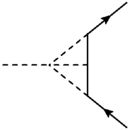


Two loop diagram from β_y



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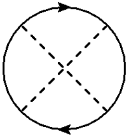

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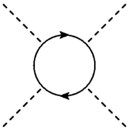

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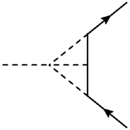

Thus, to preserve Weyl symmetry in a gauge-Yukawa theory, we must use

- the gauge beta function to $n+2$ loops,
- the Yukawa beta function to $n+1$ loops,
- the quartic beta function to n loops.

Generic beta functions

Generically, beta functions in a gauge-Yukawa theory have the form

$$\beta_{a_g} = a_g^2\big(b_1(a_g) + b_2(a_g, a_y) + b_3(a_g, a_y, a_\lambda)\big) \tag{11}$$

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Which is automatically in line with the Weyl consistency conditions!

Perturbative toy model

Antipin, Mojaza & Sannino; arXiv:1107.2932

$$\mathcal{L} = \mathcal{L}_K(G_\mu, \lambda_m, Q, \tilde{Q}, H) + \left(y_H Q H \tilde{Q} + \text{h.c}\right) - u_1 \left(\text{Tr}\left[HH^\dagger\right]\right)^2 - u_2 \text{Tr}\left[(HH^\dagger)^2\right], \quad (14)$$

Fields	$[SU(N_{TC})]$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_V$	$U(1)_{AF}$
λ_m	Adj	1	1	0	1
Q	$\square$	$\overline{\square}$	1	$\frac{N_f - N_{TC}}{N_{TC}}$	$-\frac{N_{TC}}{N_f}$
$\tilde{Q}$	$\overline{\square}$	1	$\square$	$-\frac{N_f - N_{TC}}{N_{TC}}$	$-\frac{N_{TC}}{N_f}$
H	1	$\square$	$\overline{\square}$	0	$\frac{2N_{TC}}{N_f}$
G_μ	Adj	1	1	0	0

Table : The field content of the toy model and the related symmetries

Running to the fixed points

Antipin, Di Chiara, Mojaza, EM & Sannino; arXiv:1205.6157
Antipin, Gillioz, EM & Sannino; arXiv:1303.1525

We investigate this model in the Veneziano limit of large N_{TC} and large N_f, with $x = \frac{N_f}{N_{TC}}$ fixed and rescaled couplings

$$a_g = \frac{g^2 N_{TC}}{(4\pi)^2}, \quad a_H = \frac{y_H^2 N_{TC}}{(4\pi)^2}, \quad z_1 = \frac{u_1 N_f^2}{(4\pi)^2}, \quad z_2 = \frac{u_2 N_f}{(4\pi)^2}. \tag{15}$$

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- The double trace operator $u_1 \left(\text{Tr}\left[HH^\dagger\right]\right)^2$ decouples at large N_f

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- The double trace operator $u_1 \left(\mathrm{Tr}\left[HH^\dagger\right]\right)^2$ decouples at large N_f

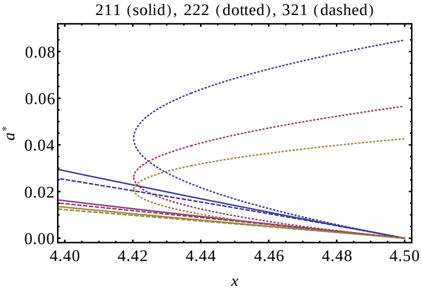
We find fixed points in three beta function counting schemes

211 to 2 loops in a_g, 1 loop in a_H and 1 loop in z_2.

222 to 2 loops in a_g, 2 loops in a_H and 2 loops in z_2.

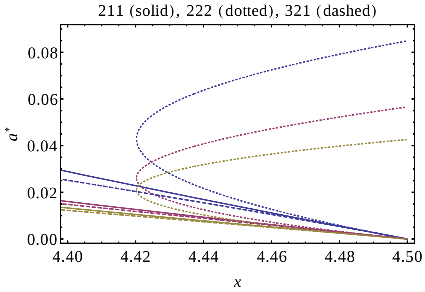
321 to 3 loops in a_g, 2 loops in a_H and 1 loop in z_2.

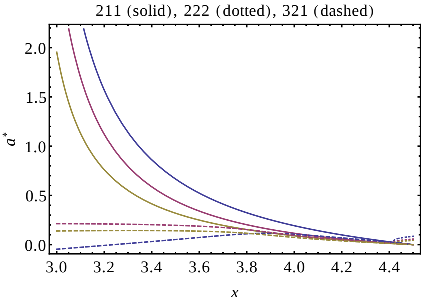
Modified fixed points



Fixed point values for a_g (blue), a_H (red) and z_2 (yellow)

Modified fixed points





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- No adjoint fermion.

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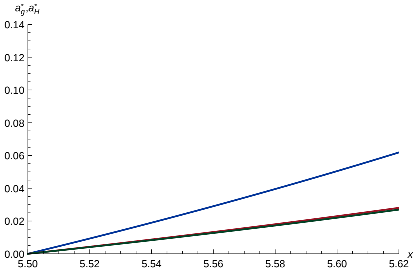

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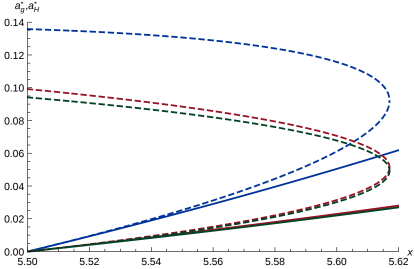
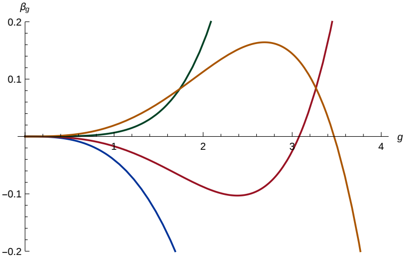


Asymptotic Safety

Litim & Sannino; arXiv:1406.2337



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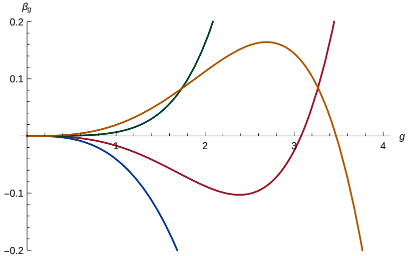

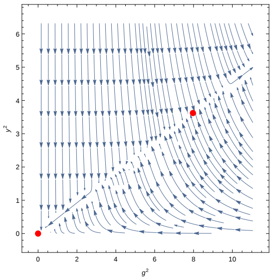

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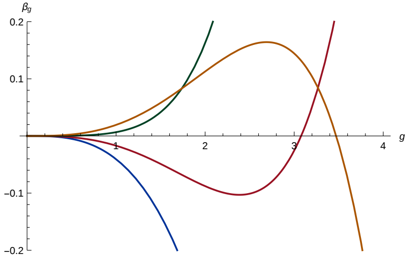

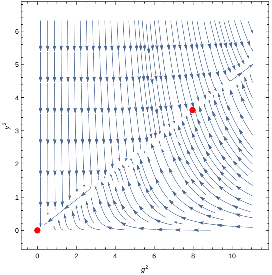

- Generalization of asymptotic freedom

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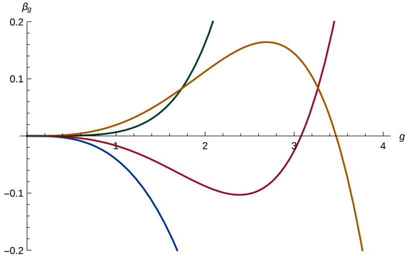

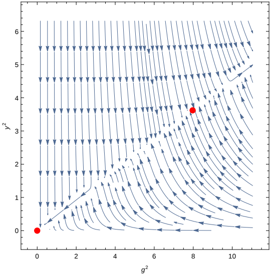

- Generalization of asymptotic freedom
- Non-perturbative renormalizability

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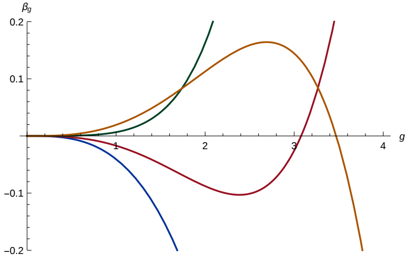

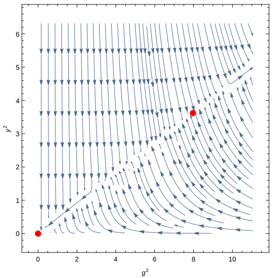

- Generalization of asymptotic freedom
- Non-perturbative renormalizability
- Fundamental QFT

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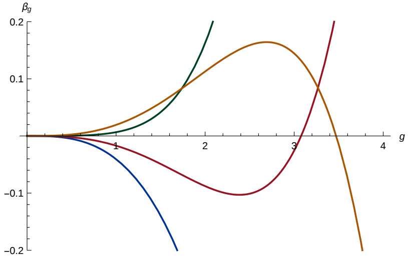

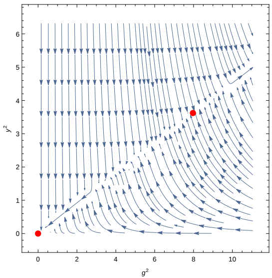

- Generalization of asymptotic freedom
- Non-perturbative renormalizability
- Fundamental QFT
- Lattice-friendly

Conclusions

- Richer fixed point structures

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  - Coupling-dependent conformal window

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  - Phenomenological