The muon g-2:
DSE status on light-by-light

Gernot Eichmann
University of Giessen, Germany

Quark Confinement and the Hadron Spectrum XI
St. Petersburg, Russia
Sep 9, 2014

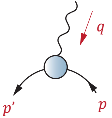
Muon g-2

- **Magnetic moment** of a fermion due to its spin:

$$\vec{\mu} = g\,\frac{e}{2m}\,\vec{S} \qquad \text{Pointlike fermion: } g = 2$$

Fermion with structure has **anomalous magnetic moment:** $a = \frac{g-2}{2}$

- **Electron & muon anomalous magnetic moments**
among the most precisely measured & theoretically calculated quantities:



$$= \; ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

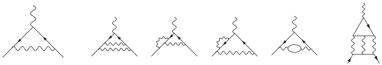
$$F_2(0) = a_{e,\mu} = \underbrace{\frac{\alpha_{\text{QED}}}{2\pi}} + \mathcal{O}(\alpha^2_{\text{QED}}) \approx 1\,‰$$

Schwinger 1948

- measured with precision 10^{-12} for electron and 10^{-10} for muon

Muon g-2

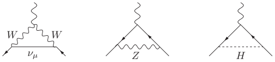
- **QED corrections:** overwhelming part, calculated up to O(α^5):



…

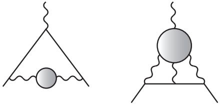
- **Electroweak** and **QCD corrections** very small: 10^{-12} for electron, 10^{-8} for muon

- **Electroweak corrections** up to 2-loop:



…

- **QCD:**



Hadronic vacuum polarization

Hadronic light-by-light scattering

$a_\mu \; [10^{-10}]$

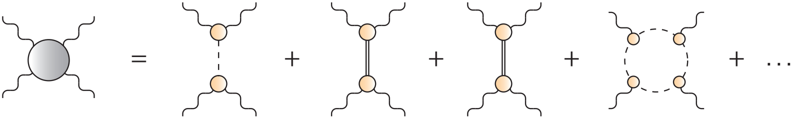
Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)
QED:	11 658 471.9	(0.0)
EW:	15.3	(0.2)
Hadronic:		
• VP (LO+HO)	685.1	(4.3)
• LBL	10.5	(2.6)
SM:	11 659 182.8	(4.9)
Diff:	26.1	(8.0)

- **Total SM prediction** deviates from measured a_μ by ~3σ: new physics?

- Theory uncertainty dominated by **QCD!** Is QCD contribution under control?

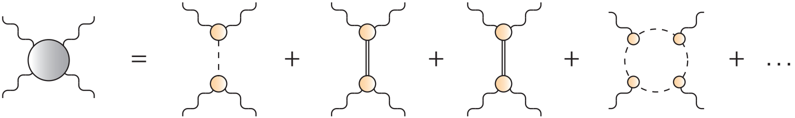
Light-by-light scattering



Model results:	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
ENJL & MD models	8 ... 11	-1	2	-2	$(\times 10^{-10})$

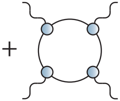
Bijnens 1995
Hakayawa 1995,
Knecht 2002,
Melnikov 2004,
Prades 2009,
Jegerlehner 2009,
Pauk 2014

Light-by-light scattering



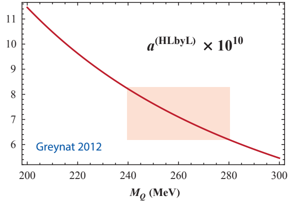
Model results:
ENJL &
MD models
Bijnens 1995
Hakayawa 1995,
Knecht 2002,
Melnikov 2004,
Prades 2009,
Jegerlehner 2009,
Pauk 2014

pseudoscalar exchange	**scalar exchange**	**axialvector exchange**	π, K **loop**	
8 ... 11	-1	2	-2	$(\times 10^{-10})$



Quark loop

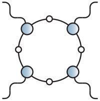
- **Constituent quark loop** known analytically: 6 ... 8



- **ENJL:** VM poles by summing up quark bubbles
  Bijnens 1995
  $$\gamma^\mu - \gamma_T^\mu \, \frac{Q^2}{Q^2+m_V^2}$$
  Large reduction: 2

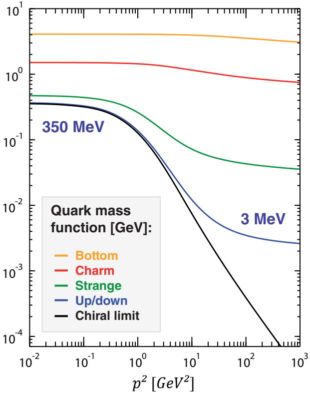
How to improve on this?

Quark loop



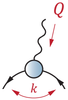
- **Quark mass is not a constant:**
  dressed quark propagator
  has nonperturbatively enhanced
  quark mass function (DSE, Lattice, ...)

$$S_0(p) = \frac{-i\not{p}+m}{p^2+m^2} \quad\longrightarrow\quad S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$



- **Quark-photon vertex is not bare!**

$$\Gamma^\mu(k,Q) = \Big[i\gamma^\mu\,\Sigma_A + 2k^\mu(i\not{k}\,\Delta_A + \Delta_B)\Big] + \Big[i\sum_{j=1}^{8} f_j\,\tau_j^\mu(k,Q)\Big]$$



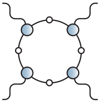
Ball-Chiu vertex,
depends only on
quark propagator
Ball, Chiu, PRD 22 (1980)

necessary for
electromagnetic
gauge invariance!

$$Q^\mu\,\Gamma^\mu(k,Q) = S^{-1}(k+\tfrac{Q}{2}) - S^{-1}(k-\tfrac{Q}{2})$$

Transverse part:
ρ-meson poles,
vanishes at $Q\to 0$,
no kin. singularities
Kizilersu et al, PRD 92 (1995);
GE, Fischer, PRD 87 (2013)

Quark loop

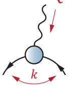


- **Quark mass is not a constant:**
  dressed quark propagator
  has nonperturbatively enhanced
  quark mass function (DSE, Lattice, ...)

$$S_0(p) = \frac{-i\not{p}+m}{p^2+m^2} \longrightarrow S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

- **Quark-photon vertex is not bare!**

$$\Gamma^\mu(k,Q) = \Big[i\gamma^\mu\,\Sigma_A + 2k^\mu(i\not{k}\,\Delta_A + \Delta_B)\Big] + \Big[i\sum_{j=1}^{8} f_j\,\tau_j^\mu(k,Q)\Big]$$



Ball-Chiu vertex,
depends only on
quark propagator
Ball, Chiu, PRD 22 (1980)

necessary for
electromagnetic gauge invariance!

$$Q^\mu\,\Gamma^\mu(k,Q) = S^{-1}(k+\tfrac{Q}{2}) - S^{-1}(k-\tfrac{Q}{2})$$

Transverse part:
ρ-meson poles,
vanishes at $Q\rightarrow 0$,
no kin. singularities
Kizilersu et al, PRD 92 (1995);
GE, Fischer, PRD 87 (2013)

Goecke, Fischer, Williams, PRD 87 (2013)

$A(p^2)$	$M(p^2)$	γ^μ	Γ_T^μ	a_μ $[10^{-10}]$
1	0.2 GeV	1	0	10
1	$M(p^2)$	1	0	10
$A(p^2)$	$M(p^2)$	1	0	5
$A(p^2)$	$M(p^2)$	Σ_A	0	10
$A(p^2)$	$M(p^2)$	Σ_A	$k=0$	4
$A(p^2)$	$M(p^2)$	Σ_A	Full	10

- **DSE result** for quark loop (including strange & charm):

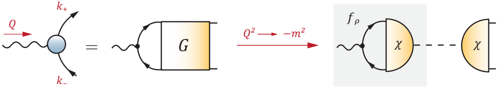
$$a_\mu = 10.7 \times 10^{-10}$$

- full Ball-Chiu vertex problematic

Quark-photon vertex

- **Quark-photon vertex:** (vector current: $J^\mu = \bar{\psi}\,\gamma^\mu \psi$)

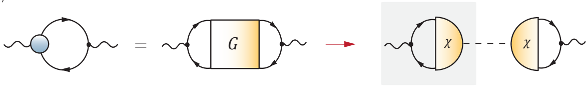
  $\langle 0|\mathrm{T}\,J^\mu(x)\,\psi(x_1)\,\bar{\psi}(x_2)|0\rangle$



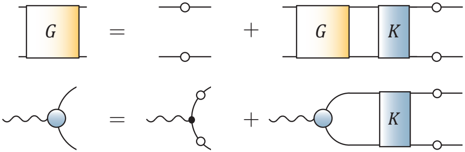
Quark-photon vertex has ρ-meson poles: 'vector-meson dominance'

- **Hadronic vacuum polarization = vector current correlator**

  $\langle 0|\mathrm{T}\,J^\mu(x)\,J^\nu(y)|0\rangle$



- BSE for **quark four-point function** & **quark photon vertex:**



determine vertex dynamically from a given $q\bar{q}$ kernel, e.g. **rainbow-ladder** (= gluon exchange)

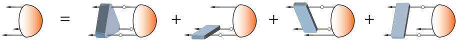
Maris & Tandy, PRC 61 (2000)

Context matters

Hadron physics from **QCD's Dyson-Schwinger & bound-state equations**

- **Mesons** from Bethe-Salpeter equation:
  meson spectra, form factors, PDFs, GPDs, ...
  Chang et al., Commun. Theor. Phys. 58 (2012), . . .

- **Baryons** from covariant Faddeev equation:
  octet & decuplet masses, nucleon & Δ form factors
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011),
  Sanchis-Alepuz, Fischer, 1408.5577, . . .

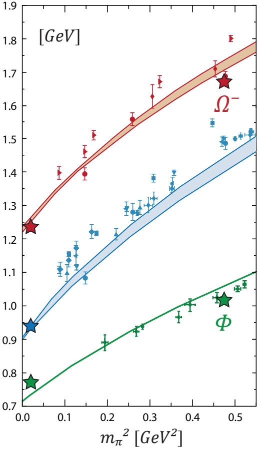


  Compton scattering, pion electroproduction, ...
  GE, Fischer, PRD 85 (2012), PRD 87 (2013)

- **Tetraquarks:**
  see talk by **Walter Heupel**

- **Beyond rainbow-ladder:**
  see talk by **Richard Williams**

Friday,
Parallel II: B9



Delta:
Sanchis-Alepuz et al., PRD 84 (2011)

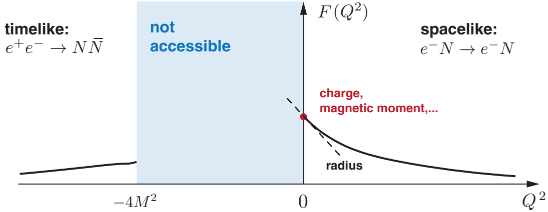
Nucleon:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010);
GE, PRD 84 (2011)

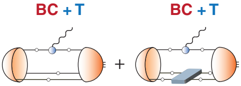
ρ–meson:
Maris & Tandy, PRC 60 (1999)

Quark-photon vertex

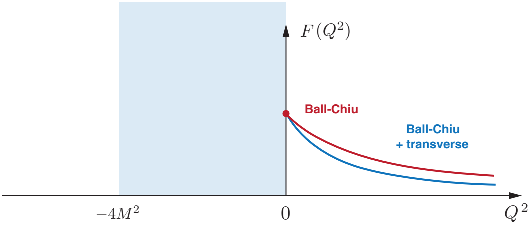
Structure of quark-photon vertex is reflected in hadron form factors GE, PRD 84 (2011)

Experimentally (sketch):





Calculated:
(Sketch)

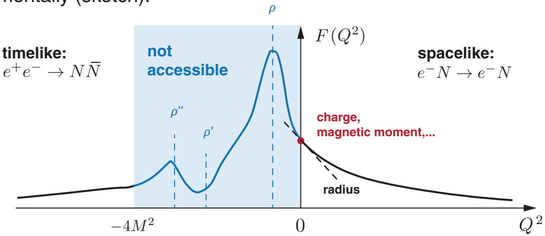


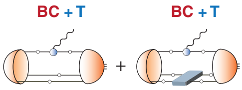
- Deriving hadronic currents: Kvinikhidze, Blankleider, PRC 60 (1999)
- Ball-Chiu part is dominant **(em. gauge invariance):** charge, magnetic moments
- Transverse part changes slope and charge radii. No pion cloud in RL ⇒ timelike ρ-**meson poles**

Quark-photon vertex

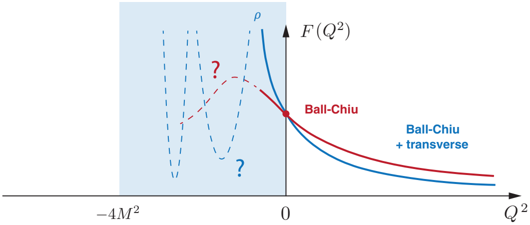
Structure of quark-photon vertex is reflected in hadron form factors GE, PRD 84 (2011)

Experimentally (sketch):



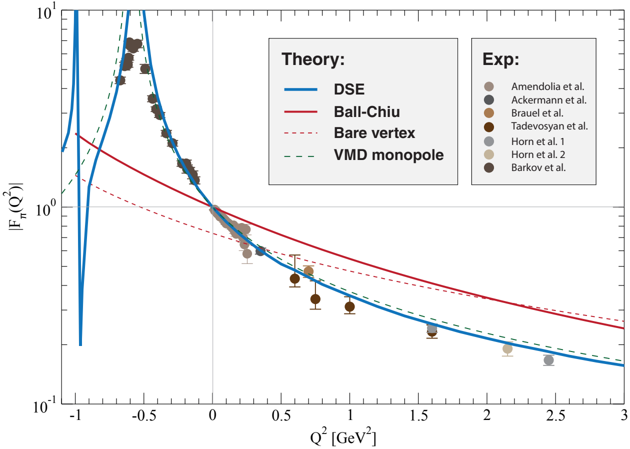


Calculated:
(Sketch)



- Deriving hadronic currents: Kvinikhidze, Blankleider, PRC 60 (1999)
- Ball-Chiu part is dominant **(em. gauge invariance):** charge, magnetic moments
- Transverse part changes slope and charge radii. No pion cloud in RL ⇒ timelike ρ-**meson poles**

Pion form factor



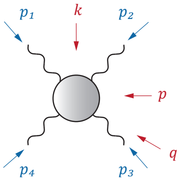
Spacelike and timelike region:

A. Krassnigg (Schladming 2010)
extension of Maris & Tandy,
Nucl. Phys. Proc. Suppl. 161 (2006)

Include **pion cloud:**

Kubrak et al., in preparation

Structure of the γγγγ amplitude



3 independent momenta:

$$p = p_2 + p_3$$
$$q = p_3 + p_1$$
$$k = p_1 + p_2$$

6 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2, \quad p\cdot q, \quad p\cdot k, \quad q\cdot k$$

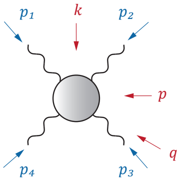
Bose symmetry:

$$\Gamma^{\mu\nu\rho\sigma}(p,q,k) = \sum_{i=1}^{136} f_i(\dots)\ \tau_i^{\mu\nu\rho\sigma}(p,q,k)$$

$\stackrel{!}{=}$ **symmetric**

S4 multiplets

Structure of the γγγγ amplitude



3 independent momenta:

$$p = p_2 + p_3$$
$$q = p_3 + p_1$$
$$k = p_1 + p_2$$

6 Lorentz invariants:

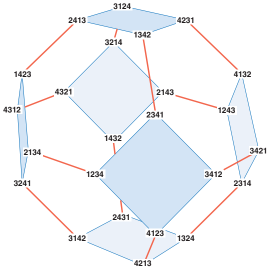
$$p^2, \quad q^2, \quad k^2, \quad p\cdot q, \quad p\cdot k, \quad q\cdot k$$

Bose symmetry:

$$\Gamma^{\mu\nu\rho\sigma}(p,q,k) = \sum_{i=1}^{136} f_i(\dots)\; \tau_i^{\mu\nu\rho\sigma}(p,q,k)$$

$\stackrel{!}{=}$ **symmetric**

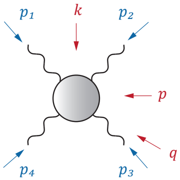
S4 multiplets



- Arrange the 24 permutations of ψ_{1234} into **multiplets:**

Singlet	Triplets	Doublets	Antitriplets	Antisinglet
$\mathcal{S}$	$\mathcal{T}_i^+ = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \end{bmatrix}$	$\mathcal{D}_j = \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}$	$\mathcal{T}_i^- = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \end{bmatrix}$	$\mathcal{A}$

Structure of the γγγγ amplitude



3 independent momenta:

$$p = p_2 + p_3$$
$$q = p_3 + p_1$$
$$k = p_1 + p_2$$

6 Lorentz invariants:

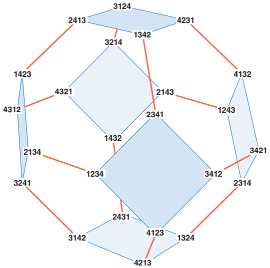
$$p^2, \quad q^2, \quad k^2, \quad p\cdot q, \quad p\cdot k, \quad q\cdot k$$

Bose symmetry:

$$\Gamma^{\mu\nu\rho\sigma}(p,q,k) = \sum_{i=1}^{136} f_i(\dots)\; \tau_i^{\mu\nu\rho\sigma}(p,q,k)$$

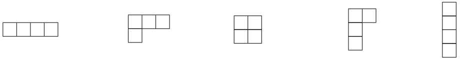
$\overset{!}{=}$ **symmetric**

S4 multiplets



- Arrange the 24 permutations of ψ_{1234} into **multiplets:**

Singlet	**Triplets**	**Doublets**	**Antitriplets**	**Antisinglet**
$\mathcal{S}$	$\mathcal{T}_i^+ = [\,\bullet\;\bullet\;\bullet\,]$	$\mathcal{D}_j = [\,\bullet\;\bullet\,]$	$\mathcal{T}_i^- = [\,\bullet\;\bullet\;\bullet\,]$	$\mathcal{A}$



- **6 Lorentz invariants** form singlet $\mathcal{S}_0$, doublet $\mathcal{D}$, triplet $\mathcal{T}^+$

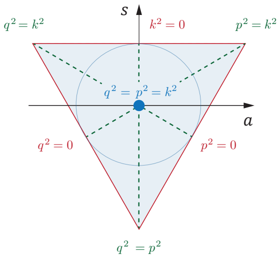
Phase space

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \frac{p^2+q^2+k^2}{4} = \frac{p_1^2+p_2^2+p_3^2+p_4^2}{4}$$

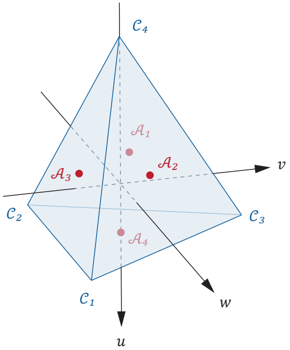
- **Doublet:** $\mathcal{D} = \begin{bmatrix} a \\ s \end{bmatrix}$

  Mandelstam triangle,
  2-photon poles (pion, scalar, axialvector, ...)



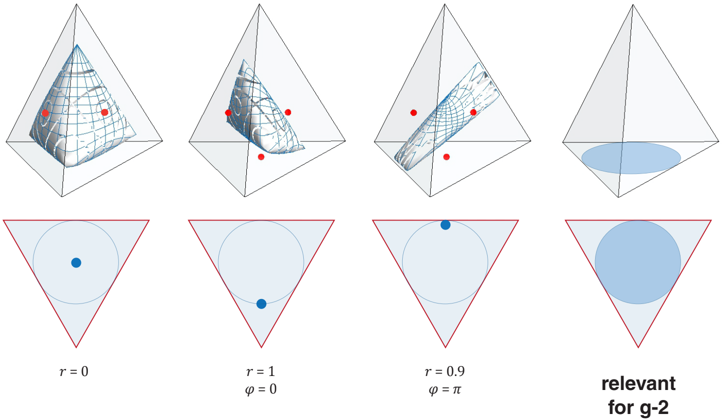
- **Triplet:** $\mathcal{T} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$

  tetrahedron bounded by $p_i^2 = 0$,
  vector-meson poles



Phase space

- fixed doublet variables $\Longrightarrow$ complicated geometric object inside tetrahedron:



Tensor basis I

- construct all possible **multiplets** from generic seed elements: 138 elements, but only **136 independent**

- removing the "wrong ones" leads to **kinematic singularities!**

- Dressing functions form multiplets too $\Longrightarrow$ expand them into **singlets:**

$$\begin{bmatrix} f_1\ (\mathcal{S}_0, \mathcal{D}_0, \mathcal{T}_0) \\ f_2\ (\mathcal{S}_0, \mathcal{D}_0, \mathcal{T}_0) \end{bmatrix} = c_1(\mathcal{S}_0) \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}_1 + c_2(\mathcal{S}_0) \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}_2 + \dots$$

Singlets depend (almost) only on $\mathcal{S}_0$, dependence on  ,  absorbed in basis

This works extremely well!

n	Seed	#	Multiplet type
0	$\delta^{\mu\nu}\delta^{\rho\sigma}$	3	$\mathcal{S},\ \mathcal{D}_1$
2	$\delta^{\mu\nu}\,k^\rho\,k^\sigma$	6	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{T}_1^+$
	$\delta^{\mu\nu}\,p^\rho\,p^\sigma$	12	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{T}_1^\pm,\ \mathcal{A}$
	$\delta^{\mu\nu}\,p^\rho\,q^\sigma$	12	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{T}_1^+,\ \mathcal{T}_2^\pm$
	$\delta^{\mu\nu}\,p^\rho\,k^\sigma$	24	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{T}_1^\pm,\ \mathcal{T}_2^\pm,\ \mathcal{T}_3^\pm,\ \mathcal{A}$
4	$p^\mu\,p^\nu\,p^\rho\,p^\sigma$	3	$\mathcal{S},\ \mathcal{D}_1$
	$p^\mu\,p^\nu\,q^\rho\,q^\sigma$	6	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{T}_1^-$
	$p^\mu\,p^\nu\,k^\rho\,k^\sigma$	10	$\mathcal{S},\ (\mathcal{D}_1,)\ \mathcal{D}_2,\ \mathcal{T}_1^\pm,\ \mathcal{A}$
	$p^\mu\,q^\nu\,k^\rho\,k^\sigma$	12	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{T}_1^+,\ \mathcal{T}_2^\pm$
	$p^\mu\,p^\nu\,p^\rho\,k^\sigma$	24	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{T}_1^\pm,\ \mathcal{T}_2^\pm,\ \mathcal{T}_3^\pm,\ \mathcal{A}$
	$p^\mu\,p^\nu\,q^\rho\,k^\sigma$	24	$\mathcal{S},\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{T}_1^\pm,\ \mathcal{T}_2^\pm,\ \mathcal{T}_3^\pm,\ \mathcal{A}$

Tensor basis I

- construct all possible **multiplets** from generic seed elements: 138 elements, but only **136 independent**

- removing the “wrong ones” leads to **kinematic singularities!**

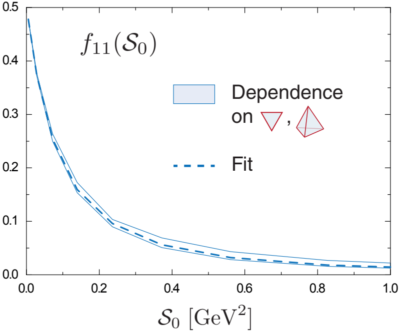
- Dressing functions form multiplets too $\Longrightarrow$ expand them into **singlets:**

$$\begin{bmatrix} f_1 \; (\mathcal{S}_0, \mathcal{D}_0, \mathcal{T}_0) \\ f_2 \; (\mathcal{S}_0, \mathcal{D}_0, \mathcal{T}_0) \end{bmatrix} = c_1(\mathcal{S}_0) \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}_1 + c_2(\mathcal{S}_0) \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}_2 + \dots$$

  Singlets depend (almost) only on $\mathcal{S}_0$, dependence on  ,  absorbed in basis

  This works extremely well!

- reproduces previous DSE results for g-2, √
  but complete Ball-Chiu vertex still erratic... ?



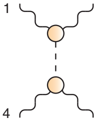
$\longrightarrow$ γγγγ amplitude calculated in **full kinematics!**

Tensor basis II

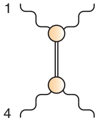
To make **gauge invariance** explicit,
split γγγγ amplitude into

$$\Gamma = \Gamma_{\text{Gauge}} + \Gamma_{\text{Transverse}}$$

- **Transverse part: 41** tensors,
  at least quartic in photon momenta,
  dominant tensors ~ pion + scalar exchange



$\sim \varepsilon_{12}^{\mu\nu}\, \varepsilon_{34}^{\rho\sigma}$



$\sim t_{12}^{\mu\nu}\, t_{34}^{\rho\sigma}$

$\varepsilon_{ab}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}\, a^\alpha\, b^\beta$ $\qquad$ $t_{ab}^{\mu\nu} = a \cdot b\, \delta^{\mu\nu} - b^\mu\, a^\nu$

- **'Gauge part': 95** tensors,
  must vanish if γγγγ amplitude gauge invariant!

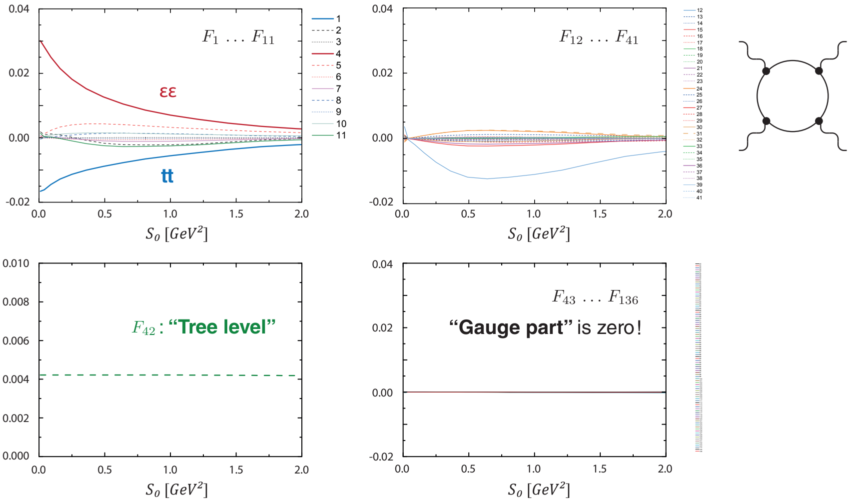
  ... but is it?

n	Seed element	#	Multiplets
4	$t_{12}^{\mu\nu}\, t_{34}^{\rho\sigma}$	3	$\mathcal{S},\, \mathcal{D}_1$
	$\varepsilon_{12}^{\mu\nu}\, \varepsilon_{34}^{\rho\sigma}$	3	$\mathcal{S},\, \mathcal{D}_1$
6	...	...	...
8	...	...	...
0	$\delta^{\mu\nu}\, \delta^{\rho\sigma}$	3	$\mathcal{S},\, \mathcal{D}_1$
2	...	...	...
4	...	...	...

Total: **136**

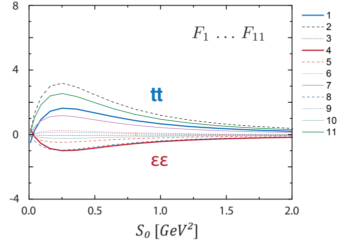
Results: quark loop with m_q = const

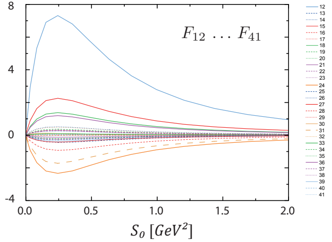
Photon four-point function: S_0 dependence for fixed doublet & triplet variables

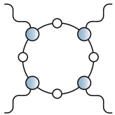


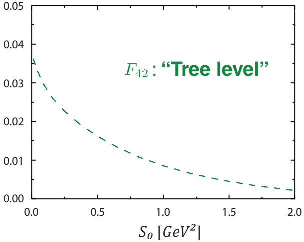
Results: quark loop from DSE

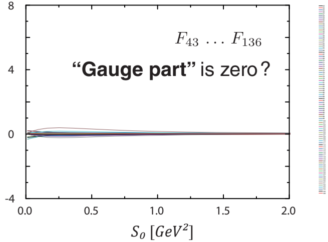
Photon four-point function: S_0 dependence for fixed doublet & triplet variables











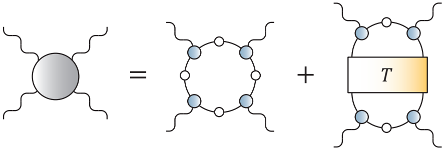
goal:

- find transverse basis w/o kin. singularities
- isolate dominant tensor structures

Gauge invariance

Full γγγγ amplitude at quark level, derived from **gauge invariance:**

GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



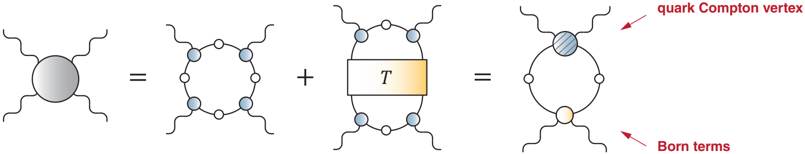
Quark loop + all **2-photon poles** from T-matrix (pion, scalar, axialvector, ...)

- no double-counting!
- gauge artifacts in quark loop must be cancelled by **offshell structure of T-matrix!**

Gauge invariance

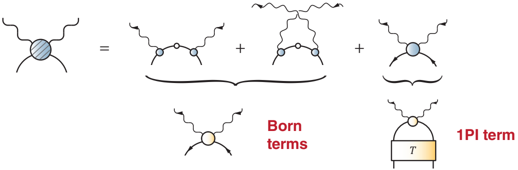
Full γγγγ amplitude at quark level, derived from **gauge invariance:**

GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



Quark loop + all **2-photon poles** from T-matrix (pion, scalar, axialvector, ...)

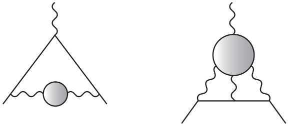
- no double-counting!
- gauge artifacts in quark loop must be cancelled by **offshell structure of T-matrix!**
- Quark Compton vertex already determined from **nucleon Compton scattering:**



GE, Fischer, PRD 87 (2013),
PoS Conf. X (2012)

Summary

Muon g-2: theory uncertainty dominated by **QCD**



$a_\mu\ [10^{-10}]$

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

LBL: need to get QCD contribution under control!

- γγγγ amplitude = **quark loop + T-matrix, no double counting, gauge invariant!**
- need to understand **structure of** γγγγ **amplitude**
- **dressed quarks & vertices** have impact, QCD prediction for LBL may change!

Electron vs. muon g-2

$a_e \; [10^{-10}]$

Exp:	11 596 521.81		
QED:	11 596 521.71 .81	(0.09) (0.08)	Cs Rb
EW:	0.00		
Hadronic:	0.02		
SM:	11 596 521.73 .83	(0.09) (0.08)	Cs Rb

$a_\mu \; [10^{-10}]$

Exp:	11 659 208.9	(6.3)
QED:	11 658 471.9	(0.0)
EW:	15.3	(0.2)
Hadronic:		
• VP (LO+HO)	685.1	(4.3)
• LBL	10.5	(2.6)
SM:	11 659 182.8	(4.9)
Diff:	26.1	(8.0)

Bijnens, Prades, Mod. Phys. Lett. A22 (2007)
Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)
Hagiwara et al., J. Phys. G 38 (2011)