True self energy function, mixing and reducibility in effective scalar theories.

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Effective theory: problem of couplings

- The theory is called effective if the corresponding Hamiltonian in the interaction picture contains all the local monomials consistent with a given linear symmetry.
- For the renormalizability ET requires presence of all types of the counterterms consistent with a given linear (algebraic) symmetry.
- Therefore in ET one needs to fix an infinite number of renormalization prescriptions.
- If this is done the ET is renormalizible.

The effective scattering theory (EST) is nothing but the effective theory only designed for S-matrix calculations. The finiteness of Green functions is not implied.
Parameters: essential and redundant.

- Essential: those defining the S-matrix elements on the mass shell.
- Redundant: all the other parameters that appear in the Green functions.

The EST only requires fixing the essential (on-shell) parameters.
Obvious problem: We need to know the 2-leg function off the mass shell!

Preliminaries.

Let us first consider the simplest effective theory: that containing only one real scalar field $\phi(x)$:

$$\phi(x) = \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2p_0} \left[a^+(p) \exp(ip\,x) + H.c.\right]. \tag{1}$$

$$[a^-(p), a^+(q)]_- = (2\pi)^3\, 2p_0\, \delta(\mathbf{p} - \mathbf{q})\,.$$

The effective Hamiltonian:

$$H(x) = \sum_{n=0}^{\infty} \left[H_n(x) + C_n\right].$$

The counterterm series:

$$C(p^2) = \left[C^{[\log]}(p^2) \cdot \log\Lambda + \sum_{n=0}^{\infty} C^{[n]}(p^2)\Lambda^{2n}\right]. \tag{2}$$

Here Λ is a cutoff parameter and

$$C^{[x]}(p^2) = \sum_{n=0}^{\infty} c_n^{[x]} p^{2n}, \qquad (x = \mathrm{Log}, 0, 1, 2, \ldots).$$

General form of vertices.

General 3-field Hamiltonian:

$$H_3 = \frac{1}{3!}\sum_{s=0} \tilde{D}^{jk;s} : \phi \left(\partial^{[s]}\phi^j\right)\left(\partial_{[s]}\phi^k\right) : \tag{3}$$

General 3-leg vertex in momentum space:

$$V(\kappa_1,\kappa_2,\kappa_3) = i(2\pi)^4\delta(k_1+k_2+k_3)\sum_{i,j,k=0}^{\infty} D^{ijk}(\kappa_1)^i(\kappa_2)^j(\kappa_3)^k \tag{4}$$

Here

$$\kappa_i = p_i^2 - m^2$$

and D^{ijk} are certain functions of $\tilde{D}^{jk;s}$ and the mass m.
The most general 4-field Hamiltonian:

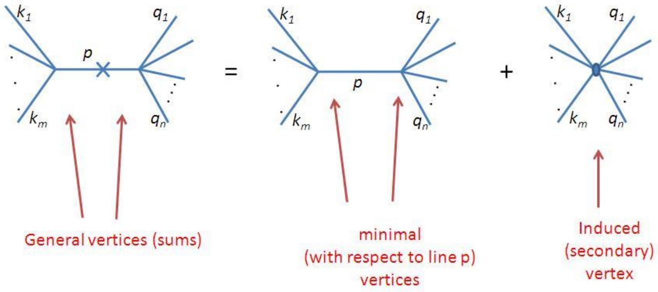
$$H_4 = \frac{1}{4!}\sum_{ijk}^{\infty}\sum_{s_1s_2s_3}^{\infty} \tilde{D}^{\,ijk;s_1s_2s_3} : \phi\left(\partial^{[s_1]}\partial^{[s_2]}\phi^i\right)\left(\partial_{[s_2]}\partial^{[s_3]}\phi^j\right)\left(\partial_{[s_3]}\partial_{[s_1]}\phi^k\right) : .$$

$$\phi^i \stackrel{\text{def}}{=} \underbrace{K\ldots K}_{i\text{ times}}\phi,$$

$$K \stackrel{\text{def}}{=} -(\partial^\mu\partial_\mu + m^2)$$

Reduction of a line

Line may be reduced:



Example: Tree graph constructed from two 3-line vertices.

From Eq. [4]:

$$V(\kappa_1, \kappa_2, \kappa_p) \sim \sum_{i,j,k=0}^{\infty} D^{ijk} (\kappa_1)^i (\kappa_2)^j (\kappa_p)^k$$

$$V(\kappa_p, \kappa_3, \kappa_4) \sim \sum_{i,j,k=0}^{\infty} D^{ijk} (\kappa_3)^i (\kappa_4)^j (\kappa_p)^k$$

Propagator: $\pi(p) = \frac{1}{\kappa_p}$ Therefore the result for the graph (sum of graphs):

$$\sum_{i,j,l,m,=0}^{\infty} (\kappa_1)^i (\kappa_2)^j (\kappa_3)^l (\kappa_4)^m \times$$

$$\left[G^{ij0} G^{lm0} \frac{1}{\kappa_p} + \sum_{r=1}^{\infty} \kappa_p^{r-1} \sum_{k=0}^{r} G^{ijk} G^{lm(r-k)} \right]$$

On the mass shell (S-matrix graph): $i = j = k = l = 0$. Therefore the contribution to S matrix reads

$$\left[G^{000} G^{000} \frac{1}{\kappa_p} + \sum_{r=1}^{\infty} \kappa_p^{r-1} \sum_{k=0}^{r} G^{00k} G^{00(r-k)} \right]$$

The propagator line turns out reduced in all terms except the first one due to the presence of nonminimal vertices.
The line is called minimal if it connects two minimal (with respect to this line!) vertices.
Definition: Graph is minimal if all its lines are minimal (reduced).

Problem: what about the notion of one-particle reducibility (1PR)?
Two kinds of 1PR: graphical (G1PR) and analytical (A1PR).
The A1PI subgraphs only require the minimal counterterms.
Definition: Graph is minimal if all its lines are minimal (reduced).

The two-leg Green function in effective single-scalar theory.

The conventional procedure:

$$G_2 = \pi + \pi S_2 \pi + \pi S_2 \pi S_2 \pi + \ldots = \frac{\pi}{1 - \pi S_2}$$

In momentum space:

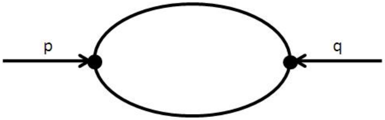
$$G_2(k) = \frac{1}{k^2 - m^2 - S_2(k)}$$

This procedure is only correct if

$$|\pi S_2| < 1, \qquad \forall p^2 : \quad 0 \leqslant p^2 \leqslant \infty.$$

In effective theories this is not the case!

The one loop two-leg function in effective theory



From the relation [3] it follows

$$S_2(p,q) = \delta(p+q) \sum_{ijklmn=0}^{\infty} D^{ijk} D^{lmn} \kappa_p^i \kappa_q^l J_{ikmn}(p^2) + C(p^2,\Lambda),$$

where

$$J_{ikmn}(p^2) = \int dr\, dt\, \delta(p+r-t)\, \delta(q+t-r) \kappa_t^{j+n-1} \kappa_r^{k+m-1}.$$

The only meaningful finite contribution corresponds to

$$J(p^2) \equiv J_{0000}(p^2) = -\delta(p+q) \int dr \, \frac{p^2 + 2rp}{(r^2-m^2)^2 \cdot [(r+p)^2 - m^2]}.$$

The others are absorbed by counterterms.
So we have:

$$S(p^2) = \sum_{il=0}^{\infty} D^{i00} D^{l00} \kappa_p^i \kappa_p^l J(p^2) + \sum_{n=0}^{\infty} c_n (p^2)^n \,.$$

So we have:

$$S(p^2) = \sum_{il=0}^{\infty} D^{i00} D^{l00} \kappa_p^i \kappa_p^l J(p^2) + \sum_{n=0}^{\infty} c_n (p^2)^n \,.$$

With

$$G^i = \sum_{k=0}^{i} D^{k00} D^{(i-k)00}$$

we obtain

$$S(p^2) = \sum_{i=0}^{\infty} G^i J(p^2) {\kappa_p}^i + \sum_{i=0}^{\infty} \widetilde{d}_i {\kappa_p}^i \,.$$

The unknown constants $\widetilde{d}_i$ are to be fixed by the corresponding RP's.
Problem: there are only two physically motivated RP's while the number of $\widetilde{d}_i$ is actually infinite.
So, $S(p^2)$ cannot be reasonably fixed.
The solution to this problem is simple: To calculate the S matrix in renormalized perturbation scheme one does not need to formulate the RP's for the constants $\widetilde{d}_i$. with $i \geqslant 2$.

The true self-energy function

The physically acceptable form of the two-point Green function reads:

$$G_2(p^2) = \frac{1}{p^2 - m^2 - \Sigma(p^2)} + Q(p^2).$$

Here it is implied that

$$Q(m^2) = 0; \qquad \Sigma(m^2) = \Sigma'(m^2) = 0,$$

and (the essential requirement of spectral representation)

$$\left.\frac{\Sigma(p^2)}{p^2}\right|_{p^2 \to \infty} \to \text{const}.$$

Such $\Sigma(p^2)$ may be called the true self energy function.

Let us rewrite:

$$S(\kappa_p) = \Sigma(\kappa_p) + \sum_{j=1}^{\infty} \kappa_p^j R^j(\kappa_p) + \sum_{j=2}^{\infty} \kappa_p^j d_j \,. \qquad (5)$$

Here

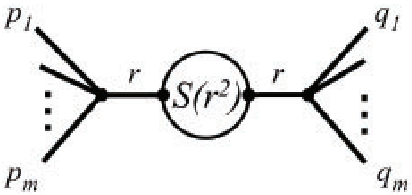
$$\overline{J}(\kappa_p) \equiv J(p^2) - J(m^2)\,, \qquad R^j(\kappa_p) \equiv G^j \overline{J}(\kappa_p)\,,$$

and

$$\Sigma(\kappa_p) \equiv G^0 \left(\overline{J}(\kappa_p) - \kappa_p \overline{J}'(\kappa_p) \right).$$

$\Sigma(\kappa_p)$ is the true SEF (TSEF).
It meets all axiomatic requirements and contains no redundant parameters. At the same time when calculating the S-matrix graphs one has to use $S(\kappa_p)$, the parameters d_i with $i \geqslant 2$ being unfixed!
In what follows I will show that these parameters only appear in the A1PI graphs with $n \geqslant 3$ legs; they can be fixed at the next steps of renormalization of one-loop graphs.
Note that in external lines the contribution from $S(\kappa)$ may be just neglected.

Internal lines.



This is G1PR graph. It contains the A1PI parts. Indeed, the corresponding analytical expression reads

$$\Gamma(p_1,\ldots,p_m;q_1,\ldots,q_n) = V_1(p_1,\ldots,p_m,r)\frac{1}{\kappa_r}S(r)\frac{1}{\kappa_r}V_2(q_1,\ldots,q_n,r)\,.$$

Let us rewrite this expression as follows:

$$\begin{aligned}\Gamma(p,q) &= V_1(p,r)\frac{1}{\kappa_r}\left[\Sigma(\kappa_r)+\sum_{j=1}^{\infty}\kappa_r^j R^j(\kappa_r)+\sum_{j=2}^{\infty}\kappa_r^j d_j\right]\frac{1}{\kappa_r}V_2(q,r) = \\ &= V_1(p,r)\frac{1}{\kappa_r}\Sigma(\kappa_r)\frac{1}{\kappa_r}V_2(q,r)+\sum_{j=2}^{\infty}d_j\left[V_1(p,r)\kappa_r^{j-2}V_2(q,r)\right]+ \\ &+\sum_{j=2}^{\infty}G^j\overline{J}(\kappa_r)\left[V_1(p,r)\kappa_r^{j-2}V_2(q,r)\right]+ \\ &+2D^{000}D^{100}\left[V_1(p,r)\overline{J}(\kappa_r)V_2(q,r)\right]\frac{1}{\kappa_r}\,.\end{aligned} \tag{6}$$

Here the first term in the RHS is just a familiar A1PR graph with 1-loop insertion, the loop being constructed from two minimal vertices of the type ϕ^3.
The second one is the sum of local vertices with the loop index $l=1$ and $(m+n)$ lines; they can be absorbed by the corresponding one-loop counterterms and fixed at the next steps of renormalization.

The third item in the RHS of (6) presents a new element which has no analog in conventional renormalizable theories. It can be treated as the *nonlocal vertex* ("countervertex" or " counter-graph) with loop index $l = 1$ and $(m+n) \geqslant 4$ lines. At last, the 4th and 5th item together show that it emerges one more type of counter-vertices with $n \geqslant 3$ legs.
Important: the TSEF $\Sigma(k^2)$ only depends on minimal parameter D^{000}; it does not require introducing the additional RP's.
If we rely upon the G1PR concept we would need to fix all the nonminimal counterterms for which we have no physical RP's. When using of A1PR concept we avoid this problem.
So, the one-loop renormalization of 2-leg graphs in effective theory is done. The problem of fixing the non-minimal parameters turned out shifted to the next step of renormalization procedure (renormalization of $n-$leg graphs with $n \geqslant 3$). It can be shown that the same phenomenon happens in renormalization of 3-leg graphs.

The diagonalization problem (mixing).

Let us now discuss another problem that occurs in multi-component effective scalar theory: mixing. Let us consider the effective theory that contains a set (possibly infinite) of real scalar fields ϕ_k $(k = 1, 2, \ldots)$:

$$\phi_c(x) = \frac{1}{(2\pi)^3} \int \frac{d^3p}{2p_0} \, [a_c^+(p) \exp(ip\,x) + H.c.],$$

with the conventional commutation relations

$$[a_r^-(p), a_k^+(q)]_- = (2\pi)^3 \, 2p_0 \; \delta_{rk}\delta(\mathbf{p} - \mathbf{q})$$

$(p_0 = \sqrt{\mathbf{p}^2 + m_k^2})$. Here m_a stands for the mass parameter of the particle a; this parameter is just the real part of the pole position of the full propagator. It is implied that $m_k > m_p$ when $k > p$.

$$H(x) = \sum_{n=0}^{\infty} \left[H_n(x) + C_n \right],$$

where $H_n(x)$ is (just as above) an infinite sum of all Lorentz-invariant monomials constructed from the fields and their derivatives of arbitrary orders, and C_n stand for the counterterms.

The most general triple interaction Hamiltonian density may be written as follows:

$$H_3 = \frac{1}{3!} \sum_{abc} \sum_{s=0} \tilde{D}^{jk;s}_{abc} : \phi_a \left(\partial^{[s]} \phi^j_b \right) \left(\partial_{[s]} \phi^k_c \right) : , \tag{7}$$

where : . . . : denotes the normal product,

$$\phi^i_a \stackrel{\text{def}}{=} K^i_a \phi_a \ ,$$

$$K_a \stackrel{\text{def}}{=} -(\partial^\mu \partial_\mu + m_a^2) \ ,$$

$$K^i_a \stackrel{\text{def}}{=} \underbrace{K_a \ldots K_a}_{i \text{ times}} ,$$

and $\tilde{D}^{jk;s}_{abc}$ are real (dimensional) coupling constants. The symbol $\sum_{abc}$ is used for the sum over the whole set of particles under consideration.
In momentum space the Feynman rules needed to write down the 2-leg graphs are constructed from the elements of bare propagator which is the diagonal matrix π:

$$\pi_{ab}(k) = \delta_{ab} \frac{1}{\kappa_a} \equiv \delta_{ab} \frac{1}{k^2 - m_a^2} \ ,$$

and the vertices of the form

$$V_{abc}(\kappa_a, \kappa_b, \kappa_c) = i(2\pi)^4 \delta(k_a + k_b + k_c) \sum_{i,j,k=0}^{\infty} D^{ijk}_{abc} (\kappa_a)^i (\kappa_b)^j (\kappa_c)^k \, . \tag{8}$$

Here

$$\kappa_x = \kappa_x(k) \equiv k^2 - m_x^2 \, ,$$

and D^{ijk}_{abc} are certain sums constructed from the coupling constants $\tilde{D}^{jk;s}_{abc}$ and masses.

The most general expression for the one-loop two point function reads (both lines a and b are considered incoming):

$$S_{ab}(k_a^2) = \sum_{ef} \left[\int dk_e dk_f \delta(k_a + k_e - k_f)\delta(k_b + k_f - k_e) \times \right.$$
$$\left. \times \sum_{\substack{ijk=0 \\ lmn=0}}^{\infty} D_{aef}^{ijk} D_{bfe}^{lmn} \frac{\kappa_a^i \kappa_e^j \kappa_f^k \kappa_b^l \kappa_f^m \kappa_e^n}{\kappa_e \kappa_f} + C_{abef}\ \delta(k_a + k_b) \right] .$$

Here the summation $\sum_{ef}$ is done over the whole set of particles that create the loop and C_{abef} stands for the counterterm series:

$$C_{abef}(q^2) = \left[C_{abef}^{[\log]}(q^2) \cdot \log\Lambda + \sum_{n=0}^{\infty} C_{abef}^{[n]}(q^2)\Lambda^{2n} \right], \tag{9}$$

where Λ is the cutoff parameter and every $C_{abef}^{[x]}(q^2)$ $(x = \log, 0, 1, \ldots)$ is just a power series in q^2.

The finite expressions for the individual items read:

$$S_{abef}(q^2) = \sum_{i,l=0}^{\infty} \kappa_a^i \kappa_b^l D_{aef}^{i00} D_{bfe}^{l00} J_{ef}(q^2) + \sum_{p=0}^{\infty} \tilde{C}_{abef}^{p}(q^2)^p . \tag{10}$$

Here

$$J_{ef}(k^2;m_e^2,m_f^2) \stackrel{\text{def}}{=} \frac{1}{2}\left[F_{ef}+F_{fe}\right], \tag{11}$$

and

$$F_{fe}(k^2;m_f^2,m_e^2) = -\int dq\, \frac{k^2+2qk}{(q^2-m_f^2)(q^2-m_e^2)[(q+k)^2-m_e^2]}\,. \tag{12}$$

Let us present the series of finite counterterms in the equivalent (though more complex) form that is most suitable for subsequent calculations:

$$\sum_{p=0}^{\infty} \tilde{C}^{p}_{abef}(q^2)^p = q^2\, C_{abef} + \sum_{i,l=0}^{\infty} S^{il}_{abef}\kappa_a^i\kappa_b^l. \tag{13}$$

Now the expression (10) can be rewritten as follows:

$$\begin{aligned} S^{[R]}_{abef}(q^2) =& \Sigma_{ab}(ef|q^2) + \sum_{i,l\geqslant 1}^{\infty}\sum_{ef}\kappa_a^i\kappa_b^l\{G^{il}_{abef}J_{ef}(q^2)+S^{il}_{abef}\} + \\ &+\sum_{i=1}^{\infty}\sum_{i=1}^{\infty}\left\{\left[\kappa_a^i(G^{i0}_{abef}J_{ef}(q^2)+S^{i0}_{abef}\right]+\left[\kappa_b^i(G^{0i}_{abef}J_{ef}(q^2)+S^{0i}_{abef}\right]\right\} \end{aligned} \tag{14}$$

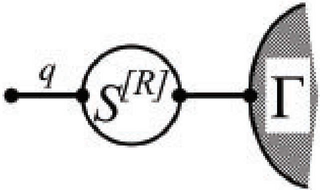
Here it is defined the object $\Sigma_{ab}(ef|q^2)$, hereafter referred to as the self-energy matrix (SEM):

$$\Sigma_{ab}(ef|q^2) \stackrel{\text{def}}{=} G^{00}_{abef} J_{ef}(q^2) + S^{00}_{abef} + q^2 C_{abef} \,. \tag{15}$$

Below it will be shown that the diagonal elements of this matrix play the role of TSE functions for the corresponding particles. To proceed further one needs to fix the unknown coefficients in (14). The above-obtained results suggest that perhaps not all the coefficients are needed for the renormalization of 2-leg insertion in the lines of S-matrix graphs. So, first we need to understand what very coefficients should be fixed. For this we should turn to the physical interpretation of the external lines of Green function graphs. The interpretation is based on the LSZ formula. In short, the external line a with the momentum q corresponds to the particle with the mass parameter m_a if the relevant Green function develops the only (simple) pole at $q^2 = m_a^2$. Once there is another pole, say, at $q^2 = m_b^2$, (or the pole at $q^2 = m_a^2$ is not simple) the interpretation becomes ambiguous. Let us consider the one-loop level of a certain Green function graph with a given external line which we would like to interpret as that corresponding to the particle with mass m_a. The interpretation problem appears when this graph is one-particle-reducible (1PR) and the line in question (the a-line with momentum q) contains the one-loop self energy insertion $S_{abef}(q^2)$. The analytical expression for the Green function graph (in fact, this is a sum of individual graphs) under consideration reads:

$$G_{a\ldots} = \frac{1}{q^2 - m_a^2} \sum_{bef} S^{[R]}_{abef}(q^2) \frac{1}{q^2 - m_b^2} \Gamma_{b\ldots} \,, \tag{16}$$

where $\Gamma_{b\ldots}$ stands for the remaining (loopless) part of the graph (ellipsis stands for the indices corresponding to another external lines). The summation indices b, e, f run over the whole set of particles.



The form (16) (with (10) taken into account) clearly demonstrates the presence of many poles in addition to that at $q^2 = m_a^2$. The extra poles arise from the terms with $l = 0$ and arbitrary i in the first item of (10). Moreover, when $i = l = 0$ the pole at $p^2 = m_a^2$ is of the second order! This means that in contrast to initially suggested identification we cannot uniquely associate the dressed external line (that with 2-leg insertion $S(q^2)$) with any concrete particle.
In the framework of the renormalized perturbation scheme the solution to this problem consists of imposing the following set of limitations on the non-diagonal elements $(a \neq b)$:

$$\begin{cases} \lim\limits_{q^2 \to m_a^2} S_{abef}(q^2) = O \\ \lim\limits_{q^2 \to m_b^2} S_{abef}(q^2) = O \end{cases} \tag{17}$$

and

$$\begin{cases} \lim\limits_{q^2 \to m_a^2} S_{aa,ef}(q^2) = O \\ \lim\limits_{q^2 \to m_a^2} \dfrac{\partial}{\partial q^2} S_{aa,ef}(q^2) = O \end{cases} \tag{18}$$

on the diagonal ones. These conditions present the diagonalizability requirements. In the case of unstable particles these limitations should be imposed on the real parts. The restrictions (17) ensure that the graph (16) does describe the interaction of the field ϕ_a associated with the particle m_a. The first one of the restrictions (18) is nothing but the conventional RP that fixes the value m_a of the particle a mass parameter. The second provides a guarantee that the wave function is properly normalized. The prescriptions (18) are also suitable when there is only one particle in a theory. The RP's (17) and (18) are necessary to assign meaning to the effective scattering theory.

The shortened notation:

$$G^{il}_{abef} = D^{i00}_{aef} D^{l00}_{bfe}.$$

Now let us turn to the expression (14) and see what very coefficients are fixed by the conditions (17) and (18). The conditions (17) give:

$$\begin{aligned}
S^{00}_{abef} &= -\, G^{00}_{abef} \frac{m_b^2 J_{ef}(m_a^2) - m_a^2 J_{ef}(m_b^2)}{m_b^2 - m_a^2}, \\
C_{abef} &= -\, G^{00}_{abef} \frac{J_{ef}(m_a^2) - J_{ef}(m_b^2)}{m_a^2 - m_b^2}, \\
S^{0i}_{abef} &= -\, G^{0i}_{abef} J_{ef}(m_a^2), \\
S^{i0}_{abef} &= -\, G^{i0}_{abef} J_{ef}(m_b^2).
\end{aligned} \tag{19}$$

Similarly, the conditions (18) give:

$$\begin{aligned}
S^{00}_{aaef} &= -\, G^{00}_{aaef} \left[J_{ef}(m_a^2) - m_a^2 J^{'}_{ef}(m_a^2) \right], \\
C_{aaef} &= -\, G^{00}_{aaef} J^{'}_{ef}(m_a^2) \,.
\end{aligned} \tag{20}$$

It can be easily shown that (20) follows from the two upper lines of (19) in the limit $m_a \to m_b$. This means that the diagonal elements of MSE, indeed, play the role of the safe energy functions corresponding to individual particles. The above-obtained results allow one to state that the insertion $S(q^2)$ in external line of S-matrix graph makes no influence on the amplitudes of physical processes. One can simply neglect them.
The influence of S^{il}_{abef} on the internal line of S-matrix graph can be analyzed precisely in the same way as in the case of the single-component theory.
It is obvious that the terms in the last line of (14) are insensitive to the constraints (17) and (18). Clearly these terms play a role which is quite similar to that of corresponding terms in the last line of (6). The finite 2-leg counterterms S^{il}_{abef} with $i, l \geqslant 1$ should be dropped because they are absorbed by the n-leg counterterms with $n \geqslant 4$ that will be fixed at the next steps of the renormalization procedure. In contrast, the terms

$$\sum_{i=1}^{\infty} \left\{ \left[\kappa^i_a (G^{i0}_{abef} J_{ef}(q^2) + S^{i0}_{abef} \right] + \left[\kappa^i_b (G^{0i}_{abef} J_{ef}(q^2) + S^{0i}_{abef} \right] \right\}$$

must be taken into account because they define the nonlocal counter-vertices with three legs. Similarly, the terms

$$\kappa^i_a \kappa^l_b G^{il}_{abef} J_{ef}(q^2)$$

with $i, l \geqslant 1$ define the nonlocal counter-vertices with $n \geqslant 4$ legs.

Thus the renormalization of 2-leg insertions in the lines of S-matrix graph of the single- and multi- component effective scalar theories is completed. The result of one-loop dressing of a line is finite and only depends on minimal parameters. It is important to stress that the problem of dependence on non-minimal parameters turns out shifted to the next step of renormalization procedure.

Conclusions.

The attractive features of effective field theories have been demonstrated already in many papers[1]. Unfortunately, the phenomenological advantage of such theories turns out strongly limited by the "problem of couplings": the number of unknown phenomenological constants catastrophically increases with the number of loops taken into account.

Meanwhile the relations obtained in [1-4] clearly demonstrate that the concept of the effective scattering theory together with the quasi-particle method result in quite reasonable sum rules (bootstrap relations) connecting among themselves the values of hadron masses and on-shell coupling constants which are nothing but the right sides of the renormalization prescriptions. This means that the solution to the problem of couplings requires developing the suitable renormalization procedure. The very first step on this way is done in the paper [1]. This talk presents just a review of the results obtained in that paper.

The main result obtained above is that there is no need in attracting the renormalization prescriptions for the higher derivatives of 2-leg graph; it turns out quite sufficient to rely upon the requirements of finiteness and diagonalizability.

Another – no less interesting – result is the demonstration of the difference between the notions of graphical and analytical irreducibility. In fact this result shows that until the complete reduction of a given graph is done there is no sense to single out the 1PI subgraphs. This, in particular, allows one to avoid the contradiction with limitations imposed by Källen-Leman representation. Of course, this preserves the correct loop counting.

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[1] The excellent review can be found in [12].