Holographic Estimates of the Deconfinement Temperature

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XIth Quark Confinement and the Hadron Spectrum
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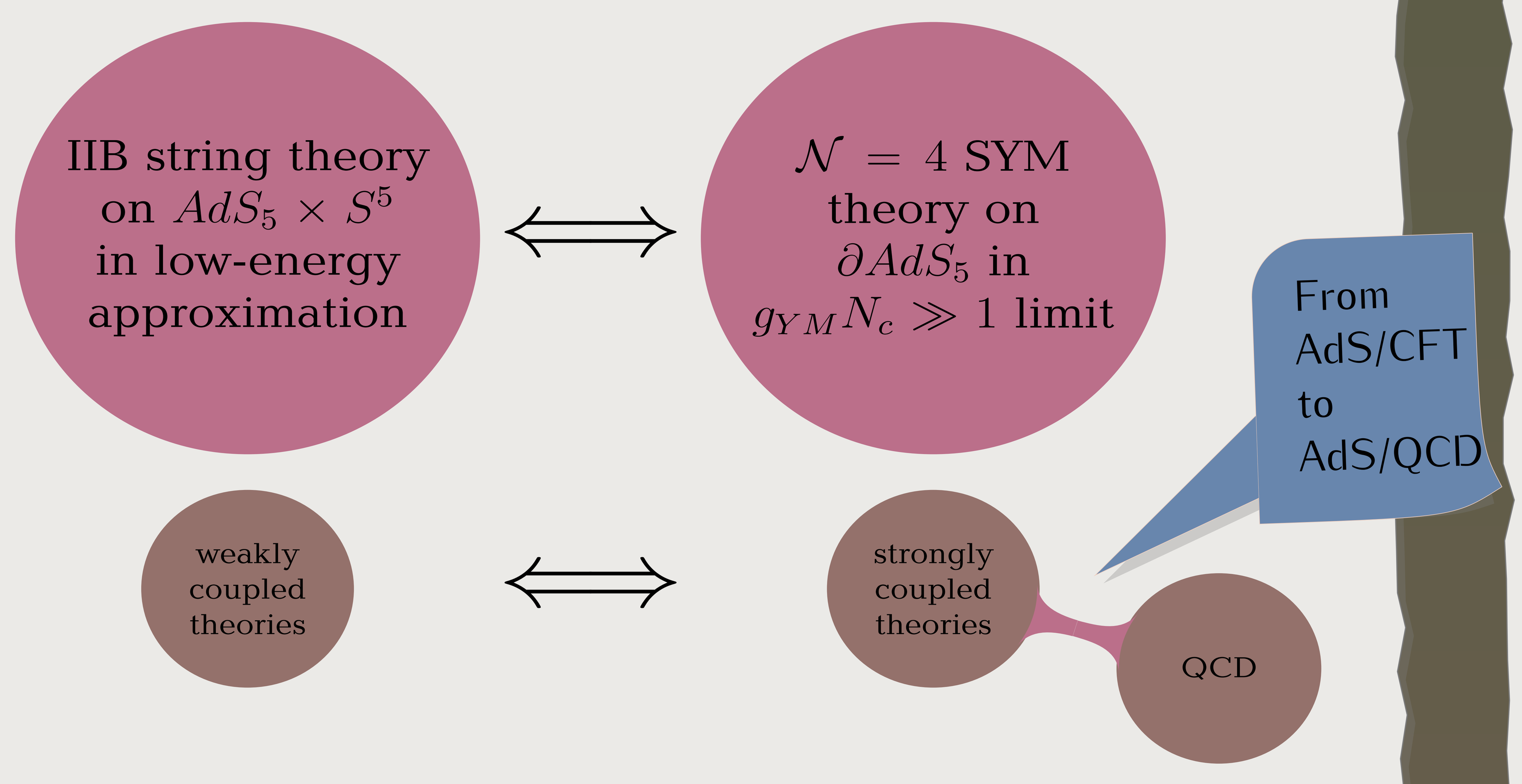
For more detailed discussion see our paper: arXiv:1408.6935 [hep-ph]

The AdS/QCD correspondence



The AdS/QCD correspondence proved to be a fruitful way to study QCD phenomena.
In particular, a rather simple method for calculating **the temperature of deconfinement T_c** was proposed by Herzog[a] within the bottom-up approach to QCD, in which the deconfinement was related to **a Hawking-Page phase transition** between a low temperature thermal AdS space and a high temperature black hole in AdS/QCD models.

[a] C. P. Herzog, Phys. Rev. Lett. **98**, 091601 (2007) [hep-th/0608151].

The Soft Wall model

In the SW model[a] the gravitational part of the action of the dual theory on-shell has the form:

$$I = \kappa \int d^4x dz e^{-\Phi} \sqrt{g},$$

where the dilaton profile $\Phi = az^2$ (Φ is assumed not to affect the gravitational dynamics of the theory). This part yields the leading contribution to the full action in the large-N_c counting ($\kappa \sim N_c^2$ while the mesonic part scales as N_c).
The on-shell gravitational action is the same for two metrics:

(1) thermal AdS: $ds^2 = \frac{L^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right)$,

(2) AdS with a black hole: $ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{f(z)}\right)$,

where $f(z) = 1 - (z/z_h)^4$ and L denotes the AdS radius.

The Hawking temperature is related to the black hole horizon z_h via the relation $T = 1/(\pi z_h)$.
The free action densities V identified with the regularized action I are:

$$V_{\text{Th}}(\epsilon) = \kappa L^5 \int_0^{\beta} dt \int_{\epsilon}^{\infty} e^{-\Phi} z^{-5} dz, \qquad V_{\text{BH}}(\epsilon) = \kappa L^5 \int_0^{\pi z_h} dt \int_{\epsilon}^{z_h} e^{-\Phi} z^{-5} dz.$$

The two geometries are compared at a radius $z = \epsilon$ where the periodicity in the time direction is locally the same $\Rightarrow \beta = \pi z_h \sqrt{f(\epsilon)}$.
The order parameter in the phase transition is defined by ΔV:

$$\Delta V = \lim_{\epsilon \to \infty} (V_{\text{BH}}(\epsilon) - V_{\text{Th}}(\epsilon)) = \frac{\pi \kappa L^5}{4 z_h^3}\left[e^{-az_h^2}(az_h^2 - 1) + \frac{1}{2} - (az_h^2)^2 \int_{az_h^2}^{\infty} \frac{dt}{t} e^{-t}\right].$$

The Hawking-Page phase transition occurs at a point where $\triangle V = 0$
Numerical calculation gives the corresponding critical temperature:

$$T_c \approx 0.49\sqrt{a}$$

[a] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [hep-ph/0602229].

Choice of the slope parameter

In the SW model the vector spectrum has the linear Regge like form:

$$m_n^2 = 4a(n+1), \qquad n = 0, 1, 2, \ldots.$$

Original Herzog's proposal:
$\sqrt{a} = 338$ MeV from the identification of the ground ($n = 0$) state with the ρ-meson, as a result

$$T_c \approx 0.246 m_\rho = 191 \text{ MeV}.$$

Our proposal: take the mean slope $4a = 1.14$ GeV^2, then

$$T_c \approx 263 \text{ MeV}.$$

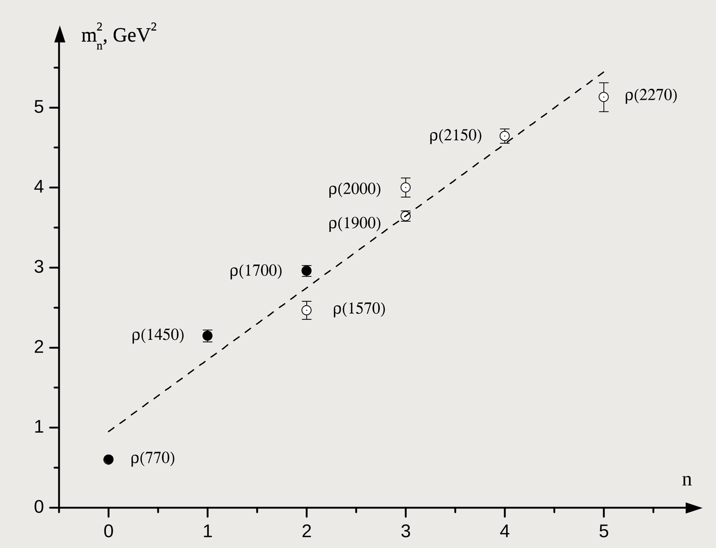


Figure 1: Our assignment of a radial number n to the ρ-mesons. Includes well established (filled) and poorly established (not filled) states from Particle Data Group, Phys. Rev. D **86**, 010001 (2012).

In the presence of massive quarks, the deconfinement phase transition at vanishing chemical potentials represents a cross-over occurring in some range of temperatures.
Recent results for T_c on the lattice with **physical quarks** converge to the range **150-170 MeV** [a] or **154 ± 9 MeV** [b].

[a] S. Borsanyi *et al.* [Wuppertal-Budapest Collaboration], JHEP **1009**, 073 (2010)
[b] A. Bazavov, T. Bhattacharya, M. Cheng, C. DeTar, H. T. Ding, S. Gottlieb, R. Gupta and P. Hegde *et al.*, Phys. Rev. D **85**, 054503 (2012).

The Generalized Soft Wall model

Realistic phenomenological spectra have the form:

$$m_n^2 = 4a(n+1+b), \qquad n = 0, 1, 2, \ldots$$

with both the slope a and the intercept b parameters.
The Generalized SW Model[a], leading to this spectrum, requires the dilaton:

$$\Phi = az^2 - 2\ln U(b, 0; az^2),$$

here U denotes the Tricomi hypergeometric function ($U(0,0;x) = 1$).
We start the analysis of the model with determination of the ΔV, which turns out to be

$$\Delta V = \frac{\pi\kappa L^5}{4z_h^3}\Big\{e^{-az_h^2}U^2(b,0;az_h^2)(az_h^2-1) + 2baz_h^2 e^{-az_h^2}U(b,0;az_h^2)U(1+b,1;az_h^2) + \frac{1}{2\Gamma(1+b)} -$$
$$-(az_h^2)^2 \int_{az_h^2}^{\infty} \frac{dt}{t} e^{-t}\left[U^2(b,0;t) + 4bU(1+b,1;t)U(b,0;t) + 2b^2U^2(1+b,1;t) + 2b(1+b)U(2+b,2;t)U(b,0;t)\right]\Big\}$$

The equation $\Delta V = 0$ yields az_h^2 at a given b, after that T_c is determined from the relation $T_c = (\pi z_h)^{-1}$.
The deconfinement temperature depends now not only on the slope parameter a but also on the intercept parameter b.
The dependance on the parameter b is practically linear $T/\sqrt{a} = 0.496 + 0.670b$ for $b \gtrsim -0.3$ as is clear from Fig. 2.
One can fix some value of T_c and find a parametric curve on the (a, b) plane corresponding to the given T_c as is shown in Fig. 3. The points on (or close to) this curve correspond to the choices of a and b at which the GSW model reproduces more or less physical value of T_c in gluodynamics.
We can also consider **the prediction of T_c from a realistic vector spectrum**. For this purpose, we need to extract the parameters a and b from the ρ or ω spectrum. The extracted values strongly depend on the choice of data and on the weight of each state in the fit. In this situation, the account for experimental errors in the mass determination is not very informative since, in practice, such errors are subleading in the final fit. We will take the central values of masses and the predicted T_c should be regarded as an estimate.
We analyze how different hypotheses on the choice of data for interpolating the linear trajectory influence on the predicted value of T_c.

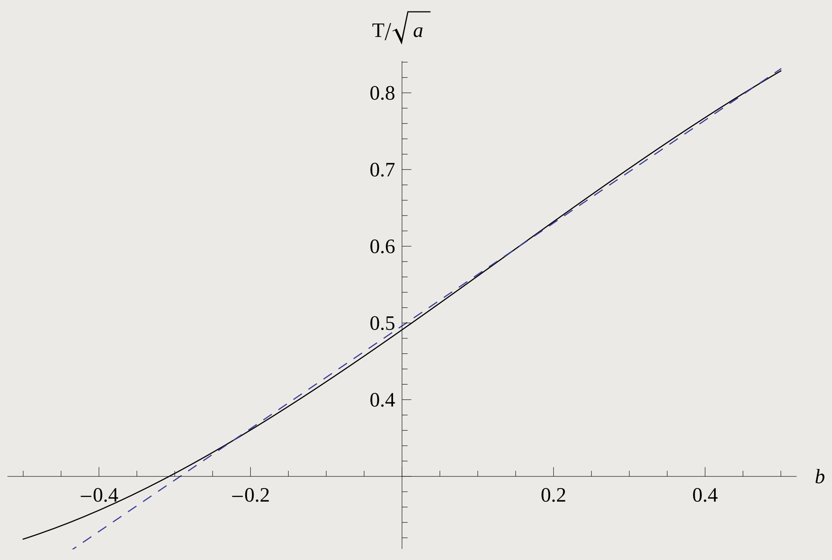


Figure 2: The dependence of $T_c/\sqrt{a}$ on b. The dotted line shows the interpolation.

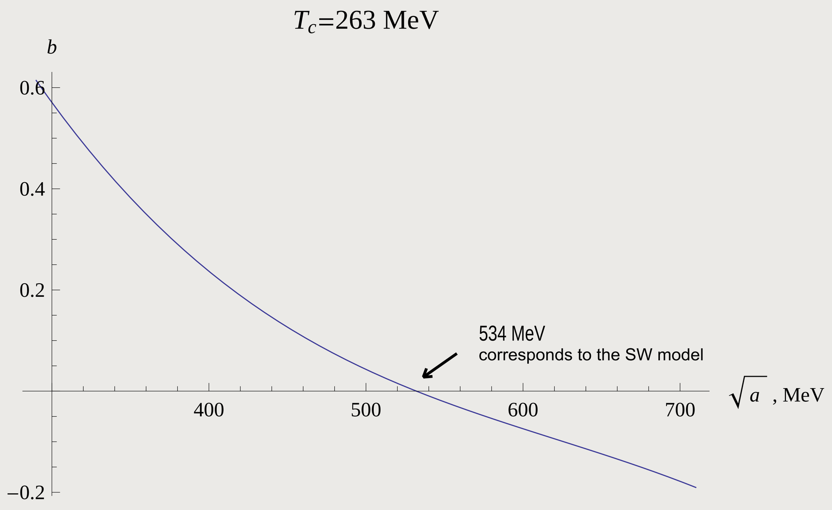


Figure 3: The parametric curve on the (a, b) plane corresponding to $T_c = 263$ MeV.

Particle	Radial states	m_n^2, GeV^2	T_c, MeV
ρ	$n = 0, 1, 2$	$1.18(n + 0.61)$	143
ω	$n = 0, 1, 2$	$1.09(n + 0.66)$	149
ρ	$n = 0, 1, 2, 3, 4$	$0.99(n + 0.89)$	207
ω	$n = 0, 1, 2, 3, 4$	$1.03(n + 0.74)$	166
ρ	$n = 0, 1, 2, 4, 5$	$0.88(n + 1.12)$	270
ω	$n = 1, 2, 3, 4$	$0.95(n + 1.04)$	255

[a] S. S. Afonin, Phys. Lett. B **719**, 399 (2013) [arXiv:1210.5210 [hep-ph]].

T_c is standardly measured in units of the string tension σ, obtained from the linear behavior of the potential between two static quarks at a large separation.

Comparison with lattice results

In AdS/QCD the gravitational part of the holographic action is dual to **pure gluodynamics** in the large-N_c limit.
$\Rightarrow$ T_c must be compared with the lattice results for gluodynamics (i.e. with **non-dynamical quarks**) extrapolated to large N_c.
Such an extrapolation was carried out[a] resulting in:

$$T_c/\sqrt{\sigma} = 0.5949(17) + 0.458(18)/N_c^2.$$

With $\sqrt{\sigma} = 420$ MeV (the value used in most lattice simulations), this extrapolation leads to **$T_c = 250$ MeV** in the large-N_c limit. For $N_c = 3$, one has **$T_c = 271$ MeV**.

[a] B. Lucini, A. Rago and E. Rinaldi, Phys. Lett. B **712**, 279 (2012) [arXiv:1202.6684 [hep-lat]].

Conclusions

The main result of this work is the prediction of the deconfinement temperature from different bottom-up holographic models. More precisely:

- We have reanalysed the simplest SW model, arguing that m_ρ seems not to be a good quantity for predicting T_c. Also, we wish to emphasize that the predicted T_c must refer to the deconfinement phase transition in the pure gluodynamics;
- We have shown that if the soft wall model is accommodated for the description of realistic vector spectra, T_c becomes ambiguous mostly because of lack of sufficient amount of reliable experimental data on the radially excited light mesons. The use of well established states results in T_c close to a cross-over transition in the lattice simulations with dynamical quarks.

Comparison (and often good agreement) of our predictions with the recent lattice results allows us to state at least that the holographic trick seems to pass an important phenomenological test.