High Energy evolution at NLO

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Inspired by Ian Balitsky

Open questions/Goals

- **What is the high energy limit of QCD?**
  many candidates: BFKL Pomeron Calculus, Lipatov's effective action, elements of Field Theory of Bartels, JIMWLK+KLWMIJ Hamiltonians for Wilson line operators....

- **How does the unitarity of QCD get manifested in high energy scattering amplitudes? Is it possible to rigorously derive an effective theory of QCD in terms of color singlet exchange amplitudes?**

- **How do gluon densities grow with energy? Do they saturate? Scales?**

- **What are applicability limits of factorization theorems?**

- **What are final states in collisions of dense objects (jets, multiplicities, correlations)?**

- **How to get thermalization in high energy collisions of very dense objects (nuclei)?**

High Energy Scattering

Target (ρ^t) **Projectile** (ρ^p)

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle$$

S-matrix:

$$\mathbf{S(Y)} \;=\; \langle \mathbf{T} \langle \mathbf{P}| \;\hat{\mathbf{S}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}})\; |\mathbf{P}\rangle\, \mathbf{T}\rangle$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \;=\; \langle \mathbf{T} \langle \mathbf{P}| \;\hat{\mathcal{O}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}})\; |\mathbf{P}\rangle\, \mathbf{T}\rangle$$

The question we pose is how these averages change with increase in energy of the process

Projectile averaged operators:

$$\langle \mathbf{P}| \; \hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \; |\mathbf{P}\rangle \;\; = \;\; \int \mathbf{D}\rho^{\mathrm{p}} \;\; \hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \;\; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

evolve with rapidity as $\qquad\qquad \mathbf{H} \;\; \rightarrow$ the HE effective Hamiltonian

$$\frac{\mathbf{d}\langle \mathbf{P}|\hat{\mathcal{O}}|\mathbf{P}\rangle}{\mathbf{dY}} \;=\; -\int \mathbf{D}\rho^{\mathrm{p}} \;\;\; \hat{\mathcal{O}}(\rho^{\mathrm{t}},\, \rho^{\mathrm{p}}) \;\; \mathbf{H}[\rho^{\mathrm{p}},\, \delta/\delta\rho^{\mathrm{p}}] \;\;\; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

or in other words

$$\frac{\mathbf{dW^{p}}}{\mathbf{dY}} \;\; = \;\; -\,\mathbf{H} \;\; \mathbf{W^{p}}$$

Spectrum of $\mathbf{H}$ defines the energy dependence of the average.

Dense/Dilute limit

$$\mathbf{H}^{\mathbf{KLWMIJ}} = \mathbf{H}^{\mathbf{RFT}}(\rho \to \mathbf{0})\,; \qquad \mathbf{H}^{\mathbf{JIMWLK}} = \mathbf{H}^{\mathbf{RFT}}(\rho \to \infty)$$

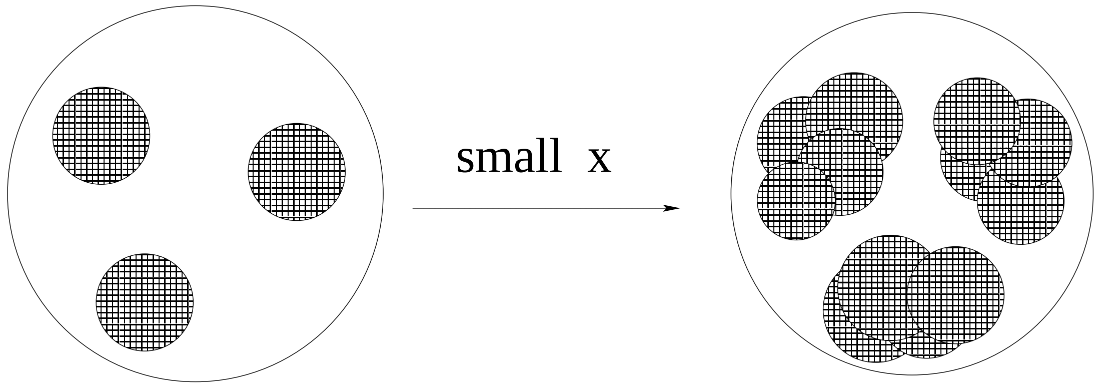
JIMWLK - Jalilian Marian, Iancu, McLerran, Leonidov, Kovner (1997-2002)

KLWMIJ - A. Kovner and M.L., Phys.Rev.D71:085004, 2005

Evolution with Pomeron Loops (model):

$$\mathbf{H}^{\mathbf{RFT}} \simeq \mathbf{H}^{\mathbf{JIMWLK}}(\rho \to \infty) \;'' + ''\; \mathbf{H}^{\mathbf{KLWMIJ}}(\rho \to \mathbf{0})$$

Dilute regime: $\delta\rho \sim \rho \quad \rightarrow \quad \rho \simeq \mathbf{e}^{\mathbf{c\,Y}}$ **BFKL** $\quad s = \exp[Y] = 1/x$



Evolution is generated by boost. Accelerated (color) charged particles radiate

Fast particles emit softer ones

High energy limit = soft gluon emission approximation

Exponential growth of gluon densities leads to unitarity violation.

At high densities the growth should be slowed down due to non-linear effects.

Transition to a non-linear regime is characterized by emergence of a new scale $\mathbf{Q_s}$, known as saturation scale.

$\mathbf{Q_s} \gg \Lambda_{\mathrm{QCD}}$ **and perturbative methods are applicable.**

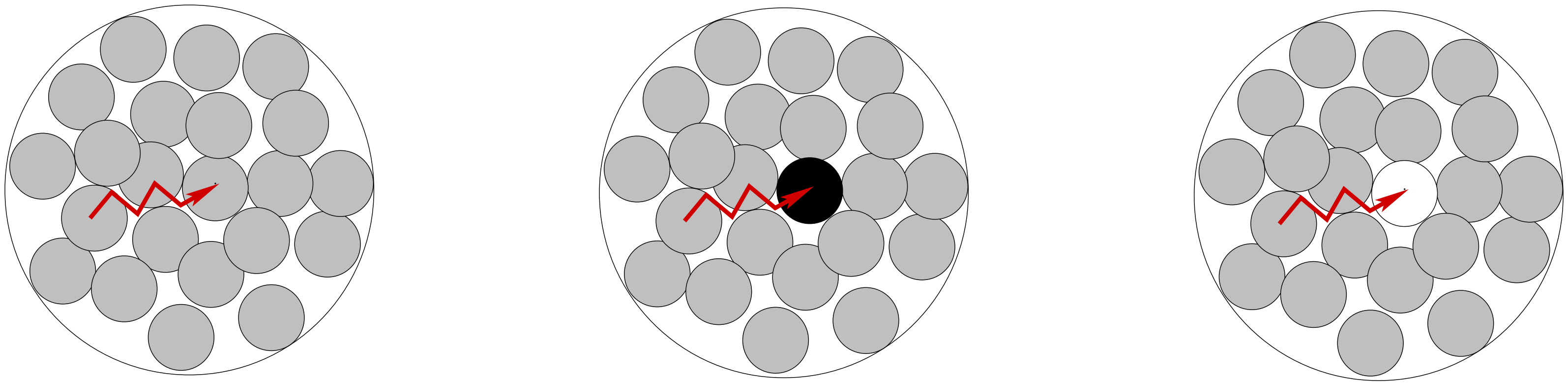
Inside Color Glass Condensate (CGC)

Dense regime:

(1) Hadron is almost black

(2) Emission probability is independent of density

(3) "Bleaching of color"

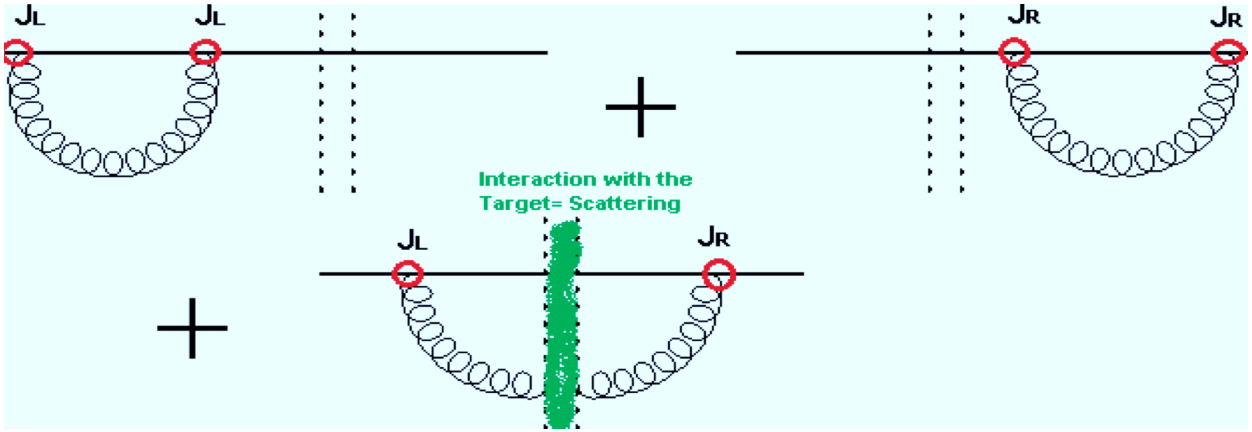


Random walk $\rho \;\sim\; \sqrt{\mathbf{Y}}$

JIMWLK Hamiltonian

The JIMWLK Hamiltonian is a limit of $\mathbf{H}$ for dilute partonic system ($\rho_p \to 0$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$$H^{JIMWLK} = \frac{\alpha_s}{2\pi^2} \int_{x,y,z} \frac{(x-z)_i(y-z)_i}{(x-z)^2(y-z)^2} \left\{ J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(x) S_A^{ab}(z) J_R^b(y) \right\}$$



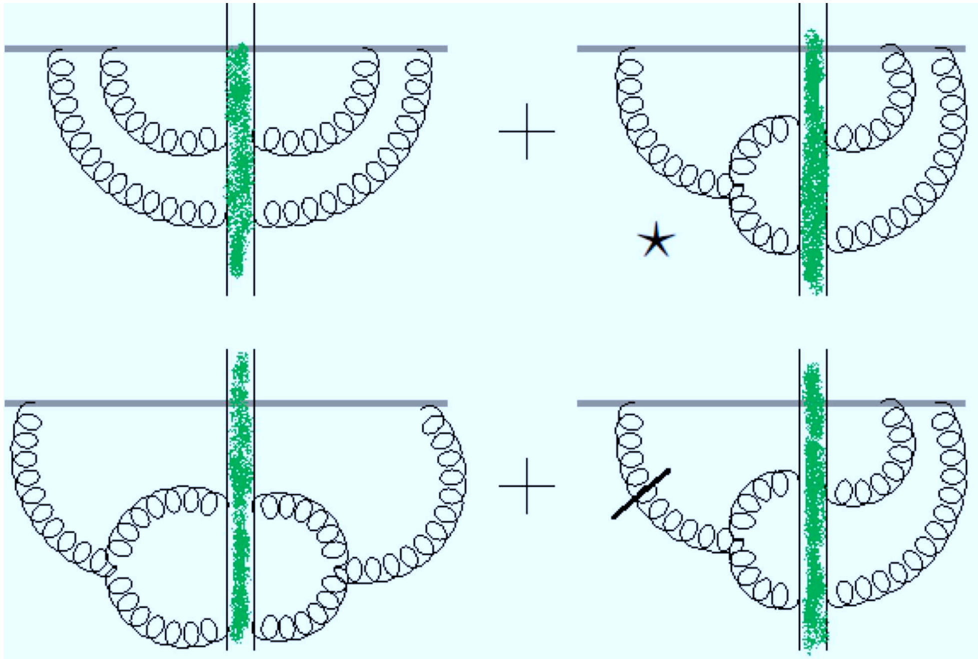
$$\mathbf{S}_A^{cd}(\mathbf{z}) \; = \; \mathcal{P} \exp \left\{ \mathbf{i} \int \mathbf{dx}^+ \, \mathbf{T}^a \, \alpha_t^a(\mathbf{z}, \mathbf{x}^+) \right\}^{cd} . \qquad \text{"}\boldsymbol{\Delta}\text{"} \alpha_t \; = \; \rho_t \quad (\mathbf{YM})$$

The left and right SU(N) generators:

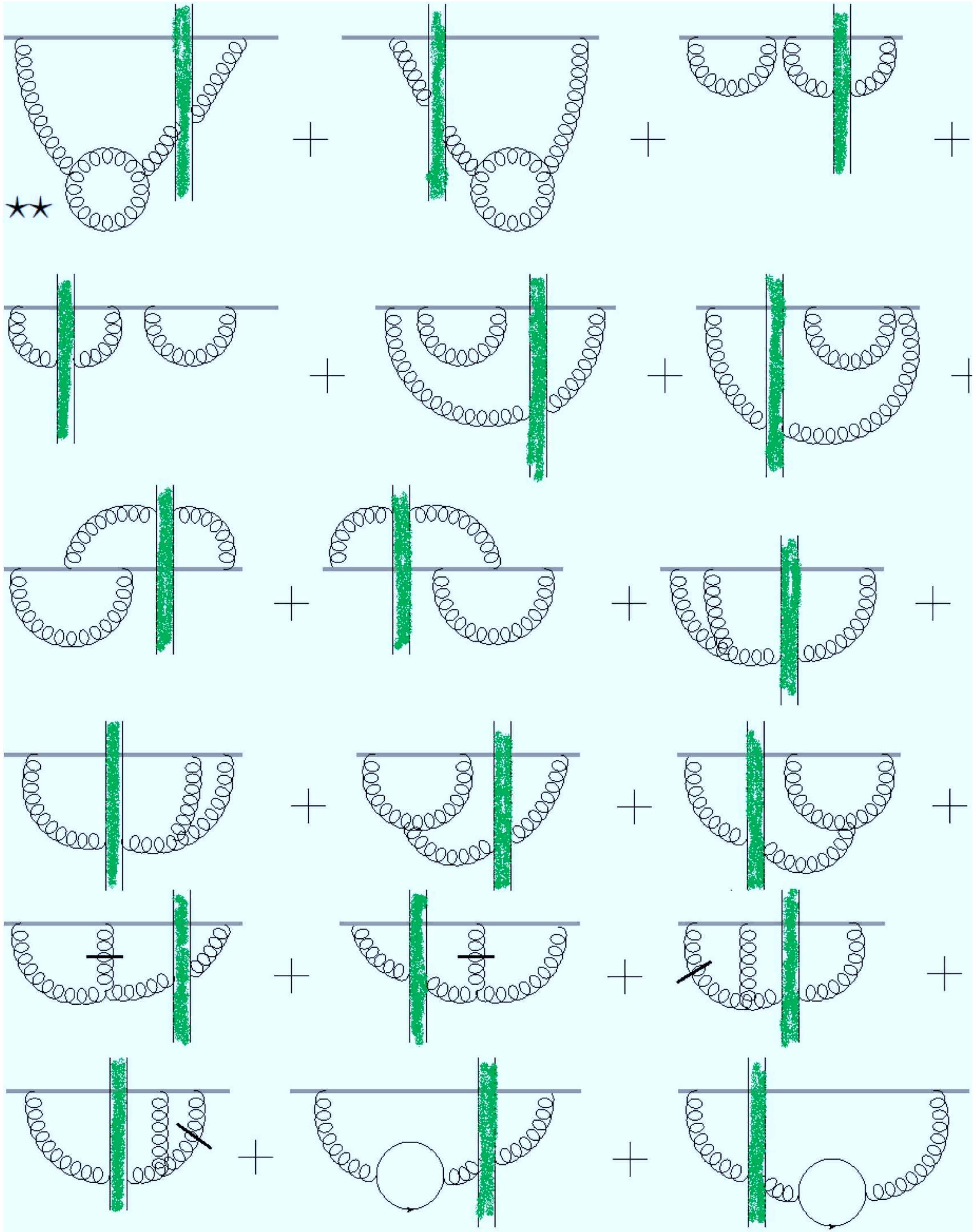
$$\mathbf{J}_L^a(\mathbf{x}) \mathbf{S}_A^{ij}(\mathbf{z}) = (\mathbf{T}^a \mathbf{S}_A(\mathbf{z}))^{ij} \, \delta^2(\mathbf{x} - \mathbf{z}) \qquad \mathbf{J}_R^a(\mathbf{x}) \mathbf{S}_A^{ij}(\mathbf{z}) = (\mathbf{S}_A(\mathbf{z}) \mathbf{T}^a)^{ij} \, \delta^2(\mathbf{x} - \mathbf{z})$$

Towards JIMWLK Hamiltonian @ NLO

Some 30 diagrams of the kind:



Symmetries: $\mathbf{SU_L(N) \times SU_R(N)}$ CPT, Unitarity



JIMWLK Hamiltonian @ NLO

$$H^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2\,J_L^a(x)\,tr[S^\dagger(z)\,t^a\,S(z')t^b]\,J_R^b(y)\; - \;J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\left[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w)\right.$$

$$\left. - J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\,\right]$$

$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\right]$$

$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\left[J_L^d(x)\,J_L^e(y)\,J_L^b(w)\; - \;J_R^d(x)\,J_R^e(y)\,J_R^b(w)\right].$$

Shortcuts to the Kernels

Step 1: Compute evolution of 3-quark Wilson loop operator in SU(3) (baryon)

$$\mathbf{B(u,v,w) = \epsilon^{ijk}\epsilon^{lmn} S_F^{il}(u) S_F^{jm}(v) S_F^{kn}(w)}$$

$$\partial_{\mathbf{Y}}\, \mathbf{B(u,v,w) = -H^{NLO\,JIMWLK}\, B(u,v,w)}$$

and compare with Grabovsky (hep-ph/1307.5414) $\rightarrow \mathbf{K_{JJSSJ},\ K_{JJSJ}}$

Step 2: Compute evolution of quark dipole operator

$$\mathbf{s(u,v) = tr[S_F(u) S_F^{\dagger}(v)]/N_c}$$

$$\partial_{\mathbf{Y}}\, \mathbf{s(u,v) = -H^{NLO\,JIMWLK}\, s(u,v)}$$

and compare with Balitsky and Chirilli (hep-ph/0710.4330) $\rightarrow \mathbf{K_{JSSJ},\ K_{JSJ},\ K_{qq}}$

NLO Kernels (for gauge invariant operators)

$$K_{JJSSJ}(w;x,y;z,z') = -i\frac{\alpha_s^2}{2\,\pi^4}\left(\frac{X_i Y'_j}{X^2 Y'^2}\right)$$
$$\times\Big(\frac{\delta_{ij}}{2(z-z')^2} + \frac{(z'-z)_i W'_j}{(z'-z)^2 W'^2} + \frac{(z-z')_j W_i}{(z-z')^2 W^2} - \frac{W_i W'_j}{W^2 W'^2}\Big)\ln\frac{W^2}{W'^2}$$

$$K_{JJSJ}(w;x,y;z) \; = \; -\,i\,\frac{\alpha_s^2}{4\,\pi^3}\left[\frac{X\cdot W}{X^2\,W^2} - \frac{Y\cdot W}{Y^2\,W^2}\right]\ln\frac{Y^2}{(x-y)^2}\,\ln\frac{X^2}{(x-y)^2},$$

$$K_{q\bar q}(x,y;z,z') = -\frac{\alpha_s^2\,n_f}{8\,\pi^4}\Big\{\frac{X'^2Y^2 + Y'^2X^2 - (x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2} - \frac{2}{(z-z')^4}\Big\}$$

$$X = x-z, \quad X' = x-z', \quad Y = y-z, \quad Y' = y-z', \quad W = w-z$$

$$K_{JSSJ}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Bigg[-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2 + X'^2Y^2 - 4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2 - X'^2Y^2]}$$

$$+\frac{(x-y)^4}{X^2Y'^2 - X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]+\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Bigg] + \tilde{K}(x,y,z,z').$$

$$K_{JSJ}(x,y;z) = -\frac{\alpha_s^2}{16\pi^3}\frac{(x-y)^2}{X^2Y^2}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2}+(\frac{67}{9}-\frac{\pi^2}{3})N_c - \frac{10}{9}n_f\Big]$$

$$-\frac{N_c}{2}\int_{z'}\tilde{K}(x,y,z,z').$$

Here μ is the normalization point, $b = \frac{11}{3}N_c - \frac{2}{3}n_f$

$$\tilde{K}(x,y,z,z') = \frac{i}{2}\Big[K_{JJSSJ}(x;x,y;z,z') - K_{JJSSJ}(y;x,y;z,z') - K_{JJSSJ}(x;y,x;z,z')$$

$$+K_{JJSSJ}(y;y,x;z,z')\Big]$$

The kernels are not unique though...

NLO Kernels for color non-singlets

"By inspection" of Balitsky and Chirilli (arXiv:1309.7644 [hep-ph])

$$K_{JSJ}(x,y,z) \to \bar{K}_{JSJ}(x,y,z) \equiv K_{JSJ}(x,y,z) +$$
$$+\frac{\alpha_s^2}{16\pi^3}\left\{\left[\frac{1}{X^2}+\frac{1}{Y^2}\right]\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c-\frac{10}{9}n_f\right]+\frac{b}{X^2}\ln X^2\mu^2+\frac{b}{Y^2}\ln Y^2\mu^2\right\};$$

$$K_{JSSJ}(x,y;z,z') \to \bar{K}_{JSSJ}(x,y;z,z') +$$
$$\equiv K_{JSSJ}(x,y;z,z') + \frac{\alpha_s^2}{8\pi^4}\left[\frac{4}{(z-z')^4}-\frac{I(x,z,z')}{(z-z')^2}-\frac{I(y,z,z')}{(z-z')^2}\right];$$

$$K_{q\bar{q}}(x,y;z,z') \to \bar{K}_{q\bar{q}}(x,y;z,z') \equiv K_{q\bar{q}}(x,y;z,z') - \frac{\alpha_s^2 n_f}{8\pi^4}\left[\frac{I_f(x,z,z')}{(z-z')^2}+\frac{I_f(y,z,z')}{(z-z')^2}\right],$$

$$I(x,z,z') = \frac{1}{X^2-(X')^2}\ln\frac{X^2}{(X')^2}\left[\frac{X^2+(X')^2}{(z-z')^2}-\frac{X\cdot X'}{X^2}-\frac{X\cdot X'}{(X')^2}-2\right];$$
$$I_f(x,z,z') = \frac{2}{(z-z')^2}-\frac{2X\cdot X'}{(z-z')^2(X^2-(X')^2)}\ln\frac{X^2}{(X')^2}.$$

Is the JIMWLK Hamiltonian Conformally invariant?

Scale invariance is trivial. Lets focus on inversion. Introduce $\mathbf{x}_\pm = \mathbf{x}_1 \pm \mathbf{i}\,\mathbf{x}_2$

$$\textbf{Inversion transformation}: \quad x_+ \to 1/x_- \,; \qquad\qquad x_- \to 1/x_+$$

A "naive" representation $\mathcal{I}_0$ of the inversion transformation is

$$\mathcal{I}_0: \quad \mathbf{S}(\mathbf{x}_+, \mathbf{x}_-) \to \mathbf{S}(\mathbf{1}/\mathbf{x}_-, \mathbf{1}/\mathbf{x}_+)\,, \qquad \mathbf{J}_{\mathbf{L},\mathbf{R}}(\mathbf{x}_+, \mathbf{x}_-) \to \frac{\mathbf{1}}{\mathbf{x}_+\mathbf{x}_-}\mathbf{J}_{\mathbf{L},\mathbf{R}}(\mathbf{1}/\mathbf{x}_-, \mathbf{1}/\mathbf{x}_+)\,.$$

Conformal invariance (in the gauge invariant sector) @LO:

$$\mathcal{I}_0\, \mathbf{H}^{\mathbf{LO\,JIMWLK}}\, \mathcal{I}_0 \;=\; \mathbf{H}^{\mathbf{LO\,JIMWLK}}$$

No (naive) Conformal invariance @NLO:

$$\mathcal{I}_0\, \mathbf{H}^{\mathbf{NLO\,JIMWLK}}\, \mathcal{I}_0 \;=\; \mathbf{H}^{\mathbf{NLO\,JIMWLK}} + \mathcal{A}$$

QCD is not conformally invariant beyond tree level, but $\mathcal{N}=4$ SUSY is.

JIMWLK Hamiltonian IS conformally invariant! (in $\mathcal{N}=4$)

S **forms a non-trivial representation of the conformal group:**

$$\mathcal{I} : S(x) \to S(1/x) + \delta S(x)\,, \qquad \mathcal{I} : H^{LO} \to H^{LO} - \mathcal{A}$$

Here δS is of order α_s. The condition is that the net anomaly cancels:

$$\mathcal{I}\,(\mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}})\,\mathcal{I} \;=\; \mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}}$$

We have constructed $\mathcal{I}$ perturbatively: $\qquad \mathcal{I} \;=\; (\mathbf{1} + \mathcal{C})\,\mathcal{I}_0\,.$

$$\mathcal{C} \;=\; -\,\frac{\mathbf{1}}{\mathbf{2}}\,\frac{\alpha_{\mathrm{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{x},\mathbf{y},\mathbf{z}} \ln\Big[\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}\mathbf{a}^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\Big]\times$$

$$\times\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\Big\{\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{x})\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{y}) + \mathbf{J}^{\mathrm{a}}_{\mathrm{R}}(\mathbf{x})\mathbf{J}^{\mathrm{a}}_{\mathrm{R}}(\mathbf{y}) - \mathbf{2}\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{x})\mathbf{S}^{\mathrm{ab}}_{\mathrm{A}}(\mathbf{z})\mathbf{J}^{\mathrm{b}}_{\mathrm{R}}(\mathbf{y})\Big\}$$

For an arbitrary operator $\mathcal{O}$ (s, B, H^{JIMWLK},...) we define its conformal extension:

$$\mathcal{O}^{conf} = \mathcal{O} + \frac{1}{2}[\mathcal{C},\,\mathcal{O}]\,; \qquad [s^{conf} \text{ by Balitsky and Chirilli (arXiv : 0903.5326)}]$$

CONCLUSIONS

- **We have constructed the JIMWLK Hamiltonian at NLO. It fully reproduces and generalizes (all?) previously known low x evolution equations at NLO, including Balitsky's hierarchy at NLO**

- **We have proven the conformal invariance of the NLO JIMWLK Hamiltonian (in $\mathcal{N} = 4$). For any operator, we can construct its perturbative extension, such that the resulting operator evolves with conformal kernels.**

- **Once expanded in the dilute limit, the NLO JIMWLK makes it possible to study evolution of any multi-gluon BKP state and transition vertices at NLO.**

Comparing with Balitsky and Chirilli (arXiv:1309.7644 [hep-ph])

Compute evolution of Wilson lines with open color indices:

$$\partial_Y \left[\mathbf{S}^{ab}(\mathbf{x})\right] = -\mathbf{H}^{\mathrm{NLO\,JIMWLK}} \left[\mathbf{S}^{ab}(\mathbf{x})\right]$$

$$\partial_Y \left[\mathbf{S}^{ab}(\mathbf{x})\mathbf{S}^{cd}(\mathbf{y})\right] = -\mathbf{H}^{\mathrm{NLO\,JIMWLK}} \left[\mathbf{S}^{ab}(\mathbf{x})\mathbf{S}^{cd}(\mathbf{y})\right]$$

$$\partial_Y \left[\mathbf{S}^{ab}(\mathbf{x})\mathbf{S}^{cd}(\mathbf{y})\mathbf{S}^{ef}(\mathbf{z})\right] = -\mathbf{H}^{\mathrm{NLO\,JIMWLK}} \left[\mathbf{S}^{ab}(\mathbf{x})\mathbf{S}^{cd}(\mathbf{y})\mathbf{S}^{ef}(\mathbf{z})\right]$$

100% agreement!