Conformal bootstrap in 4D

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based on arXiv:1406.6166 [hep-th]
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Quark Confinement and the Hadron Spectrum XI

Saint-Peterburg September 12, 2014

Outline

- Review of the conformal bootstrap
- Sum rules in CFT with global symmetry
- Example
- Conclusions

Conformal bootstrap:
Basics objects in a CFT

Conformal group:

Poincare (P_μ) + Dilatations (D) + Special conformal (K_μ)

Basic objects of the CFT:

Primary operators (elementary fields): $K_\mu \mathcal{O}(0) = 0$

Descendant operators (composite fields): $P_{\mu_1} ... P_{\mu_n} \mathcal{O}(0) = 0$

All dynamics of the descendants
fixed by those of primaries

Conformal bootstrap: CFT spectrum and OPE

CFT is specified by spectrum (dimensions and spins) of the primary operators:

$$\{O_i\} = (\Delta_i, l_i)$$

Basic property of a CFT is an existence of the operator product expansion (OPE):

$$\mathcal{O}_i(x)\,\mathcal{O}_j(y) = \sum_{k=Prim+Desc} c_{ij}^k(x-y)\mathcal{M}_k(y) \quad \rightarrow$$

Inside correlations functions, product on the LHS can be replaced by the sum on the RHS, as long as there are no other operators at smaller distances from y than |x-y|

Conformal bootstrap: OPE

OPE can be simplified by imposing conformal invariance :

$$\mathcal{O}_i(x)\mathcal{O}_j(y) = \sum_{k=Prim} C_{ij}^k \; L_k(x-y,\partial_y)\, \mathcal{O}_k(y) \frac{1}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}}$$

the differential operator L_k that encodes descendants contribution depends on the dimensions and spins of the primaries $\mathcal{O}_k$ and not on the dynamics of the CFT.

$(\Delta_i, l_i, C_{ij}^k)$ this data fixes all correlators in a CFT

- Start with n-point function and replace two operators with OPE. The n-point function is now an (infinite) sum over (n-1)-point functions.
- The (n-1)-point functions can be reduced by an OPE to sums over (n-2)-point functions and so on...
- Repeat until you get to the basic 2- and 3-point functions

$$\langle \mathcal{O}_i(x)\,\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}} \qquad \langle \mathcal{O}_i(x_1)\,\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ij}^k}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{13}|^{\Delta_1+\Delta_3-\Delta_2}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}}$$

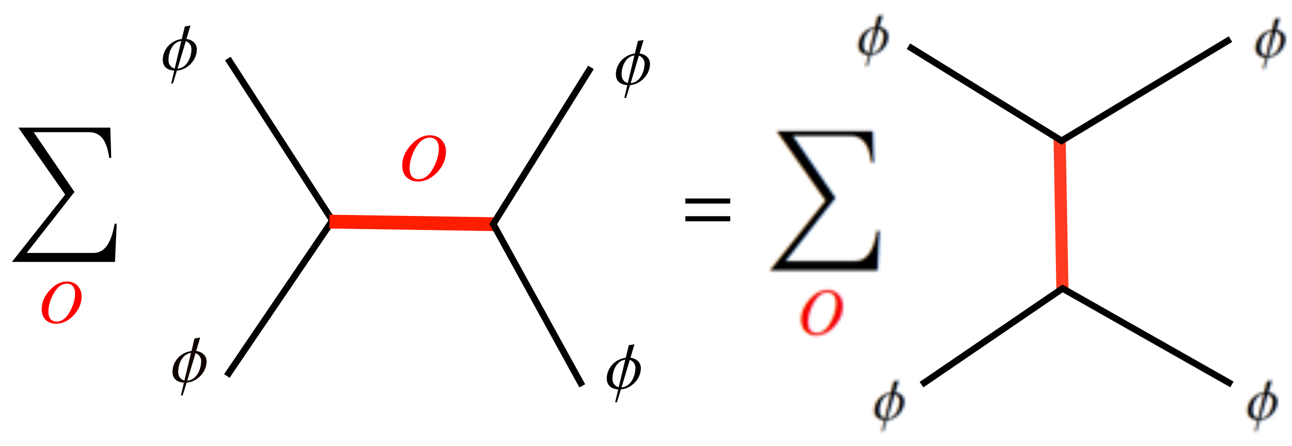
Conformal bootstrap: Crossing symmetry

This procedure has ambiguity.

Consider, 4-point function in the, e.g. (12)-(34) channel :

$$\langle \underbrace{\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)}_{\text{OPE}}\underbrace{\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)}_{\text{OPE}}\rangle = \sum_k \frac{C_{12}^k C_{34}^k L_k(x_{12}, \partial_{x_2}) L_k(x_{34}, \partial_{x_4}) \langle \mathcal{O}_k(x_2)\, \mathcal{O}_k(x_4)\rangle}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_k}\ |x_{34}|^{\Delta_3+\Delta_4-\Delta_k}}$$

Consistency requires that (12)-(34) = (14)-(23)



Conformal bootstrap: Crossing symmetry constraint

Introduce conformal blocks (kinematical info):

$$\mathbf{G}_k^{12,34}(x_1,x_2,x_3,x_4) \equiv \frac{1}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_k}} \frac{1}{|x_{34}|^{\Delta_3+\Delta_4-\Delta_k}} L_k(x_{12},\partial_{x_2}) L_k(x_{34},\partial_{x_4}) \langle \mathcal{O}_k(x_2)\, \mathcal{O}_k(x_4)\rangle$$

known

Dolan,Osborn '00,'03

(12)-(34) = (14)-(23)

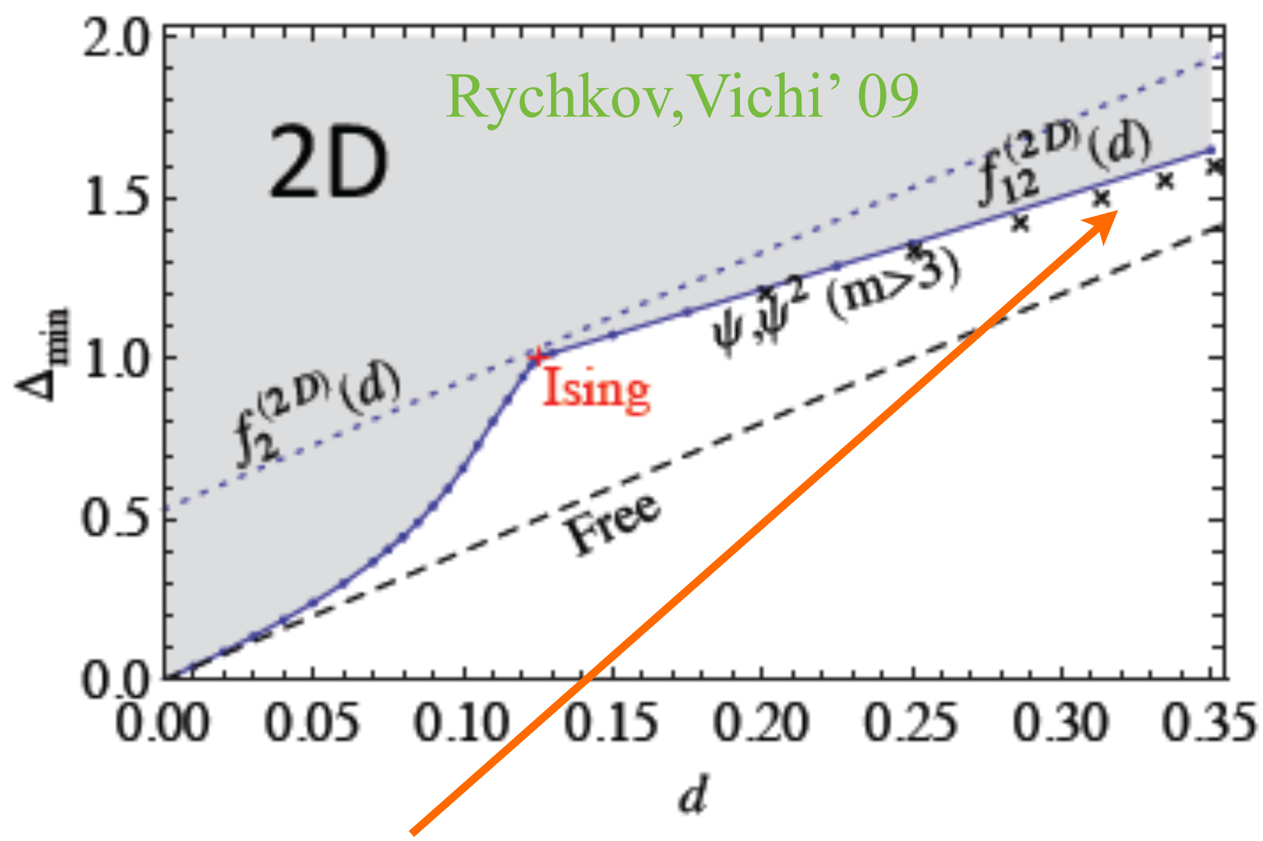
$$\sum_k C_{12}^k C_{34}^k \; \mathbf{G}_k^{12,34}(x_1,x_2,x_3,x_4) = \sum_k C_{14}^k C_{23}^k \; \mathbf{G}_k^{14,23}(x_1,x_4,x_2,x_3)$$

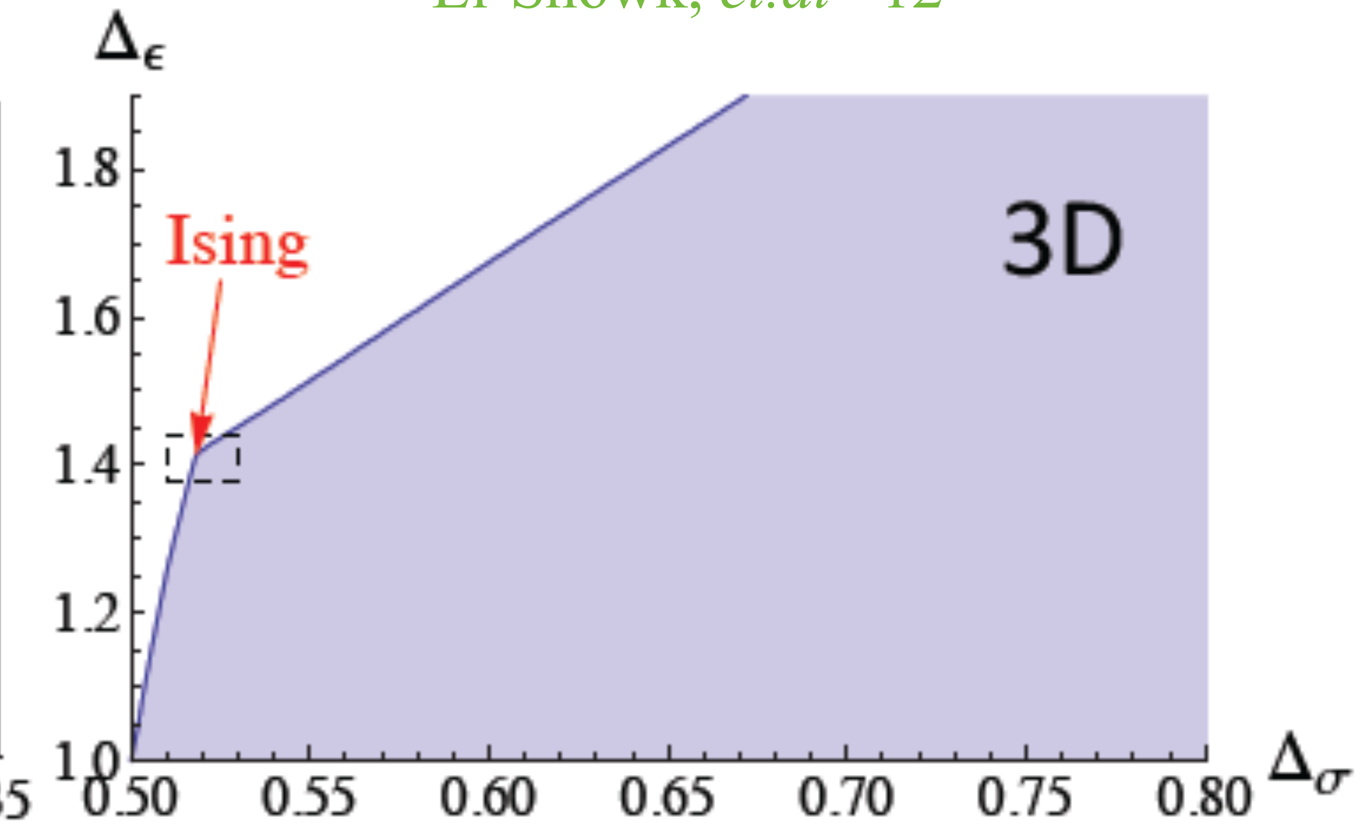
This is non-perturbative constraint on the CFT data

$$(\Delta_i, l_i, C_{ij}^k)$$

Conformal bootstrap:
Constraint in lower dimensions

$$f_{12}^{(2D)}(d) \simeq \begin{cases} 4.3d + 8d^2 - 87d^3 + 2300d^4, & d \lesssim 0.122, \\ 0.64 + 2.87d, & d \gtrsim 0.122. \end{cases}$$





2D minimal unitary models:

$$\psi \times \psi = 1 + \psi^2 + \ldots, \qquad \Delta_\psi = \frac{1}{2} - \frac{3}{2(m+1)}$$

$$\Delta_{\psi^2} = 2 - \frac{4}{m+1},$$

In 2D and 3D bound exhibit singular points which select the known CFT models

Conformal bootstrap: Constraint in 4D

In an arbitrary 4D unitary CFT conformal symmetry implies 4-pt function:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \sum_k C_{12}^k C_{34}^k \, \mathbf{G}_k^{12,34}(x_1,x_2,x_3,x_4)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\equiv \left(\frac{|x_{24}|}{|x_{14}|}\right)^{\Delta_1-\Delta_2} \left(\frac{|x_{14}|}{|x_{13}|}\right)^{\Delta_3-\Delta_4} \frac{g(u,v)}{|x_{12}|^{\Delta_1+\Delta_2}|x_{34}|^{\Delta_3+\Delta_4}}$$

For all Δ equal:

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2d}\,x_{34}^{2d}},$$

$$g(u,v) = 1 + \sum p_k\, g_k(u,v), \quad p_k \equiv (C_{\phi\phi}^k)^2 \geq 0,$$

Dolan,Osborn ‘00,’03

$$g_k(u,v) = g_{\Delta,l}(u,v) = \frac{(-1)^l}{2^l}\frac{z\bar{z}}{z-\bar{z}}\left[k_{\Delta+l}(z)k_{\Delta-l-2}(\bar{z}) - (z \leftrightarrow \bar{z})\right],$$

$$k_\beta(x) \equiv x^{\beta/2}{}_2F_1(\beta/2,\beta/2,\beta;x)\,, \qquad u = z\bar{z}, \quad v = (1-z)(1-\bar{z})$$

Sum rule

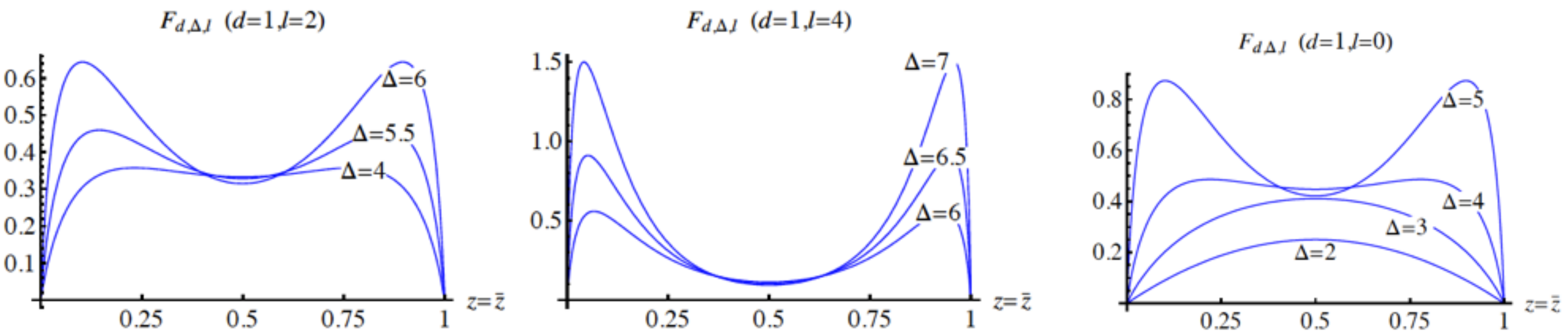
Crossing constraint : $v^d g(u,v) = u^d g(v,u)$ ⟶

$$1 = \sum p_{\Delta,l} F_{d,\Delta,l}\,, \qquad F_{d,\Delta,l} \equiv \frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}$$

Conformal bootstrap: Constraint in 4D

Sum rule

$$1 = \sum p_{\Delta,l} F_{d,\Delta,l}\,, \qquad F_{d,\Delta,l} \equiv \frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}$$



$$F''_{d,\Delta,l} > 0 \qquad \text{at } z = \bar{z} = 1/2,$$
$$l = 2, 4, 6\ldots, \qquad \Delta \geq l + 2,$$
$$1 \leq d \leq 1 + \varepsilon\,.$$

Need to have $\Delta_c < 3.6$ otherwise sum rule cannot be satisfied

Strategy: look for differential operator that gives 0 on the LHS but stays positive when applied to the F-functions on the RHS

Conformal bootstrap: Constraint in 4D

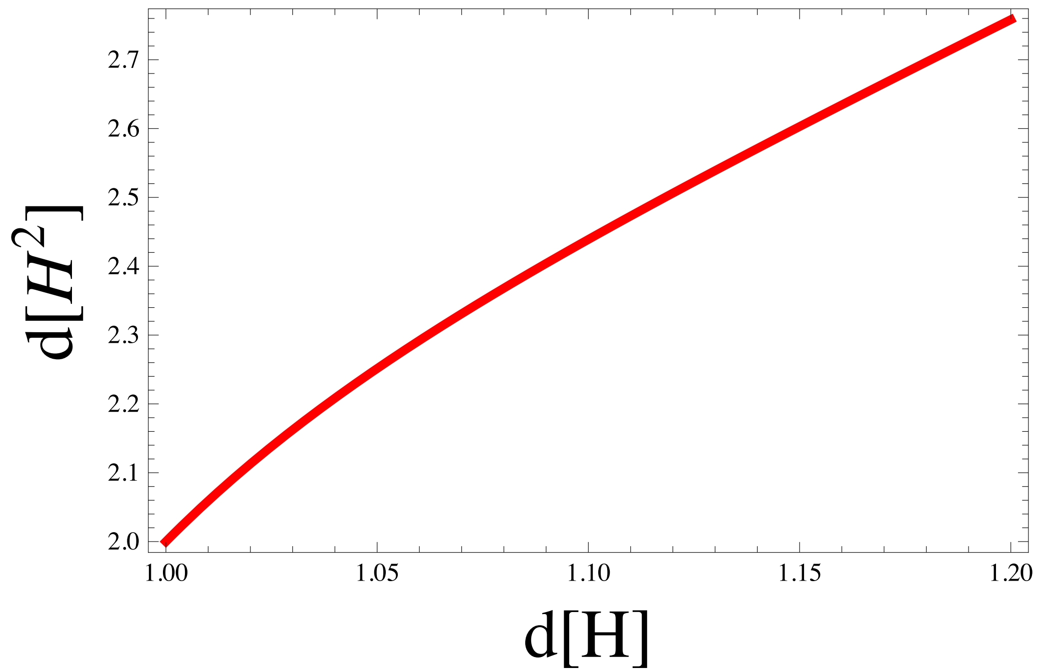
For the lowest dimension scalar primary operator ϕ in the OPE :

$$\phi(x)\phi(0) = \frac{1}{x^{2d}}(1 + C_{\phi\phi}|x|^{\Delta}\phi^2(0) + \dots)\ , \quad d \equiv \Delta_\phi, \quad \Delta = d[\phi^2]$$

the bound on Δ was derived :

Rattazzi, Rychkov, Tonni, Vichi '08
Poland, Simmons-Duffin, Vichi '11

$$\Delta \leq \Delta_{max} = 2 + 3.006(d-1) + 0.16(1 - e^{-20(d-1)})$$



Does not exhibit singular points

CFT with a global symmetry: SU(N)x SU(N)

Basic objects: $H_a^{\alpha^*} = (\mathbf{N_f}, \mathbf{N_f^*})$ and $H_{b^*}^{\beta} = (\mathbf{N_f^*}, \mathbf{N_f})$

$$(\mathbf{N_f}, \mathbf{N_f^*}) \times (\mathbf{N_f^*}, \mathbf{N_f}) = (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathrm{Adj}) + (\mathrm{Adj}, \mathbf{1}) + (\mathrm{Adj}, \mathrm{Adj})$$

Basic OPE : $H_{i\alpha}(x) \times H^\dagger_{\beta j}(0) \sim \frac{1}{|x|^{2d_H}}$

$$\left\{\delta_{ij}\delta_{\alpha\beta}\left(1 + c_S|x|^{\Delta_S} Tr[HH^\dagger](0)\right) + c_L|x|^{\Delta_L} M_{ii\alpha\beta}(0) + c_R|x|^{\Delta_R} M_{ij\alpha\alpha}(0) + c_A|x|^{\Delta_A} M_{ij\alpha\beta}(0)\right\}$$

Performing OPE twice we decompose a 4-pt function into conformal blocks

$$\langle \underbrace{H(x_1)H^\dagger(x_2)}_{\text{OPE}}\, \underbrace{H(x_3)H^\dagger(x_4)}_{\text{OPE}} \rangle =$$

$$\left[(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathrm{Adj}) + (\mathrm{Adj}, \mathbf{1}) + (\mathrm{Adj}, \mathrm{Adj})\right] \times \left[(\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathrm{Adj}) + (\mathrm{Adj}, \mathbf{1}) + (\mathrm{Adj}, \mathrm{Adj})\right] =$$

$$\mathbf{G_S}(\mathbf{1}, \mathbf{1}) + \mathbf{G_L}(\mathbf{1}, \mathbf{1_{AA}}) + \mathbf{G_R}(\mathbf{1_{AA}}, \mathbf{1}) + \mathbf{G_A}(\mathbf{1_{AA}}, \mathbf{1_{AA}})$$

CFT with a global symmetry: SU(N)xSU(N)

Requiring crossing symmetry : (12)-(34) = (14)-(23)

$$\langle H(x_1)H^\dagger(x_2)H(x_3)H^\dagger(x_4)\rangle = \langle H(x_1)H^\dagger(x_4)H(x_3)H^\dagger(x_2)\rangle$$

4 equations for 4 unknowns

$$v^{d_H}\left(G_S - \frac{1}{N_f}(G_L + G_R) + \frac{1}{N_f^2}G_A\right) = u^{d_H}\,\widetilde{G}_A\ ,$$

$$v^{d_H}\,G_A = u^{d_H}\left(\widetilde{G}_S - \frac{1}{N_f}(\widetilde{G}_L + \widetilde{G}_R) + \frac{1}{N_f^2}\widetilde{G}_A\right)\ ,$$

$$v^{d_H}\left(G_R - \frac{1}{N_f}G_A\right) = u^{d_H}\left(\widetilde{G}_L - \frac{1}{N_f}\widetilde{G}_A\right)\ ,$$

$$v^{d_H}\left(G_L - \frac{1}{N_f}G_A\right) = u^{d_H}\left(\widetilde{G}_R - \frac{1}{N_f}\widetilde{G}_A\right)\ ,$$

where

$$\widetilde{G} \equiv G(u \leftrightarrow v)$$

Example: QCD + mesons + gluinos

See also E. Molgaard talk

O.A, Mojaza, Sannino' 11

QCD+gluino — "Higgs"-sector

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + i\bar{\lambda}\not{D}\lambda + \overline{Q}i\not{D}Q + \partial_\mu H^\dagger \partial^\mu H + y_H \overline{Q}HQ\right] - u_1(\mathrm{Tr}[HH^\dagger])^2 - u_2\,\mathrm{Tr}[HH^\dagger HH^\dagger]$$

$$H_{ij} = \frac{\phi + i\eta}{\sqrt{2N_f}}\delta_{ij} + \sum_{a=1}^{N_f^2-1}(h^a + i\pi^a)T^a_{ij}$$

$$(\mathbf{1},\mathbf{1}) = \delta_{ij}\delta_{\alpha\beta}H_{i\alpha}H^\dagger_{\beta j} = Tr[HH^\dagger]$$

$$(\mathbf{Adj},\mathbf{1}) = \mathbf{H_{i\alpha}H^\dagger_{\alpha j}} = (\mathbf{HH^\dagger})_{\mathbf{ij}}$$

$$(\mathbf{1},\mathbf{Adj}) = H_{i\alpha}H^\dagger_{\beta i} = (HH^\dagger)_{\alpha\beta}$$

Notice that there is no mass term for the "H" field so that the model is classically conformal at the tree level

Symmetries

$$\mathcal{L} = \mathcal{L}_K(F_{\mu\nu}, \lambda, \psi, H; g) + y_H \bar{\psi} H \psi - u_1 \left(\mathrm{Tr} H^\dagger H\right)^2 - u_2 \mathrm{Tr}\left(H^\dagger H\right)^2$$

Veneziano limit :

$N_c \to \infty$, $N_f \to \infty$

$x = N_f / N_c$ fixed

Rescaled couplings :

$$a_g = \frac{g^2 N_c}{(4\pi)^2}, \quad a_H = \frac{y_H^2 N_c}{(4\pi)^2}, \quad z_1 = \frac{u_1 N_f^2}{(4\pi)^2}, \quad z_2 = \frac{u_2 N_f}{(4\pi)^2}$$

Model contains QCD SU(Nf) x SU(Nf) global symmetry

Fields	$[SU(N_c)]$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_V$	$U(1)_{AF}$
λ	Adj	1	1	0	1
q	$\square$	$\bar{\square}$	1	$\frac{N_f - N_c}{N_c}$	$-\frac{N_c}{N_f}$
$\tilde{q}$	$\bar{\square}$	1	$\square$	$-\frac{N_f - N_c}{N_c}$	$-\frac{N_c}{N_f}$
H	1	$\square$	$\bar{\square}$	0	$\frac{2N_c}{N_f}$
G_μ	Adj	1	1	0	0

External parameters: (x, number of gluions)

Our goal

To test the numerical bootstrap solutions within this explicit model

To achieve this:

- We have to check that this theory has a fixed point i.e. it is conformal. As we will see this fixed point will be perturbative Banks-Zaks fixed point.
- At the fixed point we need to calculate the conformal dimensions of the operators that enter the basic OPE
- Compare these conformal dimensions with the bounds from numerical solutions to the bootstrap system

Unfortunately, the bootstrap bound exploiting the full SU(N)xSU(N) global symmetry has not been obtained yet and we have to resort to the currently available bounds.

To achieve this, we will simplify the bootstrap system by using Veneziano limit...

Perturbative Bank-Zaks CFT

Proper perturbative truncation has to respect the Weyl consistency conditions (WCC) obeyed by different beta functions across the different loop orders. At the LO :

$$\frac{\partial(\chi^{jk}\beta_k)}{\partial g_i} = \frac{\partial(\chi^{im}\beta_m)}{\partial g_j}\,, \chi^{ij} \equiv \mathrm{diag}[\chi_{a_g a_g}, \chi_{a_H a_H}, \chi_{z_1 z_1}, \chi_{z_2 z_2}] = \left(\frac{N_c^2}{128\pi^2 a_g^2}\,, \frac{N_f^2}{384\pi^2 a_H}\,, 0\,, \frac{N_f^2}{192\pi^2}\right)$$

O.A., Gillioz, et al. '13
and Gillioz talk

Beta functions in the 321 scheme

$$\beta_{a_g} = -\frac{2}{3}a_g^2\Big[11 - 2\ell - 2x + (34 - 16\ell - 13x)\,a_g + 3x^2 a_H + \frac{81x^2}{4}a_g a_H - \frac{3x^2(7+6x)}{4}a_H^2 + \frac{2857 + 112x^2 - x(1709 - 257\ell) - 1976\ell + 145\ell^2}{18}a_g^2\Big]\,,$$

$$\beta_{a_H} = a_H\left[2(x+1)a_H - 6a_g + (8x+5)a_g a_H + \frac{20(x+\ell) - 203}{6}a_g^2 - 8xz_2 a_H - \frac{x(x+12)}{2}a_H^2 + 4z_2^2\right]\,,$$

$$\beta_{z_1} = 4\left(z_1^2 + 3z_2^2 + 4z_1 z_2 + z_1 a_H\right)\,, \qquad \beta_{z_2} = 2\left(2z_2 a_H + 4z_2^2 - x a_H^2\right)\,.$$

WCC-related terms are color-coded

Solution to this system of equations defines our perturbative CFT

Comparison with the bootstrap bound

The bootstrap bound exploiting the full SU(N)xSU(N) global symmetry has not been obtained yet and we have to resort to the currently available bounds...

We need to calculate the conformal dimensions of the composite operators:

$$H_{i\alpha}(x) \times H^\dagger_{\beta j}(0) \sim \frac{1}{|x|^{2d_H}} \left\{ \delta_{ij}\delta_{\alpha\beta}\left(1 + c_S |x|^{\Delta_S} Tr[HH^\dagger](0)\right) + c_A |x|^{\Delta_A} M_{ij\alpha\beta}(0) + \cdots \right\}$$

$$(1,1): \;\; \Delta_S = 2 + \gamma_S \qquad (\mathrm{Adj},\ \mathrm{Adj}): \;\; \Delta_A = 2 + \gamma_A$$

To the two-loop order and in Veneziano limit we find:

$$\gamma_S = \gamma_{Tr[HH^\dagger]} \equiv \Delta_S - 2 = 2a_H + 4(z_1 + 2z_2) - 8a_H(z_1 + 2z_2) - 20z_2^2 - 3xa_H^2 + 5a_g a_H$$

$$\gamma_A = \gamma_{Tr[T^a H T^a H^\dagger]} \equiv \Delta_A - 2 = 2a_H + 4z_2^2 - 3xa_H^2 + 5a_g a_H$$

Conformal dimension of the H: $\gamma_H \equiv d_H - 1\ ,$ $\qquad \gamma_A = 2\gamma_H$!

Comparison with the bootstrap bound

$$\gamma_A = 2\gamma_H \quad \text{implies:} \quad M_{ij\alpha\beta}(0) \sim\, :H_{i\alpha}H^\dagger_{\beta j}:(0)$$

$$H_{i\alpha}(x) \times H^\dagger_{\beta j}(0) \sim \frac{1}{|x|^{2d_H}}\left\{\delta_{ij}\delta_{\alpha\beta}\left(1 + c_S|x|^{\Delta_S} Tr[HH^\dagger](0)\right) + c_A|x|^{\Delta_A} M_{ij\alpha\beta}(0) + \cdots\right\}$$

This leads to the "generalized free scalar theory" :

$$\langle H_{i\alpha}(x_1)H^\dagger_{\beta j}(x_2)M_{ij\alpha\beta}(y)\rangle = \langle H_{i\alpha}(x_1)H^\dagger_{\alpha i}(y)\rangle \ \langle H_{j\beta}(y)H^\dagger_{\beta j}(x_2)\rangle = \frac{1}{x_{14}^{2d_H}x_{23}^{2d_H}}$$

completely specified by 2-point function:

$$\langle H_{i\alpha}(x)H^\dagger_{\alpha i}(0)\rangle = \frac{1}{|x|^{2d_H}} \Rightarrow G_A = \left(\frac{u}{v}\right)^{d_H} = \frac{x_{12}^{2d_H}x_{34}^{2d_H}}{x_{14}^{2d_H}x_{23}^{2d_H}}$$

From the 3-point function:

$$\langle H_{i\alpha}(x_1)H^\dagger_{\beta j}(x_2)M_{ij\alpha\beta}(y)\rangle = \frac{c_A}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_y}|x_{1y}|^{\Delta_1+\Delta_y-\Delta_2}|x_{2y}|^{\Delta_2+\Delta_y-\Delta_1}} = \frac{c_A}{|x_{1y}|^{2d_H}|x_{2y}|^{2d_H}}$$

$$\Rightarrow \quad c_A = 1$$

Solving the bootstrap in Veneziano limit

Recall the system

$$v^{d_H}\left(G_S - \frac{1}{N_f}(G_L + G_R) + \frac{1}{N_f^2}G_A\right) = u^{d_H}\,\widetilde{G}_A\ ,$$

$$v^{d_H}\,G_A = u^{d_H}\left(\widetilde{G}_S - \frac{1}{N_f}(\widetilde{G}_L + \widetilde{G}_R) + \frac{1}{N_f^2}\widetilde{G}_A\right)\ ,$$

$$v^{d_H}\,\left(G_R - \frac{1}{N_f}G_A\right) = u^{d_H}\,\left(\widetilde{G}_L - \frac{1}{N_f}\widetilde{G}_A\right)\ ,$$

$$v^{d_H}\,\left(G_L - \frac{1}{N_f}G_A\right) = u^{d_H}\,\left(\widetilde{G}_R - \frac{1}{N_f}\widetilde{G}_A\right)\ ,$$

Solve in the large Nf expansion:

$$G_{S,A} \equiv \sum_{\Delta,l} p^{S,A}_{\Delta,l}\, g^{S,A}_{\Delta,l}(u,v) = G^{disc}_{S,A} + \frac{G^{conn}_{S,A}}{N_f^2} + \cdots$$

$$G_{L,R} \equiv \sum_{\Delta,l} p^{L,R}_{\Delta,l}\, g^{L,R}_{\Delta,l}(u,v) = \frac{G_{L,R}}{N_f} + \cdots$$

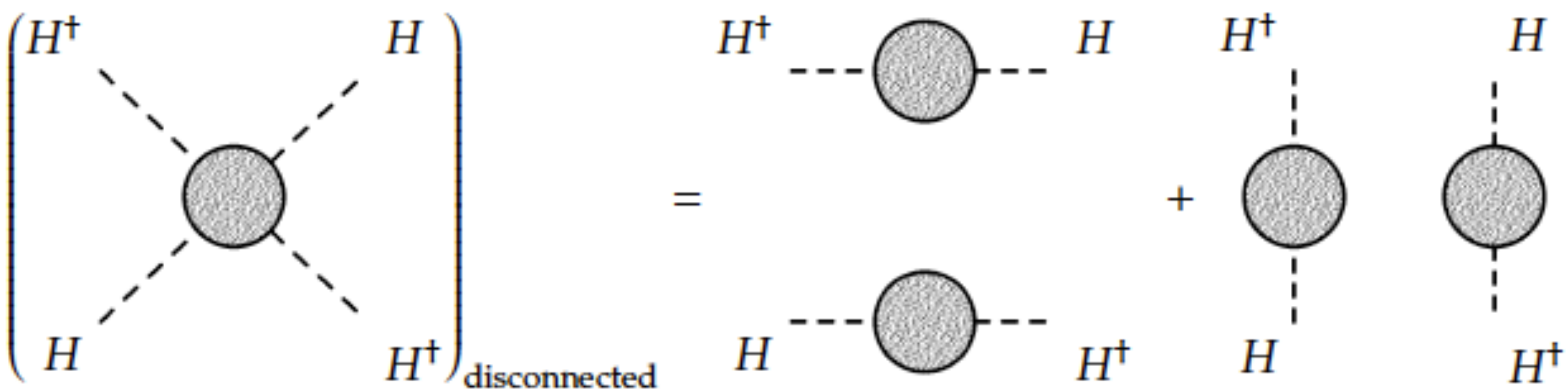
Disconnected diagrams are leading in the large N limit

$$G^{disc}_A = \left(\frac{u}{v}\right)^{d_H}$$

we found on the previous slide

Solving the bootstrap in Veneziano limit

Structure of the 4-pt function:



$$\mathcal{O}(1): \qquad u^{d_H}\widetilde{G}^{disc}_A = v^{d_H} G^{disc}_S\,, \quad \text{with} \quad \widetilde{G}^{disc}_A = \left(\frac{v}{u}\right)^{d_H}$$

$$\mathcal{O}(1): \qquad v^{d_H} G^{disc}_A = u^{d_H}\widetilde{G}^{disc}_S\,, \quad \text{with} \quad G^{disc}_A = \left(\frac{u}{v}\right)^{d_H}$$

$$\Rightarrow \qquad G^{disc}_S = \widetilde{G}^{disc}_S = 1.$$

Solving the bootstrap in Veneziano limit

$$\mathcal{O}(1/N_f^2): \qquad v^{d_H}\left(G_S^{conn} - (G_L + G_R)\right) + u^{d_H} = u^{d_H}\widetilde{G}_A^{conn}\,,$$

$$\mathcal{O}(1/N_f^2): \qquad u^{d_H}\left(\widetilde{G}_S^{conn} - (\widetilde{G}_L + \widetilde{G}_R)\right) + v^{d_H} = v^{d_H}G_A^{conn}\,,$$

$$\mathcal{O}(1/N_f^2): \qquad \left[v^{d_H}(G_L + G_R) - u^{d_H}(\widetilde{G}_L + \widetilde{G}_R)\right] = 2(u^{d_H} - v^{d_H})$$

$$\Rightarrow \qquad v^{d_H}G_S^{conn} - u^{d_H}\widetilde{G}_S^{conn} = u^{d_H}(1 + \widetilde{G}_A^{conn}) - v^{d_H}(1 + G_A^{conn})$$

after additional considerations (with some caveats)

$$v^{d_H}(G_S^{conn})^{\text{non-fact}} - u^{d_H}(\widetilde{G}_S^{conn})^{\text{non-fact}} = u^{d_H} - v^{d_H}$$

Numerical results

Banks-Zaks FP exists when 1-loop coefficient of the gauge beta function is small and the signs of the 1- and 2 -loop coefficients are opposite:

$$\beta_{a_g} = -\frac{2}{3}a_g^2\Big[11 - 2\ell - 2x + (34 - 16\ell - 13x)\,a_g + 3x^2 a_H + \frac{81x^2}{4}a_g a_H - \frac{3x^2(7+6x)}{4}a_H^2 + \frac{2857 + 112x^2 - x(1709 - 257\ell) - 1976\ell + 145\ell^2}{18}a_g^2\Big], \tag{18}$$

$$\beta_{a_H} = a_H\Big[2(x+1)a_H - 6a_g + (8x+5)a_g a_H + \frac{20(x+\ell) - 203}{6}a_g^2 - 8xz_2 a_H - \frac{x(x+12)}{2}a_H^2 + 4z_2^2\Big], \tag{19}$$

$$\beta_{z_1} = 4\left(z_1^2 + 3z_2^2 + 4z_1 z_2 + z_1 a_H\right), \qquad \beta_{z_2} = 2\left(2z_2 a_H + 4z_2^2 - x a_H^2\right). \tag{20}$$

Comparison with the bootstrap bound strategy:

- For a given FP (which means FP values of $(a_g^*, a_H^*, z_2^*, z_1^*)$ corresponding to a fixed $x \equiv N_f/N_c$ and ℓ), calculate the (γ_S, γ_A) and γ_H values
- Use the same value of γ_H to compute the $\gamma_{max} \equiv \Delta_{max} - 2$ value from and compare with the (γ_S, γ_A) values

$$\Delta \leq \Delta_{max} = 2 + 3.006(d-1) + 0.16(1 - e^{-20(d-1)})$$

Numerical results (QCD in the Veneziano limit) in the WCC (321) scheme

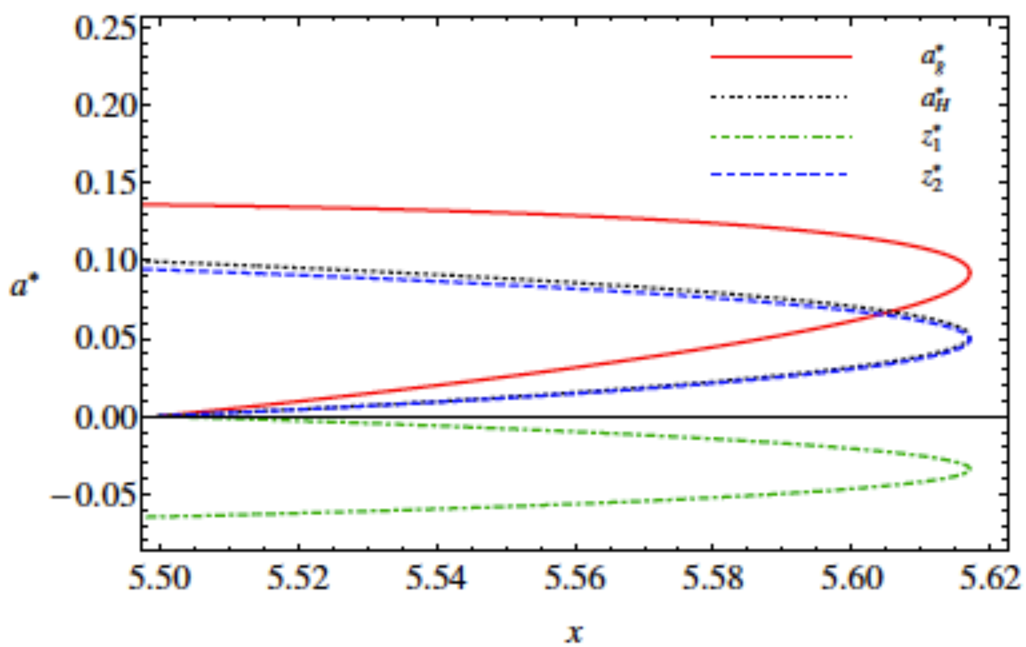
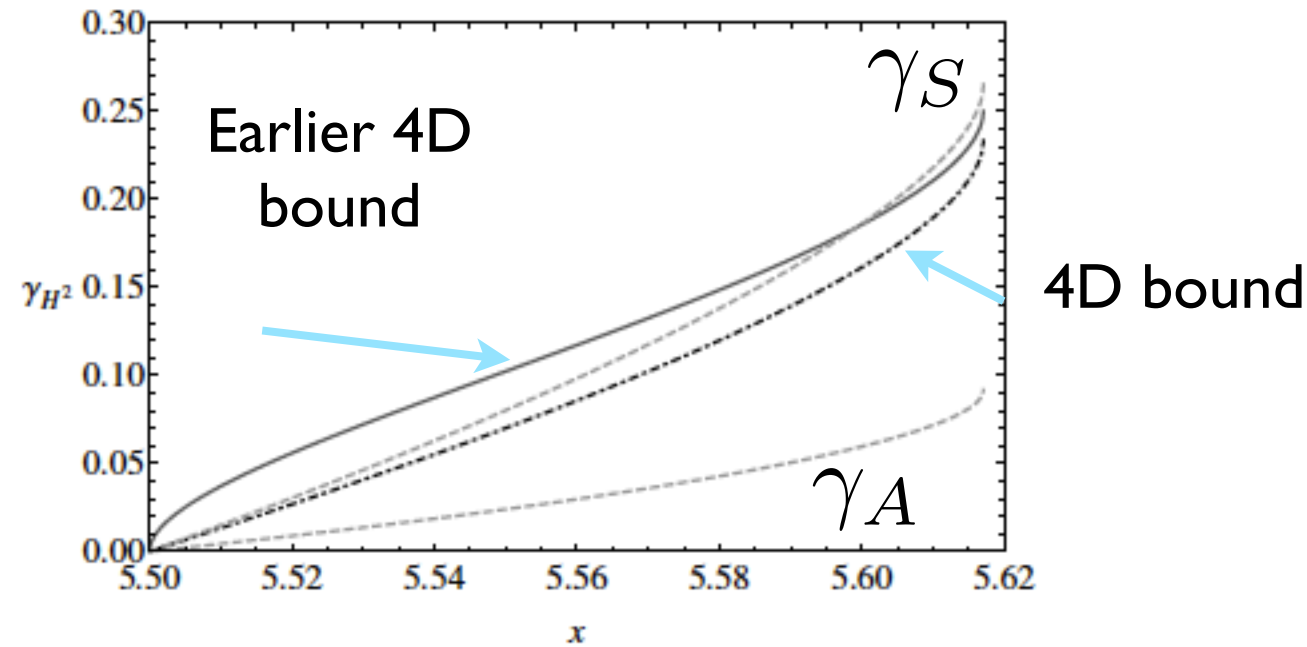
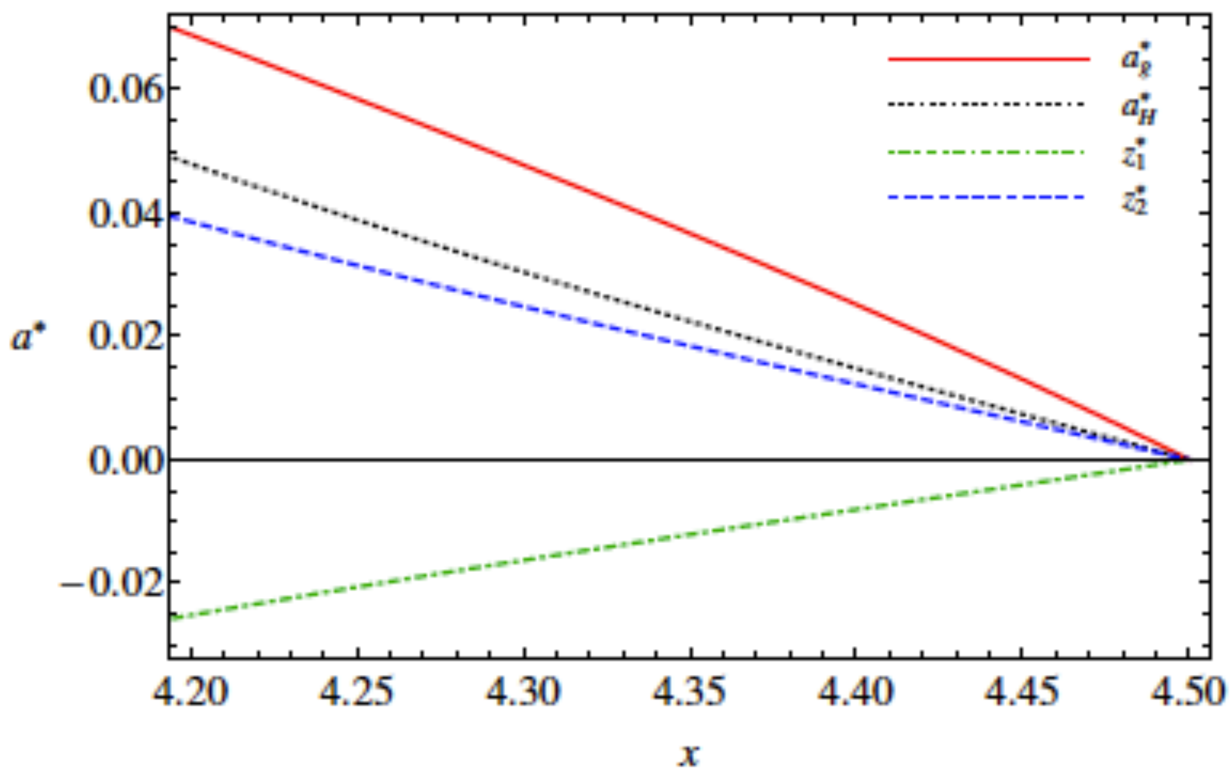


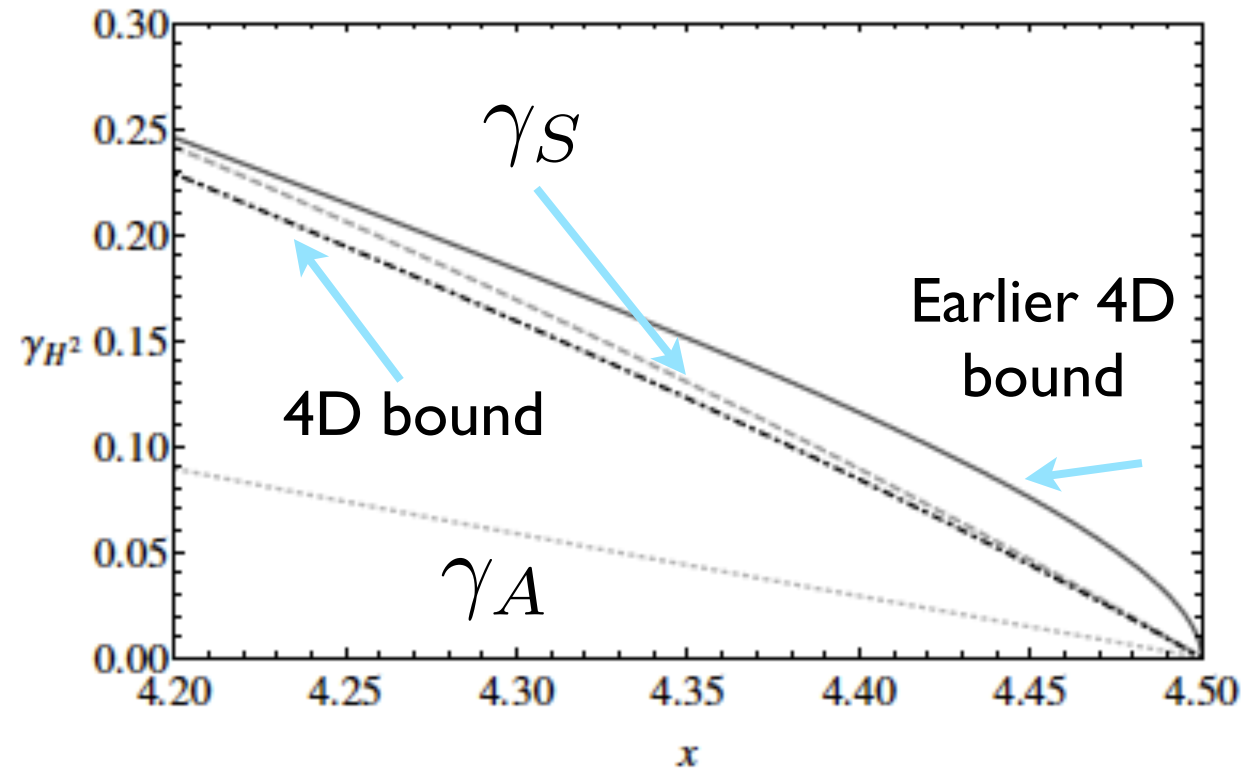
FIG. 3.a Fixed point structure of the model with $\ell = 0$. The boundary of asymptotic freedom is on the left-hand edge of the plot at $x = 5.5$, the FP value of a_g is the solid red line, a_H is the dotted black, z_1 is the dot-dashed green, and z_2 is the dashed blue.



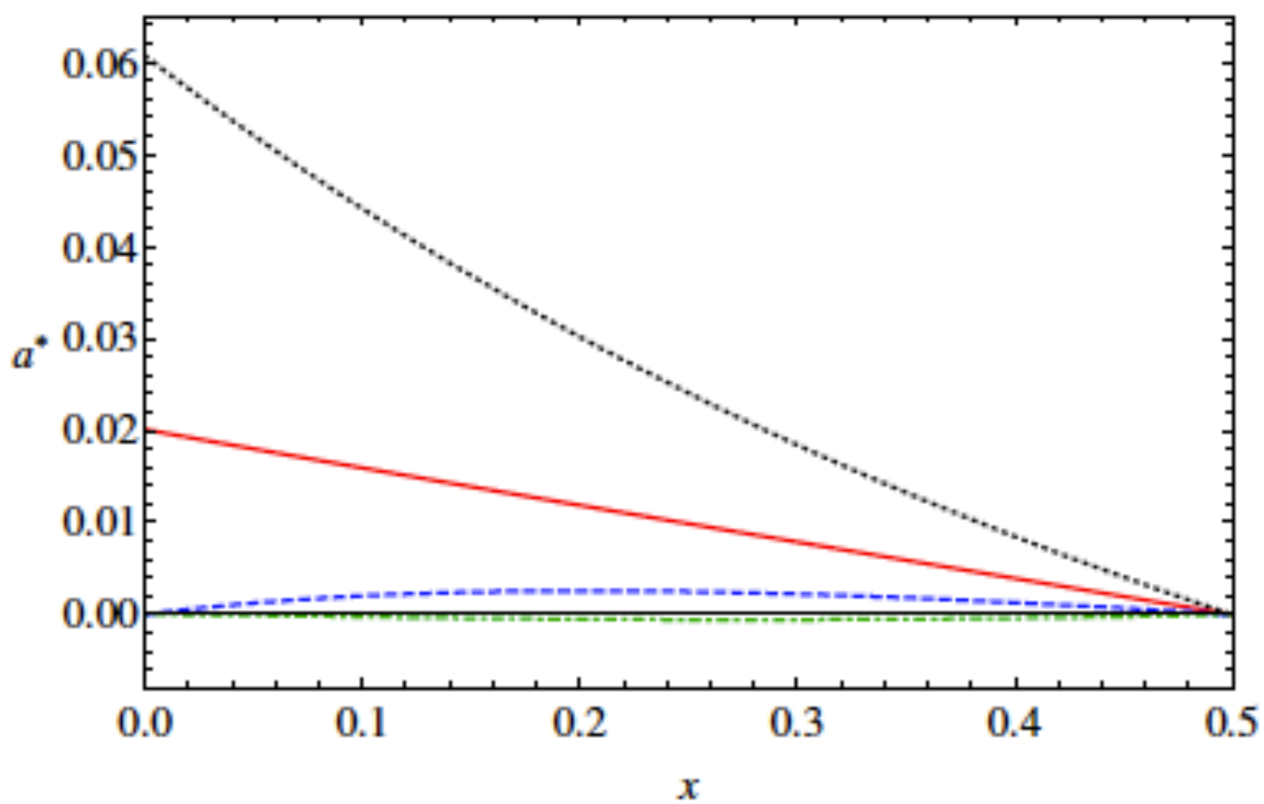
The functional form of the strongest 4D bound was chosen somewhat arbitrary and might not be the best approximation in the perturbative region

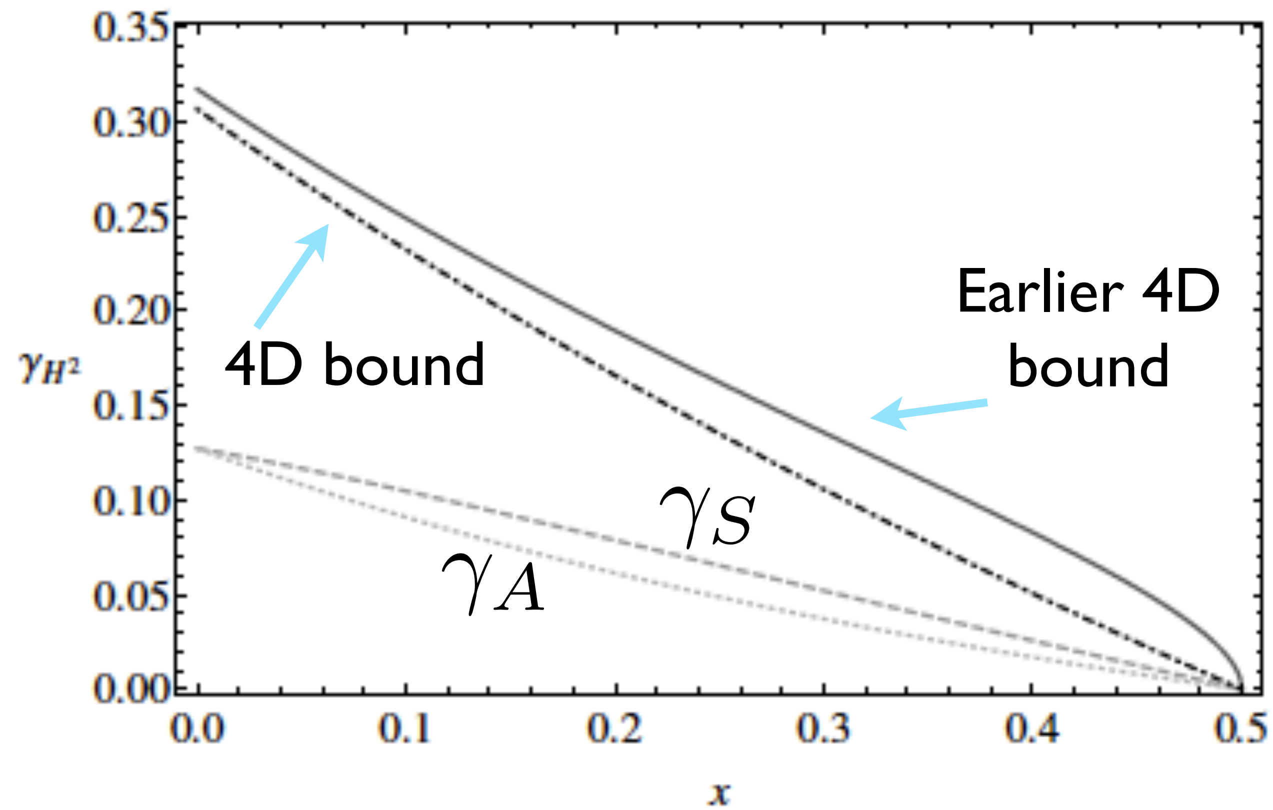
Numerical results (QCD in the Veneziano limit with one gluino) in the WCC (321) scheme





Numerical results (QCD in the Veneziano limit with five gluinos) in the WCC (321) scheme



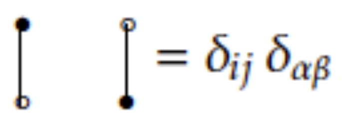


Conclusions

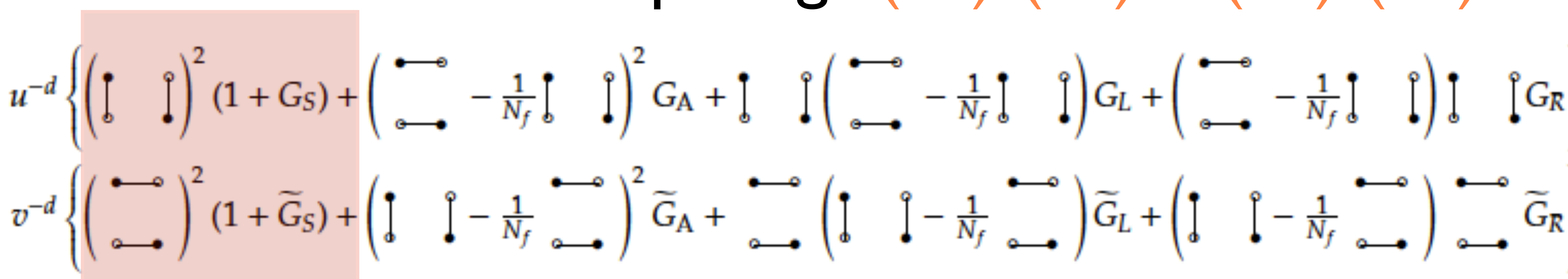
- We reviewed the 4D bound on the lowest dimension scalar in the arbitrary 4D CFT from the bootstrap equation
- We derived the crossing symmetry constraints for the QCD-like theories
- We considered the QCD in the Veneziano limit and computed anomalous dimensions appearing in the basic OPE to the 2-loop level. We found that anomalous dimension of the singlet is bigger than of the adjoint
- We showed that the OPE contains a "double trace" operator leading to disconnected correlators of "generalized free scalar field"
- We solved the QCD-bootstrap system analytically in the large-N expansion and argued that there is a part of the conformal block for singlet operator satisfying the bootstrap condition without global symmetry

Future: Solve the bootsrap system for QCD numerically and compare with the perturbative results in our perturbative model

CFT with a global symmetry: SU(N)xSU(N)

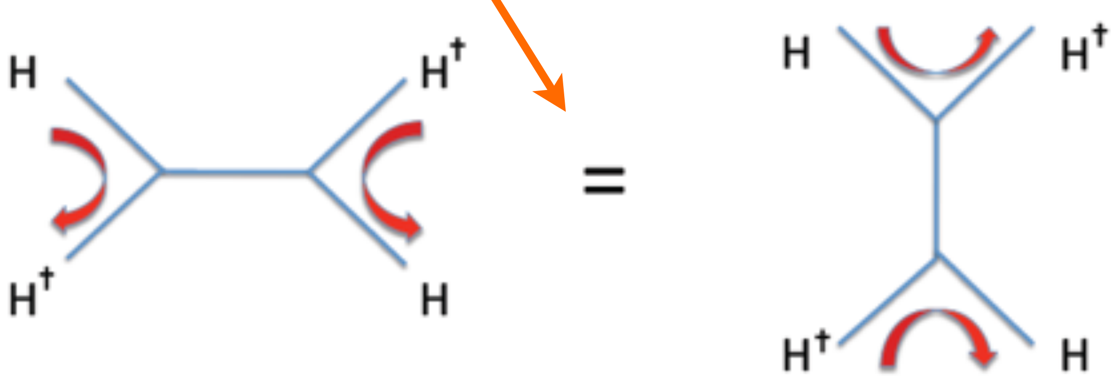


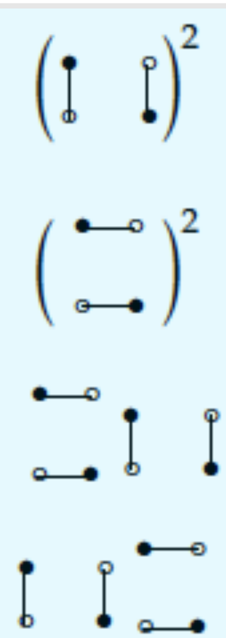
requiring : (12)-(34) = (14)-(23)



(12)-(34)

(14)-(23)





$$: \quad u^{-d}\left(1+G_S-\frac{1}{N_f}(G_L+G_R)+\frac{1}{N_f^2}G_A\right)=v^{-d}\,\widetilde{G}_A$$

$$: \quad u^{-d}\,G_A=v^{-d}\left(1+\widetilde{G}_S-\frac{1}{N_f}(\widetilde{G}_L+\widetilde{G}_R)+\frac{1}{N_f^2}\widetilde{G}_A\right)$$

$$: \quad u^{-d}\left(G_R-\frac{1}{N_f}G_A\right)=v^{-d}\left(\widetilde{G}_L-\frac{1}{N_f}\widetilde{G}_A\right)$$

$$: \quad u^{-d}\left(G_L-\frac{1}{N_f}G_A\right)=v^{-d}\left(\widetilde{G}_R-\frac{1}{N_f}\widetilde{G}_A\right),$$

In the large Nf limit :

$$u^{-d}\left[1+G_S+G_A\right]=v^{-d}\left[1+\widetilde{G}_S+\widetilde{G}_A\right] \qquad u^{-d}\left[G_L+G_R\right]=v^{-d}\left[\widetilde{G}_L+\widetilde{G}_R\right]$$