Existence of Mass-Gap & Pure Yang – Mills Theory

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Abstract

Due to the non-linearity involved in quantum chromodynamics (QCD), the required uncertainty in position of a transverse hard gluon, emitted in 3-jet event, is accommodated by allowing for the possibility that Gribov copies are created as virtual entities by quantum fluctuations of the transverse gluon energy over the brief intervals of time during which the special relativity theory and the quantum theory are merged together consistently in QCD. These Gribov copies can be ignored in perturbative sector due to asymptotic freedom of pure QCD empty space but their common characterstic i.e., zero value of Faddeev-popov operator, serves as a mathematical proof of mass-gap and color confinement properties on the boundary of the Gribov region, so-called the Gribov horizon in the non-perturbative sector of pure QCD.

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1. <u>INTRODUCTION</u>:

"The laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. In 1954, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the "mass gap": the quantum particles have positive masses, even though the classical waves travel at the speed of light. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of Yang –Mills theory & mass-gap will require the introduction of fundamental new ideas both in physics and mathematics. Here in this paper the existence of the Quantum

Yang-Mills theory & mass gap has been mathematically proved by introduction of two fundamental ideas." [24]

Firstly, the fundamental idea of establishing Gribov copies as virtual entities is introduced. "Quantum mechanics and special relativity are two great theories of twentieth-century physics. Both are very successful. But these two theories are based on entirely different ideas, which are not easy to reconcile. In particular, special relativity puts space and time on the same footing, but quantum mechanics treats them very differently. This leads to a creative tension: *At the time of its emission in 3-jet event, imagine a transverse hard gluon particle moving on average at the speed of light in perturbative sector of pure QCD, but with an uncertainty in position, as required by quantum theory. Evidently it there will be some probability for observing this transverse gluon particle to move a little faster than average, and therefore faster than light, which special relativity won't permit*."[Adapted from Ref. 1]

"Normally, in quantum electrodynamics (QED) the only known way to resolve this tension in italics involves introducing the idea of antiparticles. Very roughly speaking, the underlying linearity of QED accommodates the required uncertainty in photon position by allowing for the possibility that the act of measurement can involve the creation of several particles, each indistinguishable from the original, with different positions. To maintain the balance of conserved quantum numbers, the extra particles must be accompanied by an equal number of antiparticles. When special relativity is taken into account, quantum theory must allow for fluctuations in energy over brief intervals of time. This is a generalization of the complementarity between momentum and position that is fundamental for ordinary, non-relativistic quantum mechanics. Loosely speaking, energy can be borrowed to make evanescent virtual particles, including particle-antiparticle pairs. Each pair passes away soon after it comes into being, but new pairs are constantly boiling up, to establish an equilibrium distribution. In this way the wave function of (superficially) empty space becomes densely populated with virtual particles, and the empty space comes to behave as a dynamical medium."[Adapted from Ref. 1] "However, in case of QCD, the gluons are the quanta of Yang-mills field which obeys highly non-linear field equations. As a result, there is no known analytical approach for strong interaction physics for making predictions from first principles and developing a fundamental understanding of the theory." [25]

In view of the aforesaid non-linearity, the aforesaid tension in italics has not been resolved in this paper by the introduction of the idea of antiparticles for a hard transverse gluon emitted in 3-jet event. Rather, the technical aspects of the Hamiltonian quantization in Coulomb gauge have been taken into account in this paper for the resolution of the aforesaid tension in case of a hard transverse gluon emitted in 3-jet event. Given the fact that the non-linearization of QCD theory is due to color-charged nature of its quanta called gluon, it has been shown in this paper that due to color charged nature, the stay of a transverse gluon on any gauge orbit in configuration space is only over brief intervals of time, say ϕ and when special relativity is taken into account, quantum theory must allow for fluctuations in transverse gluon energy over these brief intervals ϕ of time. This is a generalization of the complementarity between momentum and position and accordingly, has been used in overcoming the technical complication in the derivation of the Hamiltonian operator $\hat{H}$ in the Equation (10b) of this paper. Loosely speaking, energy can be borrowed to make evanescent Gribov copies as virtual entities with different positions. These Gribov copies pass away soon after the exit of the transverse gluon from the particular gauge orbit in question but new sets of Gribov copies are constantly coming up at next gauge orbits.

Secondly, the fundamental idea of using the zero value of Faddeev-popov operator on the boundary of the Gribov region, so-called the Gribov horizon in the non-perturbative sector of pure QCD as a mathematical proof of the existence of the Quantum Yang-Mills theory and Mass-gap & color confinement properties has been introduced. A high energy hard transverse gluon, at the time t_0 of its emission, say in 3-jet event, lies somewhere in the vicinity of $A = 0$ in the *fundamental modular region* (FMR) of the configuration space where the Gribov copies can be easily ignored due to the asymptotic freedom property of pure QCD empty space. Theoretically speaking, this ignoring of Gribov copies in FMR is evident from the positive value of Faddeev – Popov (F.P.) operator in FMR as stated by Equation (16) in this paper. This positive value of F.P. operator is maintained so throughout the FMR with reference to high energy hard transverse gluon only. But on the boundary of the Gribov region, so-called the Gribov horizon, when the effective interaction strength (QCD coupling) becomes of the order of unity, the lowest eigenvalue of the F-P operator vanishes as stated by Equation (19) in this paper. For the transverse gluon stepping out of the FMR and on the boundary of the Gribov region, so-called the Gribov horizon, the vanishing of the lowest eigenvalue of the F-P operator in Equation (19) in this paper is, in-fact, an indication of replacement of the said hard transverse gluon by a set of Gribov copies pertaining to the gauge orbit on the boundary of the Gribov region, so-called the Gribov horizon in configuration space. Thus, the zero value of the F-P operator in Equation (19) in this paper refers to a set of Gribov copies pertaining to the gauge orbit on the boundary of the Gribov region, so-called the Gribov horizon in configuration space and accordingly, is a proof of 'mass-gap' property in the light of the Equation (20) in this paper.

In a Yang – Mills theory, the variable conjugate to A_0 of the gauge field A_μ ($\mu = 0,1,2,3$) does not exist. This implies that not all the four components of A_μ are independent. Unlike the well-known Weyl gauge, where the condition $A_0 = 0$ implies that the component A_0 has been independently fixed to zero without any reference to other three spatial components ($\mu = 1,2,3$) of A_μ, here the component A_0 has been treated as a dependent variable while implementing the Gauss law identically (in principle) by construction at Lagrangian level in this paper. The generalized Coulomb gauge then modifies this Gauss law to express dependent variable A_0 as a functional of the other three spatial components (μ = 1,2,3) of A_μ. During the quantization procedure, the transient nature of the generalized Coulomb gauge allows the direct imposition of the equal time canonical commutation relations $[A_i^{a\perp}(x), \pi_j^{b\perp}(y)] = \delta_{ij}\, \delta^{ab}\, \delta^3(x - y)$ involving unconstrained, independent vector potential $A_i^{a\perp}(x)$ and momenta variable $\pi_j^{b\perp}(y)$ in operator form for a transverse gluon. Consequent upon the same, the required uncertainty in position of hard transverse gluon emitted in 3-jet event is accommodated by allowing for the possibility that Gribov copies are created as virtual entities with different positions by quantum fluctuation of the transverse gluon energy over the brief intervals of time coinciding with transient existence of the generalized Coulomb gauge in full accordance with the uncertainty principle. Over and above the creation of Gribov copies as virtual entities, the pure QCD empty space, being full of particle-antiparticle pairs of virtual gluons, shows asymptotic freedom property due to which these Gribov copies can be ignored in perturbative sector but their common characterstic i.e., zero value of Faddeev-popov operator, serves as a mathematical proof of mass-gap and color confinement properties on the boundary of the Gribov region, so-called the Gribov horizon in the non-perturbative sector of pure QCD.

2. HAMILTONIAN FORMULATION:

Arthur Jaffe and Edward Witten in their paper [2] has mentioned that classically, by substituting the abelian group $U(1)$ with a more general compact gauge group $G = SU(3)$, the curvature is changed from $F = dA$ to $F = dA+A\wedge A$, and Maxwell's equations,0 = $dF = d{*}F$, are transformed to the Yang-Mills equations, $0 = d_AF = d_A{*}F$, where* is the Hodge duality operator, A is pure Yang-Mills gauge potential, F is pure Yang-Mills gauge field and d_A denotes the gauge-covariant extension of the exterior derivative. Further, these Yang-Mills equations can be validated by deriving them from the following pure Yang-Mills action L' that is not assumed to satisfy any particular gauge-condition and is more conveniently expressed as an integral of a pure Yang-Mills Lagrangian L in an appropriate time interval (t_0, t_1) i.e.,

$$L' = (-1/4) \int d^4x\, (F_{\mu,\nu}\,,\, F^{\mu,\nu}) \quad = \int_{t_0}^{t_1} dt\, L \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

where $F_{\mu,\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig[A_\mu(x), A_\nu(x)]$ and μ,ν are denoting space-time indices that take value in the range (0, 1, 2, 3) and

$$L = (-1/2) \int_V d^3x\, (\nabla_k(\boldsymbol{A_k})\, A_0 - \boldsymbol{\acute{A}_k}, \nabla^k(\boldsymbol{A^k})\, A^0 - \boldsymbol{\acute{A}^k}) - (1/4) \int_V d^3x (F_{kl}\, (\boldsymbol{A_k}), F^{kl}(\boldsymbol{A^k})) \text{\_\_\_\_\_\_\_\_(2)}$$

where $F_{kl} = \partial_k A_l(x) - \partial_l A_k(x) - ig[A_k(x), A_l(x)]$; k, l are denoting space indices ranging from 1 to 3; g is arbitrary non-vanishing real parameter; A_0 denotes the time-component of the pure Yang-Mills gauge potential A_μ & the quantity V is closed domain in R^3.

As a first step towards canonical quantization, we convert the classical Lagrangian L of Equation (2) into a Hamiltonian one. For this conversion, we follow the standard procedure by defining the canonical momenta $\pi_a^{\ \mu}$ as under:

$$\pi_a^{\ \mu} = \partial L'' / \partial \acute{A}^a_{\ \mu} = -\, F_a^{\ \mu 0} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (3)}$$

where A_μ are considered as **independent quantities** so that $\acute{A}^a_{\ \mu}$ are the **generalized velocities** and L'' denotes the Lagrangian density such that $L = \int_V d^3x\, L''$

It can easily observed that the canonical momenta $\pi_a^{\ 0}$ corresponding to the time index $\mu = 0$ in the above Equation (3) vanishes due to antisymmetry of $F^{\mu\nu}$ and accordingly, the Gauss law ($\nabla_k(A)\ \pi_a^{\ k} = 0$) is absent. Without the proper incorporation of the Gauss law into Hamiltonian formalism, no physical applications are possible.

So, for implementing the Gauss law identically (in principle) by construction at classical Lagrangian level as a constraint [7], first of all we solve the following non-abelian Gauss law,

$$\nabla_k(A)\nabla^k(A)A^0 - \nabla_k(A)\, \boldsymbol{\acute{A}^k} = 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (4)}$$

where the space indices k = 1,2,3 and $\nabla_k(A)$ is 'covariant gradient',

By treating the above Equation (4) as a system of linear, elliptic partial differential equations, the (matrix valued) potential component A_0, for given value of the space components $\boldsymbol{A}_\mathbf{k}$ & their time derivatives $\partial_0\boldsymbol{A}_\mathbf{k}$, has been determined in [7] by assuming that the unique solution A_0 as a functional of $\boldsymbol{A}_\mathbf{k}$ & their time derivatives $\partial_0\boldsymbol{A}_\mathbf{k}$ does exist i.e.,

$$A_0 = A_0\{\boldsymbol{A}_\mathbf{k}, \partial_0\boldsymbol{A}_\mathbf{k}\} \qquad (5)$$

Now, for implementing the Gauss law at classical Lagrangian level as a constraint [7] we substitute the above Equation (5) into the classical Lagrangian Equation (2) to get new Lagrangian L_0, i.e.,

$$L_0 = (-1/2) \int_V d^3x \, (\nabla_k(\boldsymbol{A}_\mathbf{k}) A_0\{\boldsymbol{A}_\mathbf{k}, \partial_0\boldsymbol{A}_\mathbf{k}\} - \boldsymbol{\acute{A}}_\mathbf{k}, \nabla^k(\boldsymbol{A}^\mathbf{k}) A^0\{\boldsymbol{A}^\mathbf{k}, \partial^0\boldsymbol{A}^\mathbf{k}\} - \boldsymbol{\acute{A}}^\mathbf{k})$$
$$- (1/4) \int_V d^3x (G_{kl}(\boldsymbol{A}_\mathbf{k}), G^{kl}(\boldsymbol{A}^\mathbf{k})) \qquad (6)$$

where $G_{kl} = \partial_k A_l(x) - \partial_l A_k(x) - ig[A_k(x), A_l(x)]$ and k, l are denoting space indices ranging from 1 to 3 & the quantity V is a finite closed volume. This new Lagrangian L_0 must reproduce the Lagrange equations of motion for k = 1,2,3 when Hamilton's action principle is invoked in the limit $V \rightarrow \infty$.

Now, we proceed to the Hamiltonian construction, for which the substitution of L_0 of the Equation (6) into the Equation (3) above leads to

$$\pi^k = \partial L_0{}^{''}/\partial \acute{A}^k = (\nabla^k(A) A^0\{\boldsymbol{A}^\mathbf{k}, \partial^0\boldsymbol{A}^\mathbf{k}\} - \boldsymbol{\acute{A}}^\mathbf{k}) \qquad (7)$$

where $L_0{}^{''}$ denotes the Lagrangian density such that $L_0 = \int_V d^3x \, L_0{}^{''}$

From the above expression for canonical momentum π^k, it seems impossible to find generalized velocity $\acute{A}^k$ in terms of π^k and $\boldsymbol{A}^\mathbf{k}$ because here A^0 is a functional of $\boldsymbol{A}^\mathbf{k}$ and their time derivatives $\boldsymbol{\acute{A}}^\mathbf{k} = \partial^0\boldsymbol{A}^\mathbf{k}$. To convert the aforesaid impossibility into possibility, we impose the generalized Coulomb gauge fixing condition at Lagrangian level on the non-abelian Gauss law of Equation (4) i.e.,

$$\nabla_k(A)\,\acute{\boldsymbol{A}}^{\mathbf{k}} = 0 \qquad (8)$$

With the substitution of Equation (8) in equation (4), we get

$$(\nabla_k(A)\,\pi_a^{\,k}) = \nabla_k(A)\nabla^k(A)A^0 = 0 \qquad (9)$$

The ‘covariant gradient’ term $\nabla_k(A)$ in Equation (8) above can be expanded as

$$\nabla_k(A)\partial_0 A^k(x) \equiv \partial_k\partial_0 A^k(x) + ig[A_k(x), \partial_0 A^k(x)] = 0 \qquad (9a)$$

Where g is arbitrary non-vanishing real parameter.

In above Equation (9a), the first term $\partial_k\partial_0 A^k(x)$ is exclusively responsible for changes in components of the time derivative of vector potential $A^k(x)$ and the second term $ig[A_k(x), \partial_0 A^k(x)]$ is exclusively responsible for ‘twisting’ the co-ordinate system with respect to the co-ordinate derivative. Consequent upon the forgoing exclusive acts, both of these ‘first term’ and ‘second term’ in Equation (9a) above are individually also equal to zero, i.e.,

$$\partial_k\partial_0 A^k(x) = ig[A_k(x), \partial_0 A^k(x)] = 0 \qquad (9b)$$

After integration with respect to time, the zero value of the first term $\partial_k\partial_0 A^k(x)$ in above Equation (9b) implies that the spatial divergence of the vector potential $A^k(x)$ remains equal to an arbitrary constant with passage of time and by taking this arbitrary constant equal to zero, we get the standard Coulomb gauge. Thus, the zero value of the spatial covariant derivative of the vector potential $A^k(x)$ is same as the zero value of the spatial divergence of the vector potential $A^k(x)$ as in standard Coulomb gauge. As such, we are left only with transversal component $A_k^{a\perp}$ and correspondingly, the transversal part of the momentum $\pi_a^{\,k}$ (i.e., $\pi_a^{\,k\perp}$) is exclusively focused upon.

Further, the longitudinal momentum component $\pi_a^{k||}$ arises from the resolution of the Gauss law in the Equation (9) above i.e.,

$$(\nabla_k(A)\, \pi_a^{k}) = \nabla_k(A)\, [\pi_a^{k||} + \pi_a^{k\perp}] = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (9c)$$

Since, in 3-jet event, the single hard transverse gluon is experimentally observed to be emitted from a color-singlet point source (i.e., quark – antiquark pair) in overall color-singlet manner [3], so, the color charge of the aforesaid single hard transverse gluon at the time t_0 of its emission from a color-singlet point source in 3-jet event must be momentarily taken to be equal to zero. As such, the longitudinal momentum component $\pi_a^{k||}$ in the above Equation (9c) is momentarily equal to zero for the aforesaid single hard transverse gluon lying somewhere in the vicinity of $A = 0$ in the *fundamental modular region* (FMR) of the configuration space at the time t_0 of its emission in 3-jet event i.e.,

$$(\nabla_k(A)\, \pi_a^{k}) = \nabla_k(A)\, [\pi_a^{k||} + \pi_a^{k\perp}] = \nabla_k(A)\, [0 + \pi_a^{k\perp}] = (\nabla_k(A)\, \pi_a^{k\perp}) = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (9d)$$

Further, in view of Equations (8) & (9), the equation (5) is accordingly modified as

$$A_0 = A_0\{\boldsymbol{A}_{\mathbf{k}}^{\perp}\} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (9e)$$

As such, one can now straightforwardly express the generalized velocity $\acute{A}^{k\perp}$ in terms of generalized co-ordinate and momenta variables by using $A_0 = A_0\{\boldsymbol{A}_{\mathbf{k}}^{\perp}\}$ in the Equation (7) i.e., $\acute{A}^{k\perp} = (\nabla^k(A)\, A^0\{\boldsymbol{A}^{\mathbf{k}\perp}\} - \pi^{k\perp})$. Therefore, the use of $\acute{A}^{k\perp} = (\nabla^k(A)\, A^0\{\boldsymbol{A}^{\mathbf{k}\perp}\} - \pi^{k\perp})$ and substitution of $L = L_0$ from the Equation (6) & substitution of the Gauss law constraint $(\nabla_k(A)\, \pi_a^{k\perp}) = 0$ from Equation (9d) in the mathematical construction $[H = \int_V (\pi_a^{k\perp}, \acute{A}_k^{a\perp})\, d^3x - L]$ of the Hamiltonian H through Legendre transformation yields

$$H = (-1/2) \int_V d^3x\, J^{-1}[\boldsymbol{A}_{\mathbf{k}}^{\perp}]\{(\nabla_k(\boldsymbol{A}_{\mathbf{k}})\, A_0\{\boldsymbol{A}_{\mathbf{k}}^{\perp}\} - \acute{\boldsymbol{A}}_{\mathbf{k}}^{\perp}\boldsymbol{\}},\, J[\boldsymbol{A}_{\mathbf{k}}^{\perp}]\{\nabla^k(\boldsymbol{A}^{\mathbf{k}})\, A^0\{\boldsymbol{A}^{\mathbf{k}\perp}\} - \acute{\boldsymbol{A}}^{\mathbf{k}\perp})\} + (1/4) \int_V d^3x(F_{kl}\, (\boldsymbol{A}_{\mathbf{k}}^{\perp}),\, F^{kl}(\boldsymbol{A}^{\mathbf{k}\perp})) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (10)$$

Where $J[\boldsymbol{A}_{\mathbf{k}}^{\perp}]$ is the Faddeev-Popov determinant, interpreted as the Jacobian of the transformation.

“A transition to the quantum version of the Hamiltonian H in the Equation (10) above, by means of the substitution of the fixed time Schrodinger quantization rule,

$$\pi_k^a \longrightarrow \hat{\pi}_k^a \equiv -i\frac{\delta}{\delta A_a^k} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (10a)}$$

in the Equation (10) yields the Hamiltonian operator $\hat{H}$ for the hard transverse gluon at the time t_0 of its emission in the 3-jet event and in the limit $V \rightarrow \infty$ in Equation (10), one is then invited to consider the following eigenvalue equation, in self-explanatory notation,

$$\hat{H}\Psi(A) = E\Psi(A) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (10b)” [17]}$$

However, the above derivation of the Hamiltonian operator $\hat{H}$ in the Equation (10b) is plagued by a **technical complication** [4] i.e., when the above generalized coulomb gauge fixing condition of the Equation (8) is in force for straightforwardly expressing the generalized velocity in terms of generalized co-ordinate and momenta variables, one cannot define canonical momentas by Equation (3) above for μ = 1,2,3 as these generalized velocities are no longer independent quantities. In order to solve this technical complication, we have derived below the transient nature of the generalized coulomb gauge fixing condition of the Equation (8).

In the Yang – Mills theory, gauge transformation ω is not global one. In figure 1, the general gauge potential A_k, valid at some time instant t_0 , transforms as a connection in the adjoint representation under local gauge transformation ω to A_{1k}, say valid at some later time instant t_1 ,[4] i.e.,

$$A_{1k} = \omega A_k \omega^{-1} - (i/g)(\partial_k \omega)\omega^{-1} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (11)}$$

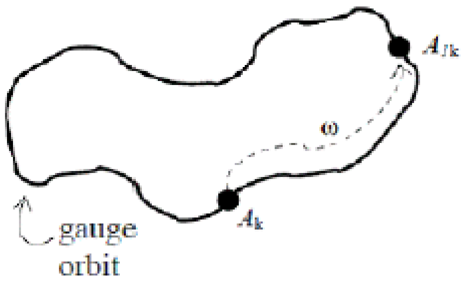

Figure 1 Illustration of the gauge orbit [11]

Differentiating above Equation (11) with respect to time t and then, applying 'covariant gradient' $\nabla_k(A)$ on both sides [7], we get

$$\nabla_k(A)\partial_0 A_{lk} = \omega[\nabla_k(A)\, \partial_0 A^{\,k} - \nabla_k(A)\nabla^k(A)\, X_0\,)]\omega^{-1} \qquad (11a)$$

where X_0 = (i/g)(ω^{-1})($\partial_0\omega$)

Thus, when the condition of equation (8) is imposed on the final potential A_{lk} by equating right hand side of the above equation to zero [4], we get

$$\nabla_k(A)\nabla^k(A)\, X_0 = \nabla_k(A)\partial_0 A^{\,k} \qquad (12)$$

If we consider any time slice at some in-between time instant t_i such that ($t_0 < t_i < t_1$), then the above elliptic linear partial differential equation (12), in-general, pertains to some in-between value of A_{ki} at the time instant t_i along the dotted gauge transformation path on the gauge orbit in above Figure 1. In other words, the Lie-algebra valued quantity X_0 = (i/g)(ω^{-1})($\partial_0\omega$) of the elliptic linear partial differential Equation (12) is exclusively defined at some in-between time instant t_i such that ($t_0 < t_i < t_1$) and remains non-zero along the dotted gauge transformation path on the gauge orbit in Figure.1 during the time–period $\phi = (t_1 - t_0)$ only. Accordingly, this gauge transform ω is uniquely determined in [4] at some fixed spatial point x by the following exponential time-integral. [4]

$$\omega(x, t_1) = [T \exp.\, (ig) \int_{t_0}^{t_1} dt\, X_0(t)]\, \omega(x, t_0) \text{ where T indicates time-ordering.} \qquad (12a)$$

Again, when the condition of equation (8) is imposed on the final potential A_{lk} by equating left hand side of the above Equation (11a) to zero [4], we get

$$\nabla_k(A)\partial_0 A_{lk} = 0 \qquad (12b)$$

Changing the order of time-derivative and spatial covariant derivative in above Equation (12b), we get

$$\partial_0\nabla_k(A) A_{lk} = 0 \qquad (12c)$$

The above Equation (12c) can be rewritten as

$$\{\partial[\nabla_k(A) A_{lk}] / \partial\omega\}\{\partial\omega / \partial t\} = 0 \qquad (12d)$$

Clearly, the second term $\{\partial\omega / \partial t\}$ in the above Equation (12d) is non-zero keeping in view the Equation (12a) above and accordingly, the above Equation (12d) can be rewritten as

$$\{\partial[\nabla_k(A) A_{lk}] / \partial\omega\} = 0 \qquad (12e)$$

Further, as per already drawn inference from Equation (9b) above, the zero value of the spatial covariant derivative of the vector potential A_{lk} is same as the zero value for the spatial divergence of the vector potential A_{lk} as in standard Coulomb gauge. As such, the above Equation (12e) can be rewritten as

$$\{\partial[\partial_k A_{lk}] / \partial\omega\} = 0 \qquad (12f)$$

The above Equation (12f) is nothing but temporal zero-modes of the Faddeev – Popov determinant that has been expressed in terms of standard Coulomb gauge on the concerned gauge-orbit. The aforesaid temporal zero-modes of the Faddeev – Popov determinant in Equation (12f) above are solely responsible for the validity of chromo-static condition $A_0 = A_0\{\boldsymbol{A}_{\mathbf{k}}^{\perp}\}$ in the above Equation (9e) that plays a decisive role while making a Legendre transform of the classical Lagrange density to the classical Hamilton density in Equation (10) above. Thus, the Faddeev – Popov determinant, which otherwise arises when considering gauge-fixing in quantum path-functional integral, gets implicitly introduced in gauge-fixing condition of Equation (8) above while making a Legendre transform of the classical Lagrange density to the classical Hamilton density in Equation (10) above because the above Equation (12f) has been ultimately derived from gauge-fixing condition of Equation (8).

Further, unlike the standard Coulomb gauge which is valid for all time & hence, is incomplete one – the gauge is only partially fixed [8, 9, 10] and where even after applying the standard Coulomb gauge, we can still perform time-dependent (spatially independent) gauge transformations, the generalized Coulomb gauge in the Equation (8) above is valid only for the time-period during which the temporal zero-modes of the Faddeev – Popov determinant in the above Equation (12f) exist because the zero value of the Equation (12d) above is solely dependent upon the time-existence of the temporal zero-modes of the Faddeev – Popov determinant in the above Equation (12f). Thus, after applying the generalized Coulomb gauge, we can still solely perform time-dependent (spatially independent) **infinitesimal** gauge transformations, as shown in above Equation (12a), that generate temporal zero-modes of the Faddeev – Popov operator on the gauge-orbit as shown in Equation (12f) above. This **infinitesimal** nature the time-dependent gauge transformation $A_k \rightarrow A_{1k}$, as illustrated in Figure.1 above, implies that the integration limits t_1 and t_0 of above time-integral, in Equation (12a), correspond to infinitesimal time–period $\phi = (t_1 - t_0)$. It is this infinitesimal nature of the time–period ϕ that leads to transient existence of X_0 in Equation (12) and hence, leads to the transient existence of $\nabla_k(A)\partial_0 A_k = 0$ as hypersurface in configuration space. Thus, the **transitory nature** of $\nabla_k(A)\partial_0 A_k = 0$ of Equation (8) has been derived from the time-dependent (spatially independent) infinitesimal gauge transformations, that generate temporal zero-modes of the Faddeev – Popov operator on the gauge-orbit. It is pertinent to mention here that Gauge tranforms are not necessarily infinitesimal – there exists non-temporal zero-modes of the Faddeev – Popov operator on the gauge-orbit called Gribov copies that have been considered elsewhere in this paper for implementing the causality for gluons. Let us now explain why the aforesaid transitory gauge-fixing condition $\nabla_k(A)\partial_0 A_k = 0$ of Equation (8) on the gauge orbit is needed a **priori**.

Since, the transverse gluon in generalized Coulomb gauge [4] evidently carries its own color charge [5, 6] and emits virtual longitudinal coulomb gluons to develop an accompanying non-abelian coulomb field at the expense of its own energy with passage of time, so, the eigenvalue E of the Hamiltonian operator $\hat{H}$ in the Equation (10b) above for the transverse gluon propagating in pure Yang-Mills gauge field is constant only for an infinitesimal time period that elapses between two consecutive emissions of virtual longitudinal coulomb gluons by the said transverse gluon. As such, the evaluation of the Hamiltonian operator $\hat{H}$ in the Equation (10b) above by considering the time-dependent (spatially independent) gauge transformation during the integration of the integrand $[X_0(t)]\ \omega(x, t_0)]$, in the above Equation (12a), over time is meaningful if and only if the time period $(t_1 - t_0)$ in Equation (12a) is taken to be infinitesimal one. In other words, we can say that the association of any transverse gluon with a particular gauge orbit is for the aforesaid infinitesimal time period $(t_1 - t_0)$ only as the Lagrangian action L' in Equation (1) is constant around the gauge orbit and hence, serves as a parameter for the identification of the gauge orbits. This explains why the transient gauge fixing condition of the Equation (8) is needed a **priori**.

In-fact, the aforesaid infinitesimal time period $(t_1 - t_0)$ expresses the uncertainty $(t_1 - t_0)$ in the time, at which the measurement of the eigenvalue E of the Hamiltonian operator $\hat{H}$ in the Equation (10b) above is made for a transverse gluon and accordingly introduces an uncertainty ΔE in the energy measurement of the transverse gluon in accordance with the **uncertainty principle** and this uncertainty ΔE when divided by speed of light ‘c’ in vacuum yields corresponding uncertainty Δp in momentum measurement of the massless transverse gluon. In accordance with the **uncertainty principle**, a transverse gluon with uncertainty Δp in momentum measurement may be assumed to be present at some space point with uncertainty Δx in position at some point of time in some inertial reference frame in which the Schrodinger formalism is manifestly valid for the transverse gluon in question. This is nothing but the imposition of the equal time canonical commutation relations $[A_i^{a\perp}(x), \pi_j^{b\perp}(y)] = \delta_{ij}\, \delta^{ab}\, \delta^3(x - y)$ for a transverse gluon such that the vector potential $A_i^{a\perp}(x)$ and momenta variable $\pi_j^{b\perp}(y)$ are in operator form, the latter being given by the Equation (10a) above.

Thus, the **transient** existence of covariant derivative null vector (i.e., $\nabla_k(A)\partial_0 A_k = 0$) allows the imposition of the equal time canonical commutation relations $[A_i^{a\perp}(x), \pi_j^{b\perp}(y)] = \delta_{ij}\, \delta^{ab}\, \delta^3(x - y)$ involving unconstrained, independent pure Yang-Mills vector potential $A_i^{a\perp}(x)$ and momenta variable $\pi_j^{b\perp}(y)$ in operator form for a transverse gluon and as such, this imposition of the aforesaid canonical commutation relations leads to the treatment of the generalized velocities $\partial_0 A_k$ as independent quantities for all intent & purpose during the transient existence of $\nabla_k(A)\partial_0\, A_{1k} = 0$. Thus, the transitory nature of $\nabla_k(A)\partial_0 A_k = 0$ of Equation (8) provides solution to the aforesaid **technical complication,** that plagues the derivation of the Hamiltonian operator $\hat{H}$ in the Equation (10b) above. Thus, the Hamiltonian operator $\hat{H}$ in the Equation (10b) above is fully valid one.

Let us assume that a high energy hard transverse gluon, at the time t_0 of its emission from a color-singlet point source (i.e., quark – antiquark pair) in 3-jet event, lies somewhere in the vicinity of $A = 0$ in the *fundamental modular region* (FMR) of the configuration space. Further, in 3-jet event, the gluon jets are identified by the particles in the hemisphere opposite to the hemisphere that is containing tagged quark & antiquark jets and is defined by a plane perpendicular to the principle event axis in the 3-jet event [3]. Consequent upon the same, in the aforesaid first hemisphere the Hamiltonian operator $\hat{H}$ in the Equation (10b) governs the time-development of wave-function of the aforesaid hard transverse gluon for infinitesimal time period $\phi = (t_1 - t_0)$ during which it undergoes time-dependent but space-independent infinitesimal gauge transformation as explained above. It is pertinent to mention here that for implementing the causality for the hard transverse gluon, Gribov copies are also generated as virtual entities with different positions by quantum fluctuations of gluon energy during the infinitesimal time period $\phi = (t_1 - t_0)$. At some time instant t_1, a virtual longitudinal coulomb gluon is emitted by the said hard transverse gluon (& also by its Gribov copies). Consequent upon this emission, the longitudinal momentum component $\pi_a^{k||}$ of the transverse gluon arises from the resolution of the Gauss law i.e.,

$$(\nabla_k(A)\, \pi_a^{k}) = \nabla_k(A)\, [\pi_a^{k||} + \pi_a^{k\perp}] = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (13a)$$

and the Lagrangian L_0, in the right hand side of the Equation (6), now contains an additional potential energy term due to the longitudinal momentum component $\pi_a^{k||}$. Accordingly, the Hamiltonian operator $\hat{H}$ in the Equation (10b) can be again derived, by making a Legendre transform of the classical Lagrange density to the classical Hamiltonian density in the light of the above Equation (13a), as

$$\hat{H}\Psi(A) = E\Psi(A) + H_{\mathrm{Coul}}\Psi(A) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (13)$$

where the additional coulomb term

$$H_{\mathrm{Coul}} = \tfrac{1}{2}\int d^3x\, d^3y\, \mathcal{J}^{-\frac{1}{2}}\rho^a(x)\, \mathcal{J}\, K^{ab}(x,y;\mathbf{A}_\perp)\, \rho^b(y)\, \mathcal{J}^{-\frac{1}{2}} \quad \text{\_\_\_\_\_\_\_} \quad (14)$$

arises from the Gauss law & is necessary to maintain gauge invariance such that $\rho = ig_s\,[\mathbf{A}_\perp, \mathbf{E}_\perp]$ is the color charge density induced by the hard transverse gluon

(& its Gribov copies) inside FMR and the Coulomb energy propagator K produces, inside FMR, an instantaneous interaction

$$K^{ab}(x, y; \mathbf{A}) = -\left[\frac{1}{\mathrm{D}[\mathrm{A}] \cdot \boldsymbol{\nabla}} \boldsymbol{\nabla}^2 \frac{1}{\mathrm{D}[\mathrm{A}] \cdot \boldsymbol{\nabla}}\right]^{ab}_{xy} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (15)}$$

It is pertinent to mention here that the sum of individual color charges ρ_1, ρ_2, ------------ induced by the hard transverse gluon and its Gribov copies individually inside FMR is equal to zero because the aforesaid high energy hard transverse gluon is experimentally observed to be emitted from a color-singlet point source (i.e., quark – antiquark pair) in 3-jet event i.e.,

$$0 = \rho_1 + \rho_2 + \text{-----------} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (15a)}$$

Further, the Hamiltonian operator $\hat{H}$ is same in both Equations (10b) and (13) because firstly, it is a constant of motion and secondly, the emission of the virtual longitudinal coulomb gluon, to develop an accompanying non-abelian coulomb field, at time t_1 takes place at the expense of the energy of the transverse gluon itself. Consequent upon the aforesaid constancy of the Hamiltonian operator $\hat{H}$, the first term $E\Psi(A)$, in the right hand side of the Equation (13), that describe the hard transverse gluon (& also its Gribov copies), now refer to a different gauge orbit as compared to the corresponding terms in Equation (10b). Thus, the Gauss law constraint of the Equation (13a) is satisfied again identically (in principle) by construction to obtain the Hamiltonian Operator Equation (13) for the aforesaid different gauge orbit and accordingly, the uncertainty in gluon position is once again referred to the uncertainty $\phi' = (t_2 - t_1)$ in the time, at which the energy measurement of the transverse gluon is made for the imposition of the equal time canonical commutation relations $[A_i^{a\perp}(x), \pi_j^{b\perp}(y)] = \delta_{ij}\, \delta^{ab}\, \delta^3(x - y)$ on the aforesaid different gauge orbit. This cycle goes on repeating itself uninterruptedly with passage of time throughout the FMR as a result of which, as experimentally observed in running coupling constant of QCD, the additional coulomb term H_{Coul} in Equation (13) gets stronger and stronger with the addition of more and more emitted virtual longitudinal coulomb gluons into the accompanying non-abelian coulomb field and the transverse gluon (i.e., the first term $E\Psi(A)$, in the right hand side of the Equation (13)), with decreasing energy, approaches the common boundary, called Gribov horizon, of FMR and the Gribov region. In the next paras, we discuss the fact that on the boundary of Gribov region, so-called Gribov horizon, any instantaneous eigenvalue of the Hamiltonian operator $\hat{H}$, in above Equation (13), represents more than one identical particles viz. the **non-temporal** zero-modes of the Faddeev – Popov operator called **Gribov copies** as virtual entities, with different positions, that can be used as a proof of the '**mass-gap**' property in non-perturbative regime of QCD.

Since, it has already been concluded shortly after Equation (9b) above that the zero value of the spatial covariant derivative of the vector potential $A^k(x)$ is same as the zero value of the spatial divergence of the vector potential $A^k(x)$ as in standard Coulomb gauge, so, the standard Coulomb gauge is automatically satisfied when the generalized Coulomb gauge condition of the Equation (8) above is satisfied. In this regard, as Gribov discussed, the standard Coulomb gauge does not fix a gauge completely, and there are equivalent gauge configurations called Gribov copies that can be ignored in FMR due to asymptotic freedom property and accordingly, the Faddeev-Popov (F-P) ghost operator $M(\vec{A}) = -(\mathbf{D}[\mathbf{A}] \cdot \boldsymbol{\nabla})$ is positive [19] for the hard transverse gluon in FMR such that

$$-\mathbf{D}[\mathbf{A}] \cdot \boldsymbol{\nabla} \Phi = \rho = ig_s [\mathbf{A}_\perp, \mathbf{E}_\perp] \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (16)$$

where $\nabla\Phi = \pi_a^{k||}$ is the longitudinal momentum component, obtained from the resolution of the Gauss law and Φ is the color-Coulomb potential.

Thus, in the interior of Fundamental Modular Region (FMR) the F-P ghost operator $M(\vec{A})$ is strictly positive, so the inverse that appears in $\Phi = M^{-1}(\vec{A})\rho$ is well-defined except possibly at some points on the boundary of FMR where $M(\vec{A})$ may have a zero eigenvalue.

"In view of the above well-defined definition of the inverse of F-P operator in $\Phi = M^{-1}(\vec{A})\rho$, the additional coulomb term H_{Coul} in the Equation (13) can be expressed in the interior of FMR as

$$H_{\mathrm{Coul}} = \tfrac{1}{2}\int d^3x\, d^3y\, \mathcal{J}^{-\frac{1}{2}}\rho^a(x)\,\mathcal{J}\,K^{ab}(x,y;\mathbf{A}_\perp)\,\rho^b(y)\,\mathcal{J}^{-\frac{1}{2}} \qquad \text{\_\_\_\_\_\_\_(17)}$$

Where $\rho = ig_s[\mathbf{A}_\perp, \mathbf{E}_\perp]$ is the color charge density induced by the hard transverse gluon (& its Gribov copies) and the Coulomb energy propagator K produces an instantaneous interaction such that

$$K^{ab}(x,y;\mathbf{A}) = -\left[\frac{1}{\mathrm{D}[\mathbf{A}]\cdot\nabla}\,\nabla^2\,\frac{1}{\mathrm{D}[\mathbf{A}]\cdot\nabla}\right]^{ab}_{xy} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_ (18)}$$ "[18]

At this juncture, it is worthwhile to mention that during the infinitesimal time-period ϕ of the transient generalized Coulomb gauge, these Gribov copies are created as virtual entities with different positions by quantum fluctuations of the transverse gluon energy during the aforesaid infinitesimal time-period ϕ for accommodating the required uncertainty in hard transverse gluon position in accordance with the uncertainty principle. In other words, the wave-character of the hard transverse gluon, defined inside FMR, is implemented in term of its Gribov copies. This means that the Equations (10b) and (13) above has got additional multiple eigen-values in the form of Gribov copies and also, the time-development of wave-function $\Psi(A)$ as outlined above starting from the last para "Let us assume that ……" at page 15 to the Equation (18) above for transverse gluon defined inside FMR, is equally valid for the Gribov copies that are defined in 2^{nd} , 3^{rd} , 4^{th} , ……. Gribov regions in the hyperspace [$\nabla_k(A)\,\acute{\boldsymbol{A}}^{\mathbf{k}} = 0$] of configuration space as depicted below.

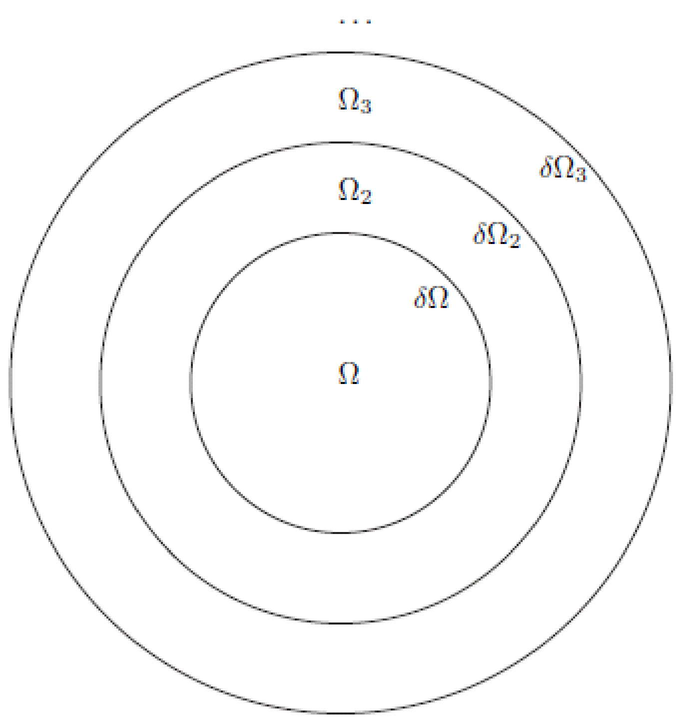

Figure 2: Different Gribov regions Ω, Ω_2, Ω_3, …. in hyperspace [$\nabla_k(A)\,\acute{\boldsymbol{A}}^{\mathbf{k}} = 0$] of Configuration space [31]

In view of the above, it is obvious that for implementing the wave-character, the outward journey of the transverse gluon, defined inside FMR, is simultaneously accompanied by the outward journey of the Gribov copies that are defined inside their respective Gribov regions Ω_2, Ω_3, -------- in Figure 2 above. For implementing the wave-character, the time instant, at which the transverse gluon approaches the common boundary $\delta\Omega$, called Gribov Horizon, of FMR and the first Gribov region, coincides with the time instants at which 1^{st}, 2^{nd}, 3^{rd}, 4^{th},

………… Gribov copies individually approaches their respective boundaries $\delta\Omega_2$, $\delta\Omega_3$, $\delta\Omega_4$, $\delta\Omega_5$ …….. of the subsequent Gribov regions Ω_2, Ω_3, -------- in Figure 2.

"On the boundary of the Gribov region, so-called the Gribov horizon, when the effective interaction strength (QCD coupling) becomes of the order of unity, the Gribov copies cannot be ignored. During aforesaid infinitesimal time-period of the transient generalized Coulomb gauge, the creation of Gribov copies as virtual entities with different positions is necessary for accommodating the required uncertainty in transverse gluon position. In other words, the wave-character of the transverse gluon is accommodated in term of Gribov copies. But there is **wave-particle duality** for the transverse gluon: **the wave-character and particle character complement each other i.e., both are never exhibited simultaneously[30]**. Consequent upon the same, the eigenvalue of the F-P operator for each and every Gribov copy vanishes on the boundary of the Gribov region, so-called the Gribov horizon i.e.,

$$-\mathrm{D}[\mathrm{A}] \cdot \nabla \Phi = 0 \qquad (19)$$

In other words, the eigen-value of the Faddeev – Popov operator in Equation (19) becomes zero for transverse gluon and all Gribov copies at the same time instant when they reach their respective Gribov boundaries $\delta\Omega$, $\delta\Omega_2$, $\delta\Omega_3$, $\delta\Omega_4$, $\delta\Omega_5$ …….. of the Gribov regions Ω, Ω_2, Ω_3, ……….. in Figure 2 above simultaneously.

It was argued by Zwanziger that entropy favors gauge configurations near the Gribov horizon and the eigenvalue distribution of the F-P operator gets concentrated near the vanishing eigenvalue compared to that in the abelian gauge theory [20]. Such an enhancement has been observed by the recent lattice simulations [21, 22]. In other words, there is concentration of the aforesaid non-temporal zero modes of $M(\vec{A})$ near its vanishing lowest eigenvalue."[23] This implies that any instantaneous eigenvalue of the

Hamiltonian operator $\hat{H}$, given by the above Equation (13), represents more than one identical particles viz. the **non-temporal** zero-modes of the Faddeev – Popov operator called **Gribov copies** corresponding to the gauge orbit on the boundary of the Gribov region, so-called the Gribov horizon in the configuration space and so, the single hard transverse gluon at the time of last emission cycle of virtual longitudinal gluon in FMR is substituted by more than one non-temporal zero modes of $M(\vec{A})$ on the boundary of the Gribov region, so-called the Gribov horizon. These non-temporal zero modes of $M(\vec{A})$ called Gribov copies are created as virtual entities with different positions on the boundary of the Gribov region, so-called the Gribov horizon in the non-perturbative sector of pure QCD by the quantum fluctuation of the transverse gluon energy during aforesaid infinitesimal time-period of the transient generalized Coulomb gauge in full accordance with the uncertainty principle.

Since, these short-lived Gribov copies as representatives of the same physical configuration cannot be detected individually at the same time instant due to aforesaid duality between wave-character & particle-character and accordingly, must present same combined physics to the outside world, so, on the boundary of the Gribov region, so-called the Gribov horizon, the common characteristic of all the Gribov copies – the zero color charge on the right hand side of the Equation (19) above – is, in-fact, the zero sum, in the Equation (15a), of individual color charges ρ_1, ρ_2, ------------ induced by these Gribov copies individually i.e.,

$$-\mathrm{D}[\mathrm{A}] \cdot \nabla \Phi = 0 = \rho_1 + \rho_2 + \text{-----------} \qquad (20)$$

and consequently, the instantaneous Coulomb energy propagator K in the Equation (18) does become singular, for each and every Gribov copy, on the boundary of the Gribov region, so-called the Gribov horizon. As such, the first term $E\Psi(A)$ in Equation (13) vanishes on the boundary of the Gribov region, so-called the Gribov horizon because the Kinetic energy of each Gribov copy, with infinite instantaneous Coulomb interaction energy, must approach zero value. As such, on the boundary of the Gribov region, so-called the Gribov horizon, only the additional coulomb term H_{Coul} given by Equation (17) is left in the spectrum of the eigenvalue Equation (13) i.e.,

$$\hat{H}\Psi(A) = H_{\mathrm{Coul}}\Psi(A) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (21)$$

which should now describe the massive gluon spectrum in the light of singular nature of the instantaneous Coulomb energy propagator K on the boundary of the Gribov region, so-called the Gribov horizon in non-perturbative regime.

Since, in Equation (17) valid inside FMR, the left hand side additional coulomb term H_{Coul} has got the dimensions of energy, so, on the right hand side of Equation (17), the color charge density $\rho = ig_s[\mathbf{A}_\perp, \mathbf{E}_\perp]$ is a dimensionless coupling constant inside FMR because the Coulomb energy propagator K has got the dimensions of energy density inside FMR. As such, the aforesaid vanishing of the first term $E\Psi(A)$ in Equation (13), on the boundary of the Gribov region, so-called the Gribov horizon, implies that the color charges ρ_1, ρ_2, ------------ induced by these Gribov copies individually, lost their meaning as dimensionless coupling constants or color charges. Instead, on the boundary of the Gribov region, so-called the Gribov horizon the product term $\rho^a(x)\,\rho^b(y)$, in the sole left out additional coulomb term H_{Coul} in Equation (21), become a dimensionful quantity, having dimension of energy density because the Coulomb energy propagator K, otherwise known as a term producing instantaneous interaction between Gribov copies, also serves as an instantaneous position indicator for the

Gribov copies & thus, destroys the wave-character of the transverse gluon on the boundary of the Gribov region, so-called the Gribov horizon.

Evidently, the transverse gluon has a dual character: the wave-character and particle character complement each other. Either character by itself is only part of the storey and can provide explanation for only certain effects [30]. In a specific event, the transverse gluon exhibits either a wave nature or a particle nature, never both simultaneously [30]. Due to this duality between particle-character and wave-character, the aforesaid destruction of the wave-character, by the Coulomb energy propagator K on the boundary of the Gribov region, so-called the Gribov horizon, is simultaneously accompanied by the sudden exhibition of the particle-character in the form of appearance of the massive single transverse gluon that is at rest in some moving inertial reference frame. This massive nature of the transverse gluon arises from the infinite value of the term K in the Equation (21) because this term K, otherwise having dimensions of energy density as the Coulomb energy propagator for the ensemble of Gribov copies inside FMR, transforms suddenly into **singular infinite term** when the zero eigenvalue of the operator $M(\vec{A})$ in the Equation (19) is substituted in the denominator of the K for the aforesaid single massive gluon on the boundary of the Gribov region, so-called the Gribov horizon. As such, on the boundary of the Gribov region, so-called the Gribov horizon, there is change in the dimensions of the product term $\rho^a(x)$ $\rho^b(y)$, in the sole left out additional coulomb term H_{Coul} in Equation (21), from dimensionless coupling constant to becoming a dimensionful quantity, having dimension of energy density because the aforesaid **singular term** being infinite cannot have the dimensions of energy density with reference to the aforesaid single massive transverse gluon.

Then, via this phenomenon of dimensional transmutation 1) one can calculate all the observables of QCD in terms of dynamically generated mass scale and there remains no adjustable parameter in QCD and 2) one can introduce a physical scale Λ_{QCD} at which 'mass-gap' property is demonstrated for each transverse gluon. Since, there is change in the dimensions of the product term $\rho^a(x)\,\rho^b(y)$, in the sole left out additional coulomb term H_{Coul} in Equation (21), from dimensionless coupling constant or 'color charge' to becoming a dimensionful quantity, having dimension of energy density with reference to the aforesaid single massive transverse gluon, so, the **color confinement** for the aforesaid massive transverse gluon also occurs at the aforesaid physical scale Λ_{QCD} on the boundary of the Gribov region, so-called the Gribov horizon in non-perturbative regime of QCD. It is pertinent to mention here that the causal propagation (i.e., the wave nature) of this massive transverse gluon, that is created in overall color singlet & electrically neutral manner on the boundary of the Gribov region, so-called the Gribov horizon and is at rest in some moving inertial reference frame, could no longer be dependent upon the Gribov copies and gauge-fixing and in-fact, the fragmentation of this massive transverse gluon occurs in laboratory reference frame shortly after its creation in overall color singlet & electrically neutral manner on the boundary of the Gribov region, so-called the Gribov horizon.

"In general, because the Gauge transformations contain arbitrary function of time, so, the usual canonical quantization procedure can only be carried out in a specific gauge. It is natural to inquire whether there are rules to ensure that quantum theories in different gauges are indeed the same. This question is closely connected with the ordering problem of operators especially in non-Aeblian Yang – Mills theory because of the non-linear nature of the Interaction"[29]. However, in the present case, the Gauge transformation does not contain arbitrary function of time but instead, as shown in above Equation (12a), is essentially time-dependent (spatially independent) **infinitesimal** gauge transformation that generate temporal zero modes of the Faddeev – Popov determinant, and as such, the usual canonical quantization procedure has been exclusively carried out in the transitory gauge-fixing condition $\nabla_k(A)\partial_0 A_k = 0$ of Equation (8) on the gauge orbit as a **priori** case keeping in view the color charged nature of the quanta of non-Aeblian Quantum Yang – Mills theory. As such, the aforesaid question "whether there are rules to ensure that quantum theories in different gauges are indeed the same" does not arise in the present case due to the color charged nature of the quanta of non-Aeblian Quantum Yang – Mills theory and accordingly, there is no operator ordering ambiguity in present case. Also, the axial gauge, where there is no Gribov problem, is simply non-applicable for the time-dependent (spatially independent) **infinitesimal** gauge transformations in present case. Further, the causal propagation of the photons in QED (where there is no Gribov problem) is implemented by the idea of antiparticles as already stated above in 'Introduction' section of this paper.

3. EXPERIMENTAL VERIFICATION:

"A gauge field describes two dynamical degrees of freedom of a massless spin-1 particle. A most economic description would have been using a two component field. However, to have Lorentz symmetry, one has to imbed the two degrees of freedom into a four-vector field A^μ, thereby introducing the gauge degrees of freedom. To ensure the gauge part do not contribute to physical observables, manifest gauge symmetry under $A^\mu \rightarrow A^\mu + \partial^\mu \chi$ is required. The gauge degrees of freedom seem to be a nuisance, it would be nice to get rid of them in actual calculations. However, this can only be done by first formulating a Lorentz-invariant theory and then imposing gauge conditions. The order of the procedure here is critically important and cannot be reversed: one cannot construct physical observables directly in term of "physical" degrees of freedom after imposing the gauge conditions. Reversely-engineered gauge symmetry is not guaranteed physical because 1) observables generally do not have proper Lorentz transformation, 2) they generally are non-local, 3) they generally have no physical measurements."[32]

In this paper, a pure Yang-Mills gauge theory has been completely defined in Hamiltonian formulation with the help of the generalized Coulomb gauge – a null covariant derivative which not only has Lorentz symmetry (i.e., by definition, in terms of a co-ordinate system the covariant derivative of a 'vector' transforms under a change of co-ordinate system 'in the same way' as the 'vector' itself) at the first instance but also, at the same time, imposes gauge fixing condition in Equation (8) above. Thus, the aforesaid order of procedure has been strictly followed in the Hamiltonian formulation of this paper. Consequent upon the same, $A_k^{\perp}$, the two "physical" degrees of freedom in the Hamiltonian formulation of this paper, are physical enough in the laboratory inertial reference frame such that any thing made out of it is 'directly' physical. The simplest gauge-invariant 'physical observable' to be made out of $A_k^{\perp}$, the two "physical" degrees of freedom, is gluon helicity, defined as the projection of gluon spin operator along the direction of the momentum.

In the Hamiltonian formulation of pure Yang-Mills gauge theory completely derived in this paper, the generalized Coulomb gauge fixing condition as a hyperspace in configuration space 'explicitly' breaks the gauge symmetry at the point of its intersection with the gauge orbit, so, the construction of the aforesaid gauge-invariant 'physical observable' (i.e., gluon helicity) during the transient existence of the generalized Coulomb gauge by using $A_k^{\perp}$, the two "physical" degrees of freedom, corresponding to any such aforesaid point of intersection is nothing but exhibition of **particle character** of gluon. Theoretically speaking, there are infinite number of such points of intersection called 'Gribov copies' for an arbitrary gauge orbit in configuration space such that $\Psi(A)$ in Equation (10b) or Equation (13) above gives the probability of any Gribov copy being selected for the construction of the aforesaid gauge-invariant 'physical observable' (i.e., gluon helicity) during the transient existence of the generalized Coulomb gauge by using $A_k^{\perp}$, the two "physical" degrees of freedom corresponding to that Gribov copy.

This is nothing but exhibition of **wave-character** of gluon. As such, the aforesaid statement “the generalized Coulomb gauge fixing condition as a hyperspace in configuration space ‘explicitly’ breaks the gauge symmetry at the point of its intersection with the gauge orbit” is nothing but wave-particle duality i.e., gluon has a dual character: the aforesaid wave character and particle character of gluon complement each other and in a specific event, gluon exhibits either a wave nature or particle nature, never both simultaneously. In other words, the Gribov copies, as special realization of a gluon, are simultaneously created as physical entities, with different positions, by quantum fluctuations of energy during the transient existence of the generalized Coulomb gauge in full accordance with the uncertainty principle such that the wave-character of a transverse gluon is implemented in term of its Gribov copies.

In Ref. [33], “Three-jet variables constructed from multi-hadronic events produced by Z^0 decays are compared to theoretical calculations assuming a vector gluon or a hypothetical scalar gluon. The data yield conclusive direct evidence for the former case. The distributions of the reduced energy of the second-most energetic jet and of the cosine of the Ellis-Karliner angle are chosen to demonstrate this effect.”[33] In this regard, it is quite obvious that the aforesaid Ellis-Karliner angle is produced by the scattering of the emitted hard gluon color charge by the color charge of the most energetic quark (or antiquark) in 3-jet events produced by Z^0 decays.

Accordingly, the hard gluon in 1st Gribov region Ω would demonstrate the cosine of the Ellis-Karliner angle near unit value and Gribov copies in higher Gribov regions $\Omega_{2,}$ $\Omega_{3,}$ .... in hyperspace $[\nabla_k(A)\ \boldsymbol{\acute{A}}^{\mathbf{k}} = 0]$ of Configuration space (see Fig. 2 above) would demonstrate their respective values of the cosine of the Ellis-Karliner angle in decreasing order because at the time of emission in 3-jet event, the color charge of the hard gluon in 1st Gribov region Ω approaches zero value and goes on increasing for the Gribov copies in higher Gribov regions $\Omega_{2,} \Omega_{3, \ldots}$ in hyperspace $[\nabla_k(A)\, \boldsymbol{\acute{A}}^{\mathbf{k}} = 0]$ of Configuration space (see Fig. 2 above). Since, the probability $\Psi(A)$ in Equation (10b) or Equation (13) above is highest for the hard gluon in 1st Gribov region Ω and goes on decreasing for the Gribov copies in higher Gribov regions $\Omega_{2,} \Omega_{3, \ldots}$ in hyperspace $[\nabla_k(A)\, \boldsymbol{\acute{A}}^{\mathbf{k}} = 0]$ of Configuration space (see Fig. 2 above), so, it is theoretically predicted that the distribution of the cosine of the Ellis-Karliner angle would be peaked near unit value. In Figure 3 of Ref. [33], the data yield conclusive direct evidence for the aforesaid theoretical prediction. This provides us the experimental verification that the Gribov copies are indeed physical entities which can be selected for the construction of the aforesaid gauge-invariant 'physical observable' (i.e., gluon helicity) during the transient existence of the generalized Coulomb gauge by using $A_k^{\perp}$, the two "physical" degrees of freedom corresponding to that Gribov copy.

Also, the lack of the clear experimental evidence for the existence of Glueballs as the leading object in the gluon jet of 3-jet event [26] experimentally verifies the postdiction that the emitted hard transverse gluon in the 3-jet event becomes a color-singlet massive transverse gluon on the boundary of the Gribov region, so-called the Gribov horizon in non-perturbative regime of QCD.

4. DISCUSSION:

The distribution of charge and invariant mass of the leading cluster Q_{lead} and M_{lead} in gluon jets beyond a rapidity gap reflect the color neutralization mechanism and with increasing rapidity gaps, the leading charges would be closer to their asymptotic distribution [26]. As per results from LEP on leading clusters as obtained by OPAL [27] and DELPHI [28], there is excess of gluon jets with $Q_{lead} = 0$ and $M_{lead} \leq 2.5$ GeV [26]. A natural explanation would be a leading gluonic system or glueball [26]. But the clear evidence for the existence of Glueballs as the leading object in the gluon jet of 3-jet event is still missing [26].

In this regard, the fragmentation of the aforesaid massive transverse gluon, that is created in overall color singlet & electrically neutral manner on the boundary of the Gribov region, so-called the Gribov horizon and is at rest in some moving inertial reference frame, easily provides explanation for the aforesaid excess of gluon jets with $Q_{lead} = 0$ and $M_{lead} \leq 2.5$ GeV. It is pertinent to mention here that the early QCD prediction of massive **Glueball** arises out of the ability of the self-interacting gluons to bind themselves to give rise to new spectroscopy of gluonic matter [26]. Bur in view of the emitted hard transverse gluon in the 3-jet event becoming a color-singlet electrically neutral massive transverse gluon on the boundary of the Gribov region, so-called the Gribov horizon in non-perturbative regime of QCD, there can be no new spectroscopy of gluonic matter or Glueballs. As on date, only the quantitative predictions of Glueballs are derived from the lattice QCD & from QCD sum rules, but the experimental verification of these Glueballs is still in doubt [26].

5. CONCLUSION:

Since, any attempt to naturally complete the all-time valid Coulomb gauge by supplying a further (spatially independent) gauge constraint leads to a contradiction with the perturbative renormalizability of the theory [8, 12, 13, 14, 15, 16], so, we cut short the all-time validity of the Coulomb gauge by using the transient generalized Coulomb gauge [4] that is valid for infinitesimal time period (t_1 - t_0) only and implicitly includes the standard Coulomb gauge also. In the process of quantizing pure QCD in this transient generalized Coulomb gauge, we can solely perform time-dependent (spatially independent) **infinitesimal** gauge transformations, as shown in above Equation (12a), that generate temporal zero-modes of the Faddeev – Popov operator on the gauge-orbit as shown in Equation (12f) above. At the same time, the non-temporal zero modes of the Faddeev-Popov determinant called the Gribov copies also implicitly gets introduced during the canonical quantization because the transient generalized Coulomb gauge [4] during the infinitesimal time period (t_1 - t_0) implicitly includes the standard Coulomb gauge also. As such, the required uncertainty in position of a transverse gluon is accommodated by allowing for the possibility that Gribov copies are created as virtual entities by quantum fluctuation of the transverse gluon energy over the brief intervals of time coinciding with transient existence of the generalized Coulomb gauge in full accordance with the uncertainty principle. These Gribov copies can be ignored in perturbative sector due to asymptotic freedom of pure QCD empty space but their common characterstic i.e., zero value of Faddeev-popov operator in Equation (20) above, serves as a mathematical proof of mass-gap and color confinement properties on the boundary of the Gribov region, so-called the Gribov horizon in the non-perturbative sector of pure QCD.

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