Existence of Mass-Gap & Pure Yang – Mills Theory

Rajan Dogra

Principal, Government Industrial Training Institute for Women, Sector 11-C, Chandigarh, India

E-mail: rajandogra_2000@yahoo.com

ABSTRACT

Due to the non-linearity involved in quantum chromodynamics (QCD), the required uncertainty in position of a transverse hard gluon, emitted in 3-jet event, is accommodated by allowing for the possibility that Gribov copies are created as virtual entities by quantum fluctuations of the transverse gluon energy over the brief intervals of time during which the special relativity theory and the quantum theory are merged together consistently in QCD. These Gribov copies can be ignored in perturbative sector due to asymptotic freedom of pure QCD empty space but their common characterstic i.e., zero value of Faddeev-popov operator, serves as a mathematical proof of mass-gap and color confinement properties on the boundary of the Gribov region, so-called the Gribov horizon in the non-perturbative sector of pure QCD.

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This manuscript basically divides the outward journey of a hard transverse gluon, emitted in 3-jet event, into following three parts;-

(1) Hamiltonian formulation for a hard transverse gluon lying somewhere in the vicinity of A = 0 in the fundamental modular region (FMR) of the configuration space at the time t_0 of its emission in 3-jet event

▪ The canonical momenta π_a^0 corresponding to the time index $\mu = 0$ in the Equation [$\pi_a^\mu = \partial L''/\partial \acute{A}^a_\mu = - F_a^{\mu 0}$,where the classical Lagrangian $L = \int_V d^3x\, L''$] vanishes due to antisymmetry of $F^{\mu\nu}$ and accordingly, the Gauss law ($\nabla_k(A)\, \pi_a^k = 0$) is absent. Without the proper incorporation of Gauss law into Hamiltonian formalism, no physical applications are possible.

▪ For implementing the Gauss law identically (in principle) by construction at classical Lagrangian level as a constraint, first of all we solve the non-abelian Gauss law [$\nabla_k(A)\nabla^k(A)A^0 - \nabla_k(A)\, \acute{A}^k = 0$, where the space indices k = 1,2,3]and obtain an unique solution $A_0 = A_0\{ A_k, \partial_0 A_k\}$ as a functional of A_k and their time derivatives $\partial_0 A_k$. Then, $A_0 = A_0\{ A_k, \partial_0 A_k\}$ is substituted into the aforesaid classical Lagrangian L to get new classical Lagrangian as L_0.

▪ For expressing generalized velocity $\acute{A}^k$ in terms of π^k and A^k, this unique solution $A_0 = A_0 \{ A_k, \partial_0 A_k\}$ is modified as $A_0 = A_0\{A_k^\perp\}$ by using the generalized Coulomb gauge $\nabla_k(A)\, \acute{A}^k = 0$ in the non-abelian Gauss law [$\nabla_k(A)\nabla^k(A)A^0 - \nabla_k(A)\, \acute{A}^k = 0$] and by expanding the 'covariant gradient' term $\nabla_k(A)$ in the generalized Coulomb gauge $\nabla_k(A)\, \acute{A}^k = 0$ as,

$$\nabla_k(A)\partial_0 A^k(x) \equiv \partial_k\partial_0 A^k(x) + ig[A_k(x), \partial_0 A^k(x)] = 0$$

it can be easily proved that standard Coulomb gauge is automatically satisfied when the generalized Coulomb gauge $\nabla_k(A)\, \acute{A}^k = 0$ is in force. As such, we are left only with transversal component $A_k^{a\perp}$ and correspondingly, transversal part of the momentum (i.e., $\pi_a^{k\perp}$) is exclusively focused upon from this point onwards.

▪ By using Gauss law constraint ($\nabla_k(A)\, \pi_a^{k\perp}$) = 0 and $\acute{A}^{k\perp} = (\nabla^k(A)\, A^0 \{A^{k\perp}\} - \pi^{k\perp})$ & substituting $A_0 = A_0\{A_k^\perp\}$ into the classical Lagrangian L_0, the classical Hamiltonian H, that contains only kinetic energy terms and is valid in the vicinity of $A = 0$ at time t_0 of hard transverse gluon emission in 3-jet event, is obtained through Legendre transformation as

$H = (-1/2) \int_V d^3x\, J^{-1}[A_k^\perp]\{(\nabla_k(A_k) - A_0\{A_k^\perp\} - \acute{A}_k^\perp\}, J[A_k^\perp]\{\nabla^k(A^k)\, A^0\{A^{k\perp}\} - \acute{A}^{k\perp})\} + (1/4) \int_V d^3x (F_{kl}(A_k^\perp), F^{kl}(A^{k\perp}))$ ________ **(A)**

▪ A transition to the quantum version of the aforesaid classical Hamiltonian H, by means of the substitution of the fixed time Schrodinger quantization rule

$$\pi_k^a \rightarrow \hat{\pi}_k^a \equiv -i\frac{\delta}{\delta A_a^k}$$

yields the Hamiltonian operator $\hat{H}$ for the hard transverse gluon at the time t_0 of its emission in the 3-jet event and in the limit $V \rightarrow \infty$ in the above Equation (A), one is then invited to consider the following eigenvalue equation, in self-explanatory notation $\hat{H}\Psi(A) = E\Psi(A)$

▪ In the above eigenvalue equation, the Hamiltonian operator $\hat{H}$ for the hard transverse gluon at the time t_0 of its emission in the 3-jet event is plagued by a technical complication i.e., when the generalized coulomb gauge fixing condition [$\nabla_k(A)\, \acute{A}^k = 0$] is in force for straightforwardly expressing the generalized velocity in terms of generalized co-ordinate and momenta variables, one cannot define canonical momentas by Equation [$\pi_a^\mu = \partial L''/\partial \acute{A}^a_\mu = - F_a^{\mu 0}$] above for $\mu = 1,2,3$ as these generalized velocities are no longer independent quantities.

▪ The aforesaid **technical complication** has been removed,-

(a) firstly, by proving the transitory nature of the generalized coulomb gauge fixing condition [$\nabla_k(A)\, \acute{A}^k = 0$] on the basis of the fact that temporal zero-modes of Faddeev-Popov determinant are generated by time-dependent infinitesimal gauge transformation;

(b) secondly, by establishing the priori need for using transient gauge fixing condition [$\nabla_k(A)\acute{A}^k = 0$] during an infinitesimal time period ϕ that elapses between two consecutive emissions of virtual longitudinal coulomb gluons by the hard transverse gluon emitted in 3-jet event and

(c) lastly, by elaborating that "the aforesaid priori need of using **transient** ($\nabla_k(A)\partial_0 A_k = 0$) during infinitesimal time period ϕ leads to energy uncertainty ΔE for a transverse gluon & thus, allows the imposition of equal time canonical commutation relations $[A_i^{a\perp}(x), \pi_j^{b\perp}(y)] = \delta_{ij}\, \delta^{ab}\, \delta^3(x - y)$ involving unconstrained, independent pure Yang-Mills vector potential $A_i^{a\perp}(x)$ and momenta variable $\pi_j^{b\perp}(y)$ in operator form and as such, this imposition of the aforesaid canonical commutation relations leads to the treatment of the generalized velocities $\partial_0 A_k$ as independent quantities for all intent and purpose during the transient existence of transient gauge fixing condition $\nabla_k(A)\partial_0 A_k = 0$."

▪ Accordingly, the validity of the Hamiltonian operator $\hat{H}$ in Equation $\hat{H}\Psi(A) = E\Psi(A)$ ________________ **(B)**

for the hard transverse gluon at the time t_0 of its emission in the 3-jet event is proved.

(2) *The time development of the wave-function $\Psi(A)$ of the hard transverse gluon throughout FMR during the time period between its emission time t_0 and the time instant at which the hard transverse gluon approaches the common boundary, called Gribov horizon, of FMR and the Gribov region.*

▪ **(a)** The Hamiltonian operator $\hat{H}$ in Equation (B) governs the time-development of wave-function $\Psi(A)$ of the hard transverse gluon for the aforesaid infinitesimal time period $\phi = (t_1 - t_0)$ during which it undergoes time-dependent but space-independent infinitesimal gauge transformation

▪ **(b)** With the emission of first virtual longitudinal gluon at time t_1 by transverse gluon emitted in 3-jet event, the longitudinal momentum component $\pi_a^{k||}$ of the transverse gluon arises from the resolution of the Gauss law i.e., $(\nabla_k(A)\, \pi_a^k) = 0 = \nabla_k(A)\, [\pi_a^{k||} + \pi_a^{k\perp}]$ ________________ **(C)**

▪ **(c)** Accordingly, the Hamiltonian operator $\hat{H}$ in the above Equation (B) can be again derived at time t_1, by making a Legendre transform of the classical Lagrange density to the classical Hamiltonian density in the light of the above Equation (C), as

$$\hat{H}\Psi(A) = E\Psi(A) + H_{\rm Coul}\Psi(A) \qquad \text{(D)}$$

▪ **(d)** The new Hamiltonian operator in the above Equation (D) acquires an additional Coulomb term $H_{\rm Coul}$ that arises from the resolution of Gauss law and is necessary to maintain gauge invariance, i.e.

$$H_{\rm Coul} = \tfrac{1}{2}\int d^3x\, d^3y\, \mathcal{J}^{-\frac{1}{2}}\rho^a(x)\, \mathcal{J}\, K^{ab}(x, y; \mathbf{A}_\perp)\, \rho^b(y)\, \mathcal{J}^{-\frac{1}{2}} \qquad \text{(E)}$$

▪**(e)**The Hamiltonian operator $\hat{H}$ is same in both Equations (B) and (D) because firstly, it is a constant of motion and secondly, the emission of the virtual longitudinal coulomb gluon, to develop an accompanying non-abelian coulomb field, at time t_1 ($> t_0$) takes place at the expense of the energy of the transverse gluon itself. Consequent upon the aforesaid constancy of the Hamiltonian operator, the1st term $E\Psi(A)$ in the right hand side of the Equation (D), that describe the hard transverse gluon, now refer to a different gauge orbit as compared to the corresponding term in Equation (B).

▪ **(f)** Then, the Gauss law constraint in Equation (C) is satisfied again identically (in principle) by construction to obtain the Hamiltonian Operator in (D) for the aforesaid different gauge orbit and so on.

▪ **(g)** This cycle goes on repeating itself uninterruptedly with passage of time throughout the FMR as a result of which, as experimentally observed in running coupling constant of QCD, the additional coulomb term in $H_{\rm Coul}$ Equation (E), containing coulomb energy propagator K also called ghost propagator, gets stronger and stronger in Equation (D) with the addition of more and more emitted virtual longitudinal coulomb gluons into the accompanying non-abelian coulomb field and the transverse gluon (i.e., the first term $E\Psi(A)$, in the right hand side of the Equation (D)), with decreasing energy, approaches the common boundary, called Gribov horizon, of FMR and the Gribov region.

▪As already stated above, the standard Coulomb gauge is automatically satisfied when the generalized Coulomb gauge condition [$\nabla_k(A)\, \acute{A}^k = 0$] is satisfied. In this regard, as Gribov discussed, the standard Coulomb gauge does not fix a gauge completely, and there are equivalent gauge configurations called Gribov copies.

▪ At this juncture, it is worthwhile to mention that during the infinitesimal time-period ϕ of the transient generalized Coulomb gauge, these Gribov copies are created as virtual entities with different positions by quantum fluctuations of the transverse gluon energy during the aforesaid infinitesimal time-period ϕ for accommodating the required uncertainty in hard transverse gluon position in accordance with the uncertainty principle. In other words, the wave-character of the hard transverse gluon, defined inside FMR, is implemented in term of its Gribov copies.

▪This means that the Equations (B) and (D) above has got additional multiple eigen-values in the form of Gribov copies

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and also, the time-development of wave-function $\Psi(A)$ as outlined above in sub-sections **2(a)-2(g)** for transverse gluon defined inside FMR, is equally valid for the Gribov copies that are defined in 2nd , 3rd , 4th , ……. Gribov regions in the hyperspace [$\nabla_k(A)\, \acute{A}^k = 0$] of configuration space as depicted below.

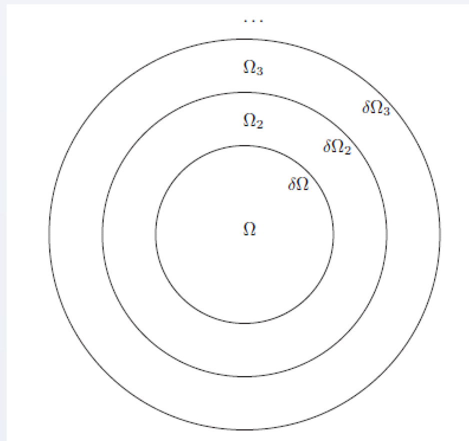

Figure 1: Different Gribov regions $\Omega, \Omega_2, \Omega_3,$ ……. in hyperspace [$\nabla_k(A)\, \acute{A}^k = 0$] of Configuration Space.

Figure Courtesy
arxiv:1202.1491v1

▪ In view of the above, it is obvious that for implementing the wave-character, the outward journey of the transverse gluon, defined inside FMR, is simultaneously accompanied by the outward journey of the Gribov copies that are defined inside their respective Gribov regions $\Omega_2, \Omega_3,$ ……. in Figure 1 above.

▪ Further, the standard Coulomb gauge is automatically satisfied when the generalized Coulomb gauge condition [$\nabla_k(A)\, \acute{A}^k = 0$] is satisfied. As such, the Gauss law in Equation (C) can be rewritten for transverse gluon defined inside FMR as

$$-\mathbf{D}[\mathbf{A}] \cdot \nabla\, \Phi = \rho = ig_s\, [\mathbf{A}_\perp, \mathbf{E}_\perp] \qquad \text{(F)}$$

Where $M(\vec{A}) = -\, (\mathbf{D}[\mathbf{A}] \cdot \nabla)$ is the Faddeev-Popov ghost operator,

ρ is the color charge density induced by the hard transverse gluon inside FMR and

$\nabla\, \Phi = \pi_a^{k||}$ is the longitudinal momentum component obtained from the resolution of the Gauss law and

Φ is the color-Coulomb potential.

▪ The above Equation (F) is equally valid for the Gribov copies also, inside their respective Gribov regions $\Omega_2, \Omega_3,$ ……. in Figure 1, so long as the transverse gluon is inside FMR. As such, the inverse that appears in $\Phi = M^{-1}(\vec{A})\, \rho$ is well-defined inside FMR and Gribov regions $\Omega_2, \Omega_3,$ ……. in Figure 1. It is pertinent to mention here that the sum of individual color charges $\rho_1, \rho_2,$ ----------- induced by the hard transverse gluon inside FMR and its Gribov copies individually inside their respective Gribov regions $\Omega_2, \Omega_3,$ ……. in Figure 1, is equal to zero because the high energy hard transverse gluon is experimentally observed to be emitted from a color-singlet point source (i.e., quark – antiquark pair) in 3-jet event i.e.,

$$0 = \rho_1 + \rho_2 + \text{-----------} \qquad \text{(G)}$$

▪These Gribov copies and their color potential Φ can be easily ignored as long as hard transverse gluon is inside FMR due to the **Asymptotic freedom** property. But on the boundary of the Gribov region, so-called the Gribov horizon, i.e., $\delta\Omega, \delta\Omega_2, \delta\Omega_3,$ …….. in Figure 1, these Gribov copies cannot be ignored as is evident from next section 3.

(3) *Appearance of 'mass-gap' and 'Color Confinement' properties at the time instant at which the hard transverse gluon approaches the common boundary $\delta\Omega$, called Gribov horizon, of FMR and the Gribov region.*

▪ For implementing the wave-character, the time instant, at which the transverse gluon approaches the common boundary $\delta\Omega$, called Gribov Horizon, of FMR and the first Gribov region, coincides with the time instants at which 1st, 2nd, 3rd, 4th, …………. Gribov copies individually approaches their respective boundaries $\delta\Omega_2, \delta\Omega_3, \delta\Omega_4, \delta\Omega_5$ …….. of the subsequent Gribov regions $\Omega_2, \Omega_3,$ ……. in Figure 1.

▪ This implies that the eigen-value of the Faddeev – Popov operator in Equation (F) becomes zero for transverse gluon and all Gribov copies at the same time instant when they reach their respective Gribov boundaries in Figure 1 above simultaneously.

▪ The substitution of this zero eigen-value of the Faddeev – Popov operator at Gribov boundaries i.e., $\delta\Omega, \delta\Omega_2, \delta\Omega_3,$ …….. in Figure 1 in the denominator of the right hand side of the coulomb energy propagator

$$K^{ab}(x, y; \mathbf{A}) = -\left[\frac{1}{\mathbf{D}[\mathbf{A}] \cdot \nabla}\, \nabla^2\, \frac{1}{\mathbf{D}[\mathbf{A}] \cdot \nabla}\right]^{ab}_{xy} \qquad \text{(H)}$$

leads to an infinite value of Coulomb term $H_{\rm Coul}$ in Equation (D) for transverse gluon and its all Gribov copies at Gribov boundaries i.e., $\delta\Omega, \delta\Omega_2, \delta\Omega_3,$ …….. in Figure 1.

▪ This infinite value of Coulomb term $H_{\rm Coul}$ in Equation (D) leads to vanishing of first kinetic energy term $E\Psi(A)$ in Equation (D) i.e., gluon suppression on the common boundary, called Gribov horizon, of FMR and the Gribov region, i.e. the Equation (D) becomes

$$\hat{H}\Psi(A) = H_{\rm Coul}\Psi(A) \qquad \text{(I)}$$

▪ At the same time, this infinite value of Coulomb term $H_{\rm Coul}$ in Equation (I) leads to destruction of the wave-character of transverse gluon because this infinite value of instantaneous Coulomb term $H_{\rm Coul}$ in Equation (I) acts as an instantaneous position indicator of transverse gluon and Gribov copies which is forbidden under wave-particle duality i.e., the wave-character and particle character complement each other and both are never exhibited simultaneously.

▪ Under the aforesaid wave-particle duality, the destruction of the wave-character of the transverse gluon is simultaneously accompanied by the appearance of single gluon particle on the common boundary $\delta\Omega$, called Gribov Horizon, of FMR and the first Gribov region.

▪ Evidently, the transverse gluon has a dual character: the wave-character and particle character complement each other. Either character by itself is only part of the storey and can provide explanation for only certain effects. In a specific event, the transverse gluon exhibits either a wave nature or a particle nature, never both simultaneously. Due to this duality between particle-character and wave-character, the aforesaid destruction of the wave-character, by the infinite value of the Coulomb energy propagator K on the boundary of the Gribov region, so-called the Gribov horizon, is simultaneously accompanied by the sudden exhibition of the particle-character in the form of appearance of the massive single transverse gluon that is at rest in some moving inertial reference frame.

▪ This massive nature of the transverse gluon arises from the infinite value of the term K in the Equation (H) because this term K, otherwise having dimensions of energy density as the Coulomb energy propagator for the Gribov copies inside their Gribov regions, transforms suddenly into **singular infinite term** when the zero eigenvalue of the Faddeev-popov operator $M(\vec{A})$ in the Equation (H) is substituted in the denominator of the K for the aforesaid single massive gluon on the boundary of the Gribov region, so-called the Gribov horizon. As such, on the boundary of the Gribov region, so-called the Gribov horizon, there is change in the dimensions of the product term $\rho^a(x)\, \rho^b(y)$,__________ (J) in the sole left out additional coulomb term $H_{\rm Coul}$ in Equation (E), from the dimensionless coupling constant to becoming a dimensionful quantity, having dimension of energy density because the aforesaid **singular term** being infinite cannot have the dimensions of energy density with reference to the aforesaid single massive transverse gluon.

▪ Then, via this phenomenon of dimensional transmutation 1) one can calculate all the observables of QCD in terms of dynamically generated mass scale and there remains no adjustable parameter in QCD and 2) one can introduce a physical scale Λ_{QCD} at which 'mass-gap' property is demonstrated for each transverse gluon. Since, there is change in the dimensions of the product term in the Equation (J) above, in the sole left out additional coulomb term in Equation (I), from dimensionless coupling constant or 'color charge' to becoming a dimensionful quantity, having dimension of energy density with reference to the aforesaid single massive transverse gluon, so, the **color confinement** for the aforesaid massive transverse gluon also occurs at the aforesaid physical scale Λ_{QCD} on the boundary of the Gribov region, so-called the Gribov horizon in non-perturbative regime of QCD.

▪ It is pertinent to mention here that the causal propagation (i.e., the wave nature) of this massive transverse gluon, that is created in overall color singlet & electrically neutral manner on the boundary of the Gribov region, so-called the Gribov horizon and is at rest in some moving inertial reference frame, could no longer be dependent upon the Gribov copies and gauge-fixing and in-fact, the fragmentation of this massive transverse gluon occurs in laboratory reference frame shortly after its creation in overall color singlet & electrically neutral manner on the boundary of the Gribov region, so-called the Gribov horizon.

▪The lack of the clear experimental evidence for the existence of Glueballs as the leading object in the gluon jet of 3-jet event experimentally verifies the prediction that the emitted hard transverse gluon in the 3-jet event becomes a color-singlet massive transverse gluon on the boundary of the Gribov region, so-called the Gribov horizon in non-perturbative regime of QCD.

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