Perfect Abelian Dominance of Confinement in SU(3) Lattice QCD

H. Suganuma (Kyoto U.) and N. Sakumichi (RIKEN)

Abstract:
We study Abelian projection of quark confinement in SU(3) lattice QCD, in terms of the dual superconductor picture.
In the maximal Abelian (MA) gauge, we perform the Cartan decomposition of the nonabelian gauge field in SU(3) quenched lattice QCD with β=6.4 (i.e., a =0.058fm) and 32^4. We investigate the quark-antiquark potential V(r), its Abelian part $V_{Abel}(r)$ and its off-diagonal part $V_{off}(r)$. As a remarkable fact, we find *almost perfect Abelian dominance* for quark confinement (the string tension) on the fine lattice with an enough large volume.
Also, we find a nontrivial summation relation of $V(r) = V_{Abel}(r) + V_{off}(r)$.

Reference:

[1] N.Sakumichi and H.S., arXiv:1406.2215 [hep-lat], "Perfect Abelian Dominance of Quark Confinement in SU(3) QCD on a Fine Lattice".

Confinement XI, September 7-13, 2014, Saint Petersburg

Confinement mechanism

To understand the quark confinement mechanism is one of the most important unsolved issues remaining in theoretical physics, since quarks were introduced to particle physics in 1960's.

It is also identified as an extremely difficult mathematical problem to derive quark confinement directly from quantum chromodynamics (QCD), which was established as the fundamental theory of the strong interaction in 1970's.

The difficulty is considered to originate from nonabelian dynamics and nonperturbative features of QCD, which are quite different from the case of quantum electrodynamics (QED).

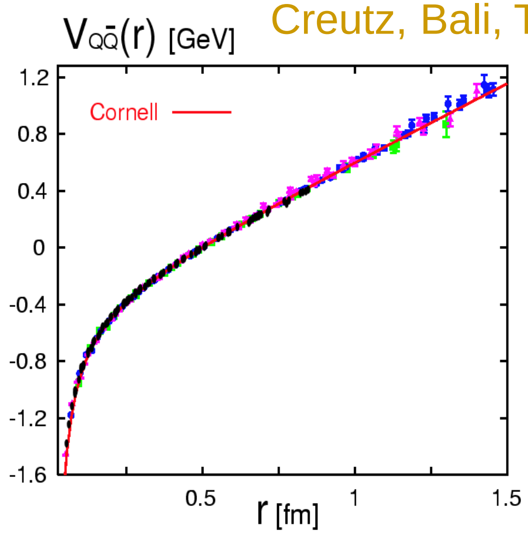
Indeed, it remains *unclear whether quark confinement is peculiar to the nonabelian nature of QCD or not*. Then, it is interesting and meaningful to investigate *Abelianization scheme* of QCD such as dual superconductor theory and center vortex theory.

Confinement has been studied in terms of inter-quark potential. Unlike QED, QCD is considered to form color-flux-tube between quarks, and this one-dimensional squeezing of color-electric field leads to linear confinement potential in the infrared region.

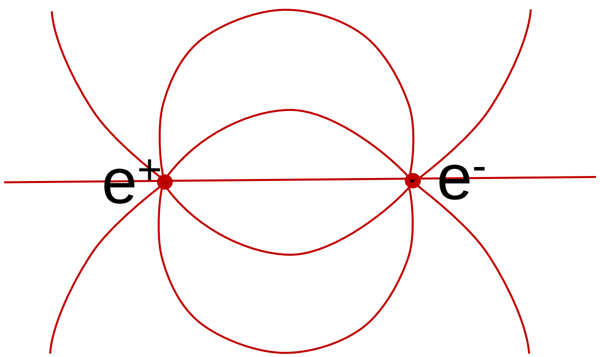
One-dimensional squeezing of color electric field

Linear confinement potential

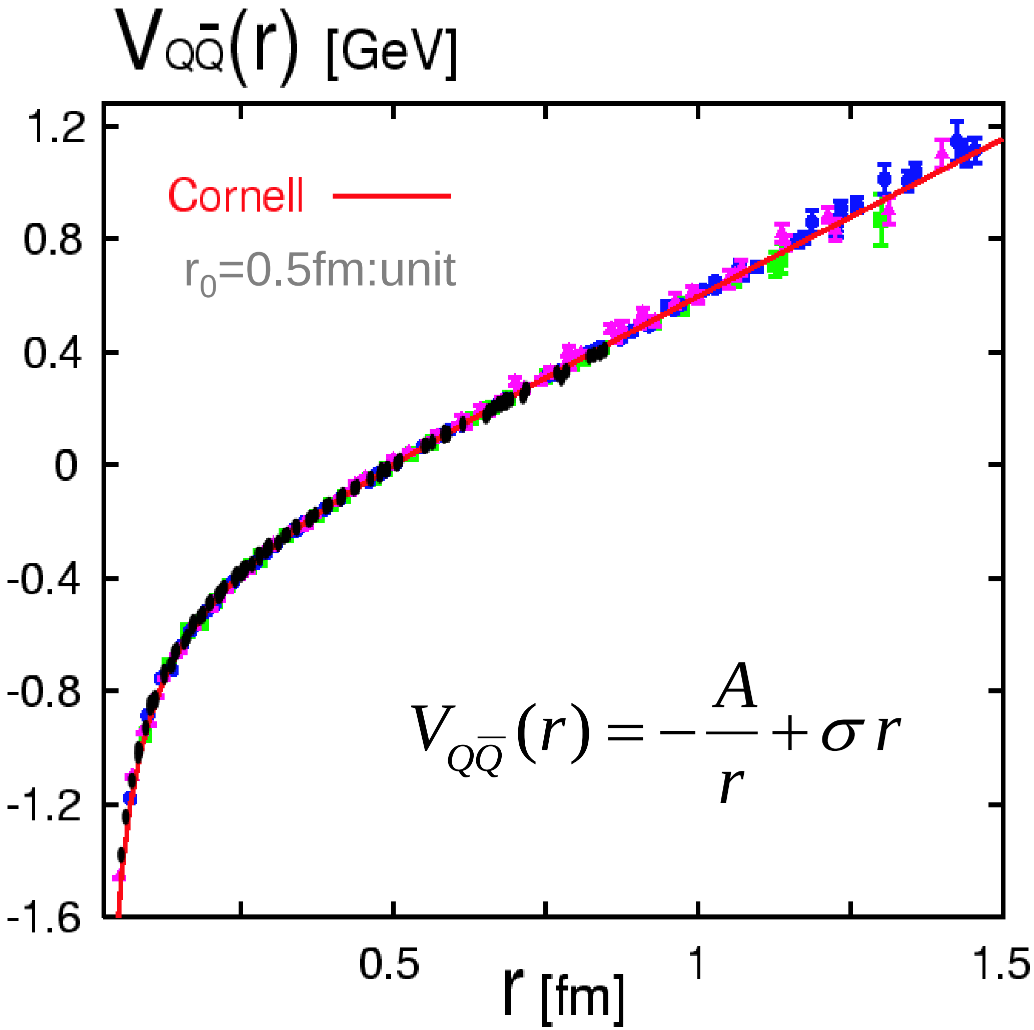
Creutz, Bali, Takahashi-H.S., JLQCD



cf QED



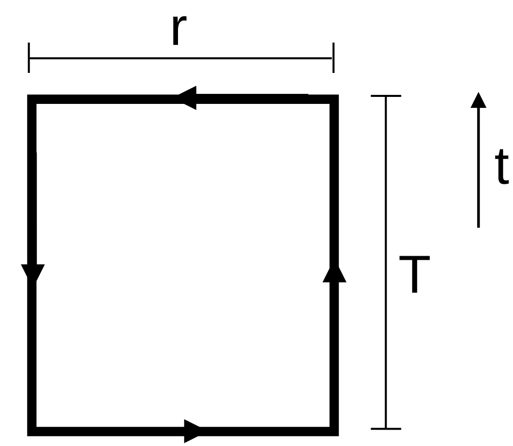
Quark-antiquark static potential in Lattice QCD



M.Creutz (1979,80)

quark **anti-quark**

Summarized lattice QCD data
G.S.Bali (2001)
Takahashi, H.S. et al. (2002)
JLQCD (2003)

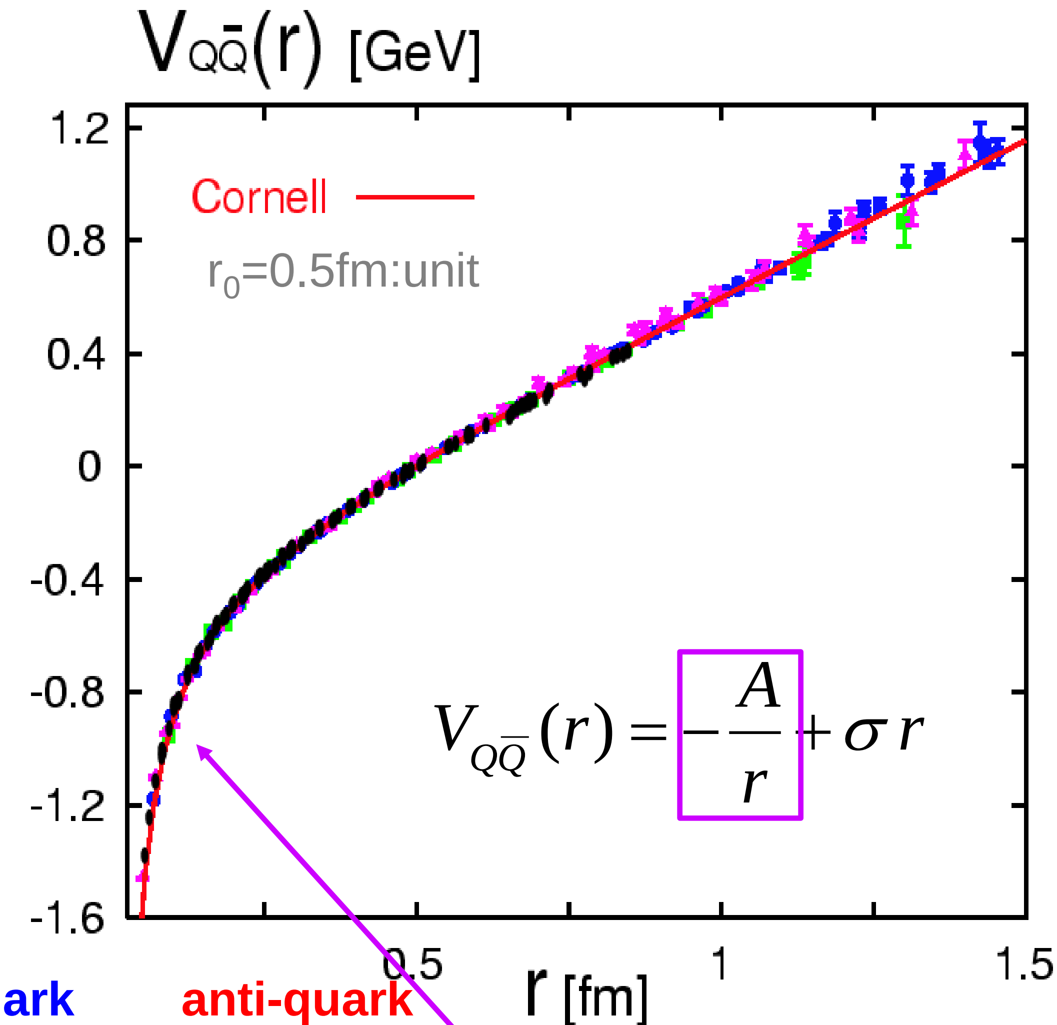


Wilson loop

$$V_{Q\bar{Q}}(r) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W \rangle_T$$

The *inter-quark potential* is obtained from the *Wilson loop* in lattice QCD.

Quark-antiquark static potential in Lattice QCD



M.Creutz (1979,80)

quark ● **anti-quark** ●

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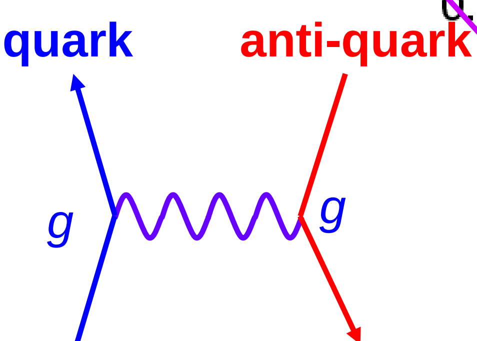
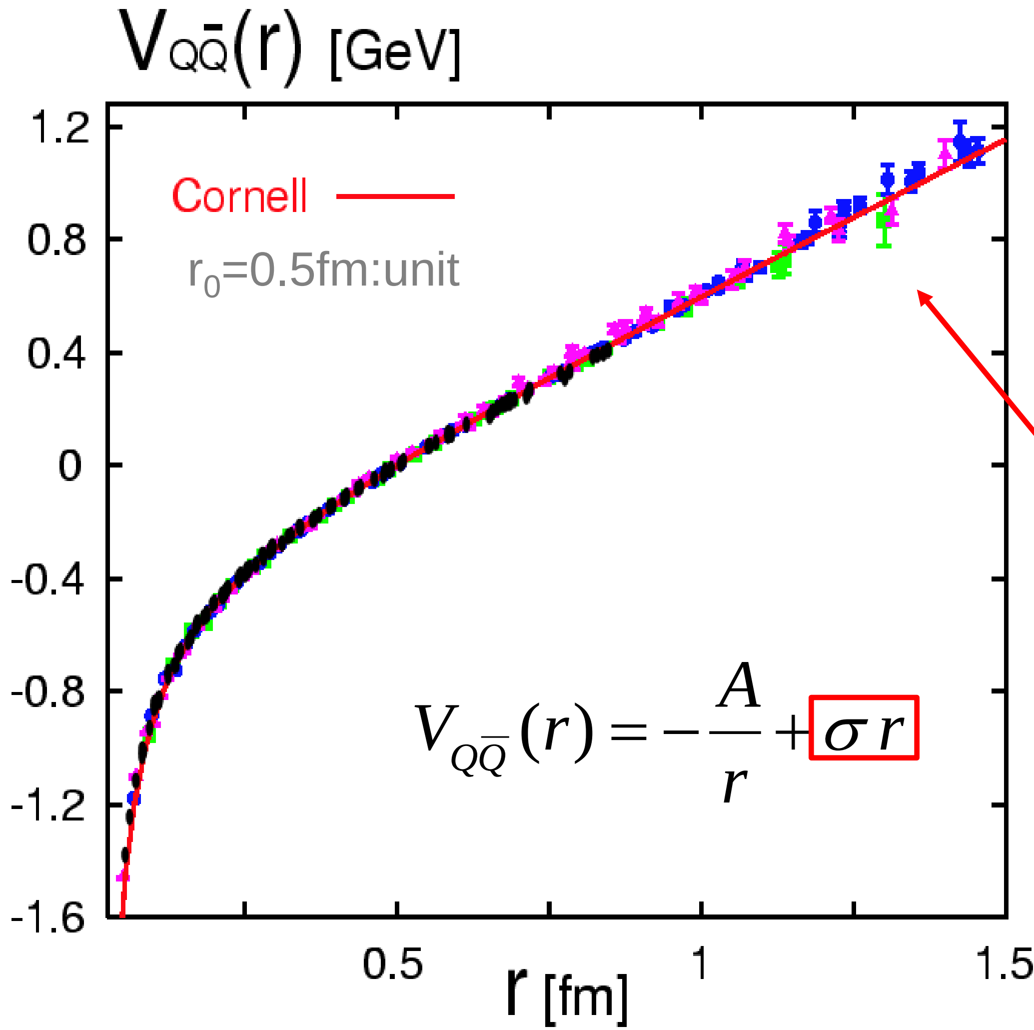


At the short distances, the Q-$\bar{\text{Q}}$ potential behaves as the Coulomb potential, which is expected from the one-gluon-exchange (OGE) process.

Quark-antiquark static potential in Lattice QCD



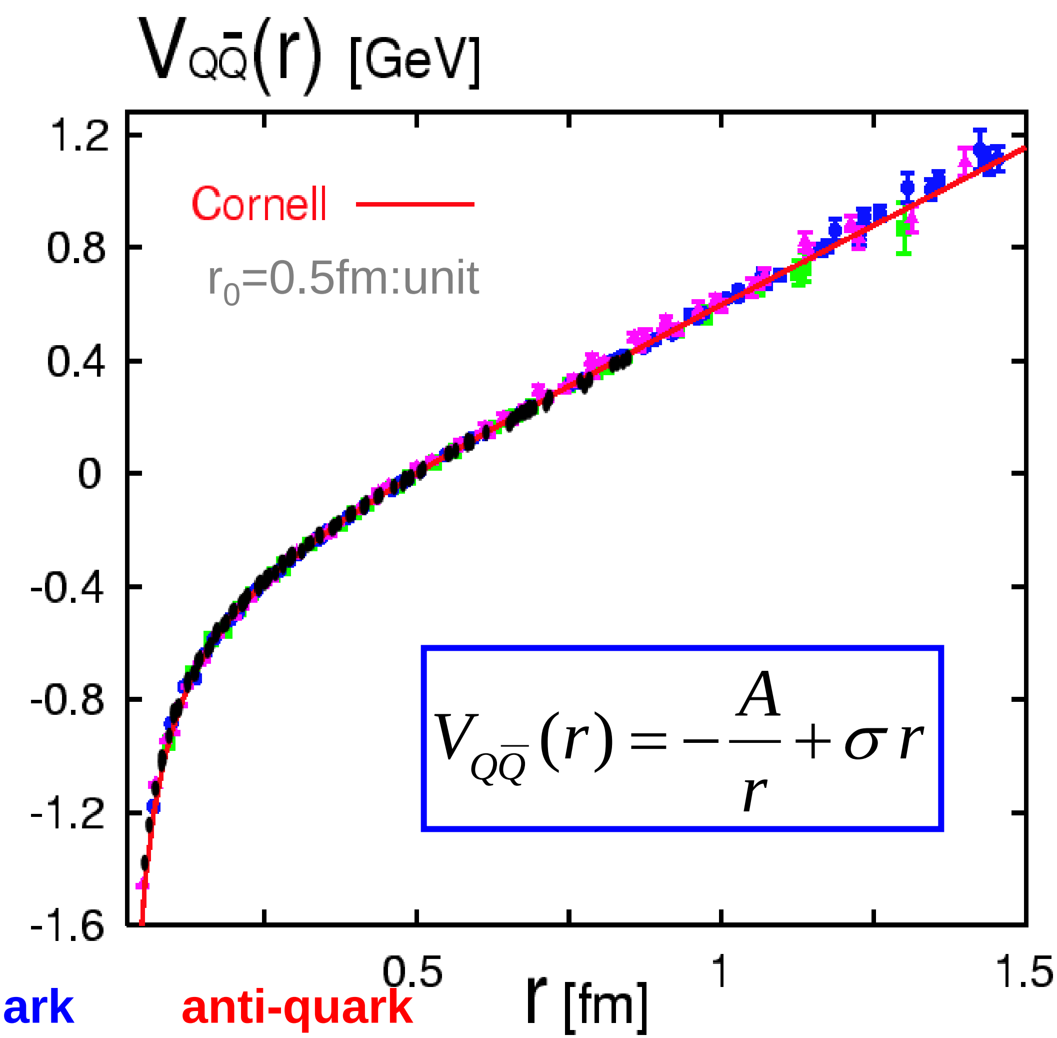
M.Creutz (1979,80)

Summarized lattice QCD data
G.S.Bali (2001)
Takahashi, H.S. et al. (2002)
JLQCD (2003)

At the long distances, the Q-$\bar{Q}$ potential behaves as a linear potential, which indicates one-dimensional squeezing of the color-electric flux between quark and antiquark.

One-dimensional squeezing of color flux between q and $\bar{q}$

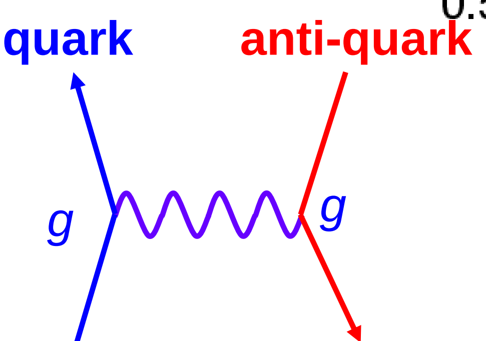
Quark-antiquark static potential in Lattice QCD



M.Creutz (1979,80)

Summarized lattice QCD data
G.S.Bali (2001)
Takahashi, H.S. et al. (2002)
JLQCD (2003)

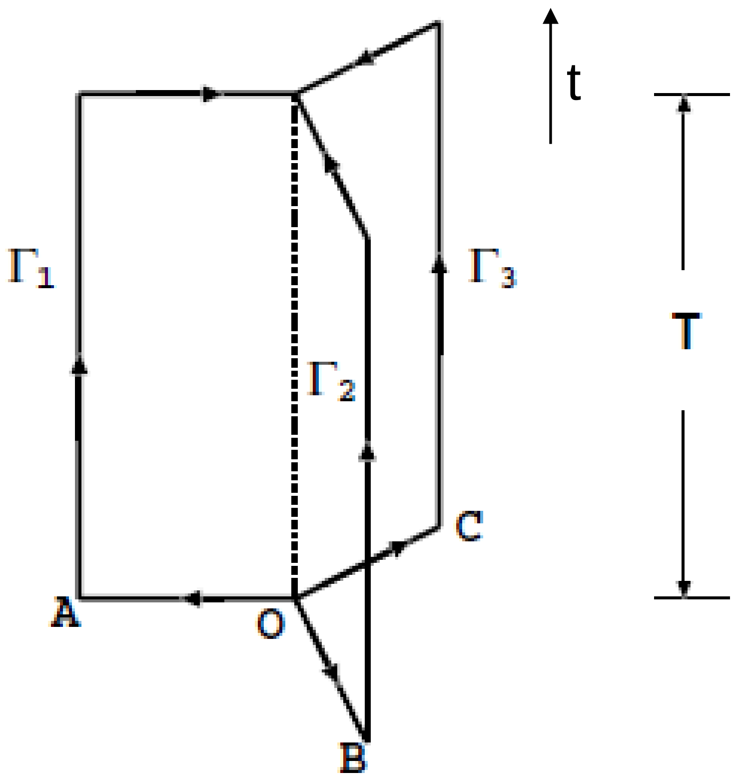
Quantitatively,
quark-antiquark potential
is well described by
a simple sum of
Coulomb plus linear.



One-dimensional squeezing of color flux between q and $\overline{q}$

Baryonic Three-Quark Potential in QCD

Similar to the QQbar potential, the 3Q potential can be calculated with the 3Q Wilson loop defined on the contour of three large staples as



N. Brambilla et al., PLB362 (1995) 113.

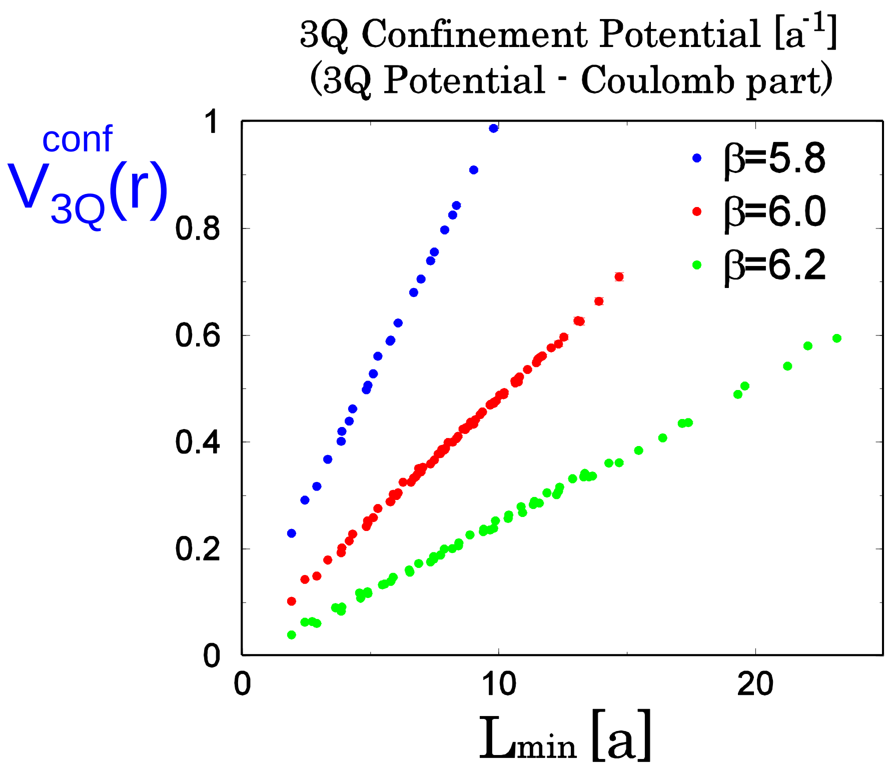
Takahashi, H.S. et al. PRL 86 (2001) 18.
Takahashi, H.S. et al. PRD65 (2002)114509.

$$V_{3Q} = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W_{3Q} \rangle_T$$

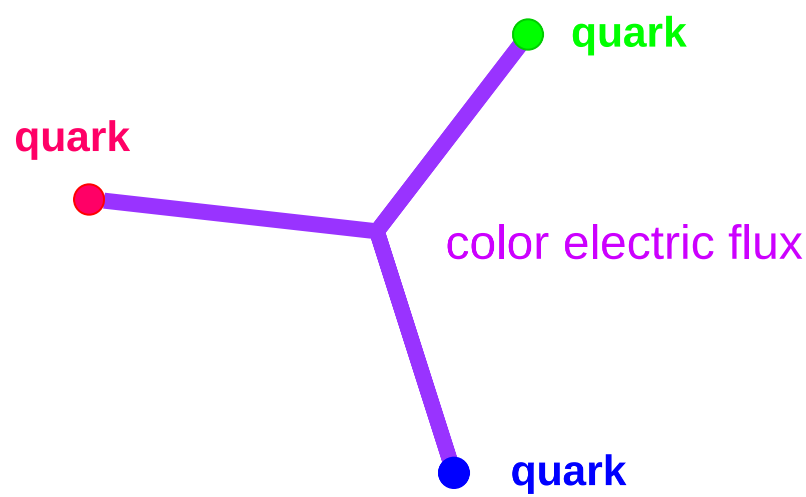
The 3Q Wilson loop physically means that
gauge-invariant 3Q state is created at $t = 0$ and is annihilated at $t = T$ with the three quarks spatially fixed in $\mathbf{R}^3$ for $0 < t < T$.

Baryonic Three-Quark Potential in Lattice QCD

Takahashi, H.S. et al. PRL 86 (2001) 18.
Takahashi, H.S. et al. PRD65 (2002)114509.
Takahashi, H.S. Phys. Rev. Lett. 90 (2003).



L_{min} : *total length of string linking three valence quarks*



$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{\left| \vec{r}_i - \vec{r}_j \right|} + \sigma L_{\min}$$

One-Gluon-Exchange Coulomb potential

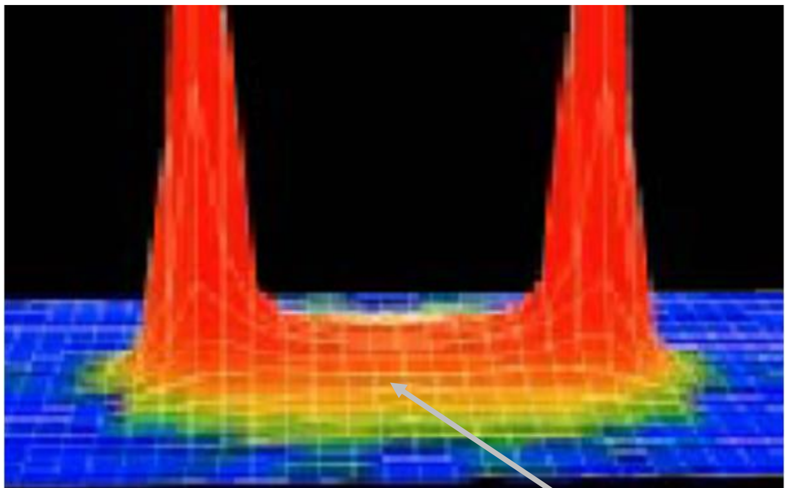
Linear potential based on string picture

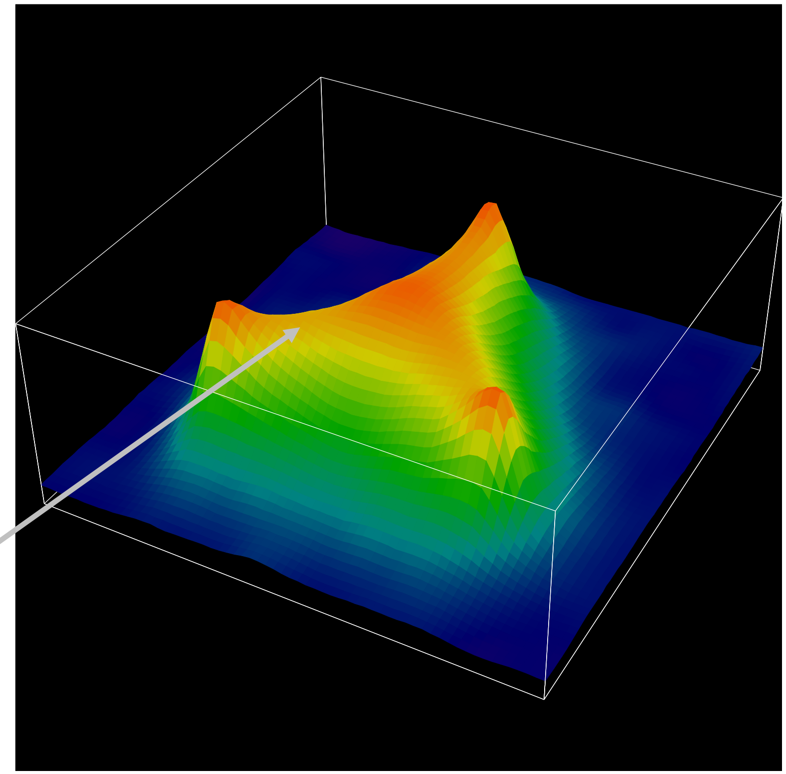
The 3Q potential is also well described by a simple sum of one-gluon-exchange (OGE) Coulomb plus string-picture linear potential.

Flux tube formation for QQbar and 3Q systems in Lattice QCD

H. Ichie et al., Nucl. Phys. A721, 899 (2003).
V.G. Bornyakov et al., PRD70, 054506 (2004).

G.S. Bali





Color–Electric Flux Tube

Non-perturbative

Actually, apart from the color-Coulomb energy around quarks, flux-tube formation has been observed in lattice QCD both for QQbar and 3Q systems, and such one-dimensional squeezing of color flux leads to linear confinement potential.

Dual Superconductor Picture for Confinement

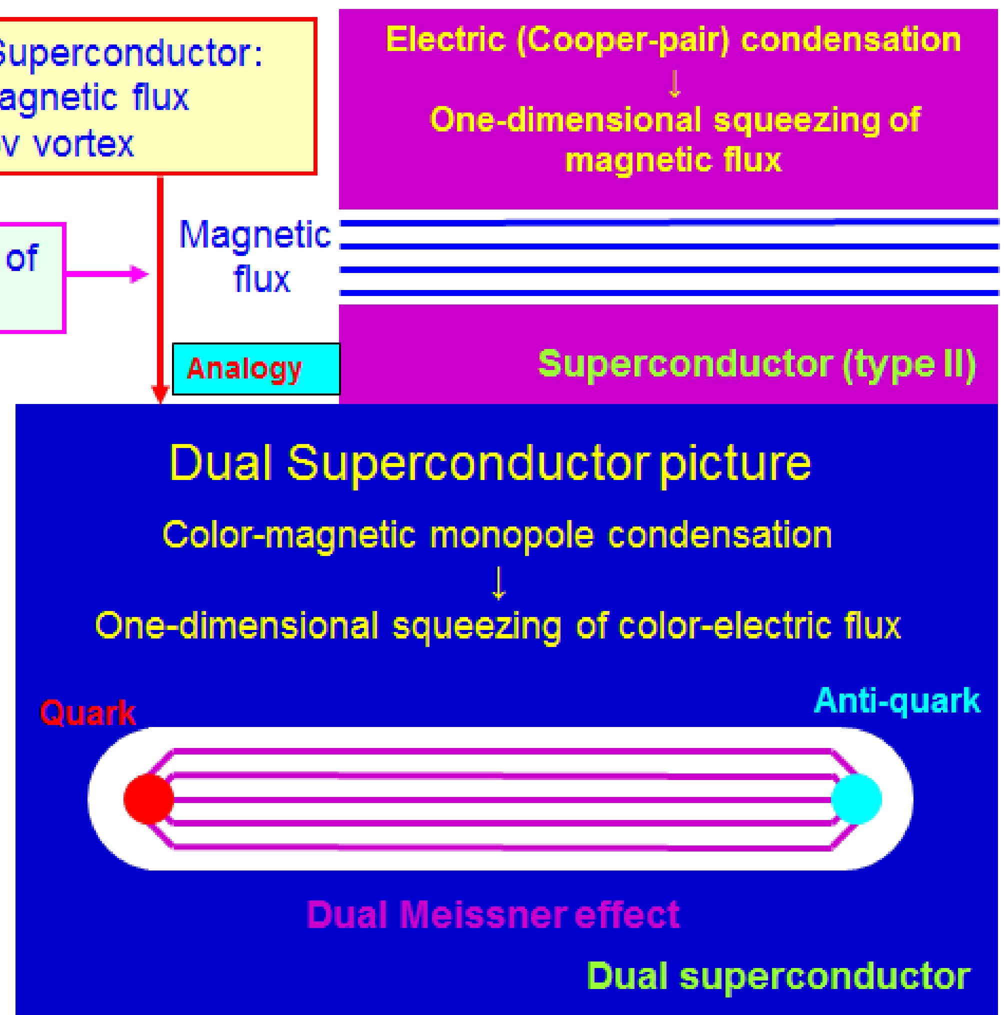
Historical Overview

In 1970's, *Nambu, 't Hooft, Mandalstam* proposed *Dual Superconductor picture* for quark confinement, based on the analogy between Abrikosov vortex in Type-II superconductor and flux-tube/string picture for hadrons.

Meissner effect in Superconductor: Exclusion of magnetic flux
$\Rightarrow$ Abrikosov vortex

Electromagnetic Duality of Maxwell equations

$$\begin{cases} \partial_\mu F^{\mu\nu} = j^\nu \\ \partial_\mu \tilde{F}^{\mu\nu} = k^\nu \end{cases}$$



Nambu,'tHooft, Mandelstam
(1974,75,76)

Dual Superconductor Theory in QCD

However, there are two large gaps between QCD and dual superconductor picture

1. The dual superconductor is based on the Abelian gauge theory subject to the Maxwell-type equations, where electro-magnetic duality is manifest, while QCD is a non-abelian gauge theory.

2. The dual superconductor requires condensation of color-magnetic monopoles as the key concept, while QCD does not include such a monopole as the elementary degrees of freedom.

In 1981, *'t Hooft* gave a possible mathematical foundation of this picture by way of *Abelian Gauge Fixing in QCD*, which is a partial gauge fixing on $SU(N)/U(1)^{N-1}$, similar to Non-Abelian Higgs theory.

By Abelian gauge fixing, QCD is reduced into *Abelian gauge theory*, and *magnetic monopoles appear* as topological objects.

Maximally Abelian Gauge

The maximally Abelian (MA) gauge is successful Abelian gauge.

In continuum Euclidean QCD, MA gauge is defined by minimizing off-diagonal gluon-field amplitude using SU(3) gauge transformation.

$$R_{\rm off}[A_\mu(\cdot)] \equiv \int d^4x \ {\rm tr}\left\{[\hat{D}_\mu, \vec{H}][\hat{D}_\mu, \vec{H}]^\dagger\right\} \propto \int d^4x \sum_\alpha |A_\mu^\alpha(x)|^2.$$

Local condition of MA gauge is expressed as

$$[\vec{H}, [\hat{D}_\mu, [\hat{D}_\mu, \vec{H}]]] = 0$$

Note that MA gauge is a partial gauge fixing on SU(N)/U(1)$^{N\text{-}1}$, since there remains Abelian gauge symmetry U(1)$^{N\text{-}1}$.

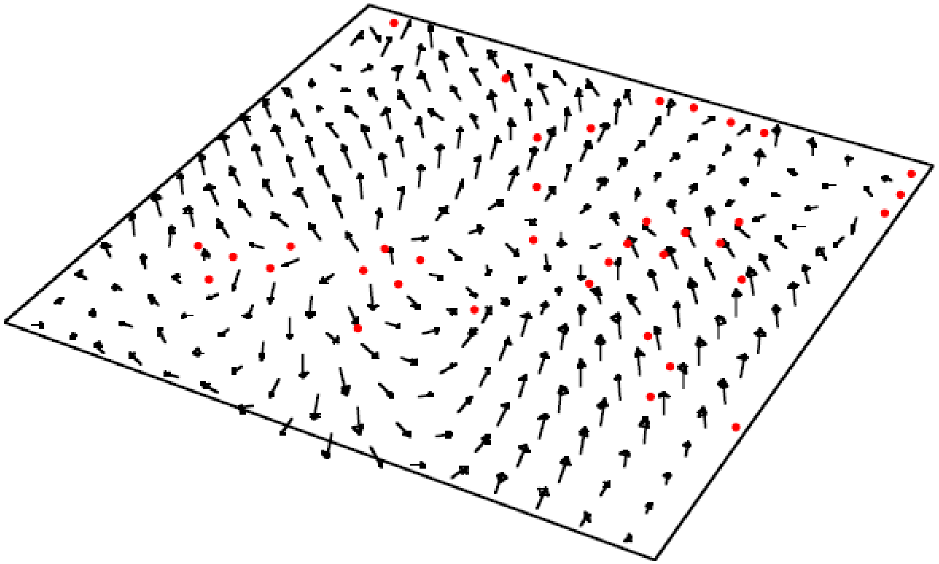
In the lattice formalism, MA gauge is defined by maximizing Abelian part of link-variables by SU(3) gauge transformation.

$$R\,[U_\mu(s)] \equiv {\rm Re} \sum_{s,\mu} {\rm Tr}\left(U_\mu^\dagger(s)\, \vec{H}\, U_\mu(s)\, \vec{H} \right)$$

In *MA gauge*, QCD is reduced into an *Abelian gauge theory* with *magnetic monopoles*. ['t Hooft, NPB190(1981)]

1. *Non-Abelian* gauge symmetry SU(3) is reduced into *Abelian* gauge symmetry $U(1)^2$, i.e., $SU(3) \rightarrow U(1)^2$. (maximal torus subgroup of SU(3))

2. There appear *magnetic monopoles* from hedgehog singularity, corresponding to the *Nontrivial Homotopy Group* $\Pi_2(SU(3)/U(1)^2) = \mathbf{Z}^2$, similar to the appearance of 't Hooft-Polyakov or GUT monopoles.



Monopoles appear around hedgehog singularities in gluon field in MA gauge (SU(2) Lattice QCD)

H. Ichie and H.S., NPB574 (2000) 70.

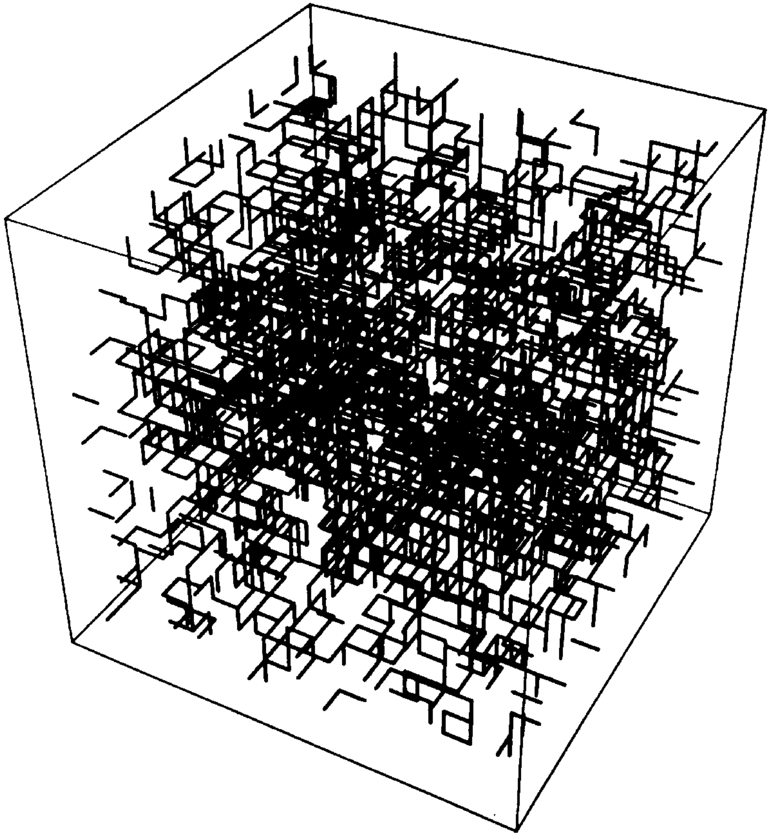
Thus, in MA gauge, QCD can be dual superconductor theory, if *Abelian dominance (inactiveness of off-diagonal gluon)* and *monopole condensation* are realized.

Monopole Current appearing in MA gauge

Monopole Current appearing in MA gauge in SU(2) lattice QCD

-Monopole clustering → A signal of monopole condensation
[A.S. Kronfeld, M.L. Laursen, G. Schierholz, U.-J. Wiese PLB (1987).]

-Magnetic Screening → An evidence of Monopole condensation
[H.S., Amemiya, Ichie, NPA (2000).]



Analysis of gluon propagators in MA gauge in SU(2) QCD

Large effective-mass generation of off-diagonal gluons in MA gauge

$M_{\mathrm{off}} \fallingdotseq 1\mathrm{GeV}$

Infrared inactiveness of off-diagonal gluons in MA gauge

Infrared Abelian Dominance

infrared quantities such as confinement and chiral symmetry breaking would be well described only with diagonal gluons in MA gauge

K.Amemiya, H.S., PRD 60 (1999) 114509.

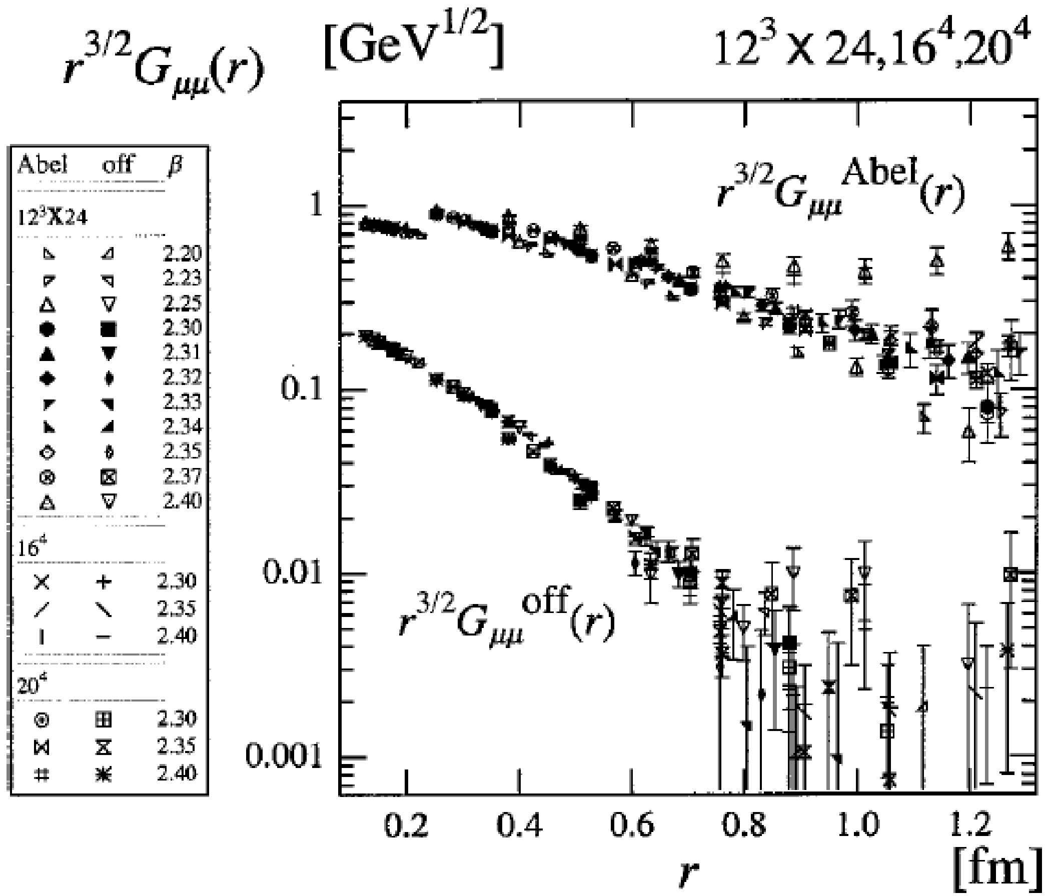
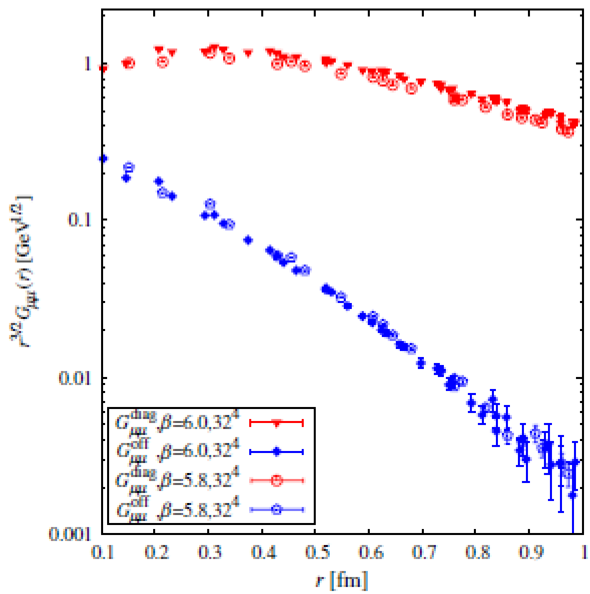


FIG. 3. The logarithmic plot of $r^{3/2}G^{\mathrm{off}}_{\mu\mu}(r)$ and $r^{3/2}G^{\mathrm{Abel}}_{\mu\mu}(r)$ as the function of the distance r in the MA gauge with the $U(1)_3$ Landau gauge fixing, using the SU(2) lattice QCD with $12^3\times 24$ $(2.2\leqslant\beta\leqslant 2.4)$, 16^4 and 20^4 $(2.3\leqslant\beta\leqslant 2.4)$. The off-diagonal gluon propagator behaves as the Yukawa-type function $G^{\mathrm{off}}_{\mu\mu}(r) \sim [\exp(-M_{\mathrm{off}}r)]/r^{3/2}$ with $M_{\mathrm{off}}\simeq 1$ GeV for $r \gtrsim 0.2$ fm. Therefore, the off-diagonal gluon seems to have a large mass $M_{\mathrm{off}}\simeq 1$ GeV in the MA gauge.

Large off-diagonal gluon mass and infrared Abelian dominance in MA gauge in SU(3) Lattice QCD

S. Gongyo, T. Iritani, H.S., PRD 86 (2012) 094018, PRD 87 (2013) 074506.

We investigate gluon propagators in MA gauge with $U(1)^2$ Landau gauge fixing in SU(3) quenched QCD, and find a *large off-diagonal gluon mass* of about 1GeV, which leads to Infrared inactiveness of off-diagonal gluons and *infrared Abelian dominance*, similar to low-energy Abelianization in the Weinberg-Salam model.



The diagonal and off-diagonal gluon propagators (log plot of $r^{3/2}$ G(r)) in SU(3) lattice QCD in the MA gauge with the $U(1)^2$ Landau gauge fixing.

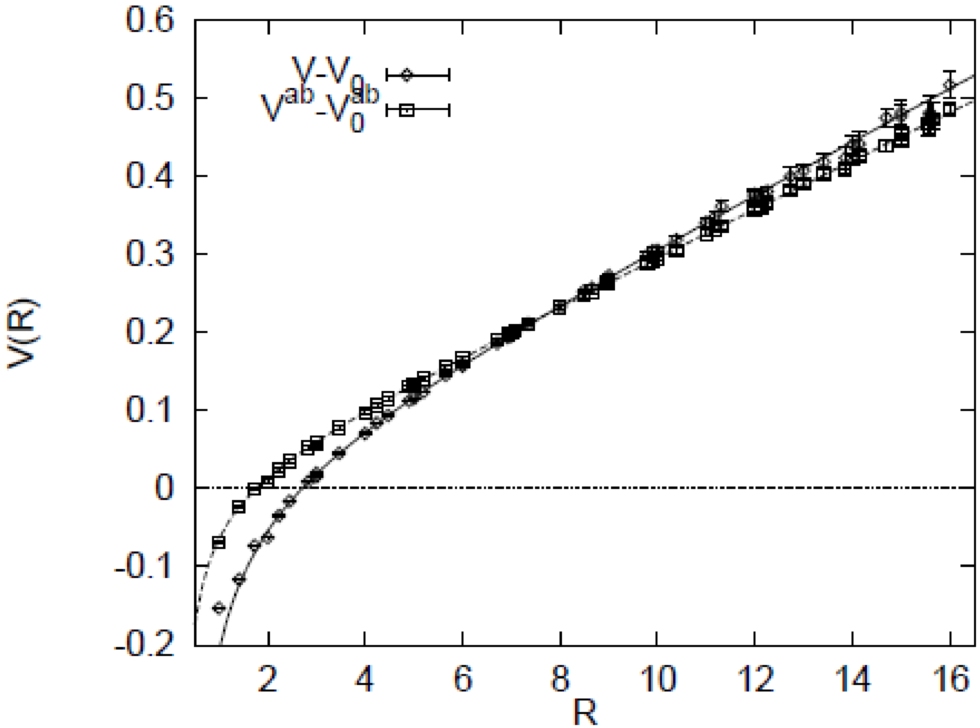
Lattice size	β	a[fm]	$M_{\rm off}$[GeV]	$M_{\rm diag}$[GeV]
32^4	5.8	0.152	1.1	0.3
	6.0	0.104	1.1	0.3

The lattice QCD result of effective masses of off-diagonal and diagonal gluons in the MA gauge. The off-diagonal gluon has a large mass of about 1GeV.

Abelian Dominance in SU(2) Lattice QCD

G.S. Bali, V. Bornyakov, M. Mueller-Preussker, K. Schilling, PRD54 (1996) 2863.

A Highlight of M.-I. Polikarpov's Plenary Review Talk at LATTICE 1996.



SU(2)

Abelian

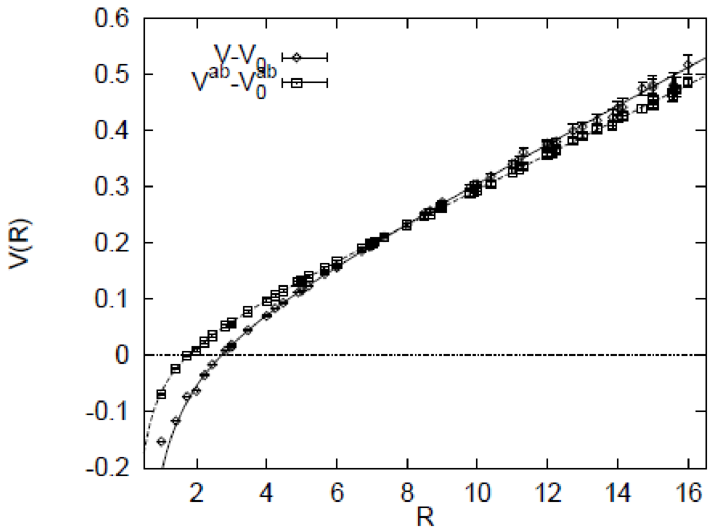
$\sigma_{Abel} \fallingdotseq 0.92\ \sigma_{SU(2)}$

MA projection of Inter-Quark Potential in Quenched SU(2) Lattice QCD
at β=2.5115 ($a \fallingdotseq$0.1fm) on 32^4 ($L \fallingdotseq$3 fm)

Abelian Dominance in SU(2) Lattice QCD

G.S. Bali, V. Bornyakov, M. Mueller-Preussker, K. Schilling, PRD54 (1996) 2863.

A Highlight of M.-I. Polikarpov's Plenary Review Talk at LATTICE 1996.



SU(2)

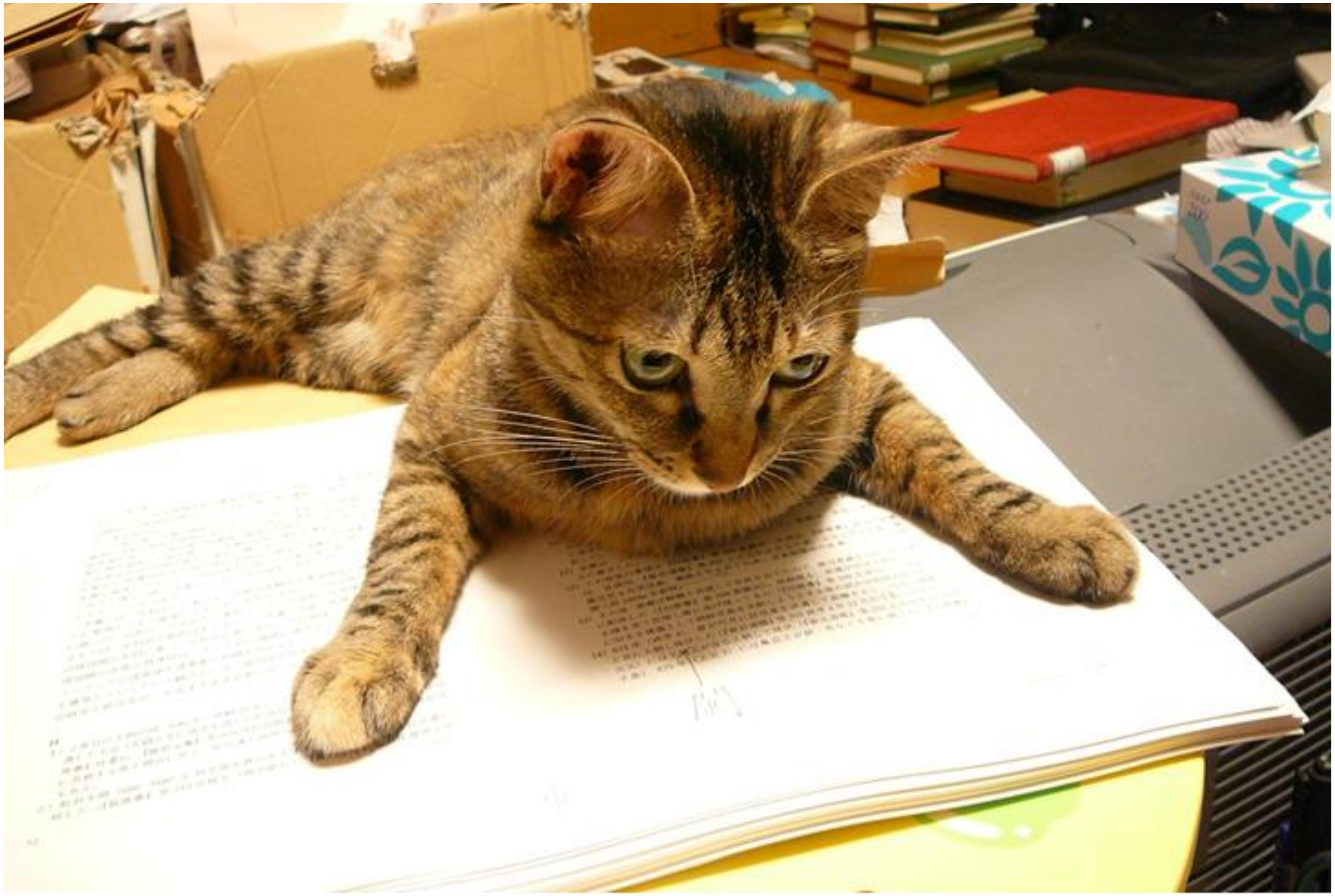
Abelian

$\sigma_{Abel} \doteqdot 0.92\ \sigma_{SU(2)}$

The differences in the slopes of the linear part of the potentials in Fig. 1 and Fig. 2 yield the following relations: $\sigma_{U(1)} \approx 92\%\,\sigma_{SU(2)}$, $\sigma_j \approx 95\%\,\sigma_{U(1)}$, where σ_j is the monopole current contribution to the string tension. It is important to study a widely discussed idea that in the continuum limit ($\beta \to \infty$) the abelian and the monopole dominance is exact (3): $\sigma_{SU(2)} = \sigma_{U(1)} = \sigma_j$.

From M.-I. Polikarpov's Paper of Plenary Talk at LATTICE 1996

Let us go to SU(3) lattice QCD study of
Abelian Dominance for Confinement



Maximally Abelian (MA) Gauge and Cartan decomposition on lattice

In the lattice formalism, MA gauge is defined by maximizing the "Abelian part of the link-variable",

$$R\,[U_\mu(s)] \equiv \mathrm{Re} \sum_{s,\mu} \mathrm{Tr}\left(U_\mu^\dagger(s)\, \vec{H}\, U_\mu(s)\, \vec{H} \right)$$

using SU(3) gauge transformation.

After the MA gauge fixing, the SU(3) link variables are factorized with respect to the Cartan decomposition of SU(3) into $U(1)^2$ and $SU(3)/U(1)^2$.

SU(3) link-variable $\quad U_\mu(s) = M_\mu(s)\, u_\mu(s) \in SU(3)$

Abelian link-variable $\quad u_\mu(s) = e^{i\vec{\theta}_\mu(s)\cdot\vec{H}} \in U(1)^2$

off-diagonal link-variable $\quad M_\mu(s) = e^{i\theta_\mu^\alpha(s) E^{-\alpha}} \in SU(3)/U(1)^2$

Residual Abelian gauge symmetry in MA gauge

In the MA gauge, there remains residual $U(1)^2$ Abelian gauge symmetry, which does not change the MA gauge condition, e.g.,

$$R[U_\mu(s)] \equiv \mathrm{Re} \sum_{s,\mu} \mathrm{Tr}\left(U_\mu^\dagger(s)\, \vec{H}\, U_\mu(s)\, \vec{H} \right)$$

Here, using Abelian gauge function $v(s) \in U(1)^2$,

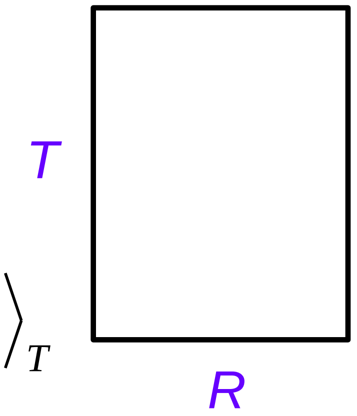
Off-diagonal link-variables behave as charged matter fields to keep the form of $SU(3)/U(1)^2$

$$M_\mu(s) \to v(s) M_\mu(s) v^+(s) \in SU(3)/U(1)^2$$

Abelian link-variables behave as Abelian gauge fields

$$u_\mu(s) \to v(s) u_\mu(s) v^+(s+\hat{\mu}) \in U(1)^2$$

Abelian Wilson loop and MA-projected quark potential



ordinary SU(3) Wilson loop

$$W[U_\mu] = Tr \prod_{i=1}^{2(R+T)} U_{\mu_i}(s_i)$$

→ SU(3) quark potential

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W[U_\mu] \rangle_T$$

Abelian Wilson loop

$$W[u_\mu] = Tr \prod_{i=1}^{2(R+T)} u_{\mu_i}(s_i)$$

→ MA-projected quark potential

$$V_{Abel}(R) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W[u_\mu] \rangle_T$$

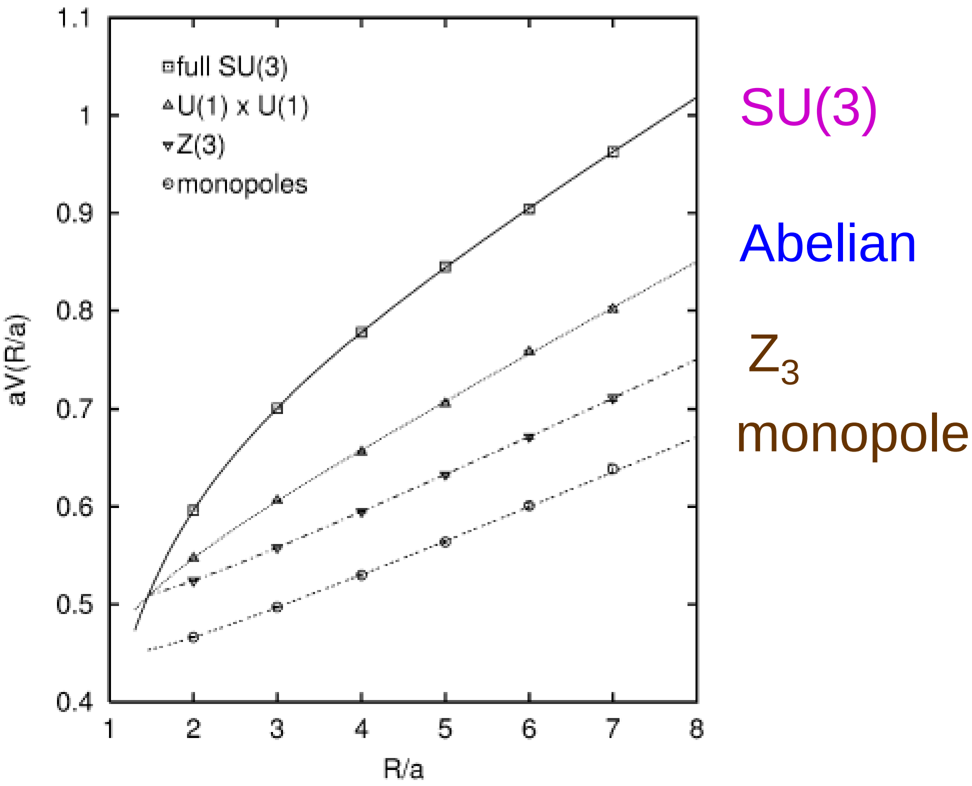
Similar to the extraction of the quark potential from the Wilson loop,
MA-projected quark potential can be obtained from Abelian Wilson loop.

N.B. The Abelian Wilson loop and MA-projected quark potential
are invariant under residual Abelian gauge transformation.

Pioneering work in SU(3) Lattice QCD 1

J.D. Stack, W.W. Tucker, R.J. Wensley, NPB639 (2002) 2013.

The maximal abelian gauge, monopoles, and vortices in SU(3) lattice gauge theory



Analysis with
Coulomb + linear Ansatz

$\sigma_{Abel} \fallingdotseq 0.9\ \sigma_{SU(3)}$

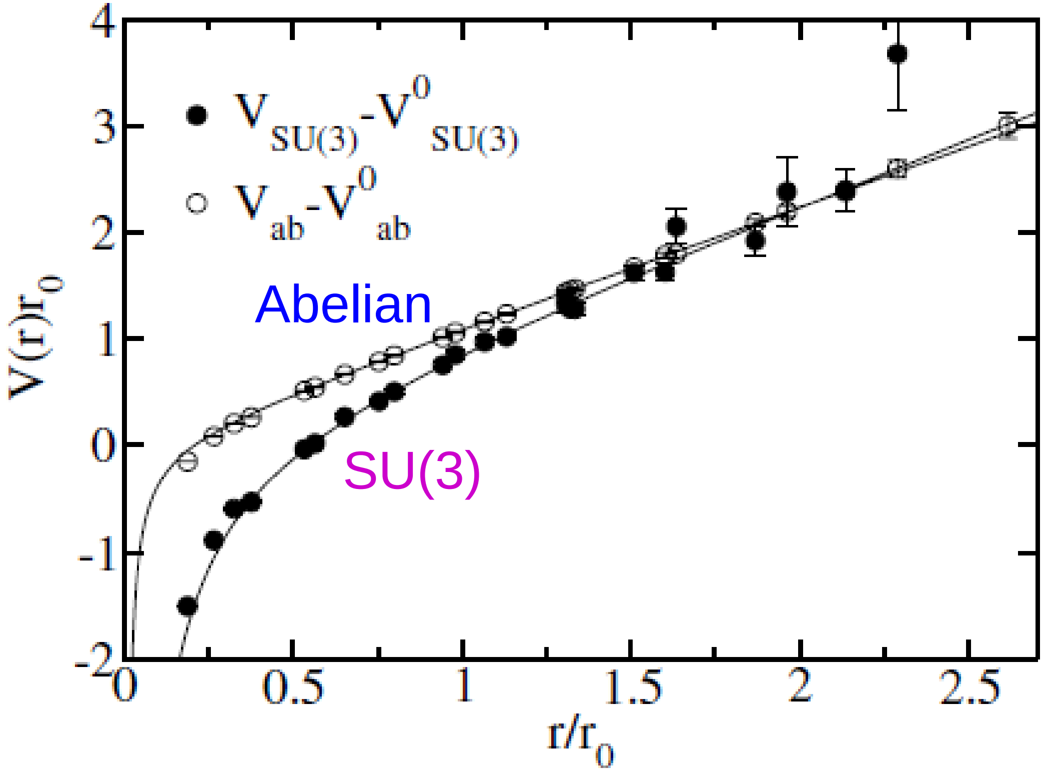
MA projection of Inter-Quark Potential in Quenched SU(3) Lattice QCD
at β=6.0 (a≒0.1fm) on 16^4 (L≒1.6fm)

Pioneering work in SU(3) Lattice QCD 2

V.G. Bornyakov et al. (DIK collaboration) PRD70 (2004) 074511.

Dynamics of monopoles and flux tubes in two-flavor dynamical QCD

V. G. Bornyakov,[1,2,3] H. Ichie,[3,4] Y. Koma,[5] Y. Mori,[3] Y. Nakamura,[3] D. Pleiter,[6] M. I. Polikarpov,[2] G. Schierholz,[6,7] T. Streuer,[6,8] H. Stüben,[9] and T. Suzuki[3]



Analysis with
Coulomb + linear Ansatz

$\sigma_{Abel} \fallingdotseq 0.83\ \sigma_{SU(3)}$

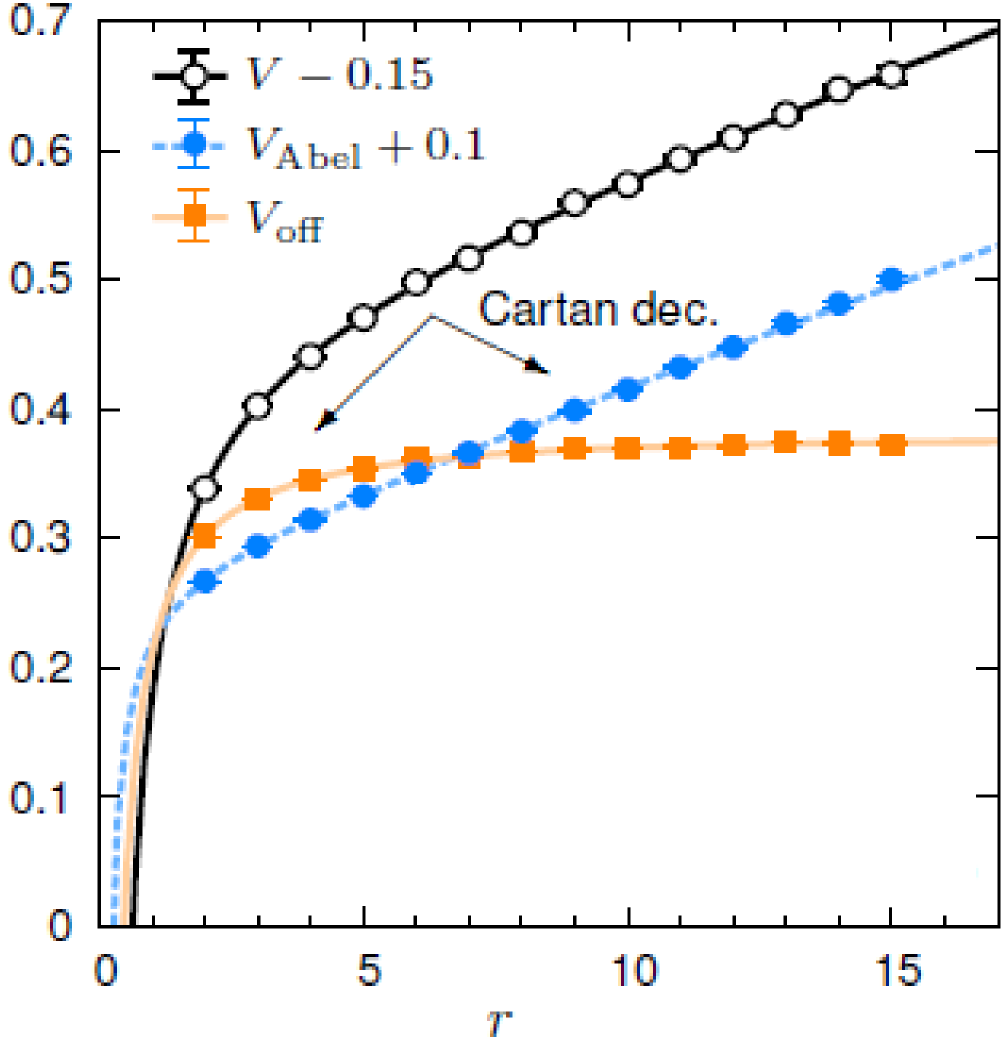
MA projection of Inter-Quark Potential in Quenched SU(3) Lattice QCD
at β=6.0 ($a \fallingdotseq$ 0.1fm) on 16^3 x 32 ($L \fallingdotseq$ 1.6fm)

(They also investigated full QCD and flux-tube formation.)

Lattice calculation condition

- SU(3) standard plaquette action at quenched level
- lattice parameter: $\beta=2N_c/g^2=6.4$,
  corresponding to fine lattice spacing: a=0.058fm
- Lattice size: $L^4=32^4$ (i.e., L=1.86fm)
- 70 gauge configurations
  (thermalization: 20000 sweeps, interval :500 sweeps)
- standard gauge-invariant smearing method
  for the sake of accurate potential calculations
- jackknife error estimate

Abelian Dominance for Confinement

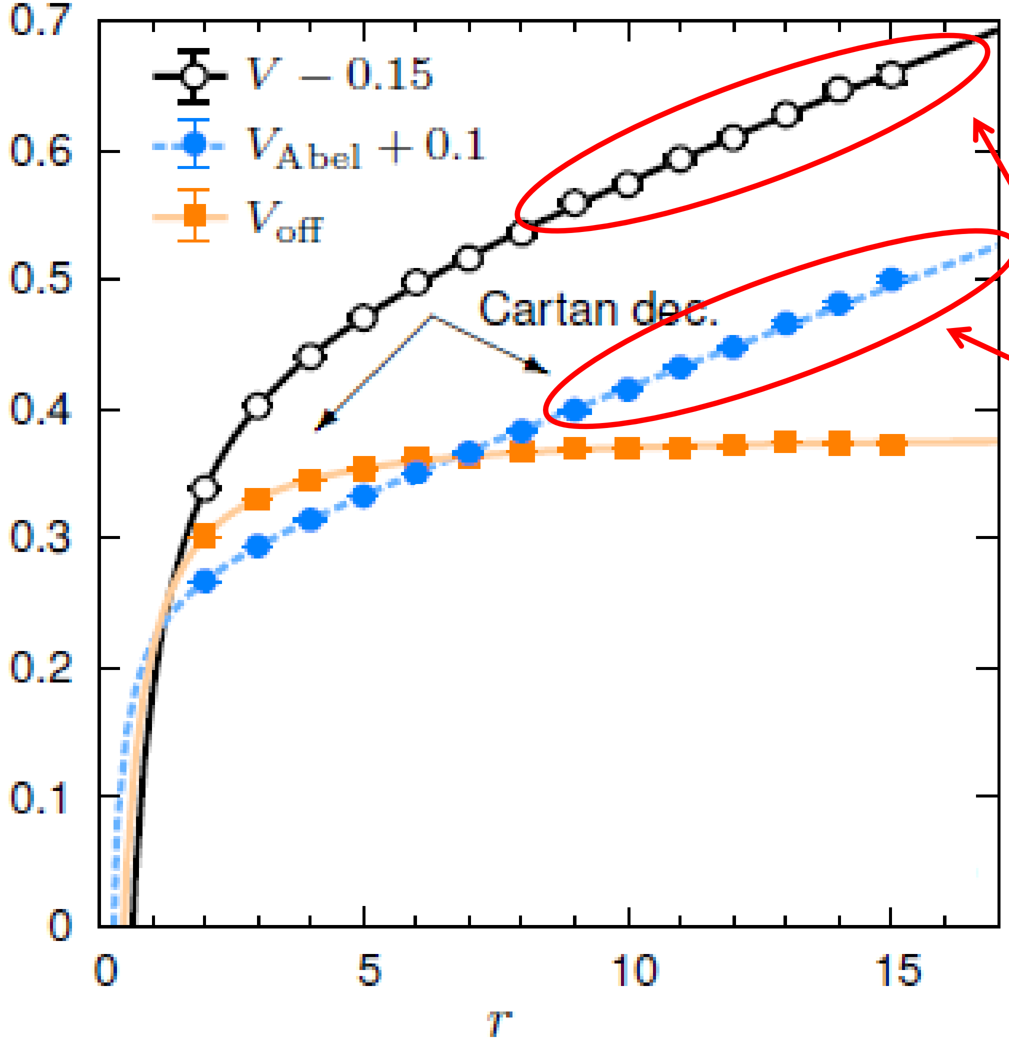


SU(3) potential V(r)

Abelian part $V_{Abel}(r)$

SU(3) potential and Abelian-projected potential
in MA gauge in lattice QCD with β=6.4 and 32^4.

Abelian Dominance for Confinement



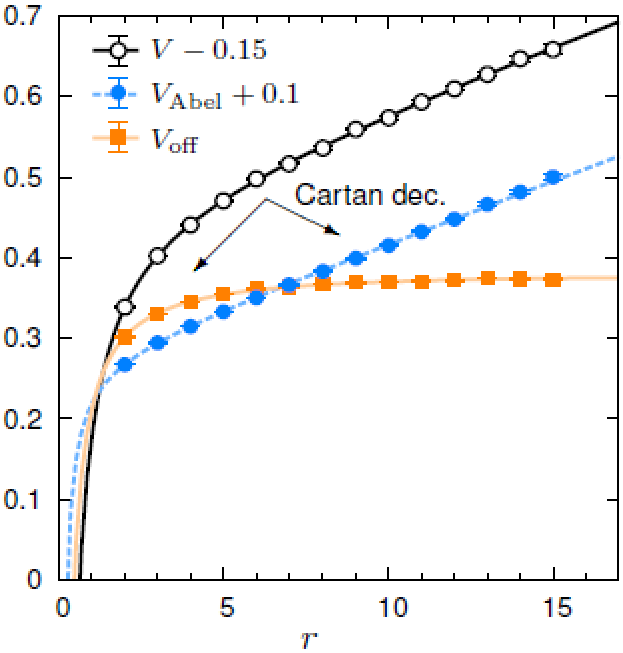
SU(3) potential V(r)

Abelian part V_{Abel}(r)

SU(3) potential and its Abelian part have almost the same slope at large distance.
⇒
Abelian Dominance for Confinement

SU(3) potential and Abelian-projected potential in MA gauge in lattice QCD with β=6.4 and 32^4.

Quantitative Analysis on Abelian Dominance for Confinement



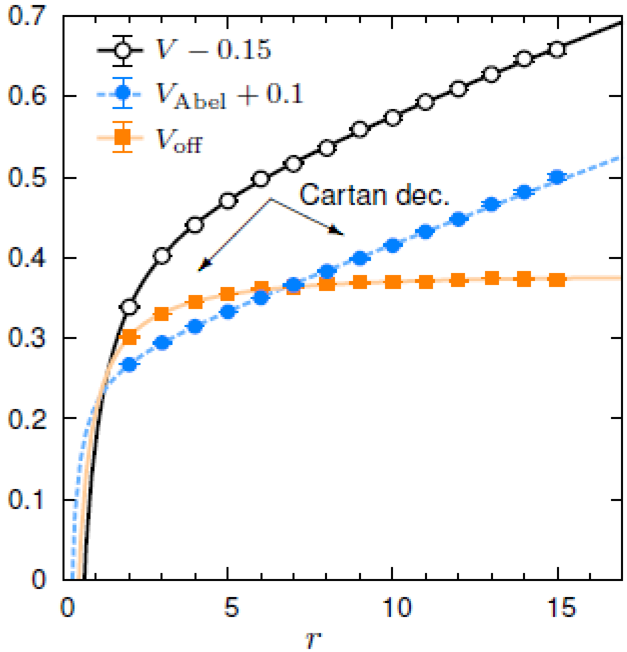
SU(3) potential V(r)

Abelian part $V_{Abel}(r)$

Fit analysis with
Coulomb-plus-linear Ansatz
for each QQbar potential

$$V(r) = -\frac{A}{r} + \sigma r + C$$

Quantitative Analysis on Abelian Dominance for Confinement



SU(3) potential V(r)

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Fit analysis with
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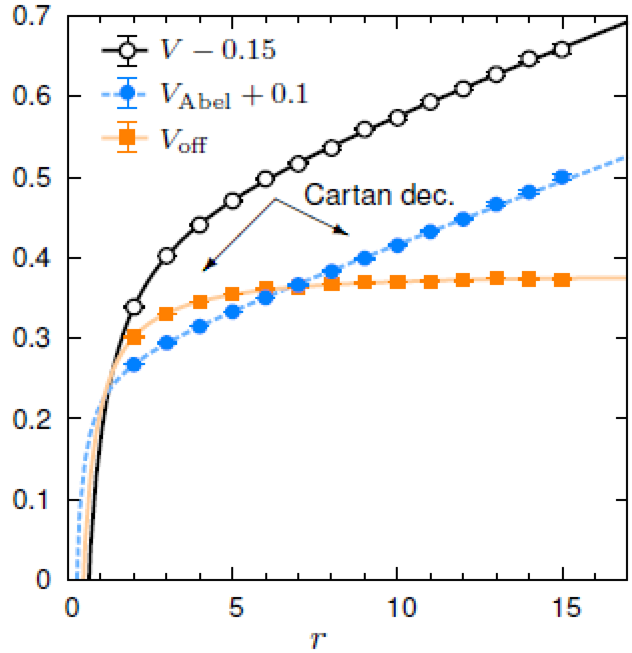
$$V(r) = -\frac{A}{r} + \sigma r + C$$

	σ	A	C	χ^2/N_{df}
V	0.01507(20)	0.290(3)	0.603(2)	1.14
V_{Abel}	0.01528(14)	0.067(2)	0.170(1)	1.17

string tension — Coulomb coefficient — irrelevant constant

Quantitative Analysis on Abelian Dominance for Confinement



SU(3) potential V(r)

Abelian part $V_{\rm Abel}(r)$

Fit analysis with
Coulomb-plus-linear Ansatz
for each QQbar potential

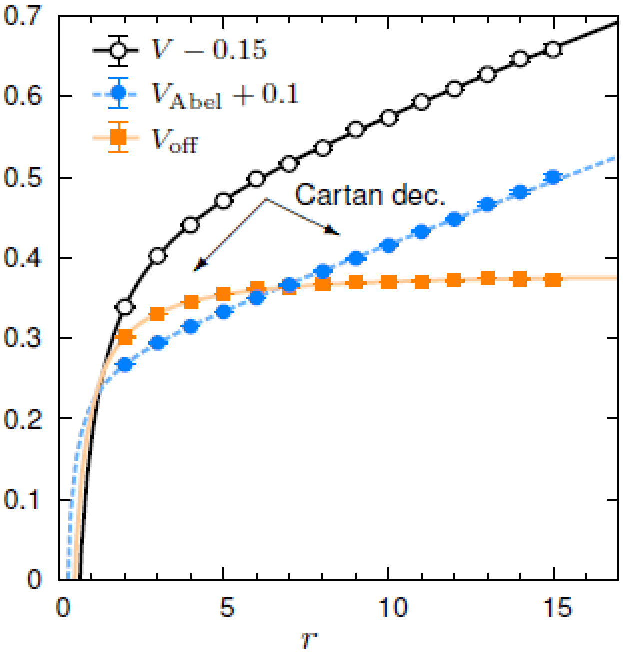
$$V(r) = -\frac{A}{r} + \sigma r + C$$

	σ	A	C	$\chi^2/N_{\rm df}$
V	0.01507(20)	0.290(3)	0.603(2)	1.14
$V_{\rm Abel}$	0.01528(14)	0.067(2)	0.170(1)	1.17

→ goodness of this fit

string tension — Coulomb coefficient — irrelevant constant

Quantitative Analysis on Abelian Dominance for Confinement



SU(3) potential V(r)

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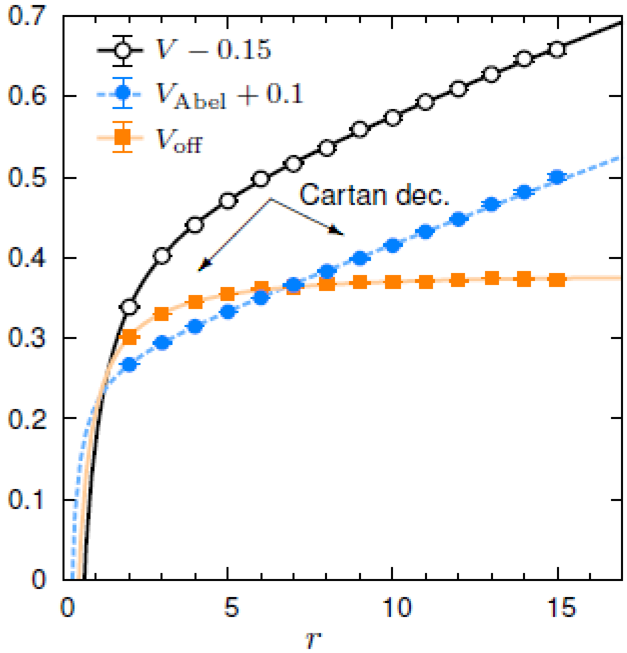
$\sigma_{\rm Abel} = \sigma_{SU(3)}$
⇒ Perfect Abelian Dominance for Confinement

Quantitative Analysis on Abelian Dominance for Confinement



SU(3) potential V(r)

Abelian part $V_{\mathrm{Abel}}(r)$

Fit analysis with
Coulomb-plus-linear Ansatz
for each QQbar potential

$$V(r) = -\frac{A}{r} + \sigma r + C$$

	σ	A	C	χ^2/N_{df}
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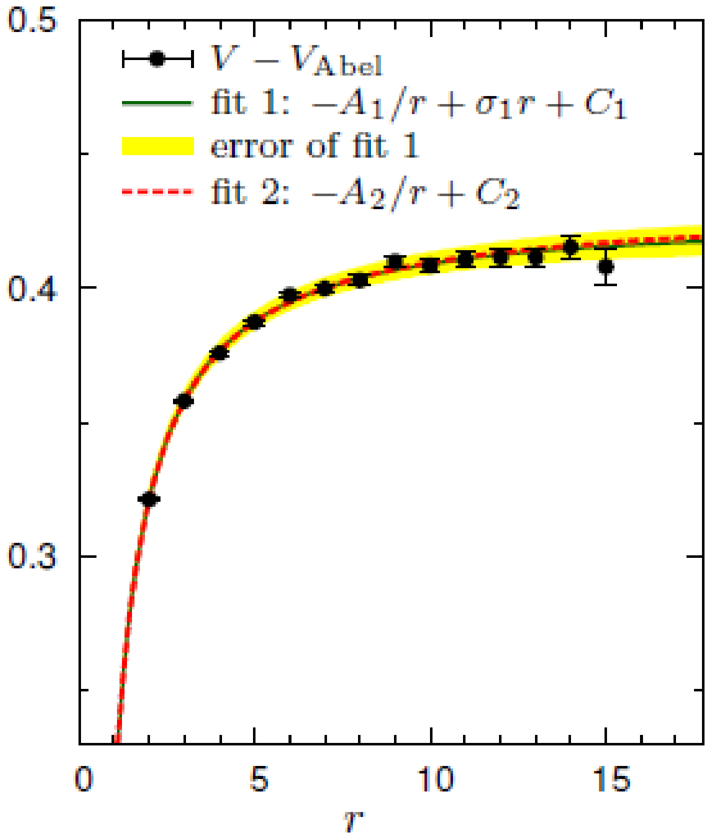
V_{Abel}	0.01528(14)	0.067(2)	0.170(1)	1.17

Coulomb coefficient: $A_{\mathrm{Abel}}/A_{SU(3)}$ = 0.231… ≒ 2/8?
Abelian gluon number / total gluon number in SU(3)

Quantitative Analysis on Abelian Dominance for Confinement

Difference between SU(3) potential $V_{SU(3)}$ and Abelian part V_{Abel}

$V_{SU(3)}(r) - V_{Abel}(r)$



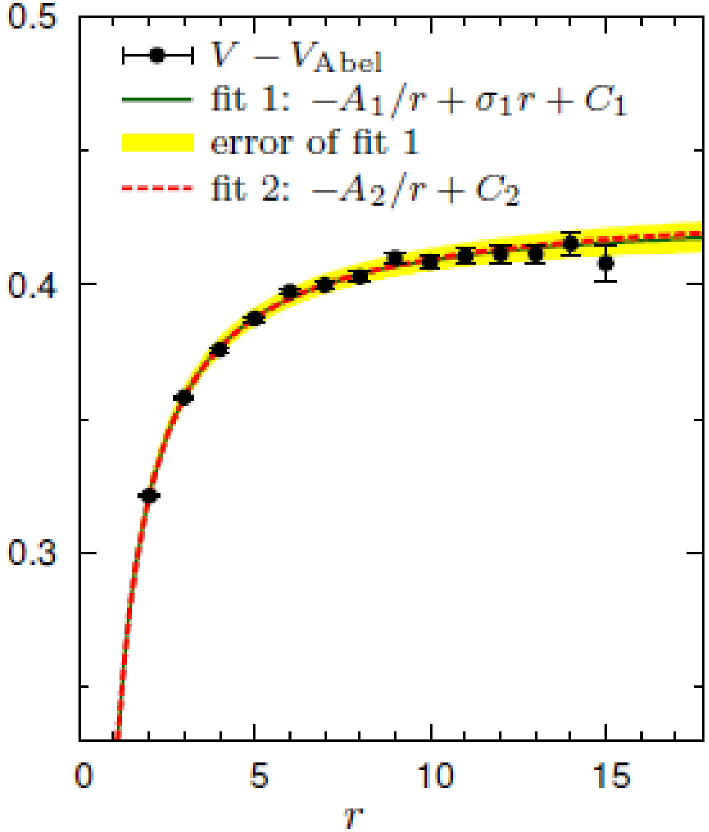
	σ	A	C	$\chi^2/N_{\rm df}$
V	0.01507(20)	0.290(3)	0.603(2)	1.14
$V_{\rm Abel}$	0.01528(14)	0.067(2)	0.170(1)	1.17
$V - V_{\rm Abel}$	−0.00017(22)	0.223(3)	0.433(2)	0.86
$V - V_{\rm Abel}$	—	0.221(2)	0.432(1)	0.84

- No string tension in the difference $V_{SU(3)}(r) - V_{Abel}(r)$.

Quantitative Analysis on Abelian Dominance for Confinement

Difference between SU(3) potential $V_{SU(3)}$ and Abelian part V_{Abel}

$V_{SU(3)}(r) - V_{Abel}(r)$



	σ	A	C	$\chi^2/N_{\rm df}$
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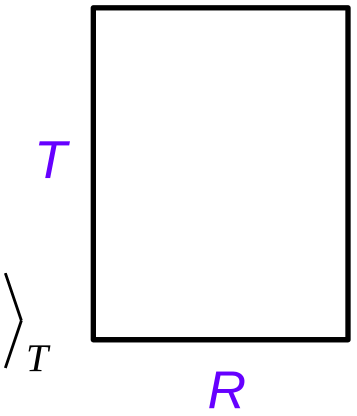
$V - V_{\rm Abel}$	−0.00017(22)	0.223(3)	0.433(2)	0.86
$V - V_{\rm Abel}$	—	0.221(2)	0.432(1)	0.84

- No string tension in the difference $V_{SU(3)}(r) - V_{Abel}(r)$.
- The difference $V_{SU(3)}(r) - V_{Abel}(r)$ can be well fitted by pure Coulomb potential.

⇒ This also suggests Perfect Abelian Dominance for Confinement

Off-diagonal part of quark potential



ordinary SU(3) Wilson loop

$$W[U_\mu] = Tr \prod_{i=1}^{2(R+T)} U_{\mu_i}(s_i)$$

SU(3) quark potential

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W[U_\mu] \rangle_T$$

Off-diagonal Wilson loop

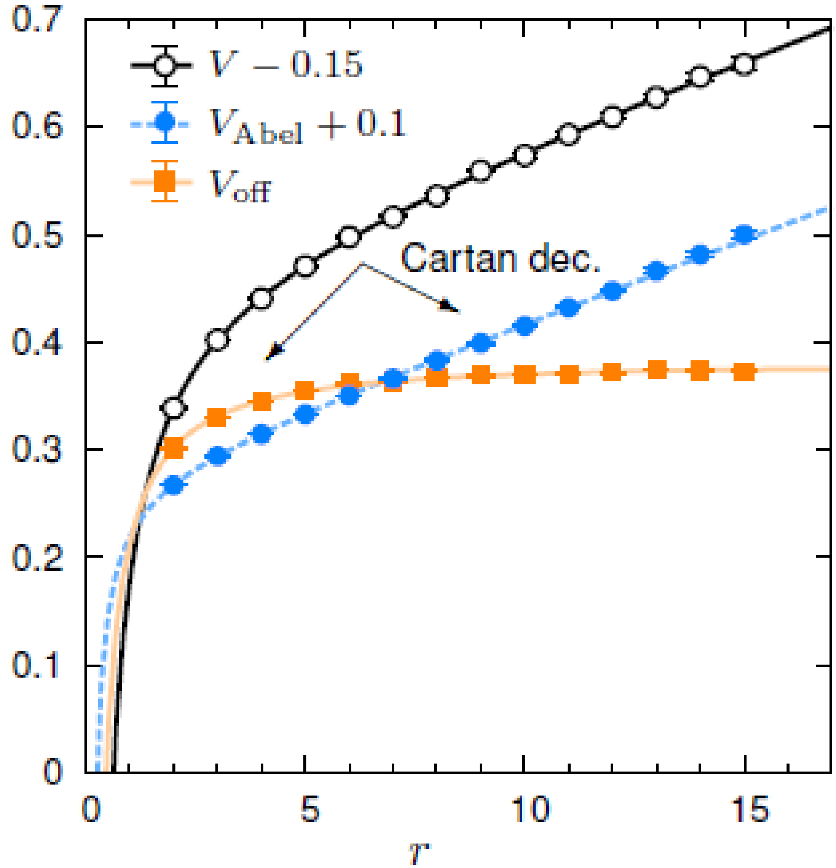
$$W[M_\mu] = Tr \prod_{i=1}^{2(R+T)} M_{\mu_i}(s_i)$$

Off-diagonal part of quark potential

$$V_{off}(R) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W[M_\mu] \rangle_T$$

Like (Abelian) Wilson loop, we consider off-diagonal Wilson loop *W*[*M*]. Since this is not invariant under residual Abelian gauge, we take the $U(1)^2$ Abelian Landau gauge. From the off-diagonal Wilson loop, we define off-diagonal part of quark potential $V_{off}(r)$.

Off-diagonal part of quark potential



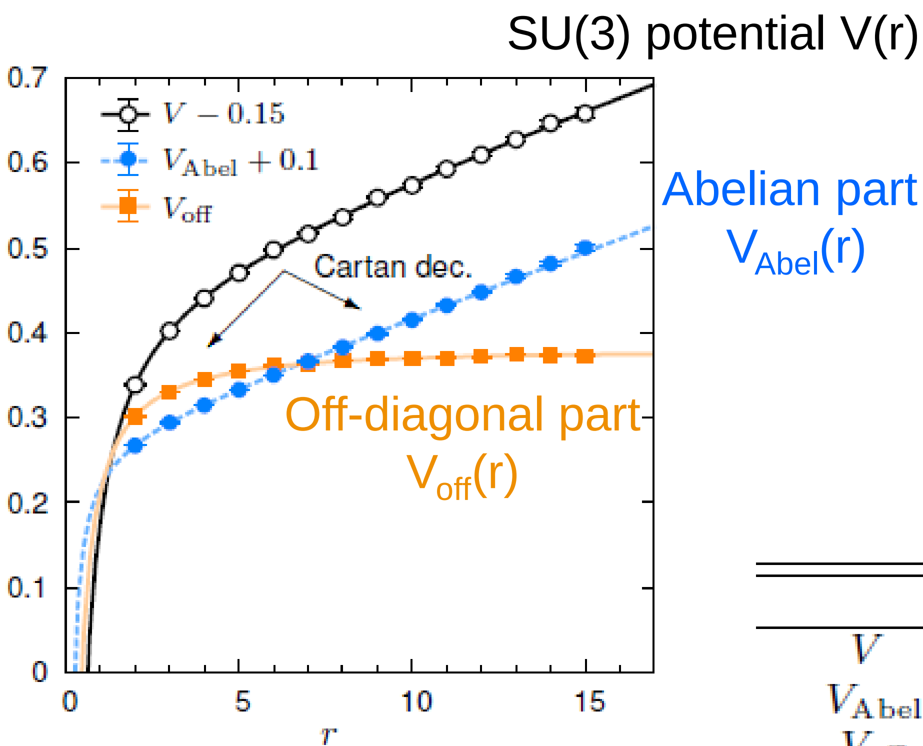
SU(3) potential V(r)

Abelian part $V_{Abel}(r)$

Off-diagonal part $V_{off}(r)$

Off-diagonal part $V_{off}(r)$ of quark potential
in SU(3) lattice QCD with β=6.4 and 32^4
in MA gauge with $U(1)^2$ Landau gauge fixing.

Off-diagonal part of quark potential



Fit analysis with Coulomb-plus-linear Ansatz for each QQbar potential

$$V(r) = -\frac{A}{r} + \sigma r + C$$

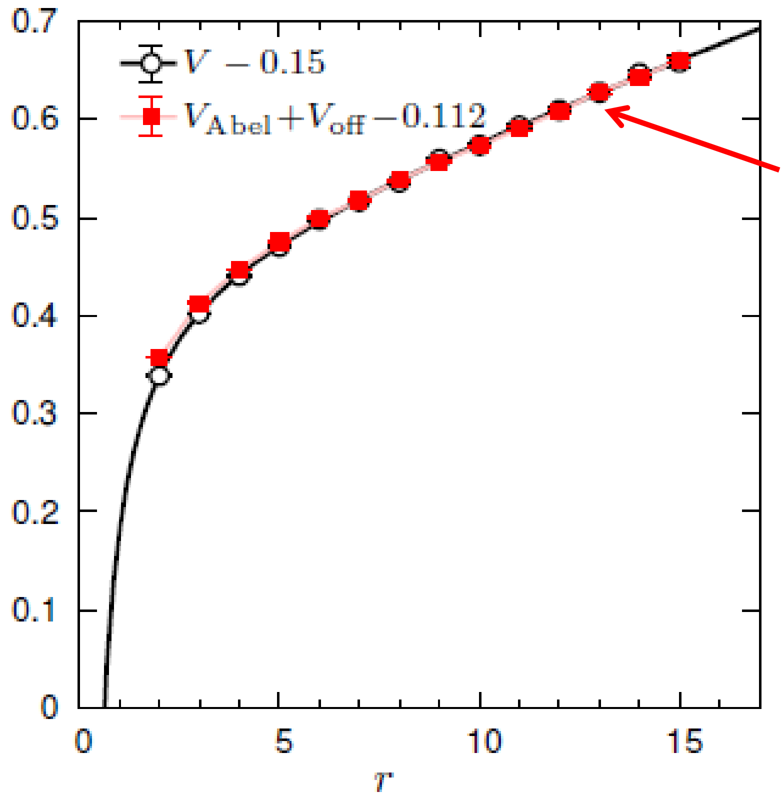
	σ	A	C	$\chi^2/N_{\rm df}$
V	0.01507(20)	0.290(3)	0.603(2)	1.14
$V_{\rm Abel}$	0.01528(14)	0.067(2)	0.170(1)	1.17
$V_{\rm off}$	−0.00038(11)	0.179(3)	0.392(1)	2.41

Almost no string tension in the off diagonal part $V_{\rm off}(r)$

⇒ Off diagonal part $V_{\rm off}(r)$ is almost pure Coulomb potential.

A summation formula for quark potential

We find a simple but nontrivial summation relation of
$V(r) = V_{Abel}(r) + V_{off}(r)$.



SU(3) potential V(r)

Abelian +off-diagonal
$V_{Abel}(r) + V_{off}(r)$

Comparison between SU(3) potential V(r) and
sum of Abelian and off-diagonal parts, $V_{Abel}(r) + V_{off}(r)$.

Non-triviality of the summation formula

$$V(r) = V_{Abel}(r) + V_{off}(r)$$

This simple summation formula is indeed nontrivial because of the nonabelian nature of gauge fields.

Consider the ordinary SU(N) Wilson loop $W[U]$ and Cartan decomposition of SU(N) link-variables U in it:

$$W[U_\mu] = Tr\{U_{\mu_1}(s_1)U_{\mu_2}(s_2)\cdots U_{\mu_N}(s_N)\}$$
$$= Tr\{\underline{M_{\mu_1}(s_1)}\underline{u_{\mu_1}(s_1)}M_{\mu_2}(s_2)u_{\mu_2}(s_2)\cdots M_{\mu_N}(s_N)u_{\mu_N}(s_N)\}$$

Here, Abelian and off-diagonal variables are not commutable.

Non-triviality of the summation formula

V(r) = $V_{Abel}(r) + V_{off}(r)$

This simple summation formula is indeed nontrivial because of the nonabelian nature of gauge fields.

Consider the ordinary SU(*N*) Wilson loop *W*[*U*] and Cartan decomposition of SU(*N*) link-variables *U* in it:

$$W[U_\mu] = Tr\{U_{\mu_1}(s_1)U_{\mu_2}(s_2)\cdots U_{\mu_N}(s_N)\}$$
$$= Tr\{M_{\mu_1}(s_1)u_{\mu_1}(s_1)\, M_{\mu_2}(s_2)u_{\mu_2}(s_2)\cdots M_{\mu_N}(s_N)u_{\mu_N}(s_N)\}$$
$$\neq Tr\{M_{\mu_1}(s_1)M_{\mu_2}(s_2)\cdots M_{\mu_N}(s_N)\cdot u_{\mu_1}(s_1)u_{\mu_2}(s_2)\cdots u_{\mu_N}(s_N)\}$$

Here, Abelian and off-diagonal variables are not commutable.

Non-triviality of the summation formula

$V(r) = V_{Abel}(r) + V_{off}(r)$

This simple summation formula is indeed nontrivial because of the nonabelian nature of gauge fields.

Consider the ordinary SU(*N*) Wilson loop *W*[*U*] and Cartan decomposition of SU(*N*) link-variables *U* in it:

$$W[U_\mu] = Tr\left\{U_{\mu_1}(s_1)U_{\mu_2}(s_2)\cdots U_{\mu_N}(s_N)\right\}$$
$$= Tr\left\{\underline{M_{\mu_1}(s_1)}\,\underline{u_{\mu_1}(s_1)}\,M_{\mu_2}(s_2)u_{\mu_2}(s_2)\cdots M_{\mu_N}(s_N)u_{\mu_N}(s_N)\right\}$$
$$\neq Tr\left\{M_{\mu_1}(s_1)M_{\mu_2}(s_2)\cdots M_{\mu_N}(s_N)\cdot u_{\mu_1}(s_1)u_{\mu_2}(s_2)\cdots u_{\mu_N}(s_N)\right\}$$

Here, Abelian and off-diagonal variables are not commutable. Then, a simple factorization of Wilson loop and a simple summation on the potential cannot be expected.

$$W[U_\mu] \neq W[M_\mu]\cdot W[u_\mu] \qquad V(r) \neq V_{Abel}(r) + V_{off}(r)$$

Summary:

We have studied Abelian projection of quark confinement in SU(3) lattice QCD, in terms of the dual superconductor picture.

In the maximal Abelian (MA) gauge, we have performed the Cartan decomposition of the nonabelian gauge field in SU(3) quenched lattice QCD with β=6.4 (i.e., a =0.058fm) and 32^4.

We have investigated the quark-antiquark potential V(r), its Abelian part $V_{Abel}(r)$ and its off-diagonal part $V_{off}(r)$. As a remarkable fact, we find *almost perfect Abelian dominance* for quark confinement (the string tension) on the fine lattice with an enough large volume.

Also, we have found a nontrivial summation relation of
$V(r) = V_{Abel}(r) + V_{off}(r)$.

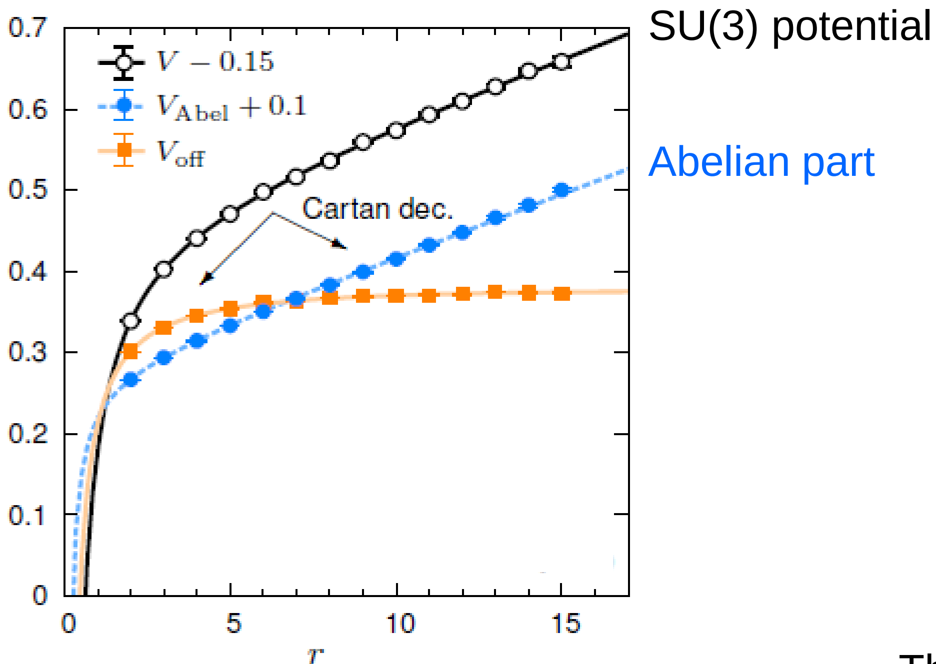
Reference:

[1] N.Sakumichi and H.S., arXiv:1406.2215 [hep-lat],
"Perfect Abelian Dominance of Quark Confinement in SU(3) QCD on a Fine Lattice".

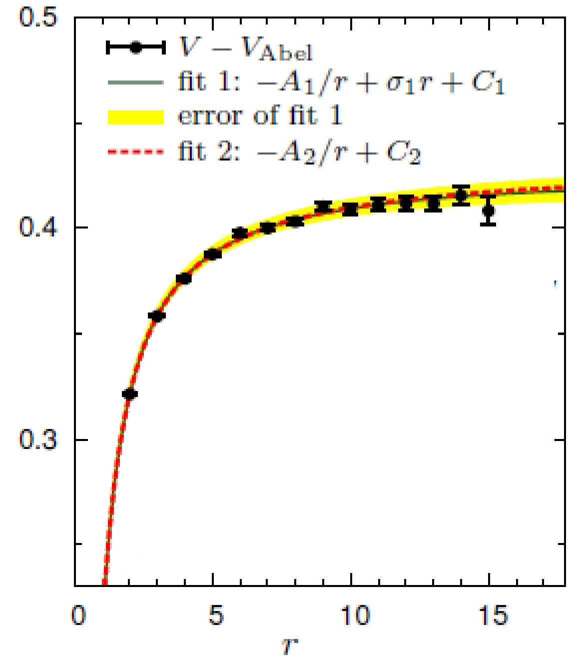
Perfect Abelian Dominance for Quark Confinement in SU(3) Lattice QCD

N. Sakumichi and H. Suganuma

Abstract: We investigate the maximally Abelian (MA) projection of the inter-quark potential in SU(3) quenched lattice QCD with β=6.4 (i.e., a =0.058fm) and 32^4, and find *almost perfect Abelian dominance* of quark confinement (the string tension).



The SU(3) potential and the Abelian part obtained from lattice QCD at β=6.4 and 32^4. They have almost the same slope at large distance.



The difference between the SU(3) potential and the Abelian part. Its form is almost the pure Coulomb form, which suggests perfect Abelian dominance of confinement.

Ref. N.Sakumichi and H.Suganuma, arXiv:1406.2215 [hep-lat], "Perfect Abelian Dominance of Quark Confinement in SU(3) QCD on a Fine Lattice".

Thank you!