Lessons from SUSY: ”Instead-of-Confinement” Mechanism

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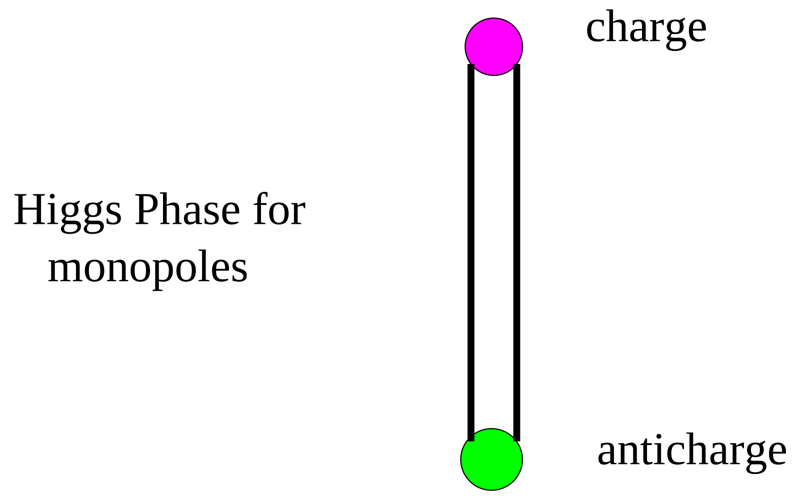
1 Introduction

Seiberg and Witten 1994 : Abelian confinement in monopole vacuum of $\mathcal{N} = 2$ QCD

Cascade gauge symmetry breaking:

- SU(N)$\to$ U(1)$^{N-1}$ VEV's of adjoint scalars
- U(1)$^{N-1}$ $\to$ 0 (or discrete subgroup) VEV's of monopoles

At the last stage Abelian Abrikosov-Nielsen-Olesen flux tubes are formed.



Deformation: mass term μ for adjoint matter

Monopole VEVs $\sim \sqrt{\mu\Lambda_{\mathcal{N}=2}}$

Seiberg-Witten scenario — Abelian confinement

In both QCD or $\mathcal{N}=1$ supersymmetric QCD there are

no adjoint fields $\rightarrow$ no Abelianization

Idea:

Increase $\mu \rightarrow \mu$-deformed theory flows to $\mathcal{N}=1$ QCD

Problem:

Weak coupling condition in the infrared-free low energy theory

$$\sqrt{\mu\Lambda_{\mathcal{N}=2}} \ll \Lambda_{\mathcal{N}=2}$$

Breaks down at large μ

Our setup:

$\mathcal{N}=2$ QCD with $U(N)$ gauge group and $N_f > N$ fundamental flavors (quarks) deformed by mass term for adjoint matter μ.

$N+1 < N_f < 3/2\, N$

$\mathcal{N}=2$ QCD: r-vacua where r quarks condense, $r \leq N$

We consider all r vacua

Weak coupling dual description at large μ only in

- $r = N$ vacuum
- zero vacua at $r < \tilde{N}$
  $\tilde{N} = N_f - N$

Reason: small or zero gaugino condensate

$r = N$ Quark vacuum

Scalar quarks condense with VEV's $\sim \sqrt{\xi}$, $\xi \sim \mu m$.

Large $\xi \to$ theory is at weak coupling

What happens if we reduce ξ and go to strong coupling?

Two steps:

- Reduce ξ at small μ (Near $\mathcal{N} = 2$ limit)
- Increase μ.

2 $r = N$ Vacuum at large ξ

$\mathcal{N} = 2$ QCD with gauge group $U(N) = SU(N) \times U(1)$ and N_f flavors of fundamental matter – quarks

The field content:
U(1) gauge field A_μ
SU(N) gauge field A^a_μ, $a = 1, ..., N^2 - 1$
complex scalar fields a, and a^a
+ fermions

Complex scalar fields q^{kA} and $\tilde{q}_{Ak}$ (squarks) + fermions
$k = 1, ..., N$ is the color index, A is the flavor index, $A = 1, ..., N_f$

Mass term for the adjoint chiral field

$$\mathcal{W}_{\rm br} = \mu \, {\rm Tr} \, \Phi^2,$$

where

$$\Phi = \frac{1}{2} \mathcal{A} + T^a \mathcal{A}^a.$$

F-terms in the potential

$$\left| \tilde{q}_A q^A + \sqrt{2}\ \frac{\partial \mathcal{W}_{\rm br}}{\partial \Phi} \right|^2 , \qquad \left| (\sqrt{2}\Phi + m_A) q^A \right|^2$$

Adjoint fields:

$$\langle {\rm diag} \Phi \rangle \approx -\frac{1}{\sqrt{2}} \left[m_1, ..., m_N \right],$$

(s)Quark VEV's

$$\langle q^{kA} \rangle \ = \ \langle \bar{\tilde{q}}^{kA} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\xi_1} & \ldots & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \sqrt{\xi_N} & 0 & \ldots & 0 \end{pmatrix},$$

$$k \ = \ 1, ..., N \, , \qquad A = 1, ..., N_f \, ,$$

where quasiclassically

$$\xi_P \approx 2\ \mu m_P , \qquad P = 1, ..., N,$$

In the equal mass limit $\mathrm{U}(N)_{\mathrm{gauge}} \times \mathrm{SU}(N_f)_{\mathrm{flavor}}$
in $r = N$ vacuum is broken down to

$$\mathrm{SU}(N)_{C+F} \times \mathrm{SU}(\tilde{N})_F \times \mathrm{U}(1)\,,$$

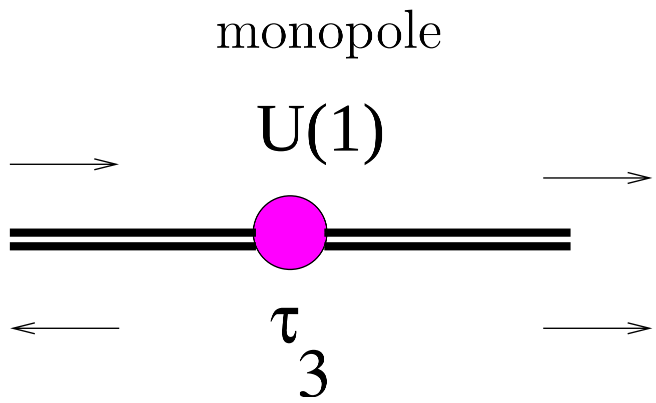
where $\tilde{N} = N_f - N$.

Quarks and gauge fields fill following representations of the global group:

$$(1,1) \quad (N^2-1,1) \quad (\bar{N},\tilde{N}) \quad (N,\bar{\tilde{N}})$$

Non-Abelian strings confine monopoles

Example in $U(2)$



String tensions

$$T_P = 2\pi|\xi_P|, \qquad P = 1, ..., N$$

3 r-Duality at small ξ

Small ξ

$$|\sqrt{\xi_P}| \ll \Lambda_{\mathcal{N}=2}\,, \qquad |m_A - m_B| \ll \Lambda_{\mathcal{N}=2}$$

Use Seiberg-Witten curve on the Coulomb branch at $\mu = 0$

- r-dual theory with gauge group

$$U(\nu) \times U(1)^{N-\nu}, \qquad \nu = \begin{cases} r, & r \leq \frac{N_f}{2} \\ N_f - r, & r > \frac{N_f}{2}, \end{cases}$$

  and N_f quark-like dyons
  (with *weight*-like electric charges)

- non-Abelian strings which

  still confine **monopoles**

  (with *root*-like electric charges)

For $r = N$ vacuum $\nu = \tilde{N} = N_f - N$

Dual gauge group

$$U(\tilde{N}) \times U(1)^{N-\tilde{N}}$$

The non-Abelian gauge factor $U(\tilde{N})$ is not broken by adjoint VEV's in the equal mass limit because this theory is infrared-free and stays at weak coupling.

Argyres Plesser Seiberg:
$SU(\tilde{N}) \times U(1)^{(N-\tilde{N})}$ was identified at the root of baryonic Higgs branch in SU(N) theory with massless quarks and $\mu = 0$.

Vacuum

Dyons

$$\langle D^{lA} \rangle = \langle \bar{\tilde{D}}^{lA} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \dots & 0 & \sqrt{\xi_1} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & \sqrt{\xi_{\tilde{N}}} \end{pmatrix},$$

$$\langle D^J \rangle = \langle \bar{\tilde{D}}^J \rangle = \sqrt{\frac{\xi_J}{2}}, \qquad J = \tilde{N}+1, ..., N\,.$$

"Vacuum leap"

$$(1, ..., N)_{\sqrt{\xi} \gg \Lambda_{\mathcal{N}=2}} \to (N+1, ..., N_f, (\tilde{N}+1), ..., N)_{\sqrt{\xi} \ll \Lambda_{\mathcal{N}=2}}\,.$$

Strong coupling

$$\xi_P = -2\sqrt{2}\,\mu\, e_P, \qquad P = 1, ..., N,$$

where e_P are the double roots of the Seiberg–Witten curve,

$$y^2 = \prod_{P=1}^{N} (x - \phi_P)^2 - 4\left(\frac{\Lambda}{\sqrt{2}}\right)^{N-\tilde{N}} \prod_{A=1}^{N_f}\left(x + \frac{m_A}{\sqrt{2}}\right) = \prod_{P=1}^{N}(x - e_P)^2$$

At small masses the double roots of the Seiberg–Witten curve are

$$\sqrt{2}e_I = -m_{I+N}, \qquad \sqrt{2}e_J = \Lambda_{\mathcal{N}=2}\,\exp\left(\frac{2\pi i}{N-\tilde{N}}J\right)$$

where

$$I = 1, ..., \tilde{N} \;\text{ and }\; J = \tilde{N}+1, ..., N\,.$$

The $\tilde{N}$ first roots are determined by the masses of the last $\tilde{N}$ quarks — a reflection of the fact that the non-Abelian sector of the dual theory is infrared-free and is at weak coupling in the domain.

4 "Instead-of-confinement" mechanism

In the equal mass limit the global group is broken to

$$\mathrm{SU}(N)_F \times \mathrm{SU}(\tilde{N})_{C+F} \times \mathrm{U}(1)$$

Now dyons and dual gauge fields fill following representations of the global group:

$$\text{small } \xi : \qquad (1,1) \quad (1, \tilde{N}^2 - 1) \quad (\bar{N}, \tilde{N}) \quad (N, \bar{\tilde{N}})$$

Recall that quarks and gauge bosons of the original theory are in

$$\text{large } \xi : \qquad (1,1) \quad (N^2 - 1, 1) \quad (\bar{N}, \tilde{N}) \quad (N, \bar{\tilde{N}})$$

$(N^2 - 1)$ of $\mathrm{SU}(N)$ and $(\tilde{N}^2 - 1)$ of $\mathrm{SU}(\tilde{N})$

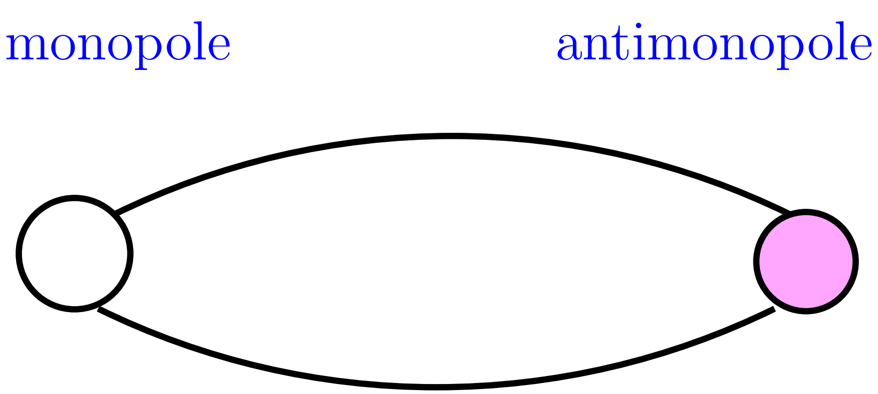
are different states

CROSSOVER

What is the physical nature of $(N^2 - 1)$ adjoints at small ξ?

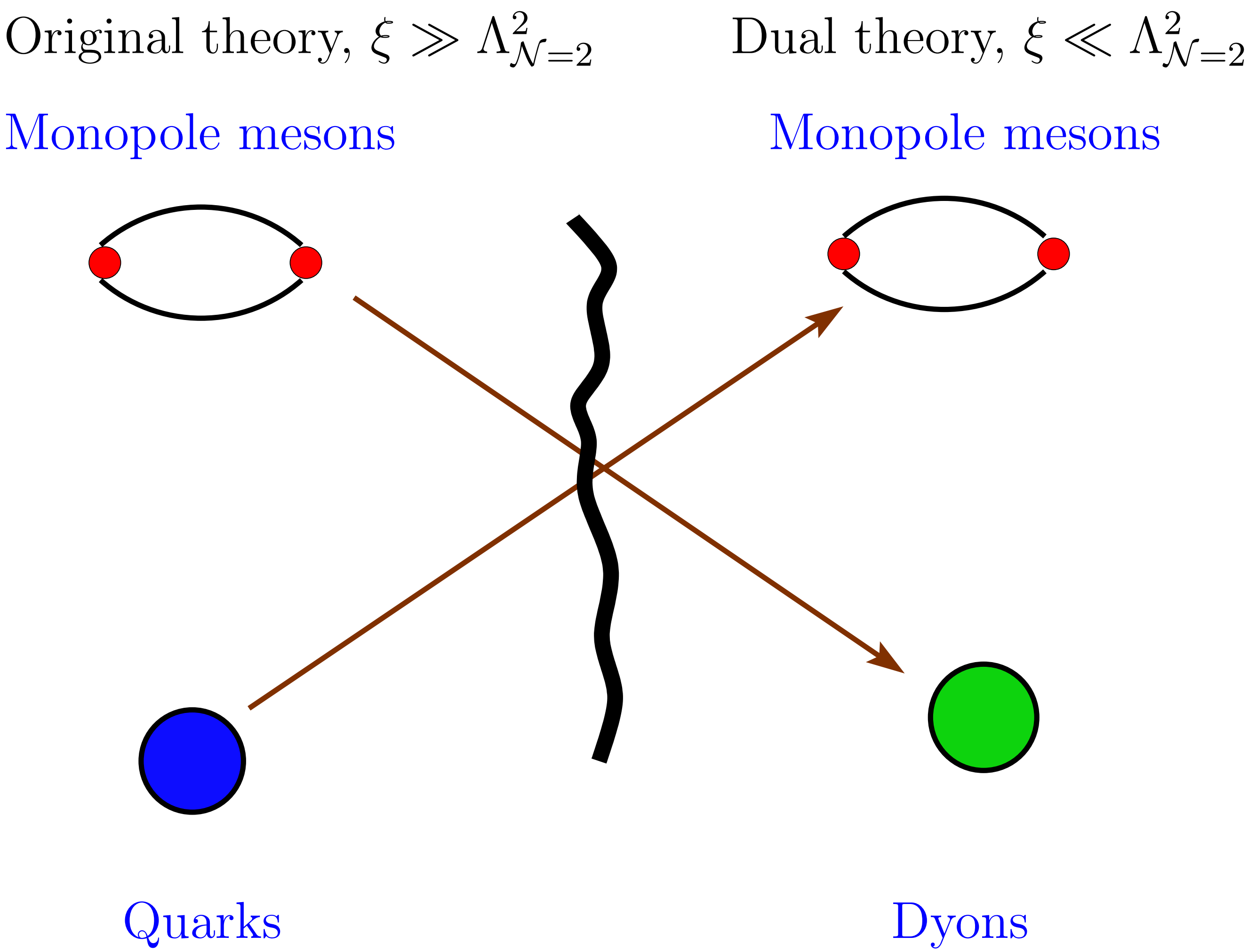
- Higgs-screened quarks and gauge bosons decay into monopole-antimonopole pairs at CMS.

At $\xi \neq 0$ monopoles are confined and cannot move apart



In the region of small ξ $(N^2 - 1)$ of SU(N) are stringy mesons formed by pairs of monopoles and antimonopoles connected by two strings

Crossover



Screened quarks evolve into monopole-antimonopole mesons

These monopole-antimonopole mesons looks like mesons in QCD

- Correct flavor quantum numbers (adjoint + singlet)
- Lie on Regge tragectories

5 r-Duality at large μ

$$\xi_P \sim \left(\xi^{\rm small}, ..., \xi^{\rm small},\ \xi^{\rm large}, ..., \xi^{\rm large}\right)$$

$$\xi^{\rm small} \sim \mu m, \qquad \xi^{\rm large} \sim \mu \Lambda_{\mathcal{N}=2}$$

Take μ large and m_A small.

$$\mu \gg m_A,$$

and

$$|\xi^{\rm small}| \ll \tilde{\Lambda}_{\mathcal{N}=1}$$

where

$$\tilde{\Lambda}_{\mathcal{N}=1}^{N-2\tilde{N}} = \frac{\Lambda_{\mathcal{N}=2}^{N-\tilde{N}}}{\mu^{\tilde{N}}}\,.$$

Infrared-free dual theory is weakly coupled

$\mathrm{U}(1)^{N-\tilde{N}}$ factors of the dual gauge group $\mathrm{U}(\tilde{N}) \times U(1)^{N-\tilde{N}}$ decouple together with Abelian dyons D_J and adjoint matter.

We are left at large μ with

$$U(\tilde{N})$$

gauge group and non-Abelian dyons D^{lA}, $l = 1, ..., \tilde{N}$, $A = 1, ..., N_f$

Superpotential

$$\mathcal{W} = -\frac{1}{2\mu}\,(\tilde{D}_A D^B)(\tilde{D}_B D^A) + m_A\,(\tilde{D}_A D^A)$$

Monopole confinement and "instead-of-confinement" phase for quarks/gauge bosons survive.

Seiberg's duality and r-duality match for $r = N$ vacuum

Seiberg's "dual quarks" are not monopoles as naive duality suggests. Instead, they are quark-like dyons appearing in the r-dual theory below crossover. Their condensation leads to confinement of monopoles and "instead-of-confinement" phase for the quarks and gauge bosons of the original theory.

6 Vacua with $r < N_f/2$

r Vacuum at large m_A

First r (s)quarks condense, $r \leq N$

F-terms in the potential

$$\left| \tilde{q}_A q^A + \sqrt{2}\, \frac{\partial \mathcal{W}_{\rm br}}{\partial \Phi} \right|^2 , \qquad \left| (\sqrt{2}\Phi + m_A) q^A \right|^2$$

Adjoint fields:

$$\langle {\rm diag}\Phi \rangle \approx -\frac{1}{\sqrt{2}} \left[m_1, ..., m_r, 0, ..., 0 \right] ,$$

For $r < N$ classically unbroken gauge group

$$U(N-r) \qquad \rightarrow \qquad U(1)^{N-r} \qquad \rightarrow \qquad U(1)$$

adjoints $\qquad$ $(N-r-1)$ monopoles

Number of isolated vacua with $r < N$

$$\mathcal{N}_{r<N} = \sum_{r=0}^{N-1} (N-r)\, C_{N_f}^{r} = \sum_{r=0}^{N-1} (N-r)\, \frac{N_f!}{r!(N_f-r)!}$$

Low energy theory at small $(m_A - m_B)$

$$U(r) \times U(1)^{(N-r)} \to U(1)^{\mathrm{unbr}}$$

r quarks + $(N-r-1)$ monopoles.

We consider $r < \frac{N_f}{2}$

Then $U(r) \times U(1)^{(N-r)}$ is infrared-free and weakly coupled if

$$\sqrt{\xi_P} \ll \Lambda_{\mathcal{N}=2}$$

Universal formula for VEVs of quarks and monopoles:

$$\xi_P = -2\sqrt{2}\,\mu\,\sqrt{e_P^2 - \frac{2S}{\mu}}, \qquad S = \frac{1}{32\pi^2}\langle \mathrm{Tr}\, W_\alpha W^\alpha \rangle, \qquad P = 1, ..., (N-1)$$

Quarks:

$$\langle q^{kA} \rangle = \langle \bar{\tilde{q}}^{kA} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\xi_1} & \ldots & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \sqrt{\xi_r} & 0 & \ldots & 0 \end{pmatrix},$$

$$k = 1, ..., r\,, \qquad A = 1, ..., N_f\,,$$

Monopoles:

$$\langle M_{P(P+1)} \rangle = \langle \bar{\tilde{M}}_{P(P+1)} \rangle = \sqrt{\frac{\xi_P}{2}}, \qquad P = (r+1), ..., N$$

7 μ-Duality in zero vacua

Gaugino condensate $\to 0$ in the limit of small m

$$S \approx \mu \, \frac{m^{\frac{N_f - 2r}{\tilde{N} - r}}}{\Lambda_{\mathcal{N}=2}^{\frac{N - \tilde{N}}{\tilde{N} - r}}} \, e^{\frac{2\pi k}{\tilde{N} - r} \, i} \ll \mu m^2, \qquad k = 1, ..., (\tilde{N} - r)\,,$$

VEVs

$$\xi_P \approx -2\mu \left(m_1, ..., m_r, \, 0, ..., 0, \, \Lambda_{\mathcal{N}=2}, ..., \Lambda_{\mathcal{N}=2} e^{\frac{2\pi i}{N - \tilde{N}} (N - \tilde{N} - 1)} \right)$$

U($\tilde{N}$) gauge group with N_f flavors of quark-like dyons

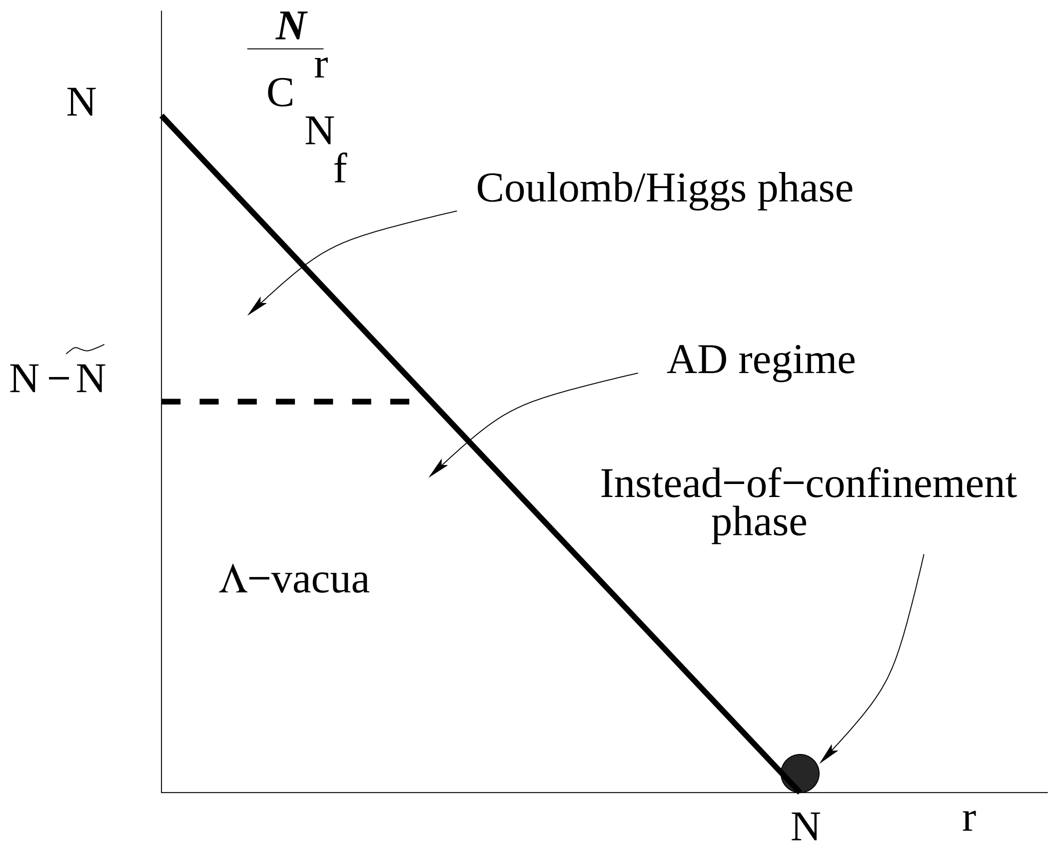
r dyons condense. Higgs/Coulomb phase

Quarks have color charges identical to quark-like dyons but differnt chiral charges.

$$q + \tilde{q} \to \bar{D} + \bar{\tilde{D}} + \lambda + \lambda$$

8 Phases of $\mathcal{N} = 1$ QCD

$$N + 1 < N_f < \frac{3}{2} N, \qquad \mu \gg \Lambda_{\mathcal{N}=1}, \qquad \sqrt{\xi} \ll \tilde{\Lambda}_{\mathcal{N}=1}$$



9 Conclusions

- There is no quark confinement phase in $\mathcal{N} = 1$ SQCD in the domain of small ξ.

  Istead of Seiberg-Witten scenario of quark confinement based on condensation of monopoles we have different scenarios:

- In zero r-vacua we have Higgs/Coulomb phase.

- In $r = N$ vacuum we have

  ”Instead-of-confinement” phase

  Higgs-screened quarks and gauge bosons evolve into monopole-antimonopole stringy mesons.

- The phase most close to what we observe in the real-world QCD is the “instead-of-confinement” phase present in the $r = N$ vacuum.

10 Connection to Seiberg's duality

Seiberg's duality is formulated for $r = 0$ (monopole) vacua. All other $r \neq 0$ vacua are runaway vacua at $\mu = \infty$

Original theory: integrate adjoint fields at large μ

$$-\frac{1}{2\mu}\,(\tilde{q}_A q^B)(\tilde{q}_B q^A) + m_A\,(\tilde{q}_A q^A)$$

Carlino, Konishi, Murayama, 2000

Generalized Seiberg's dual: $\mathrm{U}(\tilde{N})$ gauge theory with superpotential

$$\mathcal{W}_S = -\frac{\kappa^2}{2\mu}\,\mathrm{Tr}\,(M^2) + \kappa\, m_A\, M_A^A + \tilde{h}_{Al} h^{lB}\, M_B^A,$$

where M_A^B is the Seiberg neutral mesonic M field defined as

$$(\tilde{q}_A q^B) = \kappa\, M_A^B$$

Integrating out the M fields we get

$$\mathcal{W}_S^{\rm LE} = \frac{\mu}{2\kappa^2}\,(\tilde{h}_A h^B)(\tilde{h}_B h^A) + \frac{\mu}{\kappa}\,m_A\,(\tilde{h}_A h^A)\,.$$

The change of variables

$$D^{lA} = \sqrt{-\frac{\mu}{\kappa}}\,h^{lA}, \qquad l = 1, ..., \tilde{N}, \qquad A = 1, ..., N_f$$

brings this superpotential to the form

$$\mathcal{W}_S^{\rm LE} = \frac{1}{2\mu}\,(\tilde{D}_A D^B)(\tilde{D}_B D^A) - m_A\,(\tilde{D}_A D^A)\,.$$

This superpotential coincides with the superpotential of our r-dual theory

11 r-Duality at large μ

Now

$$\xi^{\rm small} \sim \mu m \ll \tilde{\Lambda}_{\mathcal{N}=1}, \qquad \mu \gg \Lambda_{\mathcal{N}=1}$$

where

$$\Lambda_{\mathcal{N}=1}^{2N-\tilde{N}} = \mu^{N}\, \Lambda_{\mathcal{N}=2}^{N-\tilde{N}}$$

’t Hooft anomaly matching:

anomaly$|_{UV}$ = anomaly$|_{IR}$

UV energy should be $E_{UV} \gg \Lambda_{\mathcal{N}=1}$, moreover, $\mu \gg E_{UV}$

UV global group:

$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_R$

At $m \ll E_{IR} \ll \xi^{\rm small}$

IR global group:

$\mathrm{SU}(N) \times \mathrm{SU}(\tilde{N}) \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{R'}$

The list of anomalies to be checked is

$$\begin{aligned}
&\mathrm{U}(1)_{R'} \times \mathrm{SU}(N)^2: \quad && -\frac{\delta^{mn}}{2} N|_{UV} = -\frac{\delta^{mn}}{2} N|_{IR}, \\
&\mathrm{U}(1)_{R'} \times \mathrm{SU}(\tilde{N})^2: \quad && 0|_{UV} = \frac{\delta^{ps}}{2} (-\tilde{N} + \tilde{N})|_{IR}, \\
&\mathrm{U}(1)_{R'} \times \mathrm{U}(1)_V^2: \quad && 0|_{UV} = 0|_{IR}, \\
&\mathrm{U}(1)_{R'}: \quad && -2N^2 + N^2|_{UV} = -N^2 = -\tilde{N}^2 - N^2 + \tilde{N}^2|_{IR}, \\
&\mathrm{U}(1)^3_{R'}: \quad && -2N^2 + N^2|_{UV} = -N^2 = -\tilde{N}^2 - N^2 + \tilde{N}^2|_{IR},
\end{aligned} \tag{1}$$

We need light M_A^B meson.

Its mass

$$m_M \sim m$$

Interpretation of Seiberg's M-meson

Our r-dual gauge group

$\mathrm{U}(\tilde{N}) \times U(1)^{N-\tilde{N}} \to \mathrm{U}(\tilde{N})$

Scale of $\mathrm{U}(\tilde{N})$

$$\xi^{\text{small}} \sim \mu m \ll \tilde{\Lambda}_{\mathcal{N}=1}$$

Weak coupling

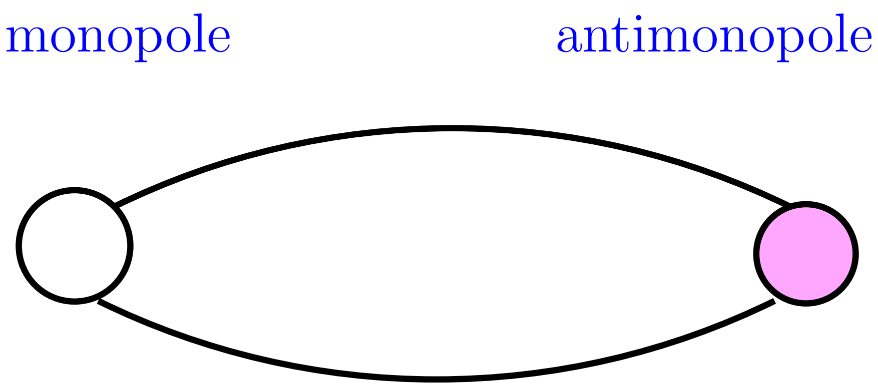
Scale of Abelian $U(1)^{N-\tilde{N}}$ is

$$\xi^{\text{large}} \sim \mu \Lambda_{\mathcal{N}=2}$$

This sector is at strong coupling

Conjecture:

Seiberg's M_A^B meson is one of monopole-antimonopole stringy mesons from Abelian $U(1)^{N-\tilde{N}}$ sector



Superpotential

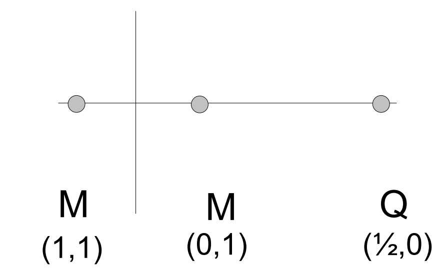
$$\mathcal{W} = \frac{\kappa^2}{2\mu}\,\mathrm{Tr}\,(M^2) - \kappa\, m_A\, M_A^A + \frac{\kappa}{\mu}\,\tilde{D}_{Al} D^{lB}\, M_B^A$$

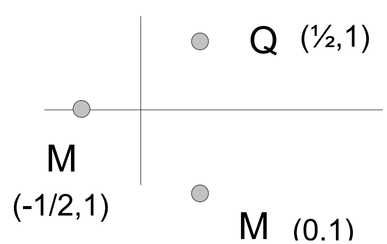
Simplest example possible: SU(2) gauge theory with $N_f = 1$

Three vacua:

monopole $(n_e, n_m) = (0, 1)$, monopole (1,1), and quark $(\frac{1}{2}, 0)$

$$m \to \infty$$





$$m = 0$$

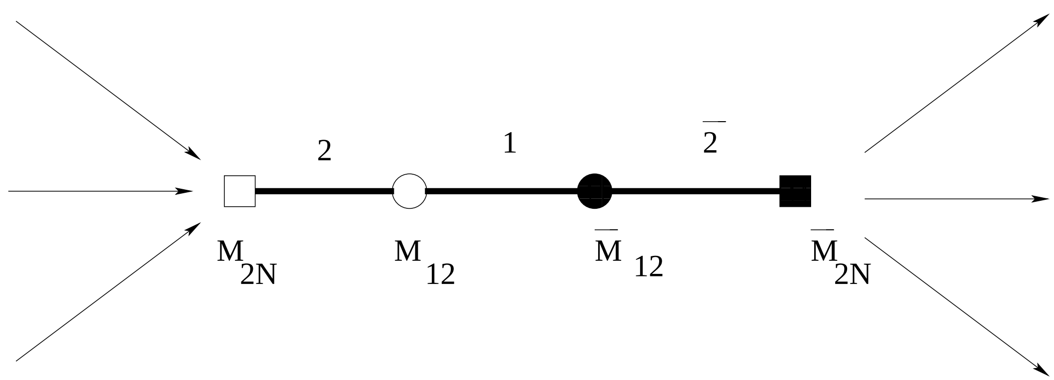
Quarks condense in the quark vacuum at any m

In the $r < N$ vacua there is a novel feature:

$$\xi_N = 0$$

One (N-th) Z_N string is absent and the associated flux of the unbroken $\mathrm{U}(1)^{\mathrm{unbr}}$ gauge factor is not squeezed into a flux tube. It is spread out in space via the Coulomb law.

Strings become metastable. They can be broken by a monopole-antimonopole pair creation of monopoles which are junctions of one of the first r Z_N-strings with the would-be N-th string (which is in fact absent).



12 Appendix B: Generalized Seiberg's duality and exact chiral rings

Cachazo-Seiberg-Witten *2003*:

$$(\tilde{q}q)_A = \frac{\mu}{2}\left(m_A + \sqrt{m_A^2 - \frac{4S}{\mu}}\right), \qquad A = 1, ..., r$$
$$(\tilde{q}q)_A = \frac{\mu}{2}\left(m_A - \sqrt{m_A^2 - \frac{4S}{\mu}}\right), \quad A = (r+1), ..., N_f$$

Here gaugino condensate S is determined by matrix model superpotential, namely:

$$S^N = \mu^N \, \Lambda_{\mathcal{N}=2}^{N-\tilde{N}} \left(\frac{m}{2} - \frac{1}{2}\sqrt{m^2 - \frac{4S}{\mu}}\right)^r \left(\frac{m}{2} + \frac{1}{2}\sqrt{m^2 - \frac{4S}{\mu}}\right)^{N_f - r},$$

where we assume the equal-mass limit for simplicity.

This imply the following equation for quark condensate:

$$\frac{1}{\mu}\,(\tilde{q}q)_A = m - \frac{1}{\mu^{\frac{N}{\tilde{N}}}\,\Lambda_{\mathcal{N}=2}^{\frac{N-\tilde{N}}{\tilde{N}}}}\,\frac{(\det\tilde{q}q)^{\frac{1}{\tilde{N}}}}{(\tilde{q}q)_A}.$$

Cachazo–Seiberg–Witten exact solution produces the same equations for the quark condensates as the continuation of the ADS superpotential to $N_f > N$.