RELATIVISTIC DESCRIPTION OF THE WEAK DECAYS OF B_s MESONS

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INTRODUCTION

Why is it interesting and important to study weak B_s decays?

- Weak B_s decays are considerably less studied both experimentally and theoretically than weak B decays
- Significant experimental progress in last few years
  - Data from Tevatron
  - Belle data collected at $\Upsilon(10860)$ resonance
  - Precise and comprehensive data are expected from LHCb (several decays are already observed for the first time)
- Additional source for determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements
- Some decay modes are preferable for studying CP violation
- Test of the standard model and "new physics models"

RELATIVISTIC QUARK MODEL

● **Quasipotential equation and $q\bar{q}$ interaction potential**

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks
M - bound state mass $(M = E_1 + E_2)$
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$
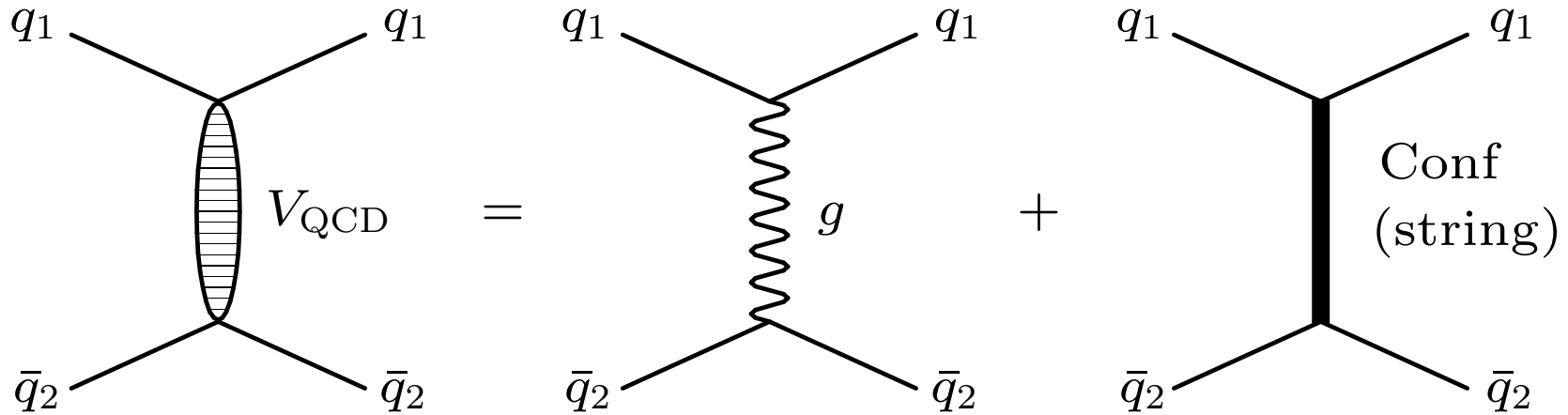
$b(M)$ - on-mass-shell relative momentum in cms:

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center-of-mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- $q\bar{q}$ **quasipotential**

(Constructed with the help of off-mass-shell scattering amplitude projected onto positive-energy states)

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + V_{\text{conf}}^{V}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu} + V_{\text{conf}}^{S}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k} = \mathbf{p} - \mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_{\mu}(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_{\mu}(\mathbf{k}) = \gamma_{\mu} + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^{\nu},$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \left(\begin{array}{c} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p) + m} \end{array} \right) \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\text{conf}} = V_{\text{conf}}^V + V_{\text{conf}}^S$

In nonrelativistic limit

$$\left. \begin{array}{rcl} V_{\text{conf}}^V(r) & = & (1 - \varepsilon)(Ar + B) \\ V_{\text{conf}}^S(r) & = & \varepsilon(Ar + B) \end{array} \right\} \quad \text{Sum}: \quad (Ar + B)$$

ε - mixing parameter

$$V_{\text{NR}}(r) = V_{\text{Coul}}(r) + V_{\text{conf}}(r)$$

$$V_{\text{Coul}}(r) = -\frac{4}{3}\frac{\alpha_s}{r}$$

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ^3P_J states and HQET

$(1 + \kappa) = 0 \implies$ vanishing long-range chromomagnetic interaction !   (flux tube model)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad \mu = \frac{2m_1 m_2}{m_1 + m_2},$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.169$ GeV

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

• Matrix elements of the electroweak current between meson states

Matrix element of weak current $J_\mu^W = \bar{c}\gamma_\mu(1 - \gamma_5)b$ for $B_s \to D_s$ weak transitions:

$$\langle D_s(p_{D_s})|J_\mu^W|B_s(p_{B_s})\rangle = \int \frac{d^3p\,d^3q}{(2\pi)^6}\bar{\Psi}_{D_s\,\mathbf{p}_{D_s}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B_s\,\mathbf{p}_{B_s}}(\mathbf{q})$$
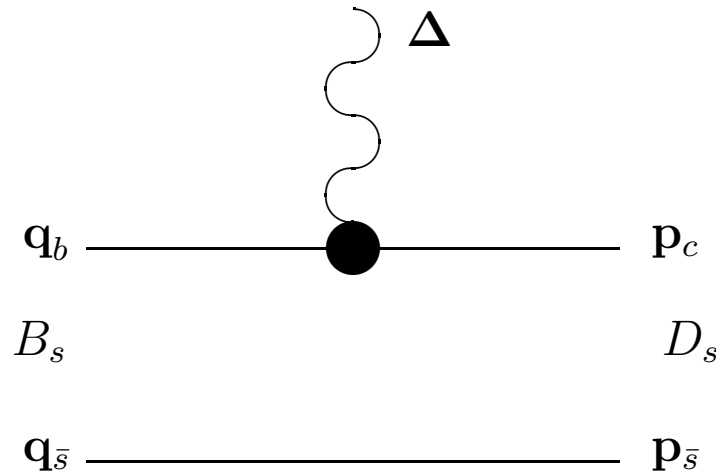

Figure 1: Lowest order vertex function $\Gamma^{(1)}(\mathbf{p},\mathbf{q})$ contributing to the current matrix element.

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q}) = \bar{u}_c(p_c)\gamma_\mu(1 - \gamma^5)u_b(q_b)(2\pi)^3\delta(\mathbf{p}_{\bar{s}} - \mathbf{q}_{\bar{s}})$$
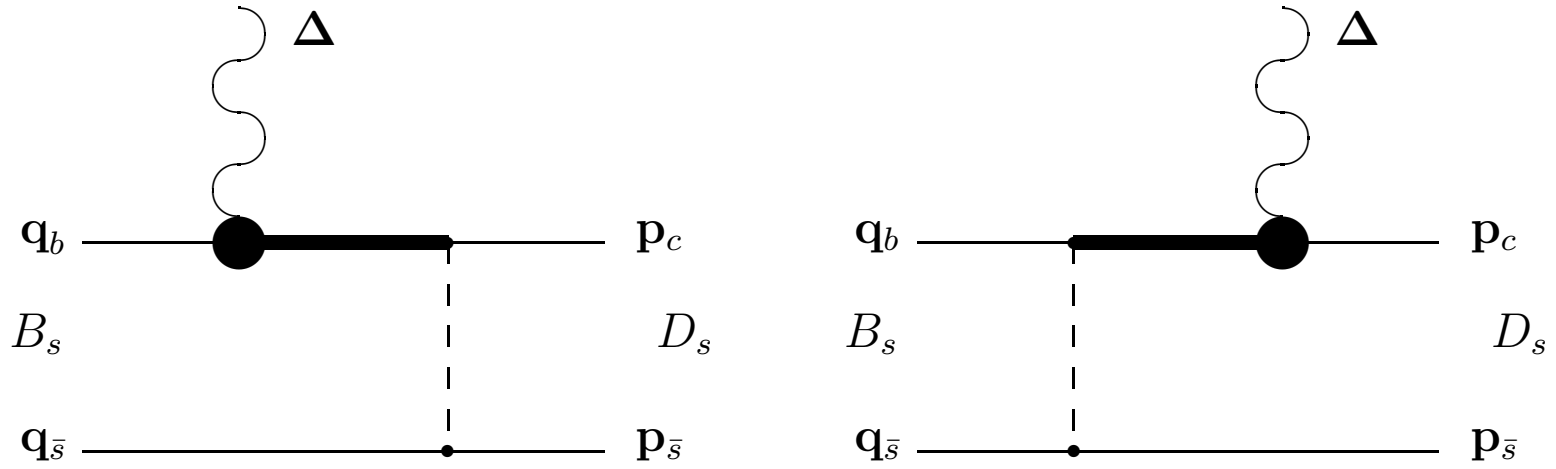
Figure 2: Vertex function $\Gamma^{(2)}(\mathbf{p}, \mathbf{q})$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}$. Bold lines denote the negative-energy part of the quark propagator.

$$\Gamma_\mu^{(2)}(\mathbf{p}, \mathbf{q}) = \bar{u}_c(p_c)\bar{u}_s(p_{\bar{s}})\left\{ \mathcal{V}(\mathbf{p}_{\bar{s}} - \mathbf{q}_{\bar{s}})\frac{\Lambda_c^{(-)}(k')}{\epsilon_c(k') + \epsilon_c(q_b)}\gamma_1^0\gamma_{1\mu}(1 - \gamma_1^5)\right.$$

$$\left. +\gamma_{1\mu}(1 - \gamma_1^5)\frac{\Lambda_b^{(-)}(k)}{\epsilon_b(k) + \epsilon_b(p_c)}\gamma_1^0\mathcal{V}(\mathbf{p}_{\bar{s}} - \mathbf{q}_{\bar{s}})\right\}u_b(q_b)u_s(q_{\bar{s}})$$

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}, \qquad \epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$$

$$\mathbf{k} = \mathbf{p}_c - \boldsymbol{\Delta}; \qquad \mathbf{k}' = \mathbf{q}_b + \boldsymbol{\Delta}; \qquad \boldsymbol{\Delta} = \mathbf{p}_{D_s} - \mathbf{p}_{B_s}$$

The wave function of a final D_s meson at rest is given by

$$\Psi_{D_s}(\mathbf{p}) \equiv \Psi_{D_{sJ}}^{JLS\mathcal{M}}(\mathbf{p}) = \mathcal{Y}^{JLS\mathcal{M}} \psi_{D_{sJ}}(\mathbf{p})$$

J and $\mathcal{M}$ — total meson angular momentum and its projection
L — orbital momentum
$S = 0, 1$ — total spin
$\psi_{D_{sJ}}(\mathbf{p})$ — radial part of the wave function

$$\mathcal{Y}^{JLS\mathcal{M}} = \sum_{\sigma_1\sigma_2} \langle L\,\mathcal{M} - \sigma_1 - \sigma_2,\ S\,\sigma_1 + \sigma_2 | J\,\mathcal{M}\rangle \langle \frac{1}{2}\,\sigma_1,\ \frac{1}{2}\,\sigma_2 | S\,\sigma_1 + \sigma_2 \rangle Y_L^{\mathcal{M}-\sigma_1-\sigma_2} \chi_1(\sigma_1)\chi_2(\sigma_2)$$

The wave function of the moving meson $\Psi_{D_s\boldsymbol{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{D_s0} \equiv \Psi_{D_s}$ by the transformation

$$\Psi_{D_s\boldsymbol{\Delta}}(\mathbf{p}) = D_c^{1/2}(R_{L_{\boldsymbol{\Delta}}}^W) D_s^{1/2}(R_{L_{\boldsymbol{\Delta}}}^W) \Psi_{D_s0}(\mathbf{p})$$

R^W — Wigner rotation
$L_{\boldsymbol{\Delta}}$ — Lorentz boost from the meson rest frame to a moving one
$D^{1/2}(R)$ — rotation matrix in spinor representation

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_{s,c}^{1/2}(R_{L_{\boldsymbol{\Delta}}}^W) = S^{-1}(\mathbf{p}_{\bar{s},c}) S(\boldsymbol{\Delta}) S(\mathbf{p})$$

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p) + m}\right)$$

$S(\mathbf{p})$ — usual Lorentz transformation matrix of Dirac spinor.

WEAK B_s DECAYS

- heavy-to-heavy decays $B_s \to D_s l \nu_l$ $b \to c$ transition   CKM favored V_{cb} $Br \sim 10^{-2}$
- heavy-to-light decays $B_s \to K l \nu_l$ $b \to u$ transition   CKM suppressed V_{ub} $Br \sim 10^{-4}$
- rare decays $B_s \to \varphi(\eta) l l$ $b \to s$ transition   penguin diagrams $V_{tb} V_{ts}$ $Br \sim 10^{-6}$

Broad kinematical range:

the square of momentum transfer to the lepton pair q^2 varies

from 0 to $q^2_{\max} \approx 10$ GeV^2 for decays to D_s

from 0 to $q^2_{\max} \approx 20$ GeV^2 for decays to K and φ mesons

$\Longrightarrow$ the explicit determination of the q^2 dependence of the decay form factors in the whole kinematical range is needed

Large recoil of the final meson requires consistent relativistic treatment (e.g. boost of the meson wave functions from the rest to the moving reference frame)

Presence of heavy quarks in B_s and D_s mesons allows one to use expansions in the inverse powers of heavy quark masses $1/m_{b,c} \Longrightarrow$ significant simplifications, heavy quark symmetry relations can be used

Light u, d, s quarks should be treated relativistically

Large recoils allow one to neglect small relative momentum ($|\mathbf{p}|$) with respect to recoil ($|\boldsymbol{\Delta}|$) in the energies of light quarks in energetic light mesons $\epsilon_q(p + \Delta) \equiv \sqrt{m_q^2 + (\mathbf{p} + \boldsymbol{\Delta})^2} \longrightarrow \epsilon_q(\Delta) \equiv \sqrt{m_q^2 + \boldsymbol{\Delta}^2}$.

Such replacement is made in subleading contribution $\Gamma^{(2)}_\mu(\mathbf{p}, \mathbf{q})$ and permits the performance of one of the integrations using the quasipotential equation. As a result, the weak decay matrix element is expressed through the usual overlap integral of initial and final meson wave functions

Heavy-to-heavy semileptonic $B_s \to D_s l \nu_l$ decays

- **Decays to ground state D_s mesons**

Weak decay matrix elements:

$$\langle D_s(p_{D_s})|\bar{c}\gamma^\mu b|B_s(p_{B_s})\rangle = f_+(q^2)\left[p^\mu_{B_s} + p^\mu_{D_s} - \frac{M^2_{B_s} - M^2_{D_s}}{q^2}q^\mu\right] + f_0(q^2)\frac{M^2_{B_s} - M^2_{D_s}}{q^2}q^\mu$$

$$\langle D_s(p_{D_s})|\bar{c}\gamma^\mu\gamma_5 b|B_s(p_{B_s})\rangle = 0$$

$$\langle D^*_s(p_{D^*_s})|\bar{c}\gamma^\mu b|B(p_{B_s})\rangle = \frac{2iV(q^2)}{M_{B_s} + M_{D^*_s}}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{B_s\rho}p_{D^*_s\sigma}$$

$$\langle D^*_s(p_{D^*_s})|\bar{c}\gamma^\mu\gamma_5 b|B_s(p_{B_s})\rangle = 2M_{D^*_s}A_0(q^2)\frac{\epsilon^* \cdot q}{q^2}q^\mu + (M_{B_s} + M_{D^*_s})A_1(q^2)\left(\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2}q^\mu\right)$$

$$- A_2(q^2)\frac{\epsilon^* \cdot q}{M_{B_s} + M_{D^*_s}}\left[p^\mu_{B_s} + p^\mu_{D^*_s} - \frac{M^2_{B_s} - M^2_{D^*_s}}{q^2}q^\mu\right]$$

At the maximum recoil point ($q^2 = 0$)

$$f_+(0) = f_0(0)$$

$$A_0(0) = \frac{M_{B_s} + M_{D^*_s}}{2M_{D^*_s}}A_1(0) - \frac{M_{B_s} - M_{D^*_s}}{2M_{D^*_s}}A_2(0)$$

Table 1: Comparison of theoretical predictions for the form factors of semileptonic decays $B_s \to D_s^{(*)} e\nu$ at maximum recoil point $q^2 = 0$.

	$f_+(0)$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
our	0.74 ± 0.02	0.95 ± 0.02	0.67 ± 0.01	0.70 ± 0.01	0.75 ± 0.02
Kramer (QM)	0.61	0.64		0.56	0.59
Blasi (SR)	0.7 ± 0.1	0.63 ± 0.05	0.52 ± 0.06	0.62 ± 0.01	0.75 ± 0.07
Chen (QM)	$0.57^{+0.02}_{-0.03}$	$0.70^{+0.05}_{-0.04}$		$0.65^{+0.01}_{-0.01}$	$0.67^{+0.01}_{-0.01}$
Li (LCSR)	$0.86^{+0.17}_{-0.15}$				
Li (QM)		$0.74^{+0.05}_{-0.05}$	$0.63^{+0.04}_{-0.04}$	$0.61^{+0.04}_{-0.04}$	$0.59^{+0.04}_{-0.04}$

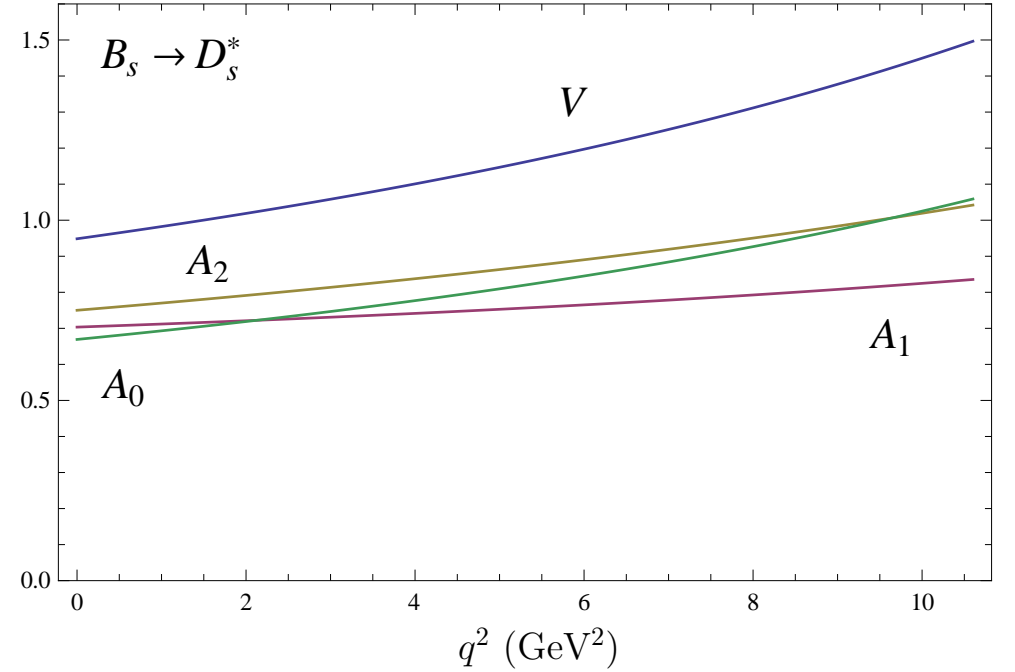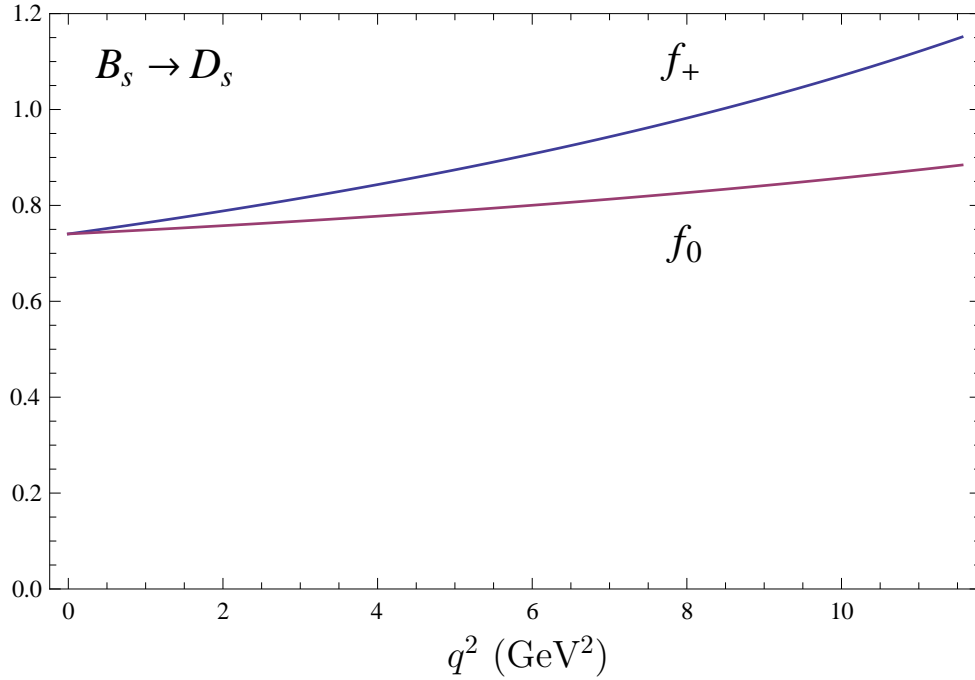Figure 3: Form factors of weak $B_s \to D_s^{(*)}$ transitions.

Table 2: Comparison of theoretical predictions for the branching fractions of semileptonic decays $B_s \to D_s^{(*)} l \nu$ (in %).

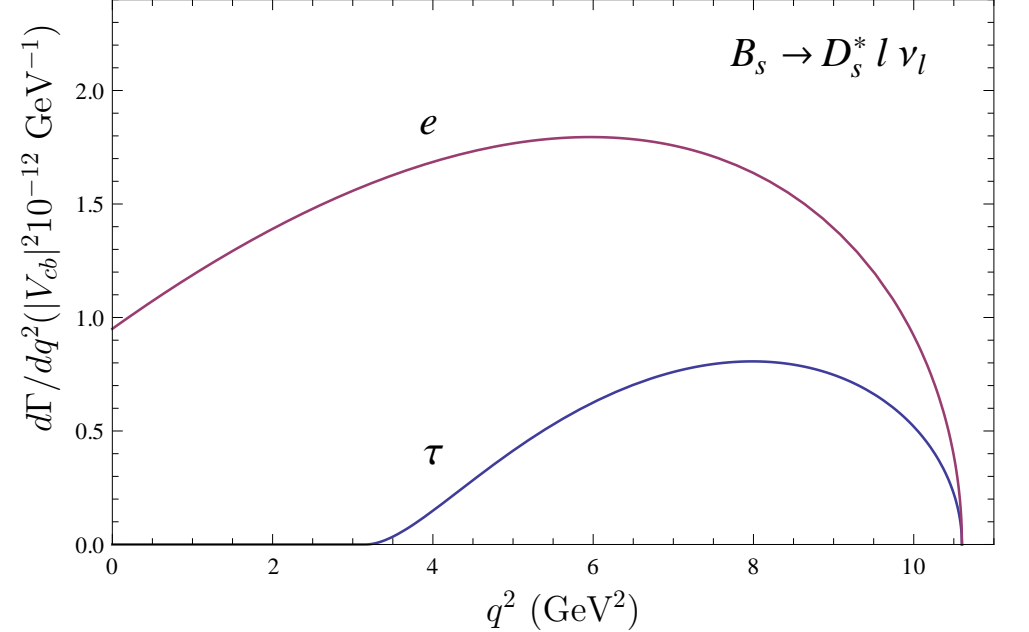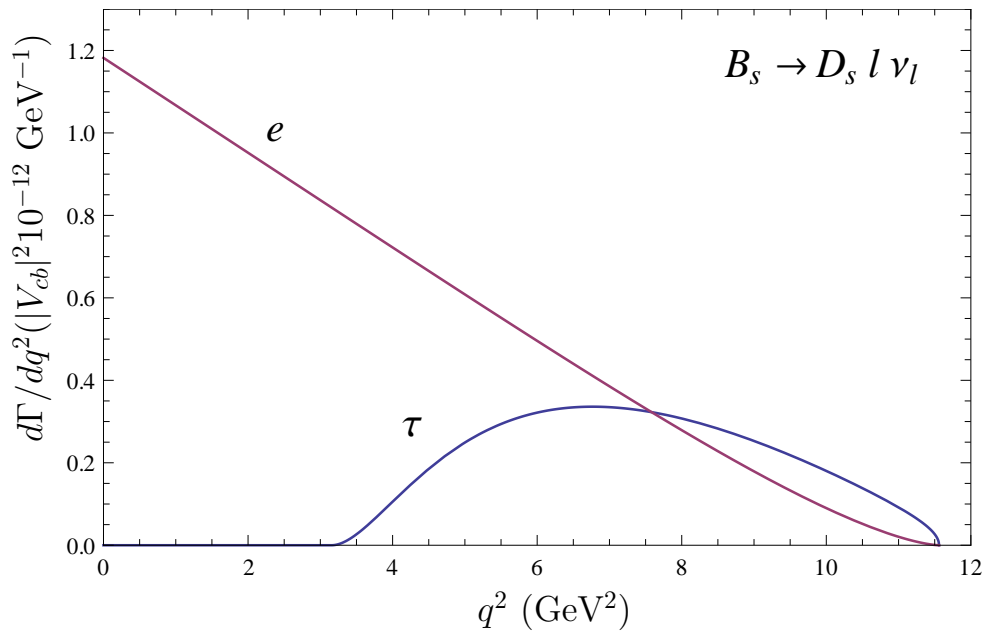| Decay	our	SR Blasi	QM Chen	LCSR Li	QM Li	QM Zhao	SR Azizi
$B_s \to D_s e \nu$	2.1 ± 0.2	1.35 ± 0.21	1.4-1.7	$1.0^{+0.4}_{-0.3}$		2.73-3.00	2.8-3.8
$B_s \to D_s \tau \nu$	0.62 ± 0.05		0.47-0.55	$0.33^{+0.14}_{-0.11}$			
$B_s \to D_s^* e \nu$	5.3 ± 0.5	2.5 ± 0.1	5.1-5.8		5.2 ± 0.6	7.49-7.66	1.89-6.61
$B_s \to D_s^* \tau \nu$	1.3 ± 0.1		1.2-1.3		$1.3^{+0.2}_{-0.1}$		

Figure 4: Predictions for the differential decay rates of the $B_s \to D_s^{(*)} l \nu$ semileptonic decays.

$$|V_{cb}| = (3.9 \pm 0.15) \times 10^{-2}$$

- **Decays to radially excited D_s mesons**

Table 3: Predictions for the branching fractions of semileptonic decays $B_s \to D_s^{(*)}(2S)l\nu$ (in %).

Decay	Br
$B_s \to D_s(2S)e\nu$	0.27 ± 0.03
$B_s \to D_s(2S)\tau\nu$	0.011 ± 0.001
$B_s \to D_s^*(2S)e\nu$	0.38 ± 0.04
$B_s \to D_s^*(2S)\tau\nu$	0.015 ± 0.002

- **Decays to orbitally excited D_s mesons**

Table 4: Comparison of the predictions for the branching fractions of the semileptonic decays $B_s \to D_{sJ}^{(*)}l\nu$ (in %).

Decay	our	$m \to \infty$ with $1/m_Q$		QM	QM	LCSR	SR	HQET+SR
	EFG	EFG	Segovia	Zhao	Li	Aliev	Huang
$B_s \to D_{s0}^* e\nu$	0.36 ± 0.04	0.10	0.37	0.443	0.49-0.571	$0.23^{+0.12}_{-0.10}$	~ 0.1	0.20
$B_s \to D_{s0}^* \tau\nu$	0.019 ± 0.002					$0.057^{+0.028}_{-0.023}$	~ 0.01	
$B_s \to D_{s1}' e\nu$	0.19 ± 0.02	0.13	0.18	0.174-0.570	0.752-0.869		0.49	0.10
$B_s \to D_{s1}' \tau\nu$	0.015 ± 0.002							
$B_s \to D_{s1} e\nu$	0.84 ± 0.09	0.36	1.06	0.477				
$B_s \to D_{s1} \tau\nu$	0.049 ± 0.005							
$B_s \to D_{s2}^* e\nu$	0.67 ± 0.07	0.56	0.75	0.376				
$B_s \to D_{s2}^* \tau\nu$	0.029 ± 0.003							

Experimental data

$$Br(B_s \to D_{s1} X \mu \nu)_{\text{D0}} = (1.03 \pm 0.20 \pm 0.17 \pm 0.14)\% \qquad \text{(D0 Collaboration 2009)}$$

$$Br(B_s \to D_{s2}^* X \mu \nu)/Br(B_s \to X \mu \nu)_{\text{LHCb}} = (3.3 \pm 1.0 \pm 0.4)\% \quad \text{(LHCb Collaboration 2011)}$$

$$Br(B_s \to D_{s1} X \mu \nu)/Br(B_s \to X \mu \nu)_{\text{LHCb}} = (5.4 \pm 1.2 \pm 0.5)\%$$

$$Br(B_s \to D_{s2}^* X \mu \nu)/Br(B_s \to D_{s1} X \mu \nu)_{\text{LHCb}} = 0.61 \pm 0.14 \pm 0.05$$

$$Br(B_s \to X \mu \nu) = (9.5 \pm 2.7)\% \qquad \text{(PDG)}$$

Theoretical prediction

$$Br(B_s \to D_{s1} \mu \nu)_{\text{theor}} = (0.84 \pm 0.09)\%$$

$$Br(B_s \to X \mu \nu)_{\text{theor}} = Br(B_s \to D_s(1S, 2S) \mu \nu) + Br(B_s \to D_s(1P) \mu \nu) = (10.2 \pm 1.0)\%$$

$$Br(B_s \to D_{s2}^* \mu \nu)/Br(B_s \to X \mu \nu)_{\text{theor}} = (6.5 \pm 1.2)\%$$

$$Br(B_s \to D_{s1} \mu \nu)/Br(B_s \to X \mu \nu)_{\text{theor}} = (8.2 \pm 1.6)\%$$

$$Br(B_s \to D_{s2}^* \mu \nu)/Br(B_s \to D_{s1} \mu \nu)_{\text{theor}} = 0.79 \pm 0.14$$

Total semileptonic $B_s \to D_s$ branching ratios

- for decays to ground state $D_s^{(*)}$ mesons

$$Br(B_s \to D_s^{(*)} e\nu) = (7.4 \pm 0.7)\%$$

$$Br(B_s \to D_s^{(*)} \tau\nu) = (1.92 \pm 0.15)\%$$

- for decays to orbitally excited $D_{sJ}^{(*)}$ mesons

$$Br(B_s \to D_{sJ}^{(*)} e\nu) = (2.1 \pm 0.2)\%$$

$$Br(B_s \to D_{sJ}^{(*)} \tau\nu) = (0.11 \pm 0.01)\%$$

- for decays to radially excited $D_s^{(*)}(2S)$ mesons

$$Br(B_s \to D_s^{(*)}(2S) e\nu) = (0.65 \pm 0.06)\%$$

$$Br(B_s \to D_s^{(*)}(2S) \tau\nu) = (0.026 \pm 0.003)\%$$

Branching fractions significantly decrease with excitation $\implies$ Considered decays give the dominant contribution to the total semileptonic branching fraction

$$Br(B_s \to D_s e\nu + \text{anything}) = (10.2 \pm 1.0)\%$$

Experimental value (PDG)

$$Br(B_s \to D_s e\nu + \text{anything})_{\text{Exp.}} = (7.9 \pm 2.4)\%$$

Heavy-to-light semileptonic $B_s \to K l \nu_l$ decays

- **Decays to ground state K mesons**

Tensor weak current matrix elements

$$\langle K(p_K)|\bar{q}\sigma^{\mu\nu}q_\nu b|B_s(p_{B_s})\rangle = \frac{if_T(q^2)}{M_{B_s}+M_K}[q^2(p^\mu_{B_s}+p^\mu_K)-(M^2_{B_s}-M^2_K)q^\mu]$$

$$\langle K^*(p_{K^*})|\bar{q}i\sigma^{\mu\nu}q_\nu b|B_s(p_{B_s})\rangle = 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{K^*\rho}p_{B_s\sigma}$$

$$\langle K^*(p_{K^*})|\bar{q}i\sigma^{\mu\nu}\gamma_5 q_\nu b|B_s(p_{B_s})\rangle = T_2(q^2)[(M^2_{B_s}-M^2_{K^*})\epsilon^{*\mu}-(\epsilon^*\cdot q)(p^\mu_{B_s}+p^\mu_{K^*})]$$

$$+T_3(q^2)(\epsilon^*\cdot q)\left[q^\mu-\frac{q^2}{M^2_{B_s}-M^2_{K^*}}(p^\mu_{B_s}+p^\mu_{K^*})\right]$$

$$T_1(0)=T_2(0)$$

Table 5: Comparison of theoretical predictions for the form factors of weak $B_s \to K^{(*)}$ transitions at maximum recoil point $q^2=0$.

|	$f_+(0)$	$f_T(0)$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$T_1(0)$	$T_3(0)$
our	0.284±0.014	0.236±0.012	0.291±0.015	0.289±0.015	0.287±0.015	0.286±0.015	0.238±0.012	0.122±0.006
LCSR	0.30±0.04		0.311±0.026	0.360±0.034	0.233±0.022	0.181±0.025	0.260±0.024	0.136±0.016
PQCD	0.24±0.05		0.21±0.04	0.25±0.05	0.16±0.04			
QM	0.31	0.31	0.38	0.37	0.29	0.26	0.32	0.23
PQCD			0.20±0.05	$0.24^{+0.07}_{-0.05}$	$0.15^{+0.04}_{-0.03}$	0.11±0.02	0.18±0.05	0.16±0.03
LCQM	0.290	0.317	0.323	0.279	0.232	0.210	0.271	0.165
LCSR	0.296±0.018	0.288±0.018	0.285±0.013	0.222±0.011	0.227±0.011	0.183± 0.010	0.251±0.012	0.169±0.008
QM	$0.260^{+0.055}_{-0.032}$		$0.227^{+0.064}_{-0.037}$	$0.280^{+0.090}_{-0.045}$	$0.178^{+0.047}_{-0.027}$			
PQCD	$0.26^{+0.05}_{-0.04}$	0.28±0.05						
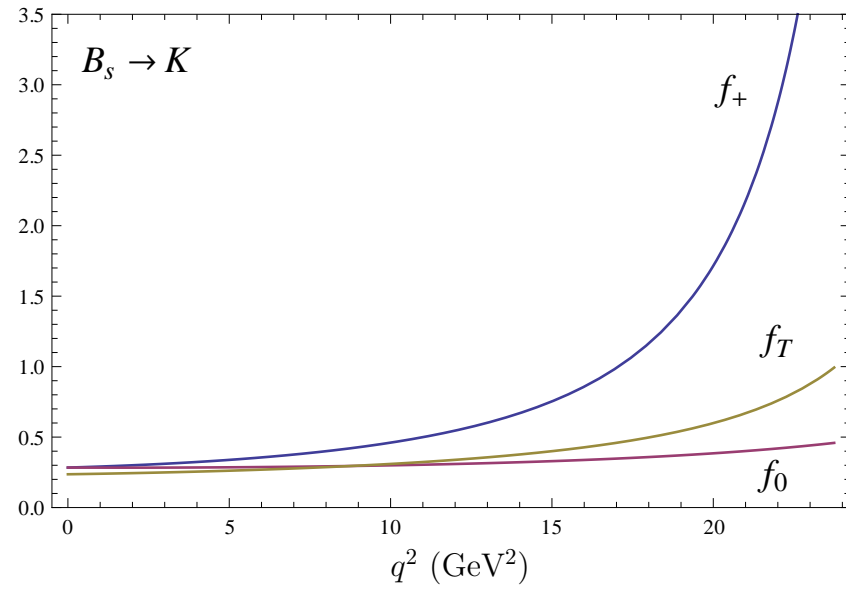
Figure 5: Form factors of the weak $B_s \to K$ transition.
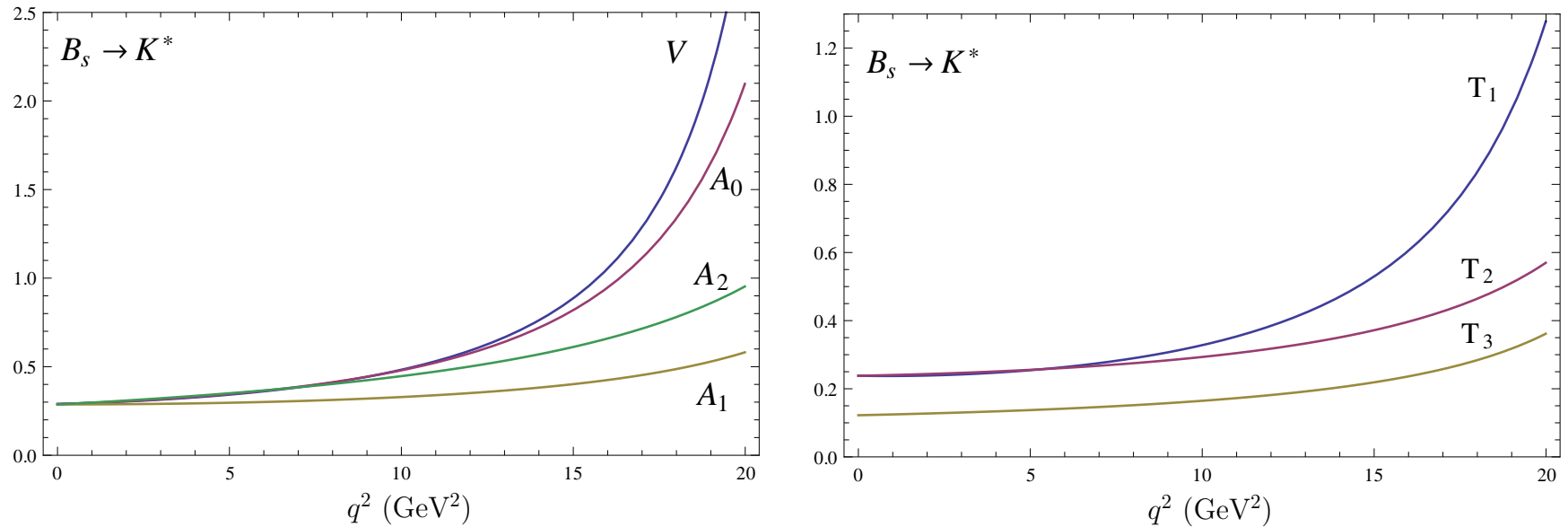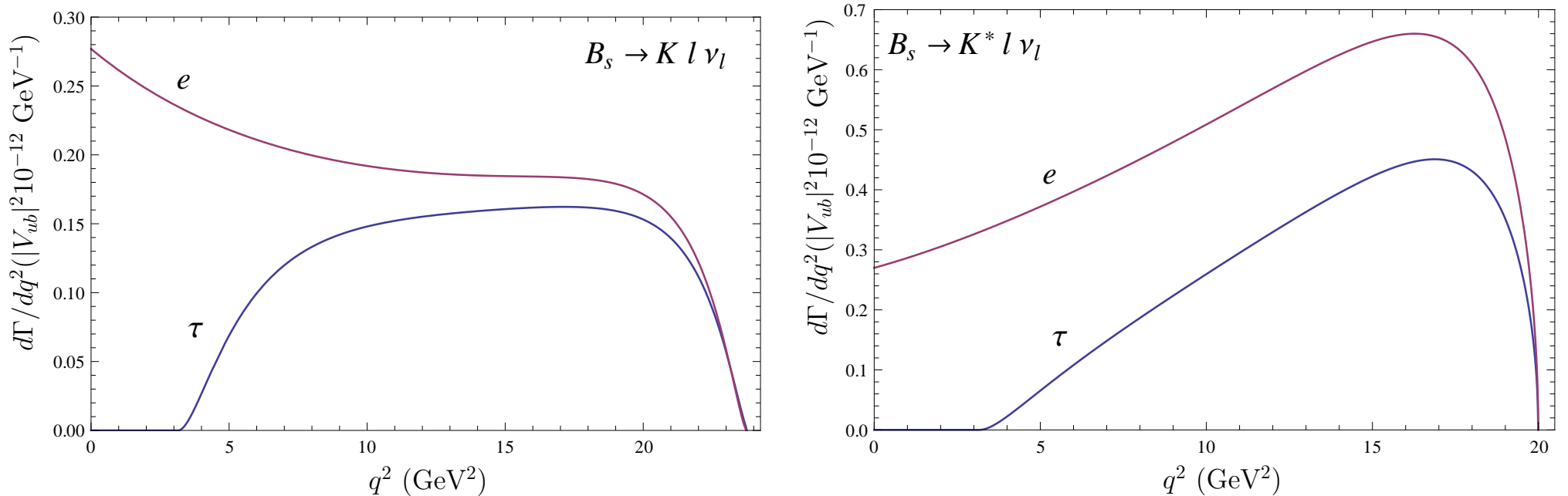

Figure 6: Form factors of the weak $B_s \to K^*$ transition.

Figure 7: Predictions for the differential decay rates of the semileptonic $B_s \to K^{(*)} l \nu_l$ decays.

Table 6: Comparison of theoretical predictions for the branching fractions of semileptonic decays $B_s \to K^{(*)} l \nu_l$ (in 10^{-4}).

Decay	our	Wang (PQCD)	Wu (LCSR)
$B_s \to K e \nu_e$	1.64 ± 0.17	$1.27^{+0.49}_{-0.30}$	1.47 ± 0.15
$B_s \to K \tau \nu_\tau$	0.96 ± 0.10	$0.778^{+0.268}_{-0.201}$	1.02 ± 0.11
$B_s \to K^* e \nu_e$	3.47 ± 0.35		2.91 ± 0.26
$B_s \to K^* \tau \nu_\tau$	1.67 ± 0.17		1.58 ± 0.13

Table 7: Comparison of theoretical predictions for the branching fractions of semileptonic decays $B_s \to K_J^{(*)} l\nu_l$ (in 10^{-4}).

Decay	our	SR Yang	LCSR Wang	PQCD Li	LCSR Yang	PQCD Li	PQCD Wang
$B_s \to K_0^* e\nu_e$	0.71 ± 0.14	$0.36^{+0.38}_{-0.24}$	$1.3^{+1.3}_{-0.4}$	$2.45^{+1.77}_{-1.05}$			
$B_s \to K_0^* \tau\nu_\tau$	0.21 ± 0.04		$0.52^{+0.57}_{-0.18}$	$1.09^{+0.82}_{-0.47}$			
$B_s \to K_1(1270) e\nu_e$	1.41 ± 0.28				$4.53^{+1.67}_{-2.05}$	$5.75^{+3.49}_{-2.89}$	
$B_s \to K_1(1270) \tau\nu_\tau$	0.30 ± 0.06					$2.62^{+1.58}_{-1.31}$	
$B_s \to K_1(1400) e\nu_e$	0.97 ± 0.20				$3.86^{+1.43}_{-1.75}$	$0.03^{+0.05}_{-0.02}$	
$B_s \to K_1(1400) \tau\nu_\tau$	0.25 ± 0.05					$0.01^{+0.02}_{-0.01}$	
$B_s \to K_2^* e\nu_e$	1.33 ± 0.27						$0.73^{+0.48}_{-0.33}$
$B_s \to K_2^* \tau\nu_\tau$	0.36 ± 0.07						$0.25^{+0.17}_{-0.12}$

- decays to ground state K mesons

$$Br(B_s \to K^{(*)} e\nu_e) = (5.11 \pm 0.51) \times 10^{-4}$$
$$Br(B_s \to K^{(*)} \tau\nu_\tau) = (2.63 \pm 0.26) \times 10^{-4}$$

- decays to first orbital excitations of K mesons

$$Br(B_s \to K_J^{(*)} e\nu_e) = (4.4 \pm 0.9) \times 10^{-4}$$
$$Br(B_s \to K_J^{(*)} \tau\nu_\tau) = (1.1 \pm 0.2) \times 10^{-4}$$

Branching fractions slightly decrease with excitation in contrast to the semileptonic $B_s \to D_s$ decays

$$Br(B_s \to K(1S, 1P) e\nu_e)_{\text{theor}} = (9.5 \pm 1.0) \times 10^{-4}$$

$$Br(B_s \to X e\nu_e)_{\text{theor}} = (10.3 \pm 1.0)\% \qquad Br(B_s \to X e\nu_e)_{\text{Exp.}} = (9.5 \pm 2.7)\% \ (\text{PDG})$$

Rare semileptonic $B_s \to \varphi(\eta)l^+l^-$ decays

Table 8: Comparison of theoretical predictions for the form factors of weak $B_s \to \eta_s$ and $B_s \to \varphi$ transitions at maximum recoil point $q^2 = 0$.

|	$f_+(0)$	$f_T(0)$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$T_1(0)$	$T_3(0)$
our	0.384±0.019	0.301±0.015	0.406±0.020	0.322±0.016	0.320±0.016	0.318±0.016	0.275±0.014	0.133±0.006
LCSR			0.434±0.035	0.474±0.033	0.311±0.030	0.234±0.028	0.349±0.033	0.175±0.018
PQCD	0.36±0.07		0.25±0.05	0.30±0.06	0.19±0.04			
RQM			0.32		0.29	0.28	0.28	
QM	0.36	0.36	0.44	0.42	0.34	0.31	0.38	0.26
PQCD			0.26±0.07	$0.31^{+0.08}_{-0.07}$	$0.18^{+0.06}_{-0.05}$	0.12±0.03	$0.23^{+0.06}_{-0.05}$	$0.19^{+0.06}_{-0.05}$
LCQM	0.288		0.329	0.279	0.232	0.210	0.276	0.170
LCSR	0.281±0.015	0.282±0.016	0.339±0.017	0.269±0.014	0.271±0.014	0.212±0.011	0.299±0.016	0.191±0.010
LFQM	0.357	0.365	0.445		0.343	0.310	0.380	
QM			$0.259^{+0.082}_{-0.037}$	$0.311^{+0.098}_{-0.049}$	$0.194^{+0.054}_{-0.029}$			
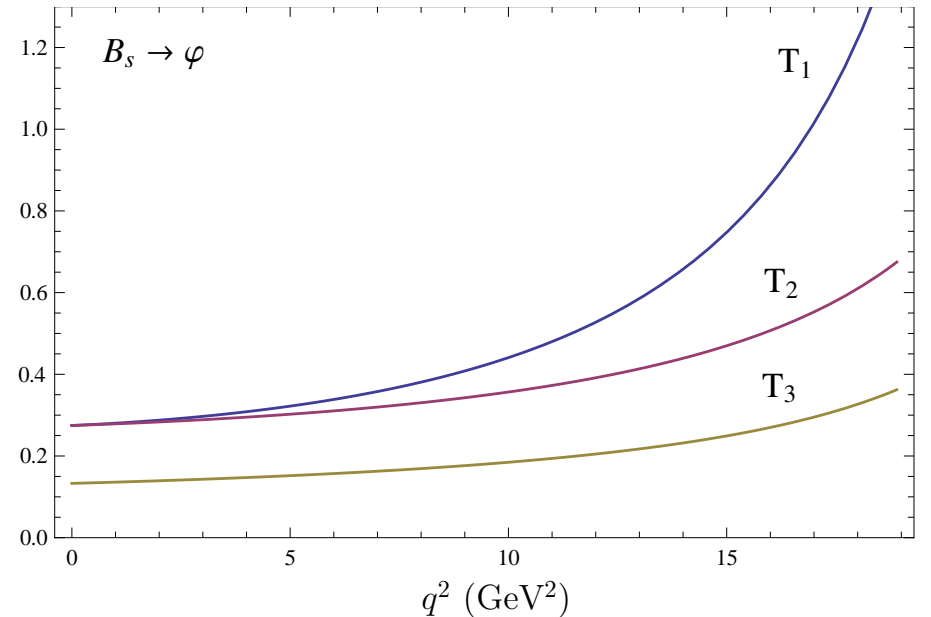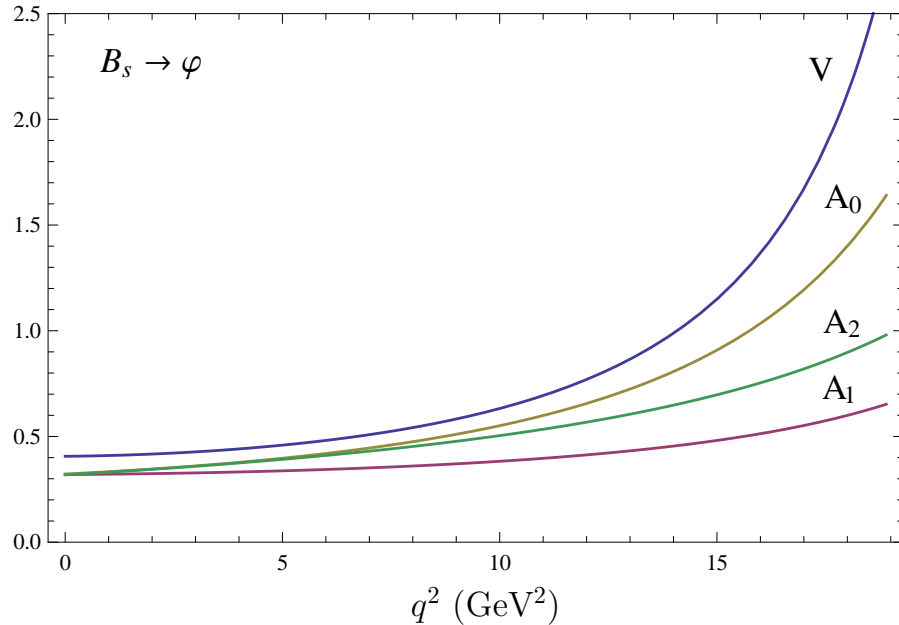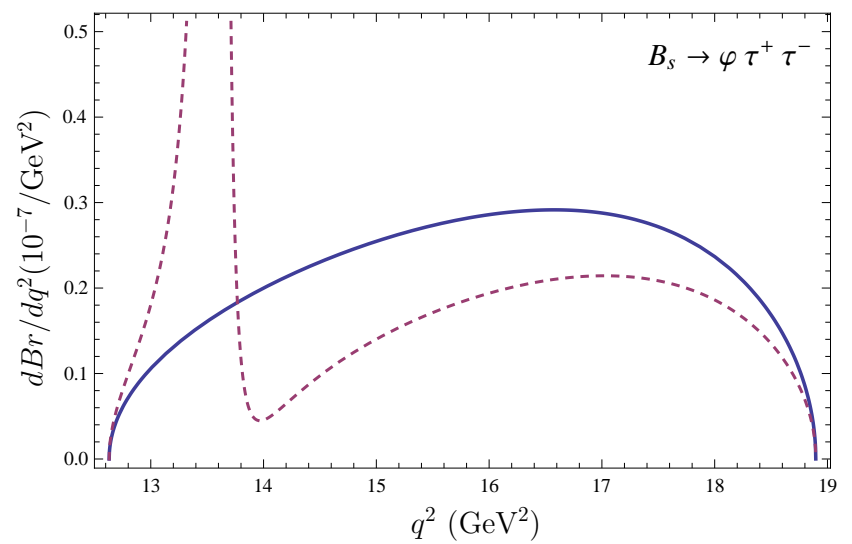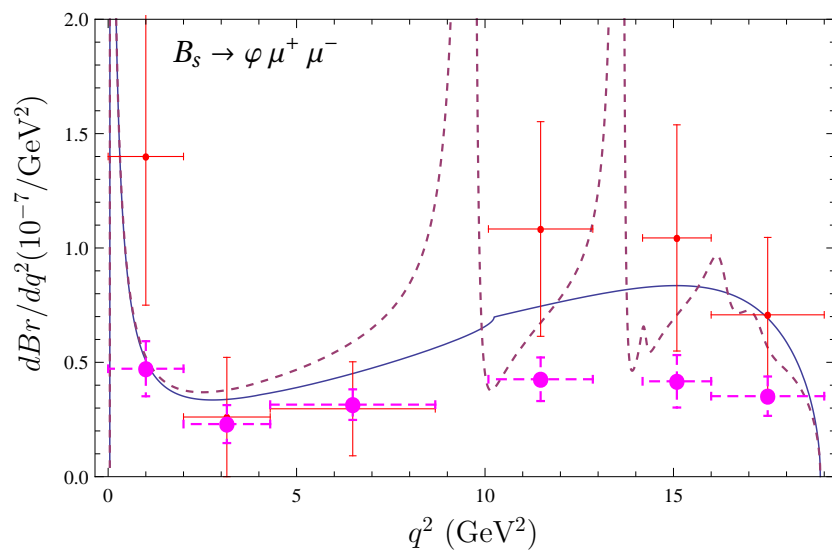
Figure 8: Comparison of theoretical predictions for the differential branching fractions $dBr(B_s \to \varphi\mu^+\mu^-)/dq^2$ with available experimental data. CDF data are given by dots with solid error bars, while LHCb data are presented by filled circles with dashed error bars.
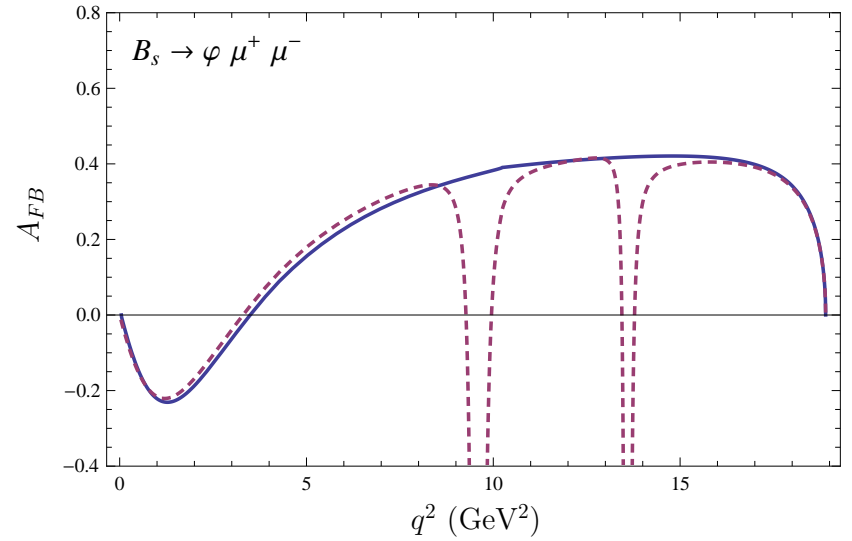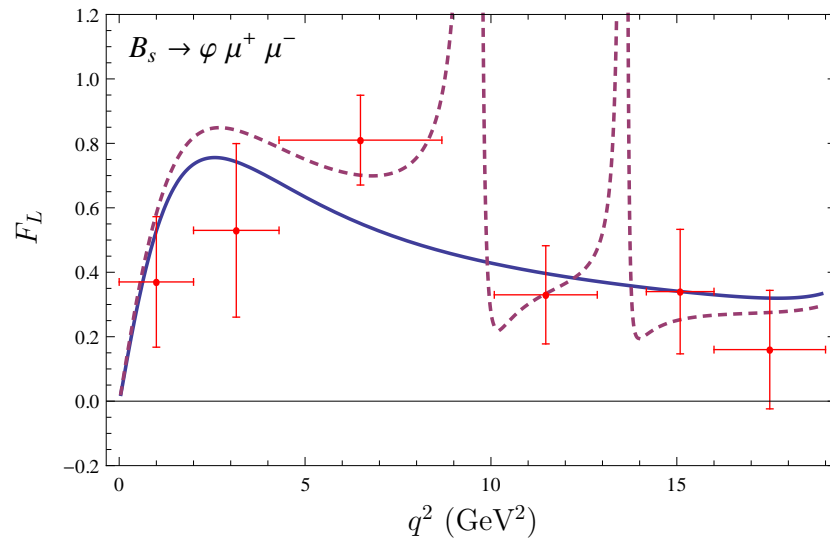

Figure 9: Comparison of theoretical predictions for the φ longitudinal polarization F_L and muon forward-backward asymmetry A_{FB} for the rare $B_s \to \varphi\mu^+\mu^-$ decays with available experimental data. LHCb data are given by dots with solid error bars.

Table 9: Comparison of theoretical predictions for the nonresonant branching fractions of the rare semileptonic B_s decays and available experimental data (in 10^{-7}).

Decay	our	LCSR Carlucci	LFQM Geng	SR Azizi	LFQM Choi	LCSR Wu	Experiment PDG	Experiment LHCb
$B_s \to \eta \mu^+\mu^-$	3.8 ± 0.4	3.4 ± 1.8	3.12	2.30 ± 0.97	2.4	1.2 ± 0.12		
$B_s \to \eta \tau^+\tau^-$	0.90 ± 0.09	1.0 ± 0.55	0.67	0.373 ± 0.156	0.58	0.34 ± 0.04		
$B_s \to \eta \nu\bar{\nu}$	23.1 ± 2.3	29 ± 15	21.7	13.5 ± 5.6	17			
$B_s \to \eta' \mu^+\mu^-$	3.2 ± 0.3	2.8 ± 1.5	3.42	2.24 ± 0.94	1.8			
$B_s \to \eta' \tau^+\tau^-$	0.39 ± 0.04	0.47 ± 0.25	0.43	0.280 ± 0.118	0.26			
$B_s \to \eta' \nu\bar{\nu}$	19.7 ± 2.0	24 ± 13	23.8	13.3 ± 5.5	13			
$B_s \to \varphi \mu^+\mu^-$	11.6 ± 1.2		16.4			11.8 ± 1.1	$11.3^{+4.0}_{-2.9}$	$7.07^{+0.97}_{-0.94}$
$B_s \to \varphi \tau^+\tau^-$	1.5 ± 0.2		1.51			1.23 ± 0.11		
$B_s \to \varphi \nu\bar{\nu}$	79.6 ± 8.0		116.5				<54000	
$B_s \to K \mu^+\mu^-$	0.24 ± 0.03				0.14	0.199 ± 0.021		
$B_s \to K \tau^+\tau^-$	0.059 ± 0.006				0.03	0.074 ± 0.007		
$B_s \to K \nu\bar{\nu}$	1.42 ± 0.14				1.01			
$B_s \to K^* \mu^+\mu^-$	0.44 ± 0.05					0.38 ± 0.03		
$B_s \to K^* \tau^+\tau^-$	0.075 ± 0.008					0.050 ± 0.004		
$B_s \to K^* \nu\bar{\nu}$	3.0 ± 0.3							

Table 10: Comparison of our predictions for the branching fractions of the rare semileptonic $B_s \to \varphi \mu^+ \mu^-$ decays in several bins of q^2 with experimental data (in 10^{-7}).

q^2 bin (GeV^2)	our		PDG	LHCb
	nonresonant	resonant	(CDF)	
$0.10 < q^2 < 2.00$	1.4 ± 0.2	1.4 ± 0.2	2.8 ± 1.3	0.944 ± 0.241
$2.00 < q^2 < 4.30$	0.79 ± 0.08	0.86 ± 0.09	0.6 ± 0.6	0.529 ± 0.191
$4.30 < q^2 < 8.68$	1.9 ± 0.2	2.6 ± 0.3	1.3 ± 0.9	1.38 ± 0.29
$10.09 < q^2 < 12.86$	2.0 ± 0.2	1.7 ± 0.2	3.0 ± 1.3	1.18 ± 0.26
$14.18 < q^2 < 16.00$	1.5 ± 0.2	1.1 ± 0.1	1.9 ± 0.9	0.759 ± 0.209
$16.00 < q^2$	1.8 ± 0.2	1.5 ± 0.2	2.3 ± 1.1	1.06 ± 0.26
$1.00 < q^2 < 6.00$	1.8 ± 0.2	2.0 ± 0.2	1.1 ± 0.9	1.14 ± 0.28
$0.10 < q^2 < 4.30$	2.2 ± 0.2	2.3 ± 0.2	3.3 ± 1.5	1.47 ± 0.23

CONCLUSIONS

- Relativistic quark model provides efficient tool for calculating weak transition matrix elements (form factors) between meson states
  - Form factors are expressed through the overlap integrals of meson wave functions
  - Wave functions are obtained as solutions of the relativistic wave equation which correctly describes meson spectroscopy
  - Relativistic transformations of the wave functions and contributions of intermediate negative-energy states are consistently taken into account
  - The momentum dependence of form factors is explicitly determined in the whole accessible kinematical range without additional assumptions and extrapolations
- Form factors of heavy-to-heavy, heavy-to-light and rare B_s decays were calculated
- On this basis semileptonic decay rates of B_s mesons to the ground-state and excited D_s and K mesons were obtained
- Rare semileptonic B_s decays to η and φ mesons were considered
- Calculated decay rates agree well with available data with the values of the CKM matrix elements V_{cb} and V_{ub} previously found in our model from weak B decays
- Obtained decay form factors can be applied for the calculation of the nonleptonic B_s decay rates in the factorization approximation