The quark masses and meson spectrum: a holographic approach

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Quark Confinement and the Hadron Spectrum XI

7-12 September 2014

Outline

- Experimental data

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- Form of spectra

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- “3-fields” model

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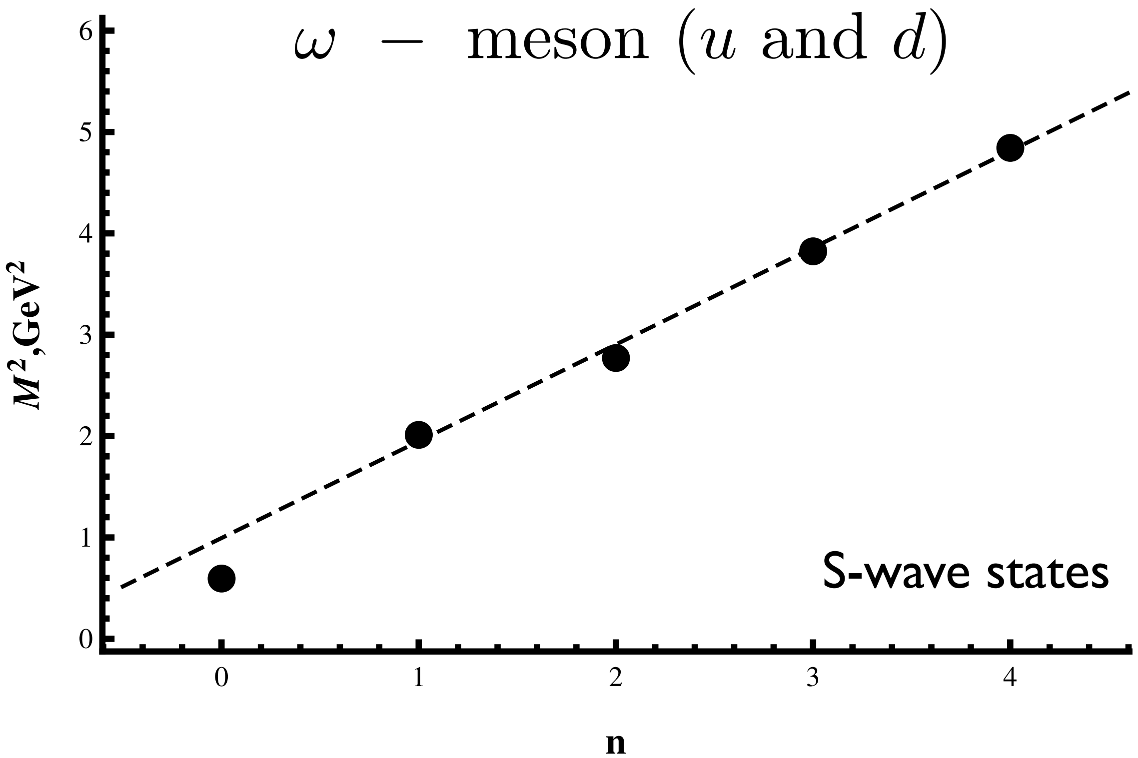
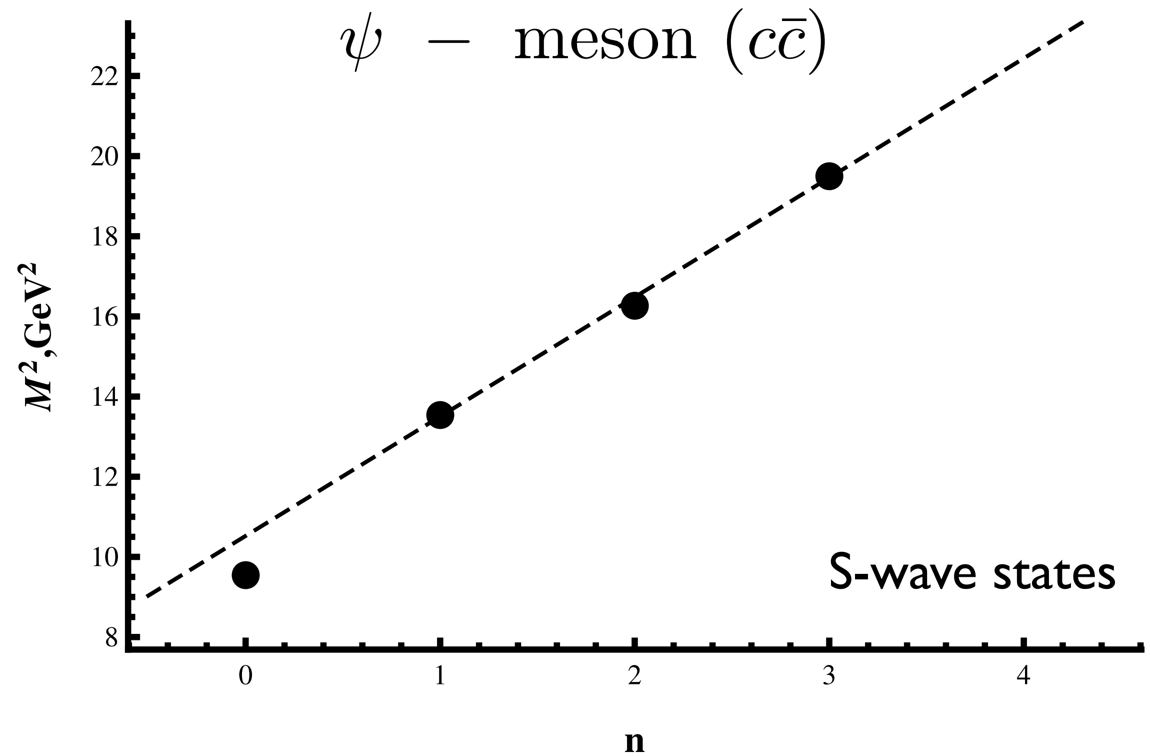
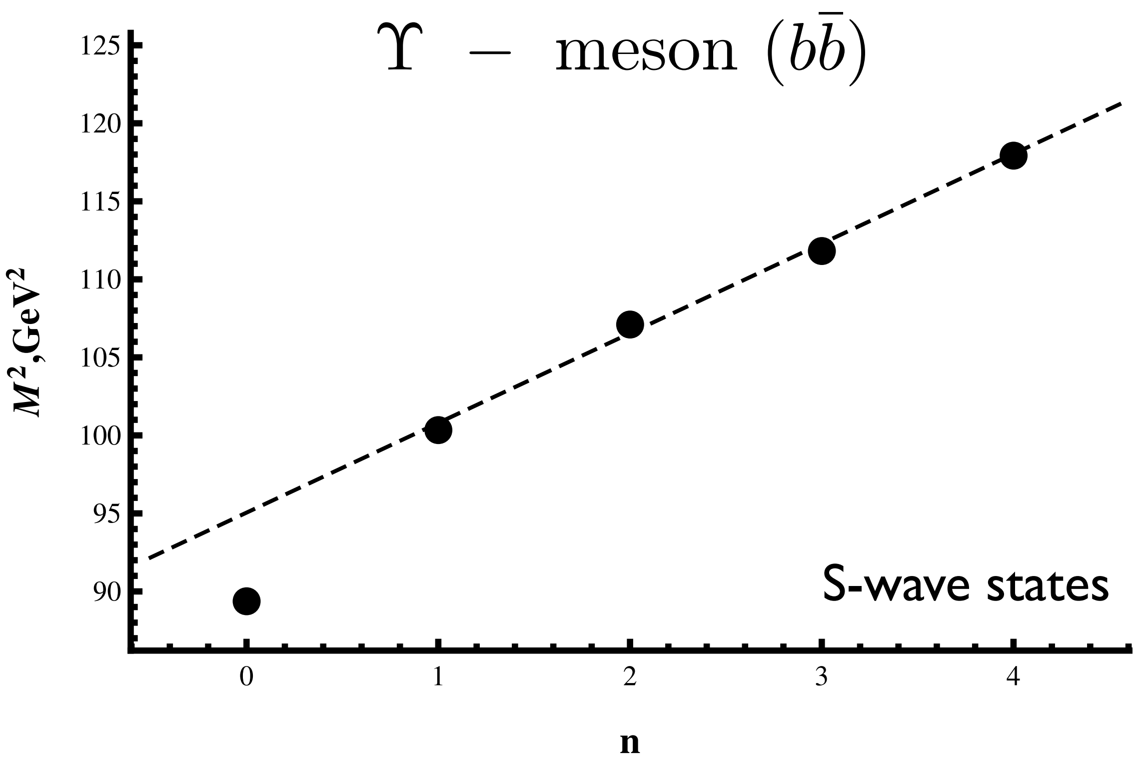
- Form of spectra
- “3-fields” model
- Fitting experiment
- Discussion and summary

Data and form of spectra

Vector sector







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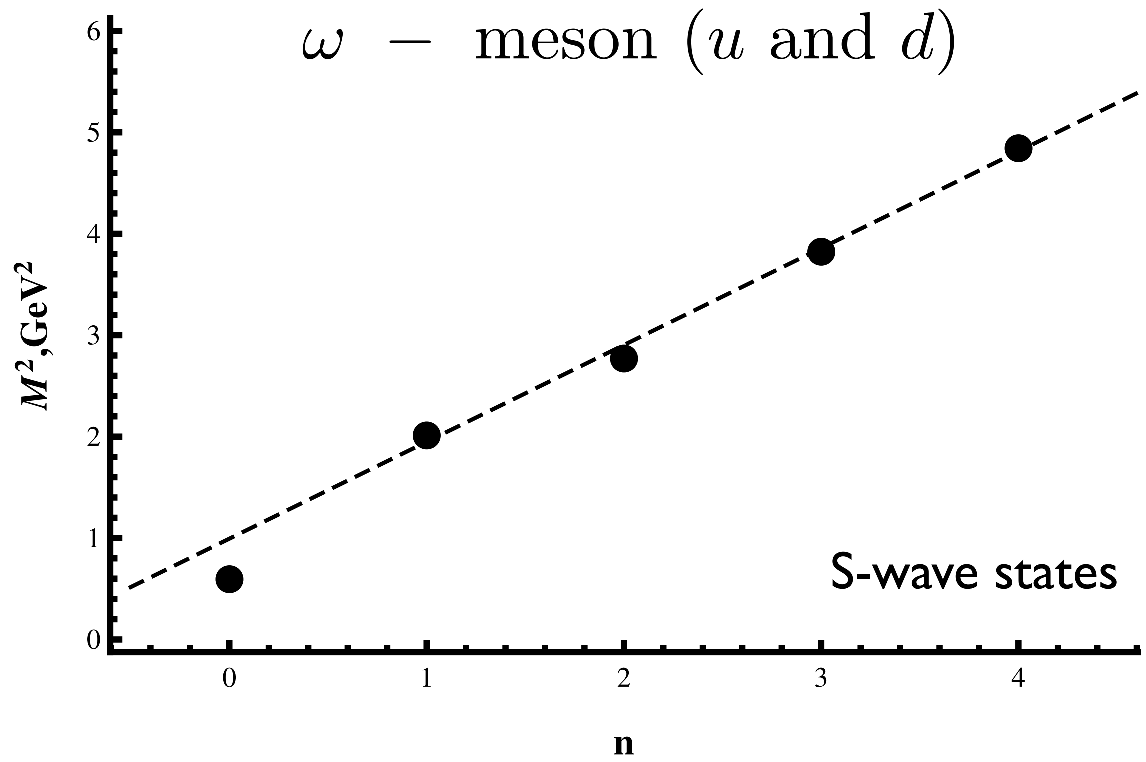

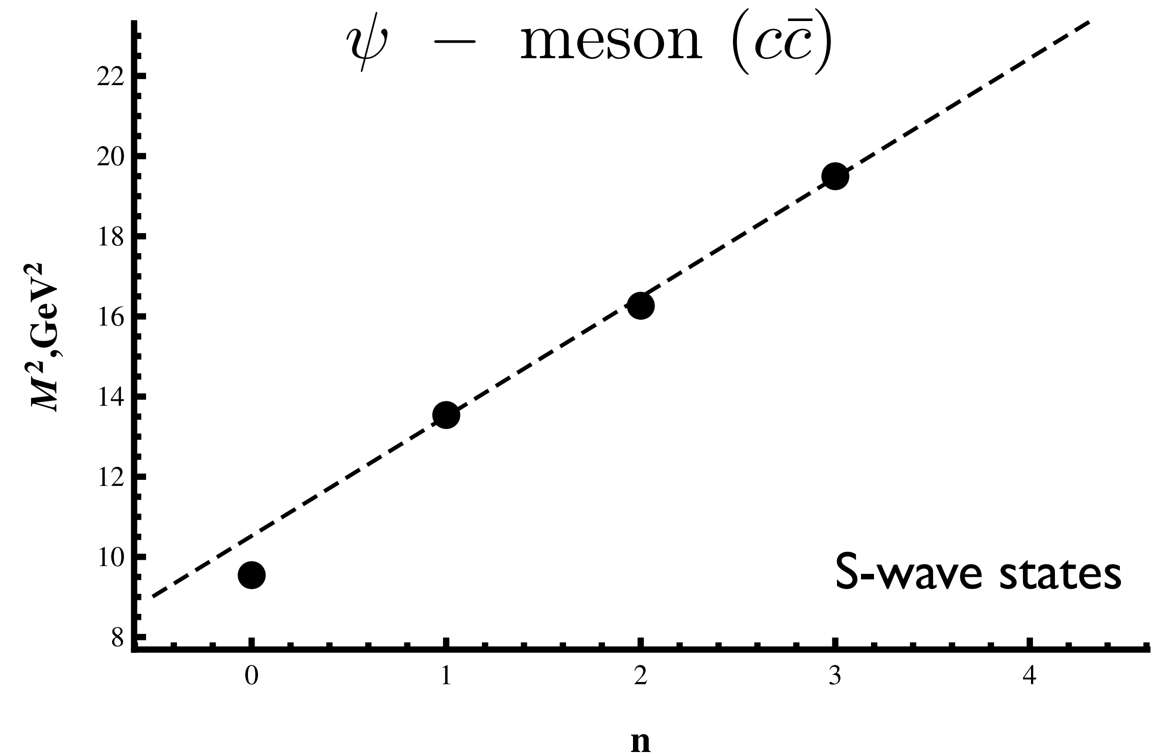

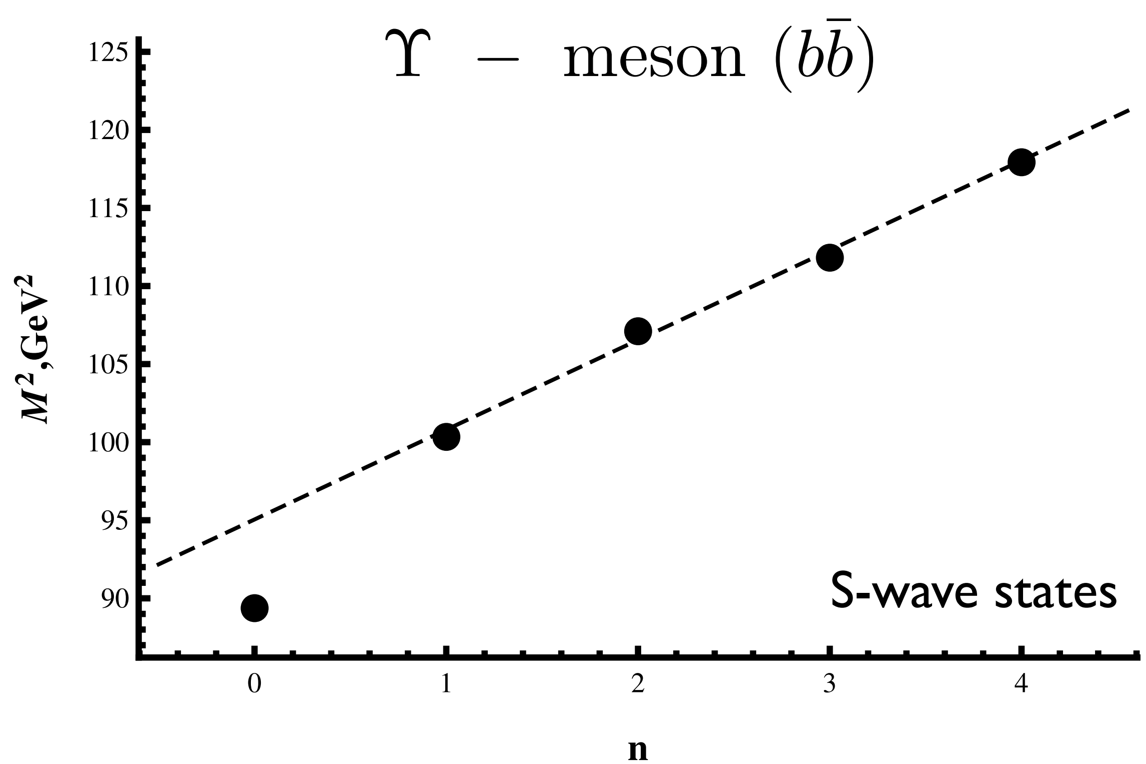

Linear behavior

$$M_n^2 = a(n+b)$$

All states → Fit I

Except ground states → Fit II

M_n^2	Fit I	Fit II
M_ω^2	$1.03(n+0.74)$	$0.95(n+1.04)$
M_ψ^2	$3.26(n+3.03)$	$2.98(n+3.53)$
M_Υ^2	$6.86(n+11.37)$	$5.75(n+16.54)$

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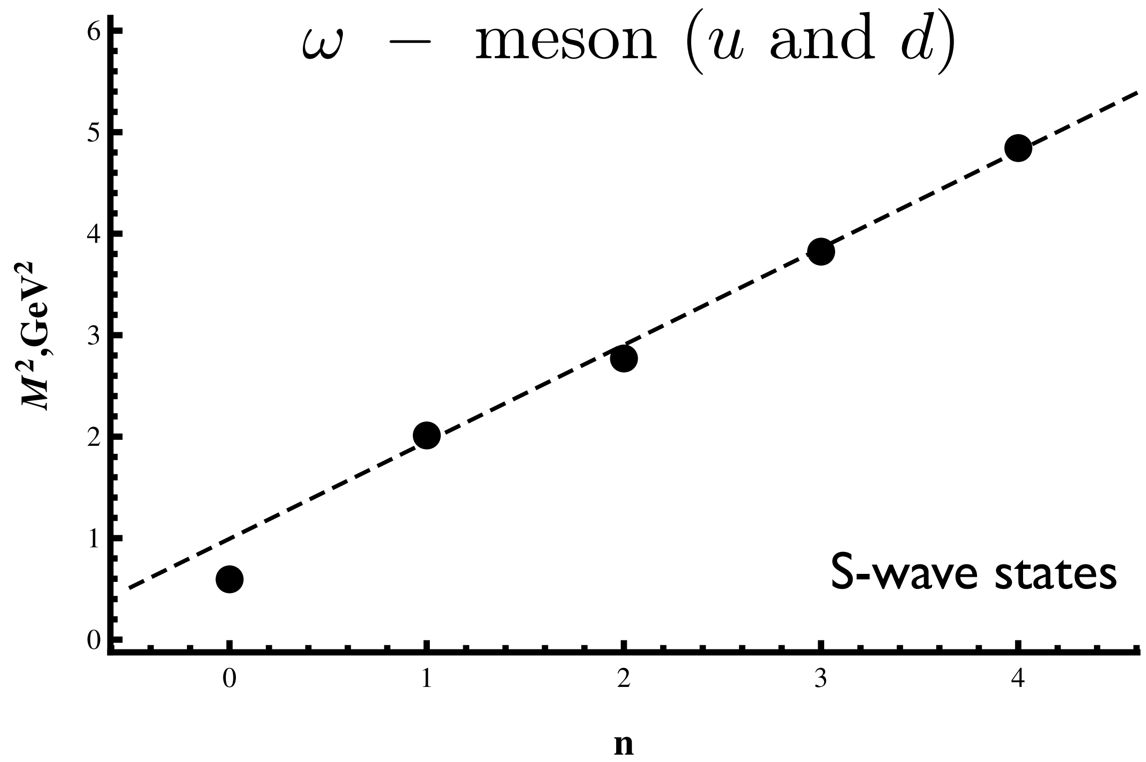

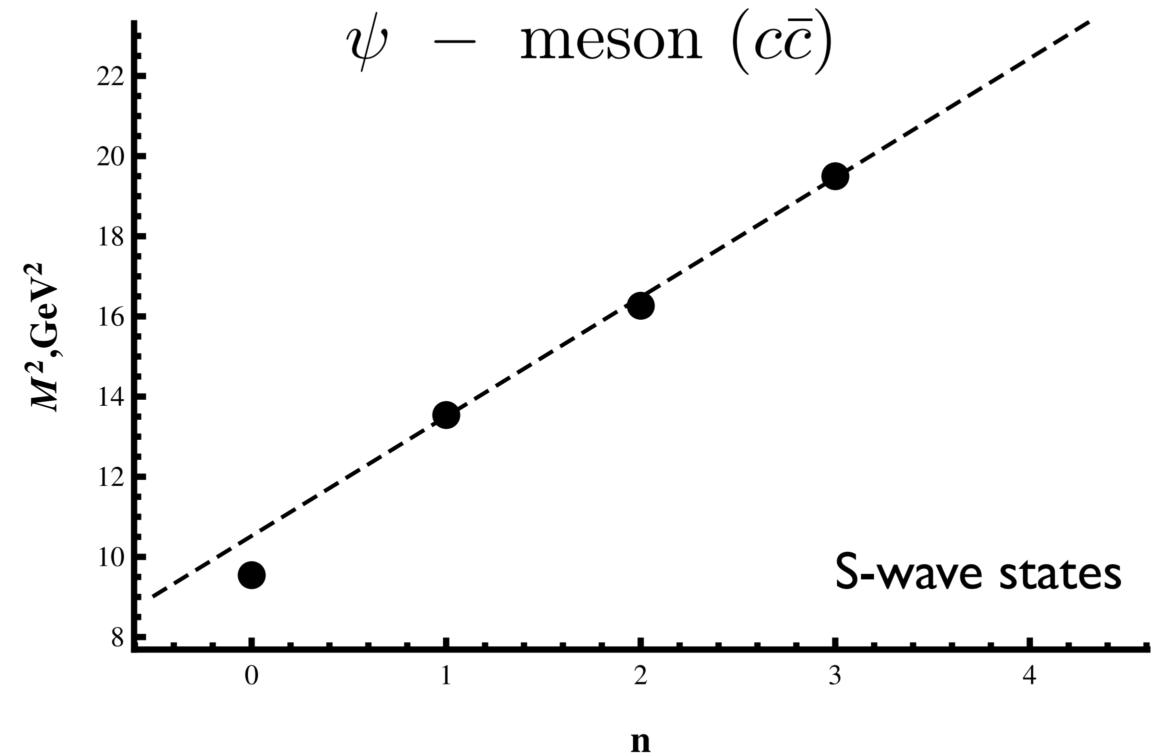

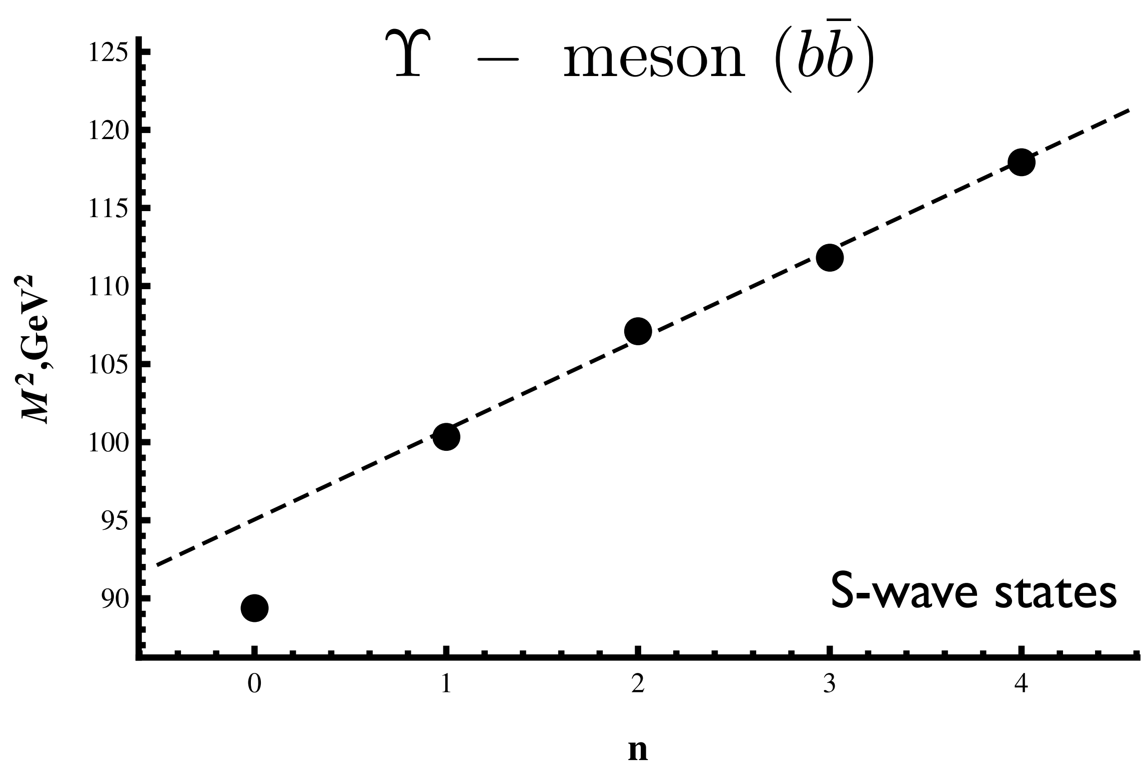

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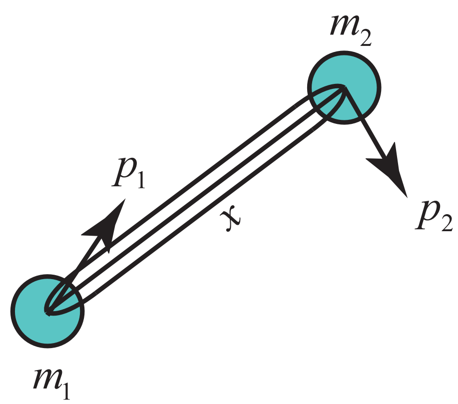
Slope ↑ Intercept ↑

Why it's linear

Hadron string model



String energy density

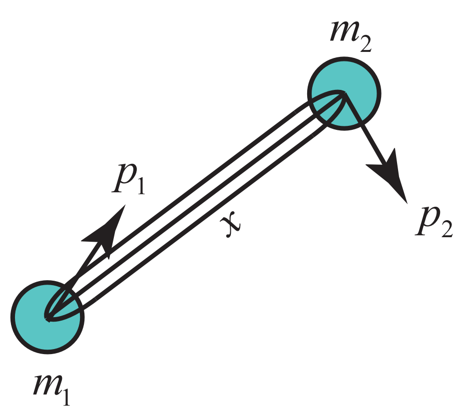
$$E_{meson} = \sqrt{p_1^2 + m_1^2} + \sqrt{p_2^2 + m_2^2} + x\sigma$$

$$\int_0^l p dx = \pi(n+b) \qquad n = 0, 1, 2, ...$$

String length

Why it's linear

Hadron string model



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String length

$$m^2 \approx 0$$

$$M^2 = 4\sigma\pi(n+b)$$

Linear for light quarks

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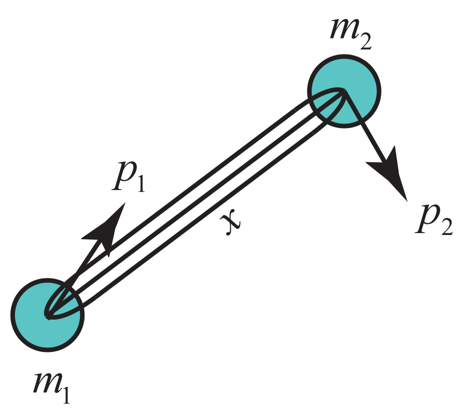
Hadron string model



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String length

$$M_n\sqrt{M_n^2 - 4m^2} + 4m^2 \ln \frac{M_n - \sqrt{M_n^2 - 4m^2}}{2m} = 4\pi\sigma(n+b).$$

NOT linear for heavy quarks

$m^2 \approx 0$

$$M^2 = 4\sigma\pi(n+b)$$

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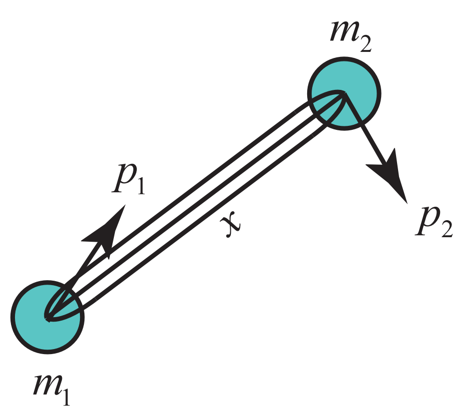

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$$M^2 = 4\sigma\pi(n+b)$$

NOT linear for heavy quarks

Linear for light quarks

Goal:

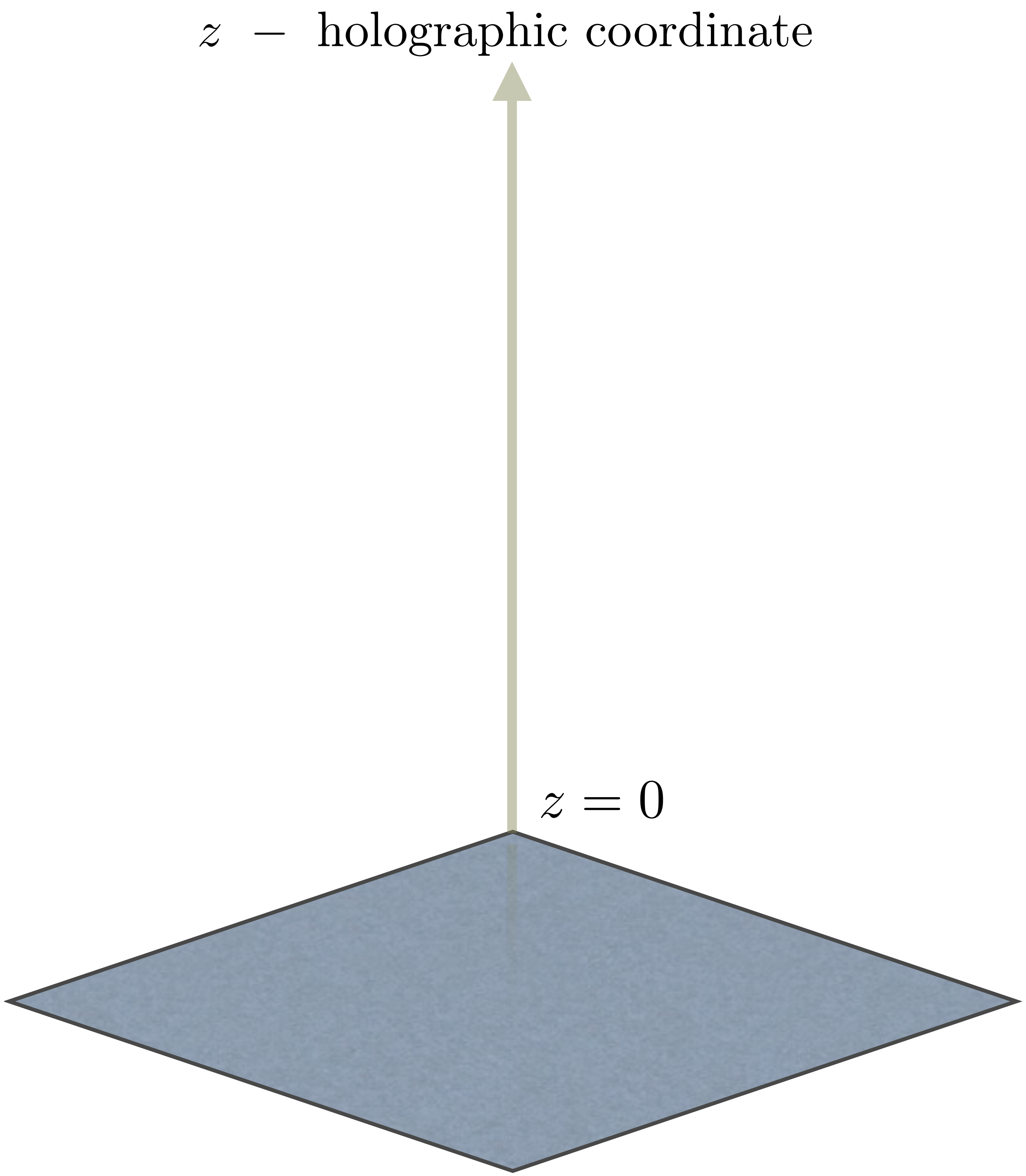
Linear spectrum for all flavors

$$M_n^2 = a[m](n + b[m])$$

Holography model*

*more details in Sergey Afonin's slides

AdS/QCD: theory content



AdS$_5$ metric

$$ds^2 = \frac{L^2}{z^2}(dx_\mu dx^\mu - dz^2)$$

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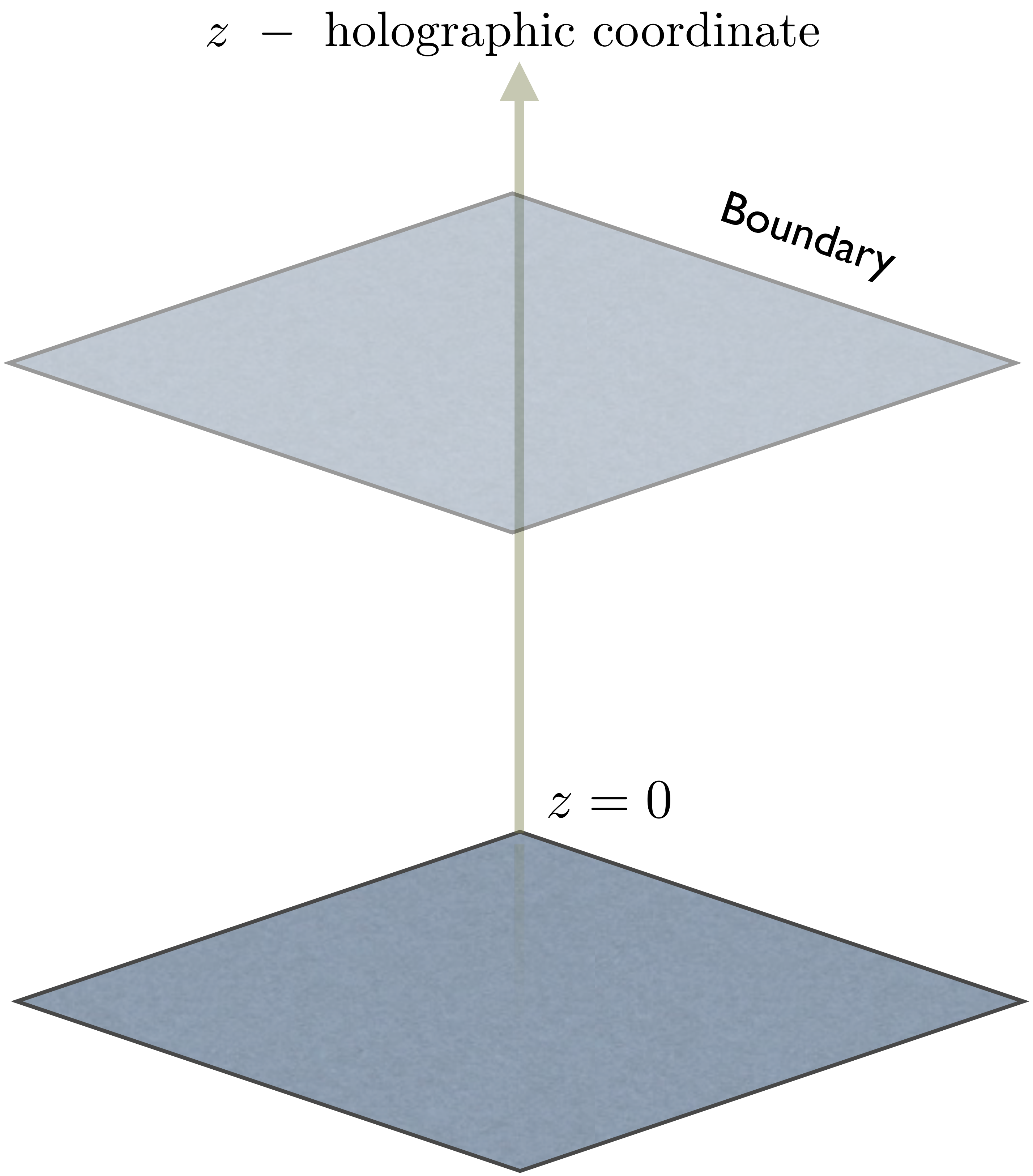

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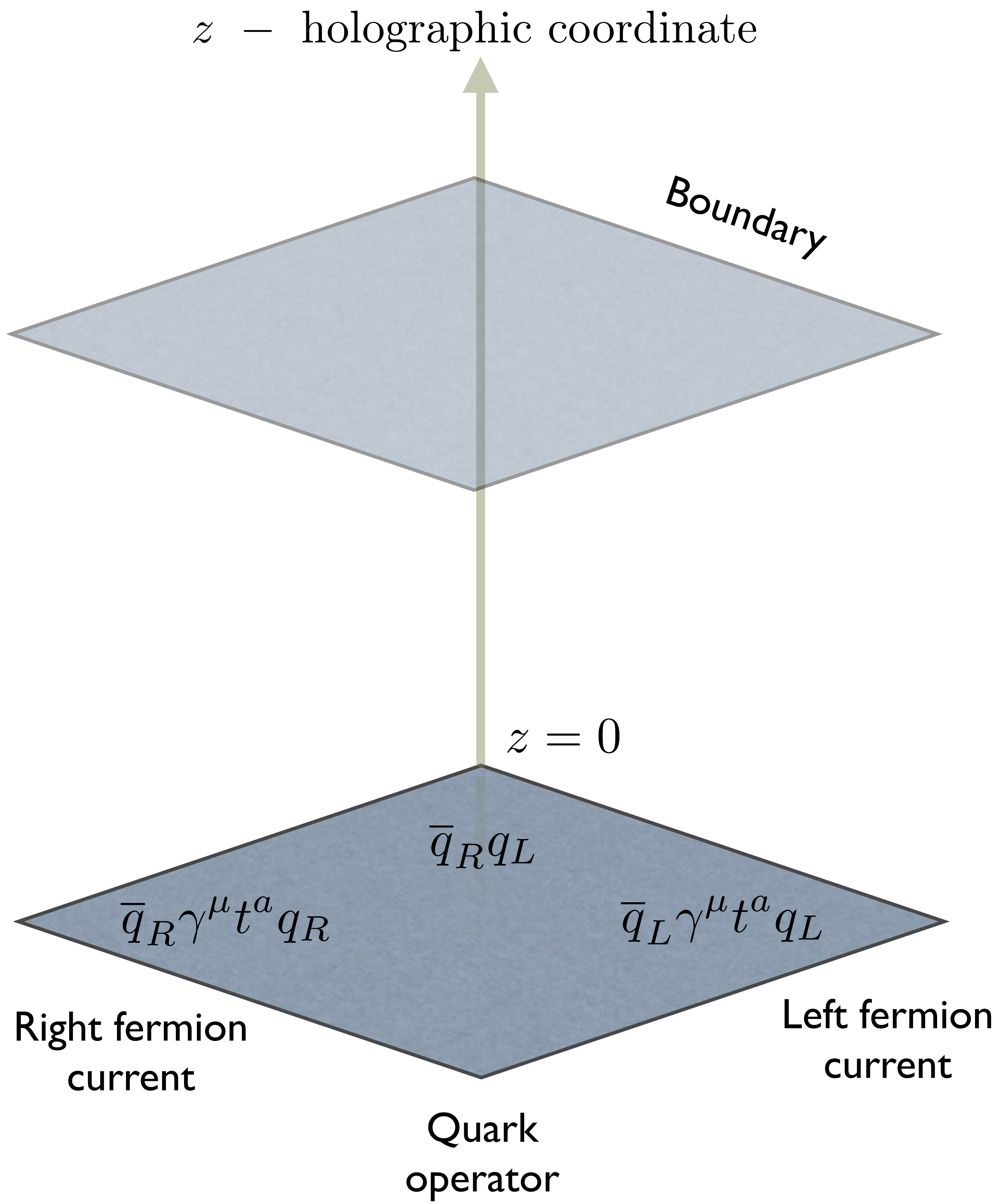

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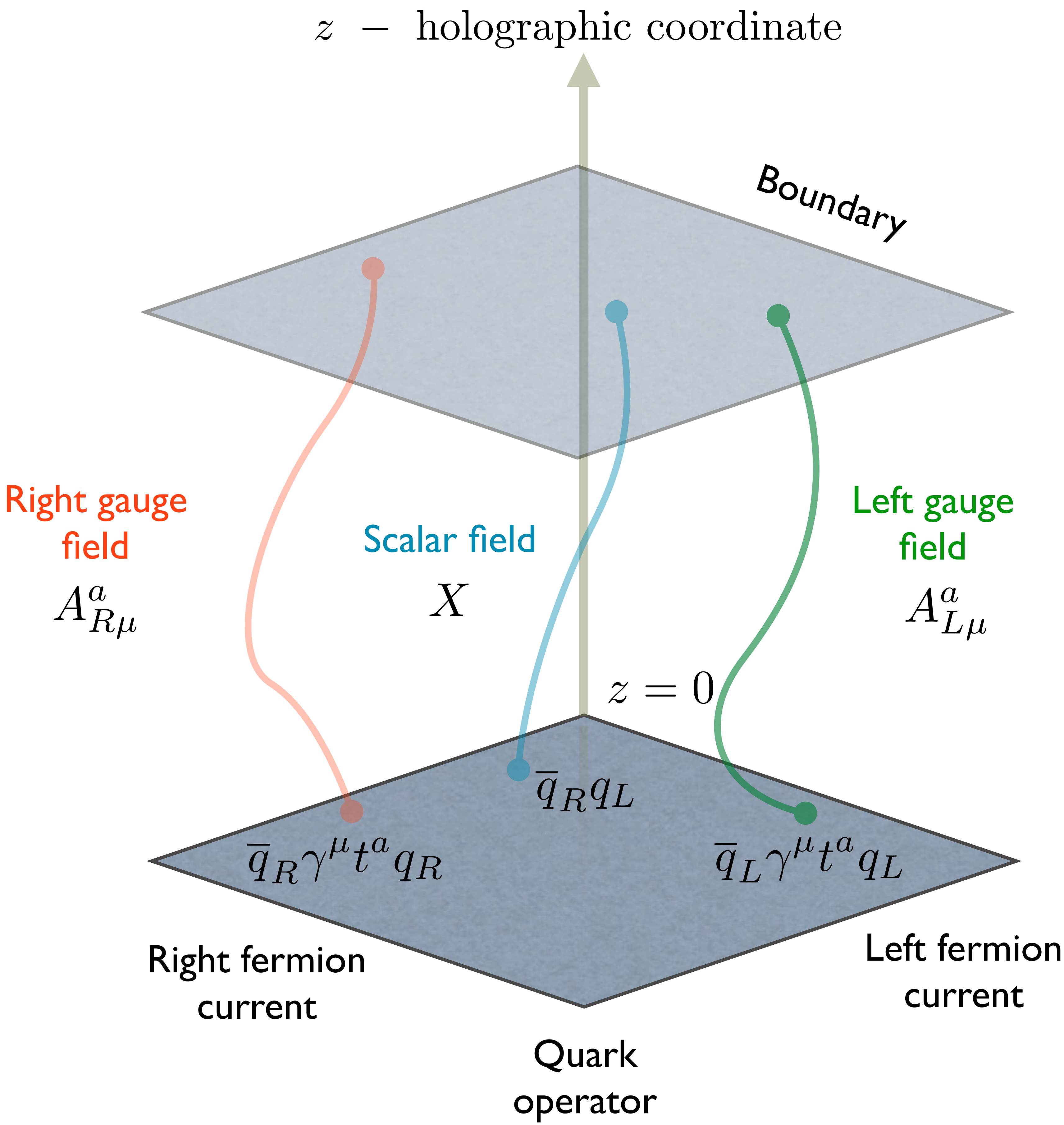

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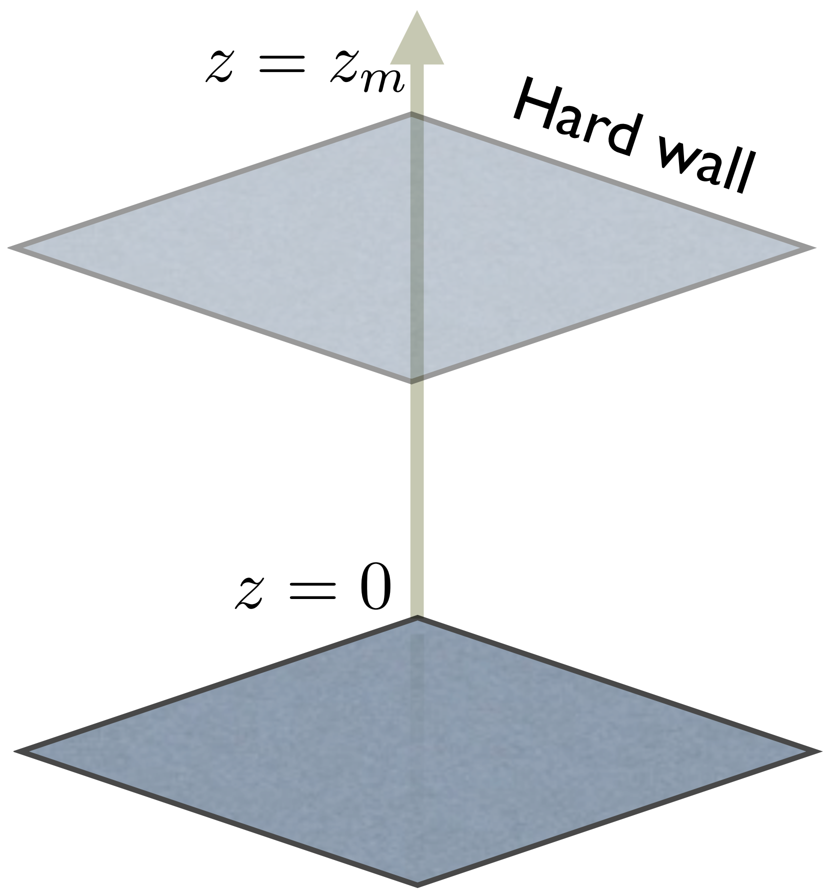
AdS/QCD: models



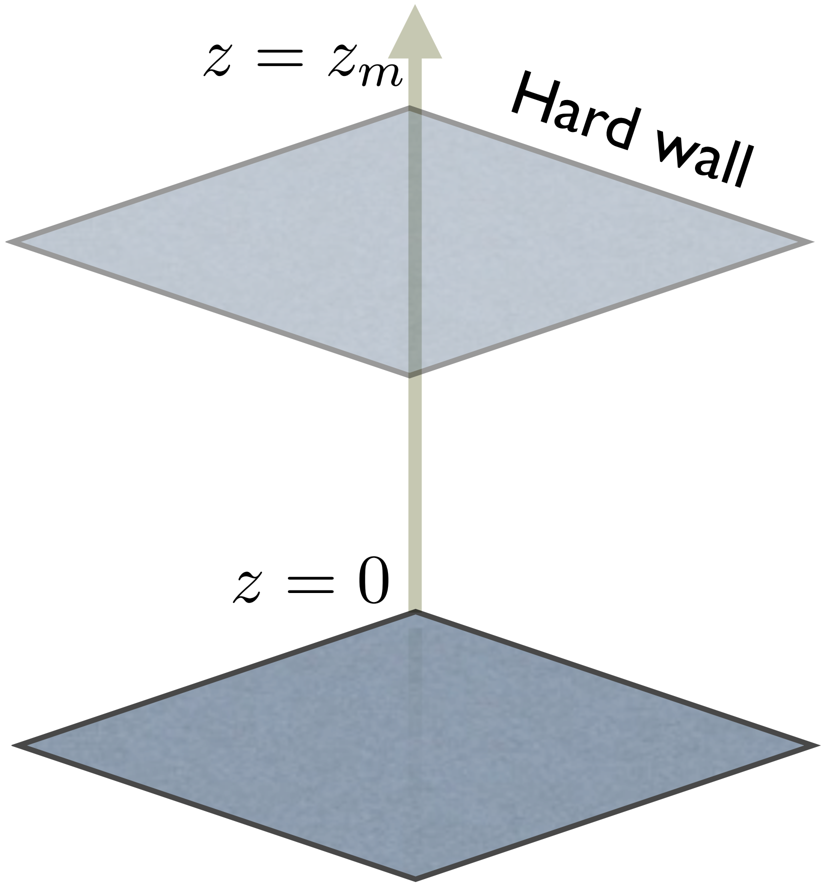
$$S^{5D} = \int_0^{z_m} d^4x dz \sqrt{g} Tr \left\{ |DX|^2 - 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

NOT linear spectra

$$M_n^2 \sim n^2$$

Erlich at al. PRL (2005)

AdS/QCD: models



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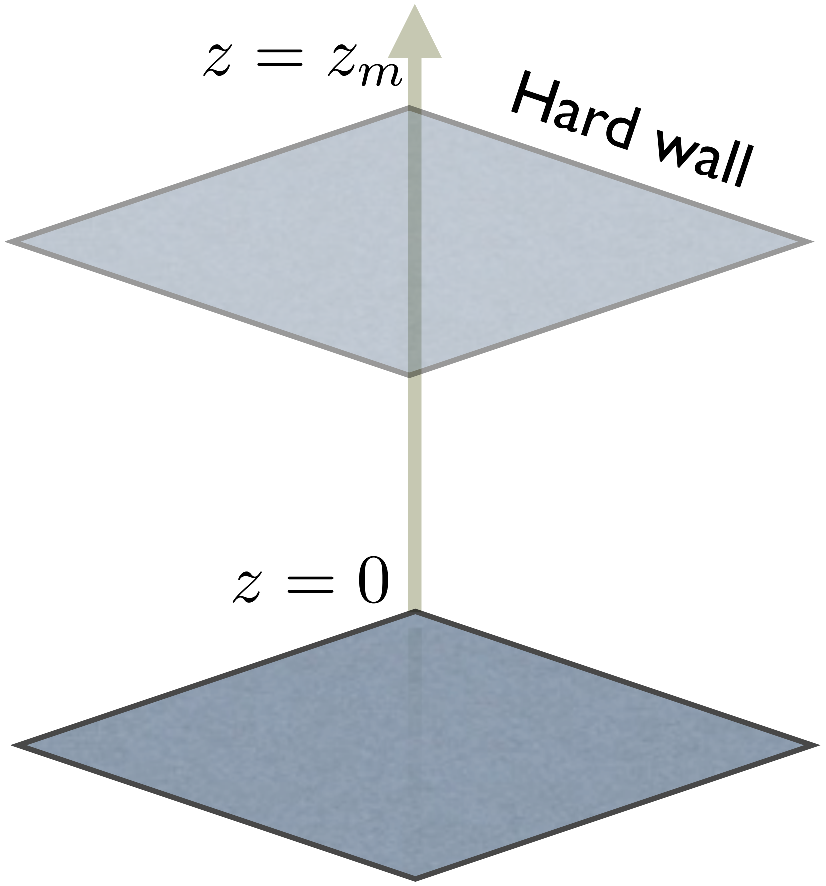

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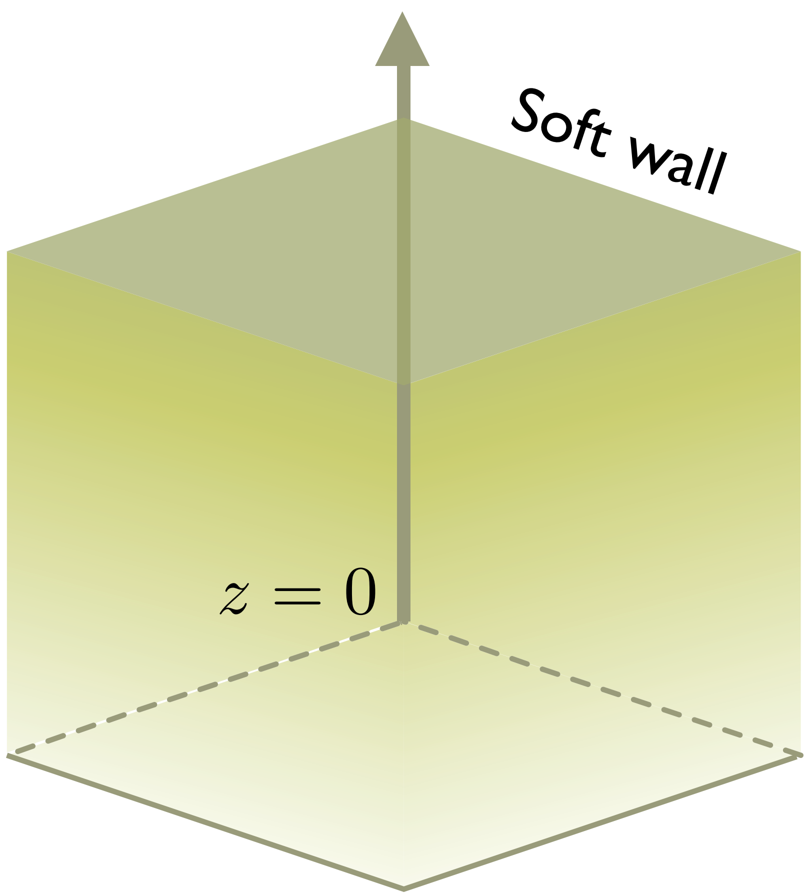

$$S^{5D} = \int d^4x dz \, e^{-\Phi(z)} \sqrt{g} \, Tr \left\{ (|DX|^2 + 3|X|^2) - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

Dilaton background

Linear spectra

$$M_n^2 = 4a(n+1)$$

Karch at al. PRD (2006)

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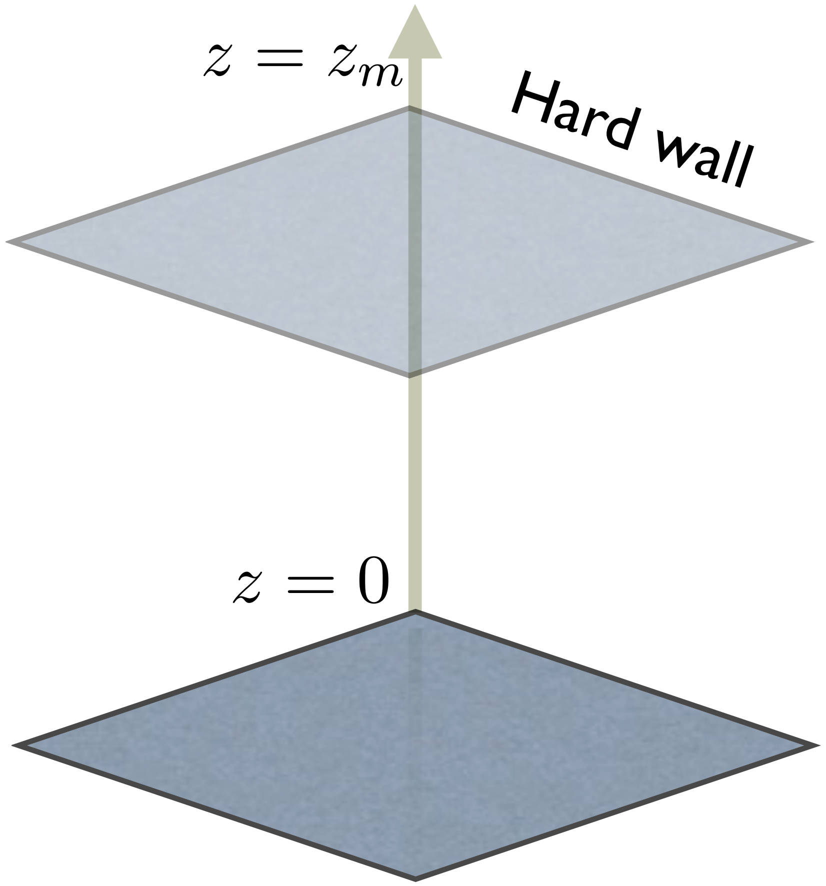

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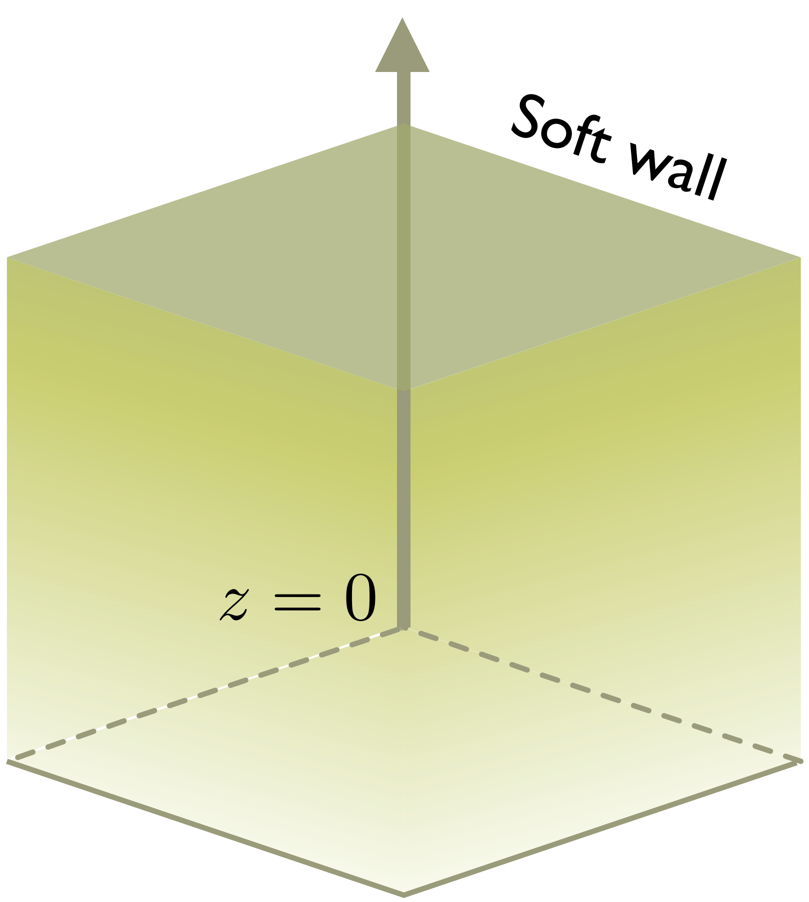

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Dilaton background

✓ **right**

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NO-wall model: trick

Soft wall action

$$S^{5D} = \int d^4x dz \; e^{-\Phi(z)} \sqrt{g} \; Tr \left\{ \left(|DX|^2 + 3|X|^2 \right) - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}$$

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Breaks Lorentz invariance → $e^{-\Phi(z)}$

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$$V = e^{\pm \Lambda^2 z^2/2} \widetilde{V}$$

$$S^{5D} = \int d^4x dz \sqrt{g} \left(\frac{\Lambda^4 z^4}{2R^2 g_5^2} \widetilde{V}_M \widetilde{V}^M - \frac{1}{4g_5^2} \widetilde{F}_{MN} \widetilde{F}^{MN} \right)$$

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massive part → gauge **non** invariant

new scalar field φ

$$S^{5D} = \int d^4x dz \sqrt{g} \left(|D\varphi|^2 - m_\varphi^2 |\varphi|^2 - \frac{1}{4g_5^2} F_{MN}F^{MN} \right)$$

✓ **gauge invariant** "No-wall" model: ~~Φ~~

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Set of scalar fields φ_i

$$S^{5D} = \int d^4xdz \sqrt{g} \left(\sum_i |D\varphi_i|^2 - m_i|\varphi_i|^2 - \frac{1}{4g_5^2} \widetilde{F}_{MN} \widetilde{F}^{MN} \right)$$

✓ gauge invariant "No-wall" model: $\not\Phi$

3 fields model: foundation

NEW 3 fields model

$$V^\mu \longleftrightarrow \bar{q}\gamma^\mu q$$

$$A^\mu \longleftrightarrow \bar{q}\gamma^\mu\gamma^5 q$$

$$\varphi_1 \longleftrightarrow G^2_{\mu\nu}$$

$$\varphi_2 \longleftrightarrow \bar{q}q$$

$$\varphi_3 \longleftrightarrow (\bar{q}q)^2$$

NEW 3 fields model

two "no-wall" fields

X in previous models

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Action

$$S = \int d^4x\, dz \sqrt{g}\,\{\mathcal{L}_{\mathrm{V}} + \mathcal{L}_{\mathrm{S}} + \mathcal{L}_{\mathrm{int}}\}$$

$$\mathcal{L}_{\mathrm{V}} = -\frac{1}{4g_5^2} F_{MN}F^{MN}$$

$$\mathcal{L}_{\mathrm{S}} = \frac{1}{2}\sum_{i=1}^{3}\left(\partial_M\varphi_i\partial^M\varphi_i - m_i^2\varphi_i^2\right)$$

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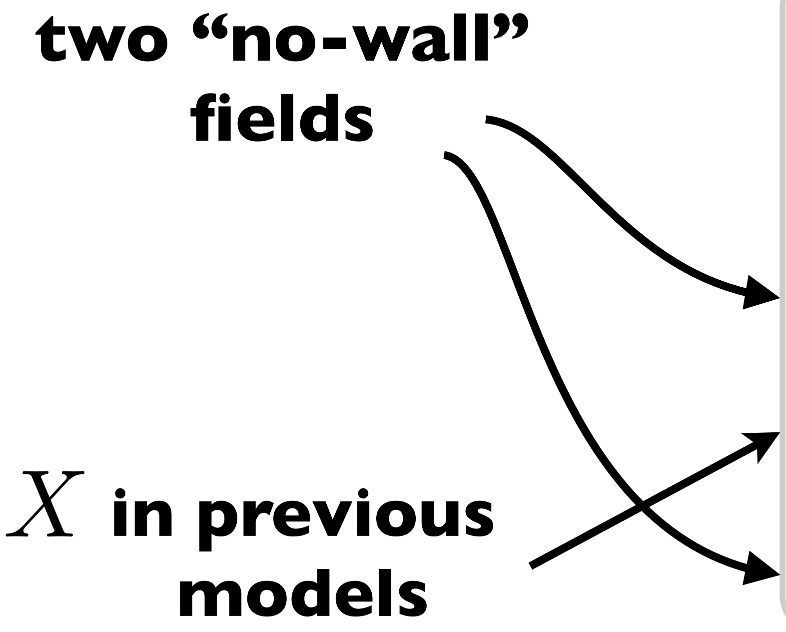

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Solving scalar equations

Mass of vector field

$$m_5^2 = z^6\left[g_2 C_2^2\frac{\sigma^2}{\xi^2} + g_3 C_{31}\right] + z^4[g_1 C_{12} + 2C_2^2\sigma m] + z^2[g_2 C_2^2 \xi^2 m^2]$$

3 fields model: vector sector

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Change of variables

$$V_\mu(q,z) = V_{0\mu}(q)\sqrt{z}\psi(q,z)$$

Schrodinger equation

$$-\psi_n'' + \left[\frac{3}{4z^2} + \frac{m_V^2(z) R^2}{z^2}\right]\psi_n = M_n^2 \psi_n.$$

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$$g_5^2 = \frac{12\pi^2}{N_c}$$

from two-point function

$$\sigma = \langle \bar{q} q \rangle$$

condensate

$$a$$

from holographic “potential”

$$\xi^2 = \frac{N_c}{4\pi^2}$$

condensate normalization factor

Fitting experimental data

3 fields model: result

Spectrum

$$M_n^2 = 4\sqrt{\alpha+\beta m}\,n + 4\sqrt{\alpha+\beta m} + \gamma m^2, \qquad n = 0, 1, 2, \dots$$

3 fields model: result

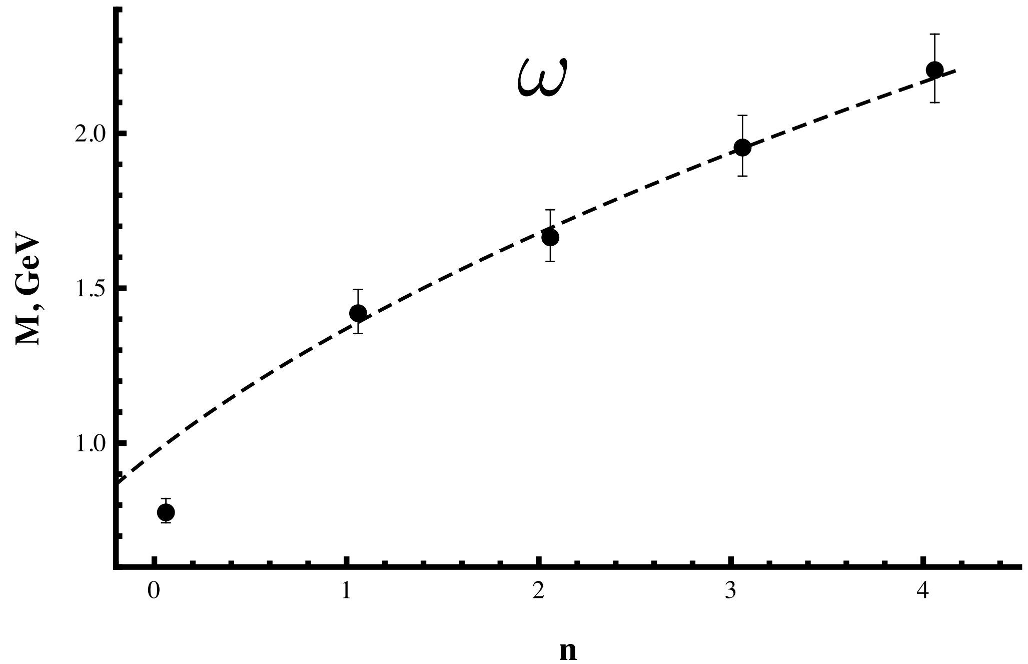
	FitI	FitII
α	0.049	0.055
β	0.530	0.382
γ	4.992	5.228
N	34.0	4.4

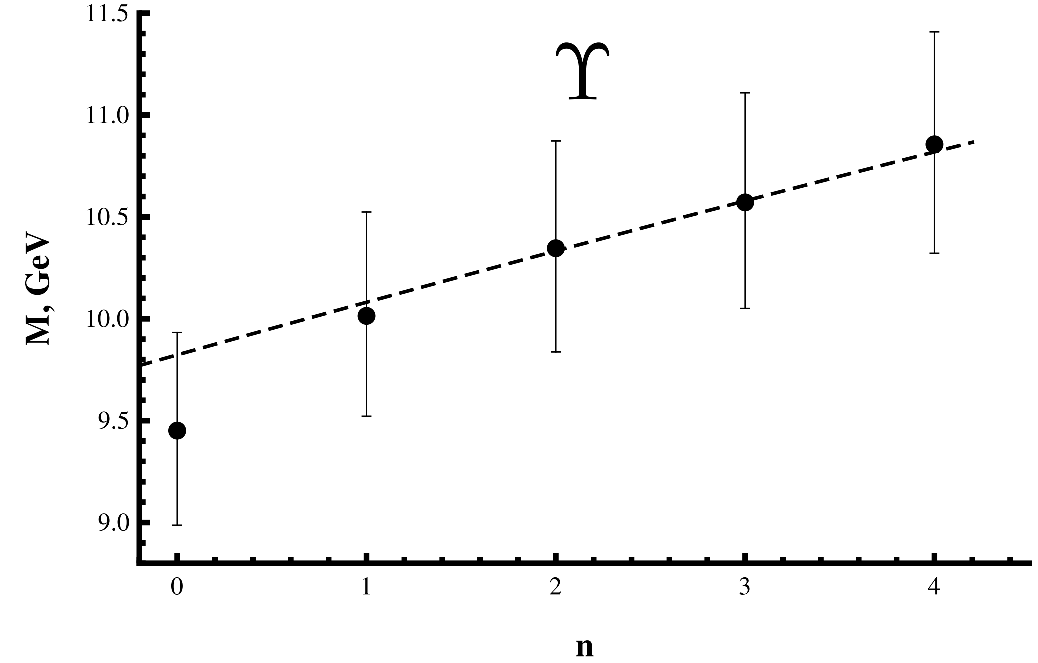
$$m_b = 4.18,\ m_c = 1.25,\ m_{u,d} \approx 0$$

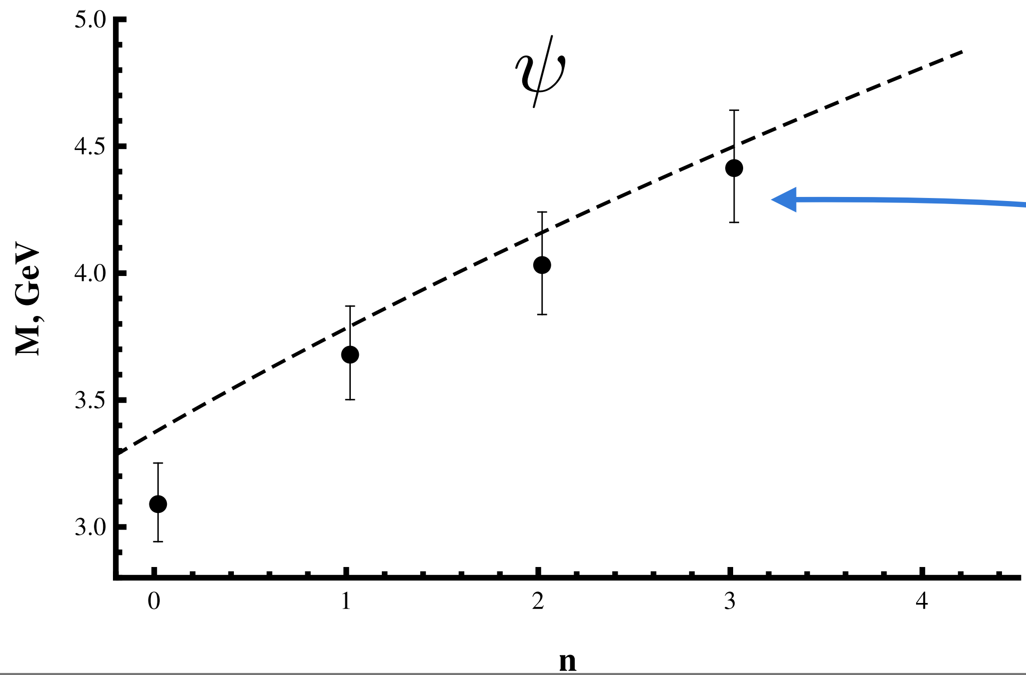
Spectrum

$$M_n^2 = 4\sqrt{\alpha + \beta m}\, n + 4\sqrt{\alpha + \beta m} + \gamma m^2, \qquad n = 0, 1, 2, \ldots$$

3 fields model: result







5% Precision gaps

	FitI	FitII
α	0.049	0.055
β	0.530	0.382
γ	4.992	5.228
N	34.0	4.4

$$m_b = 4.18,\ m_c = 1.25,\ m_{u,d} \approx 0$$

Spectrum

$$M_n^2 = 4\sqrt{\alpha + \beta m}\, n + 4\sqrt{\alpha + \beta m} + \gamma m^2, \qquad n = 0, 1, 2, \ldots$$

3 fields model: summary and discussion

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$$A \sim \sqrt{m}$$

$$B = \gamma m^2 + O(\sqrt{m})$$

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$$\gamma \approx 6$$

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shows growth of bending energy

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4. Slope with $m \to 0$.

$$4\sqrt{2} g_5 \sqrt{\sigma m + a^2} = 2m_\rho^2 - 2m_\pi^2 + O(m^2)$$

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✓ **consistent with Veneziano-like dual amplitudes**

Thank you!