The in-medium heavy-quark potential from quenched and dynamical lattice QCD

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Physics motivation

- **Heavy quarkonium:** the bound states of a heavy quark and antiquark are intensely studied probes in relativistic heavy-ion collisions. Bottomonium ($b\bar{b}$) is of particular interest as its yields have recently been obtained by the CMS collaboration with unprecedented precision. They show a suppression of excited states in the presence of a hot medium. Once we understand the real-time in-medium evolution of quarkonium it is possible to extract from the measured yields the properties of the surrounding quark-gluon plasma. [CMS collaboration] PRL 109 (2012) 222301
- **In-medium heavy quark potential**: The separation of scales $m_Q >> T$ and $m_Q >> \Lambda_{QCD}$ allows describing the interaction between heavy $Q\bar{Q}$ and its environment in the language of a Schrödinger equation with in-medium potential. Initially the potential was modeled using e.g. the color singlet free energies measured on the lattice. Modern effective field theories such as NRQCD and pNRQCD define V^{QQ} directly from a QCD observable, the real-time Wilson loop. Brambilla et. al. Rev.Mod.Phys. 77 (2005)

$$V^{Q\bar{Q}}(r)=\lim_{t\to\infty}\frac{i\partial_t W(r,t)}{W(r,t)}\qquad W(r,t)=\Big\langle \mathrm{Tr}\Big[\exp\Big(-ig\int_\square dz^\mu A_\mu(z)\Big)\Big]\Big\rangle_{QCD@T>0}$$

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017, for a brief review see A.R. MPLA 28 (2013) 133000

- **Perturbative evaluation of the potential at high T:** The hard thermal loop approximation enabled the evaluation of the real-time Wilson loop and revealed, **V^{QQ} is in general complex.** Laine et al. JHEP03 (2007) 054
  - Re[V] shows Debye screening (consistent with the classic idea of Matsui and Satz)
  - Im[V] from medium gluon scattering (Landau damping) and absorption (Singlet to Octet transition) Beraudo et. al. NPA 806:312,2008 Brambilla et. al. PRD 78 (2008) 014017
- **Goal**: Extract the values of the complex in-medium heavy quark potential non-perturbatively from lattice QCD simulations at phenomenologically relevant $T\sim T_C$
- **Challenge 1:** Potential is a real-time object but lattice QCD simulations are performed in Euclidean time. **Strategy**: Express the potential in terms of spectral functions
- **Challenge 2 :** Spectral functions are not directly accessible on the lattice, they require an analytic continuation. **Strategy**: Bayesian inference of spectra from Euclidean data

In-medium potential from lattice QCD

- V^{QQ} defined in real-time: connection to Euclidean lattice QCD via spectral functions
  - Spectral representation of the real-time Wilson loop is a simple Fourier transform and ρ positive definite A.R., T.Hatsuda & S.Sasaki PoS LAT2009 (2009) 162

$$W(r,t)=\int_{-\infty}^{\infty}d\omega\, e^{-i\omega t}\rho(r,\omega)\ \Longleftrightarrow\ W(r,\tau)=\int_{-\infty}^{\infty}d\omega\, e^{-\omega\tau}\rho(r,\omega)$$

$$V^{Q\bar{Q}}(r)=\lim_{t\to\infty}\frac{\int_{-\infty}^{\infty}d\omega\,\omega\, e^{i\omega t}\rho(r,\omega)}{\int_{-\infty}^{\infty}d\omega\, e^{i\omega t}\rho(r,\omega)}$$

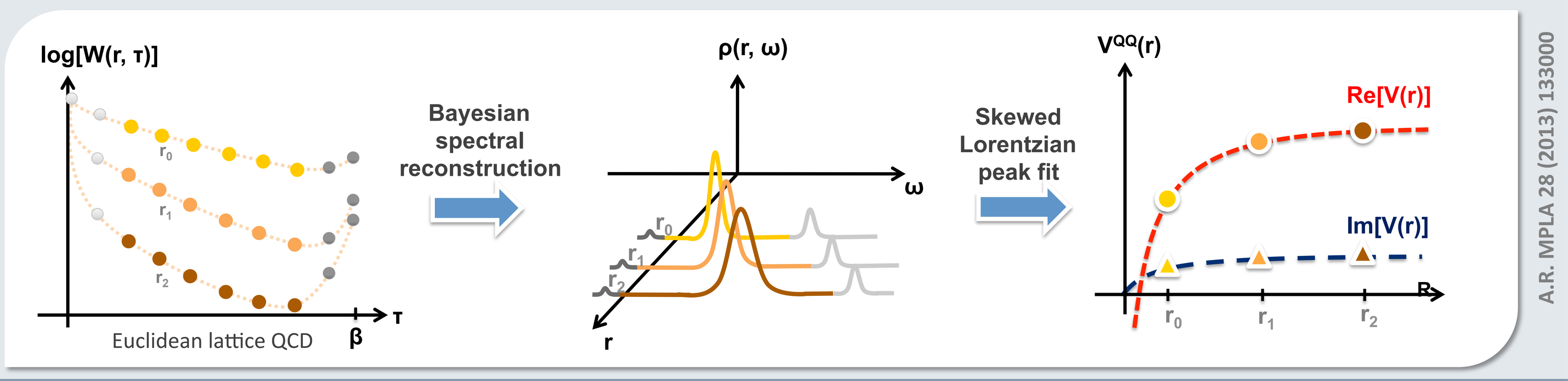
A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001 — Bayesian spectral reconstruction

  - Extracting ρ(r,ω) from W(r,τ) amounts to an inverse Laplace transform: Ill-posed problem (Bayesian inference)
- The most general form of the Wilson loop spectrum: disentangling late and early times

$$\rho(r,\omega)\propto\frac{|\mathrm{Im}V(r)|\cos[\mathrm{Re}\sigma_\infty(r)]-(\mathrm{Re}V(r)-\omega)\sin[\mathrm{Re}\sigma_\infty(r)]}{\mathrm{Im}V(r)^2+(\mathrm{Re}V(r)-\omega)^2}+c_0(r)+c_1(r)(\mathrm{Re}V(r)-\omega)+\cdots$$

Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

  - Potential represents late times physics: encoded in position and width of a low lying skewed Lorentzian peak.
- Summary: From Euclidean correlators to the in-medium heavy quark potential



Improved Bayesian spectral reconstruction

- An **ill-posed problem**: extract a spectral function $\rho(\omega_l)=\rho_l$ along N_ω frequencies from $N_\tau << N_\omega$ noisy datapoints $D(\tau_i)=D_i$ (**likelihood L** fit alone is underdetermined)

$$D_i^\rho=\sum_{l=1}^{N_\omega}\Delta\omega_l e^{-\omega_l\tau_i}\rho_l\qquad L=\frac{1}{2}\sum_{i,j}^{N_\tau}(D_i-D_i^\rho)C_{ij}^{-1}(D_j-D_j^\rho)$$

C_{ij} covariance matrix

- Bayes Theorem: Incorporation of **prior information (I) regularizes the χ^2 fit**

$$P[\rho|D,I]\propto P[D|\rho,I]\,P[\rho|I]\qquad P[D|\rho,I]=\exp[-L-\gamma(L-N_\tau)^2]$$

Since the correct ρ leads to $L\sim N_\tau$

- **Improved prior functional:** enforces (1) positive definiteness of ρ (2) independence of the result from dimension of ρ (3) smoothness of ρ, where data does not imprint peaks

$$P[\rho|I]=e^S\qquad S=\alpha\sum_{l=1}^{N_\omega}\Delta\omega_l\Big(1-\frac{\rho_l}{m_l}+\log\Big[\frac{\rho_l}{m_l}\Big]\Big)$$

m_l **default model**: correct spectrum in the absence of data. Y.Burnier, A. Rothkopf Phys.Rev.Lett. 111 (2013) 18, 182003

  - Different from Maximum Entropy Method: **S is not entropy and does not contain flat directions**
- α is integrated out analytically P[α]=1, Bayesian solution as maximum of the posterior

$$P[\rho|D,I]\propto P[D|\rho,I]\int_0^\infty d\alpha\, P[\rho|I,\alpha]\qquad \frac{\delta}{\delta\rho}P[\rho|D,I]\Big|_{\rho=\rho^{BR}}=0$$

  - Implementation via LBFGS algorithm: **no restriction of the search space** allows higher than MEM resolution
  - Due to absence of flat direction in S, true convergence to a global extremum (no artificial cutoff in step size)

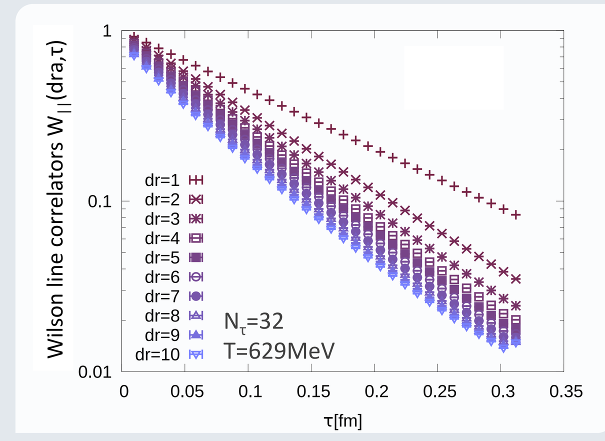
Quenched Lattice QCD

- Fixed scale approach: anisotropic naïve Wilson action β=7.0 $\xi=a_s/a_\tau=4$ a_s=0.039fm

N_τ	24	32	40	48	56	64	72	80	96
T/T_C	3.11	2.33	1.86	1.55	1.33	1.17	1.04	0.93	0.78
N_{meas}	3270	2030	1940	1110	1410	910	860	1190	1800

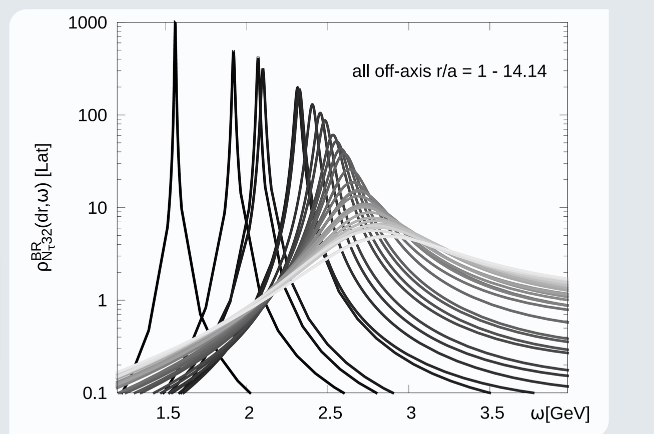
T_C=271MeV close to continuum

- Spectra from Wilson Line correlators in Coulomb gauge $W_{||}(r,\tau)$ to avoid cusp divergences in the Wilson loop

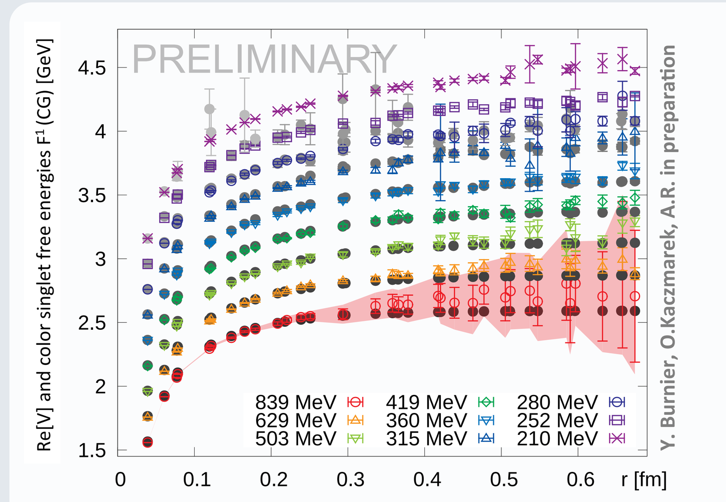


Bayesian spectral reconstruction

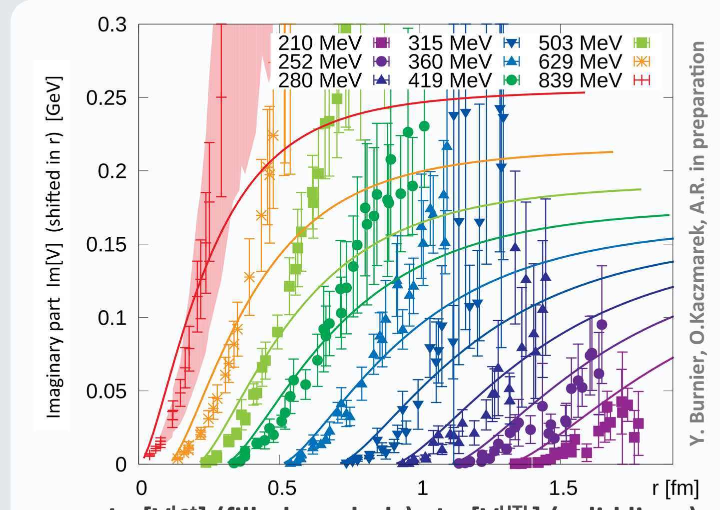
N_ω=4000, I_ω^{num}=[-13,14.5]
τ^{max}_{num}=20, m(ω)=const.
512bits prec., Δ^{min}=10^{-60}



- Wilson line correlators indicate Im[V]≠0 above T_C: curvature along τ corresponds to a finite spectral width
- Long standing question: Re[V] and Coulomb gauge color singlet free energies $F^1(r)=T\log[W_{||}(r,\beta)]$ similar ?



Re[V] (colored open symbols) F^1 (gray circles)



$\mathrm{Im}[V^{Lat}]$ (filled symbols) $\mathrm{Im}[V^{HTL}]$ (solid lines)

- Error bars denote statistical uncertainly (Jackknife), error band adds systematics (default model dependence)
- Spatial distances are corrected for lattice discretization artefacts by comparing free correlators to continuum Necco, Sommer Nucl.Phys. B622 (2002) 328-346
- **Re[V] transitions from confining to Debye screened** behavior and lies **close to the color singlet free energies** compare with: Y. Burnier et.al. , JHEP 01 (2010) 054
- **Im[V]** is reconstructed robustly at small r : **similar magnitude** as predictions from **HTL perturbation theory**
- Testing in progress: changing T by N_τ modifies reconstruction accuracy, need to quantify this N_τ dependence

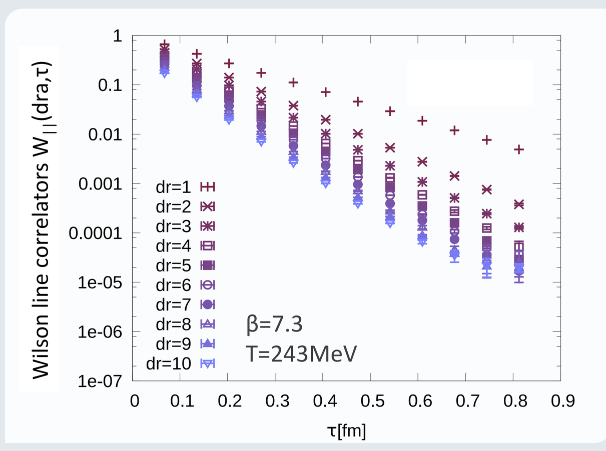
Dynamical N_f=2+1 Lattice QCD (HotQCD)

- Fixed box approach: isotropic lattices with asqtad action 48^3x12 (m_l/m_s=0.05)

β	6.80	6.90	7.00	7.125	7.25	7.30	7.48
T/T_C	0.85	0.94	1.04	1.18	1.33	1.39	1.64
a [fm]	0.111	0.100	0.090	0.080	0.071	0.068	0.057
N_{meas}	1295	1340	1015	840	1620	1150	1130

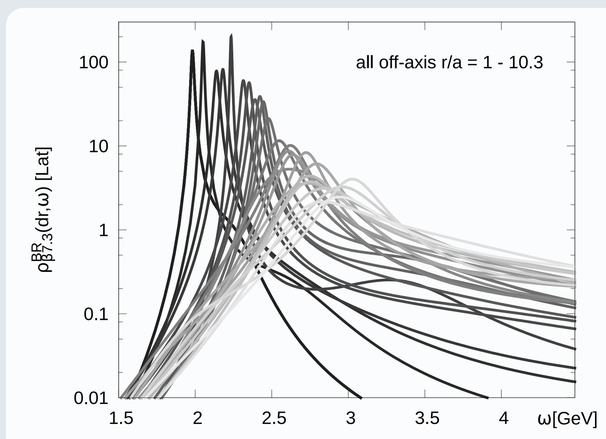
A. Bazavov et. al. PRD 85 (2012) 054503

T_C=174(3) MeV
M_π ~300 MeV

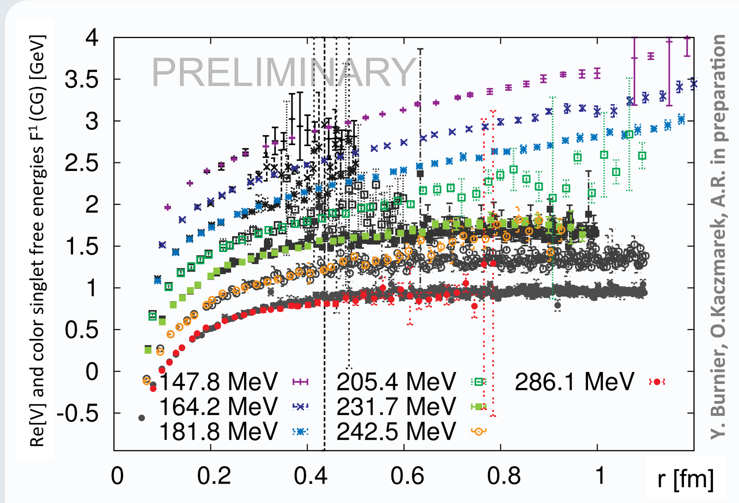


Bayesian spectral reconstruction

N_ω=4600, I_ω^{num}=[-11,12]
τ^{max}_{num}=20, m(ω)=const.
512bits prec., Δ^{min}=10^{-60}



- Wilson line correlators above T_C show increased curvature with T and R: spectral width present, thus Im[V]≠0



Re[V] (colored open symbols) F^1 (gray circles)

- Investigate Re[V] only, since N_τ=12 does not allow quantitative determination of spectral widths
- Since we use all available datapoints: stable reconstruction of linear rising part of Re[V] at low T possible (F^1 too noisy)
- Clear change from confinement to Debye screening visible, **Re[V] lies close to the color singlet free energies**
- Quantitative testing in progress: Limitations of reconstruction accuracy at different T

Conclusion

- We showed how the effective field theory based definition of the real-time in-medium heavy quark potential V^{QQ} can be used to express its values in terms of spectral functions.
- A novel Bayesian reconstruction method for spectral functions enables the controlled extraction of Re[V^{QQ}] and Im[V^{QQ}] from non-perturbative and first-principles lattice QCD simulations in Euclidean time.
- Our study on quenched and fully dynamical lattices shows that the **real-part** of the in-medium potential lies **close to the color singlet free energies** in Coulomb gauge. The extraction of the **imaginary part** reveals that its values are of the **same order of magnitude as the predictions from resummed HTL perturbation theory** down to close above the phase transition.
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