Testing the Standard Model with the lepton g-2

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Outline

- 1. μ: The muon g-2: a quick theory update
- 2. e: Testing the Standard Model with the electron g-2
- 3. τ: The tau g-2: opportunities or fantasies?

1. The muon g-2: a quick theory update

The anomalous magnetic moment: the basics

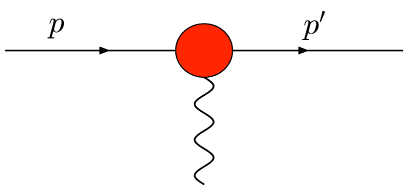
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

- Study the photon–lepton vertex:

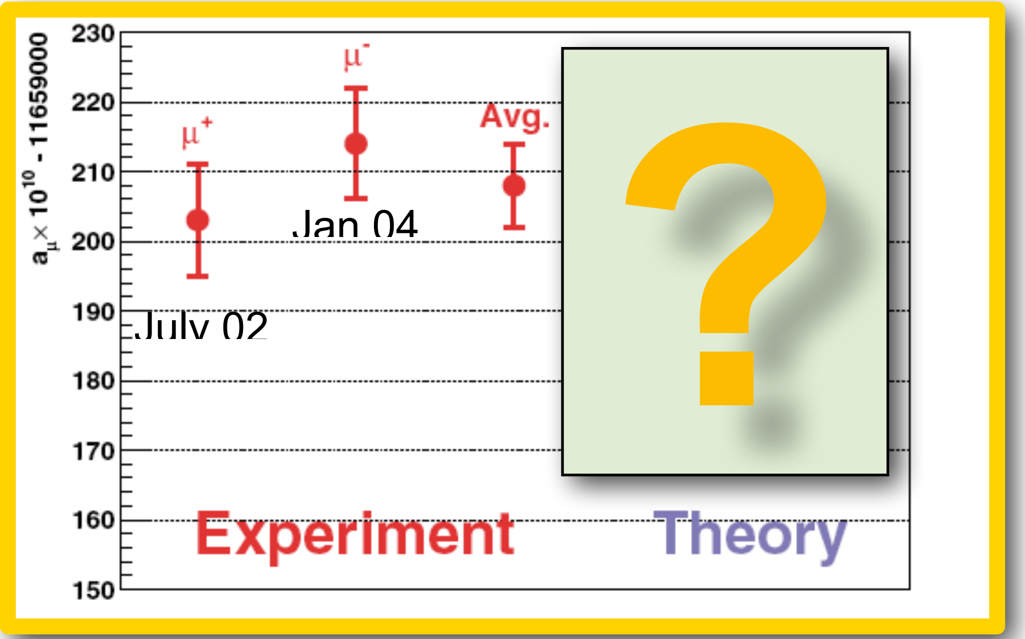


$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a_l$$

A pure “quantum correction” effect!

The muon g-2: the experimental result



- **Today:** $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- **Future:** new muon g-2 experiments proposed at:
  - Fermilab E989, aiming at $\pm 16 \times 10^{-11}$, ie 0.14ppm
  - J-PARC aiming at 0.1 ppm
- Are theorists ready for this (amazing) precision? No(t yet)

The muon g-2: the QED contribution

a_μ^{QED} = $(1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann ’57; Elend ’66; MP ’04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

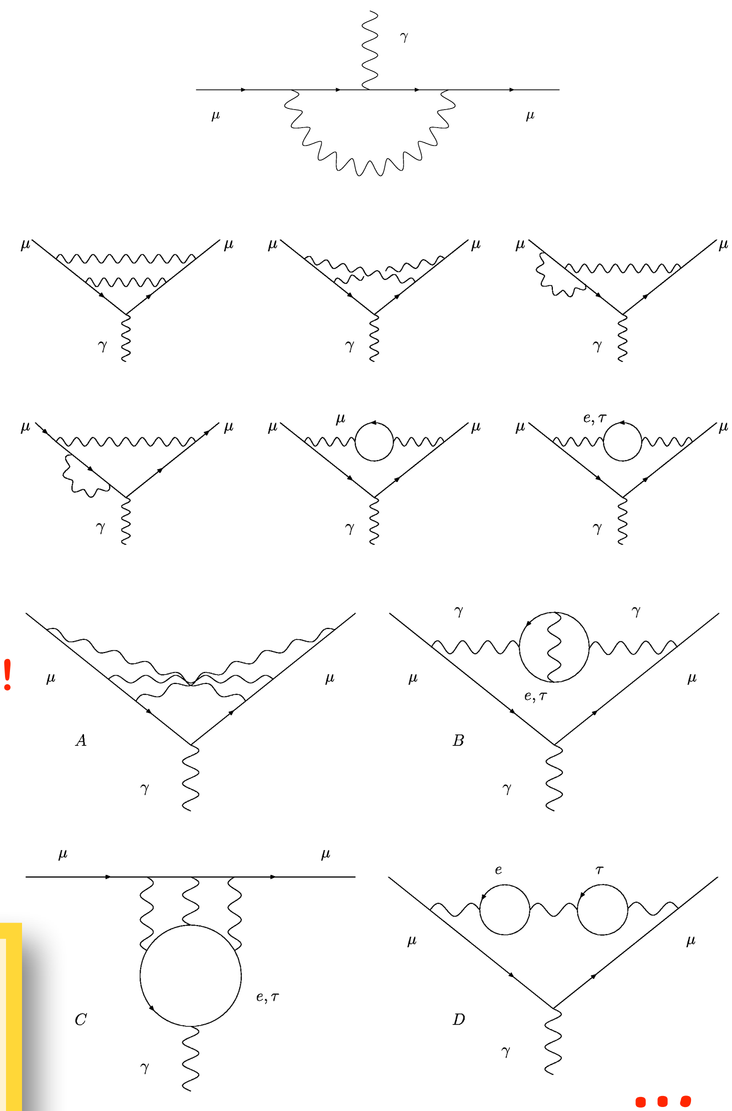
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP ’04;
Friot, Greynat & de Rafael ’05, Mohr, Taylor & Newell 2012

$+ \; 130.8796\,(63)\,(\alpha/\pi)^4$

Kinoshita & Lindquist ’81, … , Kinoshita & Nio ’04, ’05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,
Steinhauser et al. 2013 (analytic, in progress).

$+ \; 753.29\,(1.04)\,(\alpha/\pi)^5$ **COMPLETED!**

Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,…, Kataev, Kinoshita & Nio ’06, Kinoshita et al. 2012



Adding up, we get:

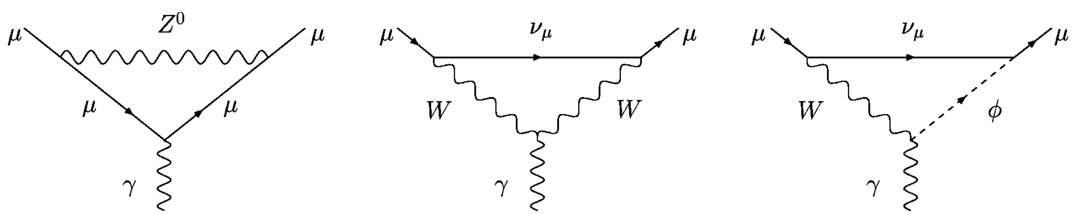
a_μ^{QED} = 116584718.951 (22)(77) x 10^{-11}

from coeffs, mainly from 4-loop unc ↲ ↳ from δα(Rb)

with α=1/137.035999049(90) [0.66 ppb]

The muon g-2: the electroweak contribution

- ## One-loop term:



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

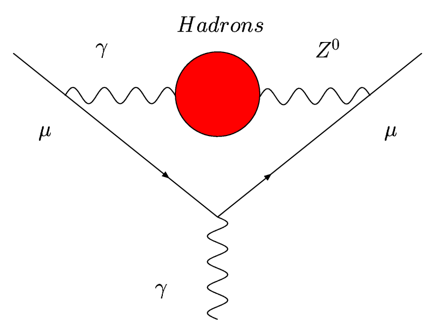
- ## One-loop plus higher-order terms:

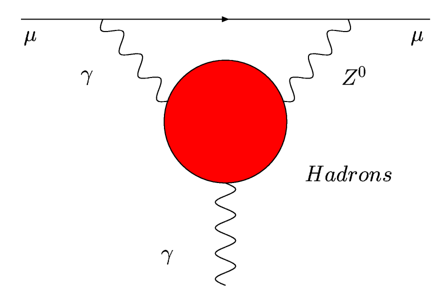
$a_\mu^{\mathrm{EW}} = 153.6\ (1) \times 10^{-11}$

with M_{Higgs} = 125.6 (1.5) GeV

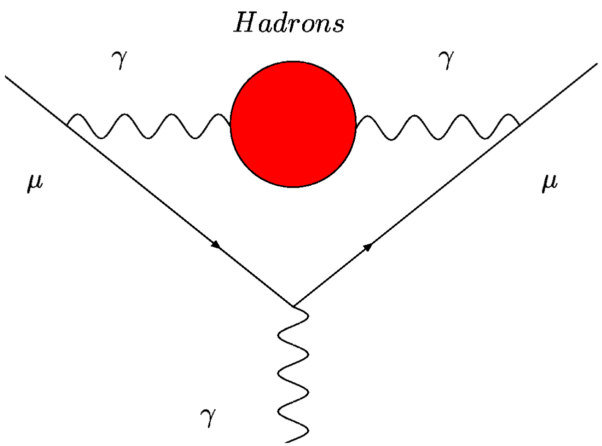
Hadronic loop uncertainties and 3-loop nonleading logs.

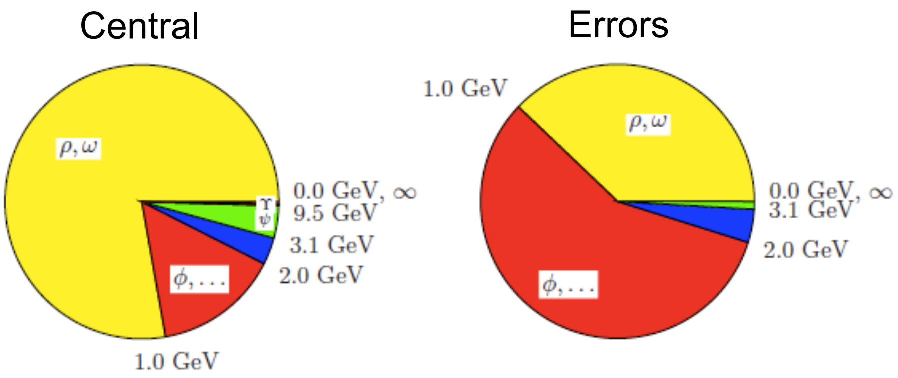
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.





The muon g-2: the hadronic LO contribution (HLO)





F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \qquad a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

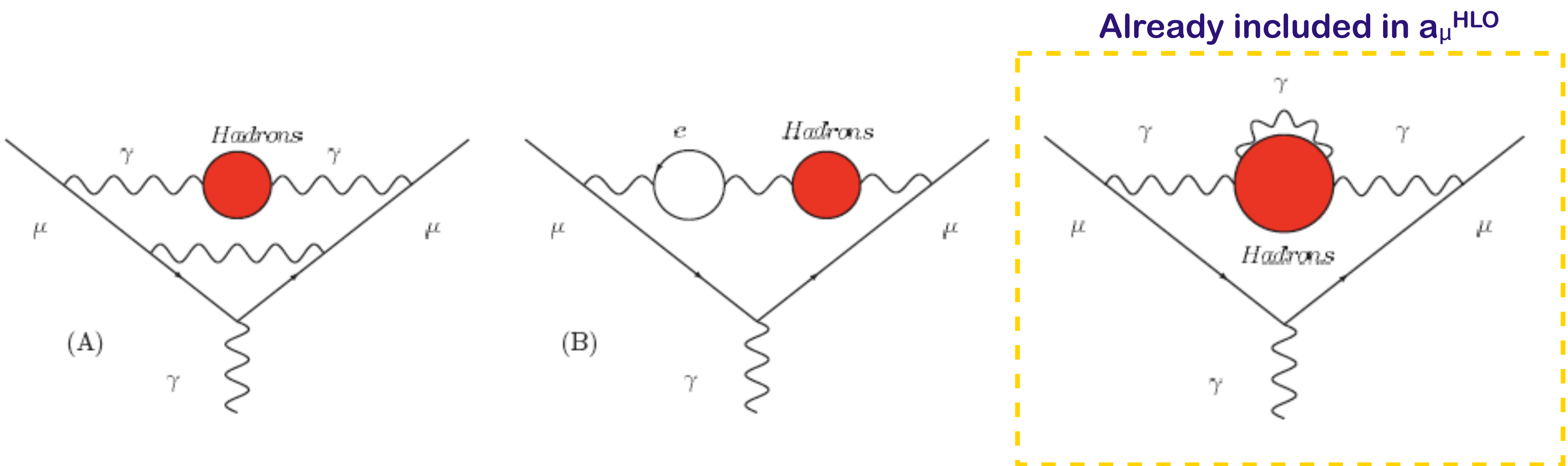
$a_\mu^{\mathrm{HLO}} = 6903\ (53)_{\mathrm{tot}} \times 10^{-11}$ F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$= 6923\ (42)_{\mathrm{tot}} \times 10^{-11}$ Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2π)

$= 6949\ (37)_{\mathrm{exp}}\ (21)_{\mathrm{rad}} \times 10^{-11}$ Hagiwara et al, JPG 38 (2011) 085003

- **Radiative Corrections are crucial!** S.Actis et al, Eur. Phys. J. C66 (2010) 585
- **New precise e^+e^- data: see Eidelman's and Lukin's talks.**

HNLO: Vacuum Polarization

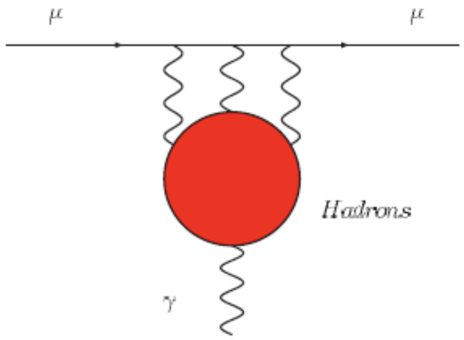


O(α^3) contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HNLO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

HNLO: Light-by-light contribution



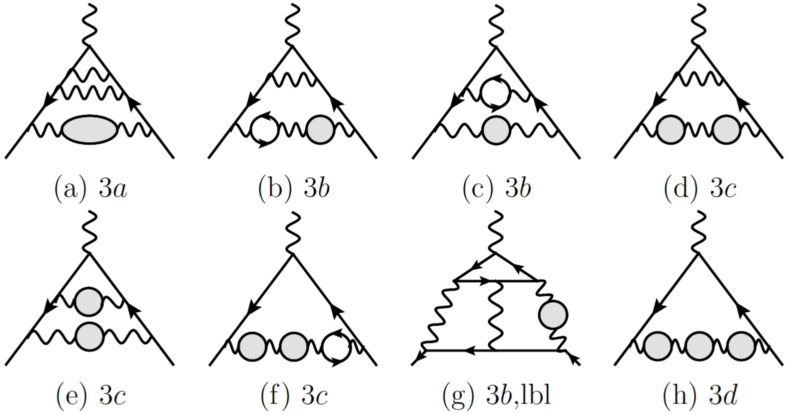
- This term had a troubled life! Latest values:

$a_\mu^{HNLO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_\mu^{HHO}(lbl) = +116\ (39) \times 10^{-11}$	Jegerlehner & Nyffeler '09

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades

- "Bound" $a_\mu^{HNLO}(lbl) < \sim 160 \times 10^{-11}$ Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11
- Dyson-Schwinger equations approach: see Gernot Eichmann's talk.
- Pion exch. in holographic QCD agrees. D.K.Hong, D.Kim '09; Cappiello, Catà, D'Ambrosio '11
- Lattice? Very hard but promising: see Tom Blum's talk.
- Dispersive method recently proposed: see Gilberto Colangelo's talk.

- **HNNLO: Vacuum Polarization**



O(α^4) contributions of diagrams containing hadronic vacuum polarization insertions:

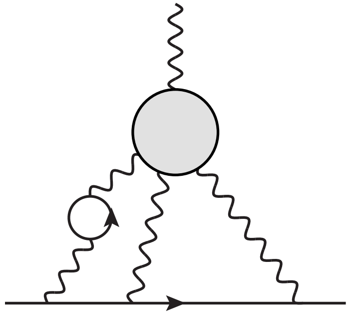
$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

- **HNNLO: Light-by-light**

$$a_\mu^{HNNLO}(lbl) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



The muon g-2: SM vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$a_\mu^{EXP} = 116592091\,(63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda = \mu_\mu/\mu_p$ from CODATA'10

$a_\mu^{SM} \times 10^{11}$	$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}$	σ
116 591 809 (66)	$282\ (91) \times 10^{-11}$	3.1 [1]
116 591 829 (57)	$262\ (85) \times 10^{-11}$	3.1 [2]
116 591 855 (58)	$236\ (86) \times 10^{-11}$	2.8 [3]

with the "conservative" $a_\mu^{HNLO}(lbl) = 116\ (39) \times 10^{-11}$ and the LO hadronic from:

[1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1
[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 2π)
[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 2π)

Note that the th. error is now about the same as the exp. one

The muon g-2: connection with the SM Higgs mass

- Δa_μ can be explained by errors in QED, EW, HNLO, g-2 EXP, HLO, or, we hope, by New Physics!
- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3}, \; s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

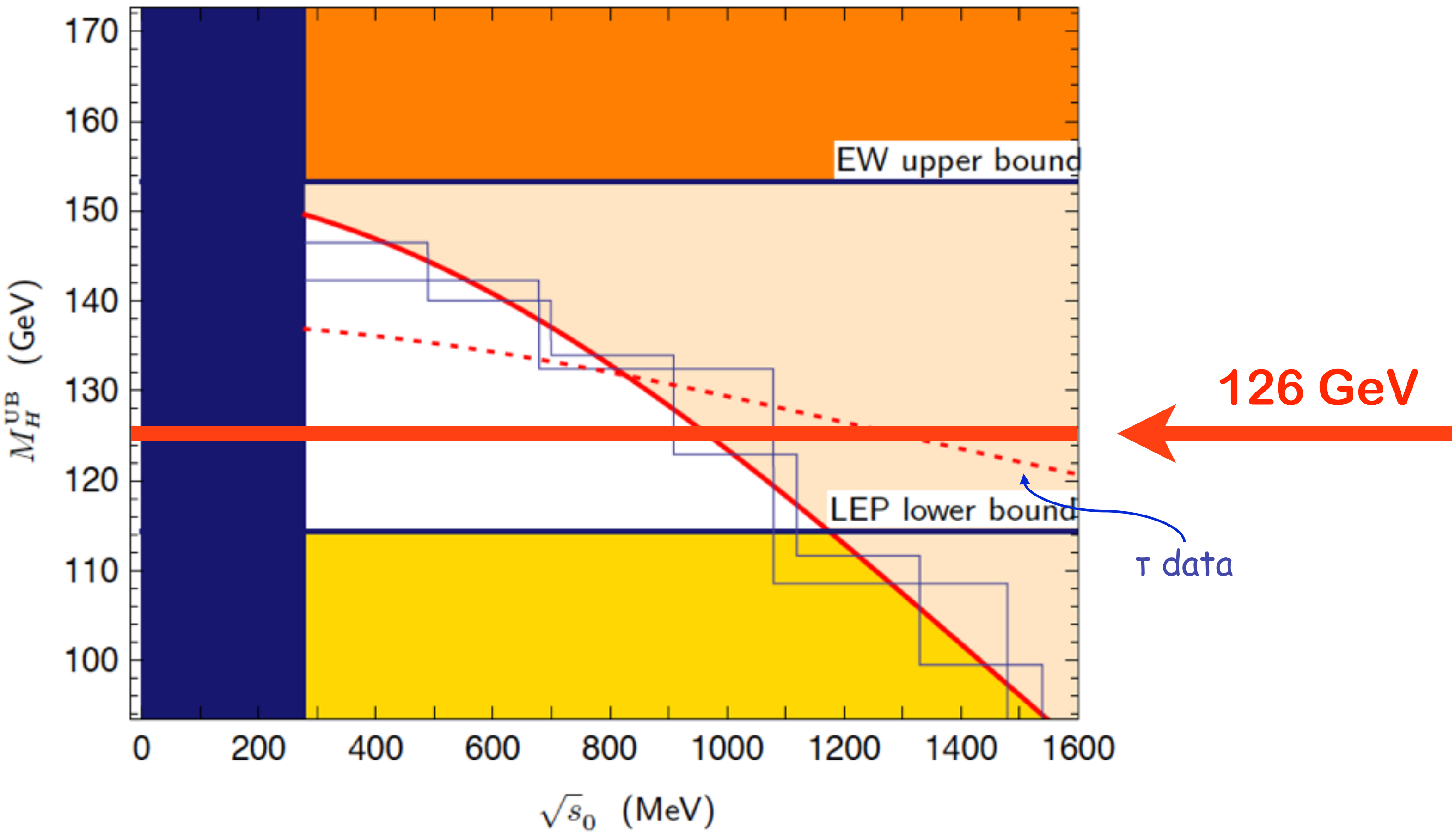
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

(ε>0), in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

The muon g-2: connection with the SM Higgs mass (2)

- How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (3)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

- Also, given a 126 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV).

- Vice versa, assuming we now have a SM Higgs with M_{Higgs} = 126 GeV, if we bridge the M_{Higgs} discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

2. Testing the SM with the electron g-2

G.F. Giudice, P. Paradisi & MP, JHEP 1211 (2012) 113

M. Fael & MP, arXiv:1402.1575 (PRD to appear)

The QED prediction of the electron g-2

$$a_e^{QED} = + (1/2)(\alpha/\pi) \;\; - \; 0.328\,478\,444\,002\,55(33)\,(\alpha/\pi)^2$$

Schwinger 1948 Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78...$$

$$A_2^{(4)}\,(m_e/m_\mu) = 5.197\,386\,68\,(26) \times 10^{-7}$$

$$A_2^{(4)}\,(m_e/m_\tau) = 1.837\,98\,(33) \times 10^{-9}$$

$O(10^{-18})$ in a_e

$$+ \; 1.181\,234\,016\,816\,(11)\,(\alpha/\pi)^3$$

$O(10^{-19})$ in a_e

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587...$$

$$A_2^{(6)}\,(m_e/m_\mu) = -7.373\,941\,62\,(27) \times 10^{-6}$$

$$A_2^{(6)}\,(m_e/m_\tau) = -6.5830\,(11) \times 10^{-8}$$

$$A_3^{(6)}\,(m_e/m_\mu, m_e/m_\tau) = 1.909\,82\,(34) \times 10^{-13}$$

$$- \; 1.9097\,(20)\,(\alpha/\pi)^4$$

$0.6\ 10^{-13}$ in a_e

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012;
Kurz, Liu, Marquard & Steinhauser 2014: analytic mass dependent part.

$$+ \; 9.16\,(58)\,(\alpha/\pi)^5$$

Complete Result! (12672 mass indep. diagrams!)

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807; work in progress to reduce the error.

$0.4\ 10^{-13}$ in a_e

NB: $(\alpha/\pi)^6 \sim O(10^{-16})$

The SM prediction of the electron g-2

- **The SM prediction is:**

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- **The EW (1&2 loop) term is:** Czarnecki, Krause, Marciano '96 [value from Codata10]

$$a_e^{EW} = 0.2973\,(52) \times 10^{-13}$$

- **The Hadronic contribution (LO+NLO+NNLO) is:**

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{HAD} = 17.10\,(17) \times 10^{-13}$$

$$a_e^{HLO} = +\,18.66\,(11) \times 10^{-13}$$

$$a_e^{HNLO} = [-2.234(14)_{vac} + 0.39(13)_{lbl}] \times 10^{-13}$$

$$a_e^{HNNLO} = +\,0.28\,(1) \times 10^{-13}$$

- **Which value of α should we use to compute a_e^{SM}?**

The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{SM}(\alpha) = a_e^{EXP}$ → best determination of alpha:

  $\alpha^{-1} = 137.035\,999\,177\ (34)$ [0.25 ppb]

- Compare it with other determinations (independent of a_e):

  $\alpha^{-1} = 137.036\,000\,0\ (11)$ [7.7 ppb] PRA73 (2006) 032504 (Cs)

  $\alpha^{-1} = 137.035\,999\,049\ (90)$ [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

The electron g-2: SM vs. Experiment

- Using α = 1/137.035 999 049 (90) [^{87}Rb, 2011], the SM prediction for the electron g-2 is

$$a_e^{SM} = 115\,965\,218\,18.1\ (0.6)\ (0.4)\ (0.2)\ (7.6) \times 10^{-13}$$

(0.6) → δC_4^{qed}; (0.4) → δC_5^{qed}; (0.2) → δa_e^{had}; (7.6) → from $\delta\alpha$

- The EXP-SM difference is (note the negative sign):

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -10.8\ (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1.3σ).
NB: The 4-loop contrib. to a_e^{QED} is $-556 \times 10^{-13} \sim 70\ \delta\Delta a_e$!
(the 5-loop one is 6.2×10^{-13})

The electron g-2 sensitivity and NP tests

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.6)_{\mathrm{QED4}}, \quad (0.4)_{\mathrm{QED5}}, \quad (0.2)_{\mathrm{HAD}},}_{(0.7)_{\mathrm{TH}}} \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\mathrm{EXP}}}$$

⬅ may drop to 0.2 or 0.3

- The $(g\text{-}2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.

→ sensitivity of 10^{-13} may be reached with ongoing exp. work

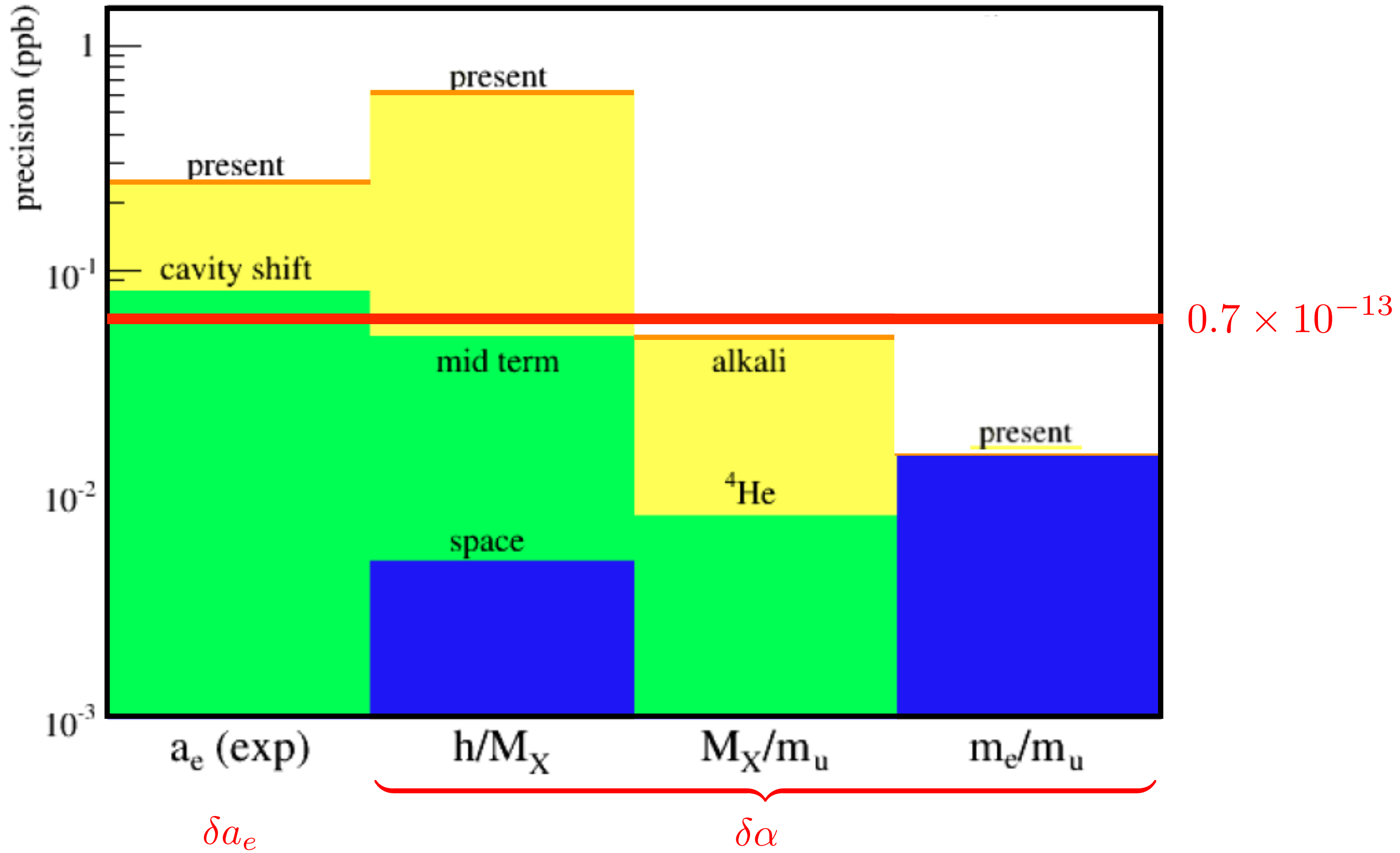
- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (2)



Summary of the exp. contributions to the relative uncertainty of Δa_e (in ppb)

F. Terranova & G.M. Tino, PRA89 (2014) 052118

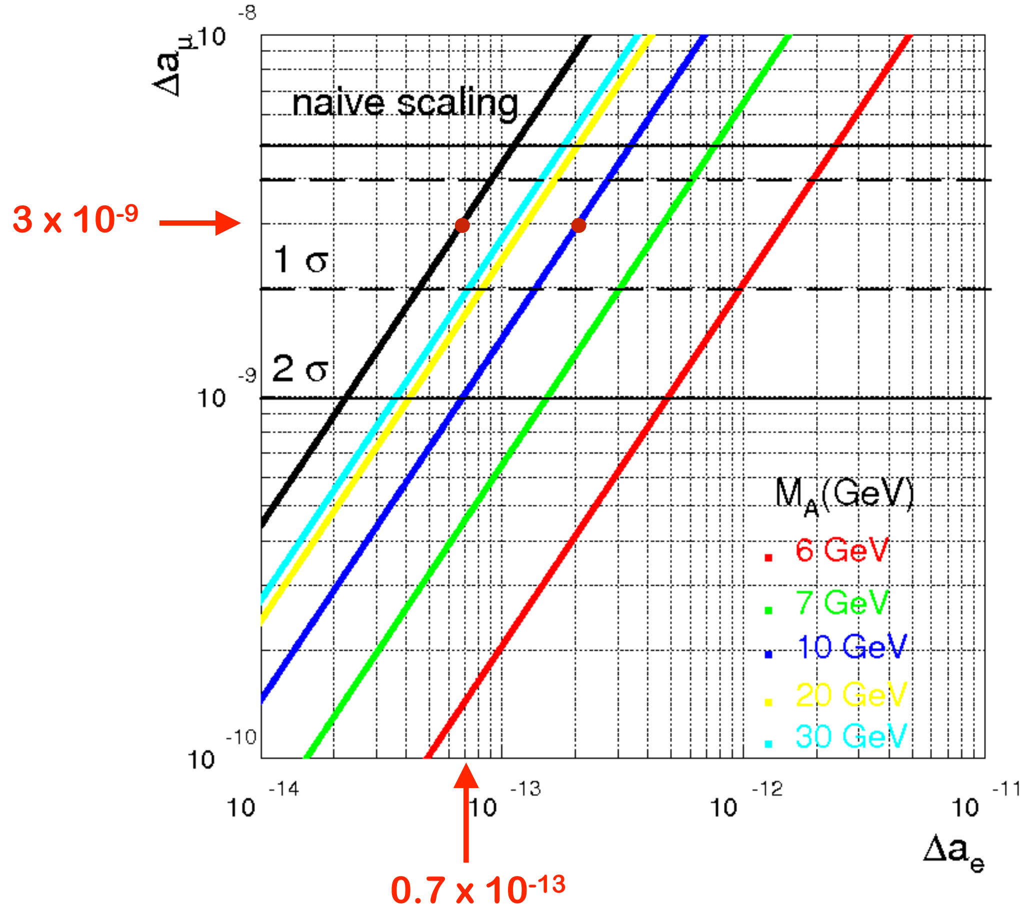
The electron g-2 sensitivity and NP tests (3)

- The experimental sensitivity in Δa_e is not very far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

- NP scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e and contributions to EDMs, LFV or lepton universality breaking observables.

- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

The electron g-2 sensitivity and NP tests (4)

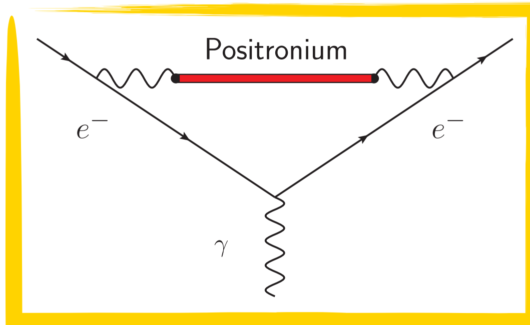


- Example: light pseudoscalars. Interplay between 1-loop and 2-loop contributions. NS violated, Δa_e lies above its naive expectation.

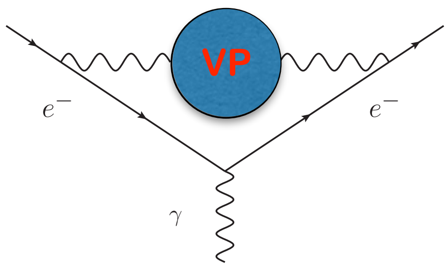
Is there a positronium contribution to the electron g-2?

- **The leading contribution of positronium to a_e comes from:**

Mishima 1311.7109; Fael & MP 1402.1575; Melnikov et al. 1402.5690; Eides 1402.5860; Hayakawa 1403.0416



- **The e^+e^- bound states appear as poles in the vac. pol. $\Pi(q^2)$ right below the branch-point $q^2 = (2m)^2$. Their contribution is:**



$$a_e(\mathrm{vp}) = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s}\,\mathrm{Im}\,\Pi(s+i\epsilon)\,K(s)$$

$$a_e^{\mathrm{P}} = \frac{\alpha^5}{4\pi}\zeta(3)\left(8\ln 2 - \frac{11}{2}\right) = 0.9\times 10^{-13} = 1.3\left(\frac{\alpha}{\pi}\right)^5$$

Mishima 1311.7109

- **Of the same magnitude of the exp. unc. of a_e & the naively rescaled muon Δa_μ. Of the same order of α as the 5-loop term!**

Is there a positronium contribution to the electron g-2? (II)

- **Melnikov, Vainshtein & Voloshin (MVV)** 1402.5690 **determined a nonpert. contrib. of the e^+e^- continuum right above threshold that cancels one-half of a_e^P:**

$$a_e(\mathrm{vp})^{\mathrm{cont,np}} = -\frac{|\alpha|^5}{8\pi}\zeta(3)\left(8\ln 2 - \frac{11}{2}\right)$$

- **In fact the total positronium poles + continuum nonperturbative contribution to a_e arising from the threshold region at LO in α is:**

$$a_e^{\mathrm{thr}}(\mathrm{vp}) = -\frac{\alpha}{\pi} K(4m^2)\,\mathrm{Re}\,A(1)$$

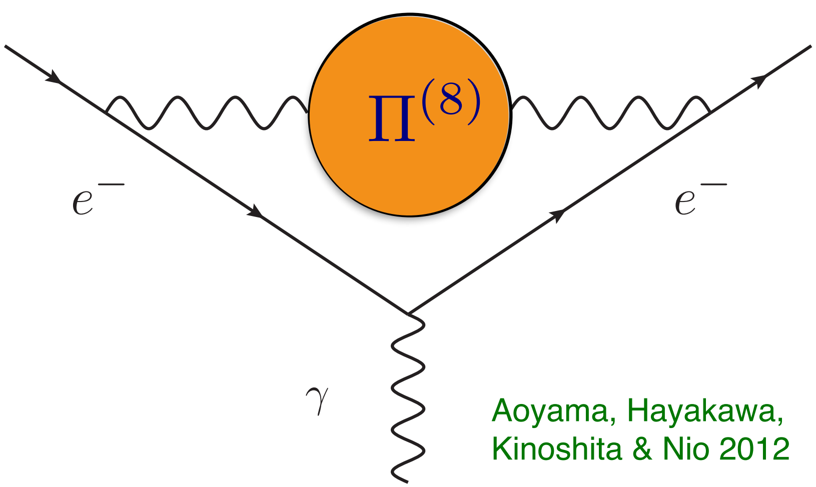
with

$$A(\beta) = -\frac{\alpha^2}{2}\left[\gamma + \psi\left(1 - \frac{i\alpha}{2\beta}\right)\right] = \frac{\alpha^2}{2}\sum_{k=1}^{\infty}\zeta(k+1)\left(\frac{i\alpha}{2\beta}\right)^k$$

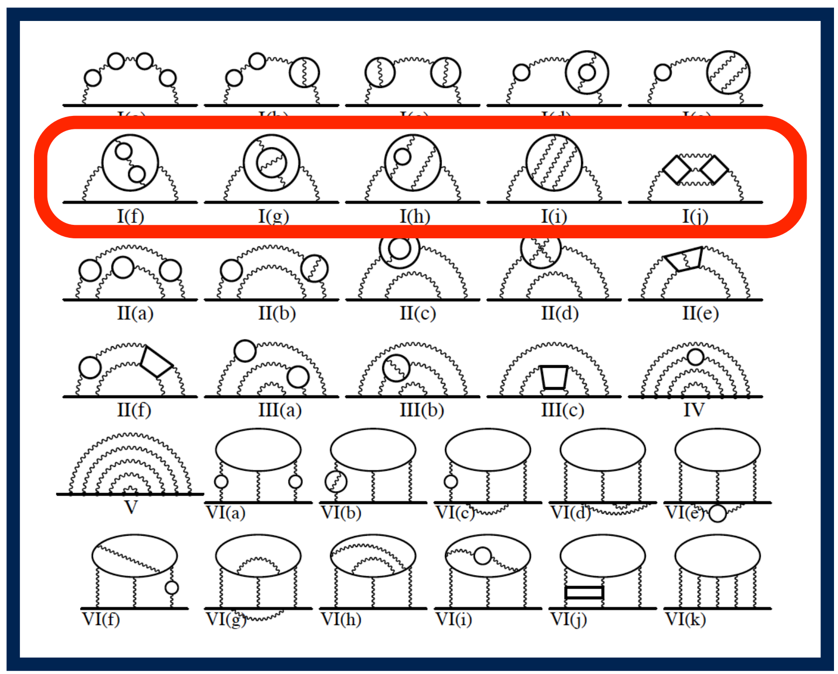
so that

$$a_e^{\mathrm{thr}}(\mathrm{vp}) = \frac{\alpha^5}{8\pi}\zeta(3)\,K(4m^2) = \frac{a_e^{\mathrm{P}}}{2}$$

- So, should we add this total threshold contribution $a_e^P/2$ to the perturbative QED 5-loop result of Kinoshita and collaborators?
- Using the Coulomb Green's function, MVV 1402.5690 argued that it is already contained in the contribution of $O(\alpha^5)$.
- Hayakawa 1403.0416 claimed that positronium contributes to a_e only through a specific class of diagrams of $O(\alpha^7)$.
- The 5-loop QED contribution to a_e arising from the insertion of the 4-loop VP in the photon line has been computed via:



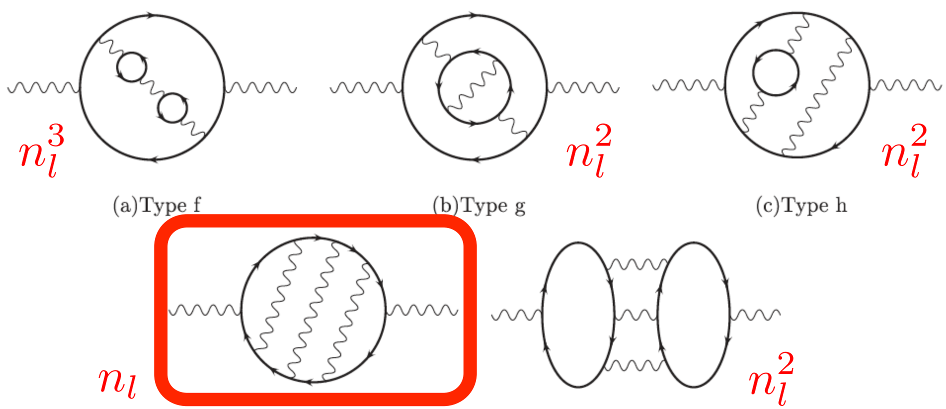
Aoyama, Hayakawa, Kinoshita & Nio 2012



- **Using explicit expressions for $\Pi^{(8)}(q^2)$ (Baikov, Maier, Marquard '13) we obtain:**

$$a_e^{(10)}(\mathrm{vp}) = -\frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Pi^{(8)}\!\left(-\frac{m^2x^2}{1-x}\right)$$

$$a_e^{(10)}(\mathrm{vp}) \;=\; n_e\,\frac{\alpha^5}{8\pi}\zeta(3)\,K(4m^2) + \cdots \;=\; \frac{a_e^{\mathrm{P}}}{2} + \cdots$$



(a)Type f (b)Type g (c)Type h

(d)Type i (e)Type j

- **$a_e^P/2$ is already included in the 5-loop contrib. of class I(i).**
- **There is no additional contrib of QED bound states beyond PT!**

M.A. Braun 1968; Barbieri, Christillin, Remiddi 1973

3. The tau g-2: opportunities or fantasies?

Work in progress in collaboration with
S. Eidelman, D. Epifanov, M. Fael, L. Mercolli

arXiv:1301.5302
arXiv:1310.1081

The SM prediction of the tau g-2

- The Standard Model prediction of the tau g-2 is:

a_τ^{SM} =		117324	(2)	x 10^{-8}	QED
	+	47.4	(0.5)	x 10^{-8}	EW
	+	337.5	(3.7)	x 10^{-8}	HLO
	+	7.6	(0.2)	x 10^{-8}	HHO (vac)
	+	5	(3)	x 10^{-8}	HHO (lbl)

$$a_\tau^{SM} = 117721\ (5) \times 10^{-8}$$

Eidelman & MP 2007

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics, and a "clean" NP test too…

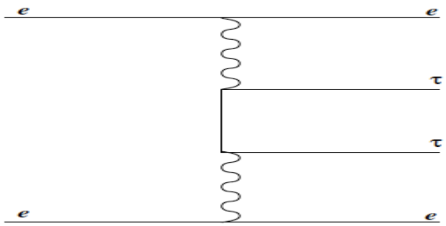
	Muon	Tau
aEW/aH	1/45	1/7
aEW/ δaH	3	10

… if only we could measure it!!

The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):



$$a_\tau = -0.018\ (17)$$

PDG 2012

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$-0.004 < a_\tau^{NP} < 0.006 \quad (95\%\ \mathrm{CL})$$

Escribano & Massó 1997

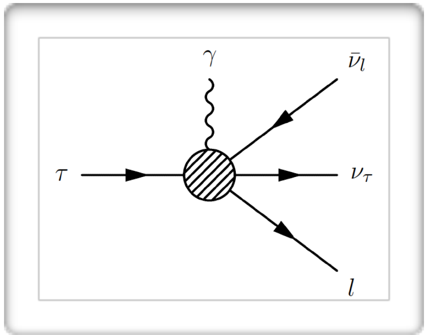
$$-0.007 < a_\tau^{NP} < 0.005 \quad (95\%\ \mathrm{CL})$$

Gonzáles-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of $F_2(q^2=M_\Upsilon^2)$ from $e^+e^- \to \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

The tau g-2 via its radiative leptonic decays: a proposal

- **Tau radiative leptonic decays at LO:**



$$\frac{d^3\Gamma}{dx\, dy\, d\cos\theta} = \frac{\alpha\, M_\tau^5\, G_F^2\, y\, \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6}\, G_0(x, y, \cos\theta, r)$$

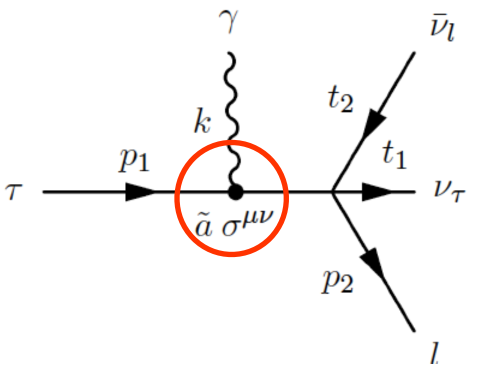
Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

$$\left.\frac{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau \gamma)}{\Gamma_{\rm total}}\right|_{E_\gamma > 10 {\rm MeV}} = 1.836\% \quad \text{vs} \quad 1.75(18)\%$$

CLEO 2000

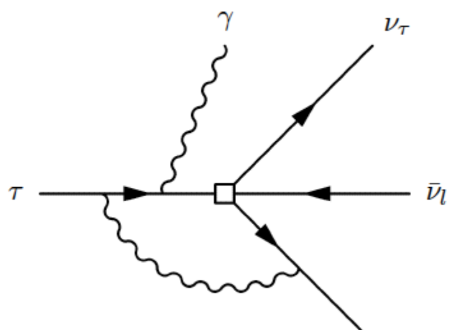
$$\left.\frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \gamma)}{\Gamma_{\rm total}}\right|_{E_\gamma > 10 {\rm MeV}} = 0.367\% \quad \text{vs} \quad 0.361(38)\%$$

$$x = \frac{2E_l}{M_\tau},\ y = \frac{2E_\gamma}{M_\tau},\ r = \frac{m_l}{M_\tau}$$



- **Add the contribution of the effective coupling and the QED corrections:**

$$G_0 \to G_0 + \tilde{a}_\tau G_a + \frac{\alpha}{\pi} G_{\rm RC}$$



(c) Diagram n.6

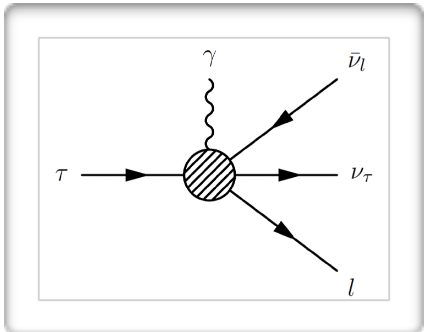
- **Measure d³Γ precisely and get $\tilde{a}_\tau$!**

[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]

- **Tau radiative leptonic decays at LO:**

$$\frac{d^3\Gamma}{dx\,dy\,d\cos\theta} = \frac{\alpha\, M_\tau^5\, G_F^2\, y\sqrt{x^2-4r^2}}{2\pi(4\pi)^6}\, G_0(x,y,\cos\theta,r)$$

Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

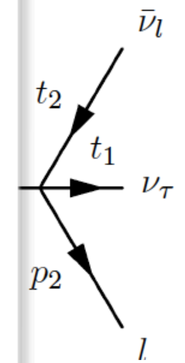


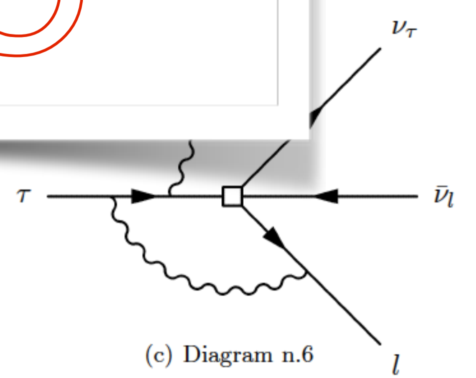
$\Gamma(\tau^-$ … , $r = \frac{m_l}{M_\tau}$

$\Gamma(\tau^-$ …

WORK IN PROGRESS

- **Add** …
coup …





(c) Diagram n.6

- **Measure d³Γ precisely and get $\tilde{a}_\tau$!**
[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]

Conclusions

- The lepton g-2 provide beautiful examples of interplay between theory and experiment.
- The muon discrepancy is $\Delta a_\mu \sim 3\div3.5\ \sigma$. Is it NP? New upcoming g-2 experiment: QED & EW terms ready for the challenge; how about the hadronic one? Future of LBL??
- Could Δa_μ be due to mistakes in the hadronic $\sigma(s)$? Very unlikely. Also, given a 126 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energies (below ~ 1GeV).
- The sensitivity of the electron g-2 has improved. It may soon be possible to test if Δa_μ manifests itself also in the electron g-2! A robust and ambitious experimental program is under way to improve α & a_e. The positronium contribution shouldn't be added.
- The tau g-2 is essentially unknown: we propose to measure it at Belle II via its radiative leptonic decays.

The End