An analysis of the nucleon spectral function in the nuclear medium from QCD sum rule

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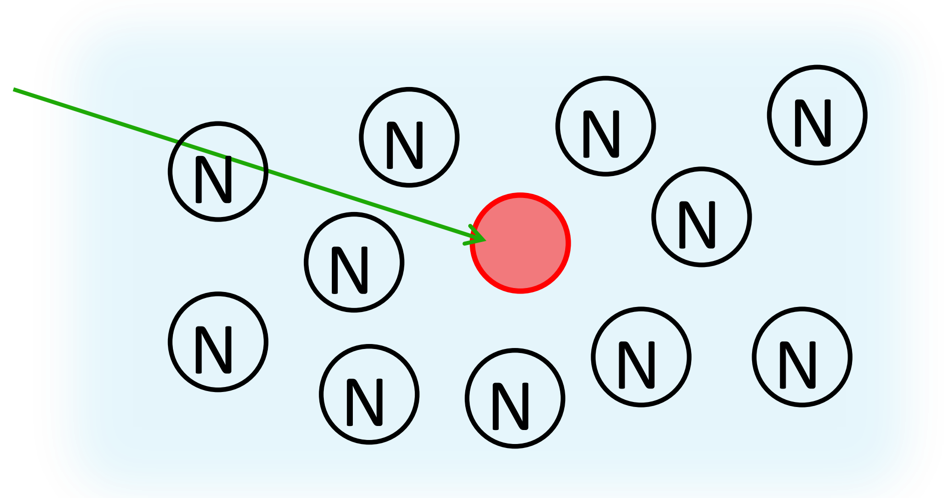
Introduction

- Hadron properties in the nuclear medium

Probe hadron



The properties of the hadron are modified in the medium.

Partial restoration of the chiral symmetry

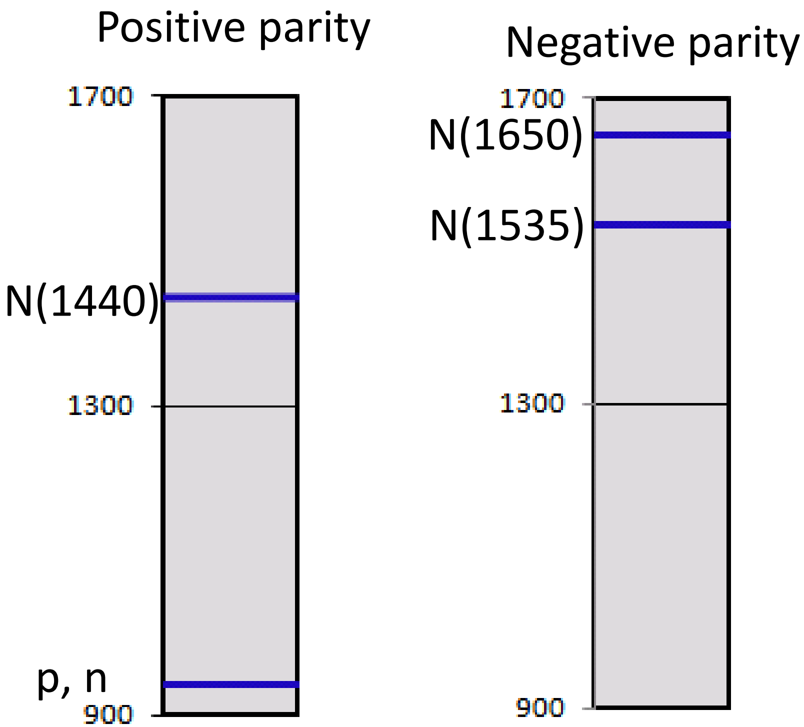
Interaction with the nucleons in the nuclear matter

Nuclear matter

We focus on the nucleon ground state and excited state.

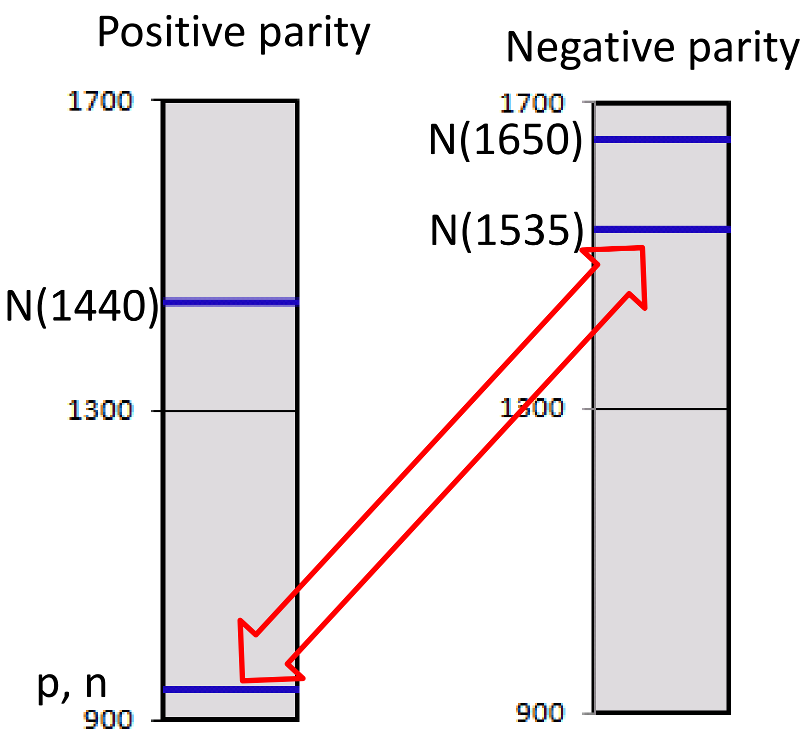
Introduction

Mass spectrum of the nucleons



Introduction

Mass spectrum of the nucleons

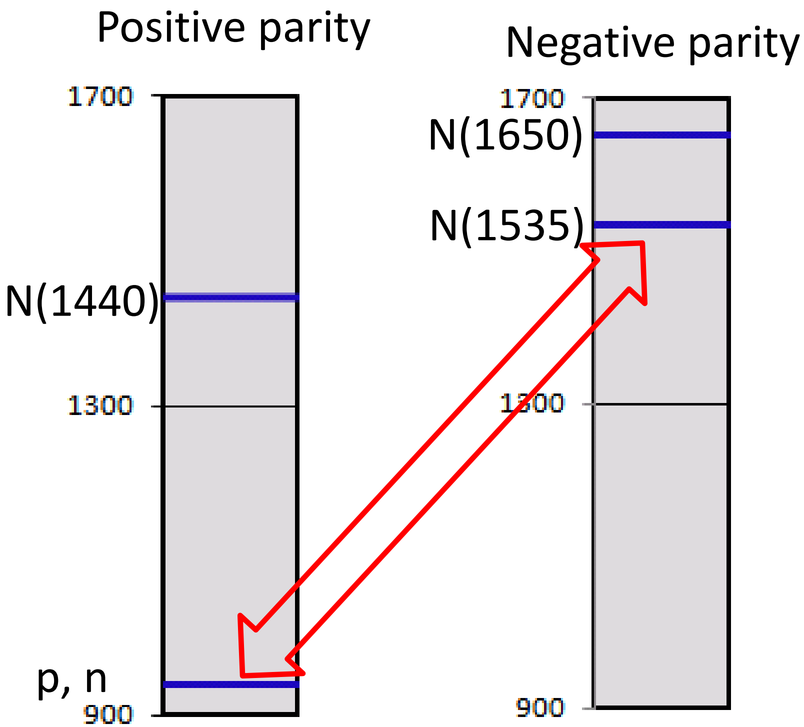


- The mass difference between nucleon ground state and N(1535) is about 600 MeV.
- It is predicted that Chiral symmetry breaking cause these difference.

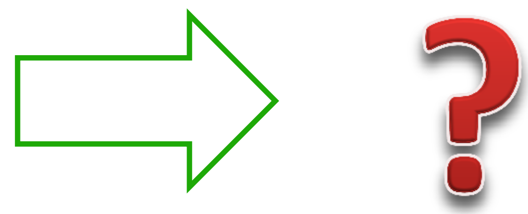
When chiral symmetry is restored, the mass spectrum will change.

Introduction

Mass spectrum of the nucleons



In the nuclear matter



When chiral symmetry is restored, the mass spectrum will change.

To investigate these properties from QCD, non perturbative method is needed.

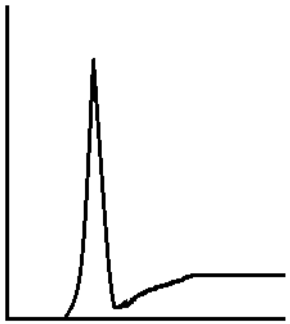
Analysis of QCD sum rule in nuclear matter

Nucleon QCD sum rule

Hadronic spectral function

$$\Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\overline{\eta}(0)] | 0 \rangle d^4x$$

$$= \int_0^\infty \frac{1}{\pi} \frac{\mathrm{Im}\Pi(t)}{t - q^2} dt = \int_0^\infty \frac{\rho(t)}{t - q^2} dt$$



is calculated by the operator product expansion (OPE)

Non perturbative contributions are expressed by some Condensates.

$\Rightarrow$ $\langle \overline{q} q \rangle$, $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ $\cdots$

An order parameter of chiral symmetry

We apply this method to the analyses in the nuclear matter.

Nucleon QCD sum rule

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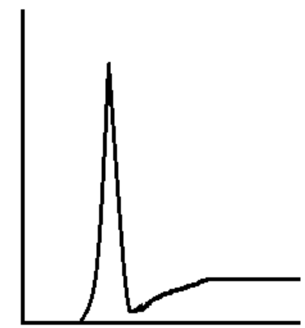

is calculated by the operator product expansion (OPE)

Application for the analyses in nuclear matter

$$i\int e^{iqx}\langle 0|T[\eta(x)\overline{\eta}(0)]|0\rangle d^4x \Rightarrow i\int d^4x e^{iqx}\langle \underline{\Psi_0}|T[\eta(x)\overline{\eta}(0)]|\underline{\Psi_0}\rangle$$

Modification: $\langle 0|O_i|0\rangle \Rightarrow \langle \Psi_0|O_i|\Psi_0\rangle$ Ψ_0 : Ground state of nuclear matter

Chiral condensate : $\langle \overline{q}q\rangle_0 \Rightarrow \langle \overline{q}q\rangle_\rho = \langle \overline{q}q\rangle_0 + \frac{\sigma_N}{2m_q}\rho + \cdots$

New condensate: $\Rightarrow \langle q^\dagger q\rangle_\rho = \frac{3}{2}\rho$

Nucleon QCD sum rule

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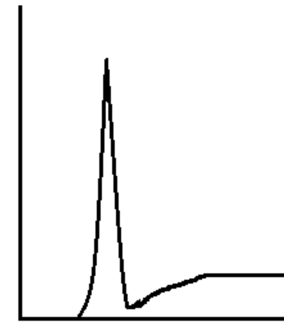


is calculated by the operator product expansion (OPE)

Gaussian sum rule

$$G(s,\tau) = \int_0^\infty \rho(\omega)\,\frac{1}{\sqrt{4\pi\tau}}\exp\Big(-\frac{(\omega^2-s)^2}{4\tau}\Big)d\omega$$

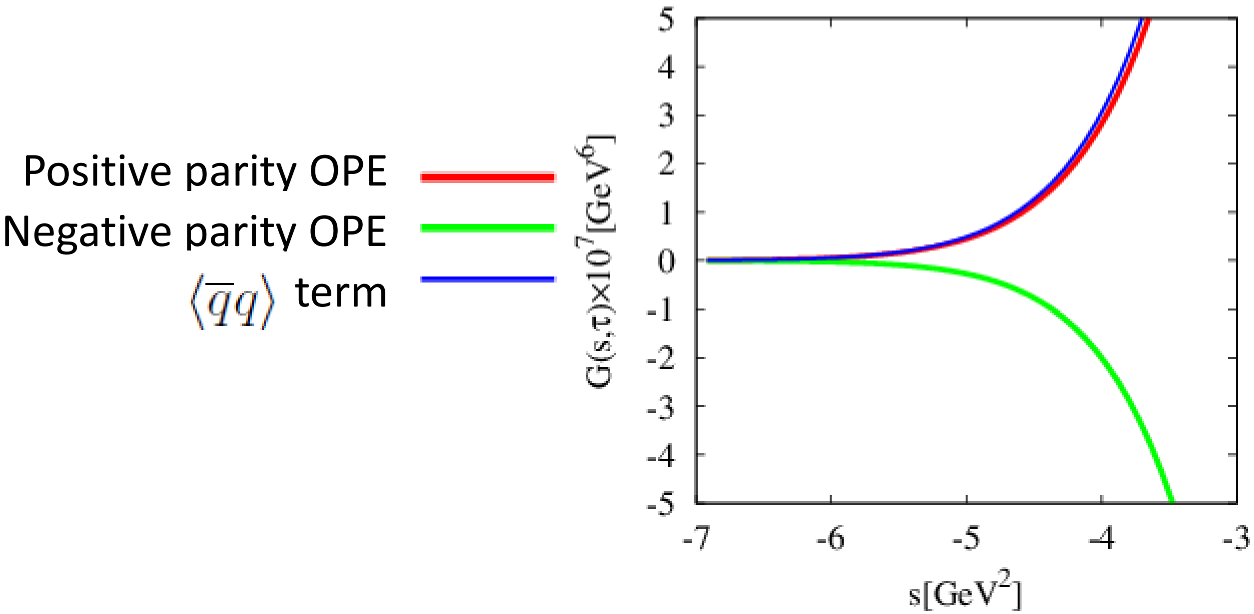
is calculated by OPE

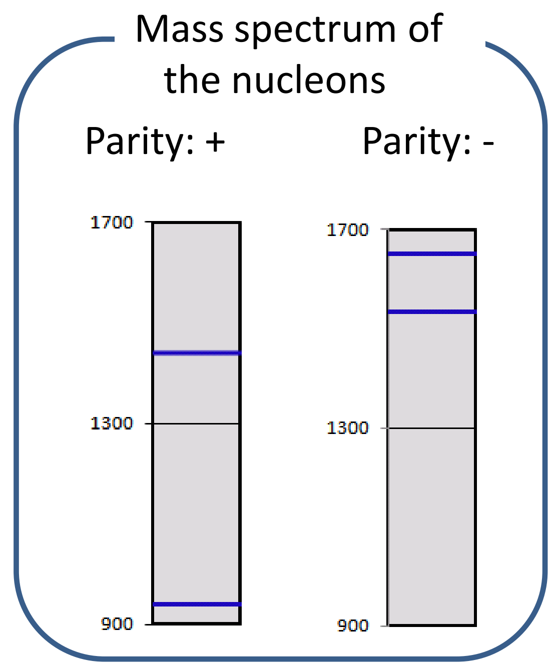
τ、s: parameter

Nucleon QCD sum rule

The behavior of the OPE data in the vacuum

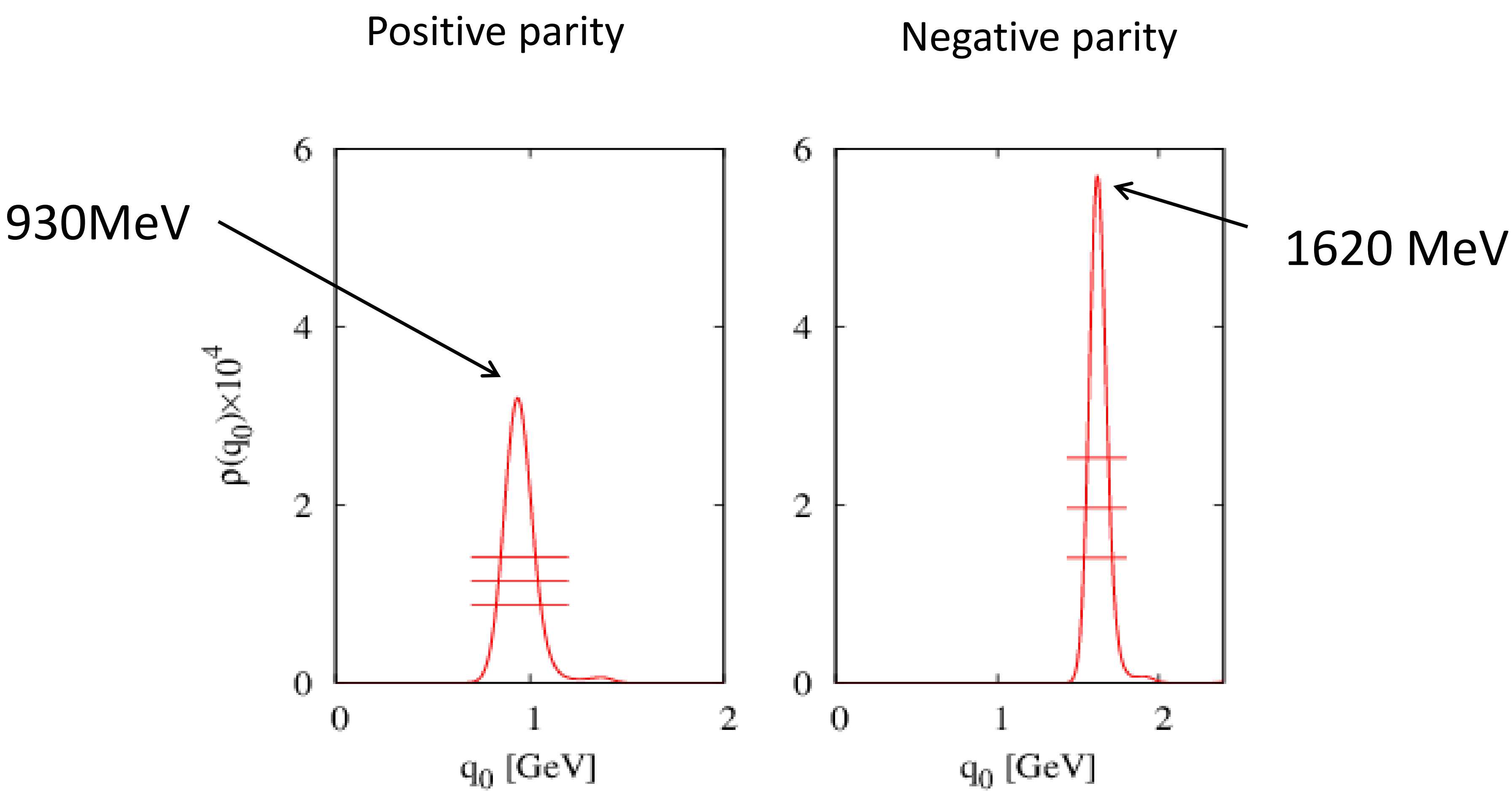
$$G^{\pm}(s,\tau) = [C_0(s,\tau) + C_4(s,\tau)\langle\frac{\alpha_s}{\pi}G^2\rangle + C_6(s,\tau)\langle\bar{q}q\rangle^2]$$
$$\underline{\pm}\,[C_3(s,\tau)\langle\bar{q}q\rangle + C_5(s,\tau)\langle\bar{q}g\sigma\cdot Gq\rangle + C_7(s,\tau)\langle\bar{q}q\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle]$$





The difference between positive parity and negative parity is mainly caused by chiral condensate term.

Nucleon QCD sum rule



In both positive and negative parity, the peaks are found.

In the negative parity analysis, the peak correspond to the N(1535) or (and) N(1650).

Nucleon QCD sum rule

The behavior of the OPE data in the nuclear matter

Vacuum

$$\Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\overline{\eta}(0)] | 0 \rangle d^4x$$

Nuclear matter

$$i \int d^4x e^{iqx} \langle \Psi_0 | T[\eta(x)\overline{\eta}(0)] | \Psi_0 \rangle$$

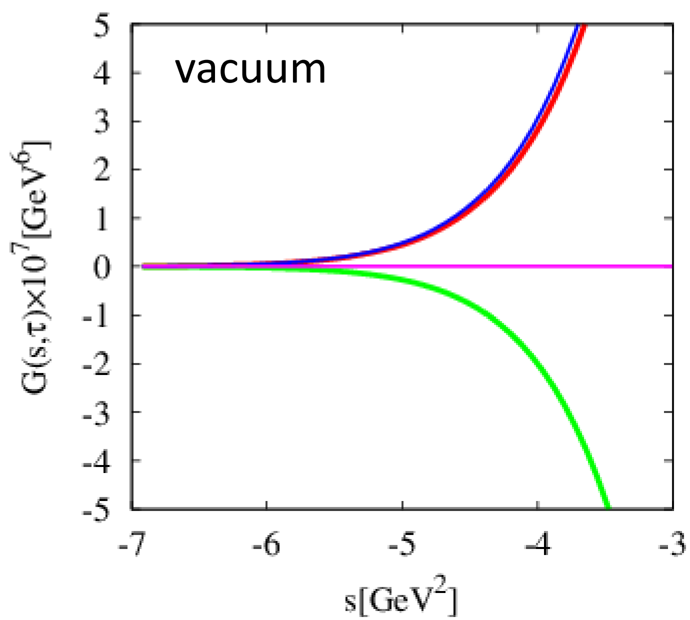
Positive parity OPE

Negative parity OPE

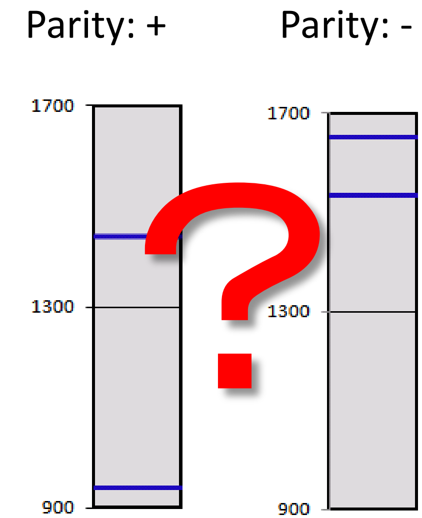
$\langle \overline{q}q \rangle_\rho$ term

$\langle q^\dagger q \rangle_\rho$ term

ρ_N: nuclear matter density



Mass spectrum of the nucleons



Positive parity: OPE data decreases.

Negative parity: OPE data does not decrease

Positive parity: $+ \; C_1 \langle \overline{q}q \rangle_\rho + C_2 \langle q^\dagger q \rangle_\rho$

Negative parity: $- \; C_1 \langle \overline{q}q \rangle_\rho + C_2 \langle q^\dagger q \rangle_\rho$

Nucleon QCD sum rule

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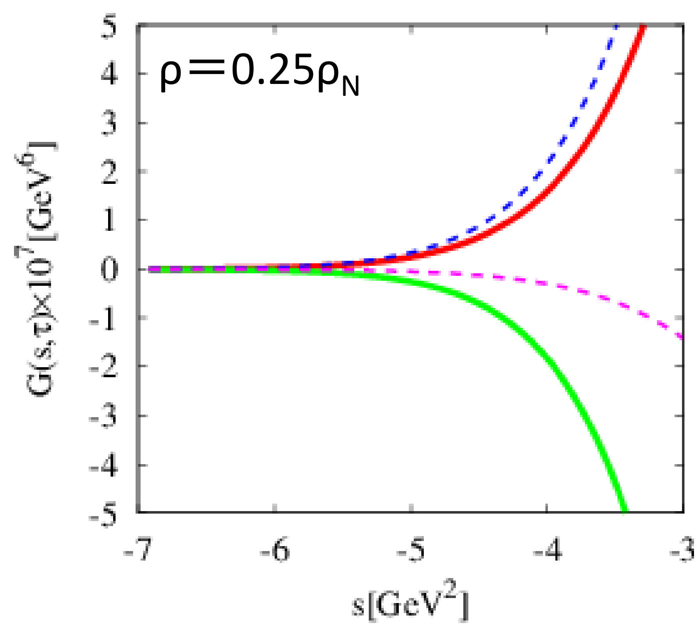

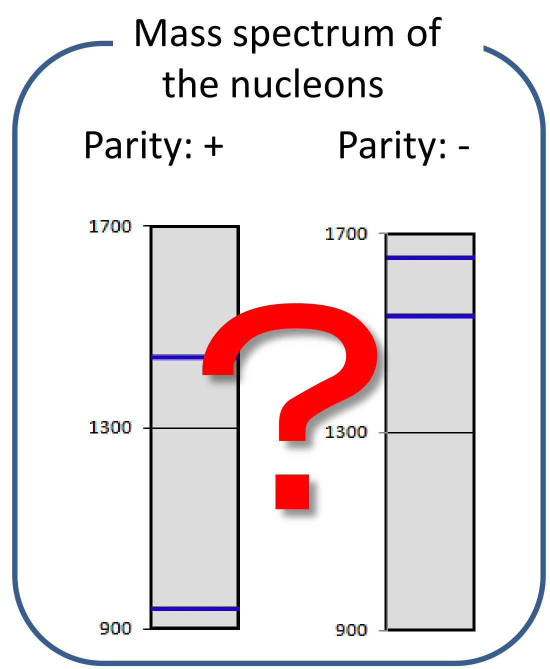

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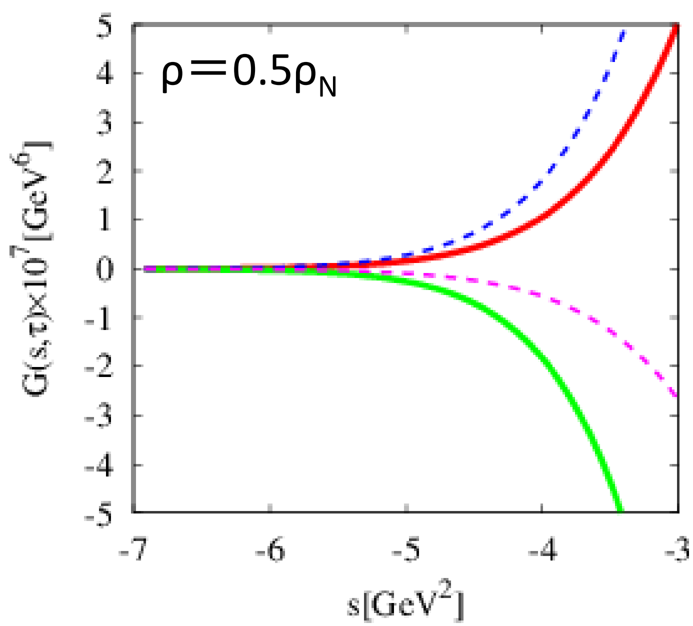

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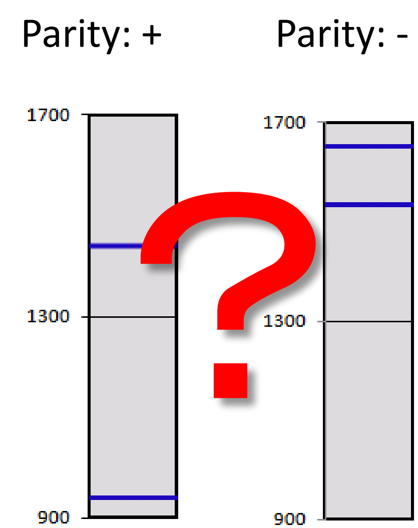

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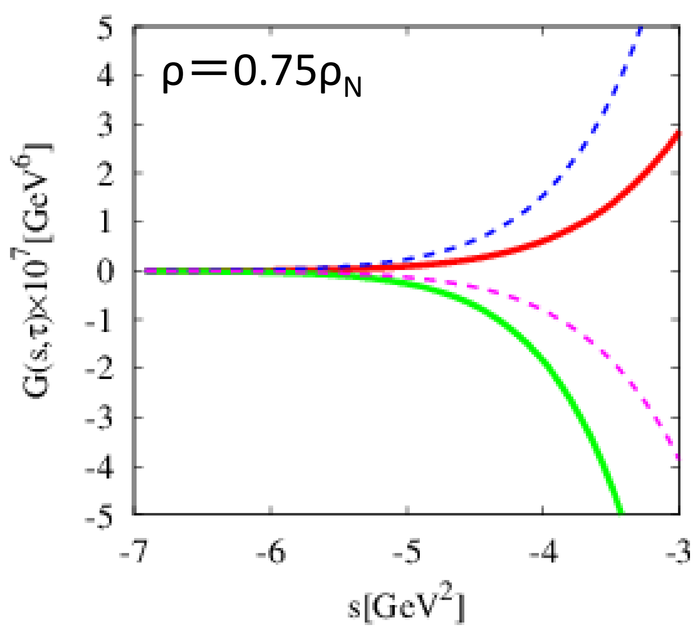

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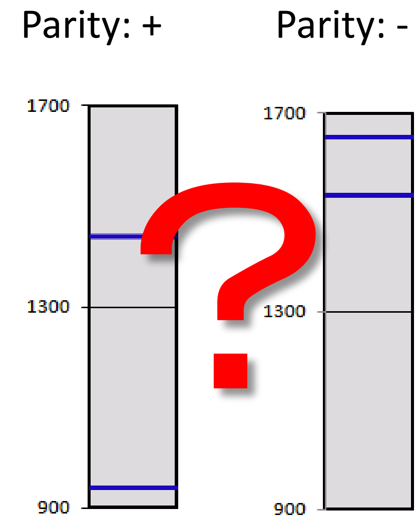

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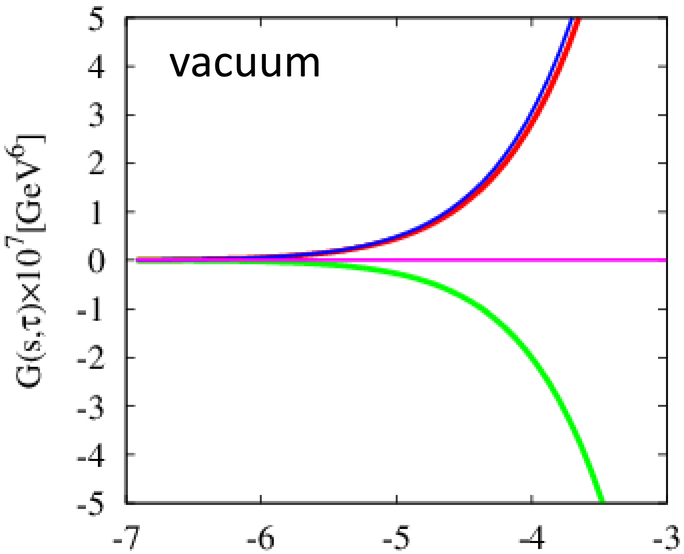
Nucleon QCD sum rule



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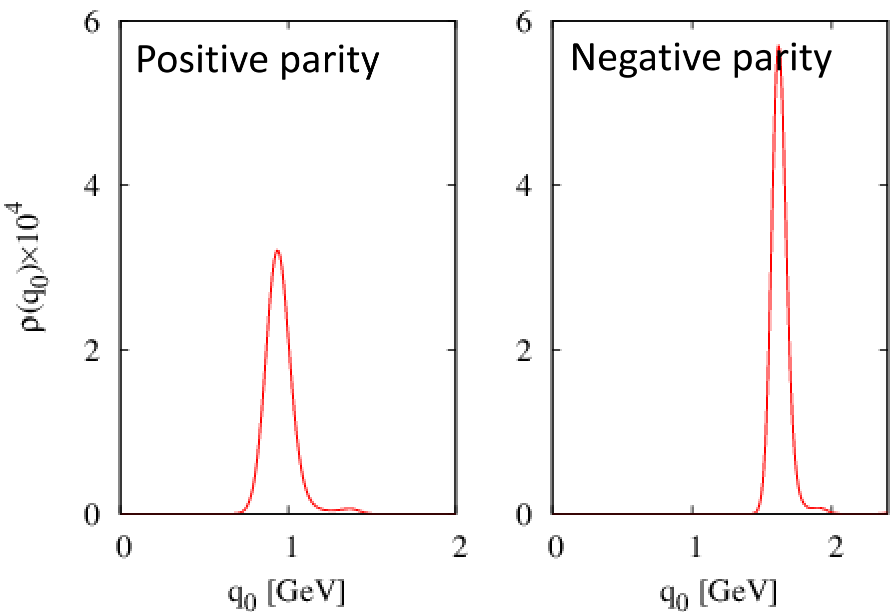
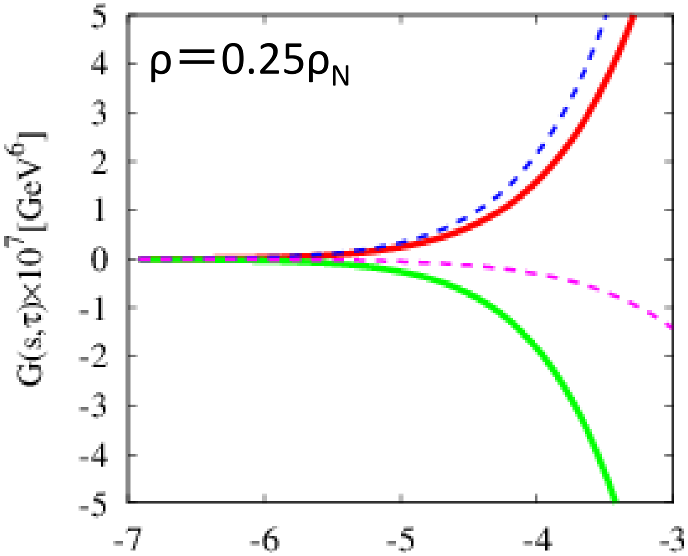


: Vacuum

Nucleon QCD sum rule



Positive parity OPE data

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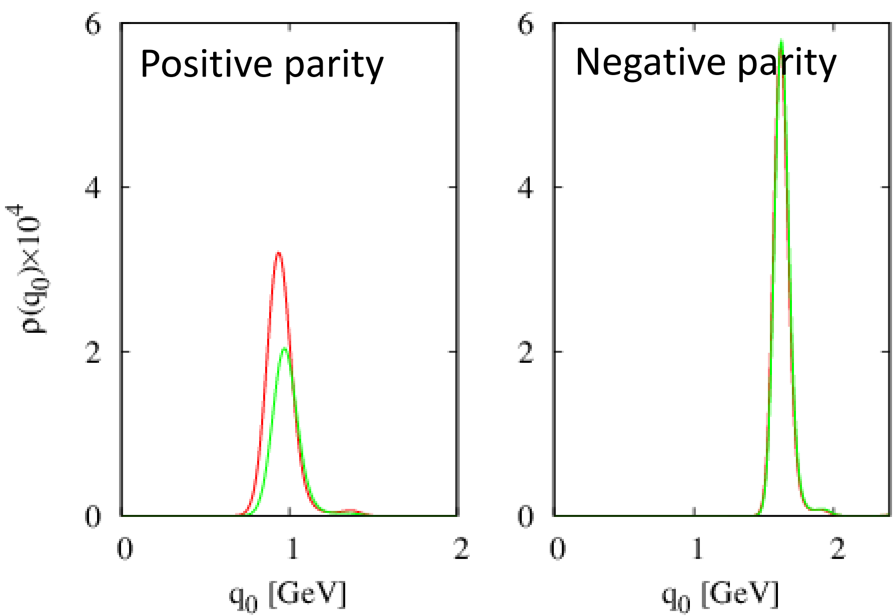
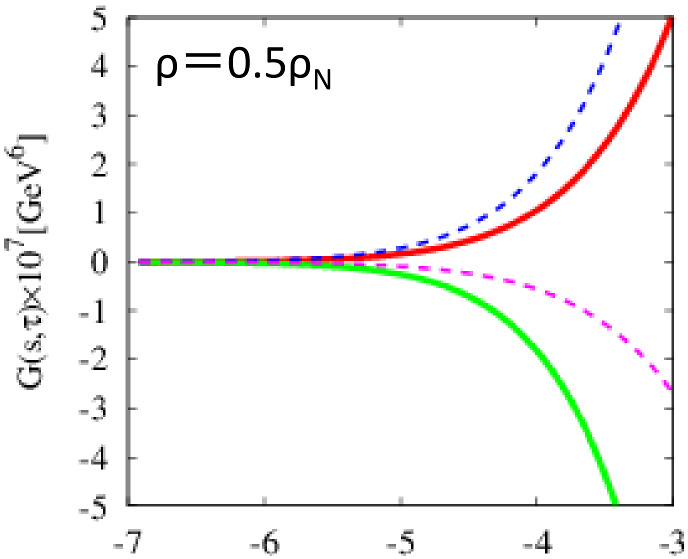


: Vacuum

: ρ=0.25ρ_0

Nucleon QCD sum rule



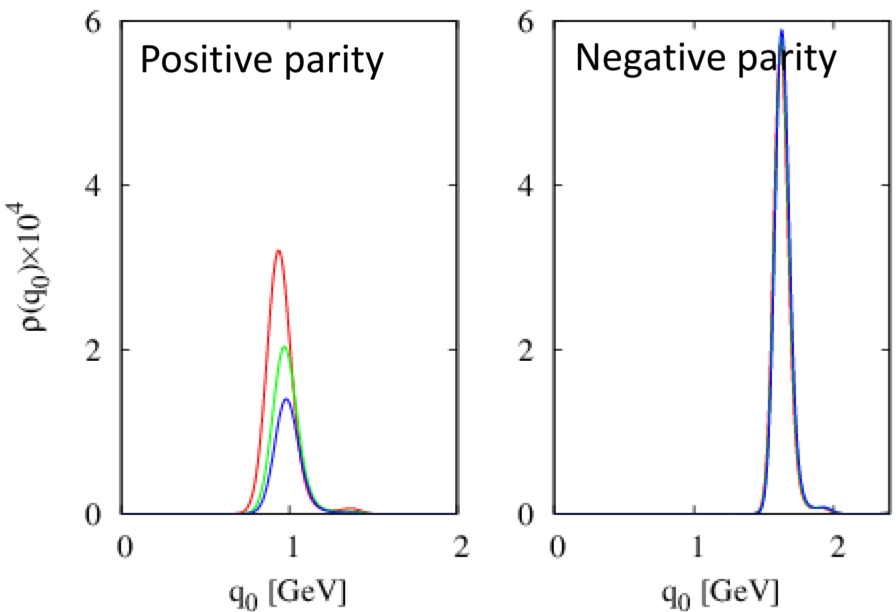
Positive parity OPE data

Negative parity OPE data

$\langle \bar{q}q \rangle_\rho$ term

$\langle q^\dagger q \rangle_\rho$ term

ρ_N : nuclear matter density

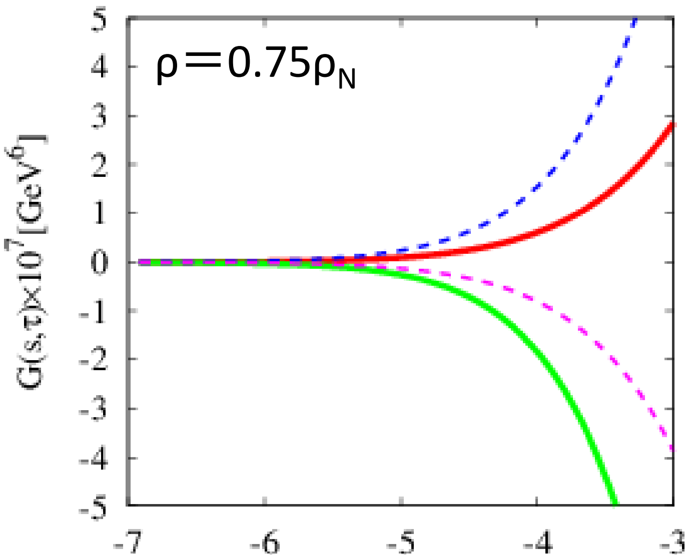


: Vacuum

: ρ=0.25ρ_0

: ρ=0.5ρ_0

Nucleon QCD sum rule



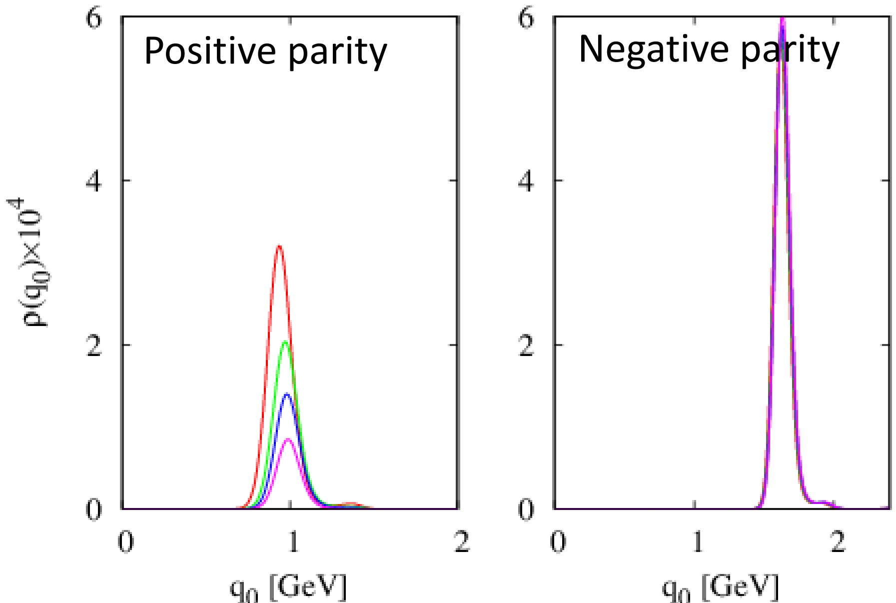
Positive parity OPE data

Negative parity OPE data

$\langle \overline{q}q \rangle_\rho$ term

$\langle q^\dagger q \rangle_\rho$ term

ρ_N : nuclear matter density



: Vacuum

: ρ=0.25ρ_0

: ρ=0.5ρ_0

: ρ=0.75ρ_0

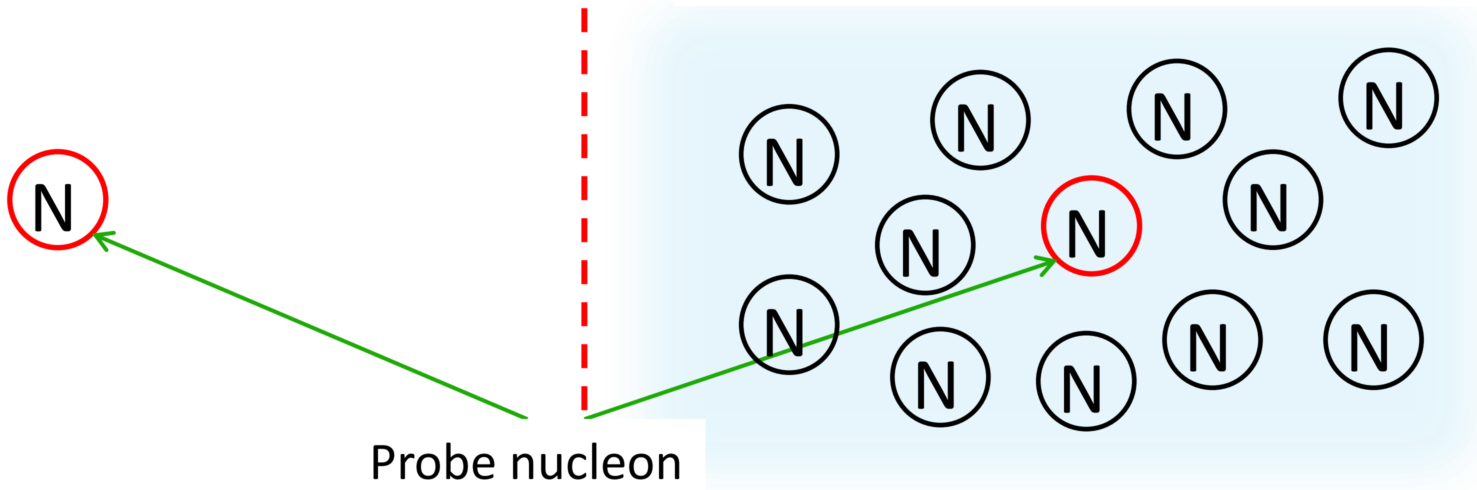
Nucleon QCD sum rule

Vacuum

$$\Pi(q) = i \int d^4x e^{iqx} \langle \underline{0} | T[\eta(x)\overline{\eta}(0)] | \underline{0} \rangle$$

Nuclear matter

$$\Pi(q) = i \int d^4x e^{iqx} \langle \underline{\Psi_0} | T[\eta(x)\overline{\eta}(0)] | \underline{\Psi_0} \rangle$$



Propagator: $\dfrac{1}{\not{q} - M + i\epsilon}$

Propagator: $\dfrac{1}{\not{q} - M - \Sigma(q) + i\epsilon}$

$$\Rightarrow \quad \frac{\not{q} - \not{u}\Sigma_v + M^*}{(q_0 - E + i\epsilon)(q_0 + \overline{E} - i\epsilon)}$$

Pole of positive energy state: $E = \sqrt{M^{*2} + \vec{q}^2} + \Sigma_v$

Effective mass: $M^* = M + \Sigma_s$

Pole of negative energy state: $-\overline{E} = -\sqrt{M^{*2} + \vec{q}^2} + \Sigma_v$

Nucleon QCD sum rule

Investigation of $M^*_{0\pm}, \Sigma^v_{0\pm}$

$$\Pi(q) = i\int d^4x e^{iqx}\underline{\theta(x_0)}\langle\Psi_0|T[\eta(x)\bar{\eta}(0)]|\Psi_0\rangle$$

$$E = \sqrt{q^2 + M^{*2}} + \Sigma^v$$

$$= \not{q}\Pi_1(q^2, q\cdot u) + \Pi_2(q^2, q\cdot u) + \not{u}\Pi_u(q^2, q\cdot u)$$

$$= \sum_n |\lambda^2_{n+}|\left(\frac{(\sqrt{M^{*2}+\vec{q}^2}\gamma_0 + M^*)}{2\sqrt{M^{*2}+\vec{q}^2}}\frac{1}{q_0 - E^+ + i\epsilon} + \frac{\gamma_i q^i}{2\sqrt{M^{*2}+\vec{q}^2}}\frac{1}{q_0 - E^+ + i\epsilon}\right)$$

+ (contribution of negative parity states)

$$q_0\Pi_1 + \Pi_2 \Rightarrow \sum |\lambda_+|^2 \frac{E^+ + M^*_+}{2\sqrt{M^{*2}_+ + \vec{q}^2}}\frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2 \frac{E^- - M^*_-}{2\sqrt{M^{*2}_- + \vec{q}^2}}\frac{1}{q_0 - E^- + i\epsilon},$$

$$q_0\Pi_1 - \Pi_2 \Rightarrow \sum |\lambda_+|^2 \frac{E^+ - M^*_+}{2\sqrt{M^{*2}_+ + \vec{q}^2}}\frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2 \frac{E^- + M^*_-}{2\sqrt{M^{*2}_- + \vec{q}^2}}\frac{1}{q_0 - E^- + i\epsilon},$$

$$\Pi_u \Rightarrow \sum |\lambda_+|^2 \frac{-\Sigma^v_+}{2\sqrt{M^{*2}_+ + \vec{q}^2}}\frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2 \frac{-\Sigma^v_-}{2\sqrt{M^{*2}_- + \vec{q}^2}}\frac{1}{q_0 - E^- + i\epsilon}.$$

By fitting the phenomenological side and OPE side, we can investigate $M^*_{0\pm}, \Sigma^v_{0\pm}$.

Nucleon QCD sum rule

ρ_N: nuclear matter density

n_0: nuclear matter density

		Vacuum	$n=0.25n_0$	$n=0.5n_0$	$n=0.75n_0$
Positive parity	M^*_{0+}	930	850	710	470
	Σ^v_{0+}	0	120	270	500
Negative parity	M^*_{0-}	1620	1630	1650	1680
	Σ^v_{0-}	0	0	-20	-50

Summary

- We analyze the nucleon spectral function by using QCD sum rules with MEM
- We find that the difference between the positive and negative parity spectral function is mainly caused by the chiral condensate.
- The information of not only the ground state but also the negative parity excited state is extracted
- We apply this method to the analyses in nuclear medium and investigate the effective masses and the vector self-energies.