Green functions of currents in the odd-intrinsic parity sector of QCD

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Motivation: Why Green functions?

- **Based on works:**
  - **[K. Kampf and J. Novotný '11 (arXiv: 1104.3137)]**
  - **[T. Kadavý '13: bachelor thesis]**
  - **[T. Kadavý '14: diploma thesis (in preparation)]**

- Theoretical objects with important physical interpretation.

- Calculations and predictions of decay widths, etc.

- Connections with the current problems of the SM.

Chiral perturbation theory (χPT)

- An effective theory in a low-energy region (≤ 1 GeV).
- The basic building block of χPT [S. Weinberg '79], [J. Gasser and H. Leutwyler '84, '85]:

$$u(\phi) = \exp\left(\frac{i}{\sqrt{2}F}\phi\right).$$

- The matrix of pseudoscalar meson fields:

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}.$$

- The lowest order Lagrangian:

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle .$$

Resonance chiral theory (RχT)

- An effective theory for an intermediate region (1 GeV $\leq E \leq$ 2 GeV).
- Massive multiplets of vector V(1^{--}), axial-vector A(1^{++}), scalar S(0^{++}) and pseudoscalar P(0^{-+}) resonances.
- A singlet and octet decomposition of the fields ($R = V, A, S, P$):

$$R = \frac{1}{\sqrt{3}} R_0 + \sum_{a=1}^{8} \frac{\lambda^a}{\sqrt{2}} R_a \,.$$

- The interaction resonance Lagrangian up to $\mathcal{O}(p^4)$ [G. Ecker, J. Gasser, A. Pich and E. de Rafael '89]:

$$\mathcal{L}_R^{(4)} = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle + \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle$$
$$+ c_d \langle S u^\mu u_\mu \rangle + c_m \langle S \chi_+ \rangle + i d_m \langle P \chi_- \rangle + \frac{i d_{m0}}{N_F} \langle P \rangle \langle \chi_- \rangle \,.$$

Odd-intrinsic parity sector of QCD

- The leading order of the pure Goldstone-boson part of the odd-intrinsic parity sector starts at $\mathcal{O}(p^4)$.
- The parameters are set entirely by the chiral anomaly.
- The Wess-Zumino-Witten Lagrangian contributes to the anomalous term and has the form [E. Witten '83]:

$$\mathcal{L}_{WZW} = \frac{N_C}{48\pi^2}\left[\int_0^1 \mathrm{d}\xi \langle \sigma_\mu^\xi \sigma_\nu^\xi \sigma_\alpha^\xi \sigma_\beta^\xi \frac{\phi}{F}\rangle - i\langle W_{\mu\nu\alpha\beta} - \widetilde{W}_{\mu\nu\alpha\beta}\rangle\right]\varepsilon^{\mu\nu\alpha\beta}\,.$$

- Resonance saturation at $\mathcal{O}(p^6)$:
  - Even sector: [V. Cirigliano, G. Ecker, M. Eidemuller, R. Kaiser, A. Pich and J. Portoles '06].
  - Odd sector.

Independent operator basis

- The antisymmetric tensor formalism.
- The independent operator basis, constructed using the large N_C approximation and relevant in the odd-intrinsic parity sector:

$$\mathcal{O}_i^X = \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta}\,\varepsilon^{\mu\nu\alpha\beta}\,.$$

- Lagrangians up to $\mathcal{O}(p^6)$:

$$\mathcal{L}_{R\chi T}^{(6,\mathrm{odd})} = \sum_X \sum_i \mathcal{O}_i^X \kappa_i^X\,.$$

- 67 operators in total.
- Except for the single resonance fields, the basis gives us the following set of the multiple field combinations:

$$X = VV, AA, SA, SV, VA, PA, PV, VVP, VAS, AAP\,.$$

Three-point Green functions

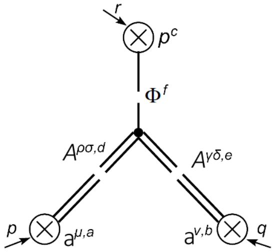
- The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the time ordered products of quantum fields.
- Definition:

$$\langle 0|T[\widetilde{\mathcal{O}}_1(p_1)\widetilde{\mathcal{O}}_2(p_2)\widetilde{\mathcal{O}}_3(0)]|0\rangle = \\ = \int \mathrm{d}^4 x_1 \int \mathrm{d}^4 x_2 \, e^{i(p_1 x_1 + p_2 x_2)} \, \langle 0|T[\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)]|0\rangle \, .$$

- Operators $\mathcal{O} = V, A, S, P$.
- Only five nontrivial Green functions in the odd-intrinsic parity sector of QCD.
  - $\langle VVP \rangle$, $\langle VAS \rangle$, $\langle AAP \rangle$, $\langle VVA \rangle$ and $\langle AAA \rangle$.

Three-point Green functions: The calculation

- The task is to calculate all contributing Feynman diagrams.
- An example: the Feynman diagram with the inner vertex contributing to the Lagrangian $\mathcal{L}_3^{AA} = \langle\{\nabla_\sigma A^{\mu\nu}, A^{\alpha\sigma}\}u^\beta\rangle\kappa_3^{AA}\varepsilon_{\mu\nu\alpha\beta}$.

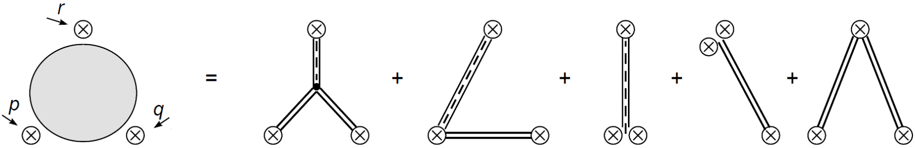


- The result:

$$
\begin{aligned}
(\Pi_2)^{abc}_{\mu\nu} &= (V_4)^{def}_{\rho\sigma\gamma\delta}(S_\chi)^{fc}(S_2(p))^{ad}_{\mu\rho\sigma}(S_2(q))^{eb}_{\gamma\delta\nu} \\
&= -\frac{4iB_0F_A^2 d^{abc}}{(p^2-M_A^2)(q^2-M_A^2)r^2}(p^2+q^2-r^2)\kappa_3^{AA}p^\alpha q^\beta\varepsilon_{\mu\nu\alpha\beta}\,.
\end{aligned}
$$

Three-point Green functions: The topology

- A general graph topology of the three-point Green functions in the antisymmetric tensor formalism:



- Double lines stand for resonances and dash lines for GB (double lines together with dash lines is the sum of both possible contributions).
- The crossing is implicitly assumed.

$\langle VVP \rangle$ Green function

- The most important example in the odd-intrinsic parity sector of QCD.
- Important phenomenological applications.
- A tensor structure:

$$(\Pi_{VVP})^{abc}_{\mu\nu}(p,q) = \Pi_{VVP}(p^2,q^2,r^2) d^{abc} p^\alpha q^\beta \varepsilon_{\mu\nu\alpha\beta} \,.$$

- OPE constraints dictate for high values of all independent momenta:

$$\Pi((\lambda p)^2,(\lambda q)^2,(\lambda r)^2)_{VVP} = \frac{B_0 F^2}{2\lambda^4} \frac{p^2+q^2+r^2}{p^2 q^2 r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right) .$$

- $\Pi^{R\chi T}_{VVP}(p^2,q^2,r^2)$: substituing the constraints into $\Pi_{VVP}(p^2,q^2,r^2)$.
- The isolation of two low-energy constants: C^W_7 and C^W_{22}.

$\langle VVP \rangle$ Green function: Decay $\pi^0 \to \gamma\gamma$

- A formfactor:

$$\mathcal{F}^{R\chi T}_{\pi^0\to\gamma\gamma}(p^2, q^2, r^2) = \frac{2r^2}{3B_0F}\Pi^{R\chi T}_{VVP}(p^2, q^2, r^2)\,.$$

- The Brodsky-Lepage behaviour for large momentum [G. P. Lepage and S. J. Brodsky '80, '81]:

$$\lim_{Q^2\to\infty} \mathcal{F}^{R\chi T}_{\pi^0\to\gamma\gamma}(0, -Q^2, m_\pi^2) \sim -\frac{1}{Q^2}\,.$$

- An amplitude: $\mathcal{A}_{\pi^0\to\gamma\gamma} = e^2\mathcal{F}_{\pi^0\to\gamma\gamma}(0,0,0)$.
- A decay width $(\pi^0(p) \to \gamma(k) + \gamma(l))$:

$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{1}{32\pi m_{\pi^0}}\sum_{\text{pol}} |\mathcal{A}_{\pi^0\to\gamma\gamma}\varepsilon^{\mu\nu\alpha\beta}k_\alpha l_\beta \epsilon^*_\mu(k)\epsilon^*_\nu(l)|^2\,.$$

$\langle VVP \rangle$ Green function: Decay $\pi^0 \to \gamma\gamma$

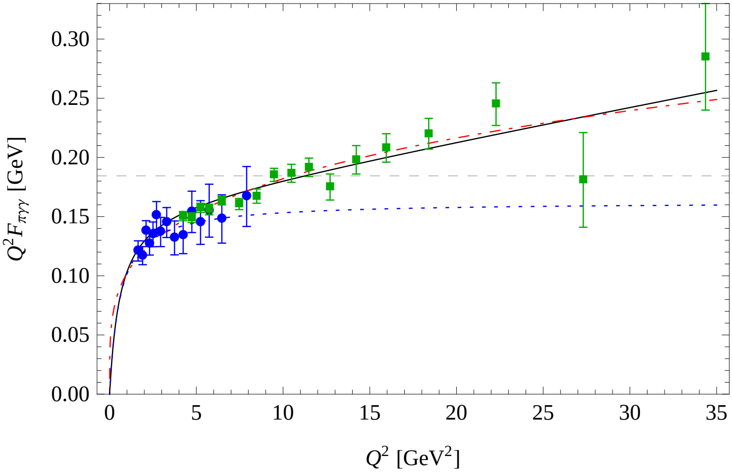


Figure : CLEO (blue points) and BABAR (green squares) data with fitted function $\mathcal{F}^{R\chi T}_{\pi^0\to\gamma\gamma}(0, -Q^2, m_\pi^2)$ [B. Aubert et al. (The BABAR Collaboration) '09].

$\langle VVP \rangle$ Green function: Decay $\pi^0 \to \gamma\gamma$ (updated)

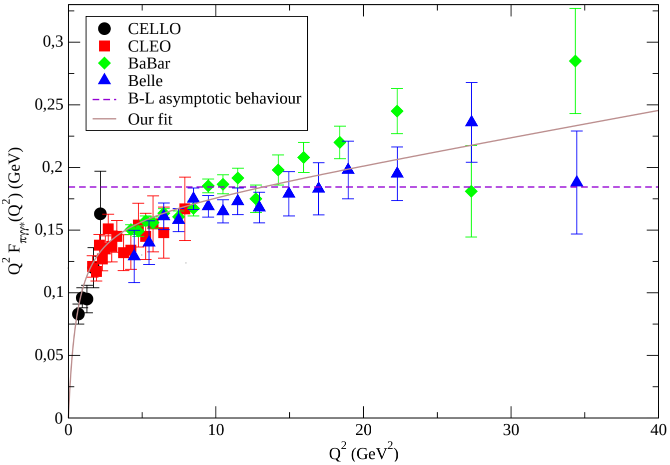


Figure : Updated data with fitted function $\mathcal{F}^{R\chi T}_{\pi^0\to\gamma\gamma}(0, -Q^2, m_\pi^2)$, [P. Roig, A. Guevara and G. L. Castro '14].

$\langle VVP \rangle$ Green function: Decay of $\rho \to \pi\gamma$

- The connection $\rho^+(q) \to \pi^+(p) + \gamma(k)$ with the previous process:

$$\mathcal{A}_{\rho^+\to\pi^+\gamma} = \frac{e}{2F_V M_V} \lim_{q^2\to M_V^2} (q^2 - M_V^2)\mathcal{F}_{\pi^0\to\gamma\gamma}(0, q^2, 0)\,.$$

- A decay width:

$$\Gamma_{\rho^+\to\pi^+\gamma} = \frac{m_\rho^2 - m_\pi^2}{48\pi m_\rho^3} \sum_{\text{pol}} |\mathcal{A}_{\rho^+\to\pi^+\gamma}\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \epsilon_\mu(p)\epsilon_\nu^*(k)|^2\,.$$

with [B. Moussallam '95]

$$\frac{2eF_V}{M_V}\left|\frac{\mathcal{A}_{\rho^+\to\pi^+\gamma}}{\mathcal{A}_{\rho^0\to\gamma\gamma}}\right| \equiv 1 + x\,.$$

- RχT: $x = -0.010 \pm 0.005$ and $\Gamma_{\rho^+\to\pi^+\gamma} = 67.0 \pm 2.3\,\mathrm{keV}$.

$\langle VVP \rangle$ Green function: Decays of $\pi(1300)$

- Two channels studied.
- The amplitudes:

$$\mathcal{A}^{R\chi T}_{\pi' \to \gamma\gamma} = e^2 \frac{8\sqrt{2}}{3} F_V \frac{2\sqrt{2}\kappa_3^{PV} M_V^2 - F_V \kappa^{VVP}}{M_V^4} \,,$$

$$\mathcal{A}^{R\chi T}_{\pi' \to \rho\gamma} = -e \frac{4\sqrt{2}}{3M_V} \frac{\sqrt{2}\kappa_3^{PV} M_V^2 - F_V \kappa^{VVP}}{M_V^2} \,.$$

- Belle collaboration [K. Abe et al. '06]: $\Gamma_{\pi' \to \gamma\gamma} < 72\,\mathrm{eV}$.
- An experimental bound on $\Gamma_{\pi' \to \gamma\gamma}$ can be used to get the estimate:

$$\kappa^{VVP} \approx (-0.57 \pm 0.13)\,\mathrm{GeV} \,.$$

$\langle VVP \rangle$ Green function: Decays of $\pi(1300)$

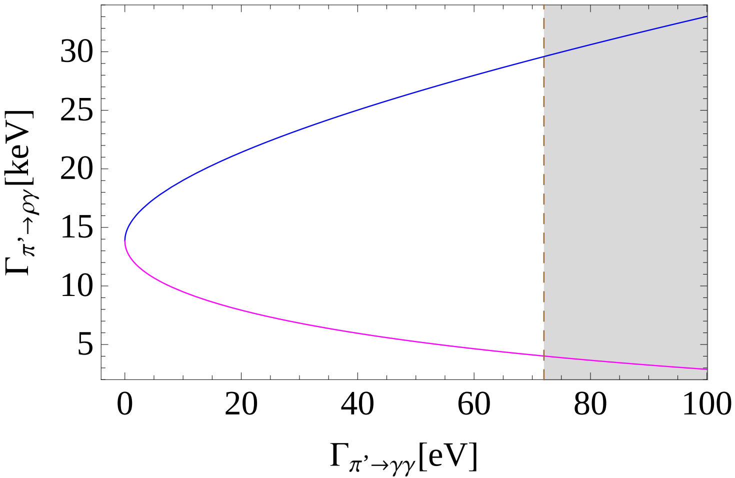


Figure : The connection of decay widths for $\pi(1300) \to \gamma\gamma$ and $\pi(1300) \to \rho\gamma$. The dashed line denotes the Belle collaboration limit. [K. Abe et al.) '06].

$\langle VVP \rangle$ Green function: The muon $g-2$ factor

- Hadronic contributions: hadronic light-by-light scattering.
  - The main source of theoretical error in the SM.
- The four point Green function $\langle VVVV \rangle$ can be simplified into:
  - $\pi^{\pm}$ and $K^{\pm}$ loops,
  - π^0, η, η' exchanges: the $\langle VVP \rangle$ case etc.
- Using the fully off-shell $\mathcal{F}^{R\chi T}_{\pi^0 \to \gamma\gamma}(p^2, q^2, r^2)$ formfactor we get:

$$a_{\mu}^{LbyL,\pi^0} = (65.8 \pm 1.2) \cdot 10^{-11} \,.$$

- The result based on AdS/QCD conjecture [L. Cappiello, O. Cata and G. D'Ambrosio '11]:

$$a_{\mu}^{\pi^0} = (65.4 \pm 2.5) \cdot 10^{-11} \,.$$

- The updated result using Belle data [P. Roig, A. Guevara and G. L. Castro '14]:

$$a_{\mu}^{\pi^0} = (66.6 \pm 2.1) \cdot 10^{-11} \,.$$

$\langle VAS \rangle$ Green function

- A tensor structure:

$$(\Pi_{VAS})^{abc}_{\mu\nu}(p,q) = \Pi_{VAS}(p^2,q^2,r^2) f^{abc} p^\alpha q^\beta \varepsilon_{\mu\nu\alpha\beta} \,.$$

- OPE constraints dictate for high values of all independent momenta:

$$\Pi((\lambda p)^2,(\lambda q)^2,(\lambda r)^2)_{VAS} = \frac{B_0 F^2}{2\lambda^4} \frac{p^2-q^2-r^2}{p^2 q^2 r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right) .$$

- At low energies, up to $\mathcal{O}(p^6)$: $\Pi(p^2,q^2,r^2) = -32 B_0 C^W_{11}$.
- An experiment: decay $K^+ \to l^+ \nu \gamma$ suggests [A. A. Poblaguev et al. '02], [R. Unterdorfer and H. Pichl '08]:

$$C^W_{11} = (0.68 \pm 0.21) \cdot 10^{-3}\, \mathrm{GeV}^{-2}$$

and

$$\kappa^{VAS} = (0.61 \pm 0.40)\, \mathrm{GeV} \,.$$

$\langle AAP \rangle$ Green function

- A tensor structure:

$$(\Pi_{AAP})^{abc}_{\mu\nu}(p,q) = \Pi_{AAP}(p^2,q^2,r^2)d^{abc}p^\alpha q^\beta \varepsilon_{\mu\nu\alpha\beta}\,.$$

- The calculations were carried out both in the vector field and antisymmetric tensor field formalisms.
- OPE constraints:

$$\Pi_{AAP}\big((\lambda p)^2,(\lambda q)^2,(\lambda r)^2\big) = \frac{B_0F^2}{2\lambda^4}\,\frac{p^2+q^2-r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right).$$

- The vector field formalism does not satisfy the OPE condition.
- The phenomenology studies are still missing (hopefully, not for long).

$\langle VVA \rangle$ and $\langle AAA \rangle$ Green functions

- The situation is a bit more complicated due to three-indices structure.

$$
\begin{aligned}
\Pi^{abc}_{\mu\nu\omega}(p,q) = {} & \Pi^{(1)} d^{abc} p^{\alpha} \varepsilon_{\mu\nu\omega\alpha} + \Pi^{(2)} d^{abc} q^{\beta} \varepsilon_{\mu\nu\omega\beta} \\
& + \Pi^{(3)} d^{abc} p^{\alpha} q^{\beta} r_{\omega} \varepsilon_{\mu\nu\alpha\beta} + \Pi^{(4)} d^{abc} p^{\alpha} q^{\beta} q_{\nu} \varepsilon_{\mu\omega\alpha\beta} \\
& + \Pi^{(5)} d^{abc} p^{\alpha} q^{\beta} p_{\mu} \varepsilon_{\nu\omega\alpha\beta} + \mathrm{index\,cycl.}
\end{aligned}
$$

- The new coupling: an axial-vector sources with a pseudoscalar ($a_{\mu}\phi$).
- The calculations were carried out only in the antisymmetric formalism.
- OPE is complicated.
  - A general analysis have been already made (for example) [M. Knecht, S. Peris, M. Perrottet, E. de Rafael '04].
  - The comparison with our calculation is currently in progress.
  - The result will give important coupling constant constraints.

Future plans

- The phenomenological studies for $\langle AAP\rangle, \langle VVA\rangle$ and $\langle AAA\rangle$.
  - Still missing (to our knowledge).
  - Necessary to subtract the dominant even contribution.
- The four-point Green functions.
  - A difficult topological structure.
  - Interesting!

Conclusion

- We have the resonance Lagrangian in the leading order in the odd-intrinsic parity sector, equivalent to the resonance Lagrangian in the even parity sector.
- A lot of coupling constants.
- Due to the anomaly the leading order of the odd sector is shifted with the respect to the even sector.
  - Even sector: LO $\mathcal{O}(p^4)$, NLO $\mathcal{O}(p^6)$.
  - Odd sector: LO $\mathcal{O}(p^6)$.
- To gather phenomenologically relevant data would be complicated.
- Thanks to the experiments, this is getting better.
- It is important to study odd sector systematically.
- We offer:
  - The most general parametrization of the resonance Lagrangians.
  - The reduction of the parameters based on the theoretical arguments (OPE, for example).
  - A phenomenology.

Thank you for your attention!

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