Sergey Afonin

Saint Petersburg State University

BOTTOM-UP HOLOGRAPHIC APPROACH TO QCD

Overview

XI Quark Confinement and the Hadron Spectrum,
Saint Petersburg, Sept. 12, 2014

A brief introduction

AdS/ CFT correspondence – the conjectured equivalence between a string theory defined on certain 10D space and a CFT (Conformal Field Theory) without gravity defined on conformal boundary of this space.

Maldacena example (1997):

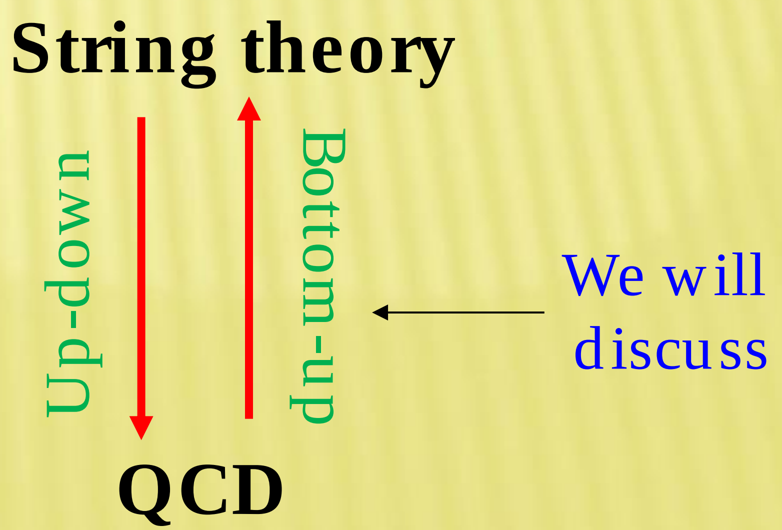
Type IIB string theory on $AdS_5 \times S^5$ in low-energy (i.e. supergravity) approximation $\Leftrightarrow$ $\mathcal{N}=4$ SYM theory on AdS boundary in the limit $g_{YM}N \gg 1$

Essential ingredient: one-to-one mapping of the following group algebras

Isometries of S^5 $\Leftrightarrow$ Supersymmetry of $\mathcal{N}=4$ Super Yang-Mills theory

Isometries of AdS_5 $\Leftrightarrow$ Conformal group $SO(4,2)$ in 4D space

AdS/ QCD correspondence – a program for implementation of such a duality for QCD following some recipies from the AdS/ CFT correspondence



AdS/CFT dictionary

boundary: field theory	bulk: gravity
energy momentum tensor T^{ab}	metric field g_{ab}
global internal symmetry current J^a	Maxwell field A_a
order parameter/scalar operator $\mathcal{O}_\mathrm{b}$	scalar field ϕ
fermionic operator $\mathcal{O}_\mathrm{f}$	Dirac field ψ
spin/charge of the operator	spin/charge of the field
conformal dimension of the operator	mass of the field
source of the operator	boundary value of the field (leading part)
VEV of the operator	boundary value of radial momentum of the field (subleading part)
(Global aspects)	
global spacetime symmetry	local isometry
temperature	Hawking temperature
chemical potential/charge density	boundary values of the gauge potential
phase transition	Instability of black holes

[Witten; Gubser, Polyakov, Klebanov (1998)]

Essence of the holographic method

$$\langle e^{\int d^dx J(x)\mathcal{O}(x)}\rangle_{\rm CFT} = \int \mathcal{D}\phi \; e^{-S[\phi,\, g]}\Big|_{\phi(x,\partial AdS)=J(x)}$$

generating functional

action of dual gravitational theory
evaluated on classical solutions

AdS boundary

$$\Pi_n \equiv \langle \mathcal{O}_{I_1}(x_1)\ldots\mathcal{O}_{I_n}(x_n)\rangle = \frac{1}{\sqrt{g}}\frac{\delta}{\delta\phi^{I_1}(x_1)}\cdots\frac{1}{\sqrt{g}}\frac{\delta}{\delta\phi^{I_n}(x_n)} S[\phi,g]$$

The output of the holographic models: **Correlation functions**

Poles of the 2-point correlator $\rightarrow$ mass spectrum

Residues of the 2-point correlator $\rightarrow$ decay constants

Residues of the 3-point correlator $\rightarrow$ transition amplitudes

Alternative way for finding the mass spectrum is to solve e.o.m. $\phi(x_\mu, z) = e^{ixp}\phi(z)$

5D Anti-de Sitter space

$$\tau^2 + y_0^2 - y_1^2 - y_2^2 - y_3^2 - y_4^2 = R^2$$

$$u = \tau + y_4 \qquad v = \tau - y_4$$

$$uv + y_\mu^2 = R^2$$

$$ds^2 = du dv + dy_\mu dy^\mu$$

Exclude v and introduce

$$z = \frac{R^2}{u}, \qquad x_\mu = \frac{z}{R} y_\mu$$

$$ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$$

invariant under dilatations

$$x_\mu \to \rho x_\mu, \quad z \to \rho z$$

4D Minkovski space at $z \to 0$

$$p_x = -i\partial_x = \frac{R}{z} p_y$$

holographic coordinate

Physical meaning of *z*: Inverse energy scale

The warped geometry is crucial in all this enterprise! For instance, it provides the hard (power law) behavior of string scattering amplitudes at high energies for holographic duals of confining gauge theories (Polchinski, Strassler, PRL(2002)).

Bottom-up AdS/QCD models

Typical ansatz:

$$S = \int d^4x dz \sqrt{g}\, F(z)\mathcal{L}$$

$$F(0) = 1$$

$$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \qquad z > 0$$

Vector mesons:

$z = \epsilon \longrightarrow 0$

$V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu q$ or $V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu \vec{\tau} q$

$A_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu\gamma_5 q$ or $A_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu\gamma_5 \vec{\tau} q$

From the AdS/ CFT recipes: $m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)$ $\quad J = 0, 1$

Masses of 5D fields are related to the canonical dimensions of 4D operators!

In the given cases: $\Delta = 3,\ J = 1 \Rightarrow m_5^2 = 0$ gauge 5D theory!

Hard wall model

(Erlich et al., PRL (2005); Da Rold and Pomarol, NPB (2005))

The AdS/ CFT dictionary dictates: local symmetries in 5D → global symmetries in 4D

The chiral symmetry: $SU_L(2) \times SU_R(2)$

The typical model describing the chiral symmetry breaking and meson spectrum:

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

$$0 < z \leq z_m$$

$$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu},\ A_{L,R} = A^a_{L,R}t^a, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

The pions are introduced via $X = X_0 \exp(i2\pi^a t^a)$ $t^a = \sigma^a/2$

$V = (A_L + A_R)/2$ $A = (A_L - A_R)/2$ $m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)$

At $z = z_m$ one imposes certain gauge invariant boundary conditions on the fields.

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	J	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

Equation of motion for the scalar field

$$\frac{1}{z^5}3X = \frac{1}{z^3}\partial_\mu\partial^\mu X - \partial_z \frac{1}{z^3}\partial_z X$$

Solution independent of usual 4 space-time coordinates

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

quark condensate

current quark mass

As the holographic dictionary prescribes

$$\Phi(x,z)_{z\to 0} = z^{4-\Delta}\Phi_0(x) + z^{\Delta}\frac{\langle O(x)\rangle}{2\Delta - 4}$$

here $\Delta = 3$

Denoting
$$X_0(z) = \frac{1}{2}v(z)\mathbf{1}, \qquad v(z) = mz + \sigma z^3$$

the equation of motion for the vector fields are (in the axial gauge $V_z=0$)

$$\left[\partial_z\left(\frac{1}{z}\partial_z V^a_\mu(q,z)\right) + \frac{q^2}{z}V^a_\mu(q,z)\right]_\perp = 0$$

where $V(q,z) = \int d^4x \;\; e^{iqx}V(x,z)$

due to the chiral symmetry breaking

$$\left[\partial_z\left(\frac{1}{z}\partial_z A^a_\mu\right) + \frac{q^2}{z}A^a_\mu - \frac{g_5^2 v^2}{z^3}A^a_\mu\right]_\perp = 0$$

The GOR relation holds $m_\pi^2 f_\pi^2 = 2M\Sigma$

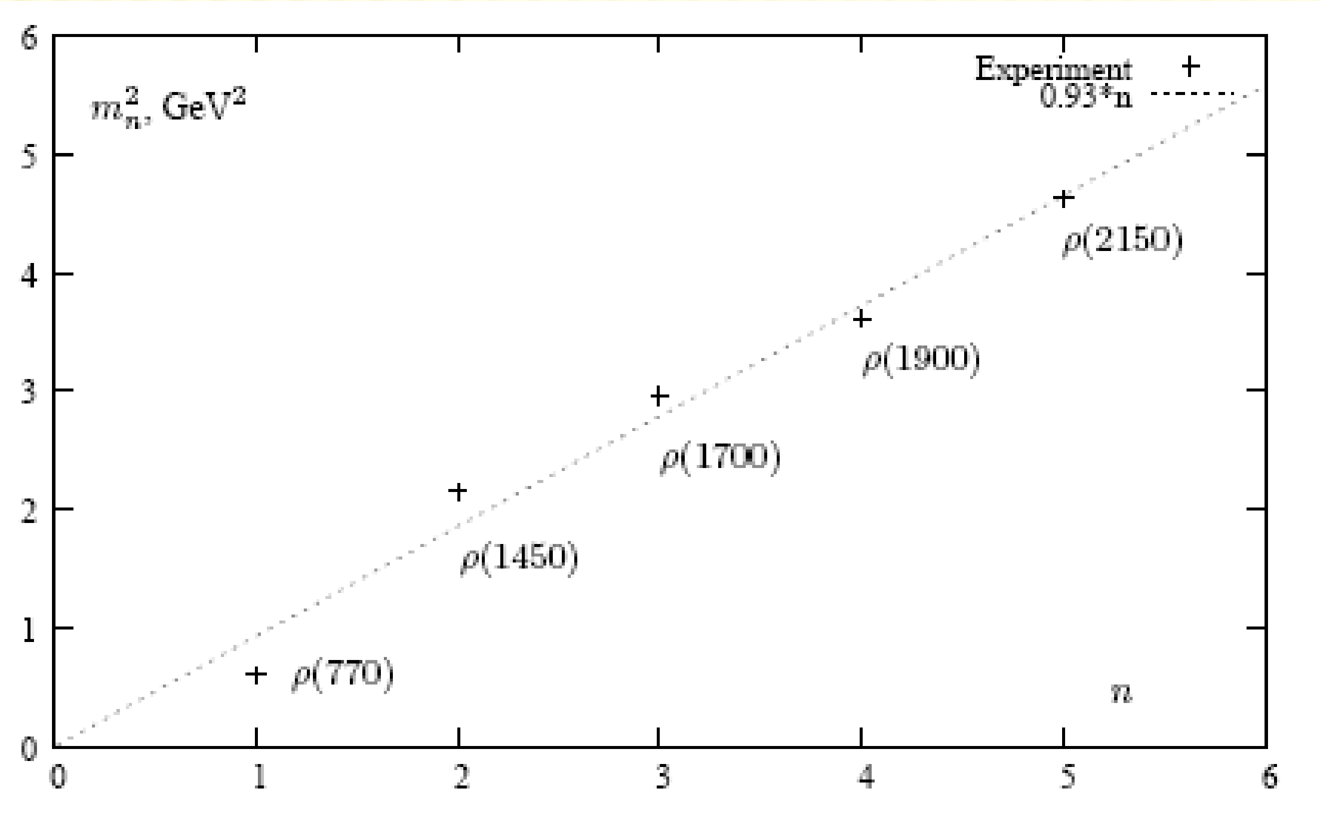
The spectrum of normalizable modes is given by $J_0(m_n z) = 0$

thus the asymptotic behavior is

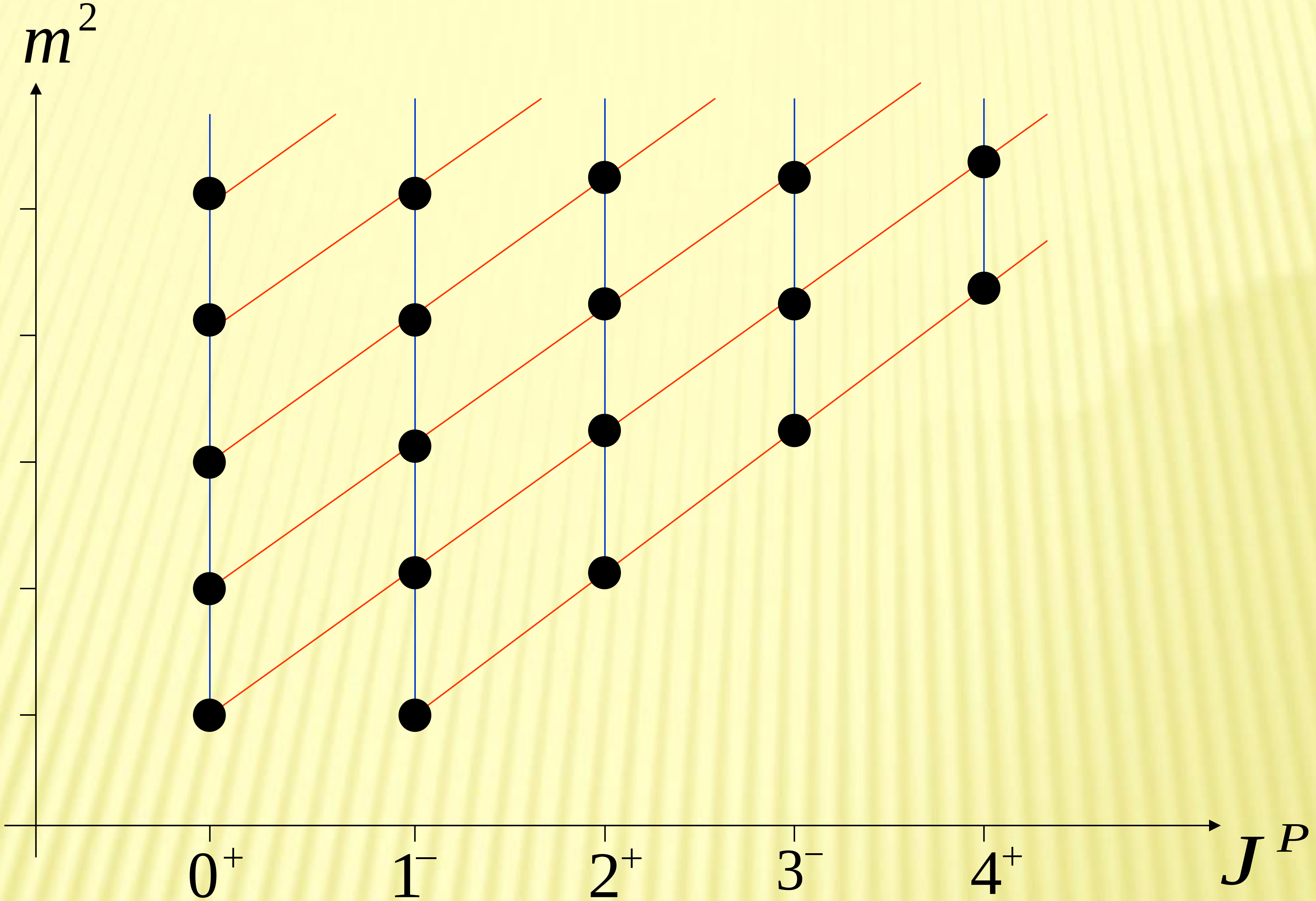
$$m_n \sim n$$

(Rediscovery of 1979 Migdal's result)

that is not Regge like $m_n^2 \sim n$



Regge and radial Regge linear trajectories

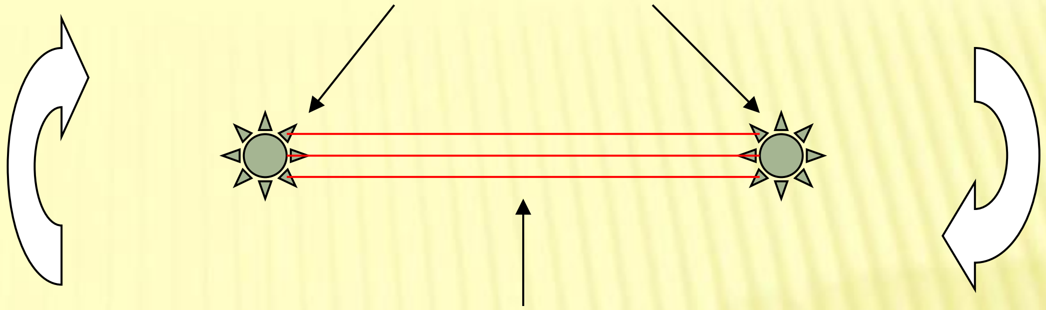


$$m^2(J) = m_0^2 + \alpha' J$$ – Regge trajectories

$$m^2(n) = \mu_0^2 + \alpha n$$ – Radial Regge trajectories

A simplistic model

Hadron string picture for mesons:



Rotating string with relativistic massless quarks at the ends

$$M^2 = 2\pi\sigma L$$

σ - string tension, L - angular momentum $(J = L, L \pm 1)$

Bohr-Sommerfeld quantization

$$\int p(r)dr = 2\pi\left(n + \frac{1}{2}\right)$$

n - radial quantum number, $p(r)$ and r are relative momentum and distance

related in the simplest case by $M = 2p + \sigma r$

Taking into account $M = l\sigma$ where ***l*** is the string length

the result is

$$M^2 = 4\pi\sigma\left(n + \frac{1}{2}\right)$$

CRYSTAL BARREL

A.V. Anisovich, V.V. Anisovich and
A.V. Sarantsev, PRD (2000)

D.V. Bugg, Phys. Rept. (2004)

Many new states in 1.9-2.4 GeV range!

Doubling of some trajectories:

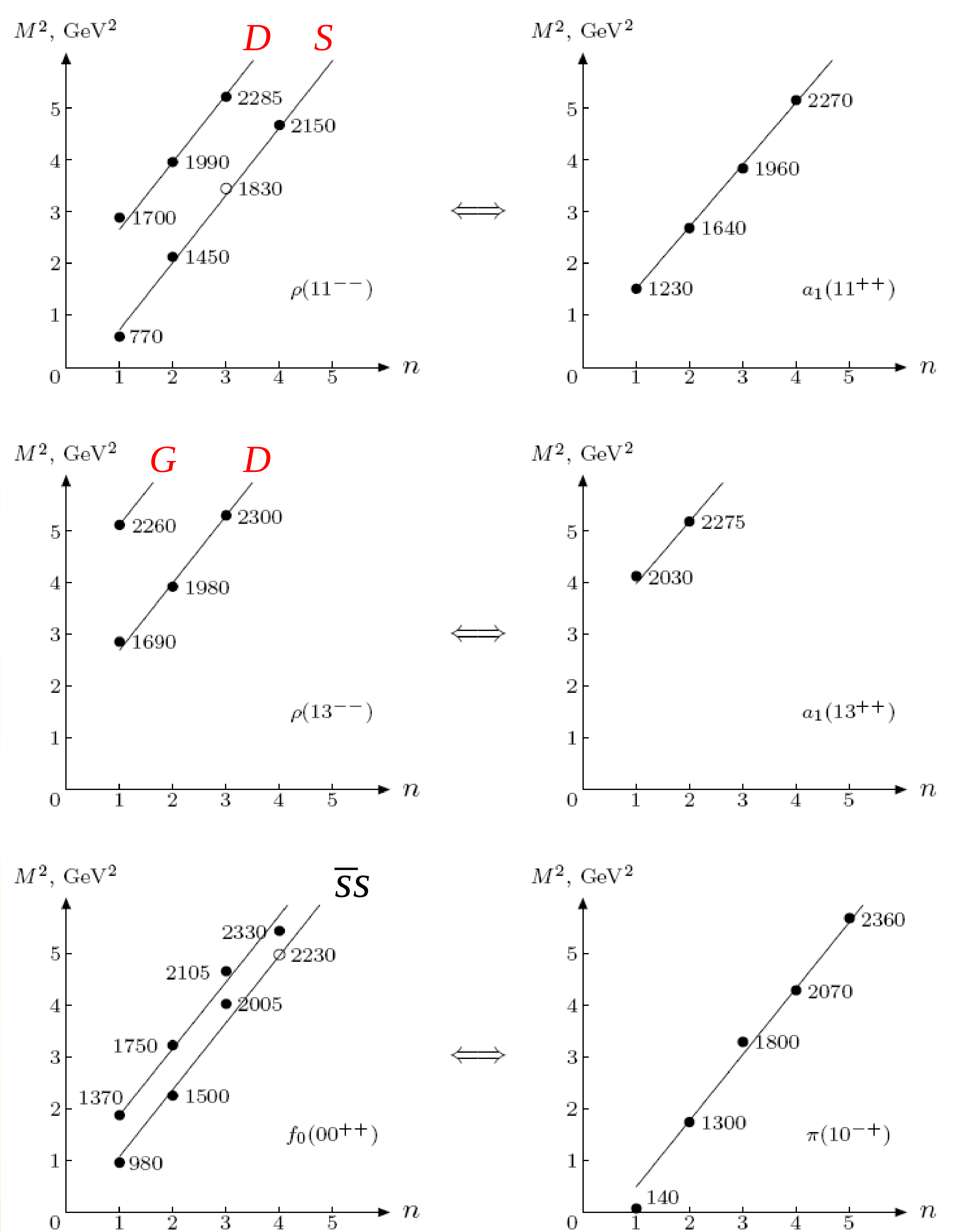
L=0 (*S*-wave): $J = \uparrow\uparrow = ½ + ½ = 1$

$q\bar{q}$

L=2 (*D*-wave): $J = \uparrow_{L}\ \downarrow\downarrow = 2 - ½ - ½ = 1$

$L\ q\bar{q}$

$\Longrightarrow$ Two kinds of ρ



Soft wall model (Karch et al., PRD (2006))

$$I = \int d^5x \sqrt{g}\, e^{-\Phi} \mathcal{L}$$

$$g_{MN}\, dx^M dx^N = e^{2A(z)}(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu) \qquad \Phi = \Phi(z)$$

The IR boundary condition is that the action is finite at $z = \infty$

Plane wave ansatz: $V_\mu(x,z) = \varepsilon_\mu e^{ipx} v(z)$ $\quad p^2 = m^2$ $\quad$ Axial gauge $V_z = 0$

E.O.M.: $\partial_z \left(e^{-B}\partial_z v_n\right) + m_n^2 e^{-B} v_n = 0$ $\quad B = \Phi(z) - A(z)$

Substitution $v_n = e^{B/2}\psi_n$

$$-\psi_n'' + U(z)\psi_n = m_n^2 \psi_n \qquad U(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B''$$

With the choice $B = \Phi - A = az^2 + \log z$ $\longrightarrow$ $U = a^2 z^2 + \frac{3}{4z^2}$

One has the radial Schroedinger equation for harmonic oscillator with orbital momentum ***L*=1**

$$-\psi'' + \left[z^2 + \frac{L^2 - 1/4}{z^2}\right]\psi = E\psi \qquad E = |a|m$$

To have the Regge like spectrum: $\Phi = az^2$

To have the AdS space in UV asymptotics: $A = -\log z$ $\longrightarrow$ $e^{2A} = \frac{1}{z^2}$

The spectrum: $m_n^2 = 4|a|(n+1)$ $\quad n = 0, 1, 2, \ldots$

The extension to massless higher-spin fields leads to (for $a > 0$)

$$m_{n,J}^2 = 4a(n+J) \qquad (\#)$$

In the first version of the soft wall model $a < 0$ (O. Andreev, PRD (2006)): $g_{MN} = \frac{e^{-az^2}}{z^2}\eta_{MN}$

A Cornell like confinement potential for heavy quarks was derived
(O. Andreev, V. Zakharov, PRD (2006))

In order to have (#) for $a < 0$, the higher-spin fields must be massive!

Generalization to the arbitrary intercept $m_n^2 = 4|a|(n+1+b)$

$$e^{-az^2} \to \Gamma(1+b)U^2(b,0;az^2)e^{-az^2}$$

(Afonin, PLB (2013))

Tricomi function

But! No natural chiral symmetry breaking!

Calculation of vector 2-point correlator: $$W_{4D}[\varphi_0(x)] = S_{5D}[\varphi(x,\epsilon)]$$

4D Fourier transform — source

$$V^\mu(q,z) = v(q,z) V_0^\mu(q)$$ $$v(q,\epsilon) = 1$$

E.O.M.:

$$\partial_z\left(\frac{e^{-az^2}}{z}\partial_z v\right) + \frac{e^{-az^2}}{z} q^2 v = 0$$

Action on the solution

$$I = \int d^4x V_0^\mu V_{0\mu} \frac{e^{-az^2}}{z} v\partial_z v\Big|_{z=\epsilon}^{z=\infty}$$

$$\int d^4x e^{iqx}\langle J_\mu(x) J_\nu(0)\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(-q^2)$$

$$\Pi_V(-q^2) = c^2 \frac{\partial_z v}{q^2 z}\Big|_{z=\epsilon}$$

$$v(q,z) = \Gamma\left(1 - \frac{q^2}{4|a|}\right) e^{(a-|a|)z^2/2} U\left(\frac{-q^2}{4|a|}, 0; |a|z^2\right)$$

$$\Pi_V(-q^2) = c^2 \left[\frac{a - |a|}{q^2} - \frac{1}{2}\psi\left(1 - \frac{q^2}{4|a|}\right)\right] + \mathrm{const}$$

$$\Pi_V(-q^2) = c^2 \left[\frac{a - |a|}{q^2} + \sum_{n=0}^{\infty} \frac{2|a|}{4|a|(n+1) - q^2}\right] + \mathrm{const}$$

$$\Pi_V(Q^2)_{Q^2\to\infty} = \frac{c^2}{2}\left[\log\left(\frac{4|a|}{Q^2}\right) - \frac{2a}{Q^2} + \frac{4a^2}{3Q^4} + \mathcal{O}\left(\frac{a^4}{Q^8}\right)\right]$$

$$Q^2 = -q^2$$

$$\Pi_V(Q^2)_{\mathrm{OPE}} = \frac{N_c}{24\pi^2}\log\left(\frac{\mu^2}{Q^2}\right) + \frac{\alpha_s}{24\pi}\frac{\langle G^2\rangle}{Q^4} + \xi\frac{\langle \bar{q}q\rangle^2}{Q^6} + \mathcal{O}\left(\frac{\mu^8}{Q^8}\right)$$

$$\Rightarrow \quad c^2 = \frac{N_c}{12\pi^2}$$

The dilaton background can be eliminated by

$$V_M \to e^{az^2/2} V_M$$

The gauge invariant action can be formulated as (No-wall model; Afonin, IJMPA (2011))

$$S = \int d^4x\, dz \sqrt{g} \left\{ |D_M \varphi|^2 - m_\varphi^2 \varphi^2 - \frac{1}{4g_5^2} F_{MN} F^{MN} \right\}$$

$$D_M = \partial_M - iV_M$$

The E.O.M. for the scalar field: $- \partial_z \left(\frac{\partial_z \varphi}{z^3} \right) + \frac{m_\varphi^2 R^2 \varphi}{z^5} = 0$

If we want to have the linear spectrum:

$\varphi_0 \sim z^2$ ⇒ $m_\varphi^2 R^2 = -4$ ⇒ $\Delta = 2$

from $m_\varphi^2 R^2 = \Delta(\Delta - 4)$

$\mathcal{O}\left(\frac{1}{Q^2}\right)$ term is absent in the OPE!

Possible extensions

- Various modifications of metrics and of dilaton background
- Alternative descriptions of the chiral symmetry breaking
- Inclusion of additional vertices (Chern-Simon, …)
- Account for backreaction of metrics caused by the condensates (dynamical AdS/ QCD)
- Construction of acceptable AdS/ QCD models from a 5D gravitational setup

Some applications

- Meson, baryon and glueball spectra
- Low-energy strong interactions (chiral dynamics)
- Hadronic formfactors
- Thermodynamic effects (QCD phase diagram)
- Description of quark-gluon plasma
- Condensed matter (high temperature superconductivity *etc.*)
- …

Deep relations with other approaches

- Light-front QCD
- Soft wall models: QCD sum rules in the large-N_c limit
- Hard wall models: Chiral perturbation theory supplemented by infinite number of vector and axial-vector mesons
- Renormgroup methods

Holographic description of thermal and finite density effects

Basic ansatz

$$A_t = A_t(z),$$
$$A_i = 0 \qquad (i = 1, \cdots, 3, z),$$
$$ds^2 = \frac{R^2}{z^2}\left(f(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{f(z)}\right)$$

- corresponds to $\bar{q}\gamma^0 q$

One uses the Reissner-Nordstrom AdS black hole solution

$$f(z) = 1 - (1+Q^2)\left(\frac{z}{z_h}\right)^4 + Q^2\left(\frac{z}{z_h}\right)^6,$$
$$A_t(z) = \mu - \kappa\frac{Q}{z_h^3}z^2,$$

where $0 \leqslant Q \leqslant \sqrt{2}$ is the charge of the gauge field.

The hadron temperature is identified with the Hawking one: $T_H = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z\to z_h} = \frac{1}{\pi z_h}\left(1 - \frac{Q^2}{2}\right)$

The chemical potential is defined by the condition $A_t(z_h) = 0$ $\quad \mu = \kappa\frac{Q}{z_h}$

Deconfinement temperature from the Hawking-Page phase transition

(Herzog, PRL (2008))

Consider the difference of free energies $\Delta V = \lim_{\epsilon \to \infty} (V_{\mathrm{BH}}(\epsilon) - V_{\mathrm{Th}}(\epsilon))$

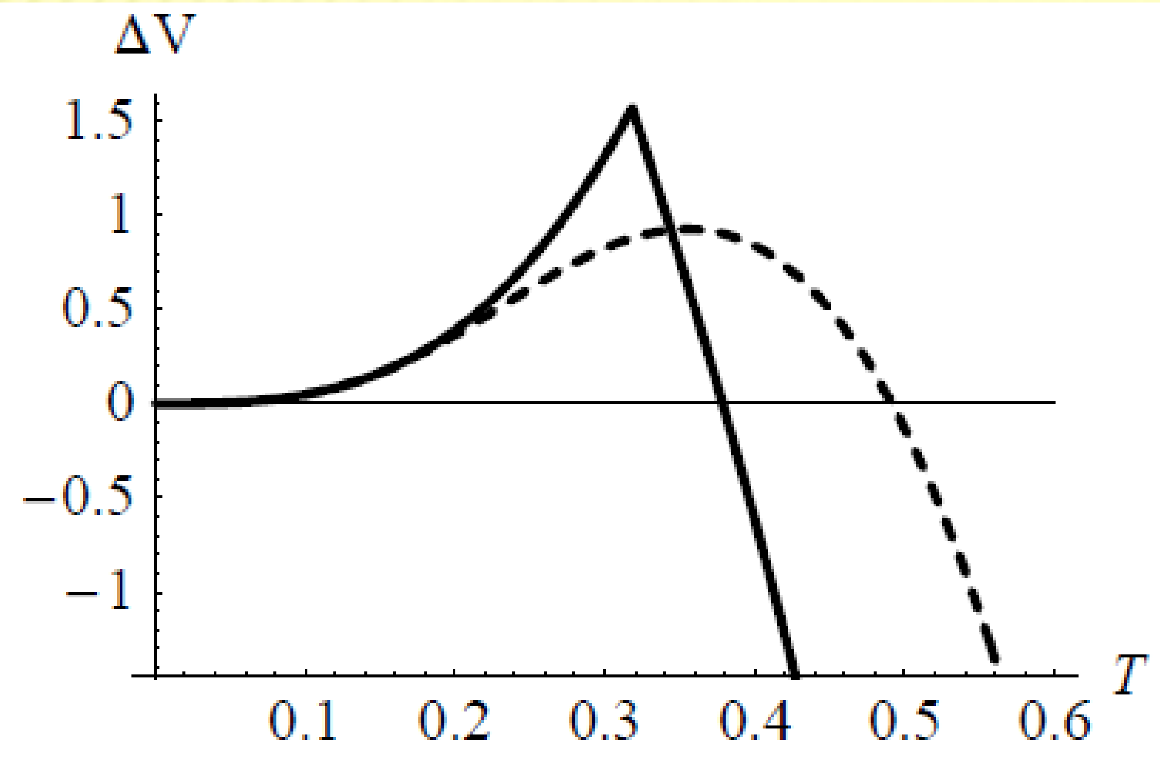


FIG. 1: The solid line is the free energy difference in the hard wall model, the dashed line the difference in the soft wall model.

HW: $T_c = \frac{2^{1/4}}{\pi z_0} \approx 0.157 m_\rho = 122\ \mathrm{MeV}$

SW: $T_c \approx 0.49\sqrt{a} \approx 0.246 m_\rho = 191\ \mathrm{MeV}$

Entropy density $\begin{cases} \mathcal{O}(1) & \text{- confined phase} \\ \mathcal{O}(N_c^2) & \text{-deconfined phase} \end{cases}$

The pure gravitational part of the SW model $I \sim \int d^5x \sqrt{g} e^{-az^2}$ where ***a*>0**

For ***a*<0**, the criterium based on the temperature dependence of the spatial string tension can be used (O. Andreev, V. Zakharov, PRD (2006))

$$T_c = \frac{\sqrt{2|a|}}{\pi} \approx 0.45\sqrt{|a|}$$

Some examples of phase diagrams

He *et al.*, JHEP (2013)

(a dynamical AdS/ QCD model)

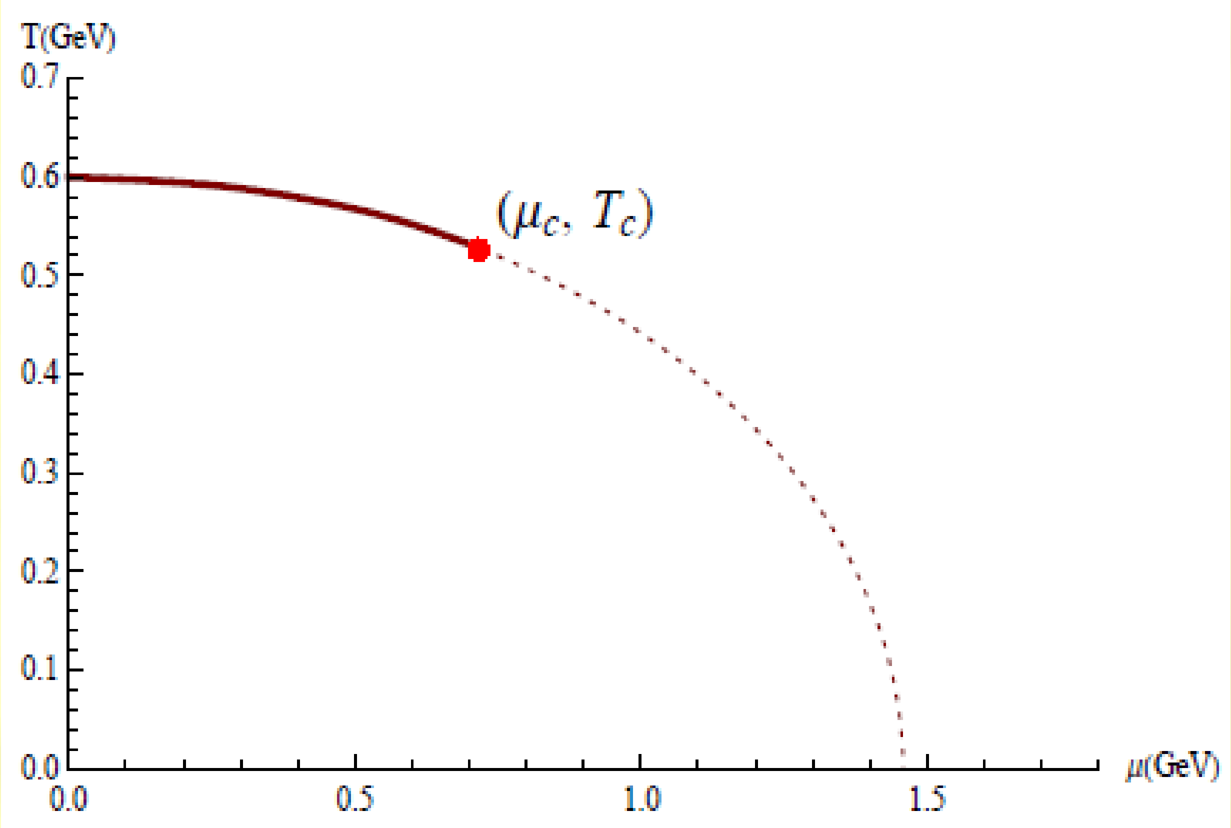
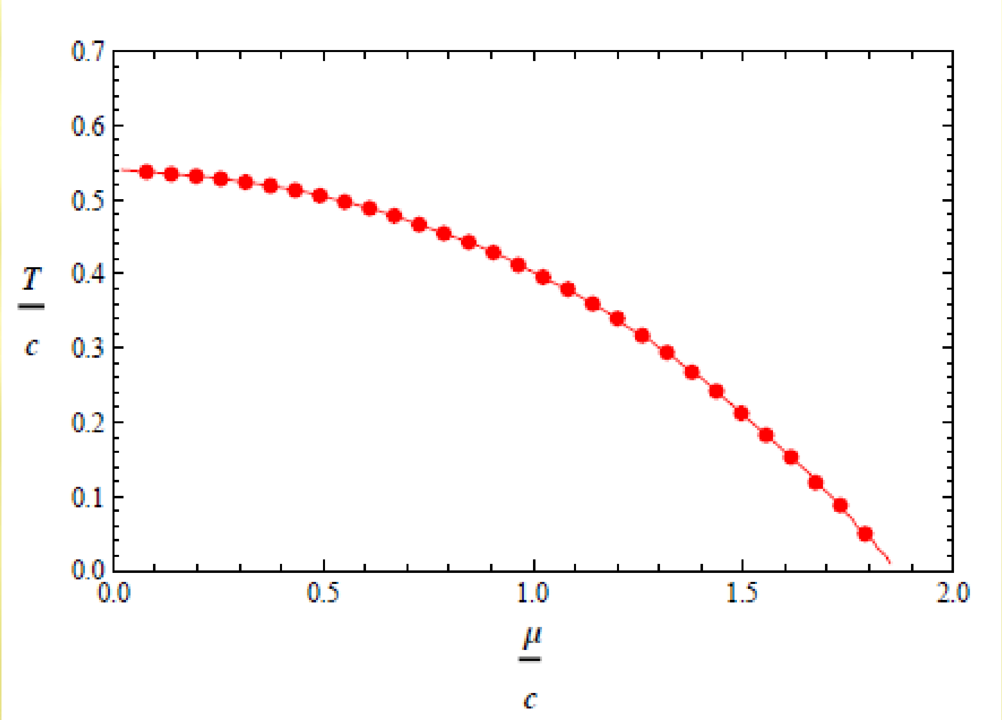


FIG. 3: The phase diagram in T and μ plane. At small μ, the system undergoes a first order phase transition at finite T. The first order phase transition stops at the critical point $(\mu_c, T_c) \simeq (0.714GeV, 0.528GeV)$, where the phase transition becomes second order. For $\mu > \mu_c$, the system weaken to a sharp but smooth crossover.

Colangelo *et al.*, EPJC (2013)

(the soft wall model)



Line in the $T - \mu$ plane where σ vanishes.

$\langle \bar{q}q \rangle \propto \sigma$

Hadronic form factors

Definition for mesons: $\langle P'|J^\mu(0)|P\rangle = (P+P')^\mu F_M(q^2)$

Electromagnetic form factor: $J^\mu = e_q \bar{q}\gamma^\mu q$

In the holographic models for QCD:

$$\int d^4x\, dz \sqrt{g}\, A^M(x,z)\Phi^*_{P'}(x,z)\overleftrightarrow{\partial}_M \Phi_P(x,z) \sim (2\pi)^4 \delta^4 (P' - P - q)\, \epsilon_\mu (P+P')^\mu F_M(q^2)$$

$\Phi_P(x,z) \sim e^{-iP\cdot x}\Phi(z)$ $\quad P_\mu P^\mu = \mathcal{M}^2$

$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q^2,z), \quad A_z = 0,$ $\quad J(Q^2=0,z) = J(Q^2, z=0) = 1$ $\quad (Q^2 = -q^2 > 0)$

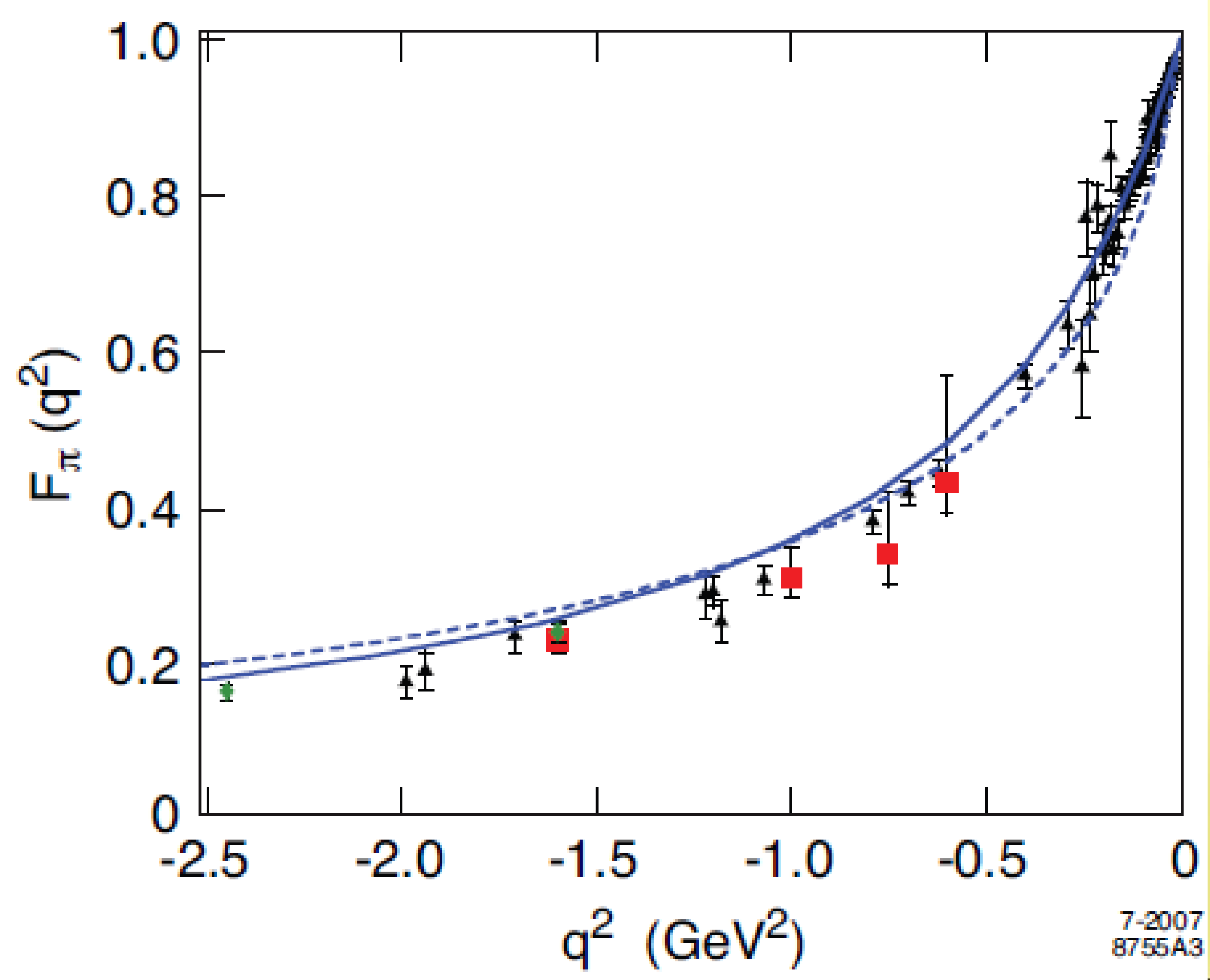


FIG. 3: Space-like behavior of the pion form factor $F_\pi(q^2)$ as a function of q^2 for $\kappa = 0.375$ GeV and $\Lambda_{QCD} = 0.22$ GeV. Continuous line: soft-wall model, dashed line: hard-wall model. Triangles are the data compilation from Baldini *et al.* [90], boxes are JLAB 1 [91] and diamonds are JLAB 2 [92].

Brodsky, de Teramond, PRD (2008)

Light-front holographic QCD

(Brodsky et al., arXiv:1407.8131, submitted to Phys. Rept.)

In a semiclassical approximation to QCD the light-front Hamiltonian equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ reduces to a Schroedinger equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta),$$

where L is the orbital angular momentum of the constituents and the variable ζ is the invariant separation distance between the quarks in the hadron at equal light-front time.

Its eigenvalues yield the hadronic spectrum, and its eigenfunctions represent the probability distributions of the hadronic constituents at a given scale. This variable is identified with the holographic coordinate z in AdS space.

Arising interpretation: z measures the distance between hadron constituents

Hard wall models: $0 < z \leq z_m$ close relatives of MIT bag models!

E.o.m. for *massless* 5D fields of arbitrary spin in the soft wall model after a rescaling of w.f.

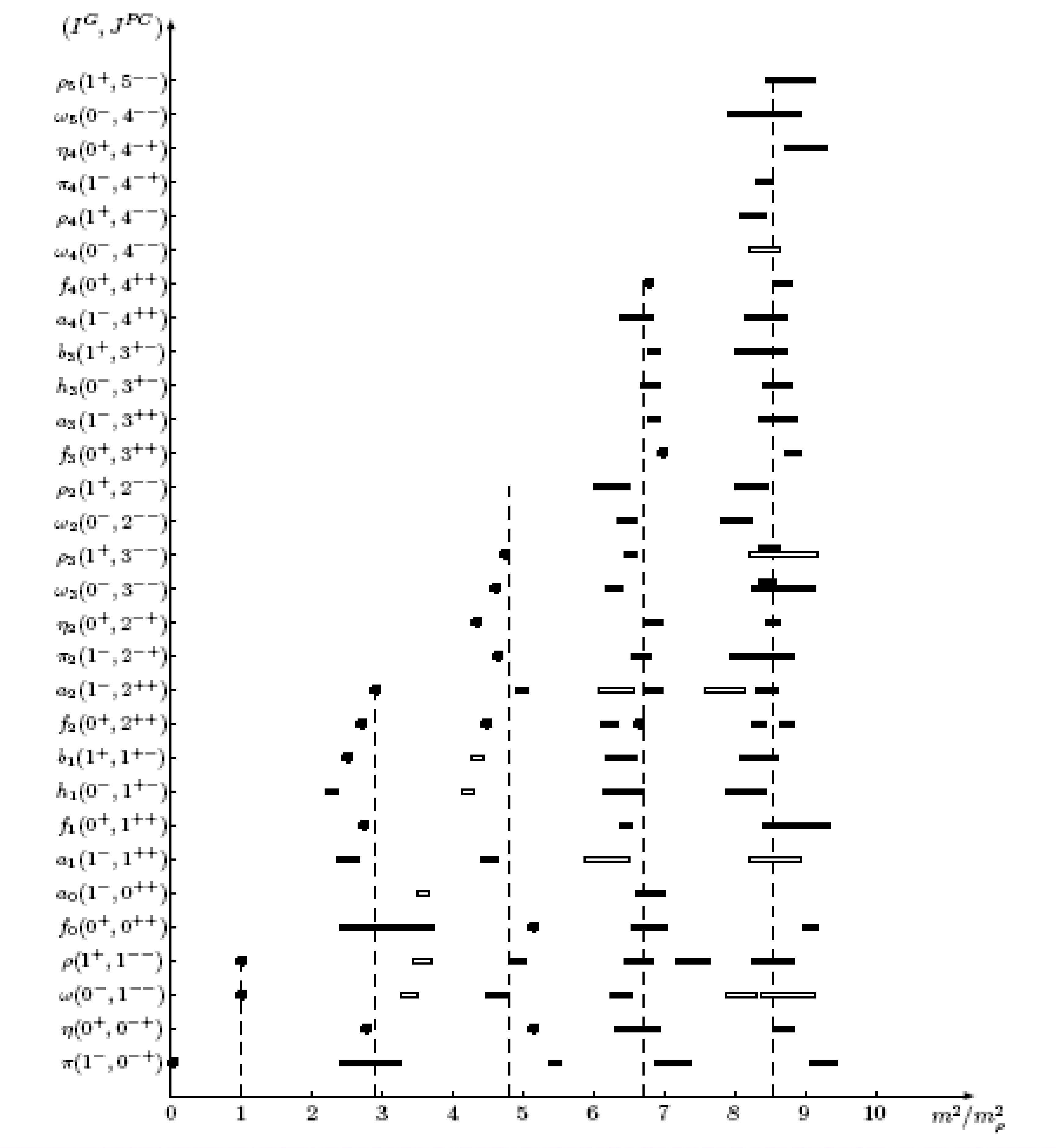
$$\left(-\frac{d^2}{dz^2} - \frac{1-4J^2}{4z^2} + a^2z^2 + 2a(J-1)\right)\phi(z) = M^2\phi(z)$$

The 5D mass from holographic mapping to the light-front QCD: $(mR)^2 = -(2-J)^2 + L^2$

The meson spectrum:

$$M^2_{n,J,L} = 4|a|\left(n + \frac{J+L}{2}\right)$$

$n, J, L = 0, 1, 2\ldots$



The light non-strange mesons from the Particle Data Group

One observes clustering of states!

Classification of light non-strange mesons in *(L,n)*

$L \backslash n$	0	1	2	3	4
0	$\pi(140)$ $\eta(548)(??)$ $\rho(770)$ $\omega(782)$	$\pi(1300)$ $\eta(1295)(??)$ $\rho(1450)$ $\omega(1420)$	$\pi(1800)$ $\eta(1760)$ $\rho(?)$ $\omega(?)$	$\pi(2070)$ $\eta(2010)$ $\rho(1900)$ $\omega(?)$	$\pi(2360)$ $\eta(2285)$ $\rho(2150)$ $\omega(2205)(?)$
1	$f_0(1370)$ $a_0(1450)(??)$ $a_1(1260)$ $f_1(1285)$ $b_1(1230)$ $h_1(1170)$ $a_2(1320)$ $f_2(1275)$	$f_0(1770)$ $a_0(?)$ $a_1(1640)$ $f_1(?)$ $b_1(1620)(?)$ $h_1(1595)(?)$ $a_2(1680)$ $f_2(1640)$	$f_0(2020)$ $a_0(2025)$ $a_1(1930)(?)$ $f_1(1971)$ $b_1(1960)$ $h_1(1965)$ $a_2(1950)(?)$ $f_2(1934)$	$f_0(2337)$ $a_0(?)$ $a_1(2270)(?)$ $f_1(2310)$ $b_1(2240)$ $h_1(2215)$ $a_2(2175)(?)$ $f_2(2240)$	
2	$\rho(1700)$ $\omega(1650)$ $\pi_2(1670)$ $\eta_2(1645)$ $\rho_2(?)$ $\omega_2(?)$ $\rho_3(1690)$ $\omega_3(1670)$	$\rho(2000)$ $\omega(1960)$ $\pi_2(2005)$ $\eta_2(2030)$ $\rho_2(1940)$ $\omega_2(1975)$ $\rho_3(1982)$ $\omega_3(1945)$	$\rho(2265)$ $\omega(2295)(?)$ $\pi_2(2245)$ $\eta_2(2267)$ $\rho_2(2225)$ $\omega_2(2195)$ $\rho_3(2300)(?)$ $\omega_3(2285)$		
3	$f_2(2001)$ $a_2(2030)$ $f_3(2048)$ $a_3(2031)$ $b_3(2032)$ $h_3(2025)$ $f_4(2018)$ $a_4(2005)$	$f_2(2293)$ $a_2(2255)$ $f_3(2303)$ $a_3(2275)$ $b_3(2245)$ $h_3(2275)$ $f_4(2283)$ $a_4(2255)$			
4	$\rho_3(2260)$ $\omega_3(2255)$ $\rho_4(2230)$ $\omega_4(2250)(?)$ $\pi_4(2250)$ $\eta_4(2328)$ $\rho_5(2300)$ $\omega_5(2250)$				

In average (in GeV^2)

$$\bar{M}^2(L,n) \approx 1.1L + 1.1n + 0.7$$

(Afonin, PRC(2007))

$\Rightarrow$ The law $M^2(L,n): \; L+n$ works!

Like in the nonrelativistic hydrogen atom:

$$E \sim \frac{1}{N^2}, \qquad N = L + n + 1$$

- principal quantum number

Potential models cannot explain the existence of “principal” quantum number!

THANK YOU!