The Critical End Point through observables

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CEP **& Critical Phenomena**

NICA / collider / FAIR / fixed target/

strong interacting matter @ high T & μ_B

Freezout point: *CEP* / QCD CP

Matter becomes weakly coupled

In the proximity of *CEP*:

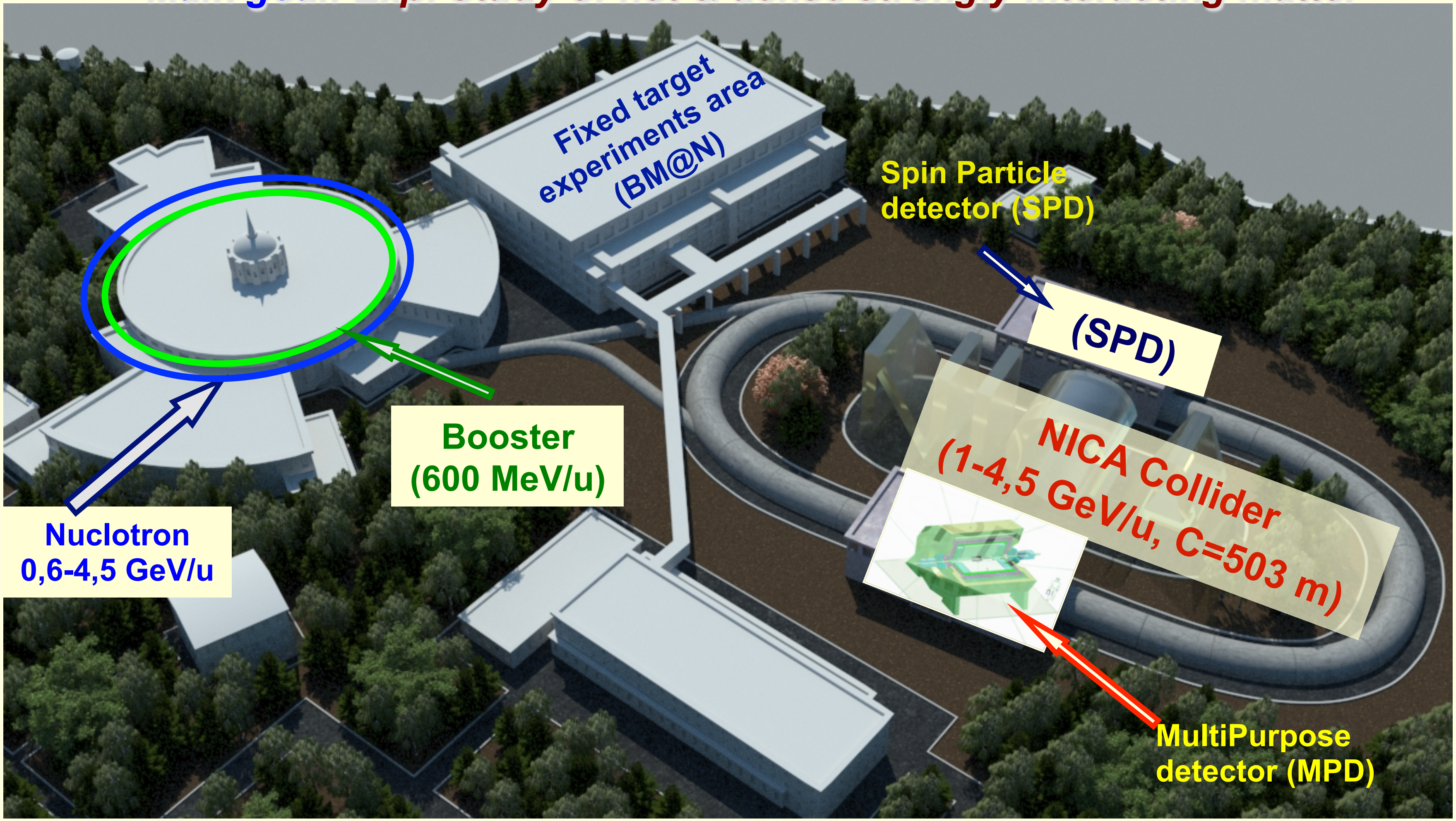
- **Color is no more confined**
- **Chiral symmetry is restored**
- **Phase transition is associated with breaking of symmetry**

Traditionally *CEP* clarified through $\left(\mu_B - T\right)$ plane

scanning of $\left(\mu_B - T\right)$ phase diagram

Superconducting accelerator complex NICA
(Nuclotron based Ion Collider fAcility)

Main goal: Exp. Study of hot & dense strongly interacting matter



Two modes in operation:
- Collider mode (MPD detector) Max momentum 13 GeV/c (protons), $L = 10^{27} cm^{-2} s^{-1}$
- Extracted beams (BM@N)

Also Spin Physics/ Polarized deutrons and protons (Energy = 26 GeV (protons))

CEP & Critical Phenomena

A few questions arise:

- ***CEP* meaning?**
- **Main observables to be measured when *CEP* achieved?**
- **New knowledge if *CEP* approached?**

Answer: QCD_T **@ large distances**

N/Perturbative phenomena: χSB **& Confinement of color**

$\downarrow$ *?relations?* $\downarrow$

Phase transition of χS **Restoration Deconfinement**

$\downarrow$ **correlations** $\downarrow$

important issue

NO correct solution (massless quarks in the theory)

Effective models, e.g., with topological defects

Topological defects (TD's)

TD's exist only in phase with SSB where $\langle \phi \rangle_{vacuum}$ emerges

Non-broken symmetry phase: ***no solution with TD's***

Minimal model: TD's (strings) arise in Abelian Higgs-like model

(Nielsen, Olesen, 1973)

$$SU(N) \xrightarrow{reduction} \left[U(1)\right]^{N-1}$$ **dual scalar thery**

breaking $\downarrow$ Higgs-like mechanism

- **MA Gauge suggests special properties of QCD vacuum**
- **Dual superconductor picture of QCD vacuum** **('t Hooft 1981)**
- **Condensation of scalar d.o.f.** **(Ezawa, Iwasaki, 1982)**

Field correlator

CEP **• Fluctuation measure • Observables**

through $\downarrow$

Fluctuations of characteristic length ξ of chiral end mode

Model: fluctuations based on the order parameter field with $m \sim \xi^{-1}$

- Deal with gauge-invariant quantities, TPCF as a function of $C_{\mu}(x)$

- Dual color string: $U_C(x,y) \sim exp\left[ig\int_y^x dz^{\mu} C_{\mu}(z)\right]$

- Bound states in terms of flux tubes

Duality & Scale symmetry breaking

$R \to \infty$: C_μ^a, instead of A_μ^a - natural variables for confinement

C_μ^a weakly coupled to $\phi_i \left(i = 1,2,3\right)$ **dilatons**
(each in the adjoint representation of magnetic gauge group)

$$Z = \int D\phi_i \exp\left\{ -\int_0^\tau d\tau \int d^3\vec{x}\; L\left(\tau, \vec{x}\right) \right\}$$

In general, $L\left(x\right) = \sum_i c_i\left(\mu\right) O_i\left(x\right), \quad \left[O_i\left(x\right)\right] = d_i$

Under scale trans.'s $x^\mu \to e^\omega x^\mu : O_i\left(x\right) \to e^{\omega d_i} O_i\left(e^\omega x\right), \quad \mu \to e^{-\omega}\mu$

Dilatation current $S^\mu\left(x\right) = T^{\mu\nu} x_\nu$

$$\partial_\mu S^\mu = T_\mu^\mu = \sum_i \left[c_i\left(\mu\right)\left(d_i - 4\right) O_i\left(x\right) + \beta_i\left(c\right) \frac{\partial}{\partial c_i} L \right], \; \beta_i\left(c\right) = \mu \frac{\partial c_i\left(\mu\right)}{\partial \mu}$$

IR Fixed Point

Slow running of α turns to smallness of $\beta(\alpha) = -\frac{b_0}{2\pi}\alpha^2 - \frac{b_1}{(2\pi)^2}\alpha^3 - \ldots$

At $Q < f$ (conformal breaking scale) to scale invariance saving:

replacement $c_i(\mu) \to \left(\frac{\phi}{f}\right)^{4-d_i} c_i\left(\mu\frac{\phi}{f}\right)$ Goldberger et al., 2008

incorporated flat direction transforms $\phi(x) \to e^{\omega}\phi(e^{\omega}x)$, $\langle\phi\rangle = f$

Theory would be nearly scale invariant if $d_i \to 4$, $\beta(\alpha) \to 0$

Breaking of chiral symmetry is triggered by the dynamics of nearly conformal sector

Effective model

$$L_{eff} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \sum_{i=1}^{3} \left[\frac{1}{2} \left| D_{\mu}^{(i)} \phi_i \right|^2 - \frac{1}{4} \lambda \left(\phi_i^2 - \phi_{0_i}^i \right)^2 \right]$$

$$G_{\mu\nu} = \partial_{\mu} C_{\nu} - \partial_{\nu} C_{\mu} - ig \left[C_{\mu}, C_{\nu} \right], \qquad D_{\mu}\phi_i = \partial_{\mu}\phi_i - ig \left[C_{\mu}, \phi_i \right]$$

C_{μ} defined by $U_C\left(x,y\right) = P \exp\left[ig \int_y^x dz^{\mu} C_{\mu}\left(z\right) \right]$

$$C_{\mu}\left(x\right) \to \Omega_C^{-1}\left(x\right) C_{\mu}\left(x\right) \Omega_C\left(x\right) + \frac{i}{g} \Omega_C^{-1}\left(x\right) \partial_{\mu} \Omega_C\left(x\right)$$

Color structure of $\phi_{0_i} = \left\langle \phi_i\left(x\right) \right\rangle, \quad i = 1,2,3$ Baker, Ball, Zachariazen 1990

$$\phi_{0_1} = \frac{f}{\sqrt{2N}} J_x, \quad \phi_{0_2} = \frac{f}{\sqrt{2N}} J_y, \quad \phi_{0_3} = \frac{f}{\sqrt{2N}} J_z, \qquad J = \frac{1}{2}\left(N - 1\right),$$

Flux tubes

Excitations above vacuum: flux tubes, $r_s \sim m^{-1}$

(in the center, $r_s \to 0$, scalar condensate vanishes)

Ensemble of a single tube system

$$\mathrm{P} = \sum_{\beta} \sum_{R} N(R) \exp\left[-\beta\, E(m,R)\right] D\left(|\vec{x}|, \beta; M\right)$$

effective action: $E(m,R) \sim m^2 R\left[a + b \ln\left(\tilde{\mu} R\right)\right]$ ***GK, 2010***

CEP: infinite fluctuation length $\xi \sim m^{-1}$

C_μ **- critical end mode!**

$$m^2(\beta) \sim g^2(\beta)\delta^{(2)}(0)$$

$$\downarrow$$

$$c/\left(\pi r_s^2\right), \quad c \sim O(1)$$

TPCF

At large distances for any correlator (observables)

$$\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim A|\vec{x}|^c D(|\vec{x}|,\beta;M) \text{ as } |\vec{x}|\to\infty$$

$$D(|\vec{x}|,\beta;M) = \exp[-M(\beta)|\vec{x}|],\ \ D(|\vec{x}|,\beta;M) \neq 0 \text{ even at } \beta=\beta_c$$

$M^{-1}(\beta)$ is the measure of screening effect of color electric field

For $SU(N=2,3)$ **, high** T**,** N_f **massless,** $\mu=0$

$$M(\beta) = M^{LO}(\beta) + N\alpha T \ln\left(\frac{M^{LO}(\beta)}{4\pi\alpha T}\right) + 4\pi\alpha T y_{n/p}(N) + O(\alpha^2 T)$$

$$M^{LO}(\beta) = \sqrt{4\pi\alpha\left(\frac{N}{3}+\frac{N_f}{6}\right)}\, T$$

Kajante et al. 1997

At $|\vec{x}| < M^{-1}(\beta)$**,** $\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim \frac{16A\pi}{3}\frac{T}{V}\sigma_{eff}(\beta)\, y_{n/p}\xi^2$ **!**

GK, 2014

String tension

$$\sigma_{eff}(\beta) \sim m^2(\beta)\alpha(\beta)$$ *GK 2010*

Flux-tube scheme:

- $\xi \sim m^{-1}$ the penetration length of color-electric field
- $\xi \sim r_s$ "string"-like radius
- $l \sim m_\phi^{-1}$ coherent length of scalar (dilaton) condensate
- $\tau = \sqrt{4/(3\alpha)}\xi$ formation time of flux tube ($\to \infty$ @ CEP)

➢ For SU(3), $m \approx 1.95\sqrt{\sigma_{eff}}$ Baker et al., 1997

✓ **Lattice:** $T_c \approx 0.65\sqrt{\sigma_{eff}}$

Effective theory applicable in deconfined phase $T_c < T < 3T_c$ **!**

Strings. Vacuum.

In $SU(3)$ gluodynamics vacuum is characterized

$$k_{GL} = \frac{\xi}{l} \sim \frac{m_\phi}{m} \begin{array}{ll} <1 & (type\ I\ vacuum,\ \ attracted) \\ >1 & (type\ II\ vacuum,\ \ repel) \end{array}$$

If $k_{GL} = 1$ parallel strings (carry the same flux) do not interact each other

NICA:

Observation of correlations between two bound states (strings) is rather useful & instructive to check the *CEP* is approached at *the Critical Point* !

NICA: sample with production $\pi^+\pi^+, \pi^-\pi^-, \pi^0\pi^0$

$$AA\ (pp) \to high\ \ T\ \ quark-gluon\ \ bubble \to hadronization \to$$
$$\to chaotic\ \ pion's\ \ production\ \ with\ \ different\ \ directions,\ \ momenta,\ \ angles$$

NICA: Bose-Einstein Correlations @ high *T*

Def.:

BEC's are the quantum effect which enhances the probability that multiple bosons be found in the same state, same position, same momentum

NICA: Bose-Einstein Correlation

- BE correlations might be measured using data collected with MPD detector at NICA in heavy-ion collisions , $\sqrt{s} = O(10GeV)$
- In the case of **no** ***CEP*** approached, the signal is observed as an enhancement of pairs of same-sign charge particles with small relative momenta ***GK 2008***

$$C_2\left(q,\beta = T^{-1}\right) = \eta(n)\left\{1+\tilde{\lambda}(\beta)e^{-q^2L_{st}^2}\left[1+\lambda_1(\beta)e^{q^2L_{st}^2/2}\right]\right\}$$

- When ***CEP*** approached:
  - **NO** signal of enhancement of pairs of same-sign charge particles is observed

! C_2 -function does not deviate from 1

$$L_{st} \to \infty \;\; as \;\; T \to T_c, \;\; \eta(n) \to 1, \;\; n \to \infty$$

NICA: Particle emission size

Theory: $L_{st} = L_{st}\left(\beta, k_T, m, \nu(n)!\right) \sim \dfrac{1}{\nu^{1/5}(n)}$ ***GK, 2009-2010***

$$\nu(n) = \frac{2 - \tilde{C}_2(0) + \sqrt{2 - \tilde{C}_2(0)}}{\tilde{C}_2(0) - 1}, \qquad \tilde{C}_2(0) = \frac{C_2(q=0)}{\eta(n)}$$

$$\langle n \rangle \geq 1 + C_2(0)/2, \quad C_2(0) \leq 2$$

CMS (2011): $\sqrt{s}$=0.9 TeV; 7 TeV L_{st} increases with $\langle n \rangle$

High T: $L_{st} \sim \left[\nu(n) k_T^2 \, T^3\right]^{-1/5}$ no μ – & m_h – dependence!

- $L_{st} \to \infty$ as $\nu(n) \to 0$ with $n \to \infty$

NICA: Expansion of particle emission size

$$L_{st}(\beta) \sim \left[\nu(n) k_T^2 \; T^3\right]^{-1/5} \to \infty \text{ as } \nu(n) \to 0 \text{ with } n \to \infty \text{ at } T \to T_c$$ ***KG 2010***

The temperature at which the signal of two-particles correlations disappears is the critical temperature at *CEP* : $C_2(q, T_c) = 1$

NICA: Dip-effect

The effect of anti-correlations (the dip-effect) is predicted at low charged-particle multiplicity n **in the event:** $C_2(\{q\}, n) < 1$ **?!** ***KG 2010***

The depth of the dip in the anti-correlation region decreases as n **increases.**

Observed by CMS at LHC [CMS Coll., JHEP 5 (2011) 029]

Proposal: at *CEP* the dip-effect disappears

Critical temperature

GK, ICHEP2014

$T_c \approx 0.28 m_{q\bar{q}} \sqrt{3\pi\left(N/3 + N_f/3\right)}$ no α_s-dependence

LO, NLO: $\alpha_s \to 0$ in the vicinity of deconfinement

$T_c = 167 MeV$ for pions; $\mu = 0.35 GeV$

NICA: Strength of BE corr's $\tilde{\lambda}(k_T,\beta)$ for incoherent particles emitted from independent sources

$$C_2(q,\beta)\approx\eta(n)\left\{1+\tilde{\lambda}(\beta)e^{-q^2L_{st}^2}\left[1+\lambda_1(\beta)e^{+q^2L_{st}^2/2}\right]\right\}$$

$$\tilde{\lambda}(\beta)=\frac{\gamma(\omega,\beta)}{\left[1+\nu(n)\right]^2},\ \nu\sim\frac{1}{n}\frac{1}{k_{GL}^2},\ \gamma(\omega,\beta)\sim O(1)$$

Measure of the *CEP*: fluctuation length $\xi\sim m^{-1}$ (of the "dual" gauge field)

Proposal: ***GK 2009-2014***

✓ $\tilde{\lambda}(k_T,\beta)$ decreases with k_T far away from the *CEP*, CMS (2011)

✓ $\tilde{\lambda}(k_T,\beta)\rightarrow 0$ **as *CEP* approached,** $k_{GL}\rightarrow\infty$ ***DECONFINEMENT***

Origin: infinite fluctuation length $\xi\rightarrow\infty$

Conclusion: Proposal for NICA/FAIR

a). $C_2^{\exp}$ is the monotonous function with the Dip-effect @ small $\langle n \rangle$, far away from *CEP*.

b). Hot emission volume: Dip disappears as $\langle n \rangle >> 1$, *CEP* signature: $C_2^{\exp} = 1$.

c). Source size L_{st} increases (smoothly) with n at low T.

d). L_{st} blows up as $T \to T_c$ due to $\nu(n) \to 0$, $m_h \to 0$; L_{st} singular @ transition point, *CEP*.

e). $\tilde{\lambda}$ decreases with k_T; $\tilde{\lambda} \to 0$ as $T \to T_c$ where $\xi \to \infty$.

Finally:

a)., c)., e). confirmed by CMS (2011)

points b).,d)., e). are subjects of NICA/FAIR.