Van der Waals forces in pNRQED and pNRQCD

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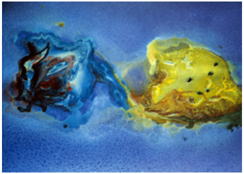
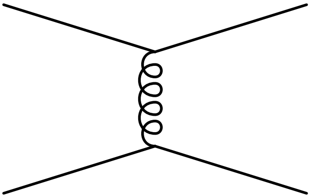


Physik-Department T30f

Outline

- The simplest possible QCD interaction between two quarks is the one-gluon exchange.
- For distances above $1/\Lambda_{QCD}$ our degrees of freedom are not quarks and gluons but hadrons.
- Since hadrons are colorless objects, one-gluon exchange between hadrons is not possible.



Can hadrons exchange more than one gluon?

- Multiple gluon exchange between instantaneous color dipoles is allowed.
- Such interaction is known under the name *QCD van der Waals forces*.
- Up to $R \sim 1\,\mathrm{fm}$ these forces can be understood as a pure gluon exchange

In fact, much work has already been done to obtain corresponding theoretical predictions by using phenomenological Lagrangians and dispersive methods

[Fujii & Mima, 1978]

[Luke et al., 1992]

[Brodsky & Miller, 1997]

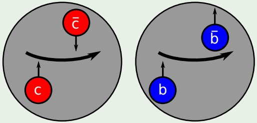
[Fujii & Kharzeev, 1999]

Reasons to study QCD van der Waals forces between charmonia or bottomonia

- Learn more about short range interactions between heavy quarkonia.
- Understand gluonic van der Waals forces in the EFT framework
- Provide precise theoretical predictions for future experiments.

Heavy quarkonium (HQ)

HQ is a $q\bar{q}$-bound state, with $q = c, b$



Well-known examples of HQ are J/Ψ ($c\bar{c}$) and Υ ($b\bar{b}$) mesons

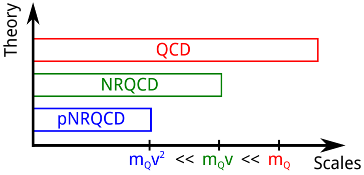
The theoretical tools are already available!

- HQ have a hierarchy of well separated scales.
- The EFT approach is widely used.
- NRQCD [Bodwin et al., 1995] and pNRQCD [Brambilla et al., 2000] are two successful EFTs of QCD.
- NRQCD arises from the systematic of expansion of $\mathcal{L}_{\rm QCD}$ in $\frac{1}{m_Q}$.
- pNRQCD arises from the systematic of expansion of $\mathcal{L}_{\rm QCD}$ in $\frac{1}{m_Q}$ and the size of the bound state r.
- The only remaining dynamical scale in pNRQCD is $m_Q v^2$.
- It is natural to study the QCD van der Waals forces in pNRQCD.

RELEVANT HQ SCALES

$m_Q \gg m_Q v \gg m_Q v^2$,
$m_Q \gg \Lambda_{\rm QCD}$

- $|\vec{p}_{\rm rel}| \sim m_Q v$
- $E_{\rm bind} = m_H - 2m_Q \sim m_Q v^2$
- $m_c \approx 1.3\,{\rm GeV}$, $m_b \approx 4.2\,{\rm GeV}$
- $v_c \approx 0.55$, $v_b \approx 0.32$



- Before we get to QCD let us look at the QED van der Waals forces first!

EM interactions between two hydrogen atoms in the ground state at distance R:

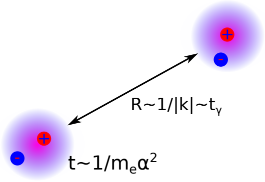
- hydrogen atoms in the ground state are neutral polarizable systems.
- no average electric dipole moments means no electric interaction at leading order.
- But: interactions between instantaneous electric dipole moments are possible.
- the interaction between magnetic dipole moments is suppressed by $1/m_e$

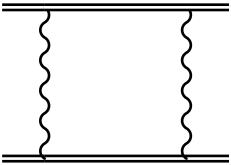
Main scales of the problem

- momentum transfer $|\mathbf{k}| \sim 1/R$
- binding energy $m_e\alpha^2 \sim 1/t$

Possible scale hierarchies

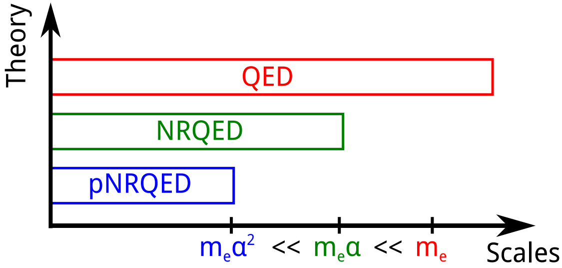
- $|\mathbf{k}| >> m_e\alpha^2$: short range interaction with $V(R) \sim 1/R^6$ (London force) [London, 1930]
- $|\mathbf{k}| << m_e\alpha^2$: long range interaction with $V(R) \sim 1/R^7$ (Casimir-Polder force) [Casimir & Polder, 1948]
- $|\mathbf{k}| \sim m_e\alpha^2$: intermediate region [Feinberg & Sucher, 1970]





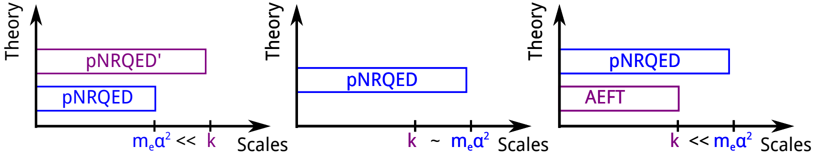
Why the different R-dependence?

- LO Feynman diagram: 2-photon exchange between two neutral fields.
- To pass the distance R, photons need time: $t_\gamma = R/c$.
- For $t_\gamma \ll t$ (i.e. $|\mathbf{k}| \gg m_e\alpha^2$) the photon exchange is instantaneous.
- For $t_\gamma \gg t$ (i.e. $|\mathbf{k}| \ll m_e\alpha^2$) the photon exchange requires finite time (retardation effects).



- In the full QED our system has more scales than just $m_e\alpha^2$ and $|\mathbf{k}|$!
- The scales m_e and $m_e\alpha$ are not relevant for the van der Waals forces.
- Integrating out the m_e scale we obtain non-relativistic QED (NRQED). [Caswell & Lepage, 1986].
- Integrating out the m_e and $m_e\alpha$ scales we obtain potential NRQED (pNRQED) [Pineda & Soto, 1998].
- NRQED and pNRQED can be rigorously derived from the full QED.
- The dynamical degrees of freedom of canonical pNRQED are ultrasoft photons $(E_\gamma, |\mathbf{p}_\gamma| \sim m_e\alpha^2)$ and the singlet field $S(\mathbf{r}, \mathbf{R}, t)$ $(E_S \sim m_e\alpha^2)$.

- After integrating out m_e and $m_e\alpha$, our intermediate theory contains not only the scale $m_e\alpha^2$ but also $|\mathbf{k}|$.
- Depending on the relative size of $|\mathbf{k}|$, three different hierarchies are possible.

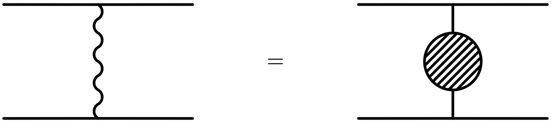


$$L_{\text{pNRQED'}} = -\frac{1}{4}\int d^3\mathbf{R}\, F^{\mu\nu}(t,\mathbf{R})F_{\mu\nu}(t,\mathbf{R})$$
$$+\int d^3\mathbf{R}\, d^3\mathbf{r}\, S^\dagger(t,\mathbf{r},\mathbf{R})\left[i\partial_0 + \frac{\nabla_{\mathbf{r}}^2}{2m_e} + \frac{\alpha}{|\mathbf{r}|} + e\mathbf{r}\cdot\mathbf{E}(t,\mathbf{R}) + c_F\, e\frac{\mathbf{S}\cdot\mathbf{B}(t,\mathbf{R})}{m}\right] S(t,\mathbf{r},\mathbf{R})$$

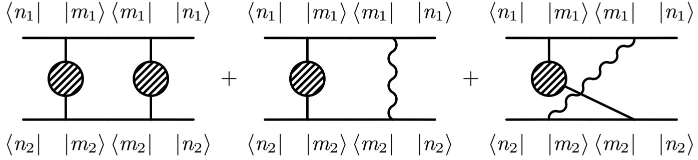
$$L_{\text{pNRQED}} = L_{\text{pNRQED'}} + \int d^3\mathbf{R}_1\, d^3\mathbf{R}_2\, d^3\mathbf{r}_1\, d^3\mathbf{r}_2\, S^\dagger S(t,\mathbf{r}_1,\mathbf{R}_1)\, V(\mathbf{R}_1-\mathbf{R}_2) S^\dagger S(t,\mathbf{r}_2,\mathbf{R}_2)$$

Matching pNRQED' to pNRQED:

- Match 1- and 2-photon exchange diagrams in pNRQED' to the potentials in pNRQED
- Tree-level: Leading order electric, magnetic and mixed potentials
- Loop-level: Subleading magnetic potential



Van der Waals interaction in pNRQED: Scattering of two S fields, where the initial and final states of each field do not change



Potentials in the isotropy approximation

- Electric : $V_{\rm LE}(\mathbf{R}) = -\frac{3e^4}{8\pi^2|\mathbf{R}|^6}\sum_{m_1,m_2}\frac{|r^1_{1nm}|^2|r^1_{2nm}|^2}{\Delta E_1+\Delta E_2}$
- Magnetic : $V_{\rm LB}(\mathbf{R}) = -\frac{3e^4}{8\pi^2|\mathbf{R}|^6}\frac{c_F^4}{m_e^2}\sum_{m_1,m_2}\frac{|S^1_{1nm}|^2|S^1_{2nm}|^2}{\Delta E_1+\Delta E_2}$

[London, 1930]

- Mixed: $V_{\rm LEB}(\mathbf{R}) = \frac{e^4}{8\pi^2|\mathbf{R}|^4}\frac{c_F^2}{m_e^2}\sum_{m_1,m_2}\frac{|r^1_{1nm}|^2|S^1_{2nm}|^2+|S^1_{1nm}|^2|r^1_{2nm}|^2}{\Delta E_1+\Delta E_2}\Delta E_1\Delta E_2$

[Feinberg, 1989]

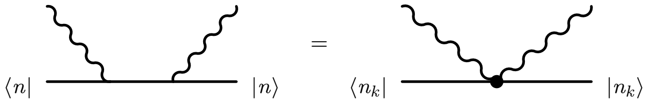
with $r^1_{inm} = \langle n_i|r^1|m_i\rangle$, $S^1_{inm} = \langle n_i|S^1|m_i\rangle$, $\Delta E_i = E_{n_i} - E_{m_i}$

$$L_{\text{pNRQED}} = -\frac{1}{4}\int d^3\mathbf{x}\, F^{\mu\nu}(t,\mathbf{x})F_{\mu\nu}(t,\mathbf{x})$$
$$+\int d^3\mathbf{R}\, d^3\mathbf{r}\, S^\dagger(t,\mathbf{r},\mathbf{R})\left[i\partial_0 + \frac{\nabla_{\mathbf{r}}^2}{2m_e} + \frac{\alpha}{|\mathbf{r}|} + e\mathbf{r}\cdot\mathbf{E}(t,\mathbf{R}) + c_F e\frac{\mathbf{S}\cdot\mathbf{B}(t,\mathbf{R})}{m}\right] S(t,\mathbf{r},\mathbf{R})$$

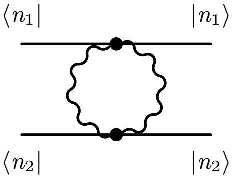
$$L_{\text{AEFT}} = -\frac{1}{4}\int d^3\mathbf{x}\, F^{\mu\nu}(t,\mathbf{x})F_{\mu\nu}(t,\mathbf{x})$$
$$+\int d^3\mathbf{R}\, d^3\mathbf{r}\sum_{n_k} S^\dagger(t,\mathbf{r},\mathbf{R},n_k)\left[i\partial_0 + E_{n_k} + c^{ij}_{(n_k)}\mathbf{E}_i\mathbf{E}_j + d^{ij}_{(n_k)}\mathbf{B}_i\mathbf{B}_j\right] S(t,\mathbf{r},\mathbf{R},n_k)$$
$$+\int d^3\mathbf{R}_1\, d^3\mathbf{R}_2\, d^3\mathbf{r}_1\, d^3\mathbf{r}_2\, S^\dagger S(t,\mathbf{r}_1,\mathbf{R}_1,n_i)\, V^{ij}(\mathbf{R}_1-\mathbf{R}_2) S^\dagger S(t,\mathbf{r}_2,\mathbf{R}_2,n_j)$$

Matching pNRQED to AEFT:

- Tree-level: Match 2-photon emission in pNRQED to the seagull vertices in AEFT.
- 1-loop: No contributions relevant at the order we are interested in.



Van der Waals interaction in AEFT: Scattering of two S fields, where the initial and final states of each field do not change



Potentials in the isotropy approximation

- Electric : $V_{\rm LE}(\mathbf{R}) = -\dfrac{23\alpha_{n_1}\alpha_{n_2}}{4\pi^2\mathbf{R}^7}$
- Magnetic : $V_{\rm LB}(\mathbf{R}) = -\dfrac{23\beta_{n_1}\beta_{n_2}}{4\pi^2\mathbf{R}^7}$
- Mixed: $V_{\rm LEB}(\mathbf{R}) = \dfrac{7(\alpha_{n_1}\beta_{n_2}+\beta_{n_1}\alpha_{n_2})}{4\pi^2\mathbf{R}^7}$

[Casimir & Polder, 1948], [Holstein, 2008]

with $\alpha_{n_k} = \dfrac{e^2}{2\pi}\sum_m \dfrac{\langle n_k|r^1|m\rangle\langle m|r^1|n_k\rangle}{E_{n_k}-E_m}$, $\beta_{n_k} = \dfrac{e^2}{2\pi}\sum_m \dfrac{\langle n_k|S^1|m\rangle\langle m|S^1|n_k\rangle}{E_{n_k}-E_m}$

- In the QCD case, the general idea remains the same, however the non-abelian nature of the theory must be taken into account (pNRQED $\rightarrow$ pNRQCD, atoms $\rightarrow$ quarkonia)
- The presence of Λ_{QCD} leads to a higher number of possible scale hierarchies

Simplest case

If Λ_{QCD} is the smallest scale, everything remains perturbative and the computations are quite similar to the pNRQED case

- pNRQCD': $|\mathbf{k}| \gg m_Q \alpha_s^2 \gg \Lambda_{\mathrm{QCD}}$
- pNRQCD: $|\mathbf{k}| \sim m_Q \alpha_s^2 \gg \Lambda_{\mathrm{QCD}}$
- EFT for quarkonium interactions: $m_Q \alpha_s^2 \gg |\mathbf{k}| \gg \Lambda_{\mathrm{QCD}}$

Non-perturbative, but same $1/R^6$ potential

- pNRQCD: $|\mathbf{k}| \gg \Lambda_{\mathrm{QCD}} \gg m_Q \alpha_s^2$, $|\mathbf{k}| \gg \Lambda_{\mathrm{QCD}} \sim m_Q \alpha_s^2$

Although pNRQCD is non-perturbative, the $1/R^6$ part of the potential which arises from the 2-potential exchange diagrams only can still be computed.

Completely non-perturbative

For $\Lambda_{\mathrm{QCD}} \gg |\mathbf{k}|$, $\Lambda_{\mathrm{QCD}} \sim |\mathbf{k}|$ one could work with chiral low energy theories.

- The simplest hierarchy corresponds to the bound state of a very heavy quarkonium with the Coulomb-type potential.

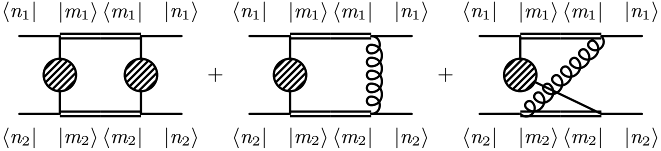
pNRQCD

In pNRQCD $Q\bar{Q}$ pairs in a particular color configuration are described by color singlet ($S \equiv \frac{1}{\sqrt{N_c}}\tilde{S}$) and color octet fields ($O \equiv \frac{T^a}{\sqrt{T_F}}\tilde{O}^a$).

$$
\begin{aligned}
L_{\mathrm{pNRQCD}'} &= -\tfrac{1}{4}\int d^3\mathbf{R}\, F^{\mu\nu}(t,\mathbf{R})F_{\mu\nu}(t,\mathbf{R}) \\
&+ \int d^3\mathbf{R}\, d^3\mathbf{r}\ \mathrm{Tr}\left\{ S^\dagger(t,\mathbf{r},\mathbf{R})\,(i\partial_0 - h_s(\mathbf{r}))\,S(t,\mathbf{r},\mathbf{R}) + O^\dagger(t,\mathbf{r},\mathbf{R})\,(i\partial_0 - h_o(\mathbf{r}))\,O(t,\mathbf{r},\mathbf{R})\right\} \\
&+ \int d^3\mathbf{R}\, d^3\mathbf{r}\, gV_A(\mathbf{r})\,\mathrm{Tr}\left\{ O^\dagger(t,\mathbf{r},\mathbf{R})\mathbf{r}\cdot\mathbf{E}(t,\mathbf{R})S(t,\mathbf{r},\mathbf{R}) + S^\dagger(t,\mathbf{r},\mathbf{R})\mathbf{r}\cdot\mathbf{E}(t,\mathbf{R})O(t,\mathbf{r},\mathbf{R})\right\} \\
&+ \int d^3\mathbf{R}\, d^3\mathbf{r}\, \tfrac{c_F g}{2m}\, V_1(\mathbf{r})\,\mathrm{Tr}\left\{ O^\dagger(t,\mathbf{r},\mathbf{R})\boldsymbol{\sigma}\cdot\mathbf{B}(t,\mathbf{R})S(t,\mathbf{r},\mathbf{R}) + S^\dagger(t,\mathbf{r},\mathbf{R})\boldsymbol{\sigma}\cdot\mathbf{B}(t,\mathbf{R})O(t,\mathbf{r},\mathbf{R})\right\} \\
&+ \int d^3\mathbf{R}\, d^3\mathbf{r}\, \tfrac{c_F g}{2m}\, V_1(\mathbf{r})\,\mathrm{Tr}\left\{ \boldsymbol{\sigma}\cdot\mathbf{B}(t,\mathbf{R})O^\dagger(t,\mathbf{r},\mathbf{R})S(t,\mathbf{r},\mathbf{R}) + (t,\mathbf{r},\mathbf{R})\boldsymbol{\sigma}\cdot\mathbf{B}(t,\mathbf{R})S^\dagger\, O(t,\mathbf{r},\mathbf{R})\right\}
\end{aligned}
$$

$$
L_{\mathrm{pNRQCD}} = L_{\mathrm{pNRQCD}'} + \int d^3\mathbf{R}_1\, d^3\mathbf{R}_2\, d^3\mathbf{r}_1\, d^3\mathbf{r}_2\, S^\dagger S(t,\mathbf{r}_1,\mathbf{R}_1)\, V(\mathbf{R}_1-\mathbf{R}_2) S^\dagger S(t,\mathbf{r}_2,\mathbf{R}_2)
$$

Van der Waals interaction in between two heavy quarkonia in pNRQCD: Scattering of two S fields, where the initial and final states of each field do not change



Potentials in the isotropy approximation

- Chromoelectric : $V_{\rm LCE}(\mathbf{R}) = -\dfrac{3g^4}{8\pi^2|\mathbf{R}|^6}\, V_A^4 \left(\dfrac{T_F}{N_c}\right)^2 (N_c^2-1) \sum_{m_1,m_2} \dfrac{|r^1_{1nm}|^2|r^1_{2nm}|^2}{\Delta E_1 + \Delta E_2}$
- Chromomagnetic :
$$V_{\rm LCB}(\mathbf{R}) = -\frac{3g^4}{8\pi^2|\mathbf{R}|^6}\, V_1^4 \left(\frac{T_F}{N_c}\right)^2 (N_c^2-1)\frac{c_F^4}{m_Q^2} \sum_{m_1,m_2} \frac{|S^1_{1nm}|^2|S^1_{2nm}|^2}{\Delta E_1 + \Delta E_2}$$
- Mixed:
$$V_{\rm LCEB}(\mathbf{R}) = \frac{g^4}{8\pi^2|\mathbf{R}|^4}\frac{c_F^2}{m_e^2}\, V_A^2\, V_1^2 \left(\frac{T_F}{N_c}\right)^2 (N_c^2-1) \times \sum_{m_1,m_2} \frac{|r^1_{1nm}|^2|S^1_{2nm}|^2 + |S^1_{1nm}|^2|r^1_{2nm}|^2}{\Delta E_1 + \Delta E_2}\Delta E_1 \Delta E_2$$

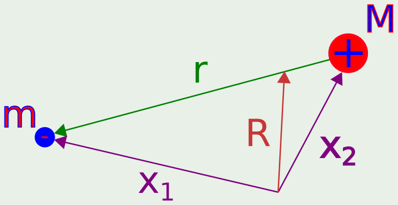
Summary

- We studied van der Waals forces between neutral atoms in pNRQED, a well established effective field theory of electromagnetic interaction, that can be rigorously derived from QED.
- We computed the electric, magnetic and mixed potentials with anisotropic polarizations.
- In the isotropy approximation we recover the well-known London and Casimir-Polder results, where the potentials correspond to the short and long distance van der Waals foces with the characteristic $1/R^6$ and $1/R^7$ behavior.
- Since the connection to the full theory (QED) is clear, higher order corrections can be studied systematically.
- Furthermore, we investigated the simplest (fully perturbative) hierarchy in QCD and obtained results very similar to the QED case.

Outlook:

- Consider other QCD hierarchies.
- Obtain predictions for the possible future experiments.

Center of mass coordinates



$$\mathbf{R} = \frac{m}{m+M}\mathbf{x}_1 + \frac{M}{m+M}\mathbf{x}_2 \approx \mathbf{x}_2$$

$$\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$$

$$\mathbf{x}_1 = \mathbf{R} + \frac{M}{m+M}\mathbf{r} \approx \mathbf{R} + \mathbf{r}$$

$$\mathbf{x}_2 = \mathbf{R} - \frac{m}{m+M}\mathbf{r} \approx \mathbf{R}$$

Multipole expansion of gauge fields at $\mathcal{O}(r^2)$

Gauge fields, covariant derivatives and gauge links

$$A^\mu(t,\mathbf{x}_1) = A^\mu(t,\mathbf{R}) + \frac{M}{m+M} r^i \nabla^i_{\mathbf{R}} A^\mu(t,\mathbf{R})$$

$$A^\mu(t,\mathbf{x}_2) = A^\mu(t,\mathbf{R}) - \frac{m}{m+M} r^i \nabla^i_{\mathbf{R}} A^\mu(t,\mathbf{R})$$

$$\mathbf{D}_{\mathbf{x}_1} = \boldsymbol{\nabla}_{\mathbf{r}} + \frac{m}{m+M} \boldsymbol{\nabla}_{\mathbf{R}} - ig\mathbf{A}(t,\mathbf{R}) - i\frac{M}{m+M} r^i (\partial_{R,i} g\mathbf{A}(t,\mathbf{R}))$$

$$\mathbf{D}_{\mathbf{x}_2} = -\boldsymbol{\nabla}_{\mathbf{r}} + \frac{M}{m+M} \boldsymbol{\nabla}_{\mathbf{R}} + ig\mathbf{A}(t,\mathbf{R}) - i\frac{m}{m+M} r^i (\partial_{R,i} g\mathbf{A}(t,\mathbf{R}))$$

pNRQED potentials

$$V_{LEE}(\mathbf{R}) = -\frac{e^4}{16\pi^2\mathbf{R}^{10}} \sum_{m_1,m_2} \times \frac{\mathbf{R}^4 \sum_i |r^i_{1nm}|^2 r^i_{2nm}|^2 - 6\mathbf{R}^2 \sum_i |r^i_{1nm}|^2 r^i_{2nm}|^2 |R^i|^2 + 9 \sum_{i,j} |r^i_{1nm}|^2 |r^j_{2nm}|^2 |R^i|^2 |R^j|^2}{\Delta E_1 + \Delta E_2}.$$

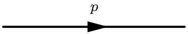
$$V^0_{\mathrm{SBB}}(\mathbf{R},\mathbf{S}_1,\mathbf{S}_2) = -\frac{c_F^2 e^2}{m^2}\left[\frac{4}{3}\mathbf{S}_{1nm}\cdot\mathbf{S}_{2nm}\delta^3(\mathbf{R}) + \frac{1}{4\pi\mathbf{R}^3}\left(\mathbf{S}_{1nm}\cdot\mathbf{S}_{2nm} - \frac{3(\mathbf{S}_{1nm}\cdot\mathbf{R})(\mathbf{S}_{2nm}\cdot\mathbf{R})}{\mathbf{R}^2}\right)\right]$$

$$V_{LBB}(\mathbf{R}) = -\frac{c_F^4 e^4}{16m^4\pi^2\mathbf{R}^{10}} \sum_{m_1,m_2} \frac{|\mathbf{R}^2(\mathbf{S}_{1nm}\cdot\mathbf{S}_{2nm}) - 3(\mathbf{R}\cdot\mathbf{S}_{1nm})(\mathbf{R}\cdot\mathbf{S}_{2nm})|^2}{\Delta E_1 + \Delta E_2}.$$

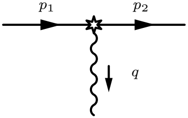
$$\begin{aligned} V_{SBB}(\mathbf{R}) &= \frac{c_F^4 e^4}{64\pi^2 m^4}(|\mathbf{S}_{1nm}\cdot\mathbf{S}_{2nm}|^2 - |\mathbf{S}^*_{1nm}\cdot\mathbf{S}_{2nm}|^2)\left(\frac{3}{\pi\mathbf{R}^5} + \left(\lambda + \frac{8}{3}\right)\left(\nabla^2\delta^{(3)}(\mathbf{R}) + \frac{81}{\pi\mathbf{R}^5}\right)\right) \\ &- \frac{c_F^4 e^4}{96\pi^2 m^4} 2\,\mathrm{Re}\,(\mathbf{S}^*_{1nm}\cdot\mathbf{k})(\mathbf{S}_{2nm}\cdot\mathbf{k})(\mathbf{S}_{1nm}\cdot\mathbf{S}^*_{2nm}) - (\mathbf{S}^*_{1nm}\cdot\mathbf{S}^*_{2nm})(\mathbf{S}_{1nm}\cdot\mathbf{k})(\mathbf{S}_{2nm}\cdot\mathbf{k})) \\ &\times \left(\frac{3}{\pi\mathbf{R}^5} + \left(\lambda + \frac{8}{3}\right)\left(\nabla^2\delta^{(3)}(\mathbf{R}) + \frac{81}{\pi\mathbf{R}^5}\right)\right) \end{aligned}$$

$$\begin{aligned} V_{LEB}(\mathbf{R}) &= \frac{c_F^2 e^4}{64m^2} \sum_{m_1,m_2} \frac{\Delta E_1 \Delta E_2}{\Delta E_1 + \Delta E_2} \\ &\times \left[\frac{1}{\pi^2\mathbf{R}^4}\left(|\mathbf{r}_{1nm}|^2|\mathbf{S}_{2nm}|^2 + |\mathbf{r}_{2nm}|^2|\mathbf{S}_{1nm}|^2\right) - \frac{1}{\pi^2\mathbf{R}^4}\sum_a\left(|r^a_{1nm}|^2|S^a_{2nm}|^2 + |r^a_{2nm}|^2|S^a_{1nm}|^2\right)\right. \\ &\left. - \sum_a\left(|r^a_{1nm}|^2 S^b_{2nm}S^{*d}_{2nm} + |r^a_{2nm}|^2 S^b_{1nm}S^{*d}_{1nm}\right)\left(\frac{\delta^{ij}}{\pi^2\mathbf{R}^4} - 4\frac{R^iR^j}{\pi^2\mathbf{R}^6}\right)\varepsilon^{iab}\varepsilon^{jad}\right]. \end{aligned}$$

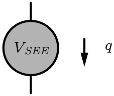
pNRQED Feynman rules



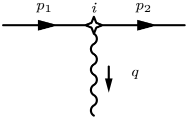
$$\frac{i}{p^0 - \hat{H} + i\varepsilon} = \sum_m \frac{i}{p^0 - E_m + i\varepsilon} \left|m\right\rangle \left\langle m\right|$$



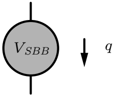
$$e r^i q^i$$



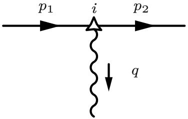
$$e^2 \frac{r_1^i q^i r_2^j q^j}{\mathbf{q}^2}.$$



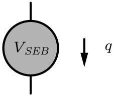
$$- e r^i q^0$$



$$- i \frac{c_F^2 e^2}{m^2} \frac{(\mathbf{q}^2 \delta^{ij} - q^i q^j)}{\mathbf{q}^2} S_1^i S_2^j$$

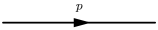


$$- \frac{c_F e}{m} \varepsilon^{ijk} S^j q^k$$

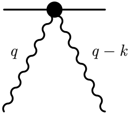


$$- i \frac{c_F e^2}{m} \frac{q^0}{\mathbf{q}^2} \left[\mathbf{q}(\mathbf{r}_1 \times \mathbf{S}_2) + \mathbf{q}(\mathbf{r}_2 \times \mathbf{S}_1)\right]$$

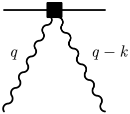
AEFT Feynman rules



$$\frac{i}{p^0 - E_{n_k} + i\varepsilon}$$



$$= 2ic^{ij}_{n_k}(-p^i_1 p^j_2 - p^0_1 p^0_2 + p^i_1 p^0_2 + p^0_1 p^j_2)$$



$$= -2id^{ij}_{n_k}\varepsilon^{ikl}\varepsilon^{jmn}p^k p^m$$

$$c^{ij}_{n_k} = -e^2 \sum_{m_k} \frac{r^i_{knm} r^{*j}_{knm}}{E_{n_k} - E_{m_k}}$$

$$d^{ij}_{n_k} = -\frac{C_F^2 e^2}{m^2} \sum_{m_k} \frac{S^i_{knm} S^{*j}_{knm}}{E_{n_k} - E_{m_k}}$$

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