All Order Linearized Hydrodynamics from Fluid/Gravity Correspondence

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based on Yanyan Bu and M.L., arXiv:1406.7222, arXiv:1408....

following some older works with Edward Shuryak

E. Shuryak and M.L., Phys.Rev.C76 (2007) 021901, D80 (2009) 065026, C84 (2011) 061901:
Generalize NS hydro by introducing ALL order dissipative terms in the gradient expansion of fluid stress tensor

$$(\nabla\nabla\mathbf{u}) \quad \text{we keep} \qquad\qquad (\nabla\mathbf{u})^2 \quad \text{we neglect}$$

Extract momenta-dependent viscosity function $\eta(\omega, \mathbf{q})$ by matching two-point correlation functions of the stress tensor with correlation functions computed from BH AdS/CFT (fluid/gravity correspondence).
We have set the problem, but at the time failed to solve it completely.
(We have done it now!)
We have done phenomenological studies of the effects of all-order gradients on entropy/multiplicity production in HI collisions

Motivation: Experiments (RHIC,LHC) probe systems with finite gradients.
Phenomenologically observed low viscosity is an "effective" viscosity measured at momentum typical for process in study.
High order gradients are very big in early stages of HI collisions
Small perturbations/correlations on top of global explosion are sensitive to high gradients. This is where our results are most applicable
Relativistic Navier-Stokes hydrodynamics is non-causal/non-stable.
Causality is supposed to be restored after summation of all orders

Relativistic Hydrodynamics

Energy momentum tensor

$$\langle \mathbf{T}^{\mu\nu} \rangle = (\epsilon + \mathbf{P})\, \mathbf{u}^{\mu}\, \mathbf{u}^{\nu} + \mathbf{P}\, \mathbf{g}^{\mu\nu} + \mathbf{\Pi}^{\mu\nu}$$

$$\mathbf{u}_{\mathrm{v}} = -1/\sqrt{1-\beta^2}, \qquad \mathbf{u}_{\mathrm{i}} = \beta_{\mathrm{i}}/\sqrt{1-\beta^2}$$

$\mathbf{\Pi}^{\mu\nu}$ – tensor of dissipations (ideal fluid: $\mathbf{\Pi}^{\mu\nu} = 0$)

Landau frame choice: $\mathbf{u}_{\mu}\, \mathbf{\Pi}^{\mu\nu} = 0.$

Navier Stokes hydro (expanding up to first order in the velocity gradient)

$$\mathbf{\Pi}_{\mathrm{ij}} = -\,\eta_0\, \sigma_{\mathrm{ij}}\,, \qquad \sigma_{\mathrm{ij}} = \frac{1}{2}\left(\partial_{\mathrm{i}}\beta_{\mathrm{j}} + \partial_{\mathrm{j}}\beta_{\mathrm{i}} - \frac{2}{3}\delta_{\mathrm{ij}}\partial\beta\right), \qquad \mathbf{\Pi}_{\mathrm{vv}} = \mathbf{\Pi}_{\mathrm{vj}} = 0.$$

$$\nabla_{\mu}\, \langle \mathbf{T}^{\mu\nu} \rangle = 0 \qquad \longrightarrow \qquad \text{Navier – Stokes Eqns.}$$

Linearized Hydro to all orders

Shuryak and M. L.: Introduce all order gradient expansion of $\langle \mathbf{T}^{\mu\nu} \rangle$:

$$\Pi_{ij} = -\left[\eta(\omega, \mathbf{q}^2)\sigma_{ij} + \zeta(\omega, \mathbf{q}^2)\pi_{ij}\right],$$

where π_{ij} is a third order tensor structure

$$\pi_{ij} = \partial_i \partial_j \partial \beta - \frac{1}{3}\delta_{ij}\partial^2 \partial \beta$$

$$\eta = \eta[\nabla^2, (\mathbf{u}\nabla)]; \qquad \zeta = \zeta[\nabla^2, (\mathbf{u}\nabla)];$$

$$\nabla^2 \to \omega^2 - \mathbf{q}^2 \text{ and } (\mathbf{u}\,\nabla) \to -\mathbf{i}\,\omega.$$

We keep the nonlinear dispersion to all orders, but

We neglect nonlinear interactions (though some terms could be recovered).

Results: Viscosities from the Fluid/Gravity correspondence

Analytical results in the hydrodynamic regime $\omega, \mathbf{q} \ll \mathbf{1}$ $(\pi\,\mathbf{T} = \mathbf{1})$:

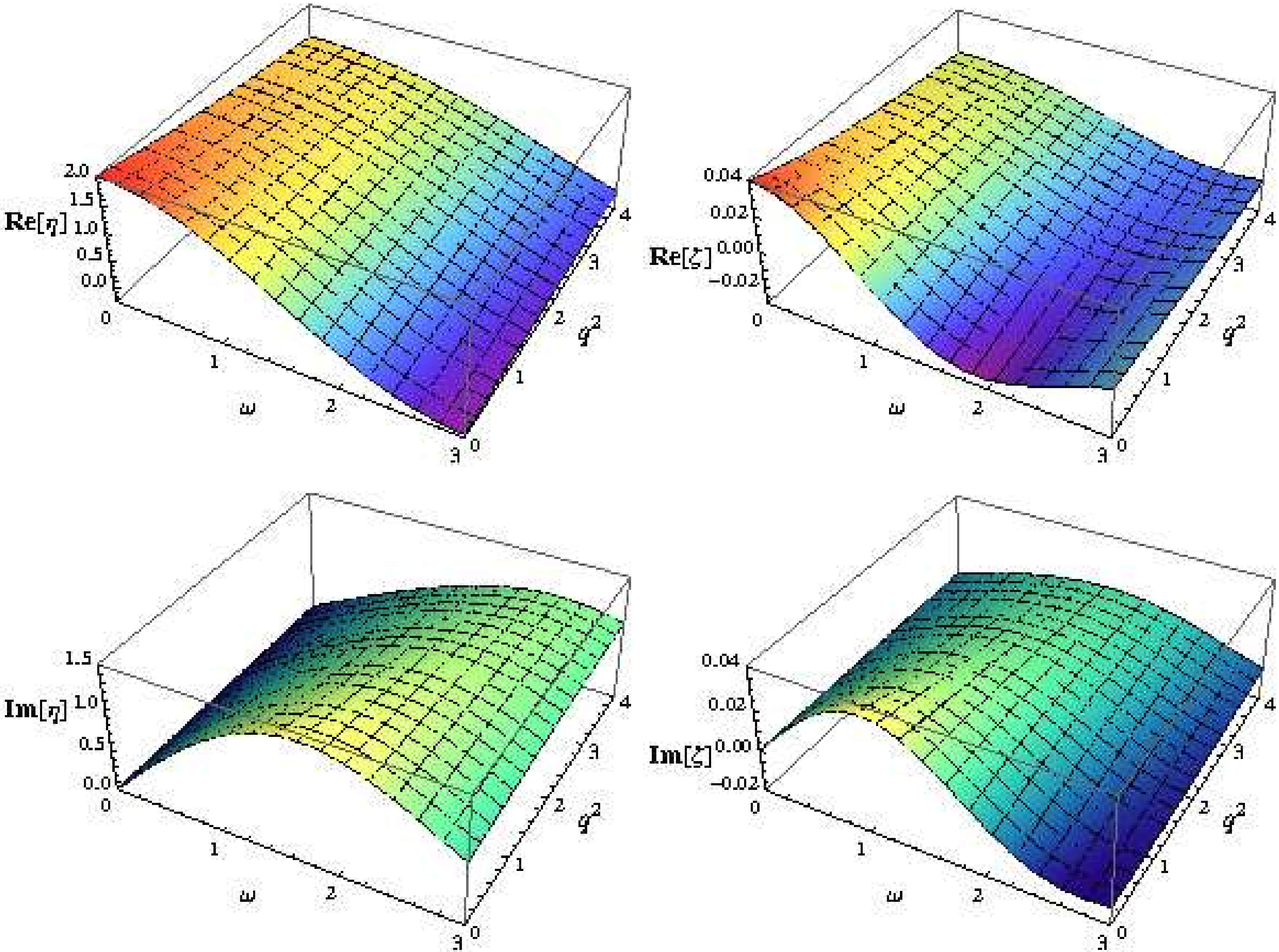
$$\eta(\omega, \mathbf{q}^2) = \mathbf{2} + (\mathbf{2} - \ln \mathbf{2})\mathbf{i}\omega - \frac{\mathbf{1}}{\mathbf{4}}\mathbf{q}^2 - \frac{\mathbf{1}}{\mathbf{24}}\left[\mathbf{6}\pi - \pi^2 + \mathbf{12}\left(\mathbf{2} - \mathbf{3}\ln\mathbf{2} + \ln^2\mathbf{2}\right)\right]\omega^2 + \cdots$$

$$\zeta(\omega, \mathbf{q}^2) = \frac{\mathbf{1}}{\mathbf{12}}(\mathbf{5} - \pi - \mathbf{2}\ln\mathbf{2}) + \cdots$$

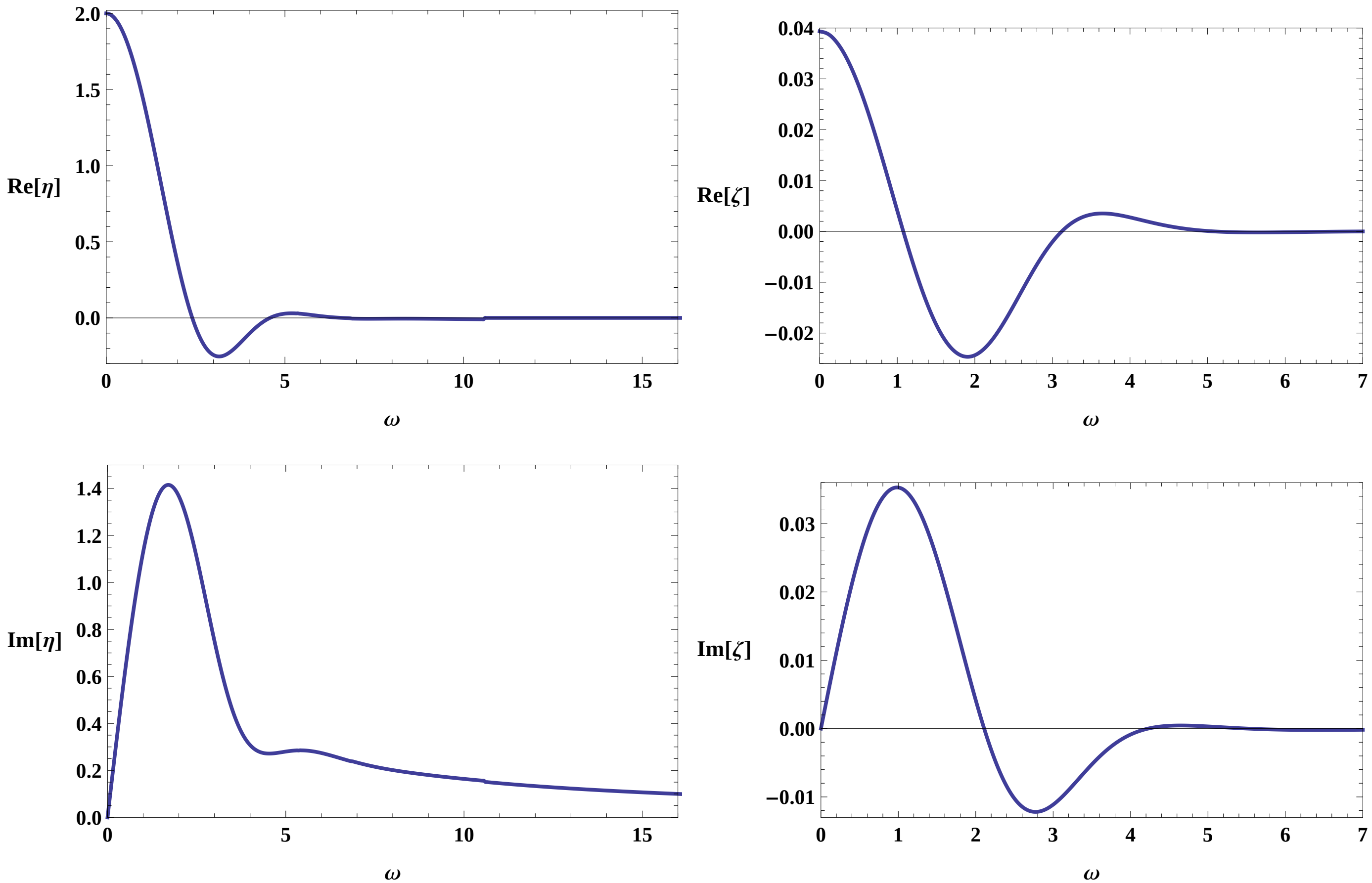
Blue terms are new!

Modified sound dispersion:

$$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6}q^2 \pm \frac{1}{24\sqrt{3}}(2\ln 2 - 1)\,q^3 + \frac{i}{288}\left(8 - \frac{\pi^2}{3} + 4\ln^2 2 - 4\ln 2\right)q^4 +$$



Re[η]
2.0
1.5
1.0
0.5
0.0
0
1
2
3
ω
0
1
2
3
4
q²
Re[ζ]
0.04
0.02
0.00
−0.02
Im[η]
1.5
1.0
0.5
0.0
Im[ζ]
0.04
0.02
0.00
−0.02

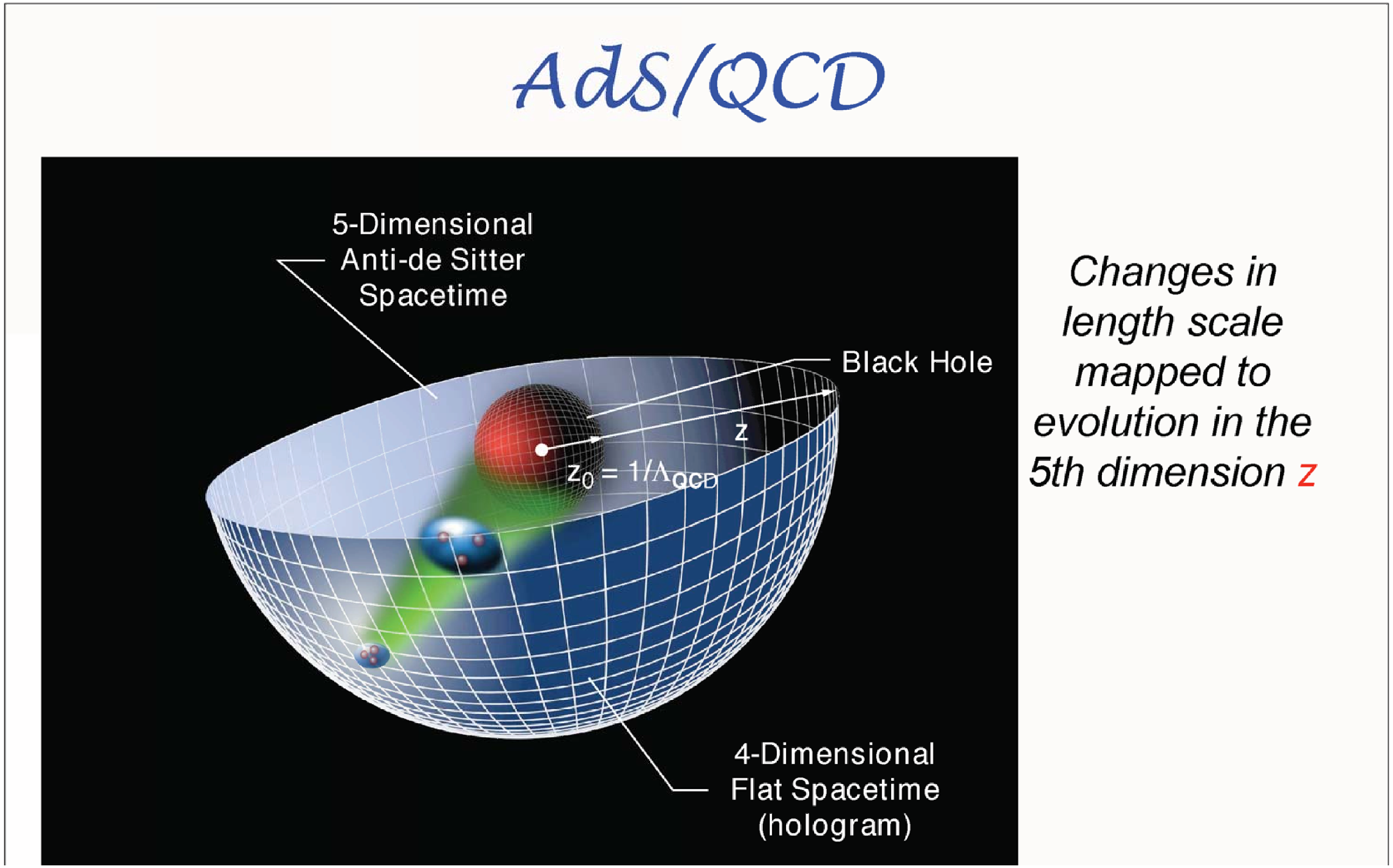


$q^2 = 0$

- **Real parts of the viscosities are decreasing functions of momenta. Oscillations are consistent with the expectations about the viscosities have infinitely many complex poles.**
- **Imaginary parts have a clear maximum near $\omega \sim 2$, introducing a (new?) transition scale.**
- **Viscosity vanish at large momenta, which is probably what is required to restore causality.**
- **ζ is always subleading vs η.**

AdS/CFT

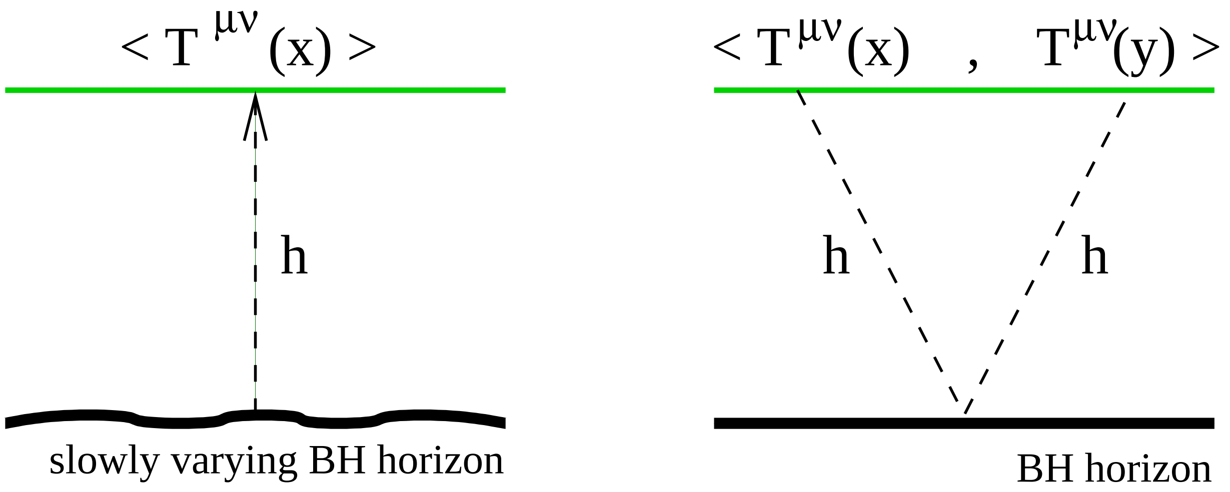
AdS/CFT correspondence: weakly coupled super-gravity in $AdS_5 \times S_5$ is "dual" to strongly coupled $\mathcal{N} = 4$ SYM gauge theory in 4d



All dissipative effects take place at the horizon. There is no dissipation in the bulk. Gravitons propagate signals from the horizon to the boundary, where the hologram is captured.
The bulk acts as a highly nonlinear dispersive medium.

First approach: Perturb metric near horizon (membrane paradigm) and read off the response of the system at the boundary.

Alternative approach: send a signal from one point at the boundary to another (two point correlations of stress energy tensor).



"Gravity", starring S. Bhattacharyya, V. E Hubeny, S. Minwalla, M. Rangamani

5d GR with negative cosmological constant:

$$\mathbf{S} = \frac{1}{16\pi G_N} \int d^5x \sqrt{-g}\,(R + 12)\,,$$

Einstein Equations

$$E_{MN} \equiv R_{MN} - \frac{1}{2} g_{MN} R - 6\, g_{MN} = 0.$$

Solution: Boosted Black Brane in asymptotic AdS$_5$

$$ds^2 = -2u_\mu dx^\mu dr - r^2 f\,(br)\, u_\mu u_\nu dx^\mu dx^\nu + r^2 \mathcal{P}_{\mu\nu} dx^\mu dx^\nu,$$

$$f(r) = 1 - 1/r^4 \quad \text{and} \quad \mathcal{P}_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$$

Hawking temperature

$$T = \frac{1}{\pi b},$$

S. Bhattacharyya, V. E Hubeny, S. Minwalla, M. Rangamani,
JHEP 0802:045,2008:

Promote β_i and b into a slowly varying functions of boundary coordinates x^α

$$\mathbf{ds^2 = -2u_\mu(x^\alpha)dx^\mu dr - r^2 f\,(b(x^\alpha)r)\,u_\mu(x^\alpha)u_\nu(x^\alpha)dx^\mu dx^\nu + r^2\mathcal{P}_{\mu\nu}(x^\alpha)dx^\mu dx^\nu},$$

Use gradient expansion of the fields $\mathbf{u(x) = u_0 + \delta x\,\nabla u}$ and $\mathbf{b(x) = b + \delta x \nabla b}$ to set up a perturbative procedure

The resulting energy momentum tensor

$$\langle \mathbf{T}^{\mu\nu} \rangle = \mathbf{T}^{\mu\nu}_{\mathbf{ideal}} + \mathbf{\Pi}^{\mu\nu}_{\mathbf{NS}} + \tau_{\mathbf{R}}\,(\mathbf{u}\,\nabla)\,\mathbf{\Pi}^{\mu\nu}_{\mathbf{NS}} + \mathbf{O}[\,(\nabla\,\mathbf{u})^2\,]$$

$$\frac{\eta_0}{\mathbf{s}} = \frac{\mathbf{1}}{\mathbf{4}\,\pi}, \qquad\qquad \tau_{\mathbf{R}} = \mathbf{2} - \log(\mathbf{2})$$

We do it somewhat differently, linearizing in the velocity amplitude

$$\mathbf{u}_\mu(\mathbf{x}^\alpha) = (-1, \epsilon\beta_i(\mathbf{x}^\alpha)) + \mathcal{O}(\epsilon^2), \quad \mathbf{b}(\mathbf{x}^\alpha) = \mathbf{b}_0 + \epsilon \mathbf{b}_1(\mathbf{x}^\alpha) + \mathcal{O}(\epsilon^2),$$

"seed" metric, i.e., a linearized version of the BH metric

$$ds^2_{seed} = 2drdv - r^2 f(r) dv^2 + r^2 d\vec{x}^2 - \epsilon\left[2\beta_i(x^\alpha) drdx^i + \frac{2}{r^2}\beta_i(x^\alpha) dvdx^i + \frac{4}{r^2} b_1(x^\alpha) dv^2\right] + \mathcal{O}(\epsilon^2),$$

$$ds^2 = ds^2_{seed} + ds^2_{corr}[\beta] \qquad \text{gauge fix} \quad g_{rr} = 0, \quad g_{r\mu} \propto u_\mu$$

$$ds^2_{corr} = \epsilon\left(-3h\, drdv + \frac{k}{r^2} dv^2 + r^2 h\, d\vec{x}^2 + \frac{2}{r^2} j_i\, dvdx^i + r^2 \alpha_{ij}\, dx^i dx^j\right)$$

$h[\beta]$, $k[\beta]$, $j[\beta]$, $\alpha[\beta]$ are to be found by solving the Einstein equations.

Boundary cond: no singularities, no modification to AdS asymptotics at $r \to \infty$

$$h < \mathcal{O}(r^0), \quad k < \mathcal{O}(r^4), \quad j_i < \mathcal{O}(r^4), \quad \alpha_{ij} < \mathcal{O}(r^0).$$

Stress tensor from the Holographic Dictionary

We consider a hypersurface Σ at constant r.

Vector n_M normal to Σ: $$n_M = \frac{\nabla_M r}{\sqrt{g^{MN}\nabla_M r \nabla_N r}}.$$

Induced metric γ_{MN} on Σ: $$\gamma_{MN} = g_{MN} - n_M n_N$$

Extrinsic curvature tensor $\mathcal{K}_{MN}$:

$$\mathcal{K}_{MN} = \frac{1}{2}\left(n^A \partial_A \gamma_{MN} + \gamma_{MA}\partial_N n^A + \gamma_{NA}\partial_M n^A\right).$$

The stress tensor for the dual fluid

$$\langle T^\mu_\nu \rangle = \lim_{r\to\infty} \tilde{T}^\mu_\nu(r)\,; \qquad \tilde{T}^\mu_\nu(r) \equiv r^4\left(\mathcal{K}^\mu_\nu - \mathcal{K}\gamma^\mu_\nu + 3\gamma^\mu_\nu - \frac{1}{2}G^\mu_\nu\right),$$

where G^μ_ν is associated with $\gamma_{\mu\nu}$. The last two terms are counter-terms which remove divergences near the boundary $r = \infty$.

$$\tilde{\mathbf{T}}^{0}_{0} = -3(1-4\epsilon b_1) + \frac{\epsilon}{2r}\Big\{-6rk + 4r^4\partial\beta - 4\partial j - r^3\partial_i\partial_j\alpha_{ij} + 18(r^5 - r)h$$

$$+6(r^6 - r^2)\partial_r h + 2r^3\partial^2 h + 6r^4\partial_v h\Big\},$$

$$\tilde{\mathbf{T}}^{0}_{i} = \frac{\epsilon}{2r^4}\Big\{2\left[4r^4\beta_i - 4(r^4-1)j_i + r^7\partial_v\beta_i - r^3\partial_i k + (r^5 - r)\partial_r j_i\right]$$

$$-r^2\left(-\partial^2 j_i + \partial_i\partial j + r^4\partial_v\partial_k\alpha_{ik} - 2r^4\partial_v\partial_i h - 3r^5\partial_i h\right)\Big\},$$

$$\tilde{\mathbf{T}}^{i}_{0} = -\frac{\epsilon}{2r^3}\Big\{2\left[4r^3\beta_i - 4r^3 j_i + r^6\partial_v\beta_i - r^2\partial_i k + (r^4 - 1)\partial_r j_i\right]$$

$$+r\left[\partial^2 j_i - \partial_i\partial j - r^4\partial_v\partial_k\alpha_{ik} - 2r^4\partial_v\partial_i h - 3(r^6 - r^2)\partial_i h\right]\Big\},$$

$$\tilde{\mathbf{T}}^{i}_{j} = \delta^i_j(1-4\epsilon b_1) + \frac{\epsilon}{2r^4}\delta^i_j\Big\{r^2\left[-\partial^2 k + (1-r^4)\partial_k\partial_l\alpha_{kl} + 2\partial_v\partial j\right]$$

$$-2\left[(1-r^4)k - 2r^7\partial\beta + 2r^3\partial j - r^3\partial_v k + (r^5 - r)\partial_r k\right] + r^6\partial^2 h$$

$$-2r^6\partial_v^2 h + 2\left[\left(3 - 12r^4 + 9r^5\right)h + (r^3 - r^7)\partial_v h + (2r - 4r^5 + 2r^9)\partial_r h\right]\Big\}$$

$$+\frac{\epsilon}{2r^2}\Big\{-2r\left[2r^4\partial_{(i}\beta_{j)} - 2\partial_{(i}j_{j)} + r^4\partial_v\alpha_{ij} + (r^6 - r^2)\partial_r\alpha_{ij}\right] - r^4\partial_i\partial_j h$$

$$+\left[\partial_i\partial_j k + (1-r^4)\partial^2\alpha_{ij} + 2(r^4-1)\partial_k\partial_{(i}\alpha_{j)k} - 2\partial_v\partial_{(i}j_{j)} + r^4\partial_v^2\alpha_{ij}\right]\Big\},$$

Approaching the boundary

$$\mathbf{j_i} \to -\mathbf{i}\omega\mathbf{r}^3\beta_\mathbf{i} - \frac{\mathbf{1}}{\mathbf{3}}\mathbf{r}^2\partial_\mathbf{i}\partial\beta + \mathcal{O}\left(\frac{\mathbf{1}}{\mathbf{r}}\right),$$

$$\alpha_\mathbf{ij} \to \left(\frac{\mathbf{2}}{\mathbf{r}} - \frac{\eta(\omega, \mathbf{q}^2)}{\mathbf{4r^4}}\right)\sigma_\mathbf{ij} - \frac{\zeta(\omega, \mathbf{q}^2)}{\mathbf{4r^4}}\pi_\mathbf{ij} + \mathcal{O}\left(\frac{\mathbf{1}}{\mathbf{r^5}}\right).$$

$$\mathbf{k} \to \frac{\mathbf{2}}{\mathbf{3}}\left(\mathbf{r}^3 + \mathbf{i}\omega\mathbf{r}^2\right)\partial\beta + \mathcal{O}\left(\frac{\mathbf{1}}{\mathbf{r^2}}\right), \quad \text{as} \quad \mathrm{r} \to \infty$$

$$\mathbf{h} = \mathbf{0}$$

The dissipative part of the stress tensor

$$\mathbf{\Pi_{ij}} = -\left[\eta(\omega, \mathbf{q}^2)\sigma_\mathbf{ij} + \zeta(\omega, \mathbf{q}^2)\pi_\mathbf{ij}\right]$$

Einstein equations for the metric corrections

Dynamical equations:

$$\mathbf{E_{rr} = 0 :} \qquad 5\,\partial_r h + r\partial_r^2 h = 0\,.$$

$$\mathbf{E_{rv} = 0 :} \qquad 3\,r^2\partial_r k = 6\,r^4\,\partial\beta + r^3\,\partial_v\partial\beta - 2\,\partial j - r\,\partial_r\partial j - r^3\,\partial_i\partial_j\alpha_{ij}$$

$$\mathbf{E_{ri} = 0 :} \qquad -\partial_r^2 j_i = \left(\partial^2\beta_i - \partial_i\partial\beta\right) + 3r\,\partial_v\beta_i - \tfrac{3}{r}\partial_r j_i + r^2\partial_r\partial_j\alpha_{ij}.$$

$$\mathbf{E_{ij} = 0 :}$$

$$\begin{aligned}
&(r^7 - r^3)\partial_r^2\alpha_{ij} + (5r^6 - r^2)\partial_r\alpha_{ij} + 2r^5\partial_v\partial_r\alpha_{ij} + 3r^4\partial_v\alpha_{ij} \\
&+r^3\left\{\partial^2\alpha_{ij} - \left(\partial_i\partial_k\alpha_{jk} + \partial_j\partial_k\alpha_{ik} - \frac{2}{3}\delta_{ij}\partial_k\partial_l\alpha_{kl}\right)\right\} \\
&+\left(\partial_i j_j + \partial_j j_i - \frac{2}{3}\delta_{ij}\partial j\right) - r\partial_r\left(\partial_i j_j + \partial_j j_i - \frac{2}{3}\delta_{ij}\partial j\right) \\
&+3r^4\left(\partial_i\beta_j + \partial_j\beta_i - \frac{2}{3}\delta_{ij}\partial\beta\right) + r^3\partial_v\left(\partial_i\beta_j + \partial_j\beta_i - \frac{2}{3}\delta_{ij}\partial\beta\right) = 0\,.
\end{aligned}$$

Holographic RG flow-type equations

j_i and α_{ij} are linear functionals of β_i. They can be uniquely decomposed as

$$j_i = a(\omega, q, r)\, \beta_i + b(\omega, q, r)\, \partial_i \partial \beta$$

$$\alpha_{ij} = 2\, c(\omega, q, r)\, \sigma_{ij} + d(\omega, q, r)\, \pi_{ij},$$

The Einstein equations reduce to ordinary diff equations

$$\begin{aligned}
r\partial_r^2 a - 3\partial_r a - q^2 r^3 \partial_r c - 3i\omega r^2 - q^2 r &= 0 \\
r\partial_r^2 b - 3\partial_r b + \frac{1}{3} r^3 \partial_r c - \frac{2}{3} r^3 q^2 \partial_r d - r &= 0 \\
(r^7 - r^3)\partial_r^2 c + (5r^6 - r^2)\partial_r c - 2i\omega r^5 \partial_r c - r\partial_r a + a - 3i\omega r^4 c + 3r^4 - i\omega r^3 &= 0 \\
(r^7 - r^3)\partial_r^2 d + (5r^6 - r^2)\partial_r d - 2i\omega r^5 \partial_r d + \frac{q^2}{3} r^3 d - 3i\omega r^4 d + 2b - 2r\partial_r b - \frac{2}{3} r^3 c &= 0\,.
\end{aligned}$$

Navier-Stokes equations

Using dynamical Einstein equations, we have constructed an "off-shell" $\mathbf{T}^{\mu\nu}$

Constraint equations

$$\mathbf{E_{vv}} = \mathbf{0} \text{ and } \mathbf{E_{vi}} = \mathbf{0}$$

are equivalent to the stress tensor conservation law

$$\partial_\mu \mathbf{T}^{\mu\nu} = \mathbf{0}$$

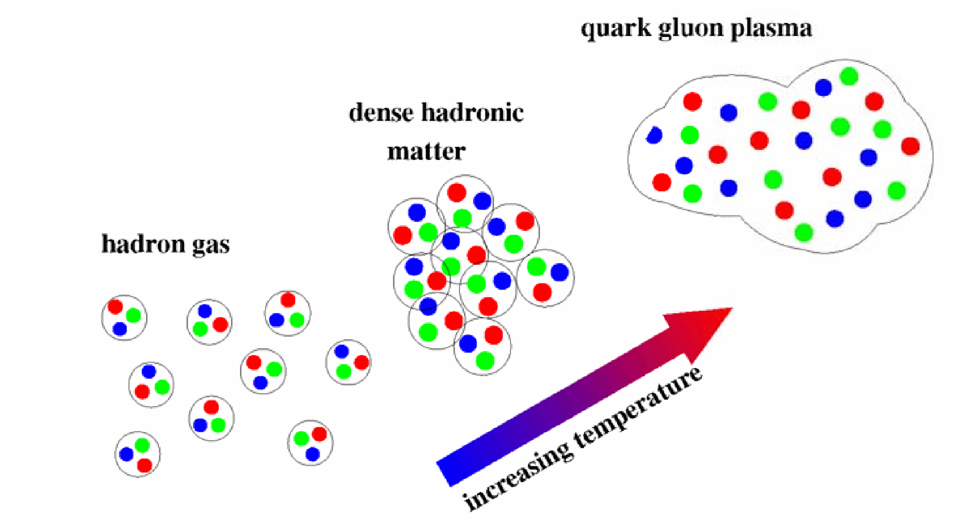
which determines the temperature and velocity profiles as functions of time, provided initial configuration is specified.

Conclusions

- **We have consistently determined the energy-momentum stress tensor of a weakly perturbed conformal fluid, whose underlying microscopic description is a strongly coupled $\mathcal{N} = 4$ super-Yang-Mills theory at finite temperature. We have found that all order dissipative terms in the fluid stress tensor are fully accounted for by two (generalized) momenta-dependent viscosity functions $\eta(\omega, q^2)$ and $\zeta(\omega, q^2)$**

- **As one of our main results, we have derived a closed form *linear* holographic RG flow-type equations for the viscosity functions. Constraint components of the bulk Einstein equationshave been shown to generalize the Navier-Stokes equations, consistently with the conservation laws of the fluid stress tensor.**

- **At large momenta, the effective viscosity is a decreasing function both of frequency and momentum. This behavior is supposed to restore causality of relativistic fluid dynamics and might be the reason behind the low viscosity observed at RHIC. It may also explain the exceptionally good survival of various hydrodynamic flows, particularly the sound waves.**



- **QGP is Deconfined**

- **QGP is strongly coupled (sQGP)**

behaves “almost” like a perfect liquid (Navier-Stokes with very small viscosity)

$$\eta \;\sim\; mean \;\; free \;\; path \;\sim 1/\sigma$$

QCD $\longrightarrow \mathcal{N} = 4$ SYM (CFT)

Strong coupling (and large N_c) $\rightarrow$ AdS/CFT $\rightarrow$ SUGRA on AdS$_5$

CFT at finite Temperature $\leftrightarrow$ AdS Black Hole

Israel-Stewart second order Hydrodynamics

Solves causality problems present in Navier-Stokes

Add extra term in the gradient expansion + non-linear terms in (∇u)

$$\mathbf{\Pi}^{\mu\nu} = (\mathbf{1} - \tau_{\mathbf{R}}\, \mathbf{u}_\lambda \nabla^\lambda)\ \mathbf{\Pi}^{\mu\nu}_{\mathbf{NS}}$$

Iterate the equation

$$(\mathbf{1} + \tau_{\mathbf{R}}\, \mathbf{u}_\lambda \nabla^\lambda)\ \mathbf{\Pi}^{\mu\nu} = \mathbf{\Pi}^{\mu\nu}_{\mathbf{NS}}$$

When thinking about small perturbations $\mathbf{u}_\lambda \nabla^\lambda \rightarrow \nabla_{\mathbf{v}} \rightarrow -\mathbf{i}\,\omega$

The IS second order hydro is equivalent (in the linear approximation) to

$$\eta_0 \rightarrow \eta(\omega) \equiv \frac{\eta_0}{\mathbf{1} - \mathbf{i}\,\tau_{\mathbf{R}}\,\omega}$$

Holographic Dictionary

$$\mathcal{Z}_{bulk}\left[\phi(\vec{x},z)|_{z=0}=\phi_0(\vec{x})\right]=\langle e^{\int d^4x\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{\text{Field Theory}}$$

Holographic renormalization:

$$\phi(\mathbf{x},\,\mathbf{z}\to\,\mathbf{0})\;=\;\phi_{\mathbf{0}}(\mathbf{x})\;+\;\mathbf{z}^{\boldsymbol{\Delta}}\,\langle\,\mathcal{O}\,\rangle_{\mathbf{FT}}\;+\cdots\;;\qquad\qquad \boldsymbol{\Delta}\;=\;-\,\mathbf{4}\;-\;\mathbf{2}\,\boldsymbol{\Delta}_{\mathcal{O}}$$

If $\mathcal{O}=T^{\mu\nu}$ **then** $\phi=g^{\mu\nu}$**:**

$$\mathbf{g}_{\mu\,\nu}(\mathbf{x},\mathbf{r}\equiv\mathbf{1}/\mathbf{z}\to\infty)\;\;=\;\;\eta_{\mu\nu}\;\;+\;\;\mathbf{0}\,\frac{\mathbf{1}}{\mathbf{r}^{\mathbf{2}}}\;\;+\;\;\frac{\mathbf{1}}{\mathbf{r}^{\mathbf{4}}}\;\langle\mathbf{T}_{\mu\nu}\rangle_{\mathbf{FT}}\;\;+\cdots$$

Two-point correlators:

$$\langle\;\mathcal{O}(\mathbf{x})\;\mathcal{O}(\mathbf{y})\;\rangle_{\mathbf{FT}}\;\;=\;\;\frac{\delta^{\mathbf{2}}\mathbf{Z}_{\mathbf{bulk}}}{\delta\phi_{\mathbf{0}}(\mathbf{x})\;\delta\phi_{\mathbf{0}}(\mathbf{y})}|_{\phi_{\mathbf{0}}=\mathbf{0}}$$