Effective Field Theories for thermal calculations in Cosmology

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9th of September, 2014

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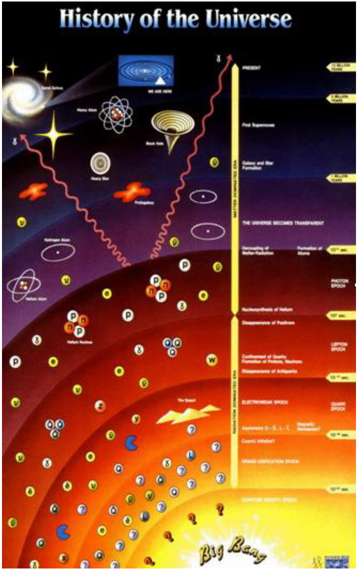
Outline

Introduction

Thermal/Non-equilibrium field theory in Cosmology



Thermal/Non-equilibrium field theory in Cosmology

Baryon asymmetry

- Needs that some particles are out of equilibrium.
- If realised throught leptogenesis needs a quite high temperature.

Dark matter

- In many models it is a particle that decouples from SM particles at some point of the thermal evolution.

Challenges in thermal field theory

Bosons. Example from QCD at finite temperature.

- Expansion parameter is g and not α_s. Resummation needed at scale gT.
- The scale $g^2 T$ is non-perturbative even if $g \to 0$.

We know how to deal with this for thermal equilibrium and static observables (combination of EFT and lattice) but for non-equilibrium it is an issue. Bad for the things we are interested in cosmology.

Another challenge comes from the doubling of degrees of freedom (lots of diagrams).

- This situation can be improved for processes involving non-relativistic particles using an EFT.

Non-relativistic particles in Cosmology

Many models that try to explain problem in cosmology have particles with very high masses.

- Heavy Majorana neutrinos

Non-relativistic particles in Cosmology

Many models that try to explain problem in cosmology have particles with very high masses.

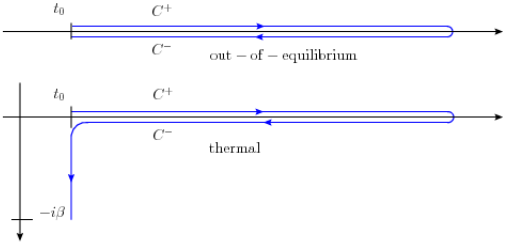
- Heavy Majorana neutrinos
- Supersymmetry
- Others...

HP in thermal field theory. Advantages

There are small parameter that lead to simplifications, double expansion.

- $\frac{T}{M}$
- $\rho_{HP} \ll 1$. For example in thermal equilibrium $e^{-M/T} \ll 1$.

Small densities and the doubling of degrees of freedom



Picture taken from T. Konstandin (2013)

Small densities and the doubling of degrees of freedom

For a static quark in thermal equilibrium

$$\begin{pmatrix} \frac{i}{k_0+i\epsilon} & 0 \\ 2\pi\delta(k_0) & \frac{-i}{k_0-i\epsilon} \end{pmatrix} + \mathcal{O}(e^{-T/M})$$

If I want to compute a time-ordered correlator of heavy particles fields I only need to consider heavy fields living in C^+.

Advantages of using an EFT for NR particles

- Even for computing thermal corrections a lot of physics happens at the scale of the heavy mass M. We can use normal QFT in the vaccum for this part.
- The number of diagrams to compute reduces a lot by taking into account that the density of NR particles is small.
- We are going to see that the information coming from the medium for the case of NR Majorana neutrinos can be encoded in equal time correlators. They are easier to compute in thermal equilibrium.

Thermal decay width of a heavy Majorana neutrino

JHEP 1312 (2013) 028

EFT for heavy Majorana neutrinos

$$\mathcal{L} = N^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) N + sub - leading$$

- Interaction is always given by higher order operators, always suppressed by powers of M.
- LO thermal corrections will always come from operators whose dimension is smaller.
- We have to respect the symmetries.

LO interaction with Higgs

$$N^\dagger N \phi^\dagger \phi$$

Operator of dimension 5

$$\delta\Gamma \propto \frac{T^2}{M}$$

LO interaction with relativistic fermions

$$N^\dagger N\bar{L}L$$

Operator of dimension 6. But in thermal equilibrium $\langle L\bar{L}\rangle = 0$.
Need to include a derivative D_0.

$$\delta\Gamma \propto \frac{T^4}{M^3}$$

LO interaction with gauge bosons

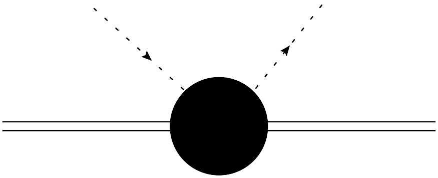
$$N^\dagger N F^{\mu\nu} F_{\mu\nu}$$

Operator of dimension 7.

$$\delta\Gamma \propto \frac{T^4}{M^3}$$

Easy way to compute thermal corrections to a process

Compute in the full theory the scattering of a heavy neutrino with a Higgs particle. Matching computation at T=0.



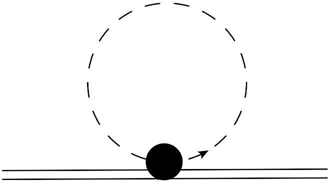
$$\delta\mathcal{L} = \frac{1}{M}\left(\Re a - \frac{3i}{8\pi}|F|^2\lambda\right) N^\dagger N \phi^\dagger \phi$$

Easy way to compute thermal corrections to a process

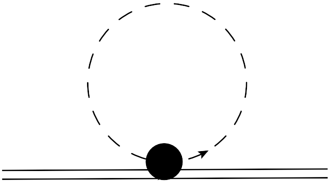
See how this operator contributes to the process you are interested

$$\delta\mathcal{L} = \frac{1}{M}\left(\Re a - \frac{3i}{8\pi}|F|^2\lambda\right) N^\dagger N\phi^\dagger\phi$$

Example, Corrections to the decay width. Tadpole diagram.



LO correction to the decay width



$$\delta\Gamma = -\frac{3|F|^2\lambda}{4\pi M}\langle\phi^\dagger\phi\rangle = -\frac{\lambda|F|^2 T^2}{8\pi M} + \mathcal{O}\left(\frac{T^4}{M^3}\right)$$

Agree with Savio, Lodone and Strumia (2011) and Laine and Schroeder (2012).

Corrections in the EFT for heavy Majorana neutrinos

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\text{N-SM}} = \;& b\,\bar{N}N\,(v\cdot D\phi^\dagger)\,(v\cdot D\phi) \\
&+c_1^{ff'}\;\left[(\bar{N}P_L\,iv\cdot DL_f)\,(\bar{L}_{f'}P_RN)\right. \\
&\qquad \left.+\,(\bar{N}P_R\,iv\cdot DL^c_{f'})\,(\bar{L}^c_fP_LN)\right] \\
&+c_2^{ff'}\;\left[(\bar{N}P_L\,\gamma_\mu\gamma_\nu\,iv\cdot DL_f)\,(\bar{L}_{f'}\,\gamma^\nu\gamma^\mu\,P_RN)\right. \\
&\qquad \left.+\,(\bar{N}P_R\,\gamma_\mu\gamma_\nu\,iv\cdot DL^c_{f'})\,(\bar{L}^c_f\,\gamma^\nu\gamma^\mu\,P_LN)\right] \\
&+c_3\,\bar{N}N\;(\bar{t}P_L\,v^\mu v^\nu\gamma_\mu\,iD_\nu t)+c_4\,\bar{N}N\;(\bar{Q}P_R\,v^\mu v^\nu\gamma_\mu\,iD_\nu Q) \\
&+c_5\,\bar{N}\,\gamma^5\gamma^\mu\,N\;(\bar{t}P_L\,v\cdot\gamma\,iD_\mu t)+c_6\,\bar{N}\,\gamma^5\gamma^\mu\,N\;(\bar{Q}P_R\,v\cdot\gamma\,iD_\mu Q) \\
&+c_7\,\bar{N}\,\gamma^5\gamma^\mu\,N\;(\bar{t}P_L\,\gamma_\mu\,iv\cdot Dt)+c_8\,\bar{N}\,\gamma^5\gamma^\mu\,N\;(\bar{Q}P_R\,\gamma_\mu\,iv\cdot DQ) \\
&-d_1\,\bar{N}N\,v^\mu v_\nu W^a_{\alpha\mu}W^{a\,\alpha\nu}\;-d_2\,\bar{N}N\,v^\mu v_\nu F_{\alpha\mu}F^{\alpha\nu} \\
&+d_3\,\bar{N}N\,W^a_{\mu\nu}W^{a\,\mu\nu}\;+d_4\,\bar{N}N\,F_{\mu\nu}F^{\mu\nu}\,.
\end{aligned}
$$

Full decay width

$$\delta\Gamma = \frac{|F|^2 M}{8\pi}\left[-\lambda\left(\frac{T}{M}\right)^2 + \frac{\lambda}{2}\frac{k^2 T^2}{M^4} - \frac{\pi^2}{80}\left(\frac{T}{M}\right)^4 (3g^2 + g'^2) - \frac{7\pi^2}{60}\left(\frac{T}{M}\right)^4 |\lambda_t|^2\right]$$

Boltzmann equation from a NR EFT

Toy model

EFT with a Majorana Dark Matter χ with mass M and some unknown physics at scale $\Lambda \gg M$

$$\mathcal{L}^{(1)}_{\rm EFT} = \frac{i}{2}\bar{\chi}\partial\!\!\!/\chi - \frac{M}{2}\bar{\chi}\chi + i\frac{a_1}{\Lambda}\bar{\chi}\gamma^5\chi\,\phi^\dagger\phi + \frac{a_2}{\Lambda}\bar{\chi}\chi\,\phi^\dagger\phi + \cdots$$

- Not a realistic model. Test for computation.
- Dark matter does not have a decay width.
- Focus on regime $M \gg T \gg M_{EW}$. In the case $M_{EW} \gg T$ we will get an EFT equivalent to the one shown in De Simone, Monin, Thamm and Urbano (2013).
- We assume $c_2 = 0$ because the computation would be analogous.

Strategy

- Integrate out the scale M to go to the EFT. $\chi \to N$. Focus on $(N^\dagger N)^2$ operators.
- Compute the evolution of the number of particles operator. $\langle N^\dagger N \rangle$.

Double expansion

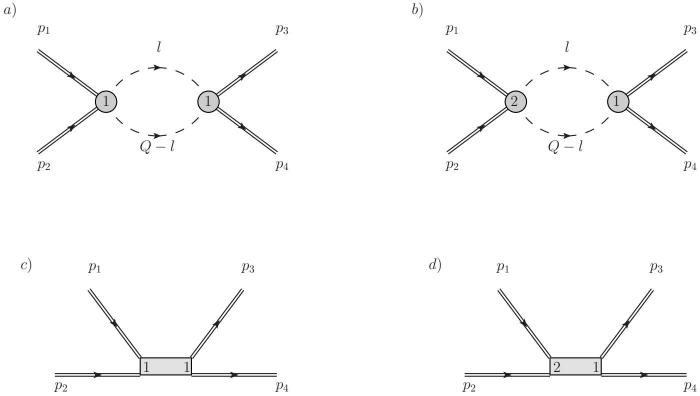
- Expansion in $\frac{T}{M}$ and $\frac{M}{\Lambda}$.
- Expansion in f_N.

$$f_N \propto \frac{\langle N^\dagger N \rangle}{\langle N N^\dagger \rangle} \ll 1$$

Boltzmann equation

In this model

$$\delta_t f_N \propto f_N^2$$



Boltzmann equation

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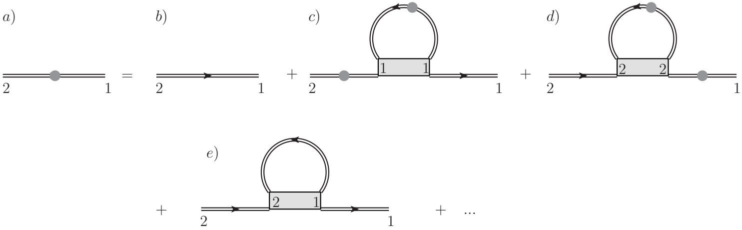

Result

Boltzmann eq. of dark matter in a background of SM particles in thermal equilibrium

$$\left(\partial_t - Hk\partial_k\right) f_\chi\left(E_k, t\right) =$$
$$\frac{1}{\pi}\left(\frac{a_1}{\Lambda}\right)^2\left[e^{-\frac{2M}{T}} e^{-\frac{k^2}{2MT}} \int \frac{d^3\vec{l}}{(2\pi)^3} e^{-\frac{l^2}{2MT}} - f_\chi\left(E_k, t\right) \int \frac{d^3\vec{l}}{(2\pi)^3} f_\chi\left(E_l, t\right)\right]$$

- Using EFT simplifies the computation a lot. Double expansion. Simple form of operators.
- In the future more complicated cases. Easy to consider the case $\frac{k^2}{2M} \sim \Gamma$.

Conclusions

Conclusions

- NR EFT can simplify a lot computations in cosmology.
- In the case of Majorana neutrinos a lot of information can be gain looking at the scattering with Higgs particles.
- Boltzmann like equations can be obtained using a double expansion in T and in the density of heavy particles.