Higher order calculations and the phase diagram of strongly coupled theories

Quark Confinement and the Hadron Spectrum XI

Saint Petersburg, 7-12 September 2014

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Pick a Theory

- Given a gauge theory what physics does it describe in the infrared?

Temperature

Chemical potential

Gauge group

# of colors

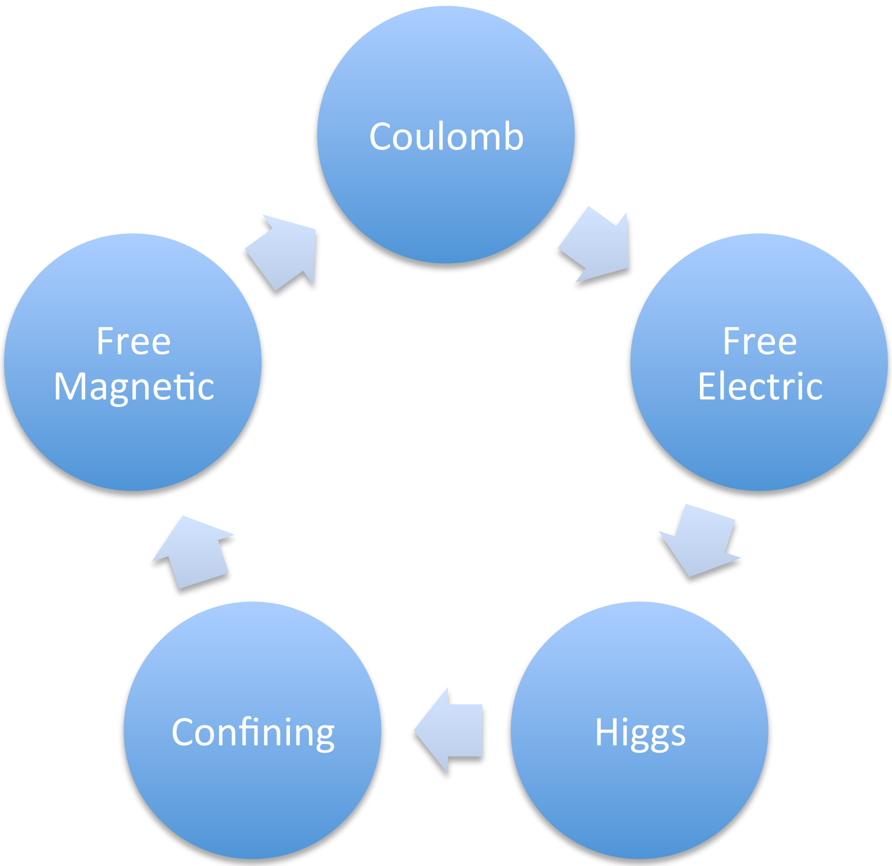
Bosons/fermions

# of flavors

Matter representation

Chiral/vector-like

N=1 Supersymmetric QCD



Lessons from SQCD (Seiberg)

- Exact NSVZ beta function

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\frac{\beta_0 - 2T(r)N_f\gamma(\alpha)}{1-\frac{\alpha}{2\pi}C_2(G)}$$ ← NSVZ beta function

$$\gamma(\alpha) = C_2(r)\frac{\alpha}{\pi} + O(\alpha^2)$$ ← Anomalous dimension

$$\beta_0 = 3C_2(G) - 2T(r)N_f$$ ← First beta function coefficient

- Note: relation between beta function and anomalous dimension.

- At zero of beta funtion

$$\gamma = \frac{3C_2\left(G\right) - 2T\left(r\right)N_f}{2T\left(r\right)N_f}$$

- Conformality requires (Mack)

$$D(\Phi\tilde{\Phi}) = 2 - \gamma \geq 1$$

- Critical number of flavors

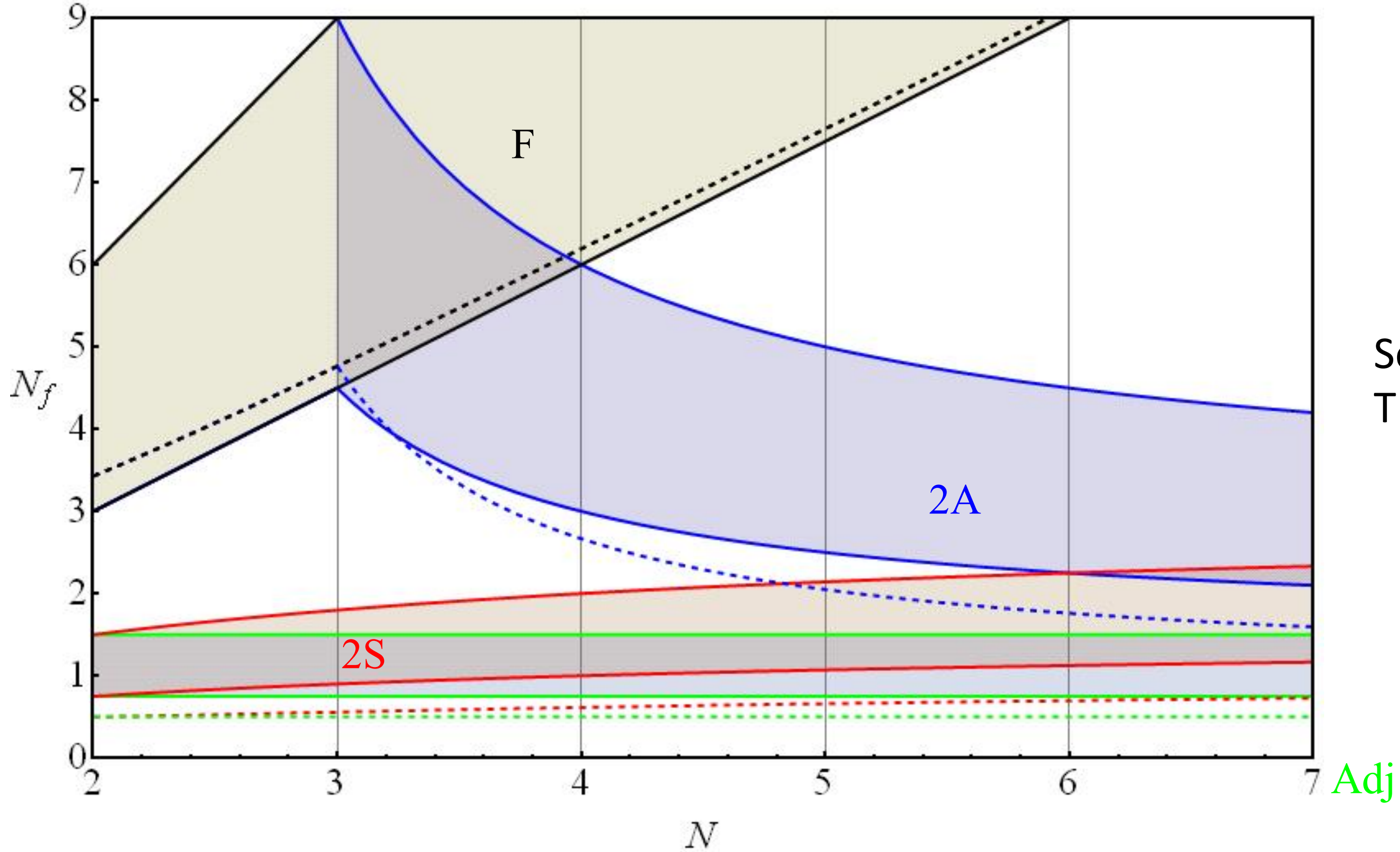
$$N_f^{critical} = \frac{3}{4}\frac{C_2(G)}{T(r)}$$

- Conformal window

Asymptotic freedom

$$\frac{3}{4}\frac{C_2(G)}{T(r)} < N_f^{critical} < \frac{3}{2}\frac{C_2(G)}{T(r)}$$

SUSY conformal window



Seiberg
TR, Sannino

- Can we match approximate techniques to the exact results of Seiberg?
- The ladder approximation does not do a good job. (Appelquist, Nyffeler, Selipsky)

SUSY three loop beta function and anomalous dimension

- Three loop beta function and anomalous dimension are known in DRbar scheme

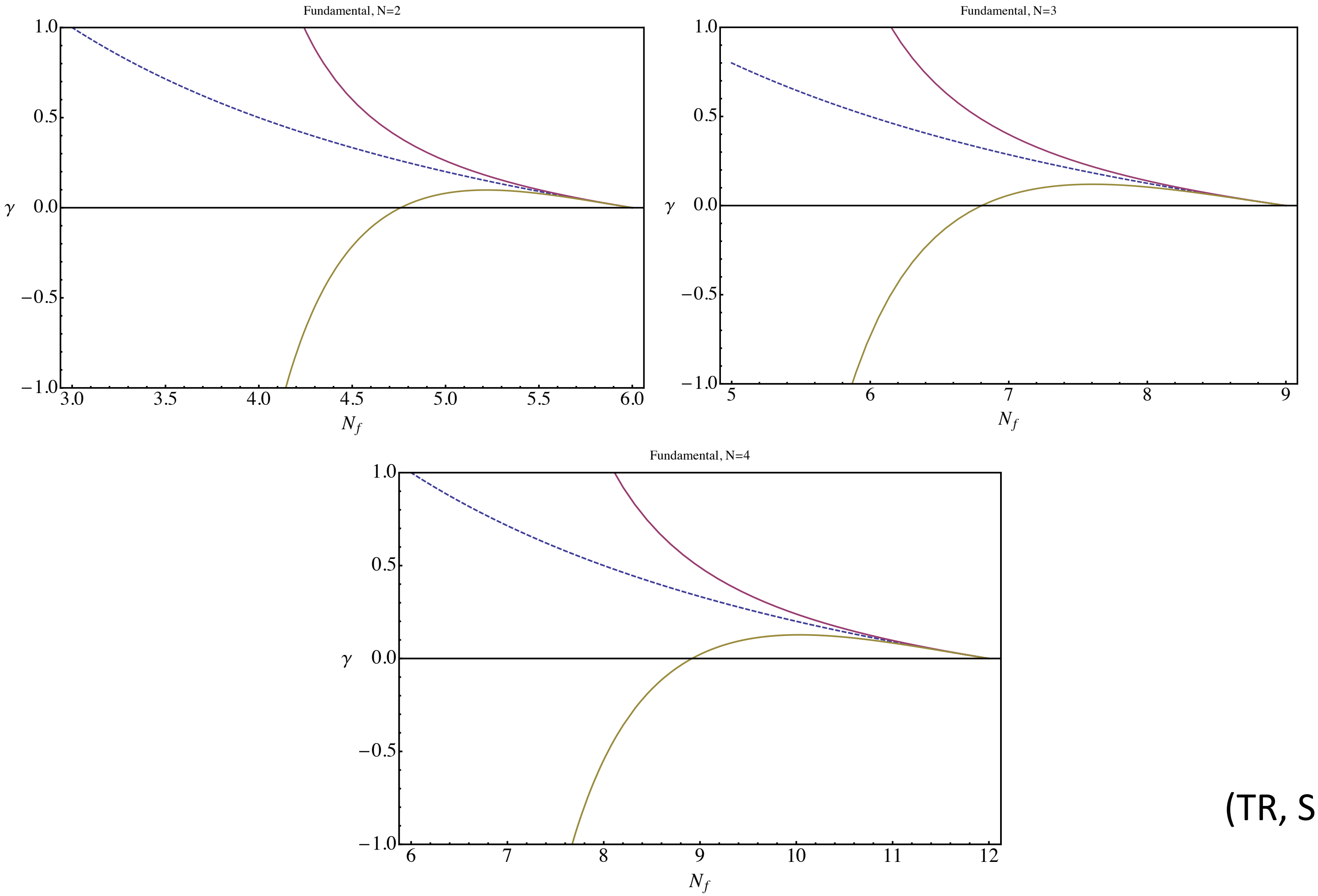
$$\beta_{3-loop}\left(\alpha\right) = -\beta_0 \frac{\alpha^2}{\left(2\pi\right)^1} - \beta_1 \frac{\alpha^3}{\left(2\pi\right)^2} - \beta_2 \frac{\alpha^4}{\left(2\pi\right)^3} + O(\alpha^5)$$

$$\gamma_{3-loop}\left(\alpha\right) = \gamma_0 \left(\frac{\alpha}{\pi}\right) + \gamma_1 \left(\frac{\alpha}{\pi}\right)^2 + \gamma_2 \left(\frac{\alpha}{\pi}\right)^3 + O(\alpha^4)$$

- Search for an infrared fixed point of the beta function.
- Evaluate the anomalous dimension at the infrared zero of the beta function.

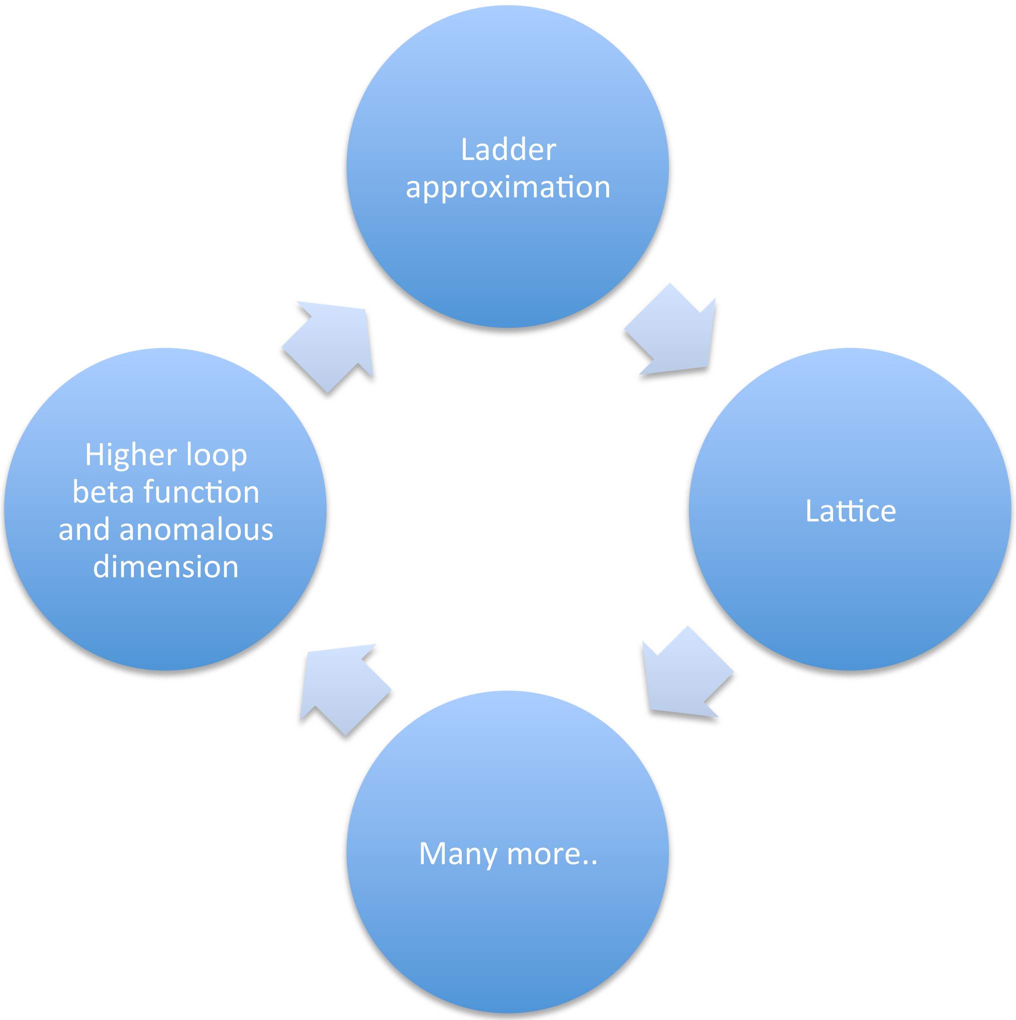
(TR, Shrock)

SUSY anomalous dimension vs. number of flavors



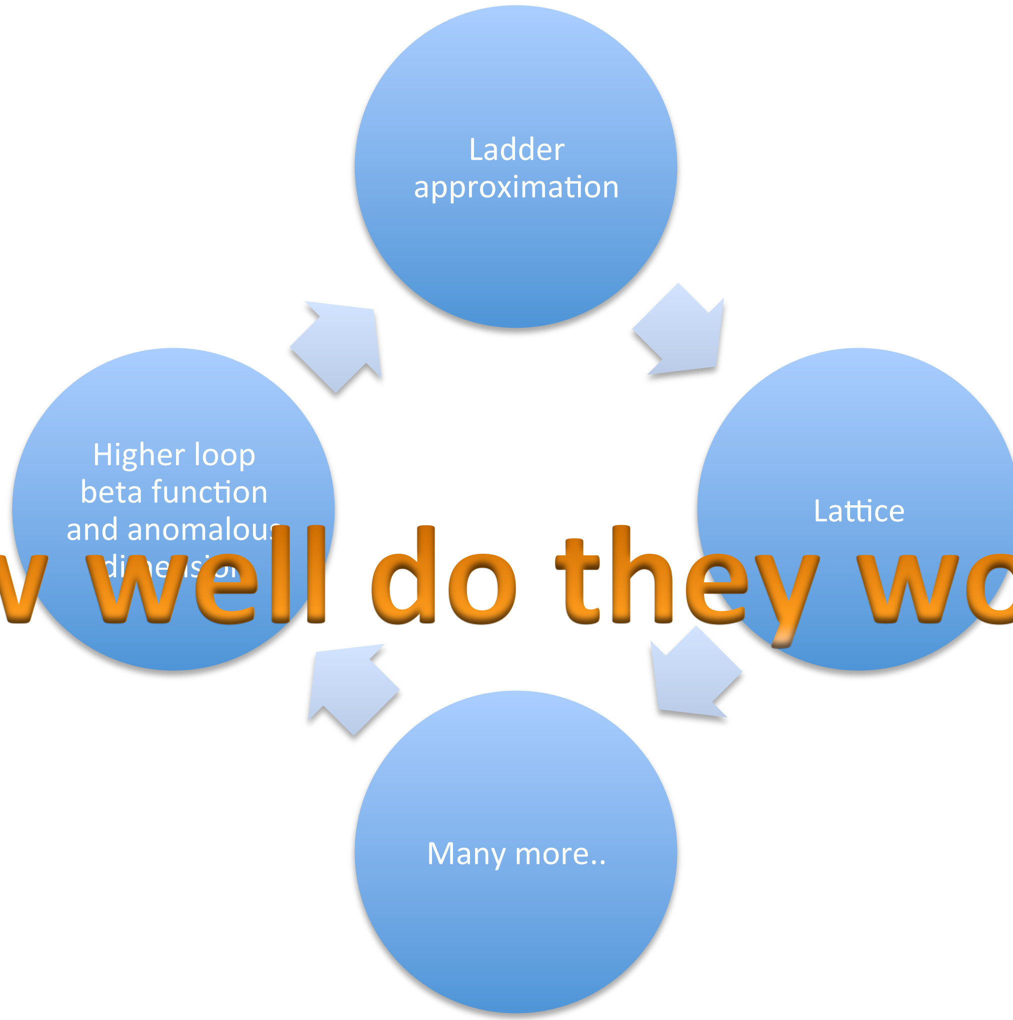
(TR, Shrock)

QCD-like



Ladder approximation
Lattice
Many more..
Higher loop beta function and anomalous dimension

QCD-like



How well do they work??

Ladder Approximation

- Two loop fixed-point coupling

$$\alpha_{IR} = -4\pi \frac{\beta_0}{\beta_1}$$

- α_{IR} becomes large as $\beta_1 \to 0$

- Chiral symmetry breaking could be triggered before α_{IR} is reached.

- The gap equation has a solution for the dynamically generated mass when the coupling reaches the value

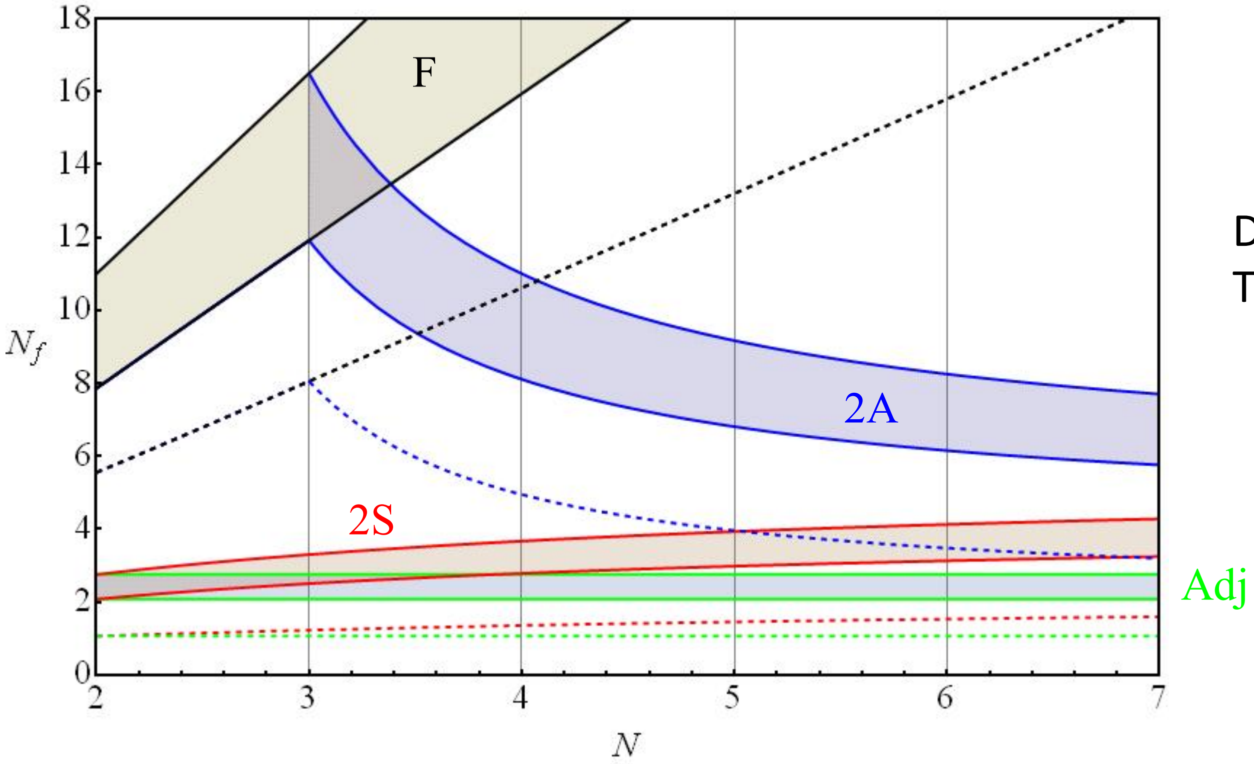
$$\alpha_c = \frac{\pi}{3C_2(r)}$$

(Higher orders ≈ 20% corrections)

- Critical number of flavors

$$\alpha_c = \alpha_{IR} \qquad \Rightarrow \qquad N_f = \frac{17C_2(G)+66C_2(r)}{10C_2(G)+30C_2(r)} \frac{C_2(G)}{T(r)}$$

Phase Diagram (ladder approximation)



Dietrich, Sannino, Tuominen

- Ladder approximation: Chiral symmetry breaking is triggered at

$$\gamma \sim 1$$

Four loop beta function and anomalous dimension

- Four loop beta function and anomalous dimension are known in MSbar scheme

(Ritbergen, Vermaseren, Larin)

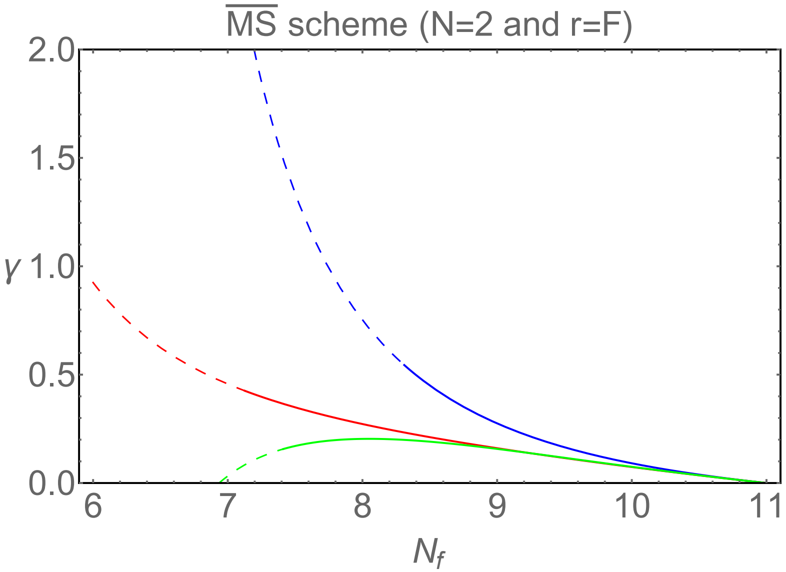
$$\beta_{4-loop}(\alpha) = -\beta_0 \frac{\alpha^2}{(2\pi)^1} - \beta_1 \frac{\alpha^3}{(2\pi)^2} - \beta_2 \frac{\alpha^4}{(2\pi)^3} - \beta_3 \frac{\alpha^5}{(2\pi)^4} + O(\alpha^6)$$

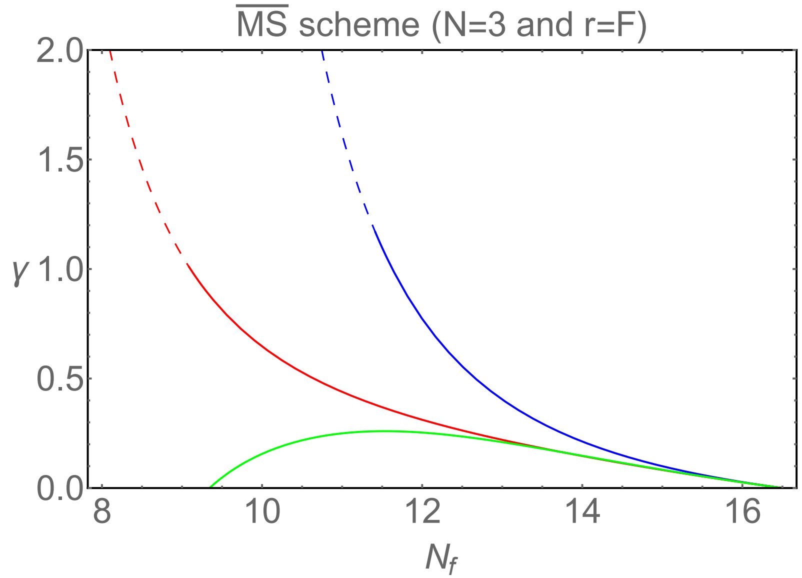
$$\gamma_{4-loop}(\alpha) = \gamma_0 \left(\frac{\alpha}{\pi}\right) + \gamma_1 \left(\frac{\alpha}{\pi}\right)^2 + \gamma_2 \left(\frac{\alpha}{\pi}\right)^3 + \gamma_3 \left(\frac{\alpha}{\pi}\right)^4 + O(\alpha^5)$$

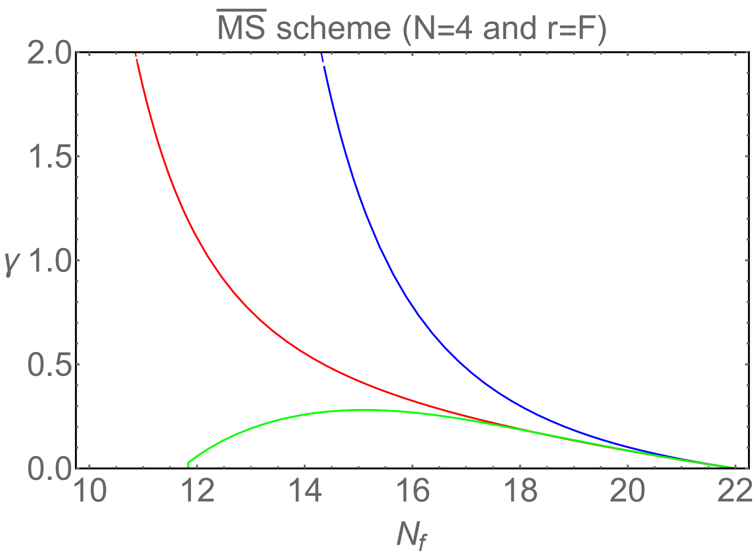
- New group invariants enter at the four loop level.
- Search for an infrared fixed point of the beta function.
- Evaluate the anomalous dimension at the infrared zero of the beta function

(Pica, Sannino
TR, Shrock)

Anomalous dimension vs. number of flavors







$\gamma \leq 2$

Solid: $\alpha<1$, Dashed: $\alpha>1$

Blue: 2-loop
Red: 3-loop
Green: 4-loop

Observations

- There is only one positive IR zero.
- Higher loop orders are smaller relative to two loops.
- Same for other representations (adjoint, 2S, 2A)
- Seems like same pattern as for SUSY.
- Warning: Sometimes different loop orders differ significantly even though $\alpha<1$.
- How about scheme dependence?

Other Schemes?

- Four loop beta function and anomalous dimension are known in

RI’ scheme

(Gracey, and many others)

mMOM scheme

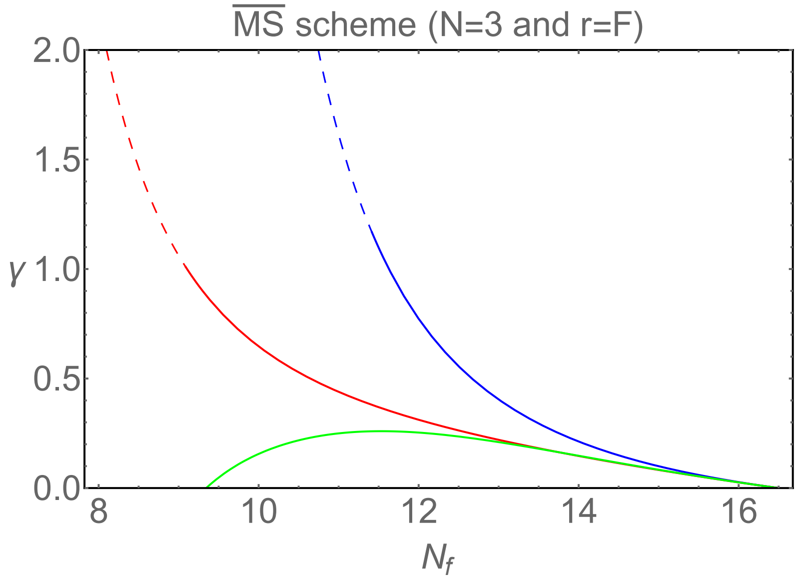
- They agree at the UV fixed point. Not necessarily at the IR fixed point.
- In both schemes there is dependence on the gauge parameter

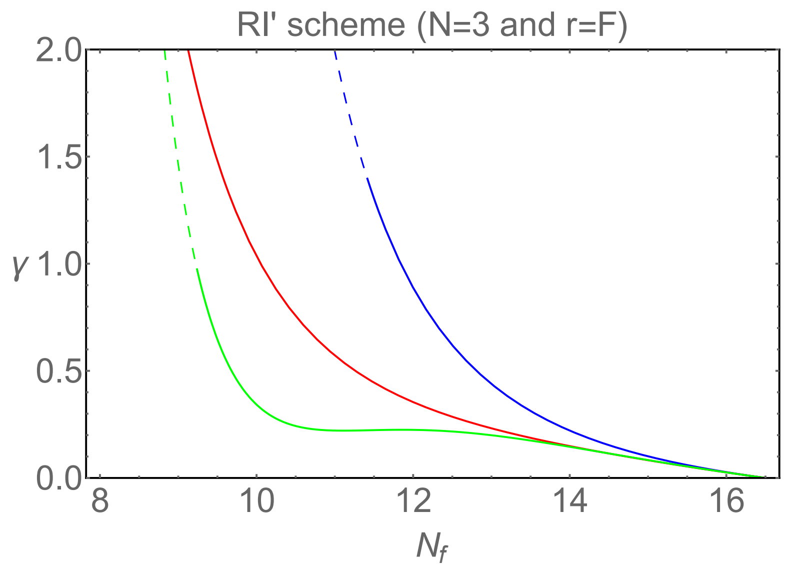
$$\beta_\alpha(\alpha,\xi)=0$$
$$\beta_\xi(\alpha,\xi)=0$$

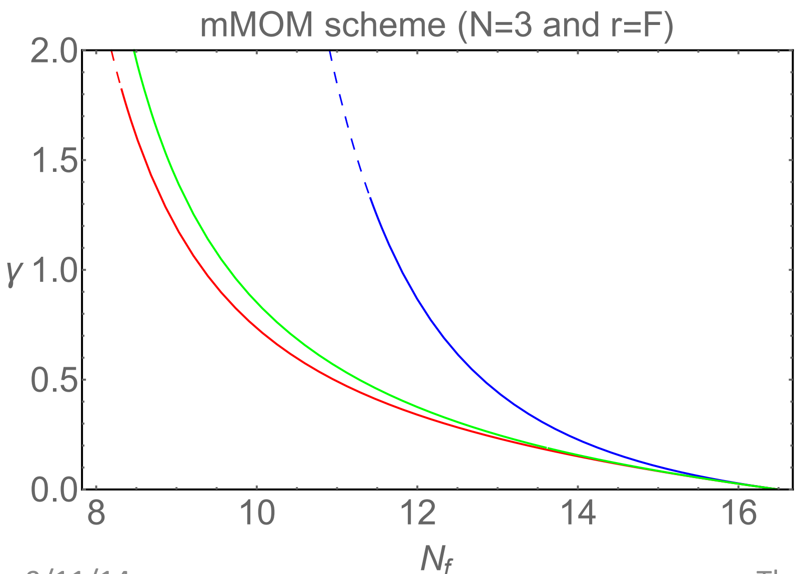
- Pick Landau gauge solution $\xi=0$.

$\beta_\alpha(\alpha,0)=0$, plot $\gamma(\alpha,0)$ (TR)

N=3 and Fundamental rep.







$\gamma \leq 2$

Solid: $\alpha < 1$, Dashed: $\alpha > 1$

Blue: 2-loop
Red: 3-loop
Green: 4-loop

Lattice

- 3 colors and 12 flavors in fundamental:

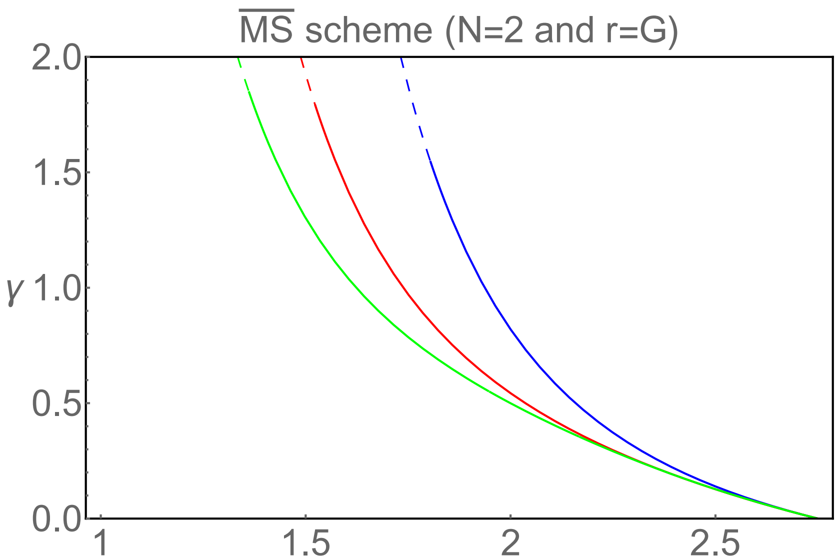
Appelquist, et. al.

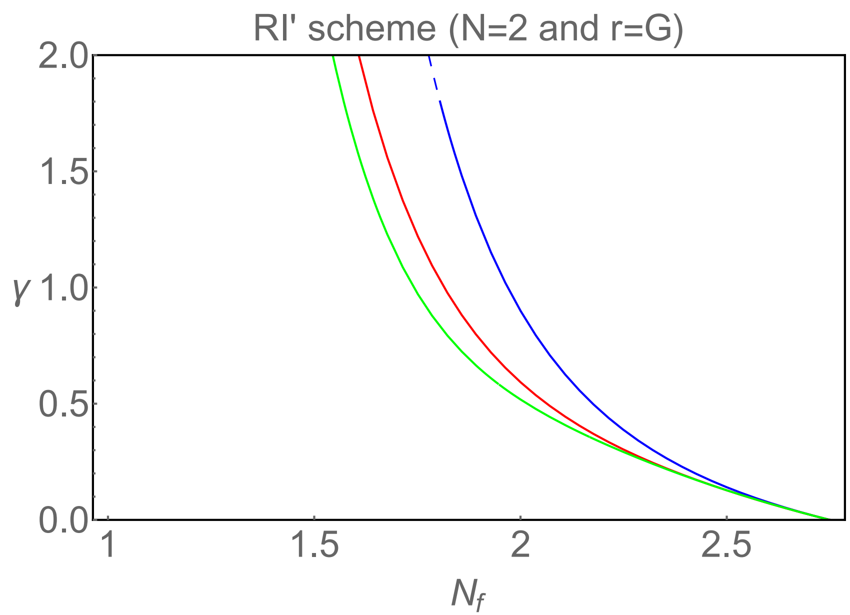
$$\gamma_{\text{lattice}} \approx 0.386 \pm 0.010$$

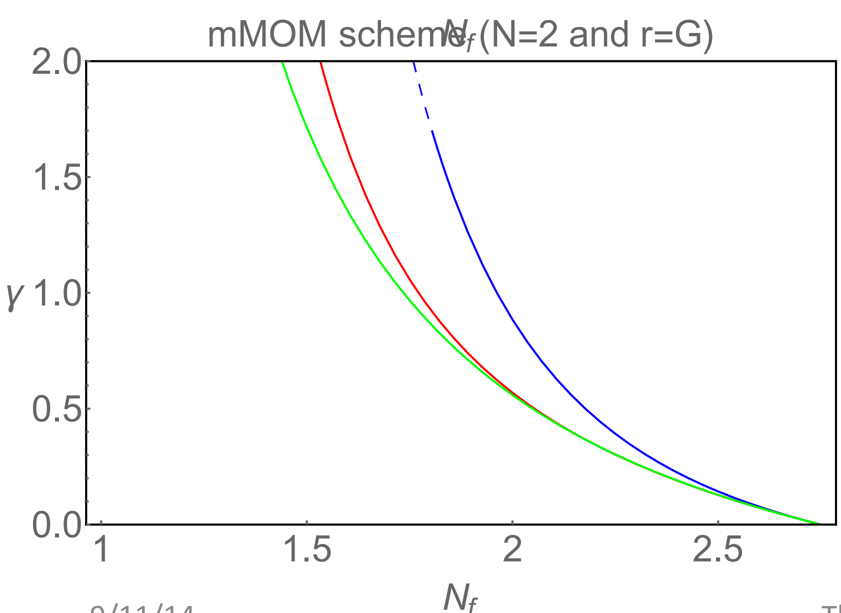
- 3 and 4 loops in all three schemes:

$$\gamma_{\text{higher orders}} \approx 0.225\text{-}0.375$$

N=2 and Adjoint rep.







$\gamma \leq 2$

Solid: $\alpha<1$, Dashed: $\alpha>1$

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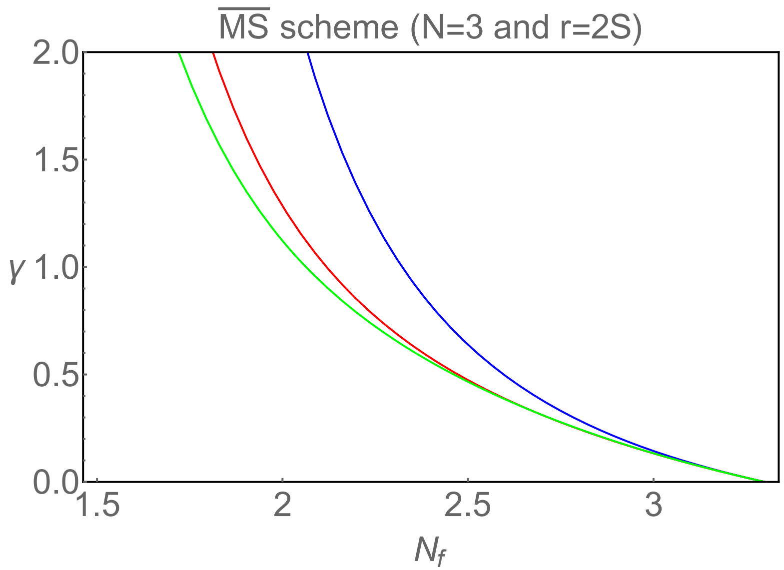
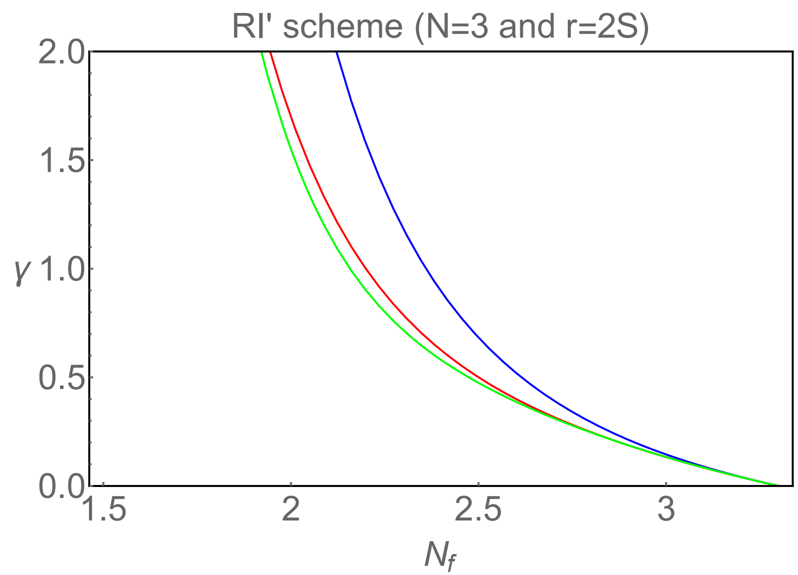
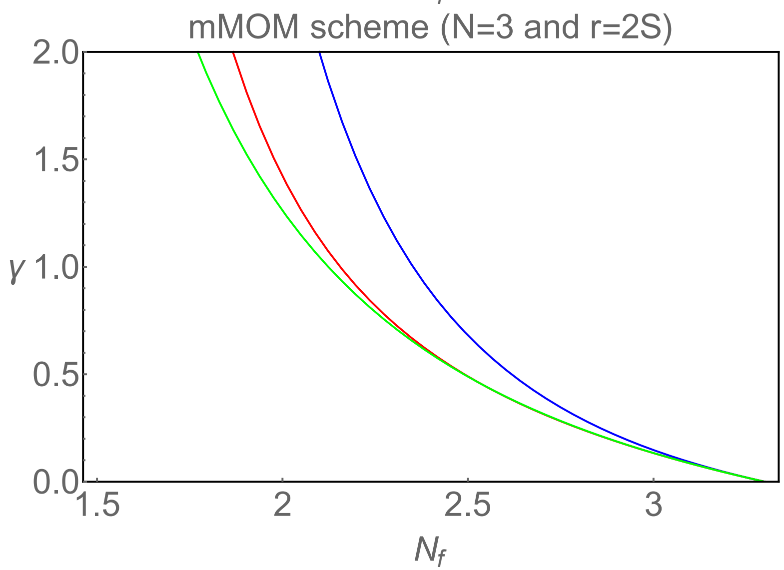
Green: 4-loop

N=3 and 2S rep.







γ≤2

Solid: α<1, Dashed: α>1

Blue: 2-loop
Red: 3-loop
Green: 4-loop

Lattice

- 2 colors and 2 flavors in adjoint:

$\gamma_{\text{Catterall, et al.}} < 0.6$, $\gamma_{\text{higher orders}} \approx 0.500\text{-}0.593$

- 3 colors and 2 flavors in 2S:

$\gamma_{\text{Kuti, et al}} > 1$, $\gamma_{\text{DeGrand, et al}} \leq 0.45$, $\gamma_{\text{higher orders}} \approx 1.12\text{-}1.70$

Conclusion

- Anomalous dimension at three and four loops is smaller than at two loops in all three schemes.
- There is only one positive IR zero in all three schemes.
- According to the Ladder approximation the results indicate that the boundary of the conformal window is lowered.