

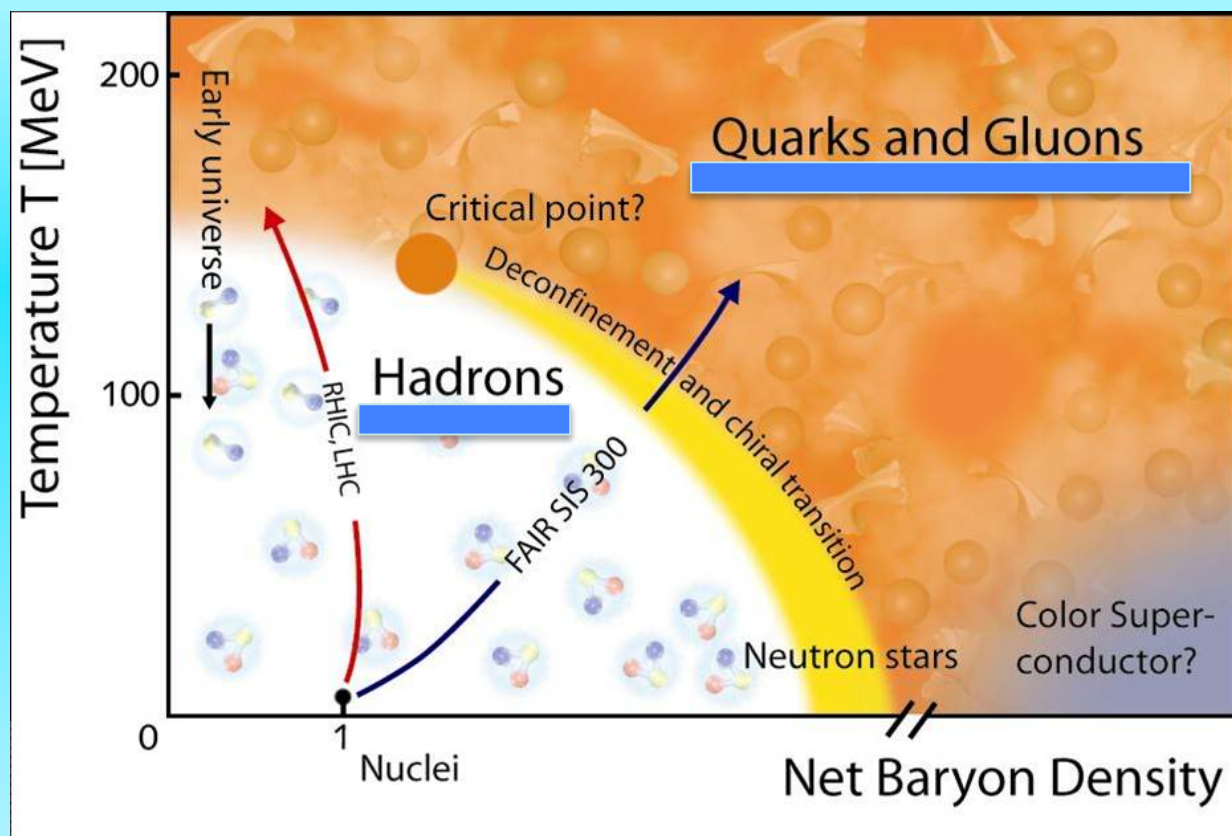
Models of Quark-Hadron Matter and Compact Stars

OUTLINE

- hadronic SU(3) model
- adding quarks
- some results for $\mu = 0$ and heavy-ions
- dense matter, compact stars
- conclusions, to-do list

*V. Dexheimer, R. Negreiros, P. Rau, J. Steinheimer, SWS
FIAS Frankfurt, Kent State, Fluminense,*

the usual phase diagram (sketch) of strong interactions



connect both worlds
in some reasonable way

Practical model useful for heavy-ion simulations and compact star physics

correct asymptotic degrees of freedom

reasonable description on a quantitative level for high T down to nuclei

possibility of studying first-order as well as cross-over transitions

hadronic SU(3) approach based on non-linear realization of $\sigma\omega$ model

Lowest multiplets

$$B = \{ p, n, \Lambda, \Sigma^{\pm/0}, X^{-/0} \} \quad \text{baryons}$$

$$\text{diag}(V) = \{ (\omega + \rho) / \sqrt{2}, (\omega - \rho) / \sqrt{2}, \phi \} \quad \text{vector mesons}$$

$$\text{diag}(X) = \{ (\sigma + \delta) / \sqrt{2}, (\sigma - \delta) / \sqrt{2}, \zeta \} \quad \text{scalar mesons}$$

Mean fields generate scalar attraction and vector repulsion

$$\text{Scalar self interaction } L_0 = -\frac{1}{2} k_0 I_2 + k_1 (I_2)^2 + k_2 I_4 + 2 k_3 I_3 + L_{\text{ESB}}$$

$$\text{invariants} \quad I_1 = \text{Tr}(X) \quad I_2 = \text{Tr}(X)^2 \quad I_3 = \det(X)$$

$$+ \text{dilaton field } L_\chi = -k_4 \chi^4 - \frac{1}{4} \chi^4 \ln(\chi^4/\chi_0^4) + \frac{\delta}{3} \chi^4 \ln(I_3/\langle X \rangle)$$

hadronic SU(3) approach ... continued

SU(3) interaction

$$L_{BW} = -\sqrt{2} g_8^W (\alpha_W [\bar{B}OBW]_F + (1 - \alpha_W) [\bar{B}OBW]_D) - g_1^W / \sqrt{3} \text{Tr}(\bar{B}OB) \text{Tr}(W)$$

V(M) $\langle \sigma \rangle = \sigma_0 \neq 0$ $\langle \zeta \rangle = \zeta_0 \neq 0$

$$\sigma \sim \langle \bar{u}u + \bar{d}d \rangle \quad \zeta \sim \langle \bar{s}s \rangle \quad \delta^0 \sim \langle \bar{u}u - \bar{d}d \rangle$$

explicit breaking $\sim \text{Tr} [c \sigma]$ ($\sim m_q \bar{q}q$)

fix scalar parameters to

baryon masses, decay constants, meson masses

Nuclear Matter and Nuclei

binding energy $E/A \sim -15.2 \text{ MeV}$ saturation $(\rho_B)_0 \sim .16/\text{fm}^3$

compressibility $\sim 223 \text{ MeV}$ asymmetry energy $\sim 31.9 \text{ MeV}$

parameter fit to known nuclear binding energies and hadron masses

1d to 3d code 2d calculation of all measured (~ 800) even-even nuclei

good charge radii $\delta r_{\text{ch}} \sim 0.5 \%$ (+ LS splittings)

error in energy $\varepsilon (A > 50) \sim 0.17 \%$ (NL3: 0.25 %)

$\varepsilon (A > 100) \sim 0.12 \%$ (NL3: 0.16 %)

+ correct binding energies of hypernuclei

relativistic nuclear
structure models

new fit in the works by T. Schürhoff
currently $\varepsilon \sim 0.3$, $\kappa < 300 \text{ MeV}$, $M \sim 2 M_\odot$

stellar crust calculations in progress

hadrons, quarks, Polyakov loop and excluded volume

Include modified distribution functions for quarks/antiquarks

$$\Omega_q = -T \sum_{j \in Q} \frac{\gamma_j}{(2\pi)^3} \int d^3k \ln \left(1 + \Phi \exp \frac{E_j^* - \mu_j}{T} \right)^* \quad \Phi \quad \text{confinement order parameter}^*$$

Following the parametrization used in PNJL calculations

$$U = -\frac{1}{2} a(T) \Phi \Phi^* + b(T) \ln[1 - 6 \Phi \Phi^* + 4 (\Phi \Phi^*)^3 - 3 (\Phi \Phi^*)^2]$$

$$a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2 \quad , \quad b(T) = b_3 T_0^3 T$$

The switch between the degrees of freedom is triggered by excluded volume corrections

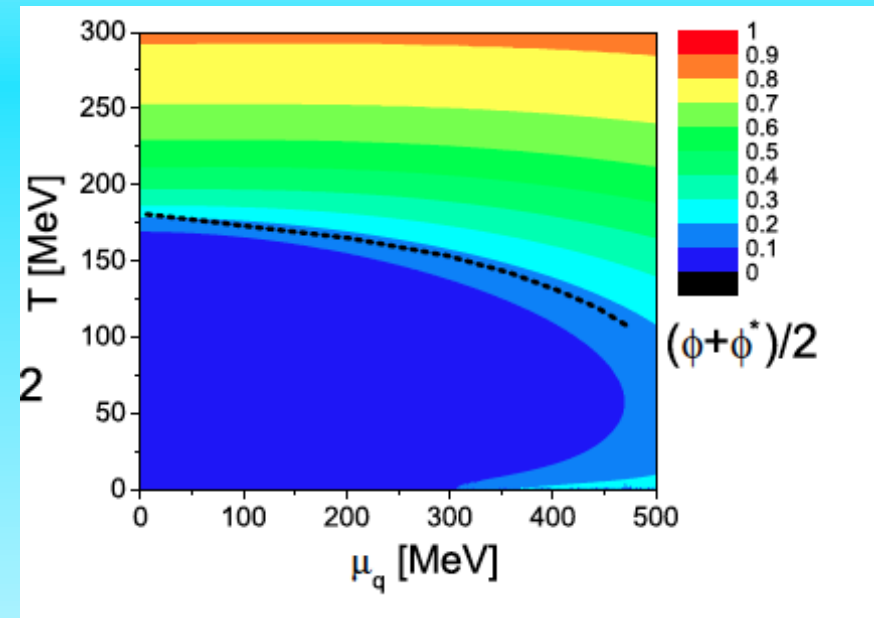
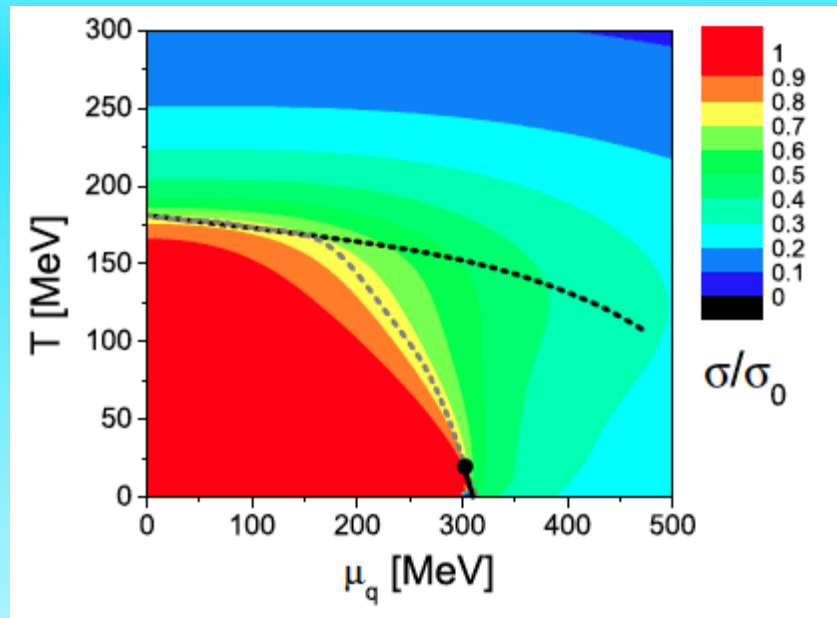
thermodynamically consistent -

no reconfinement!

$$\begin{aligned} V_q &= 0 \\ V_h &= v \\ V_m &= v / 8 \end{aligned} \quad \tilde{\mu}_i = \mu_i - v_i P \quad e = \tilde{e} / (1 + \sum v_i \tilde{\rho}_i)$$

equation of state stays causal!

Order parameters for chiral symmetry and confinement in μ and T

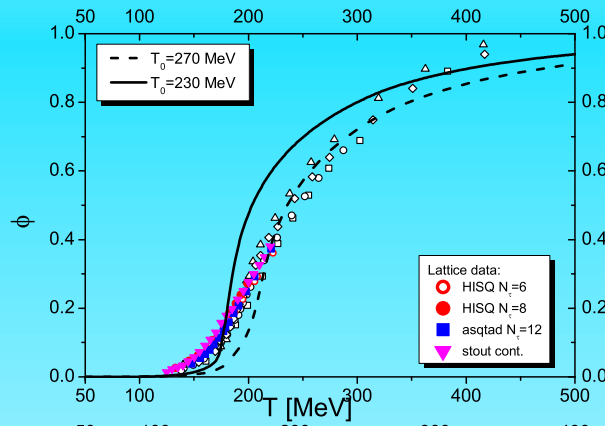


except for liquid-gas no first-order transition

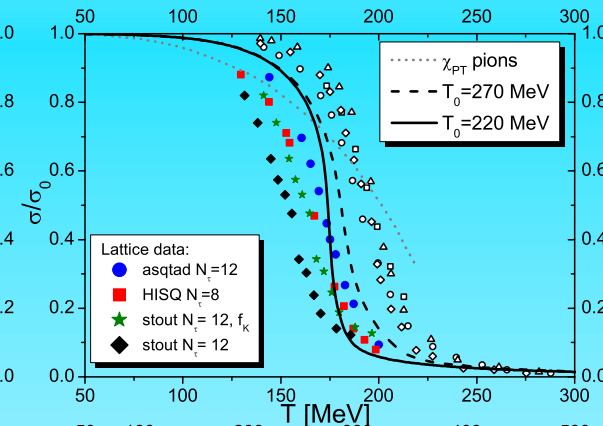
results for hot matter at vanishing chemical potential

points are various lattice results

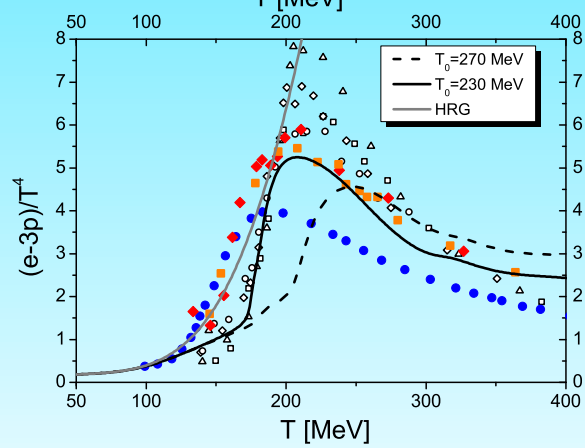
Polyakov loop



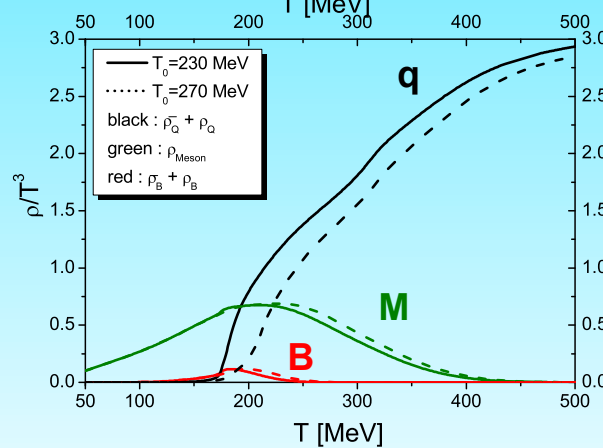
scalar condensate

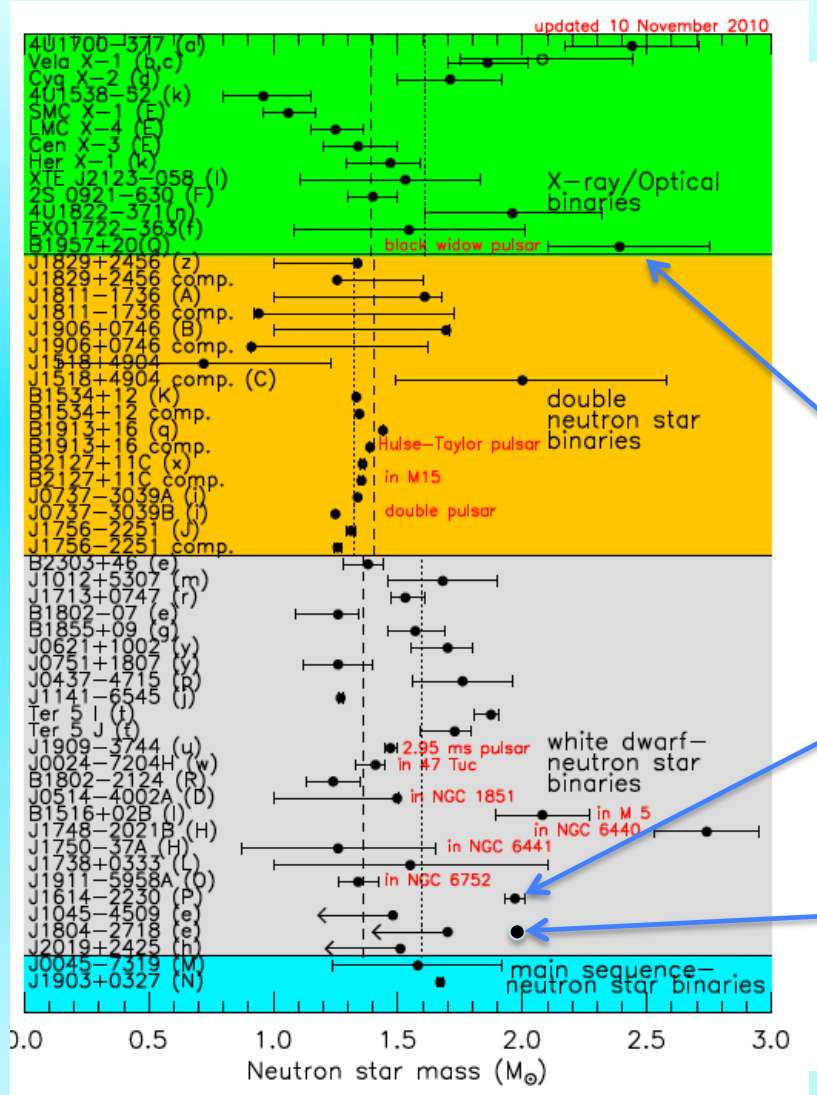


Interaction measure



densities





Lattimer, Prakash, astro-ph:1012.3208

Masses of Neutron Stars

Masses of radio pulsars

Kiziltan, Kottas, Thorsett, astro-ph:1011.4291

no signature for mass cut off

$M = (2.4 \pm 0.12) M_{\odot} ?$

van Kerkwijk et al., ApJ 728, 95 (2011)

current benchmark for NS models

$M = (1.97 \pm .04) M_{\odot}$

Demorest et al. Nature 467, 1081 (2010)

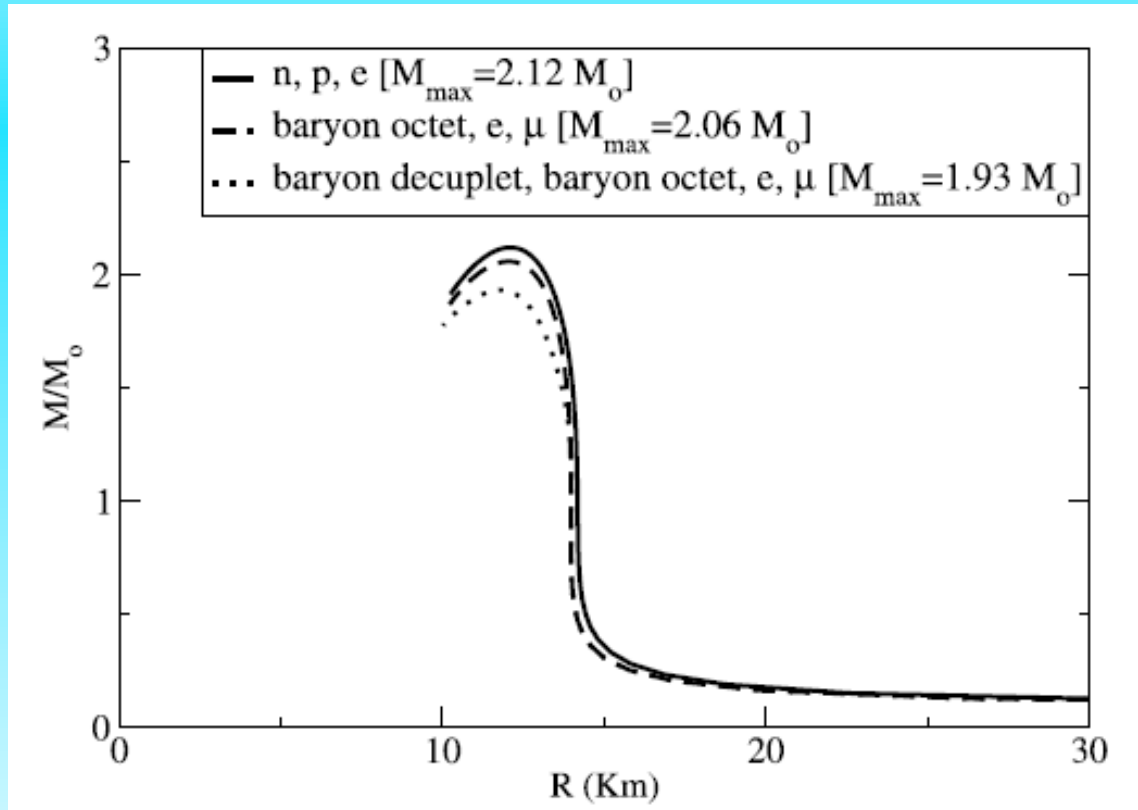
new observation PSR J0348+0432

$M = (2.01 \pm .04) M_{\odot}$

Antoniadis et al. Science 340, 448 (2013)

well established - heavy neutron stars

Neutron star masses including different sets of particles



Tolman-Oppenheimer-Volkov equations, static spherical star

changing masses with degrees of freedom

large star masses even with spin 3/2 resonances

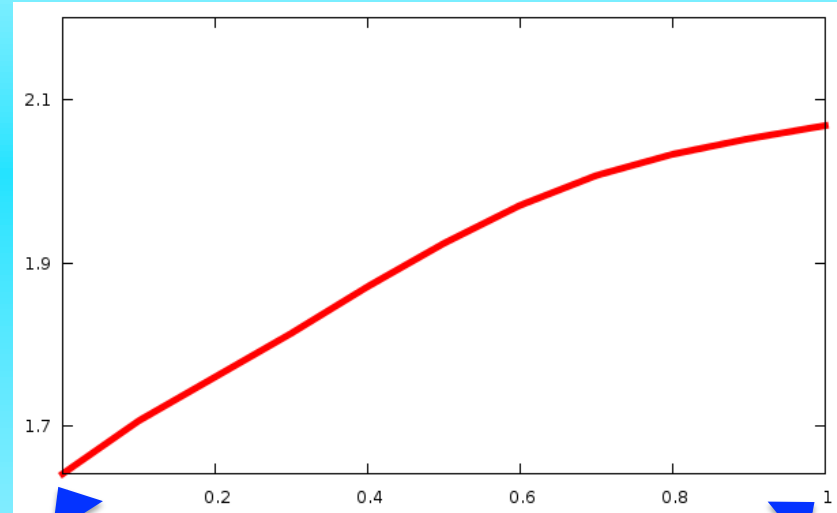
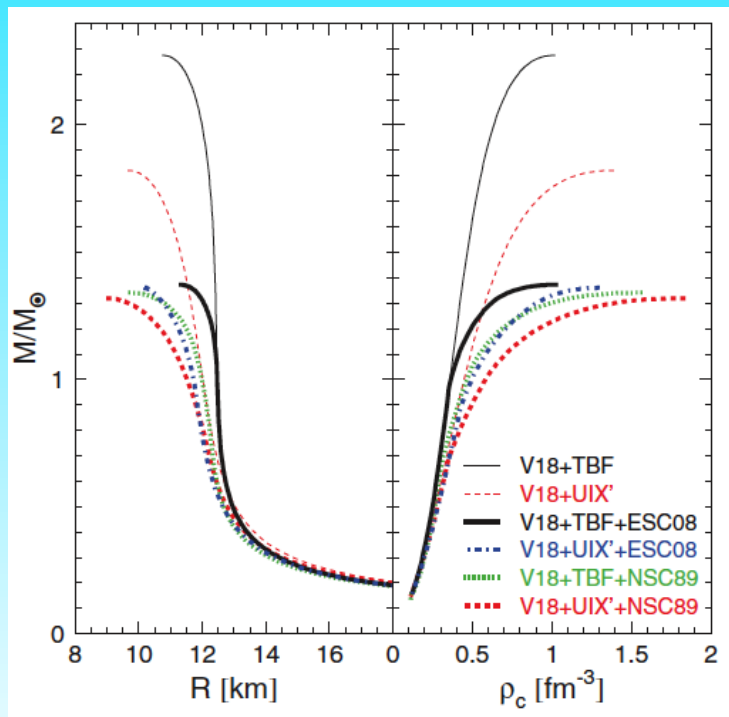
Impact of Φ field

rescale $g_{B\Phi}$ coupling parameters , $f_s(\text{core})$ varies between 0.1 and 1

$M_{\text{max}} [M_{\odot}]$

2

Nijmegen, no YY interaction



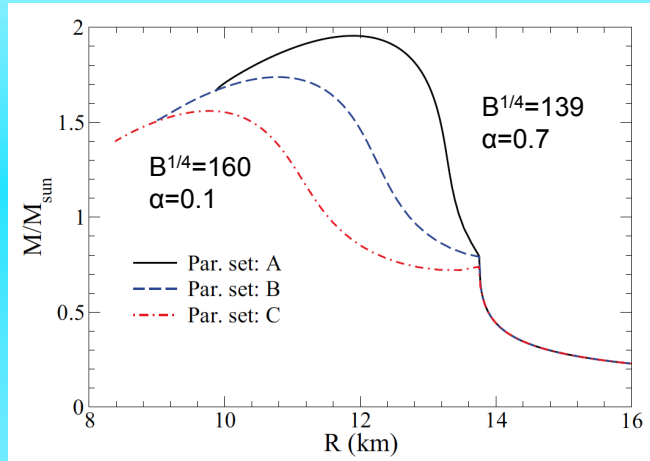
1.7

no coupling

standard fit SU(6)

$$f_s = n_s / n_B$$

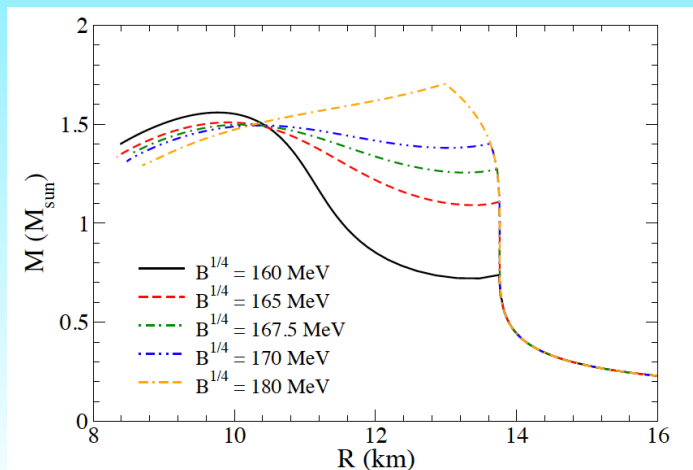
Hybrid Stars, Quark Interactions



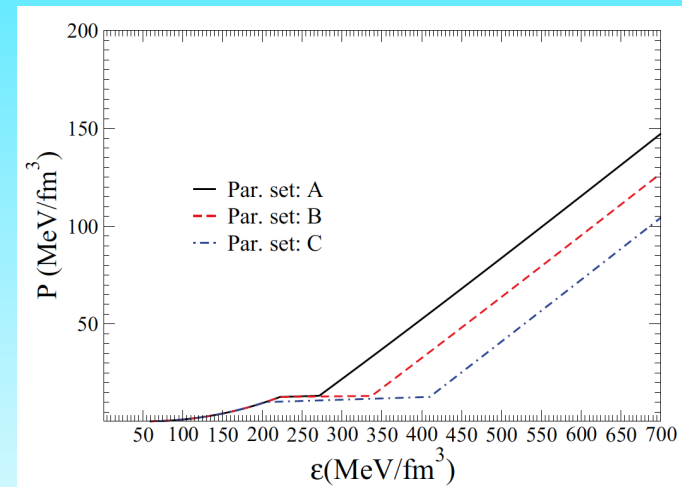
baryons alone $M_{\text{max}} \sim 1.8 M_{\text{solar}}$

ingredients –
Standard baryonic EOS (G300)
plus MIT bag model + α_s corrections

Fast cooling in the quark core
need gaps in the quark phase

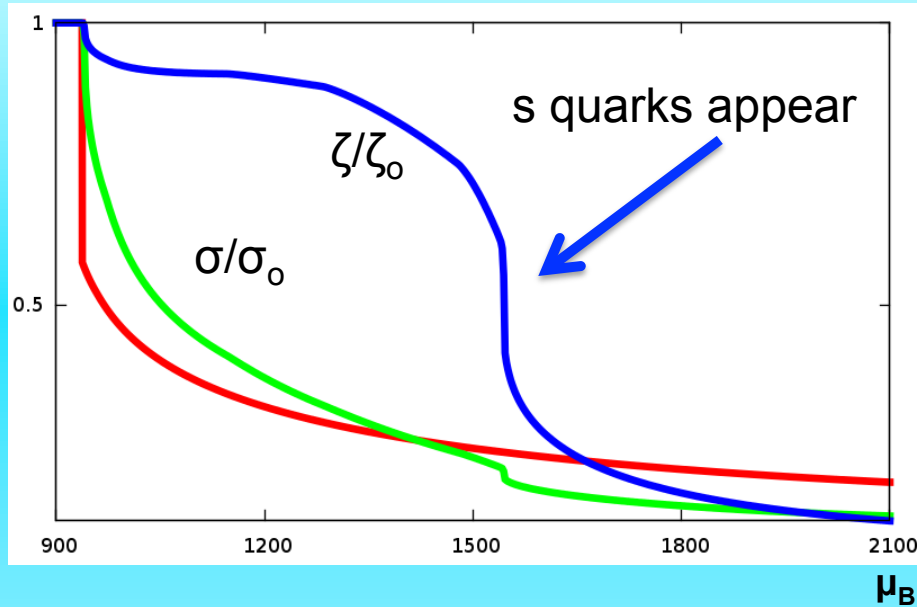


no α_s



Negreiros, Dexheimer, SWS, PRC 035805 (2012)

star matter in beta equilibrium in QH approach

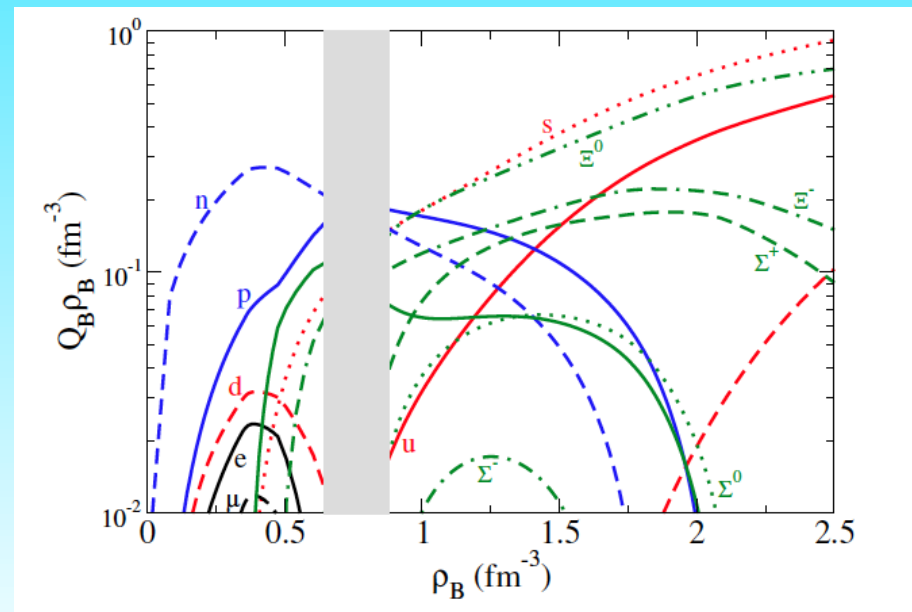


1st order phase transition
in star matter possible

cross over in symmetric matter

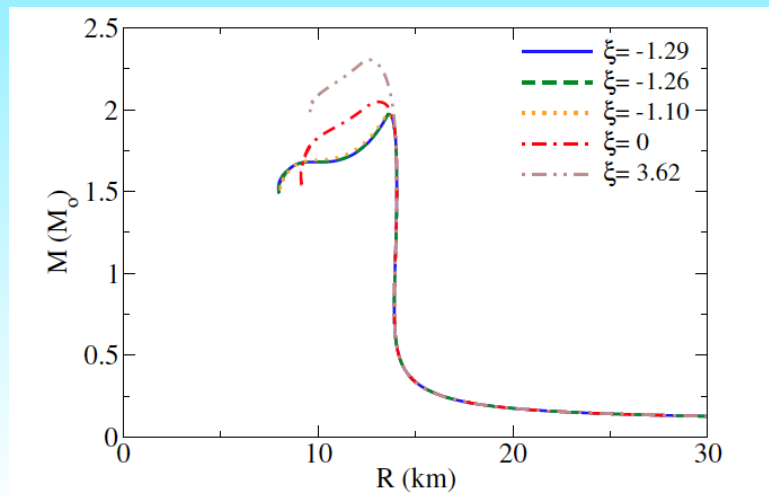
$f_s(\text{core})$ jumps to ~ 1

particle cocktail



Mass $\sim 2 - 2.3 M_\odot$ Radius $\sim 13 \text{ km}$

star masses M_\odot varying quark interactions

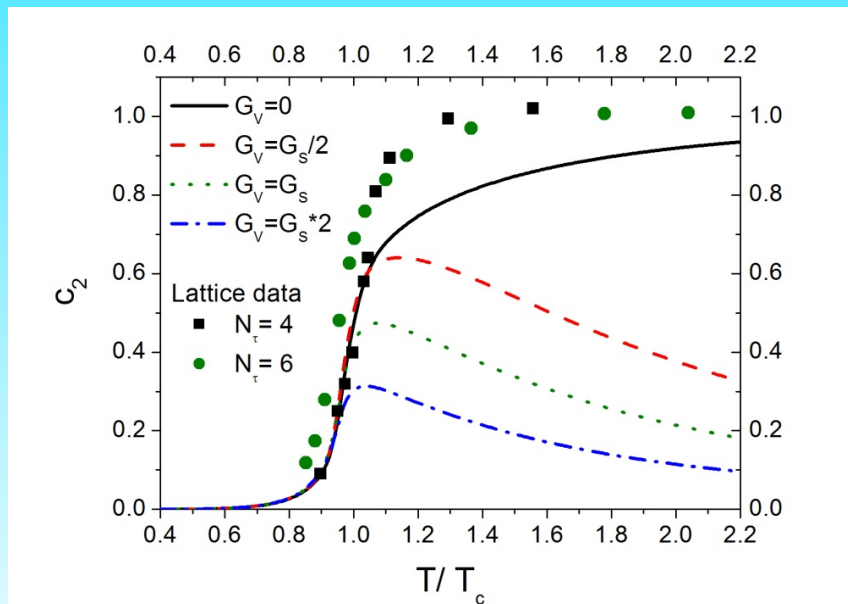


Susceptibility c_2 in PNJL and QH model for different quark vector interactions

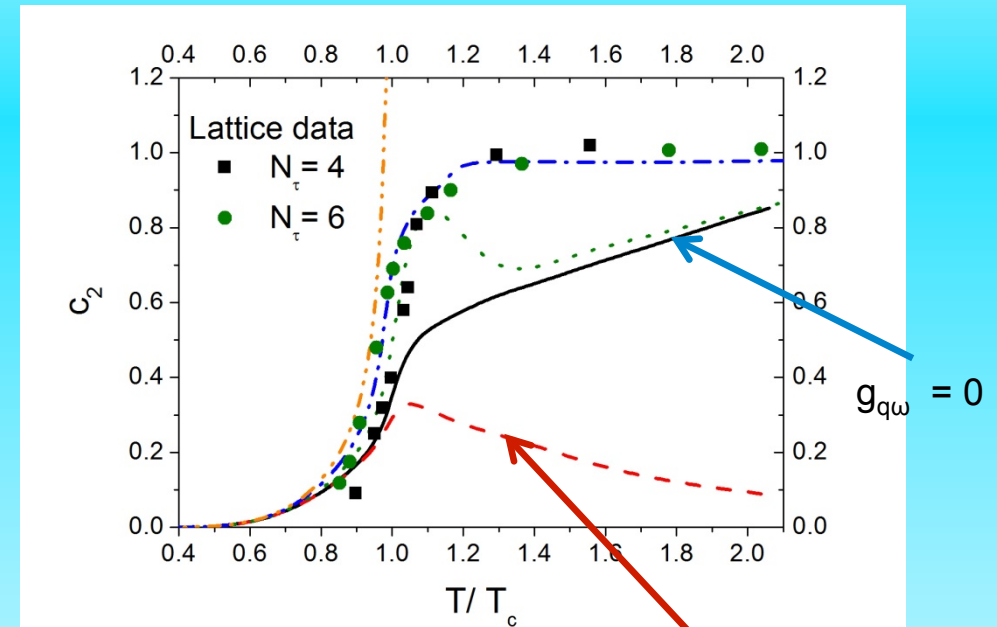
$$P(T, \mu) = P(T) + c_2(T) \mu^2 T^2 + \dots$$

small quark vector repulsion !!

PNJL

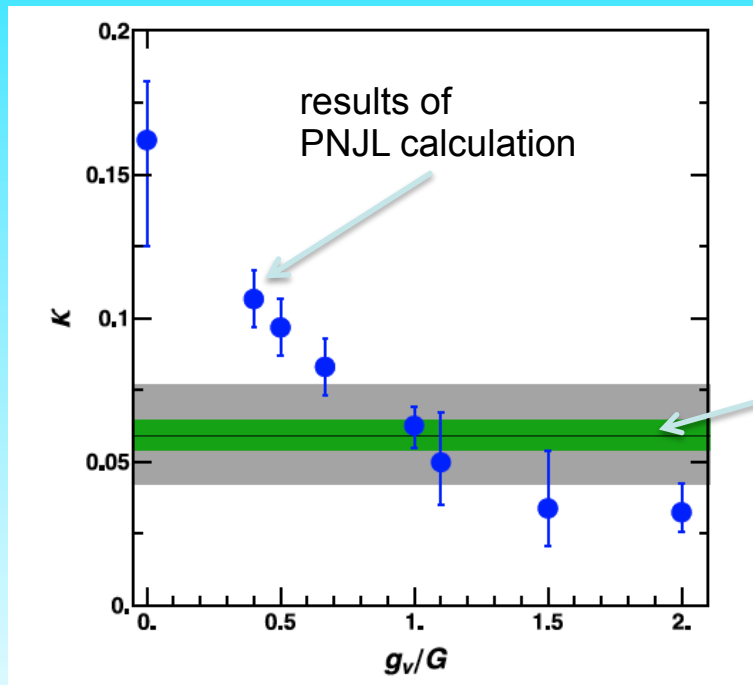


QH



signs of vector repulsion in $T_c(\mu)$ behavior

curvature of transition line $\kappa = -T_c \left. \frac{dT_c(\mu)}{d\mu^2} \right|_{\mu=0}$

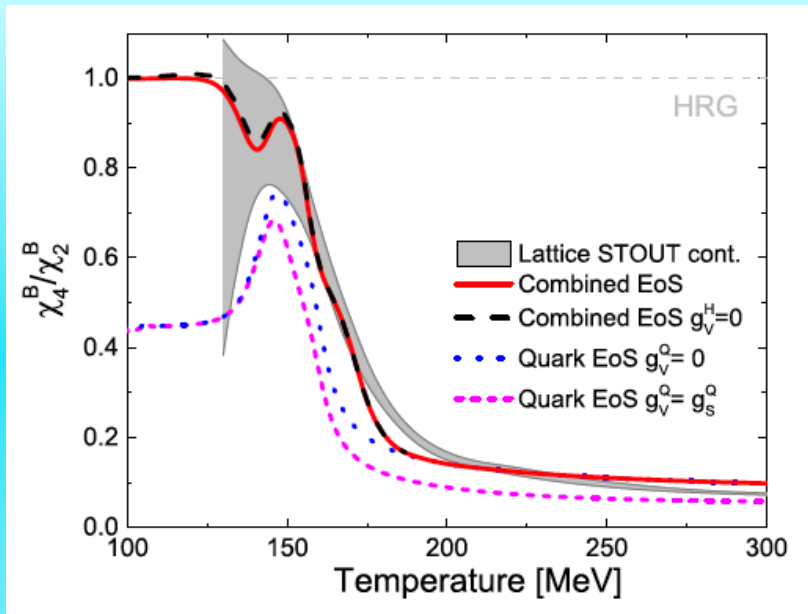


Plot taken from
Bratovic, Hatusda, Weise, PLB 719, 131 (2013)

Lattice results Kacmarek et al
PRD 83, 014504 (2011)

large quark vector repulsion?

T_c rather difficult to determine



scalar field as function of μ
for fixed $T = 140$ MeV

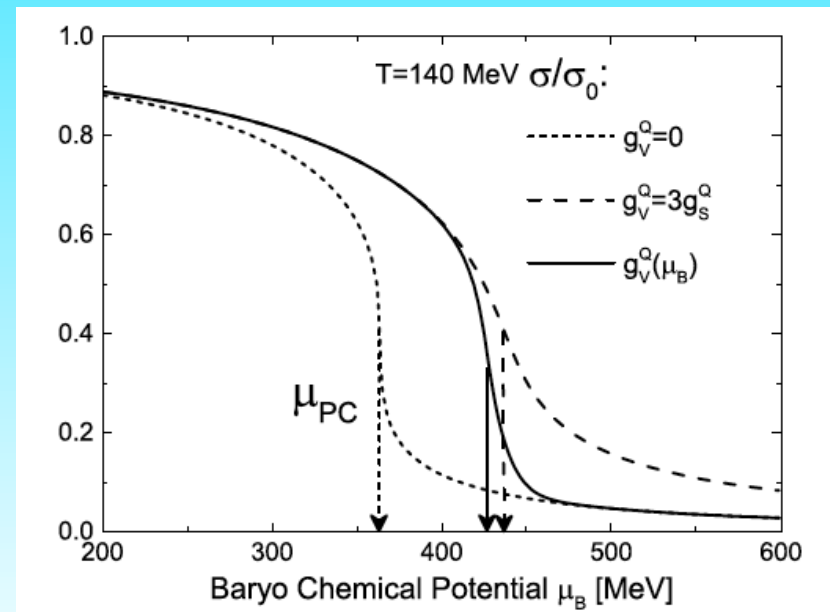
Conclusion:

quark interaction should be small
in the hadron sector either heavy baryons
and/or repulsion (liquid-gas, nuclei)

ratio of susceptibilities

turn on/off repulsion
or quarks and baryons

$$g_V^Q(\mu_B) = g_V^Q(\mu_B = 0) \cdot (1 + \exp(\mu_B - \mu_B^{PC})/\delta_\mu)^{-1}$$



only quarks here

Condensation of charged higher spin particles?

Heavy-ion collisions can generate very large B fields

W boson condensation at LHC? *Ambjørn, Olesen, PLB257, 201 (1991)*

however, see SWS, Müller, A. Schramm, PLB 277, 512 (1992)

ρ mesons? Simple estimate requires $B \sim 10^{20}$ G

SWS, Müller, A. Schramm MPLA 7, 9773 (1992)

heavy-ion collisions – bind away the whole mass of the particle

Chernodub, Phys. Rev. Lett. 106, 142003

Hidaka, Yamamoto PRD87, 094502

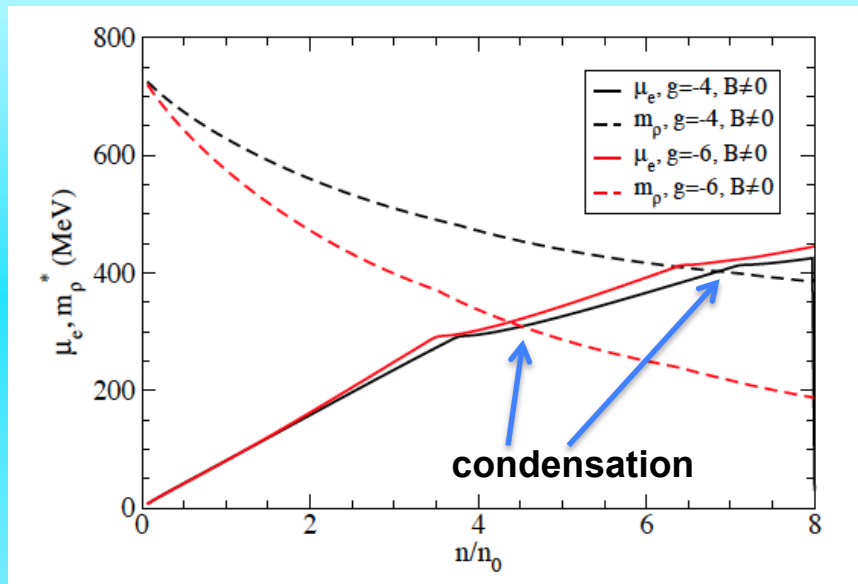
Advantage: high spin – strong interaction with magnetic field

Landau levels of the rho meson

$$E_{n,S_z}^2 = p^2 + m^2 + (2n - 2S_z + 1)eB$$

$$m_{\rho^-}^{2*} = m_{\rho^-}^2 - eB.$$

charge chemical potential and effective rho mass as function of density



Use standard hadronic model

GM3 parameterization

B value: 7×10^{18} G

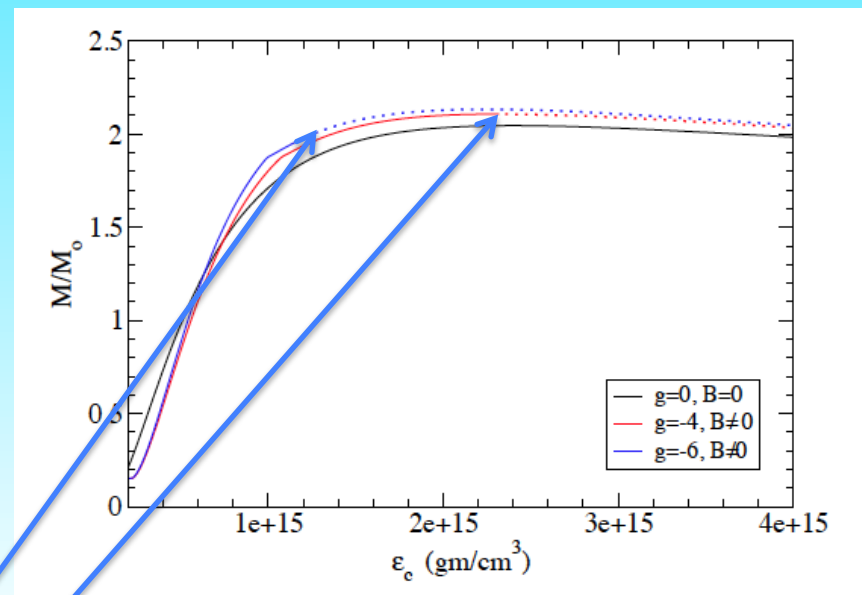
slight change of star masses
faster cooling

density dependence of rho mass ?

simple estimate $m_\rho^* = m_\rho - g \sigma$

readjust coupling to correct
asymmetry energy (32.5 MeV)

range of g limited by L, ...



onset of condensation

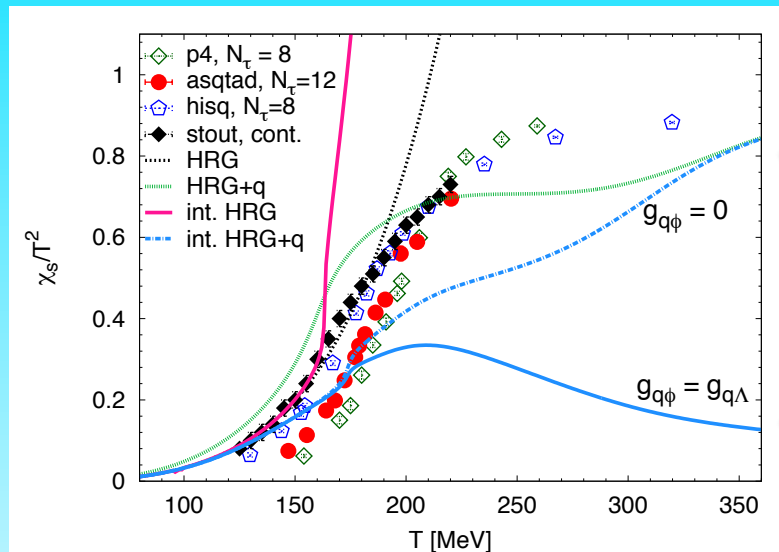
Conclusions, Outlook

- coherent phenomenological model including correct asymptotic degrees of freedom
- useful input for heavy-ion simulation (extracting interesting signals of PT hard work)
- heavy compact stars / hyper stars - little strangeness
- hybrid stars: stiff equation of state for quarks
- what about lattice susceptibilities?
- possibility of rho meson condensates in compact stars
- *comprehensive equation of state for a wide range of densities/temperatures (supernovae, mergers)*
- *couple hydro and kinetic equations for fields*

Many thanks to the organizers!

analogous behaviour of strange susceptibility

$$\chi_s = d^2(P) / (d\mu_s)^2 |_{\mu_B, \mu_S = 0}$$



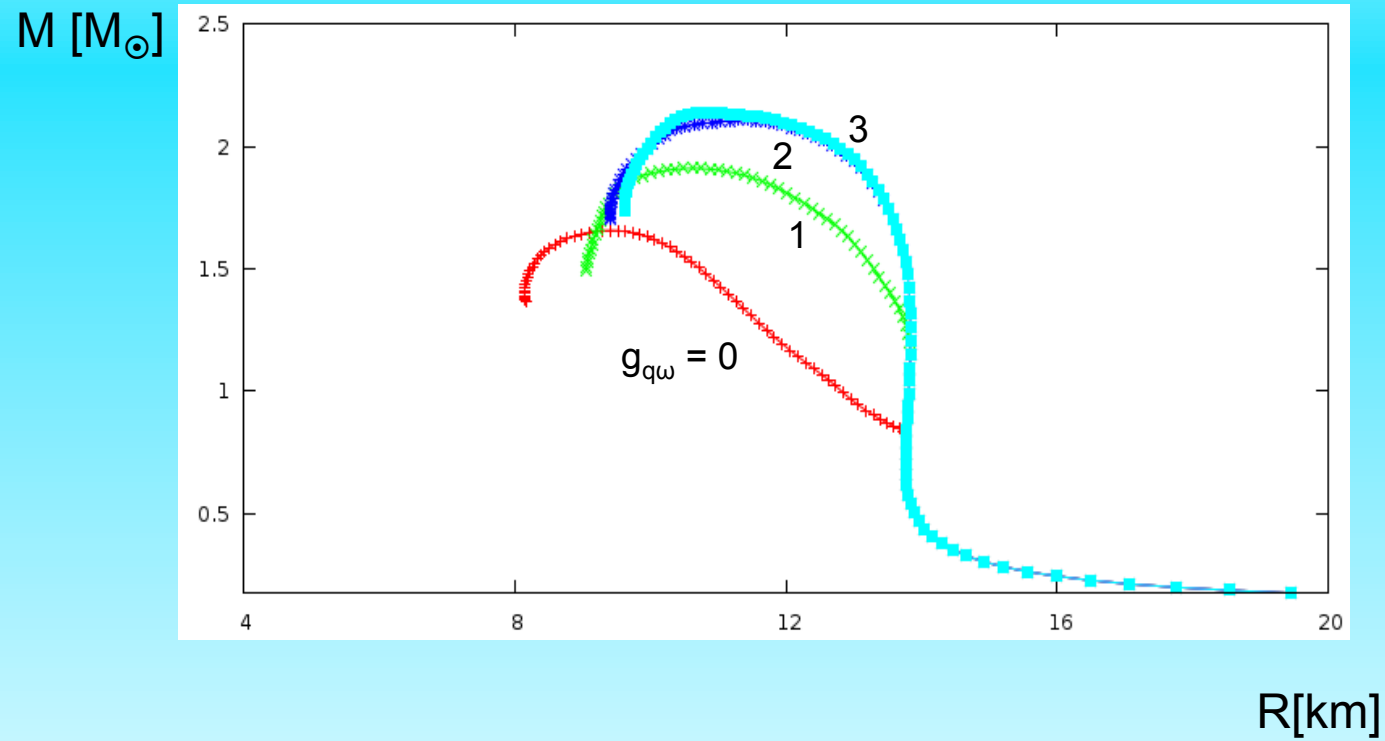
off

strange quark repulsion

on

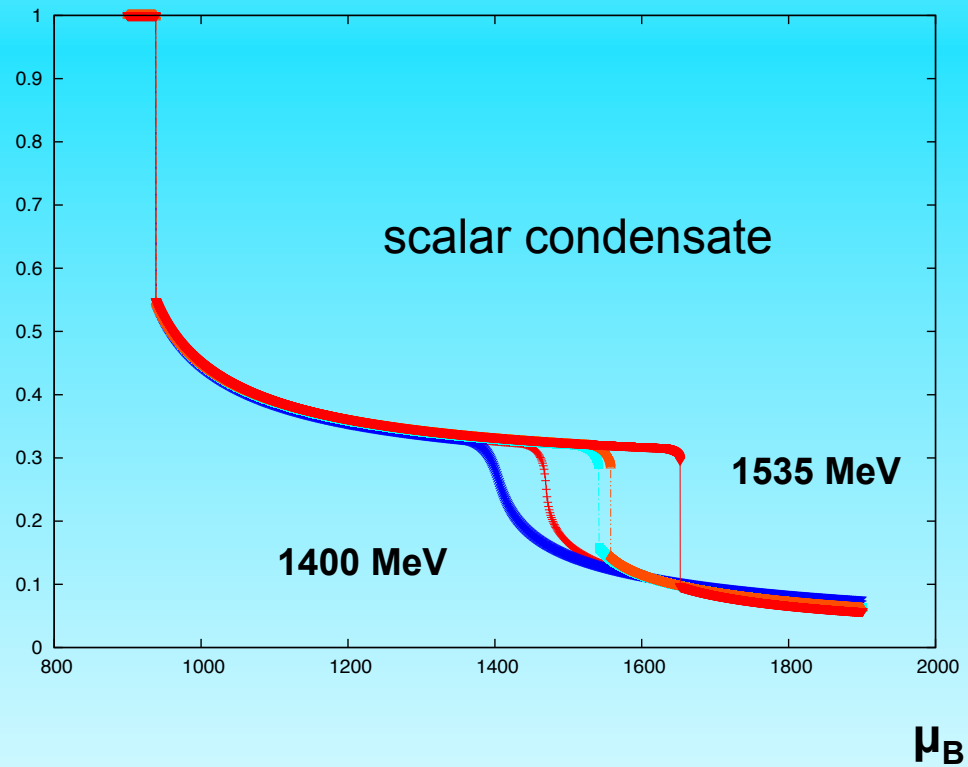
calc. by Philip Rau

Including vector interaction for quarks



increase M / R , potential problems at $\mu = 0$

scalar condensate for different masses m_{N^*}



First order transition for masses ≥ 1470 MeV, below crossover

Extension of the parity model to SU(3)

Baryon SU(3) multiplet + parity doublets

Similar approach, SU(3)-invariant potential for scalar fields

single particle energies
$$E_{\pm} = \sqrt{(g_1\sigma + g_2\zeta)^2 + m_0^2} \pm (g'_1\sigma + g'_2\zeta)$$

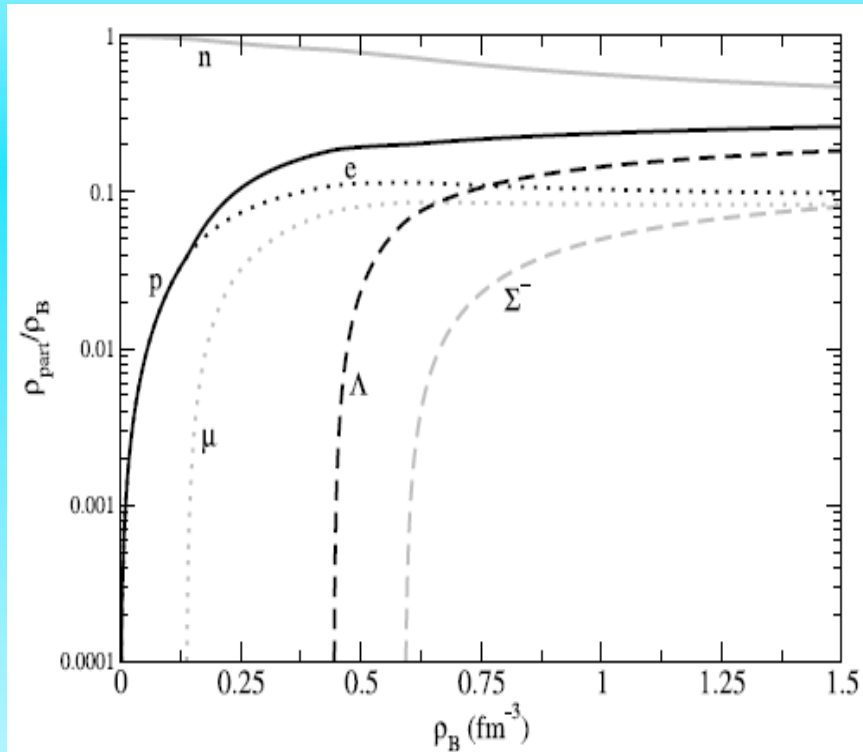
simplify investigation – same mass shift for whole octet

Candidates – $\Lambda(1670)$, $\Sigma(1750)$, Ξ (?) overall unclear

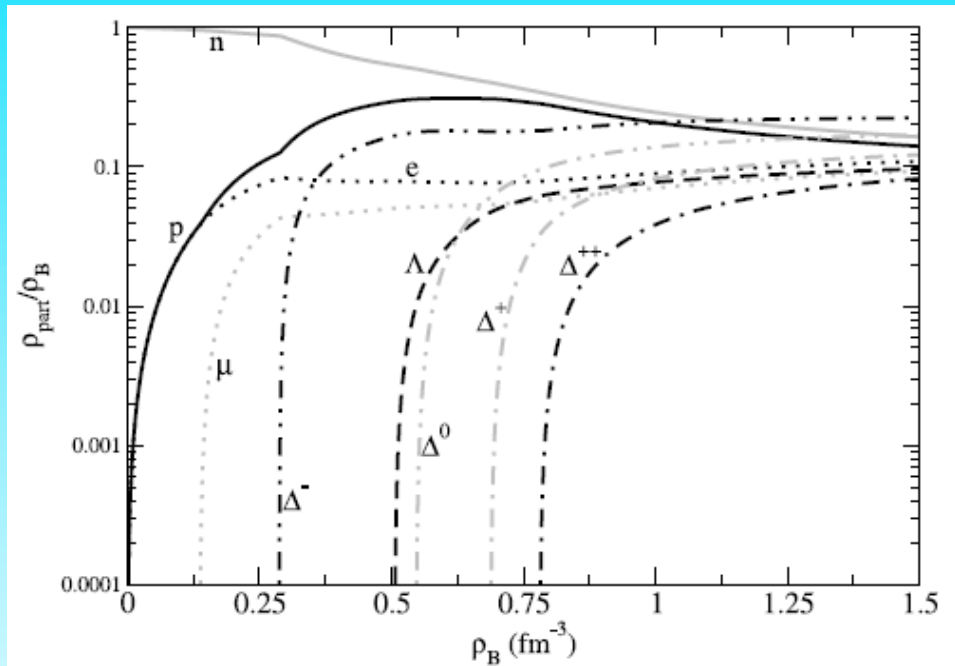
Steinheimer, SWS, Stöcker, JPhysG 38, 035001 (2011)

first study - Nemoto et al. PRD 57, 4124 (1998)

particle densities inside of the star

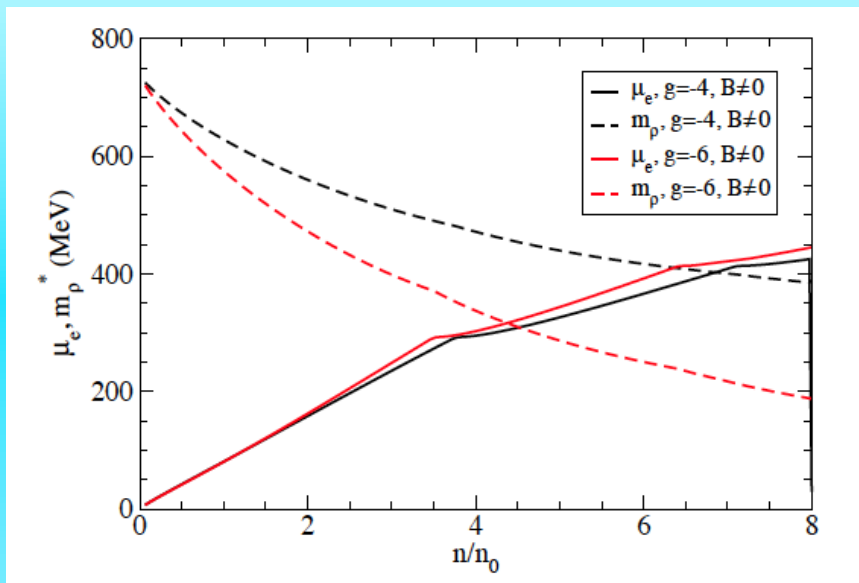


particle abundancies – no decuplet



particle numbers as function of density
uncertainties from $g_{3/2}$ coupling

charge chemical potential and effective rho mass as function of density



Use standard hadronic model

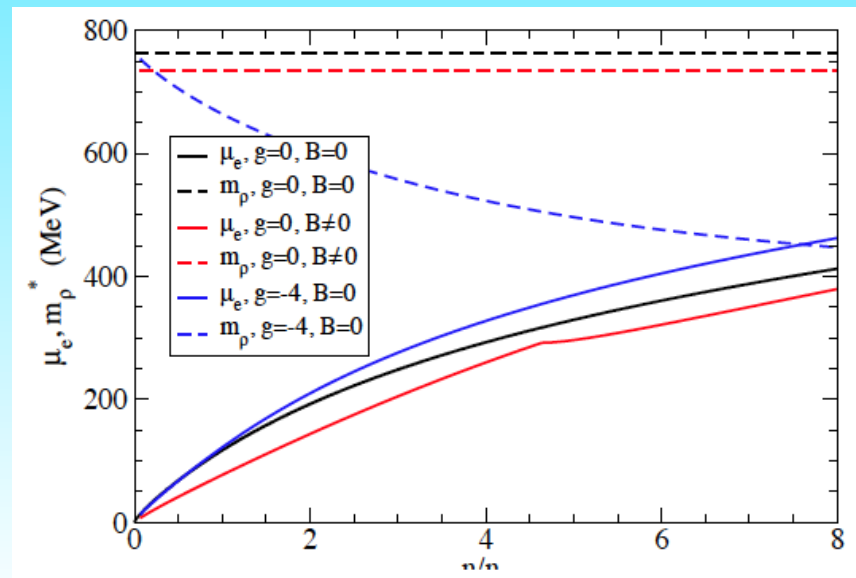
GM3 parameterization

B value: 7×10^{18} G

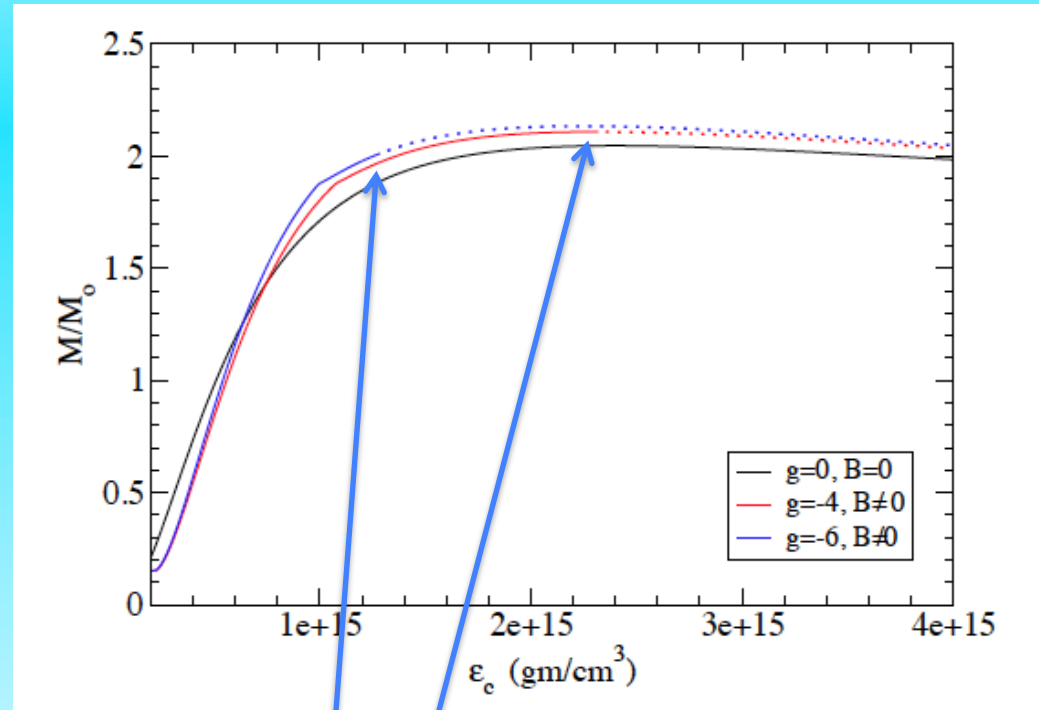
density dependence of rho mass ?

simple estimate $m_\rho^* = m_\rho - g \sigma$

readjust coupling to correct asymmetry energy (32.5 MeV)



Neutron star masses as function of central energy density



onset of condensation

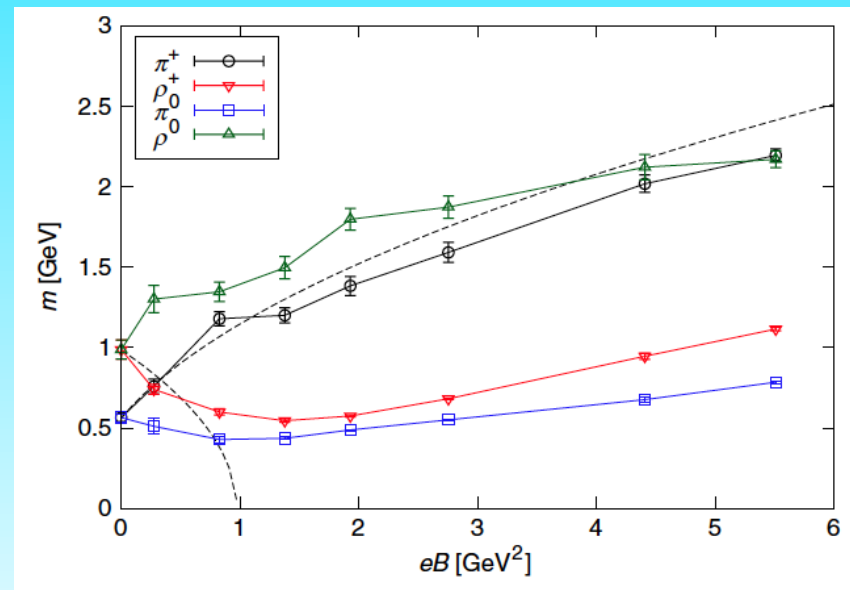
slight change of star masses, faster cooling

Different in compact stars

Advantages - long time scales (beta equilibrium)
large size
potentially large magnetic fields

quenched lattice calculations from

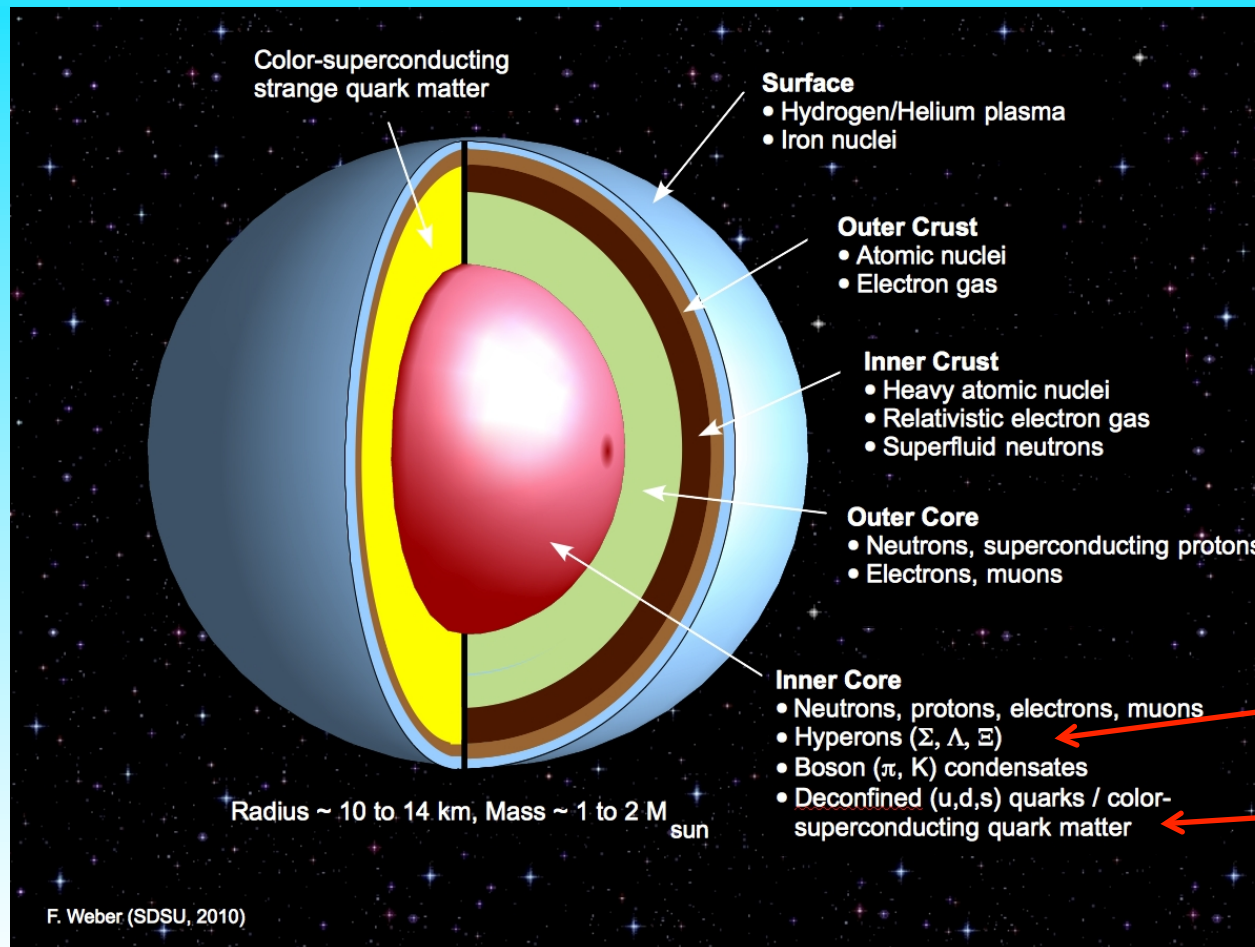
Hidaka, Yamamoto PRD87, 094502



neutron stars are remnants of Type II supernovae

1 to 2 solar masses, radii around 10 - 15 km
maximum central densities 4 to 10 ρ_0

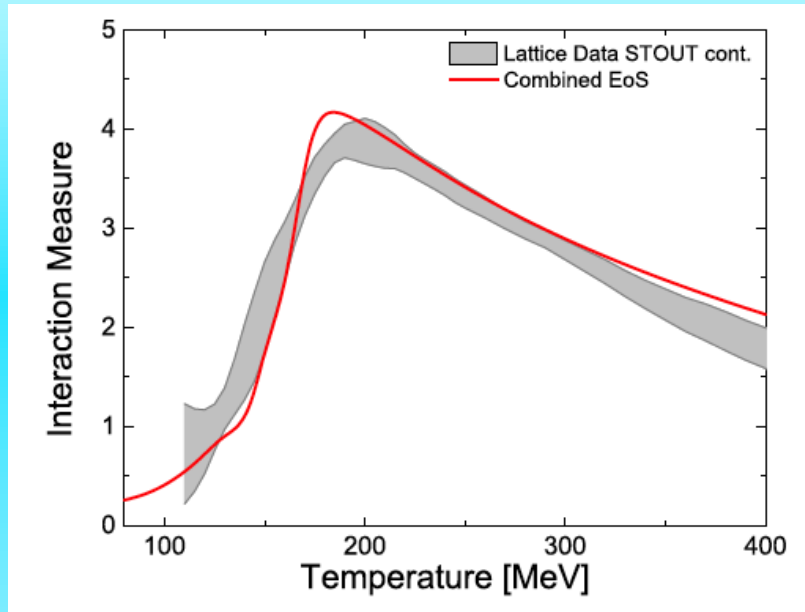
about 2000 known neutron stars



hyper star

hybrid star

after fine-tuning parameters



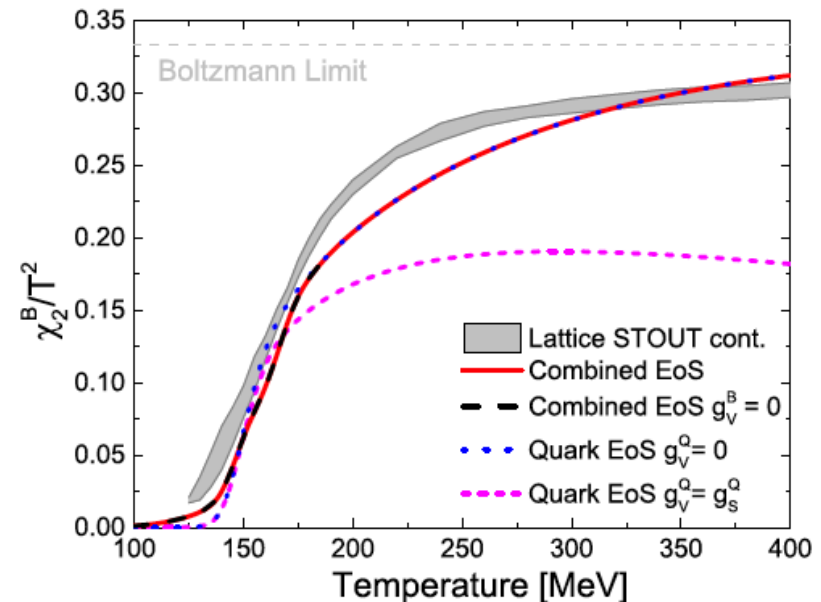
“interaction measure” $(e - 3p)/T^4$

Lattice (STOUT) and model

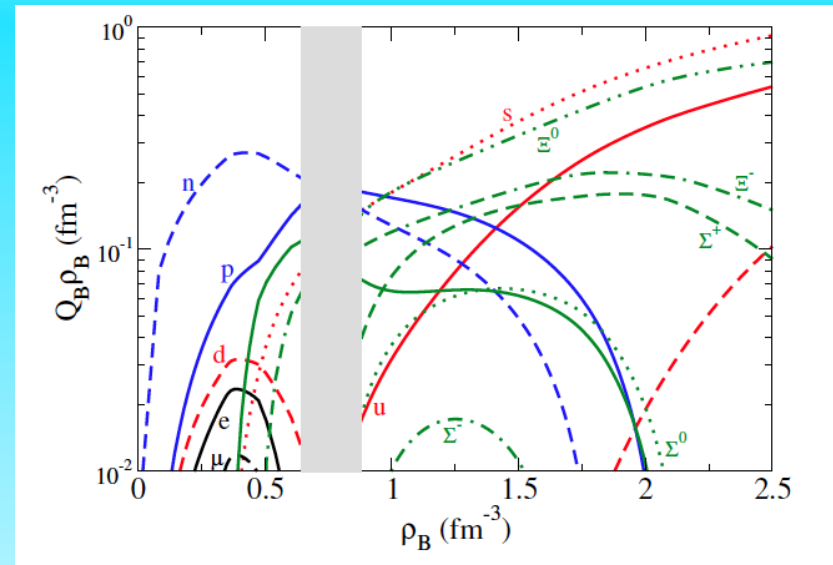
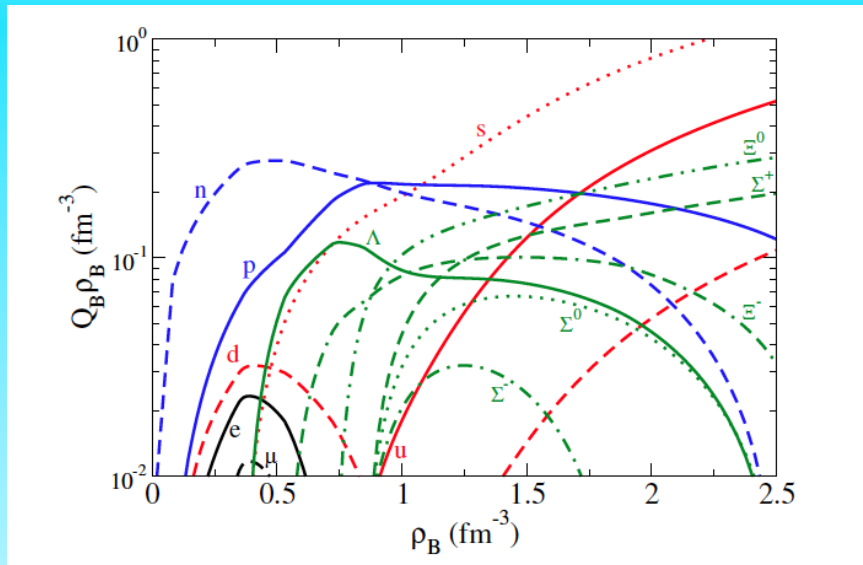
χ_2 for different vector couplings

susceptibilities χ_n, c_n

$$\frac{\chi_n^B}{T^2} = n!c_n^B(T) = \frac{\partial^n(p(T, \mu_B)/T^4)}{\partial(\mu_B/T)^n}$$

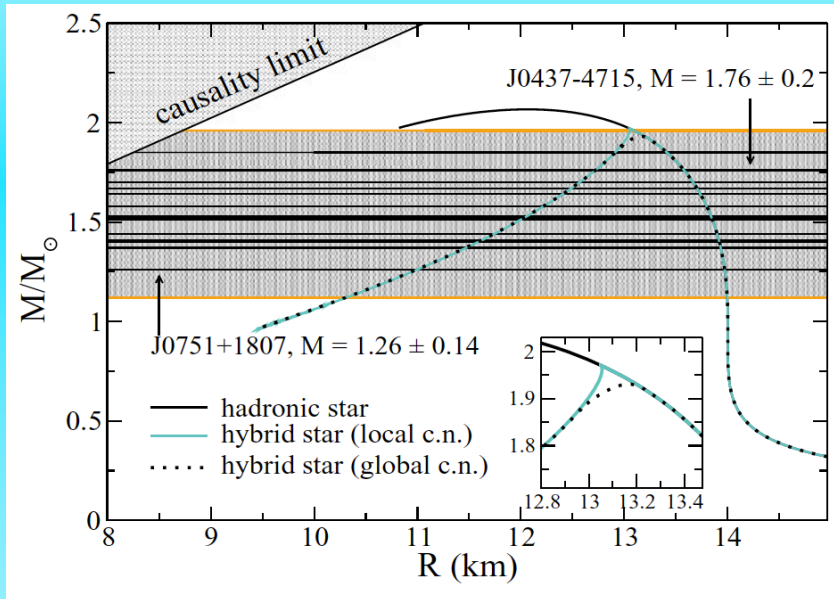


particle “cocktail” as function of density



with first-order transition

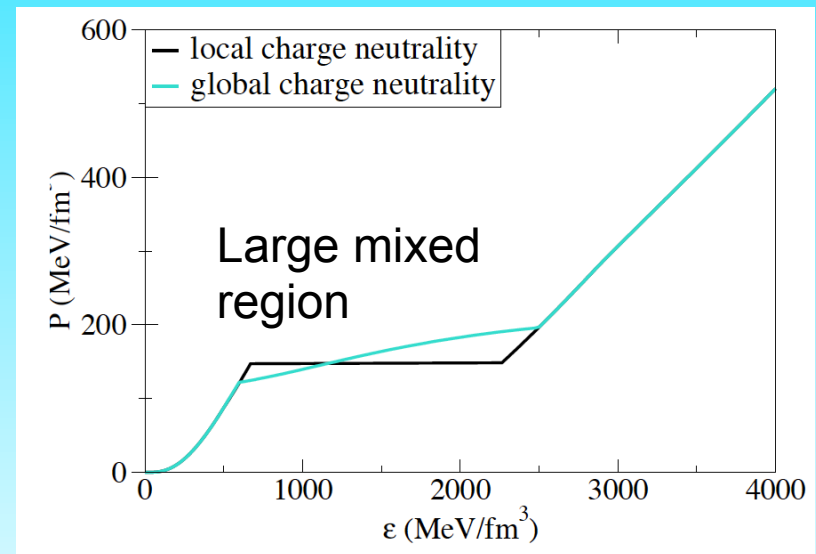
Hybrid Stars



Maxwell / Gibbs construction for
local / global charge neutrality

M-R diagram in QH model

baryonic star with a 2km core of quarks

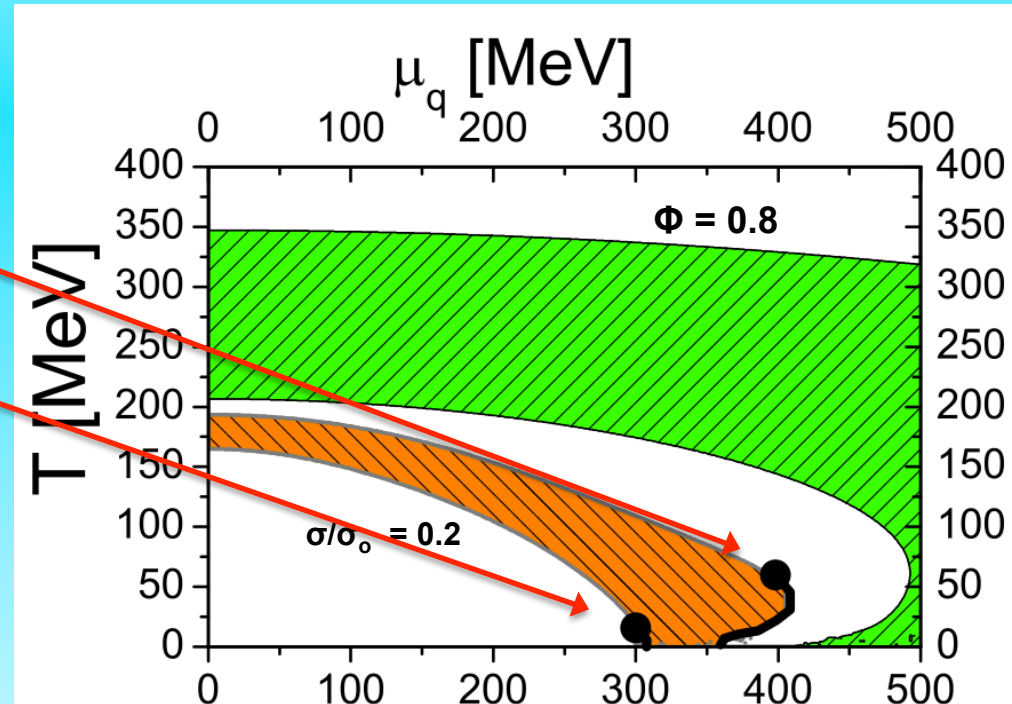


Excited quark-hadron matter in the parity-doublet approach

Chiral transition

Liquid-gas phase transition

2 different values for T_0



single particle energies

$$E_{\pm} = \sqrt{(g_1\sigma + g_2\zeta)^2 + m_0^2} \pm (g'_1\sigma + g'_2\zeta)$$