Chiral Symmetry Breaking & Quantum Hall effect in monolayer graphene

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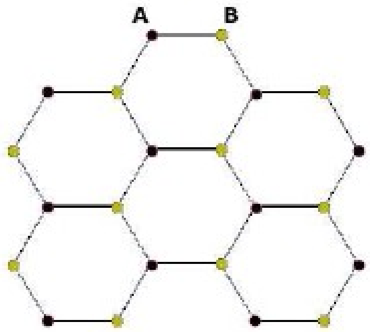
Graphene (half-filled or intrinsic): Dirac liquid

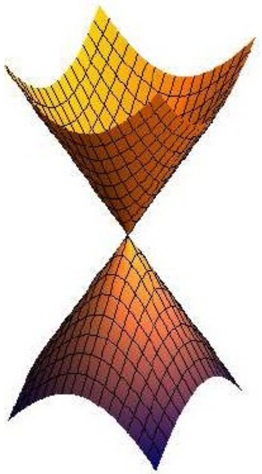


Two triangular sublattices: A and B; one electron per site (half filling)

Tight-binding model (t = 2.5 eV):

$$H_0 = -t \sum_{\vec{A},i,\sigma=\pm 1} u^\dagger_\sigma(\vec{A}) v_\sigma(\vec{A} + \vec{b}_i) + H.c.$$

(Wallace, PR , 1947)



Energy spectrum:

$$E(\vec{k}) = \pm t \left| \sum_i \exp[\vec{k} \cdot \vec{b}_i] \right|$$

The sum is complex → two equations for two variables for zero energy → Dirac points (no Fermi surface)

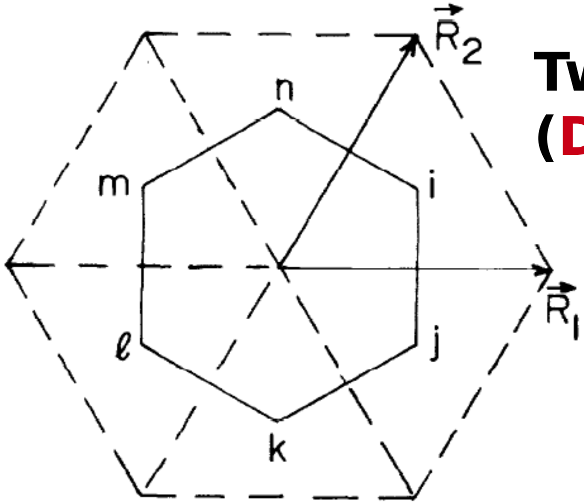
Crucial: *lack of inversion symmetry* in lattice (unlike square lattice)

Remote hoppings are too weak: spectrum is *almost* particle-hole symmetric

Dirac points can be accessed: using *GATE*

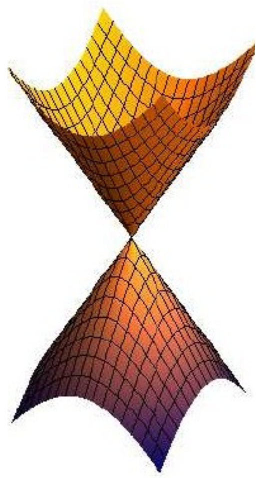
Low energy theory: emergent Dirac fermions

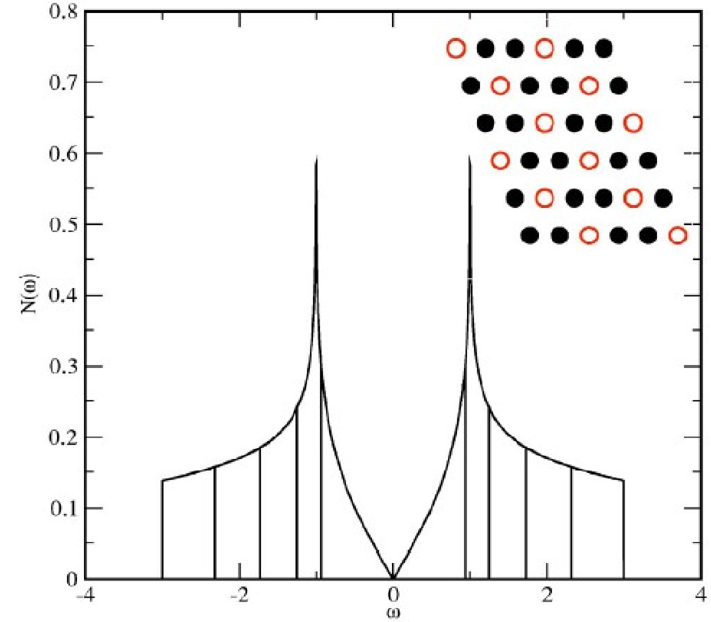
Brillouin zone:



Two inequivalent (Dirac) points at

+K and **-K**





$$\Psi^\dagger_\sigma(\vec{x},\tau) = T\sum_{\omega_n}\int^{\Lambda}\frac{d\vec{q}}{(2\pi a)^2}\,e^{i\omega_n\tau + i\vec{q}\cdot\vec{x}}\left(u^\dagger_\sigma(\vec{K}+\vec{q},\omega_n),\, v^\dagger_\sigma(\vec{K}+\vec{q},\omega_n),\, u^\dagger_\sigma(-\vec{K}+\vec{q},\omega_n),\, v^\dagger_\sigma(-\vec{K}+\vec{q},\omega_n)\right)$$

8- component Dirac fermion

"Low-energy" Hamiltonian: $H_0 = i\gamma_0\gamma_j\left(-i\partial_j - A_j\right)$ $\quad j = 1, 2$

γ **-matrix algebra:** $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ $\quad \mu, \nu = 0, 1, 2, 3, 5$

Fermi velocity v = c/300 = 1, in our units (unless mentioned)

Emergent chiral symmetry: $SU(2) : \{i\gamma_3\gamma_5, \gamma_3, \gamma_5\}$

$i\gamma_3\gamma_5 = \tau_3 \otimes s_0$: generator of translation

PRB, **79**, 085116 (2009)

Experimental detection of Dirac fermions

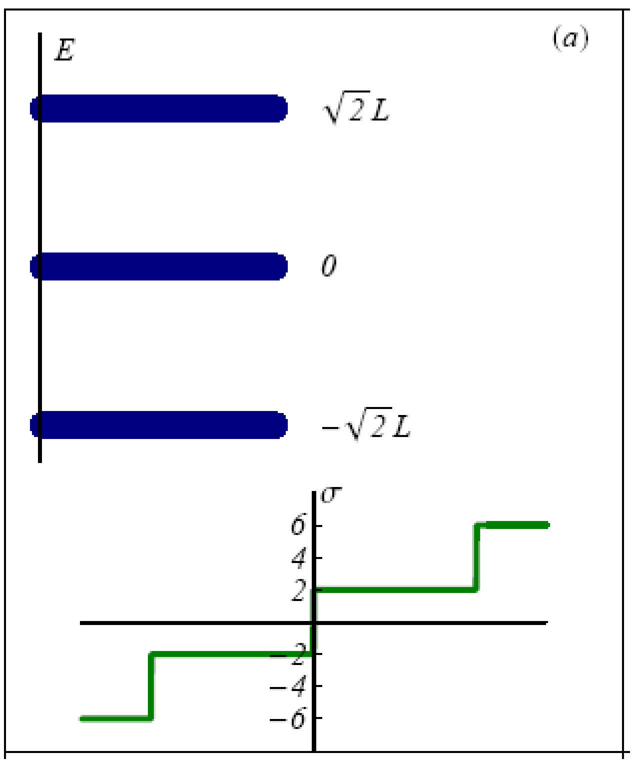
Quantum Hall effect: Dirac fermions + magnetic field

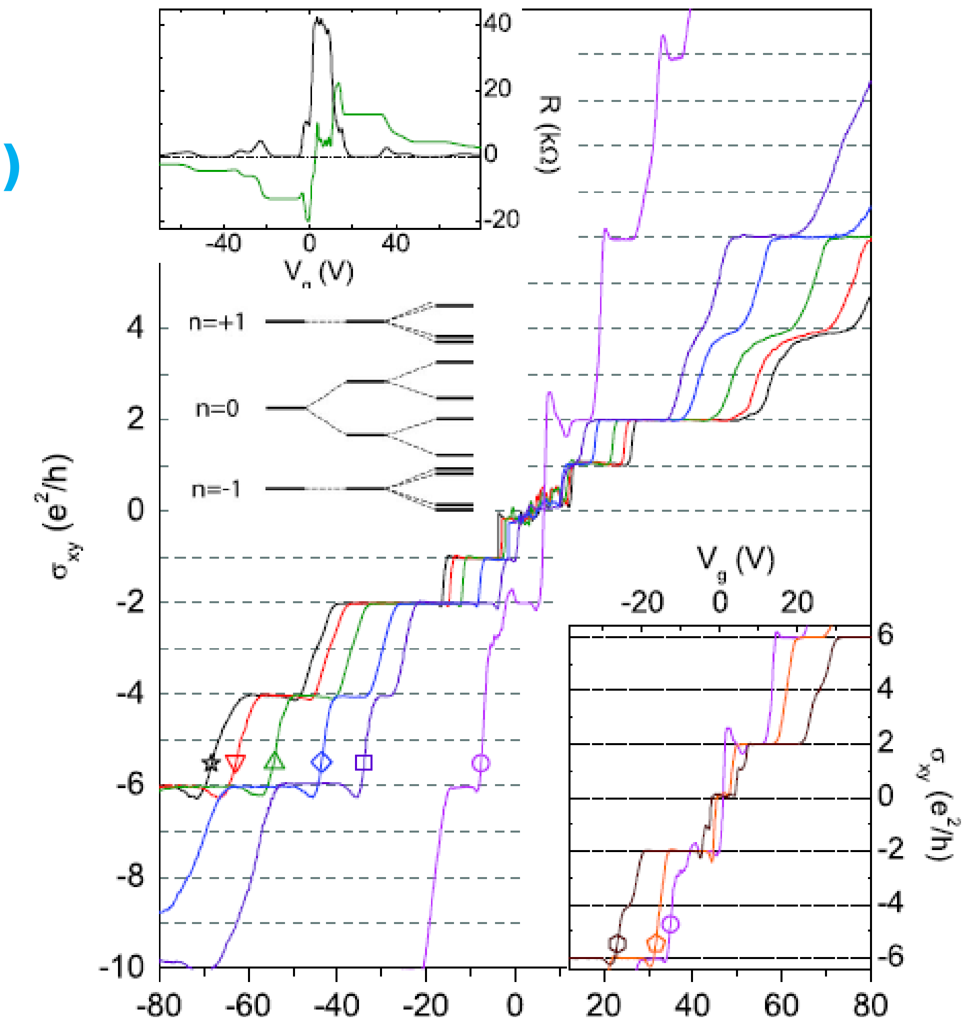
→ Landau levels @ $E_n = \pm\sqrt{\hbar v_F^2 |2nB|/c}$

Each LL: 2 (spin) x 2 (valley) x eB/hc degenerate $n = 0, 1, 2, \cdots$

→ quantization in steps of four !

(Gusynin and Sharapov, PRL, 2005)





(Y. Zhang, et. al. PRL, 2006)

Strong interaction: symmetry breaking

Mass generation for Dirac fermions (quasi-particle spectrum: gapped)

Condensed Matter analog of ***Higg's mechanism***

chiral(sublattice)-symmetry breaking: *charge-density-wave* $(j=0)$, *anti-ferromagnet* $(j=1,2,3)$ etc.

$$H_{SP}^{CSB}=\sigma_0\otimes H_0+m_j\left(\sigma_j\otimes\gamma_0\right)$$

$$\gamma_0=\tau_0(valley)\otimes s_3(sublattice) \qquad \{\gamma_0,\gamma_3\}=0=\{\gamma_0,\gamma_5\}$$

Time reversal symmetry breaking: *quantum anomalous* $(j=0)$ *or spin* $(j=1,2,3)$ *Hall insulator* (but chiral scalar)

$$H_{SP}^{TRSB}=\sigma_0\otimes H_0+\tilde{m}_j\left(\sigma_j\otimes i\gamma_1\gamma_2\right)$$

$$i\gamma_1\gamma_2=\tau_3\otimes s_3 \qquad [i\gamma_1\gamma_2,\gamma_3]=[i\gamma_1\gamma_2,\gamma_5]=[i\gamma_1\gamma_2,i\gamma_3\gamma_5]$$

In either case the spectrum becomes gapped

$$\varepsilon_\pm(\boldsymbol{p})=\pm\sqrt{|\boldsymbol{p}|^2+|\Delta_0|^2} \qquad \Delta_0=m_j,\tilde{m}_j$$

All the phases @ extremely strong coupling (unphysical)

due to vanishing density of states

All the masses break sublattice inversion symmetry: $\gamma_2=\tau_0\otimes s_1$

Microscopic Origin: short-range interactions

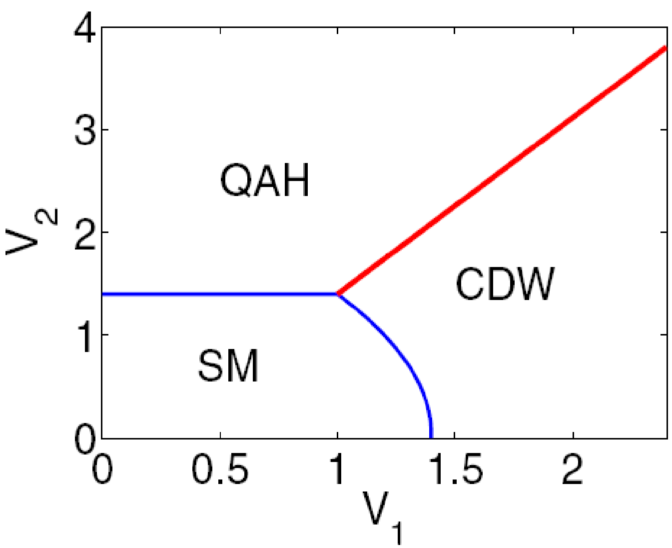
- Onsite Hubbard repulsion *(U)*: *anti-ferromagnet (AFM)*

  I. Herbut, PRL, 2006; F. F. Assaad, I. H., PRX, 2013

- Nearest-Neighbor repulsion (V_1): *charge-density-wave (CDW)*

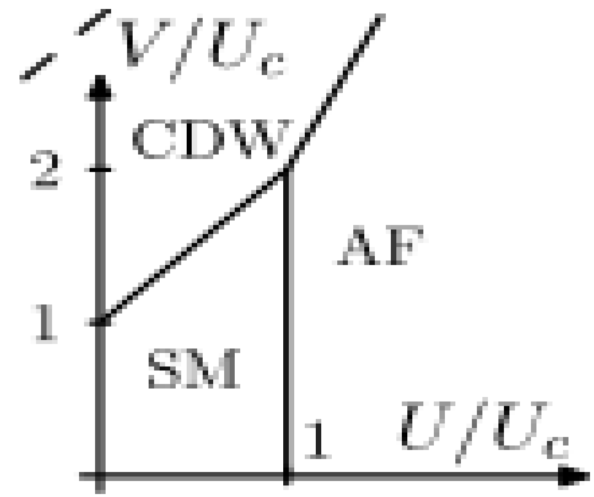
  I.H., PRL, 2006; Weeks & Franz, PRB 2010; Grushin et. al., PRL 2011

- Next-nearest-neighbor (V_2): *quantum anomalous/spin Hall insulators* **Raghu et al, PRL, 2008; B.R., I.H., PRB, 2013**



In graphene: $U > V_1 > V_2$

PRL, **106**, 236805 (2011)



Raghu et al, PRL, 2008 & I. Herbut, PRL, 2006

Chiral-symmetry-breaking masses: dominant instabilities in graphene

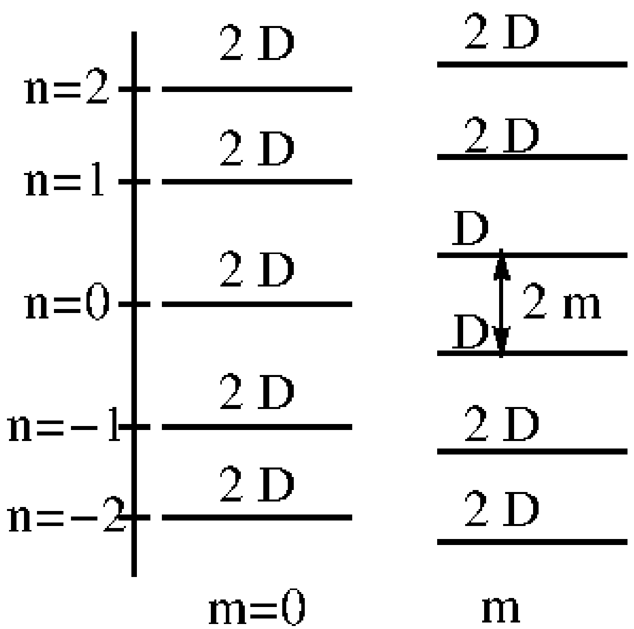
Magnetic catalysis

- Triggers the formation of CSB orders @ *infinitesimal* interaction
- Zeroth LL is simultaneously sublattice and valley polarized:

  Each spin projection: ZLL near $+(-)\vec{K}$ valley lives on $A(B)$ sublattice

- CDW/AFM $\rightarrow$ gap @ Dirac point for weak enough V_1/U

- Take the simplest example of spinless fermions:



CDW splits ZLL and pushes down all filled LLs

(ZLL on one sublattice is filled)

$\Rightarrow$ CSB orders: energetically advantegeous

@ weak coupling in magnetic fields

Magnetic catalysis

V. P. Gusynin et. al. PRL, 73, 3449 (1994)

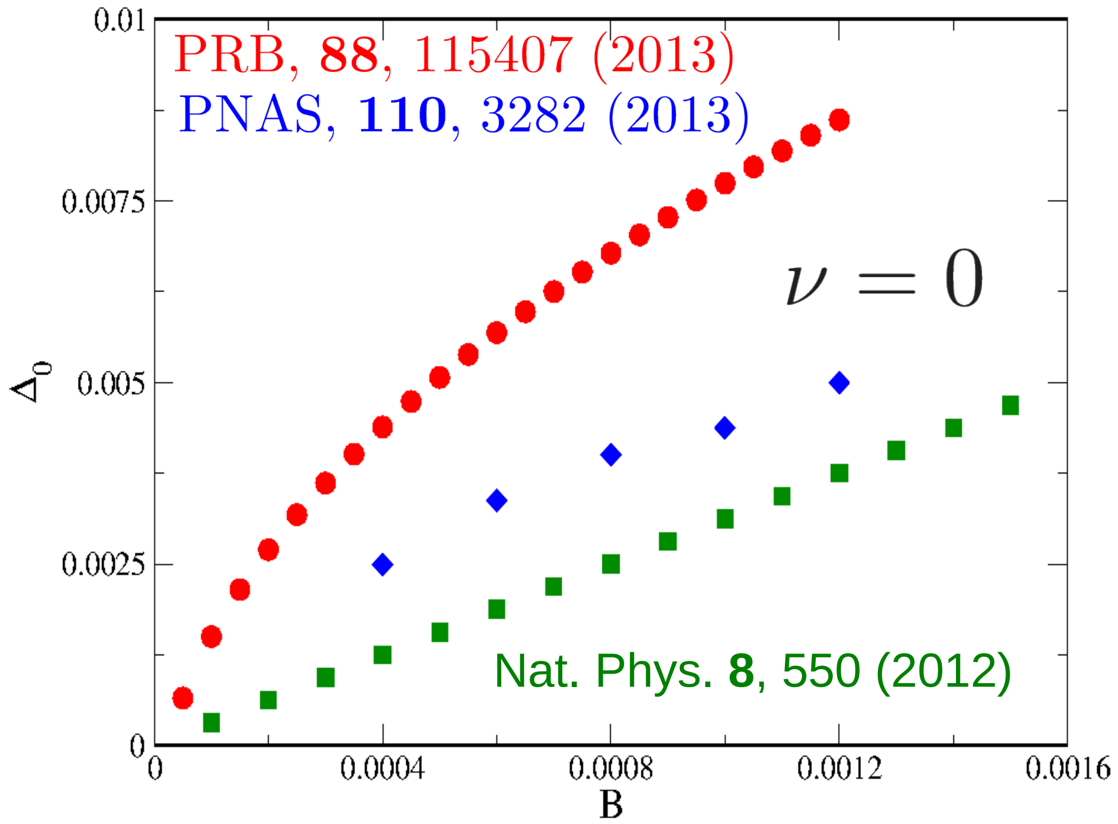
gives rise to Hall state @ $\nu = 0$

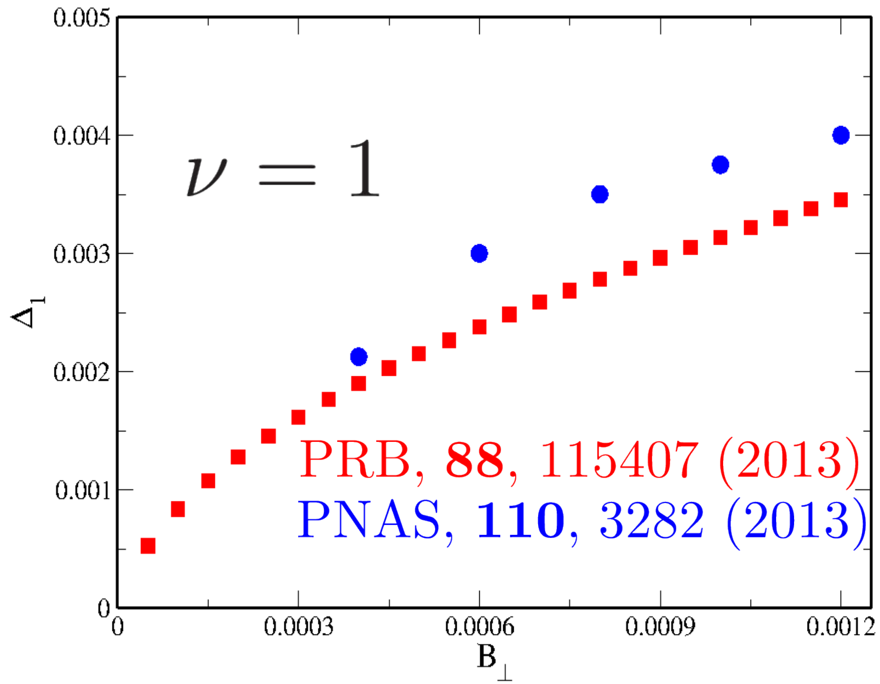
- With spin restored: onsite U drives AFM (dominant instability)

  ZLL on opposite sublattice with opposite spin projection: filled

  I. Herbut, PRB, 2007; Jung et al. PRB, 2009

Experimental status





gaps in units of $\Lambda v \sim 3$ eV in graphene & field (B) in $B_0 \sim \Lambda^2 \sim 10^4$ Tesla

- Features: @ $\nu = 0$: cross over from linear $\rightarrow$ sublinear $\rightarrow$ almost $\sqrt{B}$
  @ $\nu = 1$: roughly sublinear

Substrate (exp. method): vacuum or suspended graphene (compressibility),

boron nitride (capacitance), boron nitride(ressitivity)

Other theoretical models & limitations

- ZLL approximation: quantum Hall ferromagnet (Δ_{QHFM})

$$\sigma_0(spin) \otimes \tau_3(valley) \otimes s_0(sublattice)$$

Barlas et. al. Nano. Tech. 2012

- Splits not only ZLL, but also filled Landau levels : $-\sqrt{2nB} \pm \Delta_{QHFM}$ for $n = 0, 1, 2, \cdots$

  QHFM gains energy only by splitting ZLL

  Thus energetically inferior to CSB masses within ZLL

- Strong LL mixing by CSB order: ZLL approximation forbidden

  (Contrasting situation with non-relativistic systems : GaAs heterostructure)

- Long range Coulomb interaction: scales as $\sim \frac{\alpha}{l_B} \sim \sqrt{B}$ (always)

  $\alpha = \frac{e^2}{8\pi v_F \epsilon}$: fine structure constant (ϵ: dielectric constant)

  cannot explain the crossover of scaling for activation gap @ $\nu = 0$

- Appropriate short-range interactions: good agreement

Zeeman coupling & canted AFM

- Weak onsite-U $\rightarrow$ AFM instablity near Dirac point $\rightarrow$ $\nu = 0$ Hall state
- Zeeman coupling: restricts AFM to easy-plane ($\perp$ *to* $\vec{B} = B\hat{z}$)

& develops ferromagnet (FM) order along $\vec{B}$

- LL spectrum: $E_{n,\sigma} = \pm\sqrt{N_\perp^2 + [(N_3^2 + 2nB)^{1/2} + \sigma(m+\lambda)]^2}$

$\sigma = \pm$ (spin projections), $n = 0, 1, 2, \cdots$

energy of filled Dirac LL sea: maximally lowered $N_3 = 0, N_1, N_2, m \neq 0$

canted or easy-plane anti-ferromagnet

I. Herbut, PRB, 2007

- Free energy @ Dirac point or $\nu = 0$:

$$F_0 = \frac{N_\perp^2}{4g_a} + \frac{m^2}{4g_f} - D \sum_{\sigma=\pm} \left[\frac{1}{2} E_{0,\sigma} + \sum_{n\geq 1} E_{n,\sigma} \right]$$

$$N_\perp = \sqrt{N_1^2 + N_2^2}$$

$$N_3 = 0$$

microscopic origin: $g_a, g_f \sim U$ in magnetic fields: $g_a(l_B) \neq g_f(l_B)$

Gap equations & regularization

- gap @ $\nu = 0$: $\Delta_0 = \sqrt{N_\perp^2 + (\lambda + m)^2}$
- Minimizing F_0 w.r.t. $|N_\perp| \Rightarrow$ $$\frac{1}{g_a} = \frac{B}{\pi} \sum_{\sigma=\pm} \left[\frac{1}{2E_{0,\sigma}} + \sum_{n\geq 1}^{N_{max}} \frac{1}{E_{n,\sigma}} \right]$$

  displays UV divergence as $N_{max} \to \infty$ as $N_\perp$ pushes down filled LLs
- Min. F_0 w.r.t. $m \Rightarrow$ $$\frac{1}{g_f} = \frac{B}{\pi} \sum_{\sigma=\pm} \left[\frac{(m+\lambda)}{2E_{0,\sigma}} + \sum_{n\geq 1}^{N_{max}} \frac{(m+\lambda) + \sigma\sqrt{2nB}}{E_{n,\sigma}} \right]$$

  No UV divergence as FM spin splits all filled LLS
- Regularization: $\delta_a = \pi\left[(g_a \Lambda)^{-1} - (g_c^a \Lambda)^{-1}\right]$ $\qquad (g_c^a)^{-1} = \int_{\Lambda^{-1}}^{\infty} ds/s^{3/2}$

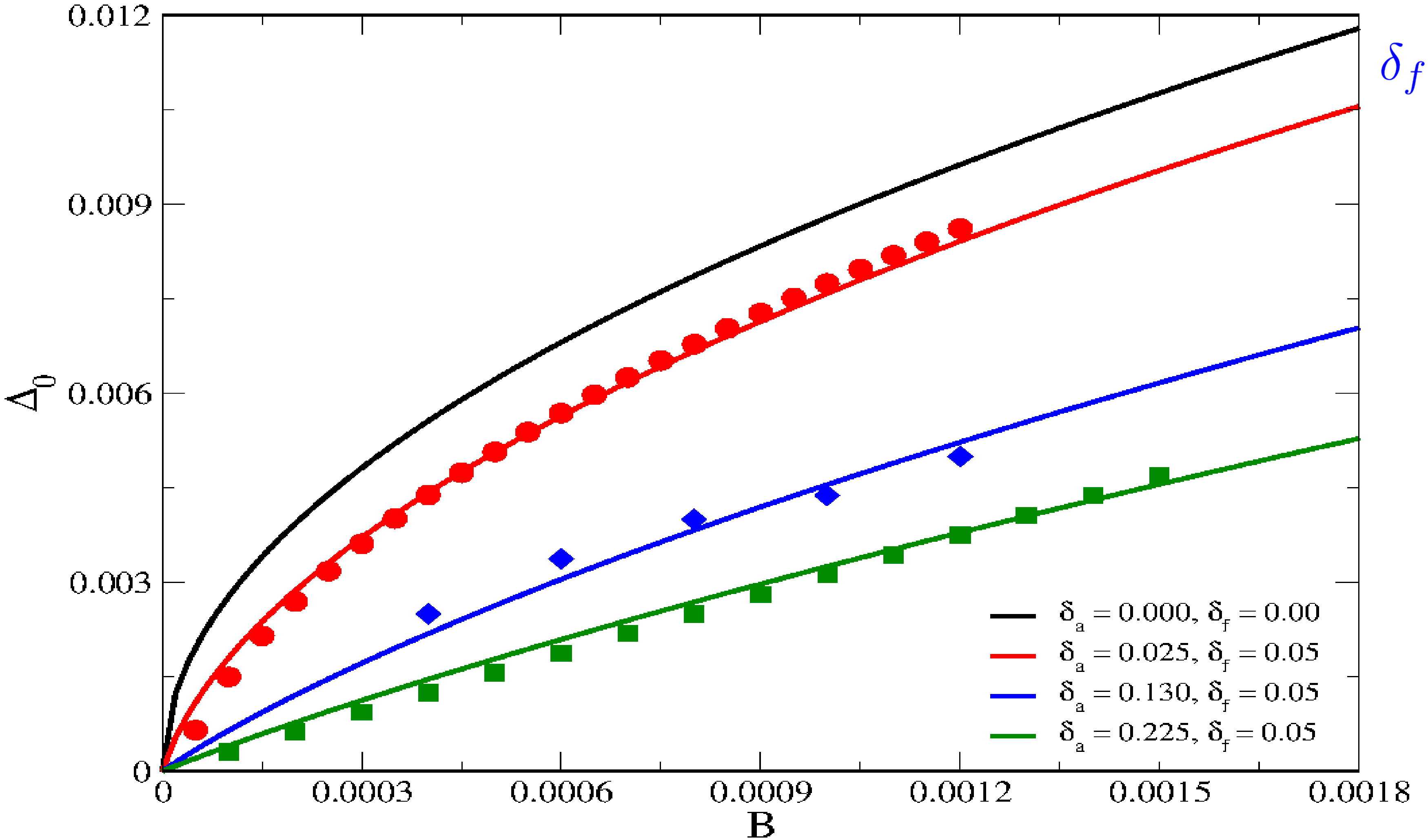
  I. F Herbut, BR, PRB, 2008

  measuring the distance from zero field AF quantum critical point (g_c^a)

  $\delta_a > 0$ subcritical interaction, $\delta_a < 0$ above critical interaction
- Physical observables: $N_\perp, m$ are cutoff (Λ) independent

  enough LLs (~ 100) within $v_F \Lambda$ to account for strong LL mixing

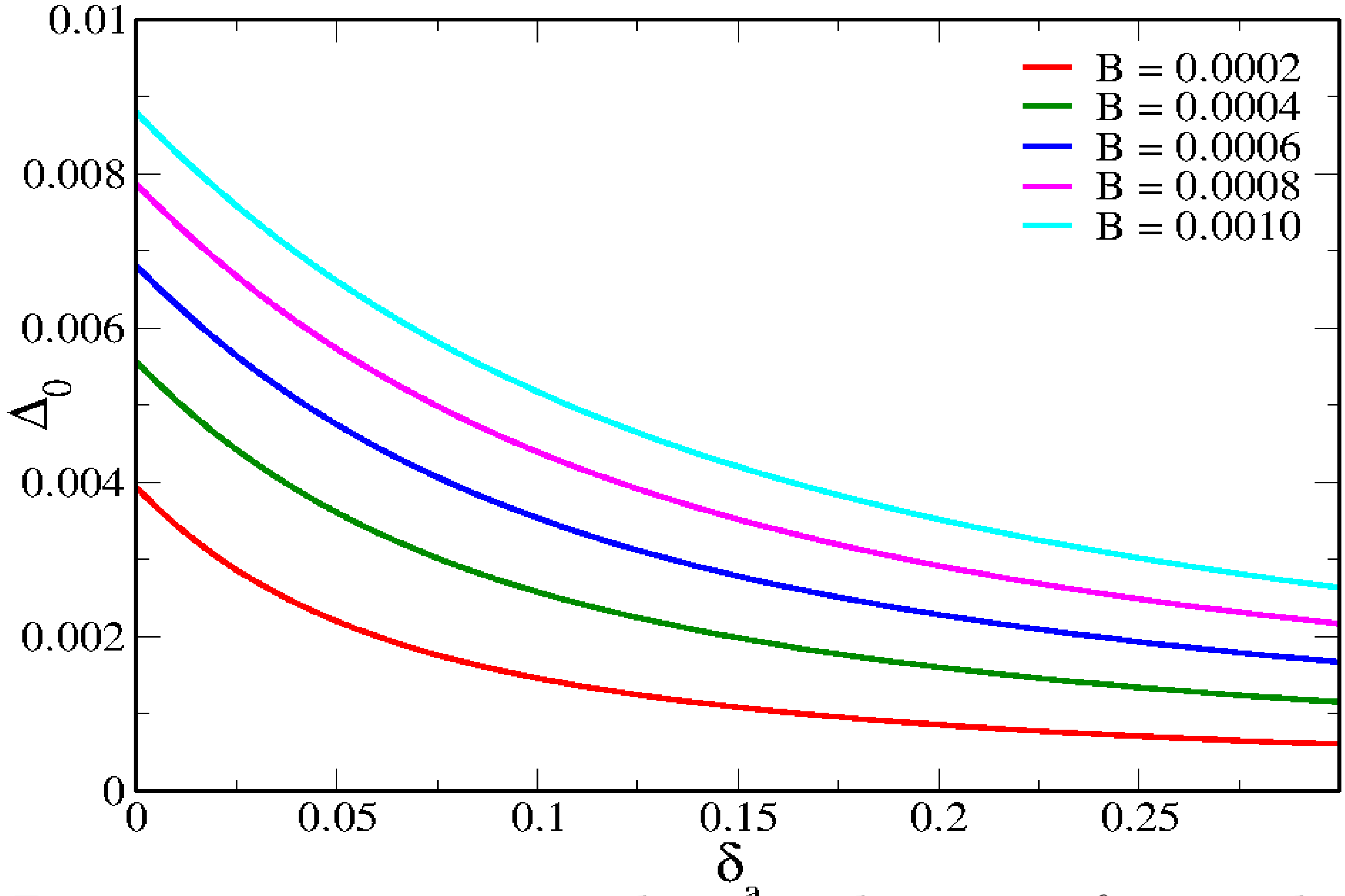
Results



$\delta_f = \pi/g_f$

- Interactions are subcritical ($\delta_a > 0$): No ordering when $B = 0$
- Interactions strength/gap size: large in suspended graphene (less screening)
- Smooth crossover: linear $\rightarrow$ sublinear $\rightarrow$ almost $\sqrt{B}$ as interaction increases
- Fits are insensitive to δ_f or m: FM set by Zeeman $\ll$ U; & $m/|N_\perp| \sim 10^{-2}$

Test of the theory

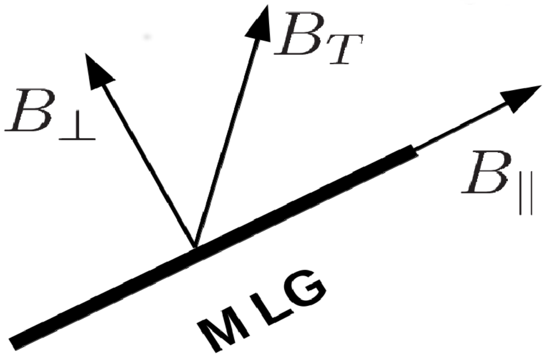


- Fixed B as interaction gets weaker gap decreases & more linear with B
- Extraction of activation gaps with different substrate
- Through gating with second graphene layer :

  closer the second MLG stronger the screening

  smaller gap and linear activation gap with B

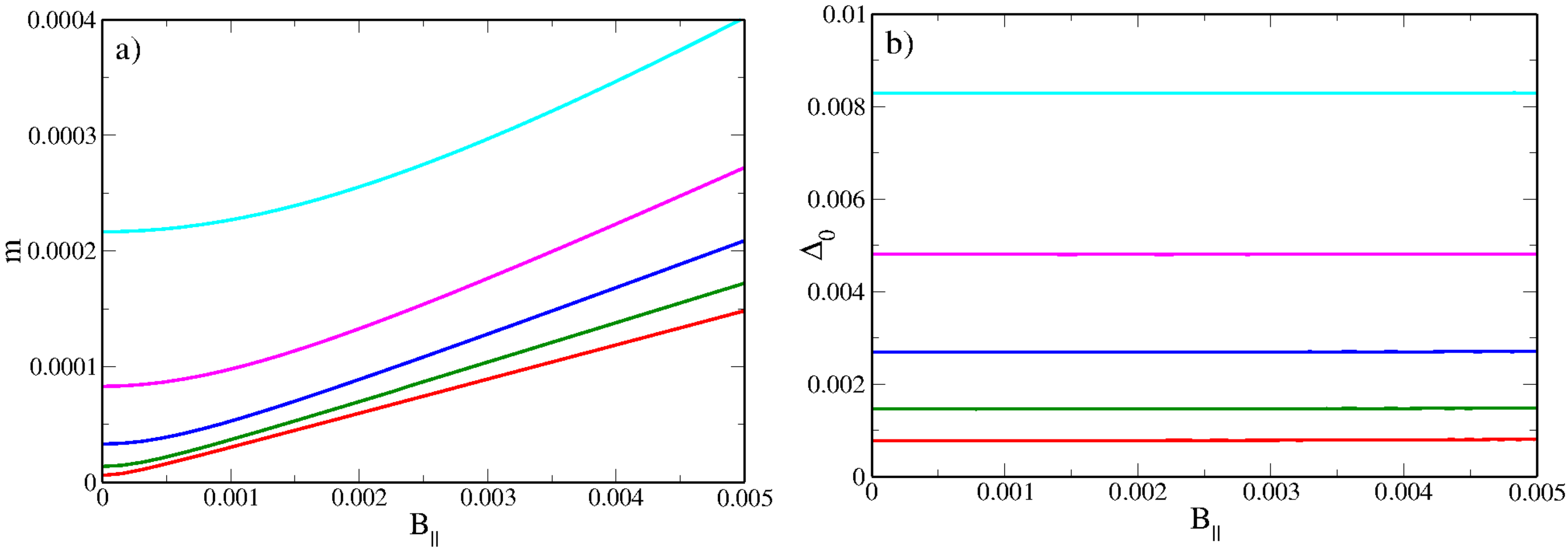
Easy-plane AFM in tilted magnetic fields



Zeeman coupling: $\lambda \sim B_T$ Dirac LL $\sim \sqrt{B_\perp}$

For fixed $B_\perp$ as $B_{||}$ increases $m \sim B_{||}$ for $B > 2$ T

$N_\perp, \Delta_0$ do not scale with $B_{||}$



$B_\perp$ = 2, 4, 8, 16, 32 Tesla δ_a = 0.225 δ_f = 0.05

No phase transition easy-plane AFM $\rightarrow$ FM

Easy-plane AFM in tilted field- II

- Pure FM supports gapless counter propagating edge states

$$\Rightarrow \sigma_{xy} = 2e^2/h$$

Abanin et. al. PRL, 2007

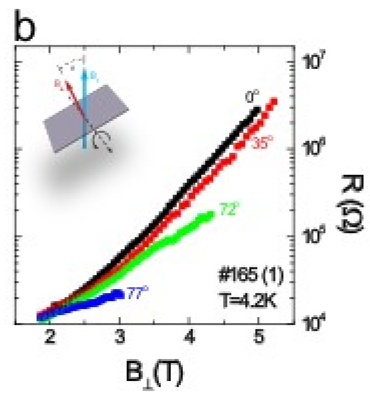
- Pure AFM or canted AFM: fully gapped edge modes

$$\Rightarrow \sigma_{xy} = 0$$

In two terminal measurements

- As FM increases in canted AFM gap for edge mods decreases

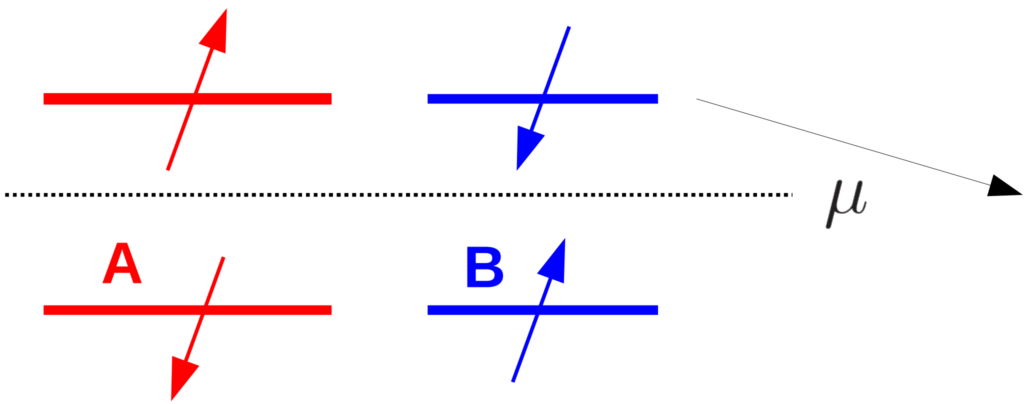
$\Rightarrow R_{xx}$ decreases: observed experimentally



Nat. Phys. **8**, 550 (2012)

Quite difficult to destroy CSB ground state

Nu=1 Hall state



$\nu = 0$ Hall state

$\nu = 1$ Hall state

Placing chemical potential Close to the first excited state

- Simultaneous sublattice & staggered spin degeneracy lifting
- Sublattice degeneracy lifting: charge-density-wave (CDW)

  $\Rightarrow$ charge gap $\Delta_1^{ch} \equiv C$ (scales only with $B_\perp$)
- Staggered spin degeneracy lifting: easy-axis AFM (N_3)

  $$\Rightarrow \text{ spin gap } \Delta_1^{sp} = 2(\lambda+m)\frac{N_3}{\Delta_0} + \mathcal{O}(N_3^2) \approx 2(\lambda+m)\frac{N_3}{|\vec{N}_\perp|} + \mathcal{O}(N_3^2)$$

Coexistence of two chiral symmetry breaking orders

Nu=1 Hall state in tilted magnetic field

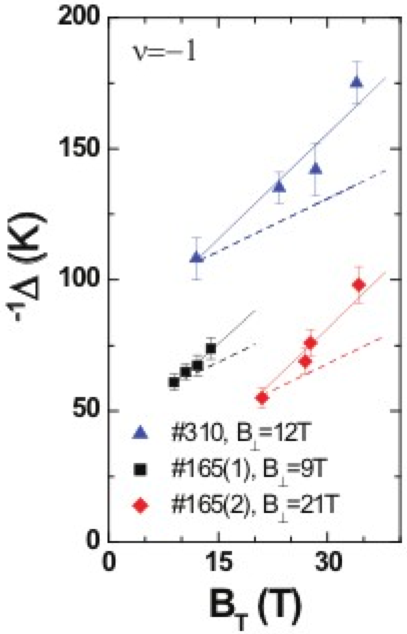
- Half(1/4) ZLL contributes to $N_\perp(N_3) \Rightarrow \dfrac{N_3}{N_\perp} < 1$

$N_3, N_\perp$ scales only with $B_\perp$

C or charge gap (Δ_1^{ch}) does not scale with $B_\parallel$

$\Rightarrow$ In tilted field $\Delta_1^{sp} \sim (m + \lambda) \sim B_T$

but with a slope bigger than λ (Zeeman)



- In graphene: $C \sim V_1 \sim U/2 \rightarrow C \gg (\lambda + m)$

$\Rightarrow$ In perpendicular B field: $\Delta_1^{ch} = C \gg \Delta_1^{sp}$

Nat. Phys. **8**, 550 (2012)

Self consistent theory for CDW @ nu=1

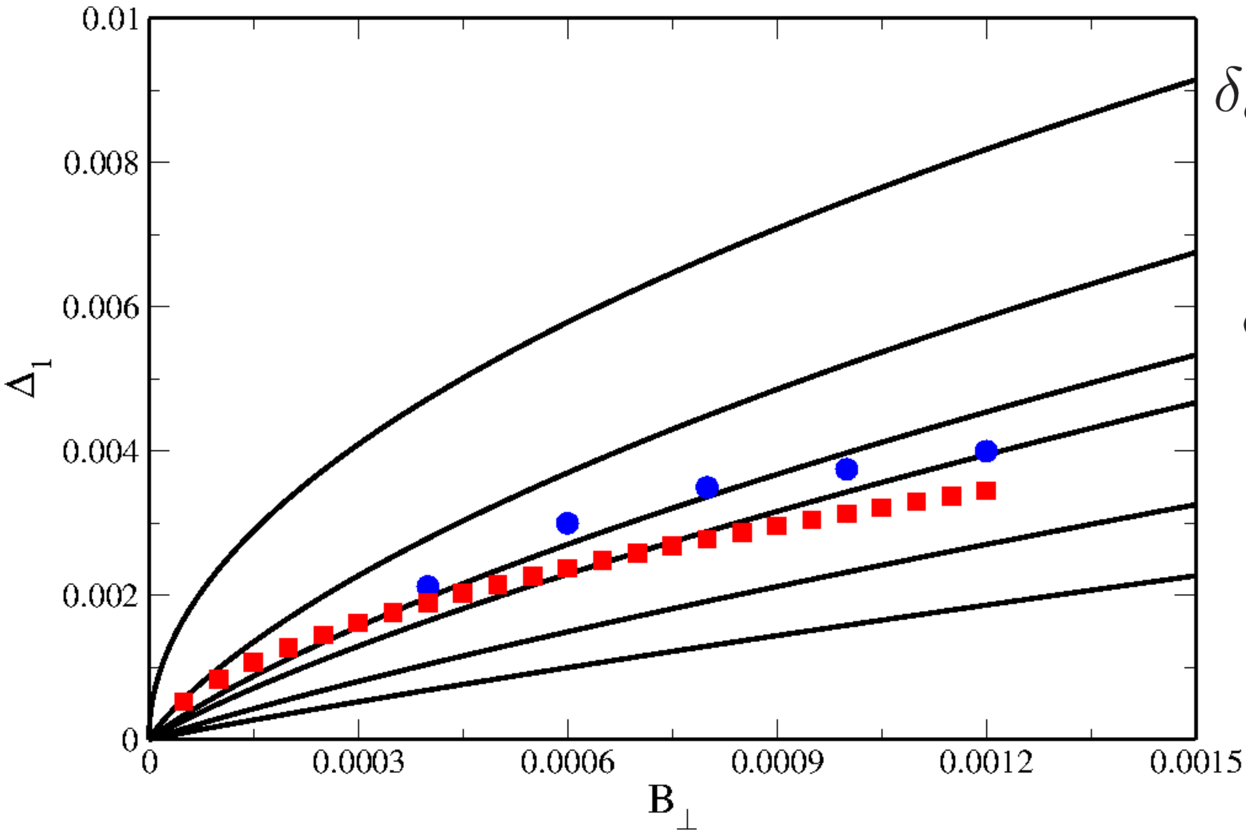
- Free energy: $F_1 = \frac{C^2}{4g_c} - D\left[\frac{C}{2} + 2\sum_{n\geq 1} E_n^C\right]$

$$g_c \sim V_1$$
$$E_n^C = \sqrt{2nB + C^2}$$

- Minimize F_1 w.r.t $C \Rightarrow$ gap equaiton for C : displays UV divergence
- Regularization: $\delta_c = \sqrt{\pi}\left[(g_c\Lambda)^{-1} - (g_c^c\Lambda)^{-1}\right]$

$$(g_c^c)^{-1} = \int_{\Lambda^{-1}}^{\infty} ds\, s^{-3/2}$$

:distance from zero field CDW quantum criticality (g_c^c)



$\delta_c = 0, 0.03, 0.06, 0.08, 0.15, 0.25$ from top to bottom

- $\delta_c = 0.06$, $\delta_c = 0.08 \Rightarrow$ reasonable aggrement with experiments
- But range of interaction: *slightly longer*

Correction due to long-range Coulomb tail

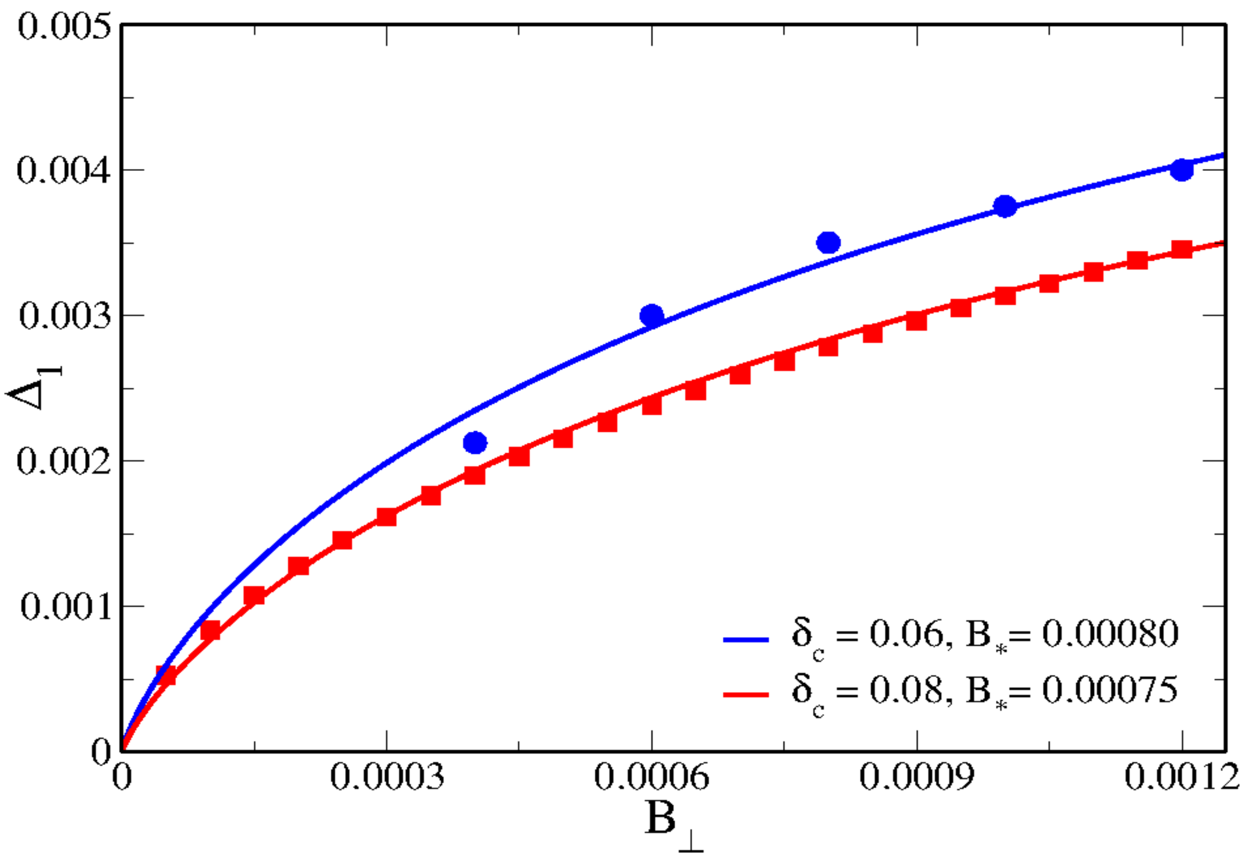
- Long range Coulomb interaction: *logarithmic correction to v*

$$v_F = v_F^0 \left[1 + \frac{e^2}{8\epsilon v_F^0} \log\left(B_*/B\right) \right]$$

v_F^0 : bare Fermi velocity

$1/\sqrt{B_*}$: characteristic length scale for the measured value of v_F

- Δ_1 : measured in units of $v_F \Lambda \Rightarrow$ gap acquires logarithmic correction



- Experimental detection:

Charge-density-wave :
sublattice resolved STM

Easy-axis AFM (N_3):
spin resolved STM

Summary & Future directions

- In the absence of magnetic fields monolayer graphene is susceptible to *chiral-symmetry-breaking* mass generations, such as ***charge-density-wave, antiferromagnet*** etc. @ strong couplings.
- Magnetic fields: conducive to formation of CSB vacuum even for *weak repulsive interactions*.
- Subcritical ***onsite*** repulsion → AFM, but Zeeman coupling → ***easy-plane AFM + easy-axis FM***: ground state for nu=0 Hall state.
- As *interaction gets stronger*: smooth crossover from ***linear*** → ***sublinear*** → $\sqrt{B}$: excellent agreement with multiple experiments.
- Weak ***nearest-neighbor*** repulsion → ***CDW***, ***onsite-U*** → ***easy-axis AFM*** for nu=1 Hall state: agreement with experiments in *perpendicular and tilted* magnetic fields (respectively).
- Search for an explanation of experimental data naturally leads to CSB orders as a minimal explanation.
- Generalization for *fractional Hall states* (composite Dirac fermions).
- Similar mechanism applicable for graphene-based layered systems; ***bilayer & trilayer graphene*** **(BR, PRB, 2014)** **Weyl semimetals** **(1406.4501)**