

# Review of Minimal Walking Theories

**Kimmo Tuominen**



University of Helsinki  
&  
Helsinki Institute of Physics



Confinement XI

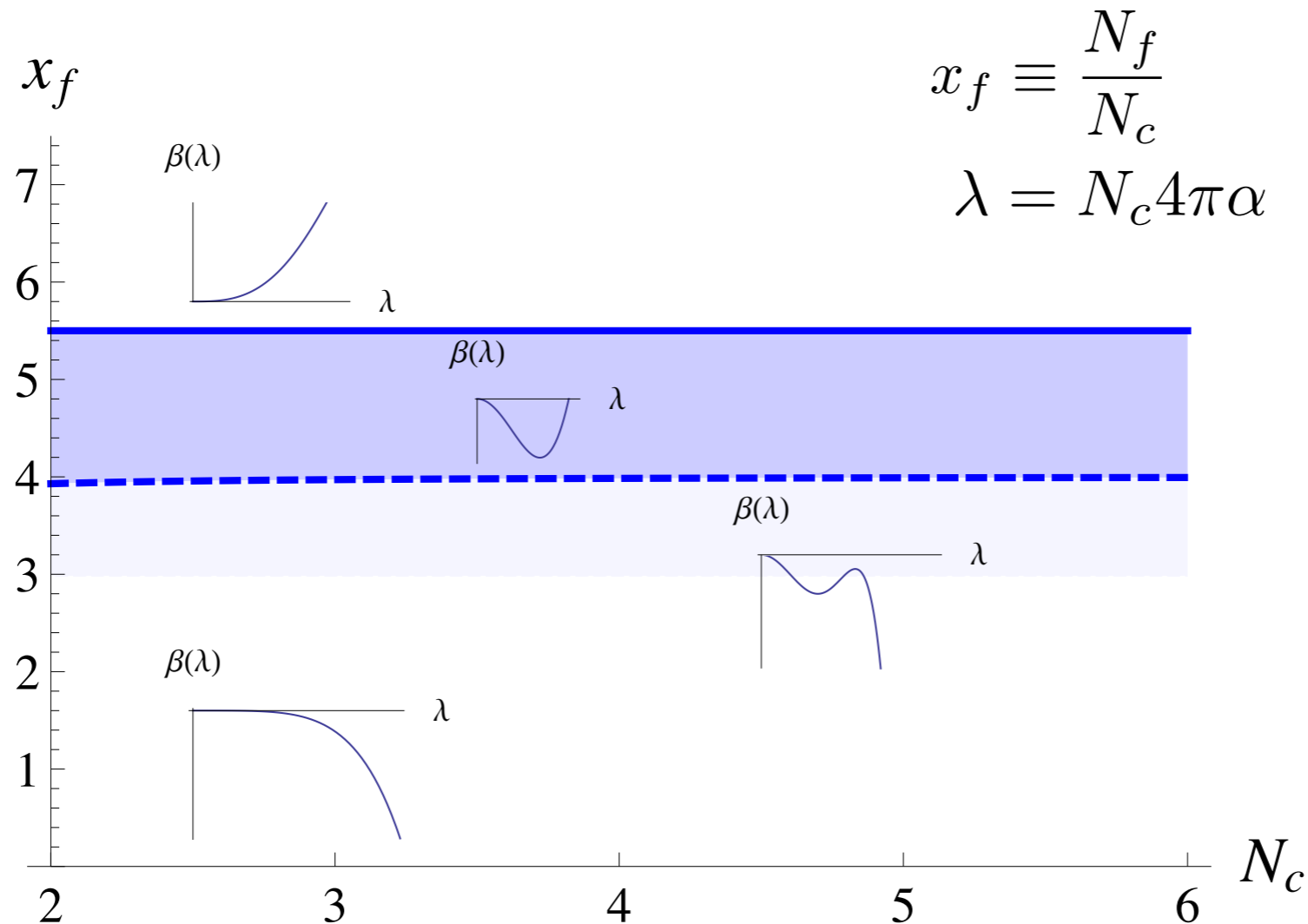
11. 9. 2014 St. Petersburg

# Outline

1. Strong dynamics in isolation: Vacuum phases
2. Coupling with the Standard Model: A light scalar
3. Other directions: Dark matter

# 1. Strong dynamics in isolation: Vacuum phases

# Theory motivation: phases of gauge theories



Given a gauge theory and its matter content,

- Where are the borderlines between different behaviors?
- Is there a quasi-conformal region (walking)?
- What are the relevant scales, excitation spectrum etc.?

(Also talks by T. Rytov and E. Mølgaard)

# The traditional approach: ladder approximation

Miransky, Yamawaki, 1997; Appelquist, Terning, Wijewardhana 1996

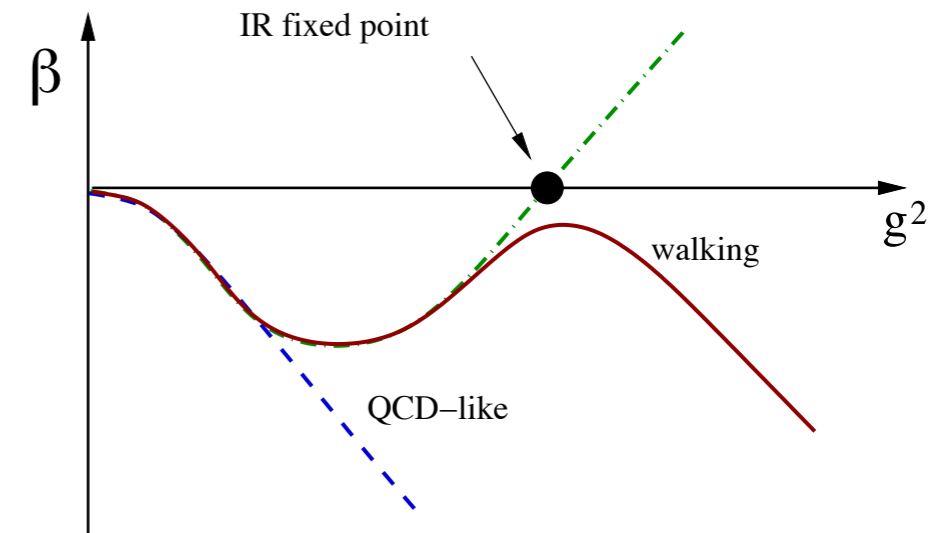
Fixed point from 2-loop betaf.  $\alpha^* = -\frac{\beta_0}{\beta_1}(4\pi)$

Critical coupling for chiral breaking from SD-equ.

$$\alpha_c = \frac{\pi}{3C_2(R)}$$

Conformal window:

$$\alpha^* \leq \alpha_c$$



Depends on  $N_c$ ,  $N_f$  and fermion representation  $R$

Alternatives:

Holography, e.g. Kiritsis, Järvinen 2012; Alho, Evans, Tuominen 2014; (see Järvinen's talk)

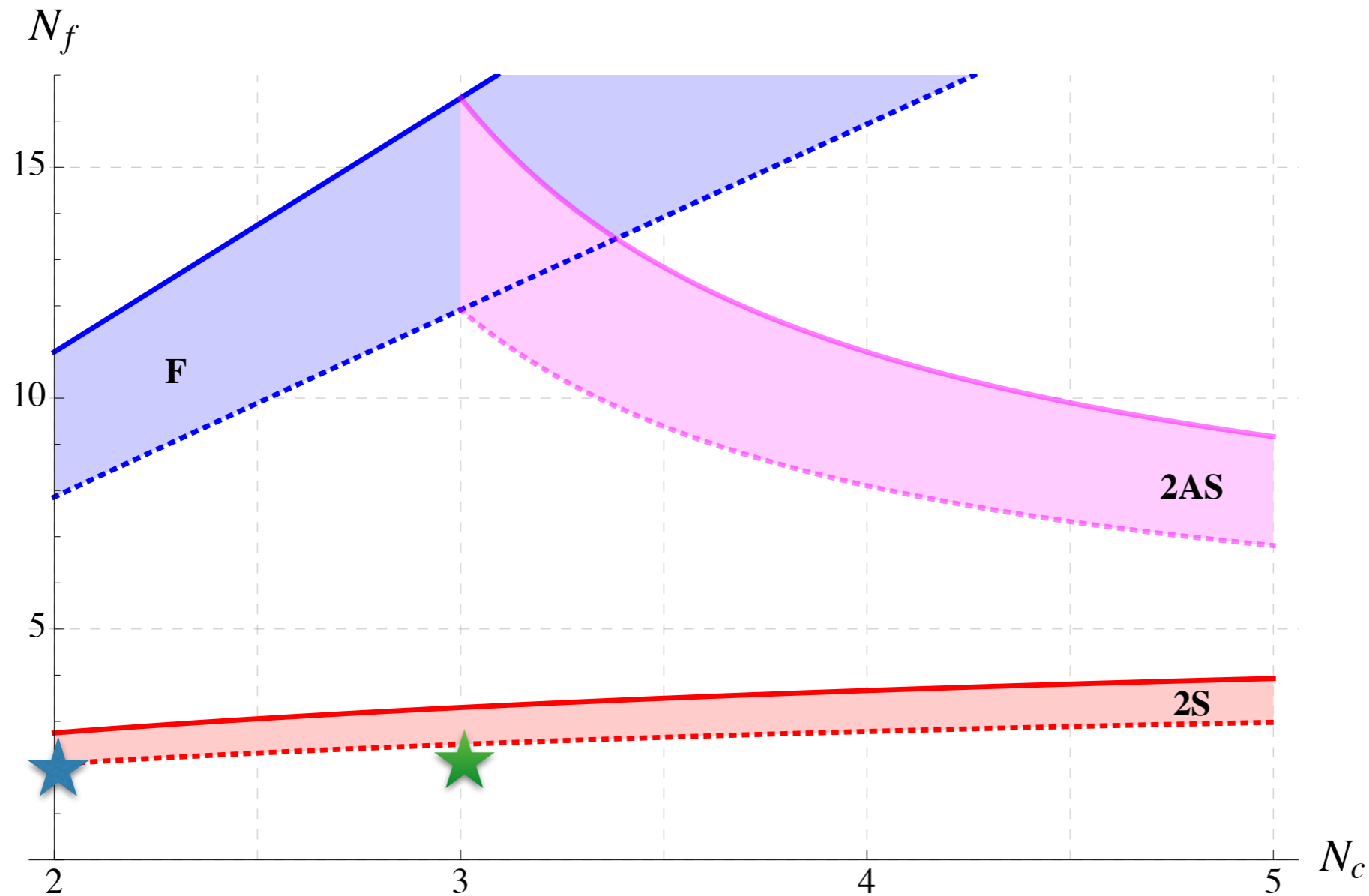
Beta function ansätze, e.g. Rytov, Sannino 2009; Antipin, Tuominen 2009

Thermal dof count, Appelquist, Cohen, Schmaltz 1999

...

All in quantitative agreement with the ladder appro.

# Ladder results (Sannino, Tuominen PRD 71 (2004) hep-ph/0405209)



Two minimal models:

★  $SU(2) + 2$  adjoint flavors:  
 $SU(2)$ -Minimal Walking Theory ( $SU(2)$ MWT)

★  $SU(3) + 2$  sextet flavors:  
 $SU(3)$ MWT

# A perfect program for lattice studies:

Lots of efforts during last 5 years.

SU(2) adjoint: (Catteral et al., Hietanen et al., Del Debbio et al.,...)

SU(2) fundamental: (Del Debbio et al., Karavirta et al.,...)

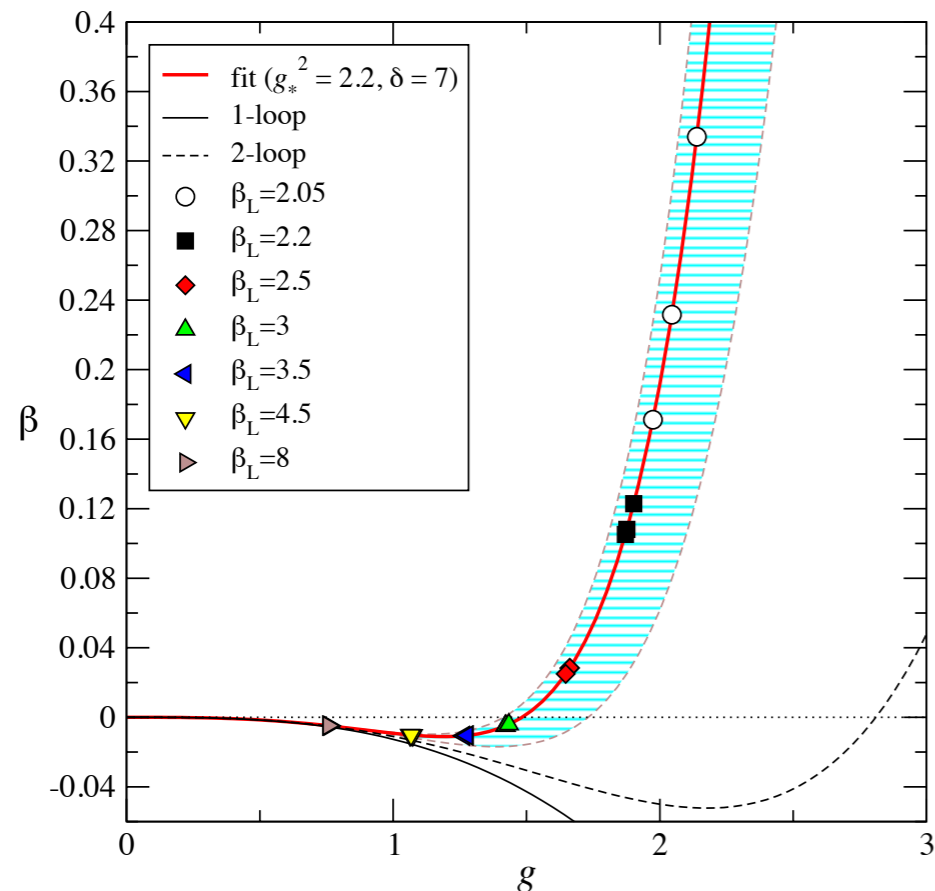
SU(3) fundamental: (Appelquist et al., Kuti et al.,...)

SU(3) sextet: (De Grand et al.,...)

## First large-scale simulations:

A. Hietanen et al. JHEP (2009), 0812.1467

A. Hietanen et al. PRD 80 (2009), 0904.0864



SU(2), 2 adjoint flavors

- IR Conformal
- Confirmed by other groups.

# Phenomenology motivation: dynamical EWSB

Vintage compositeness: replicate QCD

Weinberg '79,  
Susskind '79

Higgs mechanism as usual from  
SSB+gauge symm.

$$\langle \bar{Q}_L Q_R \rangle = \Lambda_{\text{TC}}^3, \quad \Lambda_{\text{TC}} \simeq 1 \text{ TeV}$$

The Higgs is composite.

$$\pi^\pm, \pi^0 \rightarrow W_L^\pm, Z_L \quad M_W = \frac{g F_{\text{TC}}}{2}, \quad F_{\text{TC}} \simeq 250 \text{ GeV}$$

**Phenomenological tensions, S-parameter, ETC, etc.  
suggest non QCD-like dynamics instead (i.e. walking?)**

Plenty of pheno studies based on  $SU(2)_{\text{MWT}}$  and  $SU(3)_{\text{MWT}}$ .  
- Oblique corrections, vector mesons, coupling to SM fermions,...

Dietrich, Sannino, Tuominen 2005, Foadi et al. 2007, Belyaev et al. 2009, ...

**Coupling with SM fields changes properties  
from those observed in isolation.**





## 2. Coupling with the Standard Model: A light scalar

Recall Pagels-Stokar:

$$F_{\Pi}^2 \simeq 4NM^2 \ln \left( \frac{\Lambda_{\text{TC}}^2}{M^2} \right)$$

Setting  $F_{\pi} = 246 \text{ GeV}$  and  $\Lambda_{\text{TC}} \simeq 2 \dots 10 \text{ TeV}$ , this implies that the dynamical mass is constrained:  $M \simeq 0.5 \dots 1 \text{ TeV}$

So how to get 125 GeV?

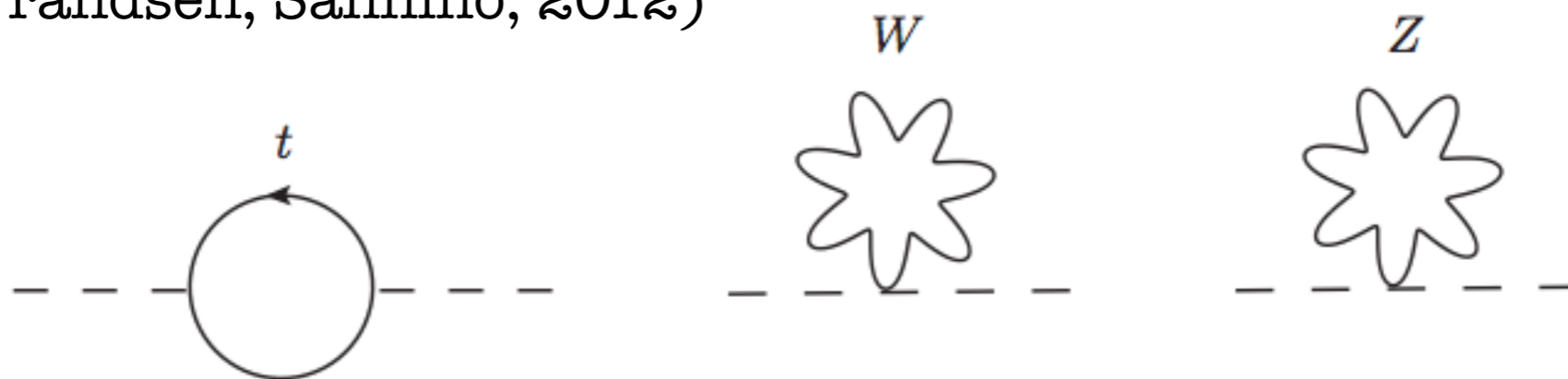
Example: the mass difference of neutral and charged pions in QCD:

$$m_{\pi^+}^2 - m_{\pi^0}^2 \simeq (35 \text{ MeV})^2 \propto \alpha_{\text{EM}}$$

This is small,  $\mathcal{O}(0.1 f_\pi)$ , because  $\alpha_{\text{EM}}$  is small.

Take  $\alpha_{\text{EM}} \sim 1/(4\pi)$  and the effect is  $\mathcal{O}(f_\pi)$ .

In SM couplings are not small, e.g. top Yukawa  
(Foadi, Frandsen, Sannino, 2012)



We will now consider this in a setting where all  
(nonzero) masses are generated dynamically

Consider a model (Di Chiara, Foadi, Tuominen (2014))

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{TC}} + \mathcal{L}_{\text{ETC}}$$

A very simple extended dynamics to generate top mass:

$$M_{\text{ETC}} \quad \mathcal{L}_{\text{ETC}} = 2G (\bar{q}_L t_R \bar{U}_R Q_L + \text{h.c.}) \quad G \sim \frac{1}{M_{\text{ETC}}^2}$$

$$\Sigma = \exp(i\Pi^a \tau^a / v)$$

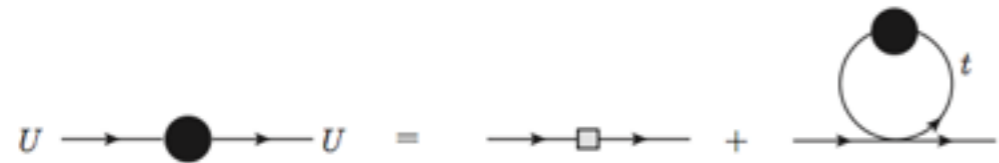
$$\Lambda_{\text{TC}} \simeq 4\pi F_{\Pi} \quad \mathcal{L}_{\text{TC}} = \bar{Q}_L i\not{D}Q_L + \bar{U}_R i\not{D}U_R + \bar{D}_R i\not{D}D_R \\ - M \left( 1 + \frac{y}{v} H + \dots \right) (\bar{Q}_L \Sigma Q_R + \bar{Q}_R \Sigma^\dagger Q_L) - \frac{m^2}{2} H^2 + \dots$$

U,D in  $N$  dimensional rep. of a new strong gauge group.

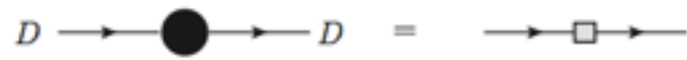
$$F_{\Pi} \simeq 246 \text{ GeV}$$

Question: what is the mass of the composite scalar  $H$  ?

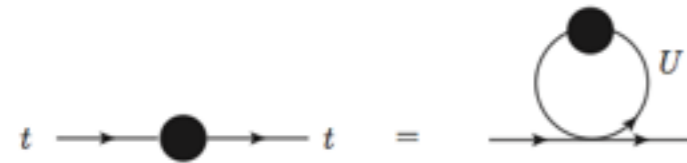
The masses determined by gap eqs:



$$M_U = \frac{N_c G M_t}{4\pi^2} \left( M_{\text{ETC}}^2 - M_t^2 \ln \frac{M_{\text{ETC}}^2 + M_t^2}{M_t^2} \right)$$



$$M_D = M$$



$$M_t = \frac{N G M_U}{4\pi^2} \left( \Lambda_{\text{TC}}^2 - M_U^2 \ln \frac{\Lambda_{\text{TC}}^2 + M_U^2}{M_U^2} \right)$$

Work to leading order in  $N$  and  $N_c$  assuming both large and  $N/N_c$  finite.

Consistency checks:

Evaluate

$$i\Pi_{\Pi^-\Pi^+} = \text{---} \left( \begin{array}{c} U \\ \circlearrowleft \\ U \end{array} \right) \text{---} + \text{---} \left( \begin{array}{c} D \\ \circlearrowleft \\ D \end{array} \right) \text{---} + \text{---} \left( \begin{array}{c} U \\ \circlearrowleft \\ D \end{array} \right) \text{---} + \text{---} \left( \begin{array}{c} U \quad t \quad U \\ \circlearrowleft \quad \circlearrowleft \quad \circlearrowleft \\ D \quad b \quad D \end{array} \right) \text{---} + \dots$$

$$i\Pi_{\Pi^0\Pi^0} = \text{---} \left( \begin{array}{c} U \\ \circlearrowleft \\ U \end{array} \right) \text{---} + \text{---} \left( \begin{array}{c} D \\ \circlearrowleft \\ D \end{array} \right) \text{---} + \text{---} \left( \begin{array}{c} U \\ \circlearrowleft \\ U \end{array} \right) \text{---} + \text{---} \left( \begin{array}{c} D \\ \circlearrowleft \\ D \end{array} \right) \text{---}$$

$$+ \text{---} \left( \begin{array}{c} U \quad t \quad U \\ \circlearrowleft \quad \circlearrowleft \quad \circlearrowleft \\ U \quad t \quad U \end{array} \right) \text{---} + \dots$$

Using the gap equations, one proves  $M_{\Pi^0} = M_{\Pi^\pm} = 0$

Also: transversality of W and Z vacuum polarisations can be shown.

To match with EW, need pion decay constant

$$iq_\mu \mu_{\Pi A} = \begin{array}{c} U \\ \circlearrowleft \\ D \end{array} \text{---} \begin{array}{c} U \\ \circlearrowleft \\ D \end{array} \begin{array}{c} t \\ \circlearrowleft \\ b \end{array} \text{---} \begin{array}{c} U \\ \circlearrowleft \\ D \end{array} \begin{array}{c} t \\ \circlearrowleft \\ b \end{array} \begin{array}{c} U \\ \circlearrowleft \\ D \end{array} \text{---} \dots$$

$$F_\Pi = \lim_{q^2 \rightarrow 0} \frac{\mu_{\Pi A}(q^2)}{\sqrt{\Sigma'_{\Pi^- \Pi^+}(q^2)}}$$

In the limit  $G \rightarrow 0$  this is just Pagels-Stokar:  $F_\Pi^2 \simeq 4NM^2 \ln \left( \frac{\Lambda_{\text{TC}}^2}{M^2} \right)$

Using known values of  $M_t$  and  $F_\Pi$ , everything expressed in terms of  $\Lambda_{\text{TC}}$  and  $M_{\text{ETC}}$

Expect:  $\Lambda_{\text{TC}} \simeq 3 \text{ TeV}$        $M_{\text{ETC}} \simeq 5 \text{ TeV}$

The scalar mass:  $\Sigma_{HH}(q^2) = q^2 - m^2 - \Pi_{HH}(q^2)$

$$i\Pi_{HH} = \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \dots$$

First, set  $G = 0$  and trade  $m$  with the dynamical mass  $M_{H0}$  via

$$\Sigma_{HH}(q^2 = M_{H0}^2) = 0$$

Then, at  $G \neq 0$  solve for  $M_{H0}$  by setting

$$\Sigma_{HH}(q^2 = M_H^2) = 0 \quad @ \quad M_H^2 = (125)^2 \text{ GeV}^2$$

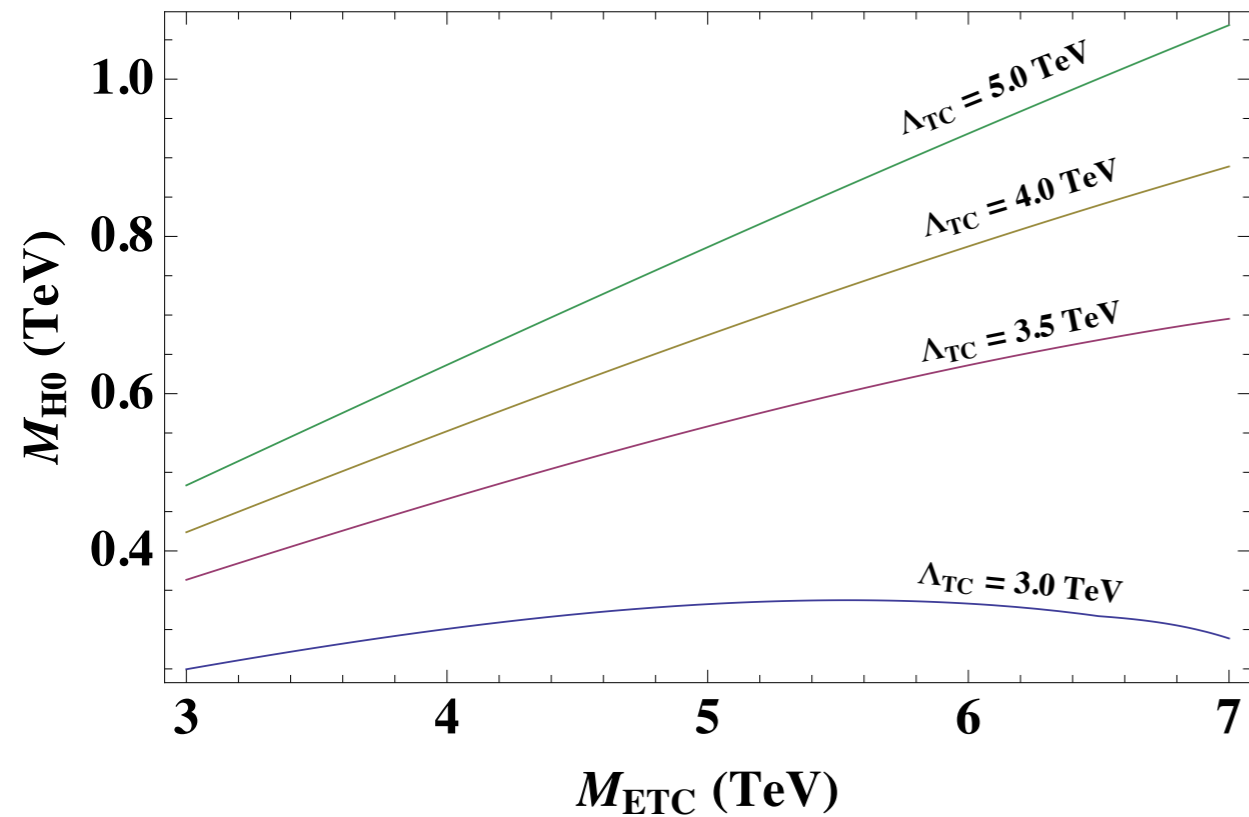
In the limit  $M_{H0} \ll \Lambda_{\text{TC}} \ll M_{\text{ETC}}$  the dynamical mass is given by

$$M_{H0}^2 \simeq \frac{1}{\ln(\Lambda_{\text{TC}}^2/M^2)} \frac{\frac{N_c}{N} \frac{M_t^2}{M_U^2}}{1 - \frac{N_c}{N} \frac{M_t^2}{M_U^2} \frac{M_{\text{ETC}}^2}{\Lambda_{\text{TC}}^2}} M_{\text{ETC}}^2$$

So  $M_{H0}$  can be large even if the physical Higgs is light.

# Numerical evaluation:

$N = 3$

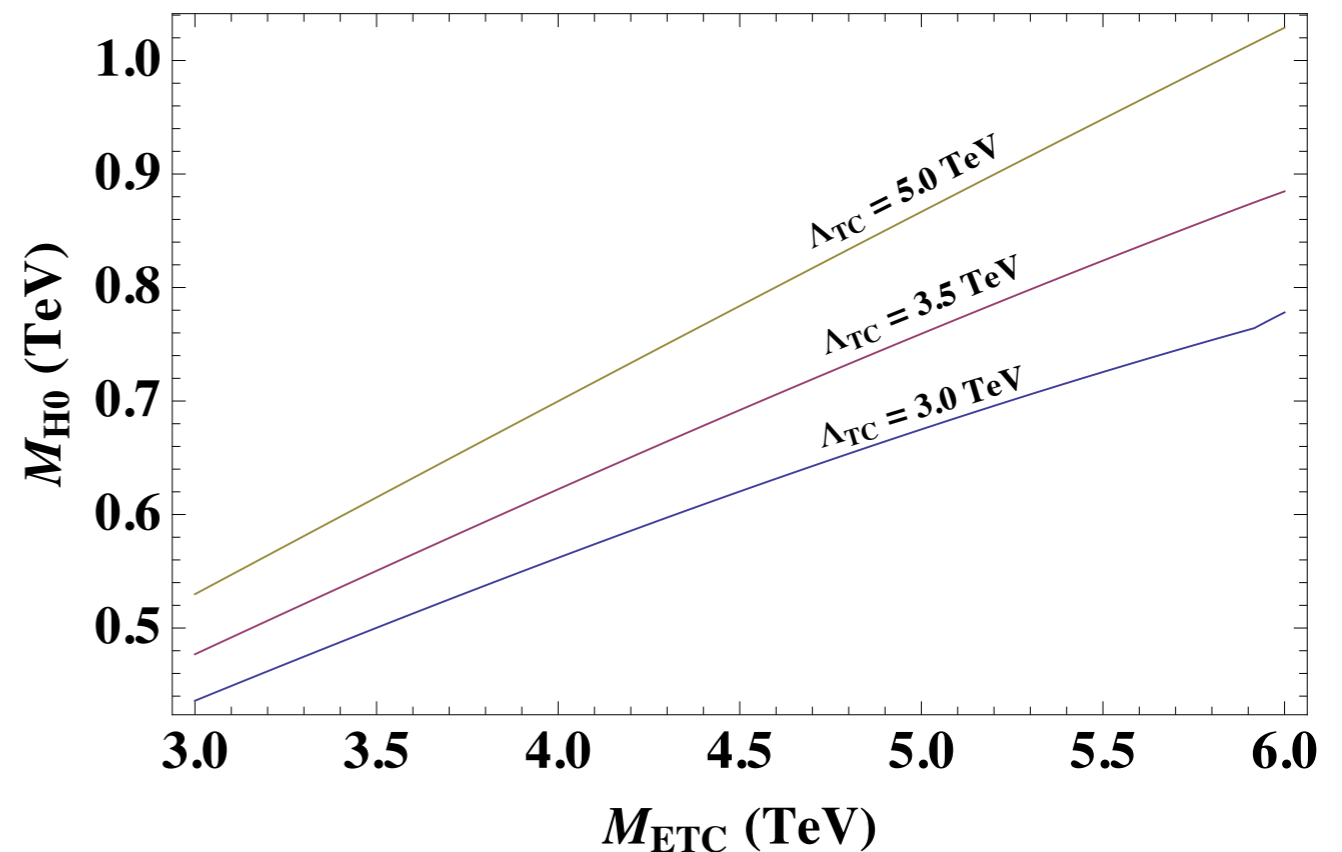


e.g. SU(3) gauge theory and  
U, D fermions in 3-dim. rep.  
(Scaled-up QCD)

SU(3) gauge theory and  
U, D fermions in 6-dim. rep.

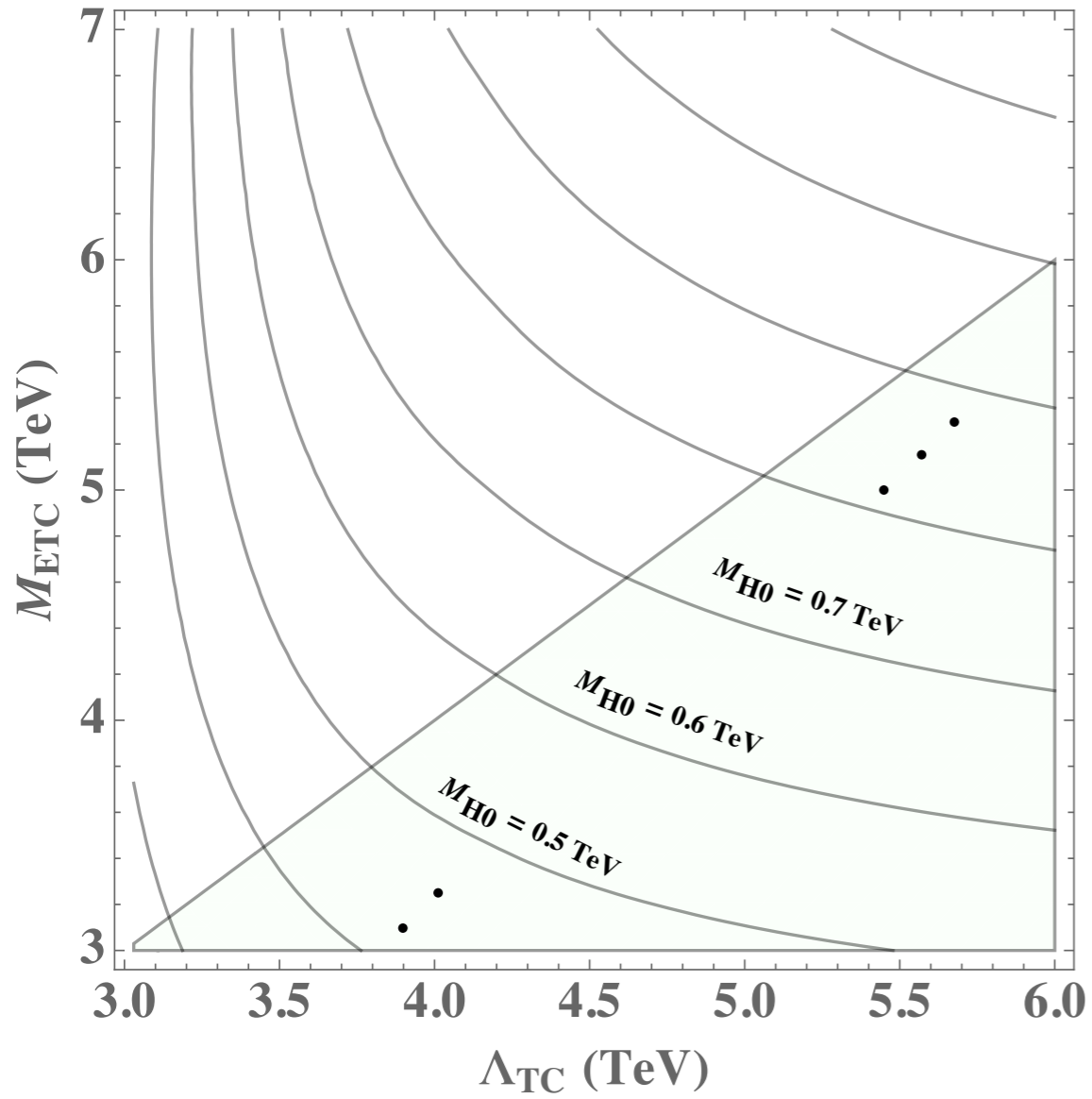
e.g. SU(3)MWT

$N = 6$





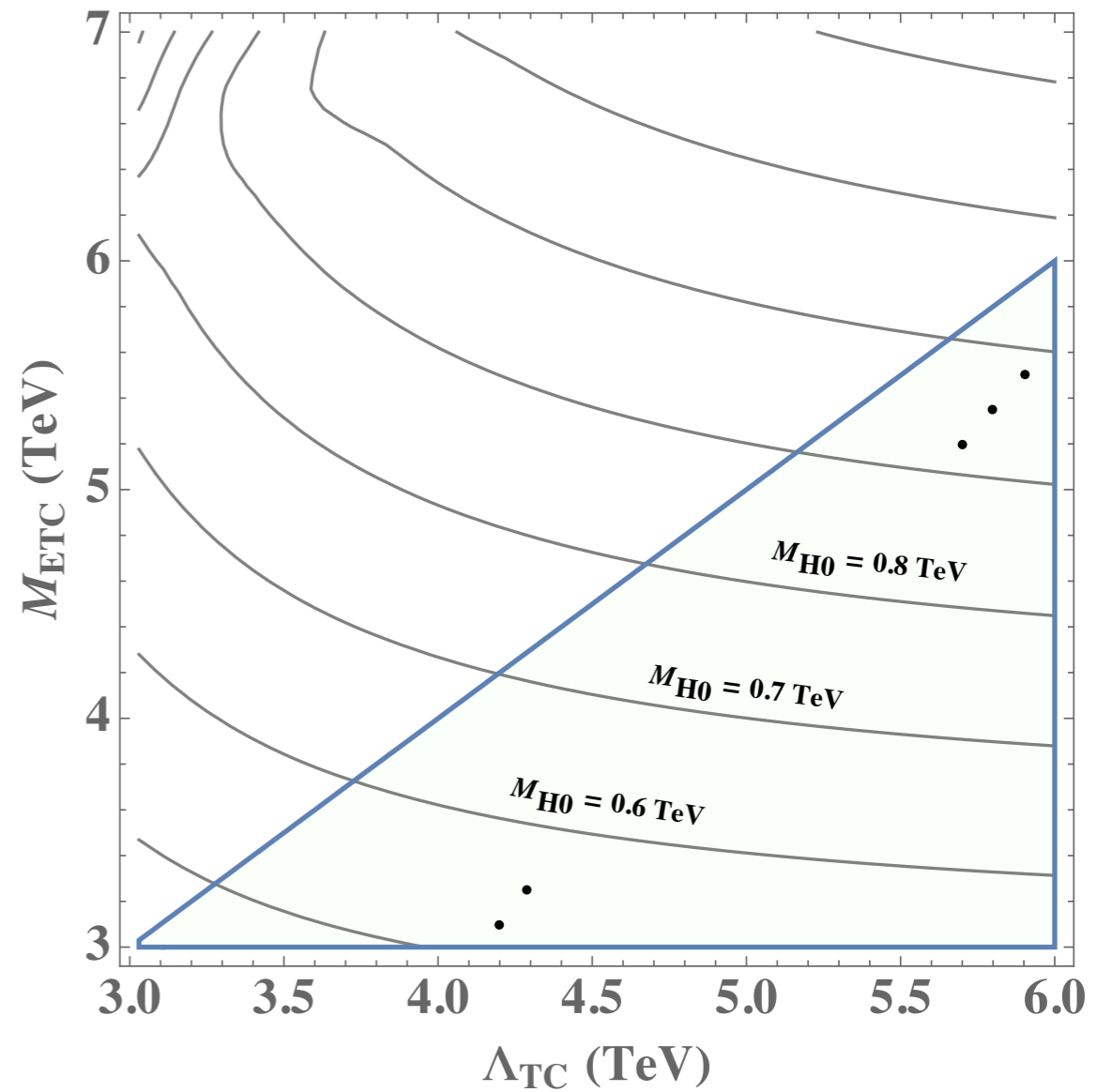
**N = 3**



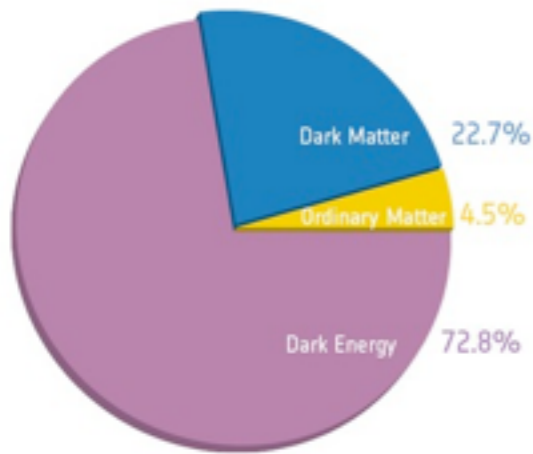
Inside the shaded region

$$M_{\text{ETC}} < \Lambda_{\text{TC}}$$

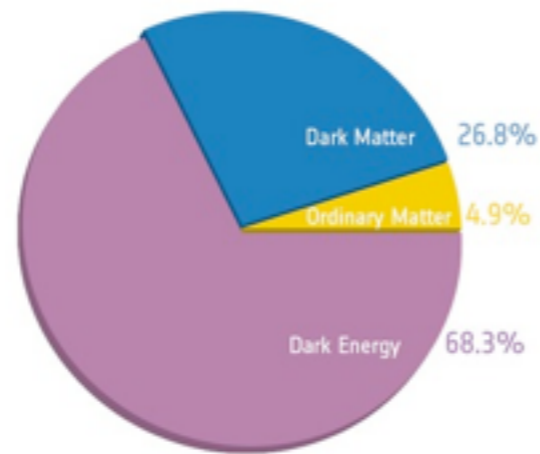
**N = 6**



### 3. Other directions: Dark matter



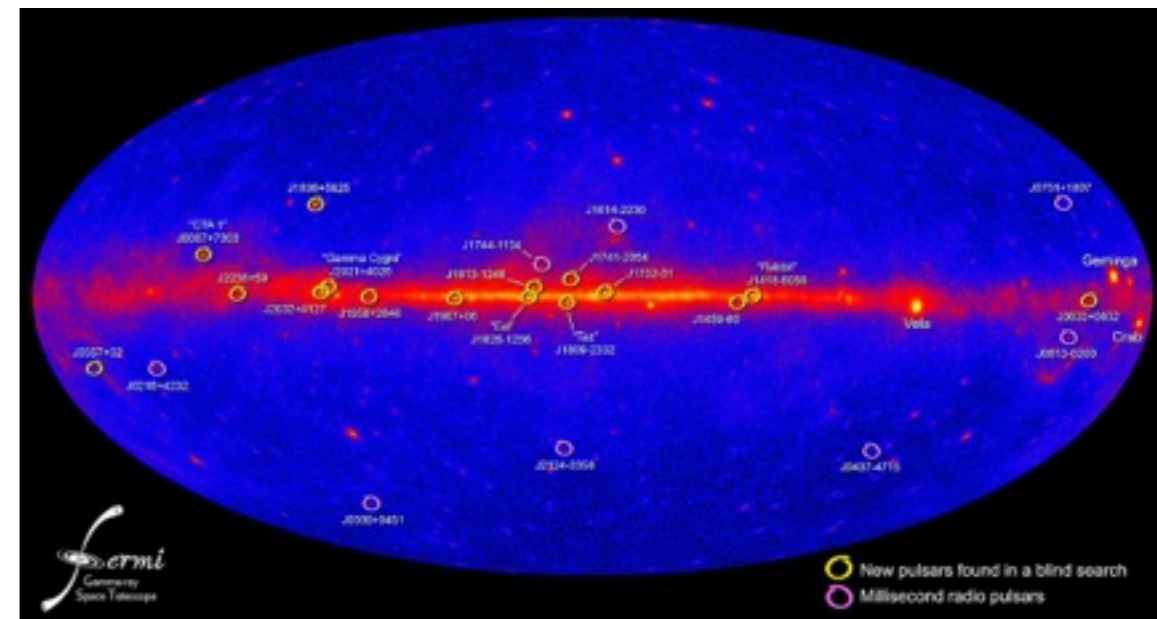
Before Planck



After Planck

$SU(3) \times SU(2) \times U(1)$   
 SM to explain just 5%

What new gauge structures provide the 95% ?



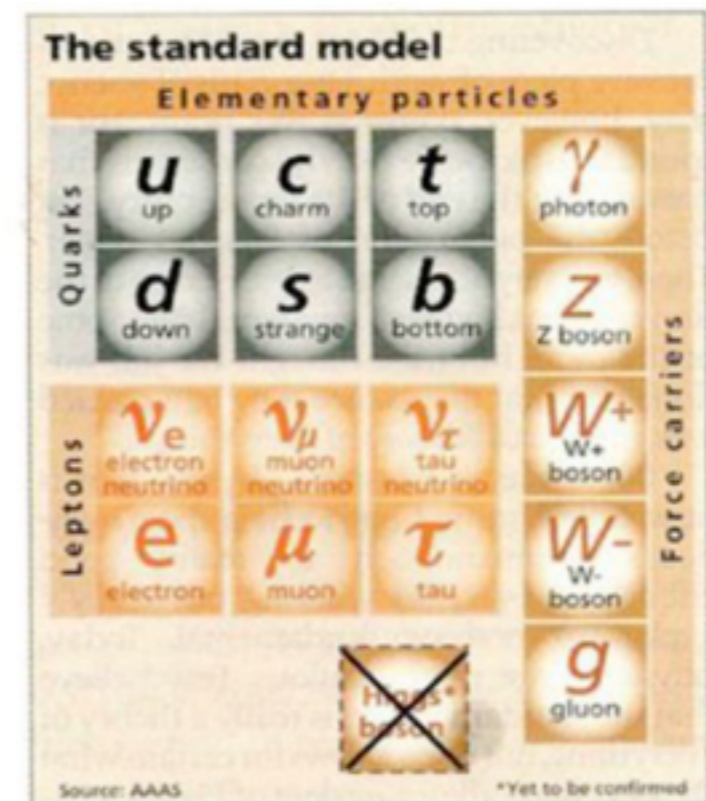
# SU(2)MWT

One weak doublet in 3 of SU(2): Witten anomaly.

$$Q_L^a = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L, \quad U_R^a, \quad D_R^a, \quad a = 1, 2, 3$$

Cure by introducing one doublet of TC/QCD singlet fermions

$$L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L, \quad N_R, \quad E_R$$



U(1)

SU(2)

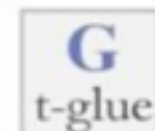
SU(3)

$$Y(Q_L) = \frac{y}{2},$$

$$Y(L_L) = -3\frac{y}{2},$$

$$Y(U_R, D_R) = \left( \frac{y+1}{2}, \frac{y-1}{2} \right),$$

$$Y(N_R, E_R) = \left( \frac{-3y+1}{2}, \frac{-3y-1}{2} \right)$$



SU(2)

# Two immediate DM candidates:

$$y = 1/3$$

4th neutrino:

Kainulainen, Tuominen, Virkajärvi JCAP 1002 (2010)

---

$$y = 1$$

A lightest neutral technibaryon. It is also a PGB.

Scattering of DM on nuclei via scalar and/ or photon exchange

$$\frac{d\sigma}{dE} \sim A \frac{\langle R_E^2 \rangle^2}{M_{\text{DM}}^2} + B \frac{\kappa^2}{M_{\text{DM}}^2}$$

From lattice: form factors of composite states.  
(talk by R. Lewis)

## Summary:

Minimal walking theories in isolation

- vacuum phase diagrams
- spectra of composite states

Minimal walking theories coupled with SM

- dynamical EWSB
- a light Higgs
- dark matter candidates

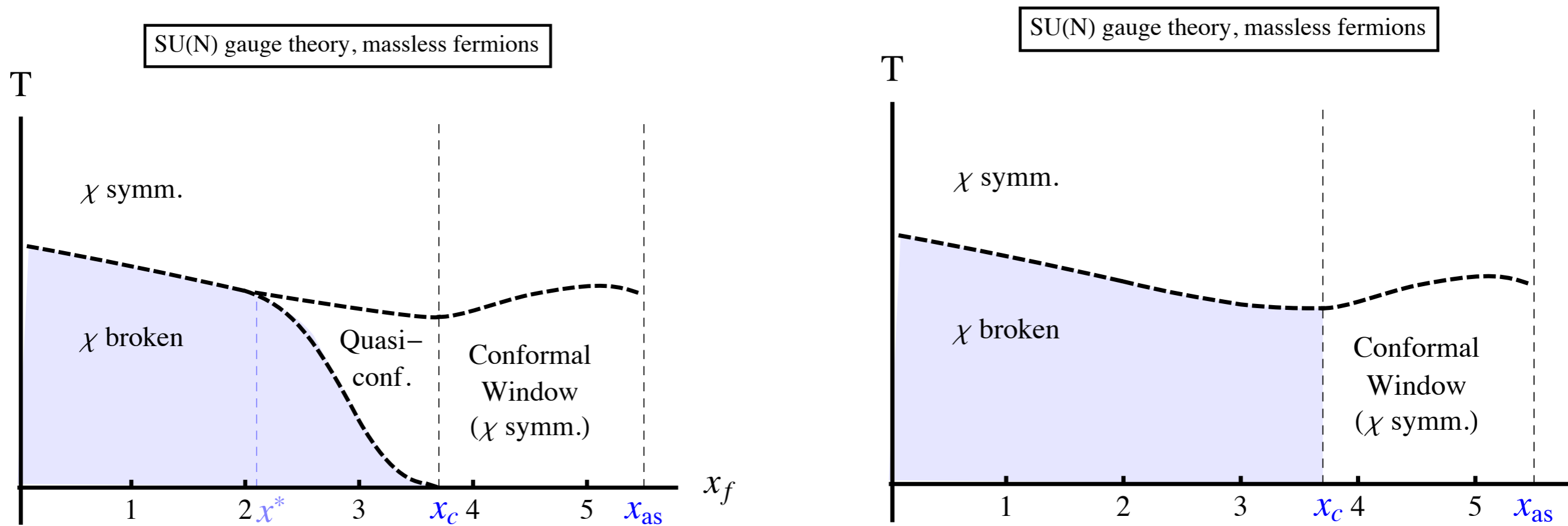
## Outlook:

Combination of model building,  
phenomenology  
and lattice results.

Extra slides

# There may not exist walking at all....

(finite T phase diagrams: K. Tuominen, Phys. Rev. D87 (2013), 1206.5772)





# A light scalar from strong dynamics:

(Dietrich, Sannino, Tuominen (2005))

$$\theta_{\mu}^{\mu} = -M_{\sigma}^2(N_f)\sigma^2 \simeq -M_{\sigma}^2(N_f)\Lambda^2 = -\frac{\beta}{2g}\Lambda^4$$

$$\beta \simeq -c(\alpha_c - \alpha^*) \rightarrow 0$$

$$M_{\sigma}^2(N_f) \simeq (N_{fc} - N_f)\Lambda^2 \quad \text{Parametrically light scalar}$$

(Finely tuned, near conformal)

In principle this is seen at LHC:

- A light scalar (observed),
- Parametrically heavy meson spectrum (not seen yet).






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# ATLAS Sept 2012

$\sqrt{s} = 7 \text{ TeV}$

$R_1, R_2 \rightarrow \ell\ell$

$ee: \int L dt = 4.9 \text{ fb}^{-1}$      $\mu\mu: \int L dt = 5.0 \text{ fb}^{-1}$

-  Dilepton 95% Exclusion
-  Dilepton 95% Expected limit
-  Dilepton 95% Expected limit  $\pm 1\sigma$
-  Running regime
-  EW precision test

