Impedance of Holes and Slots

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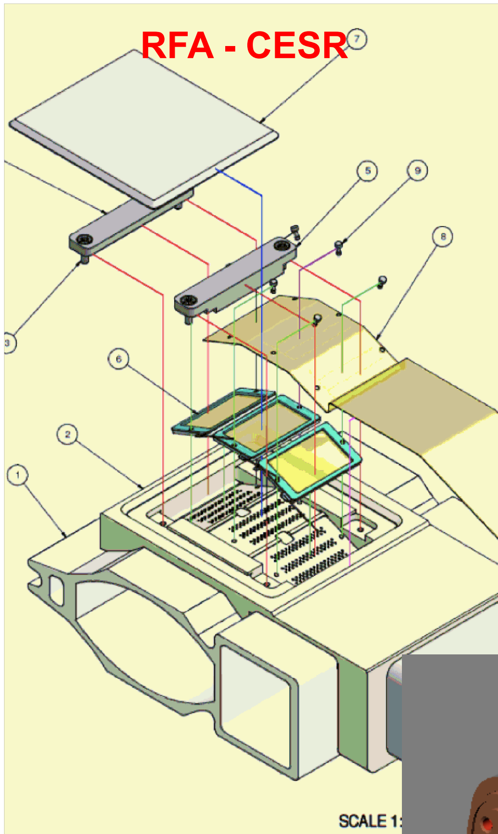
Why is there a talk on the coupling impedance of holes and slots ?

The electromagnetic interaction between accelerated beams and holes/slots on the vacuum chamber has been the object of extensive studies dating back to the 70's. Holes and slots are ubiquitous in accelerators (vacuum pumping, BPM, RFA, SPU), but their impedance is small. Yet

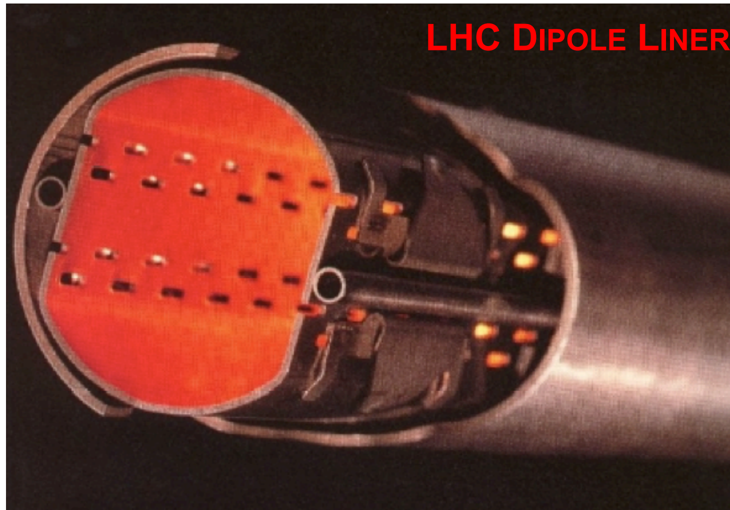
- **There can be many of them: Large accelerators like PEP, SSC, LHC.**
- **Coupling between nearby holes: Avoiding constructive interference**
- **It is formally a simple problem. It could be tackled with EM simulations codes available at the time, by analytical and experimental methods: Characteristic dimensions < λ (hole), or at least all of them except one (long slot)**

Examples of Holes and Slots

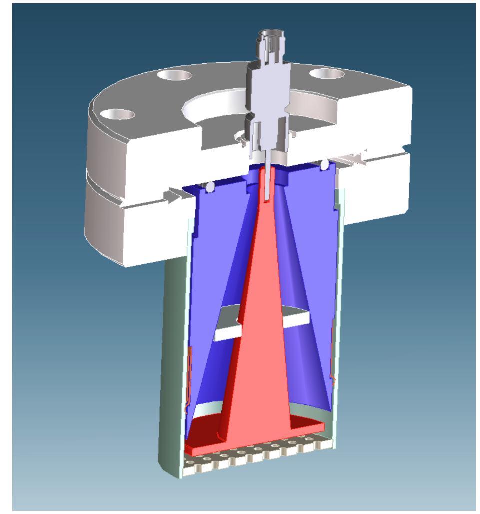
RFA - CESR



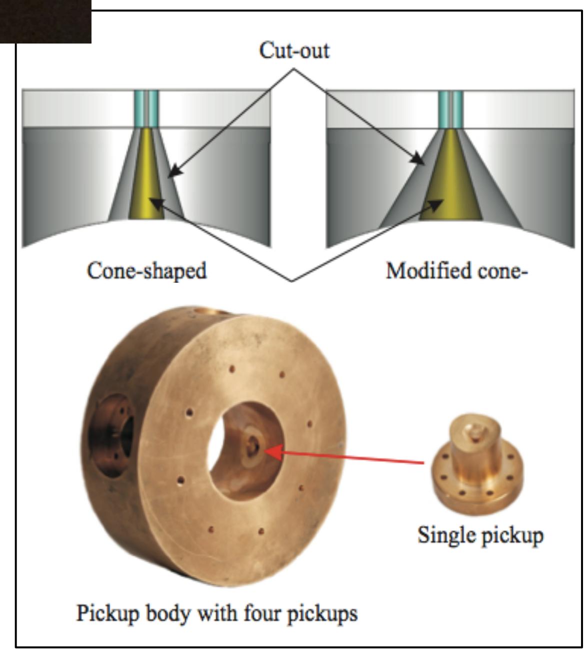
LHC Dipole Liner



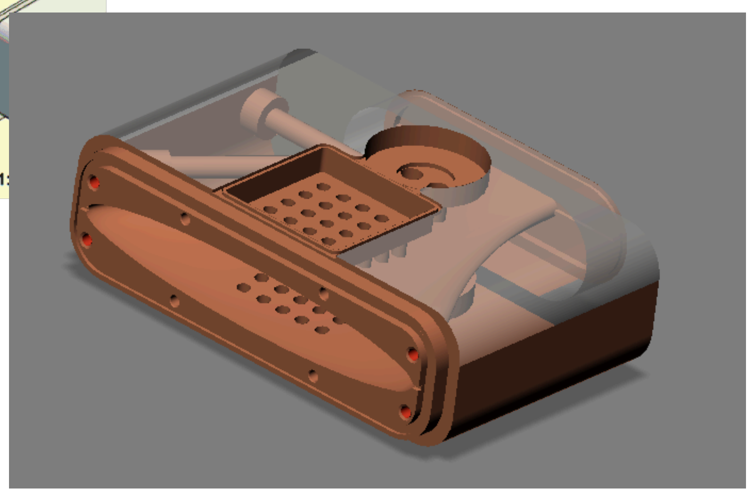
SPU - CESR



BPM - FLASH



Photon Stop - ALS



Summary

- **Impedance Calculation/Measurements Methods**
  - EM Simulation Codes
  - Bench Measurements
  - Analytical Methods
- **A Few Case Studies**
  - LHC Liner
  - Waveguide BPM (TTF)
  - Coaxial Cavity
  - Photon Stop (ALS)
- **Conclusions**

Electromagnetic Simulation Codes

Enormous Performance Advances

- Faster machines/Parallel codes -> Number of geom. elements, time intervals
- Improved Algorithms -> Precise representation of structures to simulate

Purely EM Codes (HFSS, Microwave Studio, ACE3P)

- "Virtual" bench measurements – Coaxial wire method
- Limiting factors: HOM cutoff, wire presence, SNR

Codes with Beam Excitation (Particle Studio, T3P)

- Direct wake potential measurement
- Limiting factors: bunch length, non-gaussian bunches

Limiting Factors

Virtual coaxial wire measurement for the real impedance of a hole a few millimiters large in a perfectly conducting pipe:

$$\mathrm{Re}\left(Z_{hole}\right) = \mathrm{Re}\left(2Z_c \frac{1-S_{21}}{S_{21}}\right)$$ but Re(Z_{hole}) ≈ 1 μΩ and Z_c≈ 150 Ω → $S_{21} = 1 - \frac{10^{-8}}{3}$

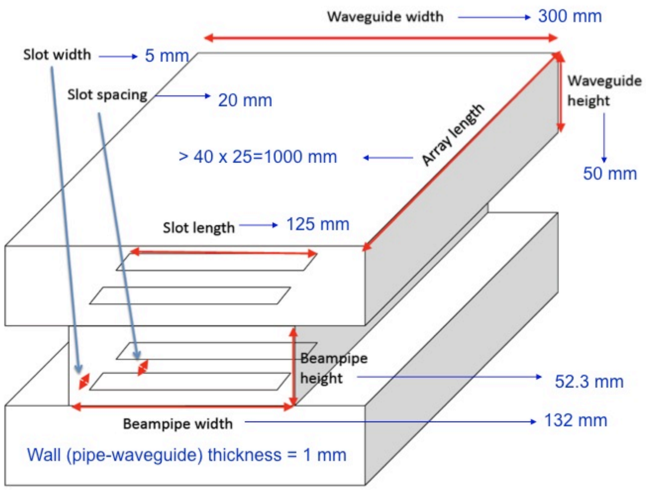
Hopeless, due to SNR (numerical noise) !

Measured longitudinal wake potential of a bunch

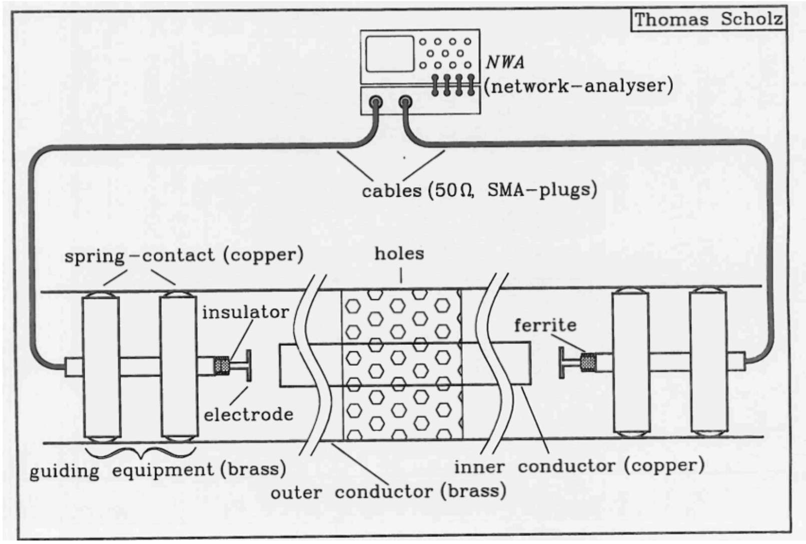
$$W_z(t) = \frac{1}{q_b}\int_{-\infty}^{+\infty} i_b(\tau) w_z(t-\tau)\, d\tau \rightarrow Z_{//}(\omega) = \frac{F\left(W_z(t)\right)}{I_b(\omega)}$$

A gaussian bunch has a well-behaved current spectrum, but it decrease by 1/e as the bunch gets longer $f^{1/e}_{[GHz]} = \frac{6.75}{\sigma_{[cm]}}$ For example a 5 cm long bunch is blind above 4 GHz.

Bench Measurement Techniques



Slotted line pickup/kicker



LHC Liner measuring setup – Courtesy of F. Caspers

Single Holes/Slots

- Can be an extremely low impedance: SNR!

Multiple Holes/Slots

- Coaxial wire may introduce a coupling mechanism between holes which is not there in the actual accelerator and could be something we are purposefully trying to avoid!

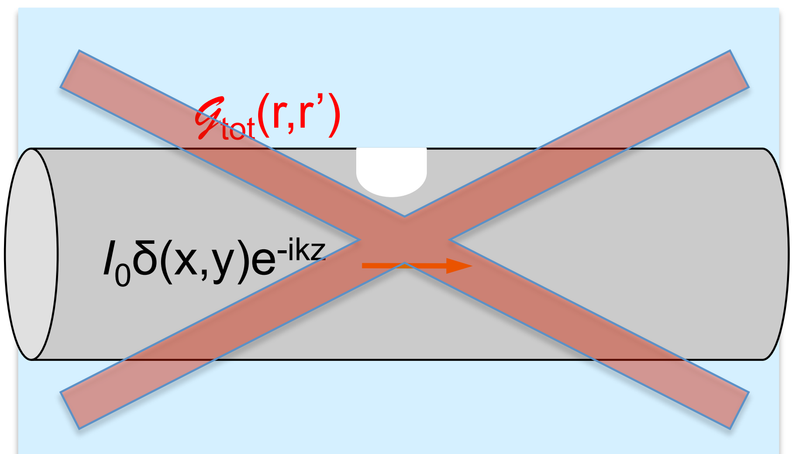
Analytical Methods

"Exact" solutions

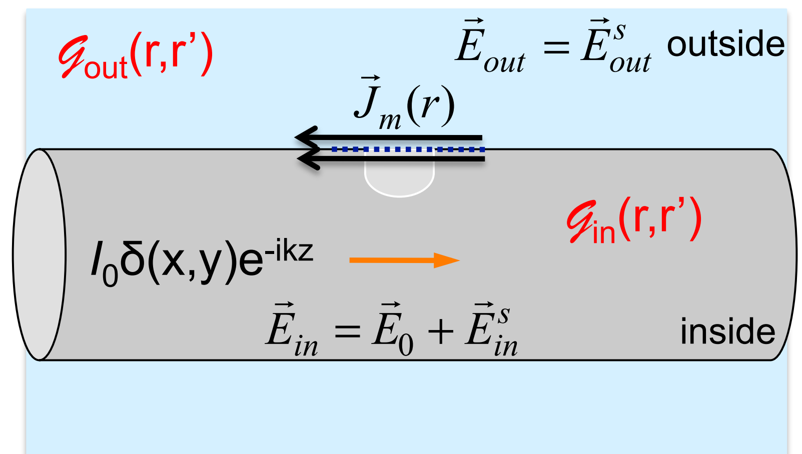
- Field matching techniques, integral equations
- Must be resolved numerically, except for a few simplest cases

unknown

we want to solve $$Z_{//}^{hole}(k) = \frac{1}{|I_0|^2}\int_{hole} \vec{n}\times\vec{E}_{hole}\cdot\vec{H}_0\, dS$$ (Gluckstern)



Total Green's function $\mathcal{G}_{tot}$ usually unknown



Separate problems more manageable. Suitable for application of variational techniques

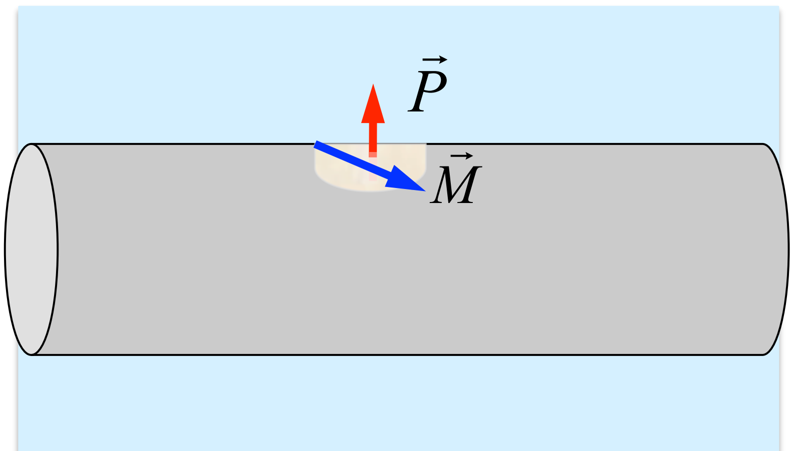
Good references: Yong-Chul Chae and A. Fedotov PhD thesis

Simpler Analytical Methods

Perturbation Methods

- Bethe's diffraction theory
- The difficult part is hidden in the hole polarizabilities

we want to solve $$Z_{//}^{hole}(\omega) = -i\frac{\omega Z_0}{2\pi q r_p}\left(\frac{1}{c}M_\varphi + P_r\right)$$



The hole is equivalent to an electric and a magnetic dipole radiating outside and inside the beampipe. Truncated Taylor series for H_T on the hole surface.

$$\begin{cases} \vec{M} = \overleftrightarrow{\alpha}_m \cdot \vec{H}_0 \\ \vec{P} = \varepsilon \overleftrightarrow{\alpha}_e \cdot \vec{E}_0 \end{cases}$$

1st approximation: incident fields are those in the pipe with no hole.
2nd approximation: dipole moments are the same regardless of outside world

Bethe's Diffraction Theory Applications

- **M. Sands**
  - Original formulation. Multiple holes, interaction length. PEP-253 (1977)
- **S. Kurennoy**
  - More formal framework. Part. Acc Vol. 39 (1992)
  - Annular slots, BPM. SSCL-636 1993
- **F. Caspers**
  - Application to LHC dipole liner, comparisons with measurements. CERN-SL-95-76-AP 1995
- **G. Stupakov**
  - Real part of $Z_{//}$ from propagating beampipe HOM's
  - "Differential polarizabilities" for a long slot
  - Interference effects in arrays of slots. Phys. Rev. E vol. 1995

Modified Bethe's Diffraction Theory

Single hole. Original theory: $Z_{//} = -i\dfrac{Z_0\omega}{4\pi^2 c}\dfrac{(\alpha_{m\perp}+\alpha_e)}{r_p^2}$

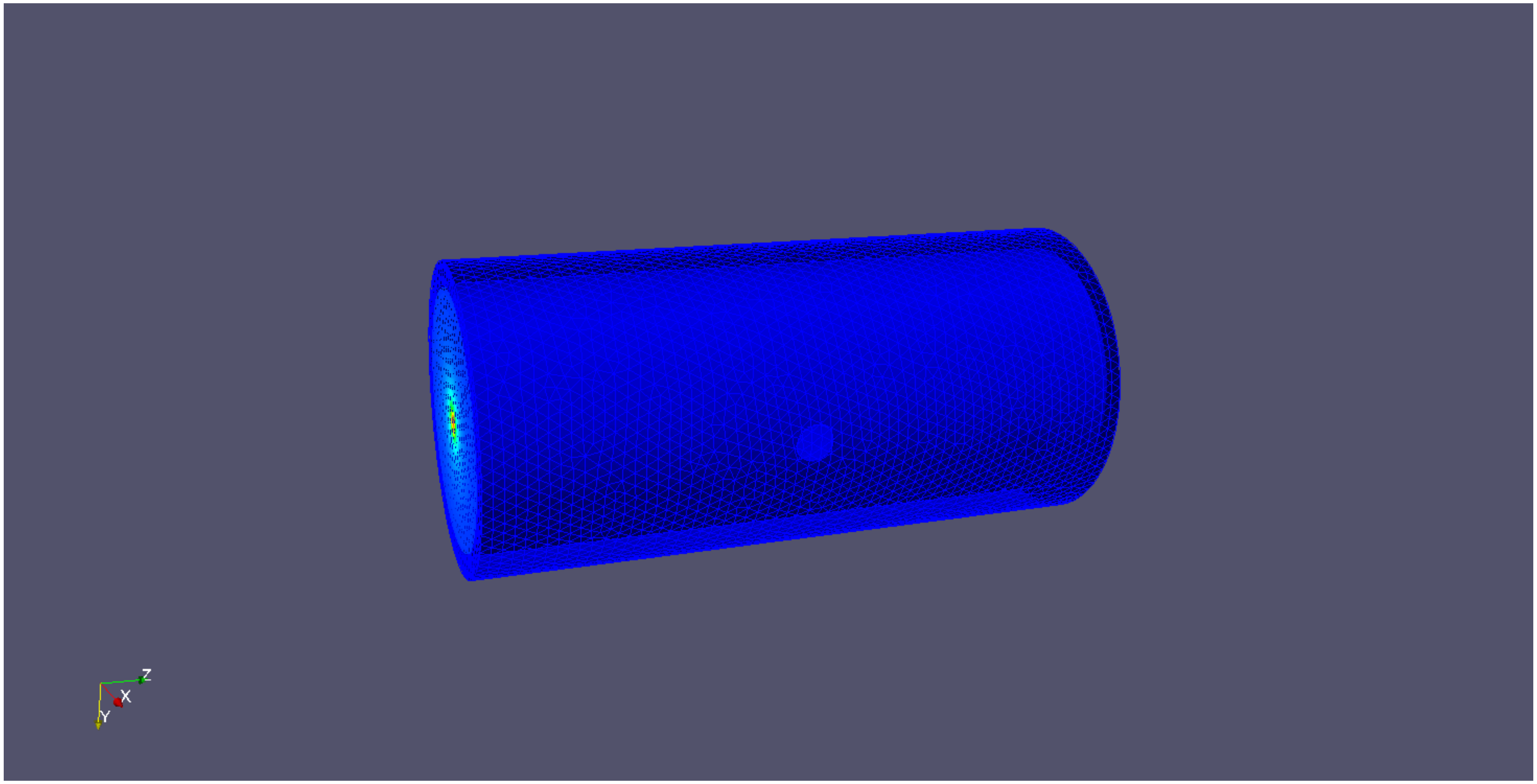
Real part is always zero, although loss factor and an "effective" real part can be calculated considering propagating modes coupling to the equivalent dipole moments and the power radiated away from the hole $\mathrm{Re}(Z_{//}) = 2P/I_0^2$
Dipole moments stay the same, regardless of the radiated power.

$$\begin{cases} \vec{M} = \vec{\vec{\alpha}}_m \cdot (\vec{H}_0 + \vec{H}_{in} - \vec{H}_{out}) \\ \vec{P} = \varepsilon \vec{\vec{\alpha}}_e \cdot (\vec{E}_0 + \vec{E}_{in} - \vec{E}_{out}) \end{cases}$$

To go around this discrepancy, we introduce reaction fields in the dipole moments calculations. Sort of a variational technique, <u>but limited to propagating fields</u>

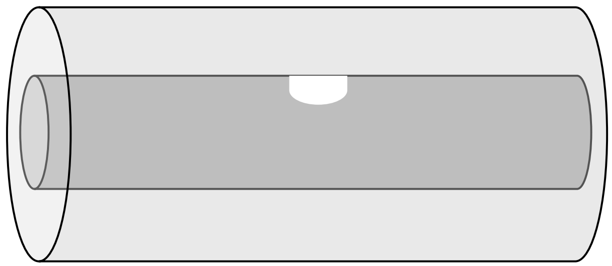
Hole in a coaxial pipe. Modified theory: $Z_{//} = \dfrac{Z_0\omega}{4\pi^2 c r_p^2}\left[-i(\alpha_{m\perp}+\alpha_e) + \dfrac{\omega(\alpha_{m\perp}^2+\alpha_e^2)}{4\pi c r_p^2 \ln(r_{out}/r_p)}\right]$

Modified Bethe's Diffraction Theory



6 mm hole, 1 cm bunch, in a coaxial pipe.
TEM wave has slight directional properties due to finite length of the hole (bunch enters on one side and leaves from the other).
Non propagating HOM's oscillate near the hole for a longer time

Modified Theory vs. Simulations (M. Zobov)



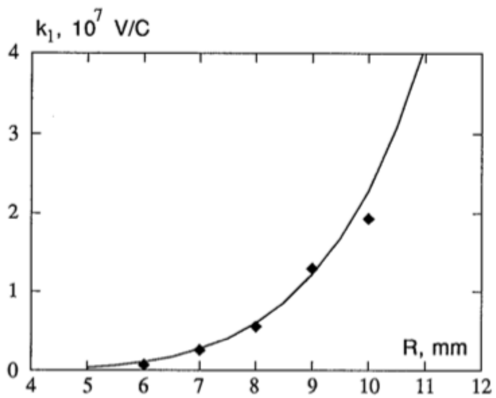


Fig. IX.1
Dependence of the loss factor on the hole radius

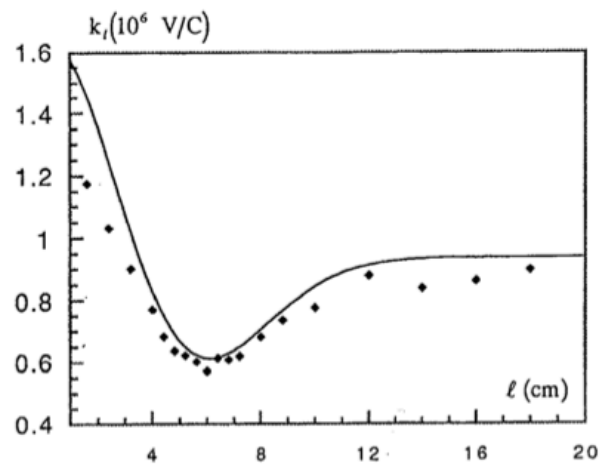


Fig. VII.4
Two holes loss factor (b=20 mm, d=24 mm, R=6 mm, σ=50 mm).

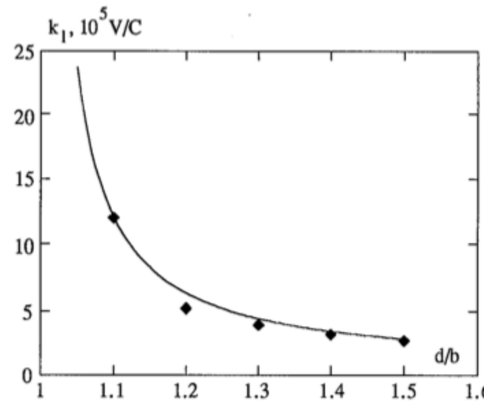


Fig. IX.2
Dependence of the loss factor on the pipe radii ratio.

Modified Theory – Differential Form

The Modified BDT allows to calculate the interaction between multiple apertures (if they are distant enough), but it cannot be used when the aperture dimensions are larger than the wavelength.

If this is the case, one can resort to use dynamic polarizabilities, that do not depend on the aperture geometry only and requires complex calculations, or subdivide the aperture in infinitesimal elements still satisfying the requirements for using the BDT and then take into account their interaction.

The dipole moments are replaced by differential dipole moments

$$d\vec{M} = d\ddot{\alpha}_m \cdot (\vec{H}_0 - \vec{H}_s),\ d\vec{P} = \varepsilon d\ddot{\alpha}_e \cdot (\vec{E}_0 - \vec{E}_s)$$

and integral equations take linear system place. In the case of a long narrow slot:

$$\frac{dM_\varphi}{dz} = \frac{\alpha_m}{L}\left[H_{0\varphi} - j\frac{\omega\mu h_{0\varphi}^2}{2}\int_{-L/2}^{L/2}\frac{dM_\varphi}{d\xi}e^{-jk_0|z-\xi|}d\xi + j\frac{\omega h_{0\varphi}e_{0r}}{2}\int_{-L/2}^{L/2}sign(\xi - z)\frac{dP_r}{d\xi}e^{-jk_0|z-\xi|}d\xi\right]$$

$$\frac{dP_r}{dz} = \frac{\varepsilon\alpha_e}{L}\left[E_{0r} - j\frac{\omega\mu e_{0r}^2}{2}\int_{-L/2}^{L/2}\frac{dP_r}{d\xi}e^{-jk_0|z-\xi|}d\xi + j\frac{\omega\mu h_{0\varphi}e_{0r}}{2}\int_{-L/2}^{L/2}sign(\xi - z)\frac{dM_\varphi}{d\xi}e^{-jk_0|z-\xi|}d\xi\right]$$

LHC Liner – A. Mostacci (PRST-AB, 1999)

$$P_L = q_b^2 k(\sigma)\frac{c}{T_b} / L$$

Applied the Modified Bethe's Theory to calculate the power dissipated per unit length in a liner model in presence of attenuation

Short device

$$P_{lin} = \frac{\sqrt{\pi}}{128\pi^4}\frac{Z_0 Q_b^2 c^2}{\sigma^3 S_b}\frac{(\alpha_m+\alpha_e)^2}{b^4 D\ln(d/b)} L$$

Long device

$$P_\infty = \left\{\frac{Z_0 Q_b^2 c^2}{\sigma^3 S_b}\left[\frac{\sqrt{\pi}}{8} + \Gamma(5/4)\frac{\sqrt{\sigma\omega/c}}{2\alpha D}\right]\right\}\frac{(\alpha_m+\alpha_e)^2}{b^4 D\ln(d/b)}$$

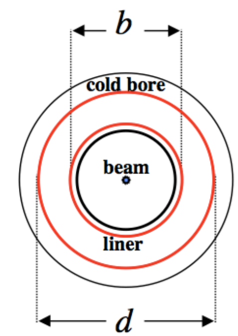
Two different asymptotic regimes were identified, in accordance with Sands and Stupakov papers. In the "linear" regime the device is much shorter than the attenuation length so that all the holes can interfere by way of the TEM mode. The power per unit length grows linearly with the device length i.e. the power loss is proportional to the number of holes squared.

In the long device regime a fixed number of holes is interacting and the power lost per unit length is constant i.e. proportional to the device length, or the number of holes.

LHC Liner (cont.)

Approx: distributed "dielectric" losses

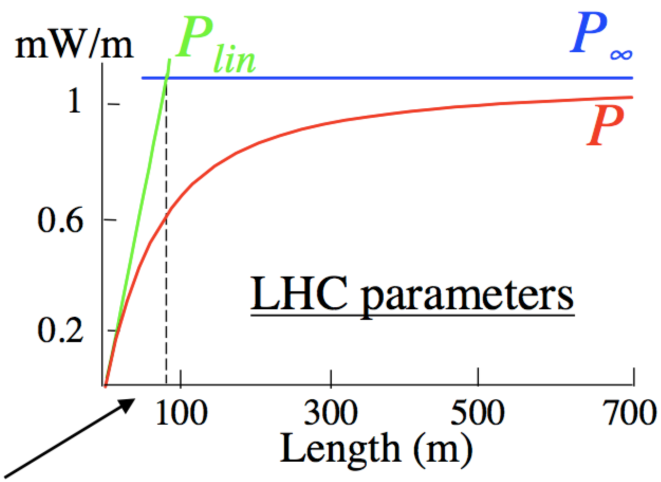
$$\alpha(\omega) = \frac{\sqrt{\rho}}{2Z_0 \ln(d/b)}\left(\frac{1}{b}+\frac{1}{d}\right)\sqrt{\frac{\mu}{2}}\sqrt{\omega}$$



P_{lin} no attenuation

P_∞ limit value

P "exact" formula



$$L_\alpha = \frac{4}{\sqrt{\pi}}\Gamma\left(\frac{5}{4}\right)\frac{1}{\alpha(\omega_c)}$$

ω_c Bunch cut-off angular frequency

$$Z_{\mathrm{Re}}(\omega) = Z_0 \frac{\omega^2}{16\pi^3 b^4 \ln(d/b) c^2}\left[N(\alpha_m^2 + \alpha_e^2) + (\alpha_m + \alpha_e)^2 F(D,N,\alpha) + (\alpha_m - \alpha_e)^2 B(D,N,\alpha,\omega)\right],$$

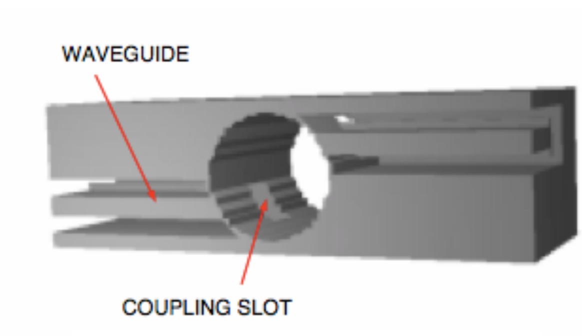
$$F(D,N,\alpha) = \sum_{h=1}^{N}\sum_{k=1}^{h-1} e^{\alpha(z_k - z_h)} = \sum_{h=1}^{N}\sum_{k=1}^{h-1} e^{\alpha(k-h)D}, \qquad B(D,N,\alpha,\omega) = \sum_{h=1}^{N-1}(N-h)e^{-\alpha h D}\cos\left(2h\frac{\omega}{c}D\right).$$

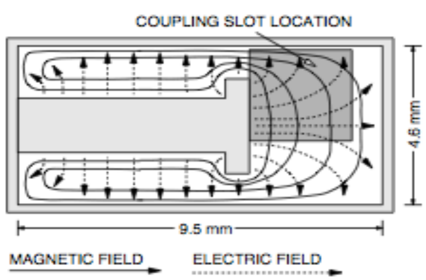
Waveguide BPM – T. Kamps (PAC 2001)

$$\left|\frac{H_{(0,1)}}{H_{\varphi}}\right| = \frac{2\alpha_m k_z/(ab)}{\sqrt{1+\left(2\alpha_m k_z/(ab)\right)^2}}$$

$$\alpha_m = 0.67\frac{\pi}{16}\ell w^2 e^{-\pi T/w}$$

Small-sized BPM used in the TTF FEL undulator to ensure overlap of electron beam and seed laser (10 modules).
12 GHz design frequency. Ridged waveguide coupled to the vacuum chamber through a 2.5x2.5 mm slot.





Simplified model

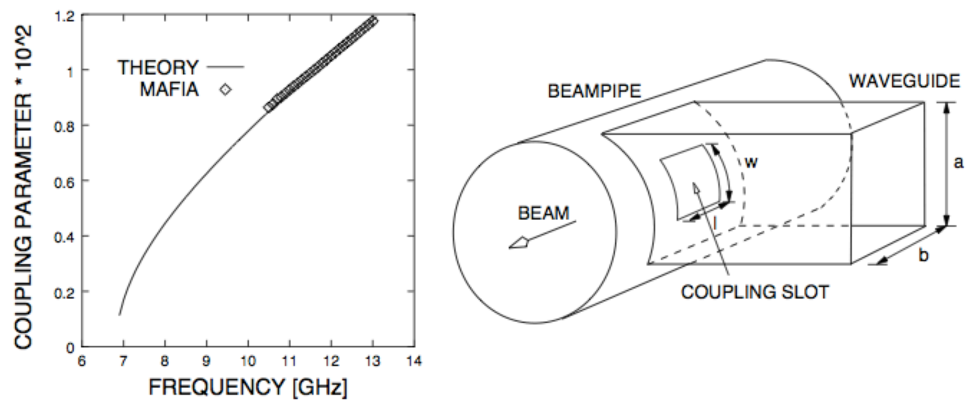
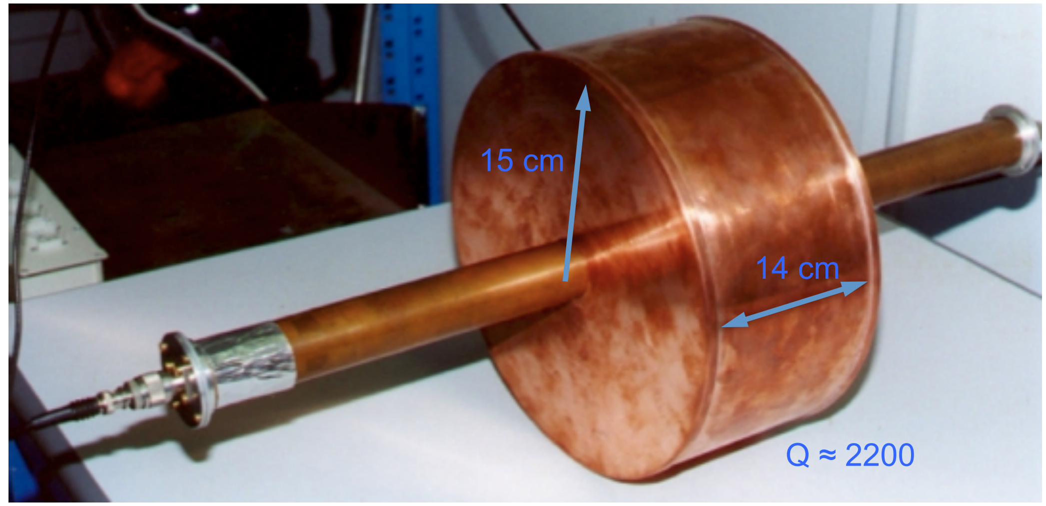
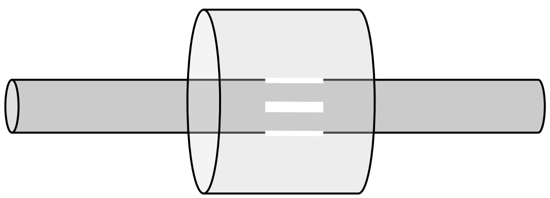


Figure 3: *Left: Comparision chart for MAFIA S_{21} simulations against analytical calculations. Right: Geometry used for calculations.*

Coaxial Cavity – A. Argan (EPAC 2000)

Coaxial cavity coupled through 4 narrow slots to the beampipe. Coaxial wire measurements were compared to the impedance value obtained applying the modified Bethe's theory.





$$P_r = \alpha_e \varepsilon_0 \left[E_{0r} - \frac{N}{\tilde{k}} \left(-j\omega\mu_0 k_n e_{rn} h_{\varphi n} \alpha_{m\perp} H_{0\varphi} + \right. \right.$$
$$\left. \left. + \omega^2 \mu_0 \tilde{q} e_{rn}^2 \alpha_e \varepsilon_0 E_{0r} \right) \right]$$

$$M_\varphi = \alpha_{m\perp} \left[H_{0\varphi} + \frac{N}{\tilde{k}} \left(j\omega k_n e_{rn} h_{\varphi n} \alpha_e \varepsilon_0 E_{0r} + \right. \right.$$
$$\left. \left. + k_0^2 h_{\varphi n}^2 \alpha_{m\perp} H_{0\varphi} \right) \right] \quad (10)$$

Equivalent dipole moments

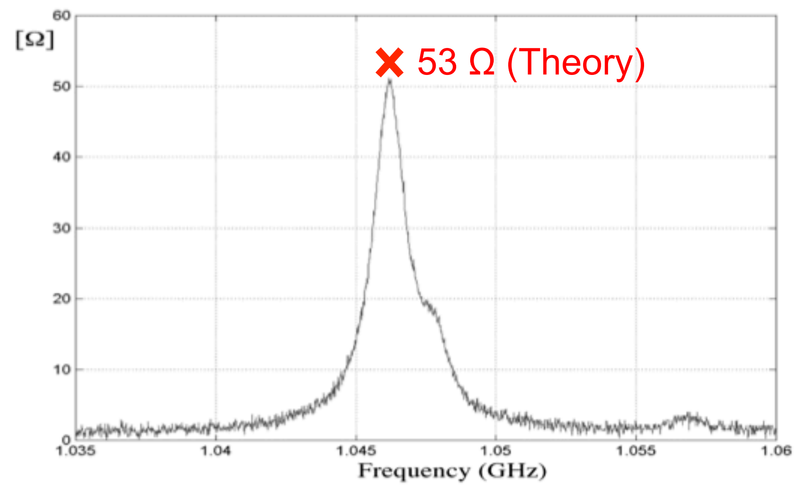
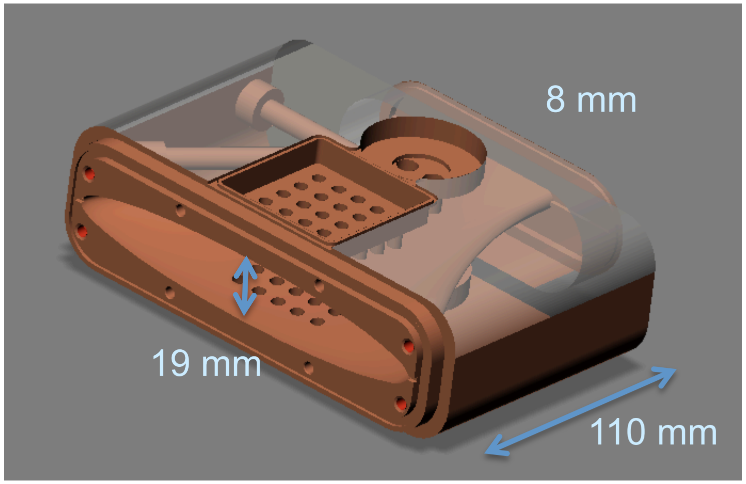


Figure 3: Long slots. Real part of the impedance.

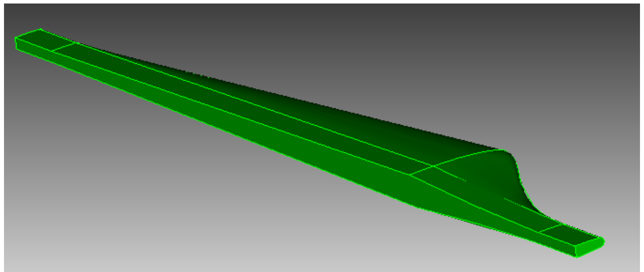
ALS Photon Stop – T. Luo (April 2014)

In real life holes do not usually come on long lengths of uniform cylindrical pipe, but are there together with other elements with their own impedance.
Theoretical methods become almost but impossible to apply (complicated boundary conditions).
3D simulations are not simple to interpret: which part of the impedance is due to one element, or to the other ? Which one to their interaction ?



Taper + Pumping Holes

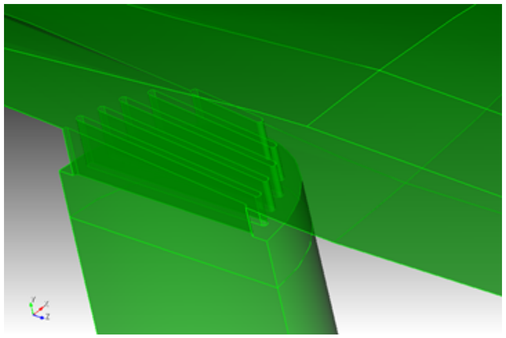
We run T3P on a NERSC cluster with 1280 CPU's. A 30,000 tetrahedral elements (2nd order) model over 30,000 time steps run takes ~30 mins.

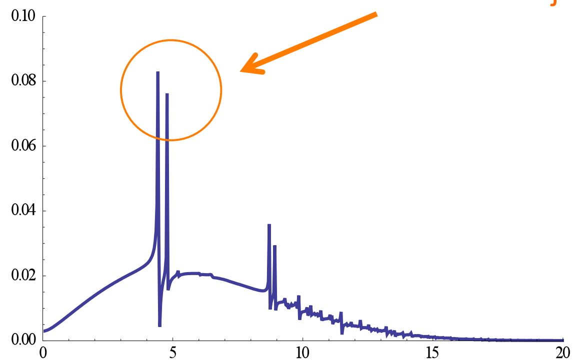


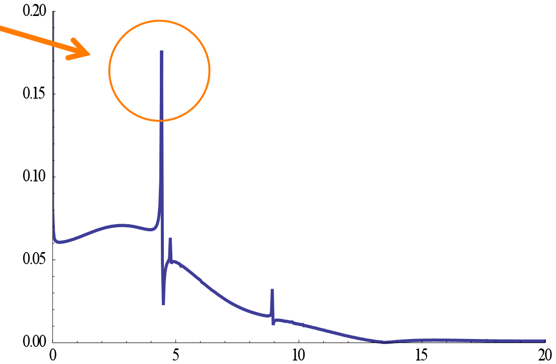
T3P can only run with identical ports. We added a long smooth taper (and subtract the asymptotic value for the step-out impedance).

ALS Photon Stop – Slots or Holes ?

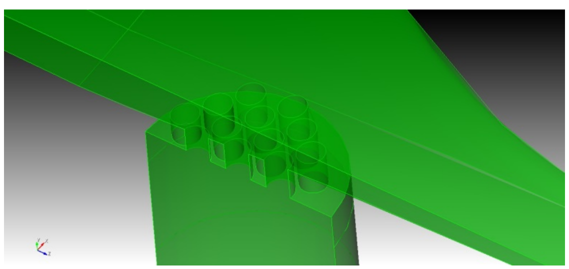
Resonances between adjacent slots

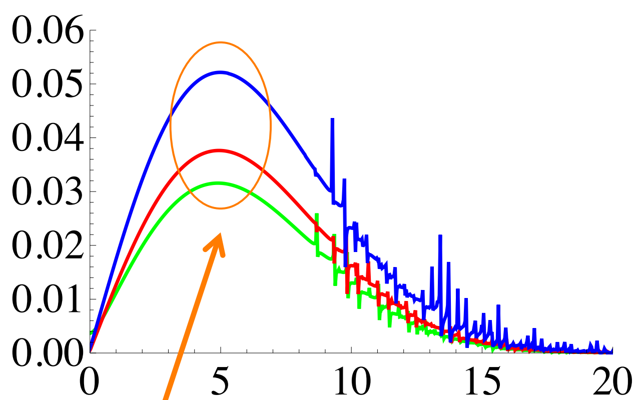


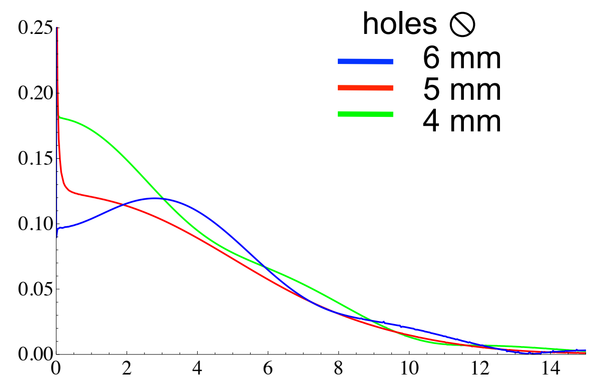




Fourier Transform of Longitudinal and Transverse Wake Potentials (σ = 1 cm)







Impedance seems only dependent ~linearly on R

Conclusions

- **Well Established Theory For Small Holes and Slots**
  - But it only gives broad guidelines at best in many real life situations
- **Ever Powerful Simulation Codes**
  - Is there still a need for further theoretical research ?
- **Possible Topics For Further Investigation**
  - Ultra-short bunches (FEL's): <100 μm -> bunch cutoff ~ THz
  - Non-gaussian bunches: THz/IR sources, HHC lengthened
  - Small vacuum chambers: ultra-low emittance rings, $r_p \approx$ few mm