Analytical methods for inserts of finite length as benchmarkers

ICFA mini-Workshop on
"Electromagnetic wake fields and impedances in particle accelerators"

Nicolò Biancacci

26th April 2014, Erice, Italy

Acknowledgements: V.G.Vaccaro, E.Métral, B.Salvant, M.Migliorati, L.Palumbo.

Outline

Motivation

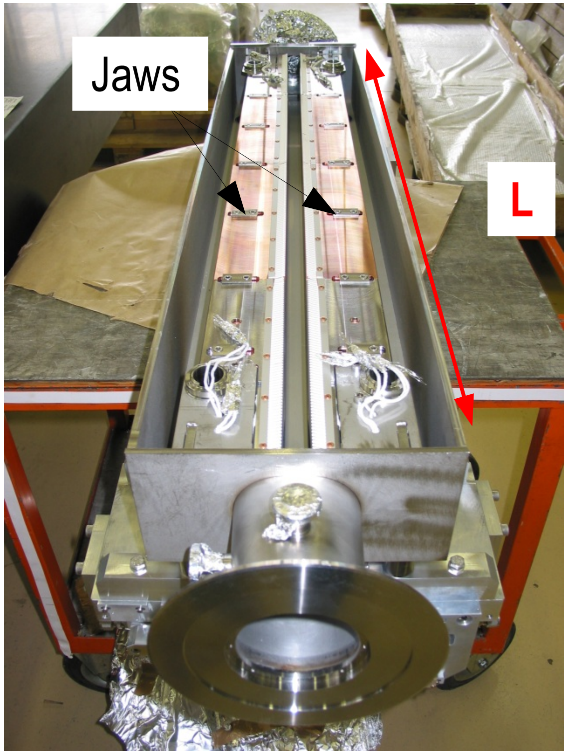
The Mode Matching method

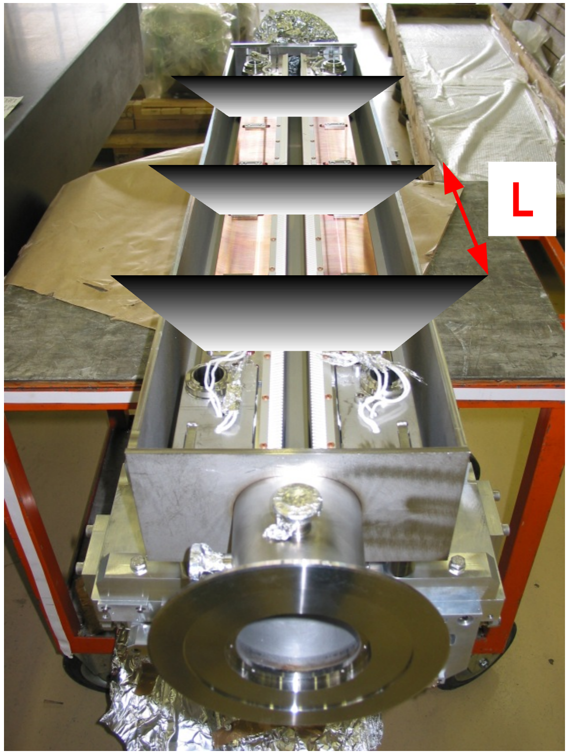
Benchmarks

Applications

Motivation

CERN → Studies of collimator impedance reduction by segmentation in modules.

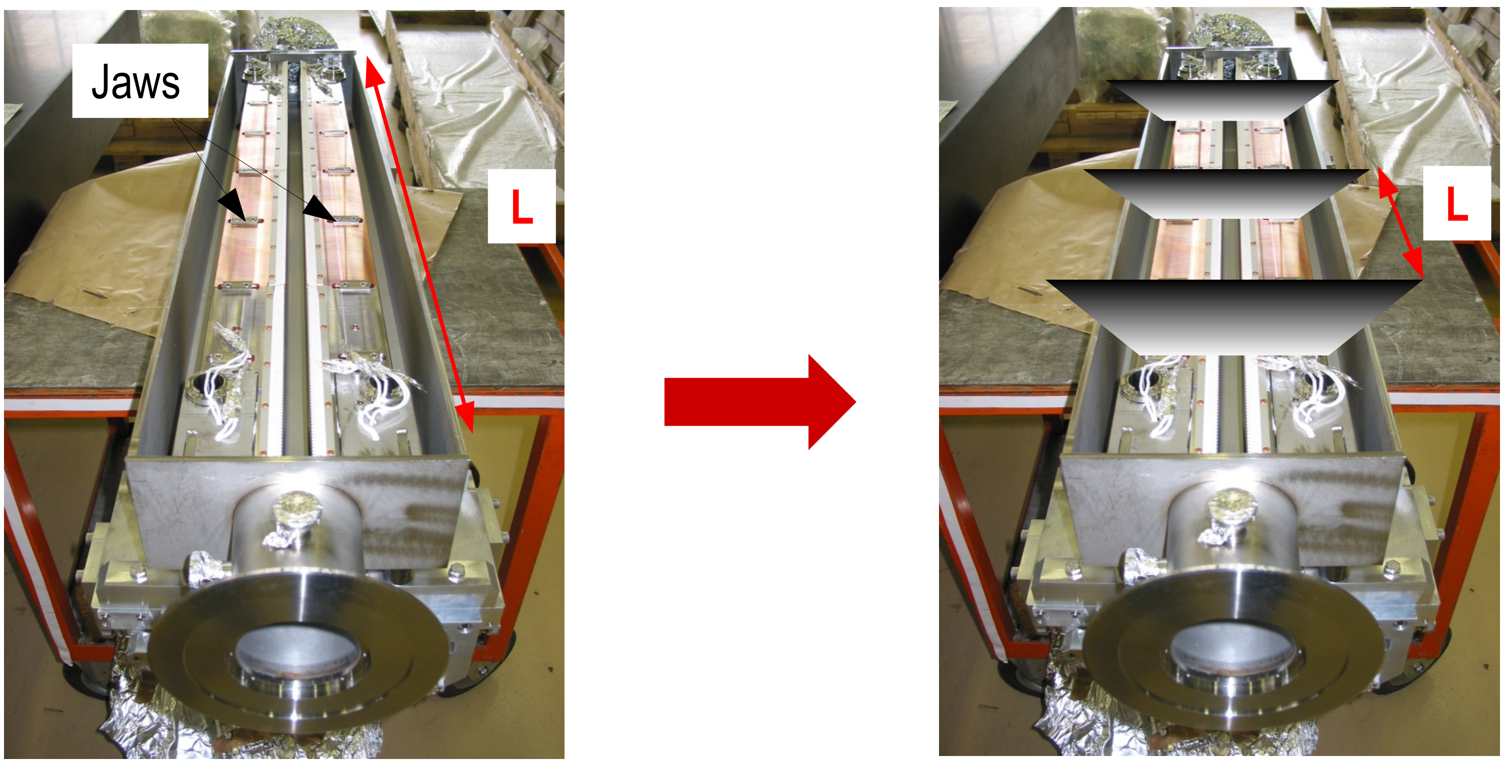




Motivation

CERN → Studies of collimator impedance reduction by segmentation in modules.

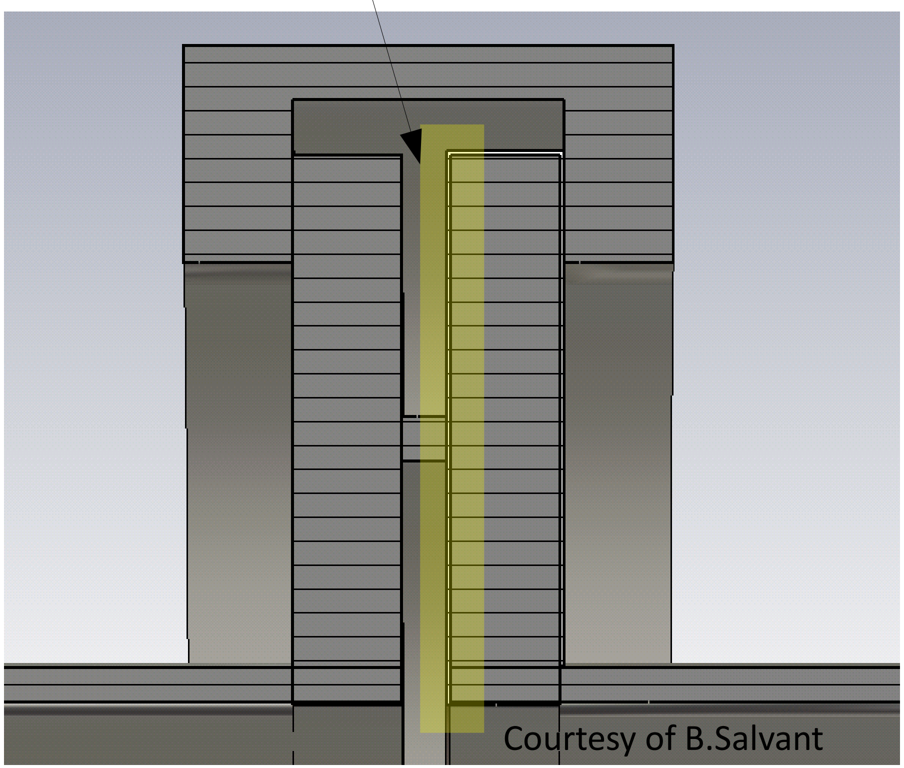
What is the dependence of the impedance on length?



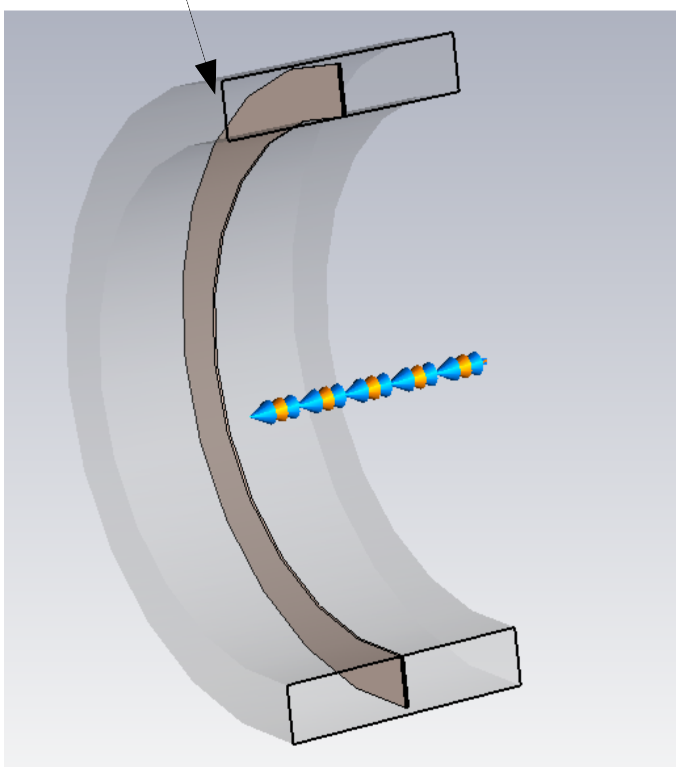
Motivation

Pipe flanges → Thin inserts → numerical simulators encounter meshing problems!

Beam pipe flanges



CST model

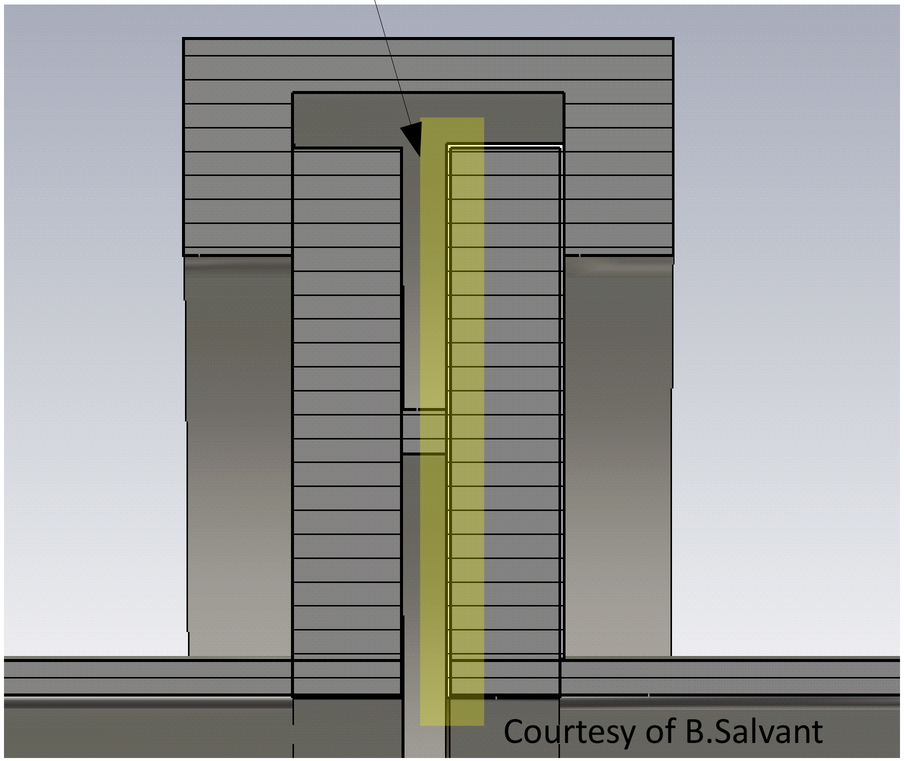


Motivation

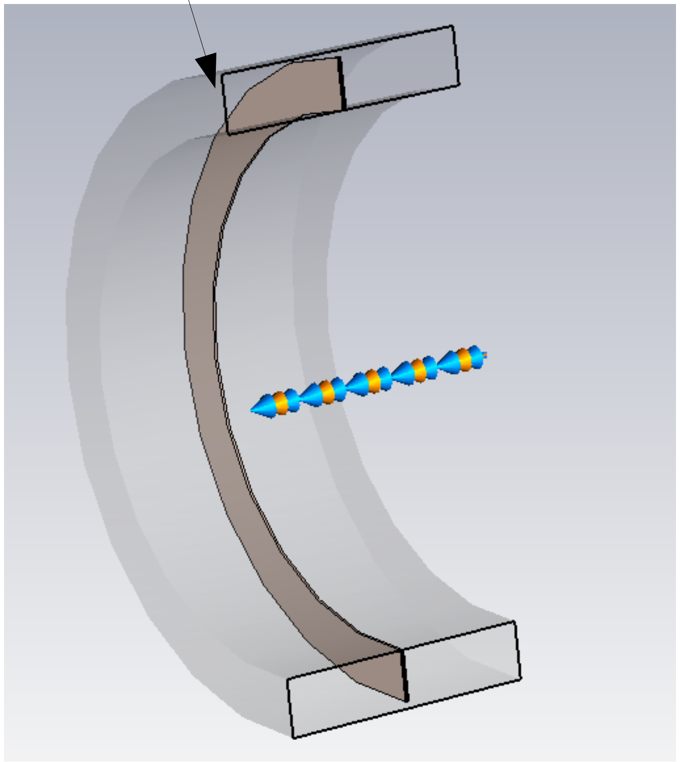
Pipe flanges → Thin inserts → numerical simulators encounter meshing problems!

Impedance of thin inserts?

Beam pipe flanges

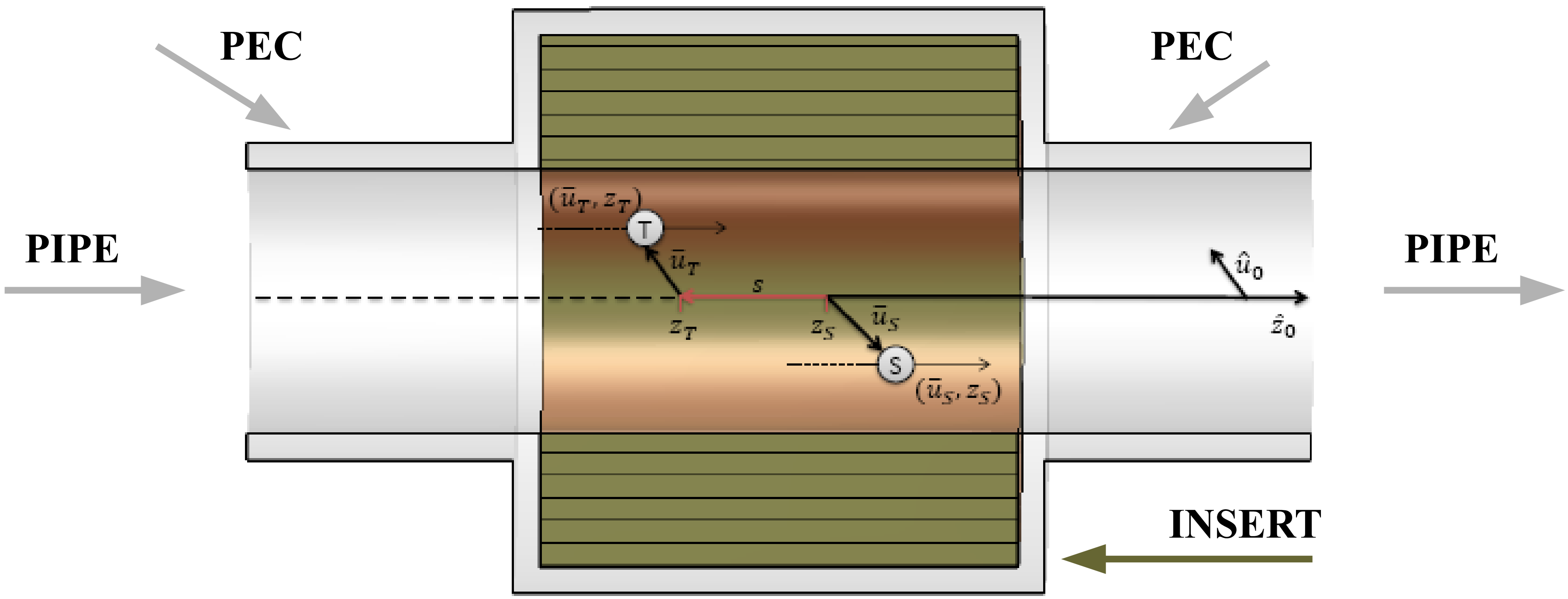


CST model



Model

Model → Cavity loaded with a toroidal insert connected to the beam pipes.



Assumptions →
- ✔ Any relativistic β
- ✔ Any insert length, pipe/cavity radius
- ✔ Any frequency range
- ✔ Linear, isotropic, homogeneous, dispersive material

Targets →
- ✯ Longitudinal and transverse dipolar (or driving) impedances

Past work

Past studies already done in order to assess the impedance of finite length devices.

Year	Authors	Note
2004	S. Krinsky et al., Phys. Rev. ST Accel. Beams 7.	Leontovich approx.
2005	G. Stupakov, Phys. Rev. ST Accel. Beams 8.	Leontovich approx.
2006	R.L. Gluckstern and B. Zotter, unpublished.	Unpublished
2009	Y. Shobuda et al. Phys. Rev. ST Accel. Beams 12.	Short length approx.

Past work

Past studies already done in order to assess the impedance of finite length devices.

Year	Authors	Note
2004	S. Krinsky et al., Phys. Rev. ST Accel. Beams 7.	Leontovich approx.
2005	G. Stupakov, Phys. Rev. ST Accel. Beams 8.	Leontovich approx.
2006	R.L. Gluckstern and B. Zotter, unpublished.	Unpublished
2009	Y. Shobuda et al. Phys. Rev. ST Accel. Beams 12.	Short length approx.

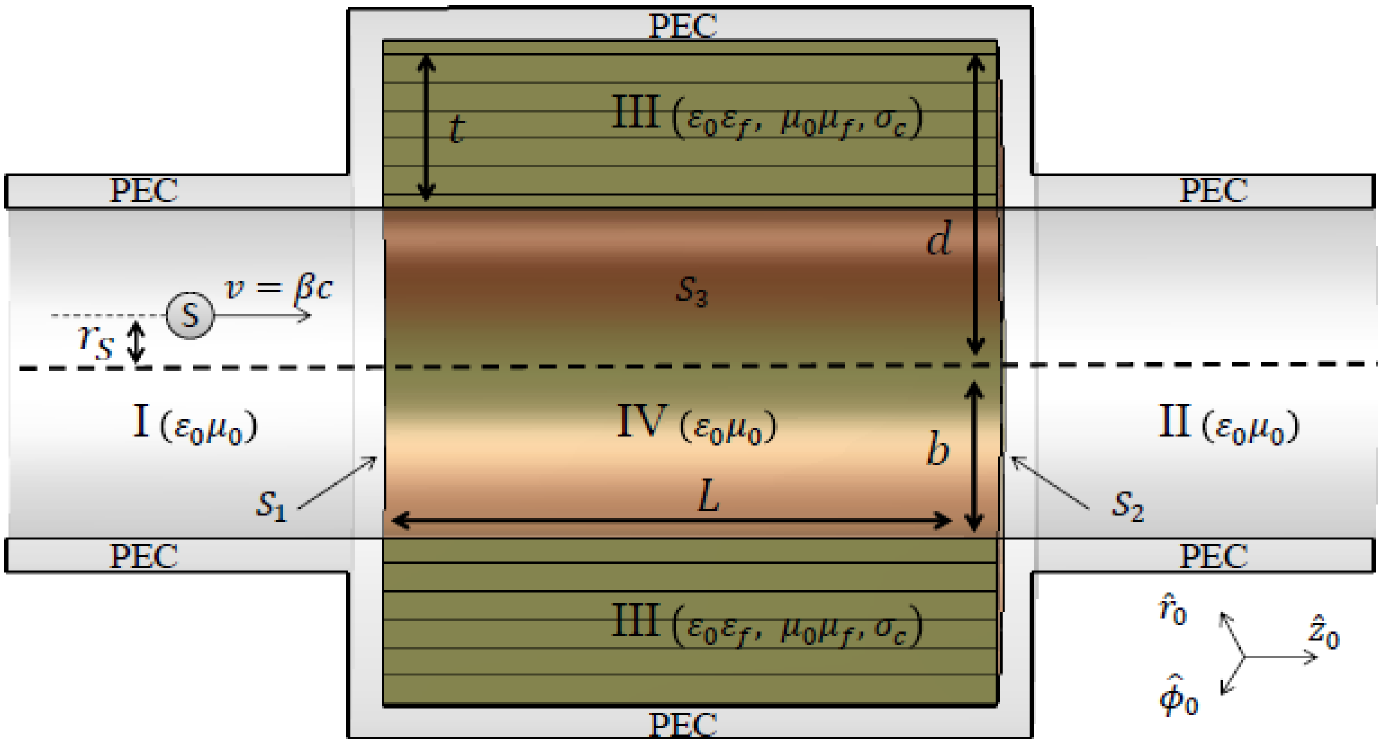
See also previous talk
"Impedance of a ceramic break
and its resonance structure"
by Y. Shobuda

We solved it applying the...

Mode Matching method

Splitting volumes:

We have 4 unknowns for the 4 volumes → We need 4 matching conditions!



Mode Matching method

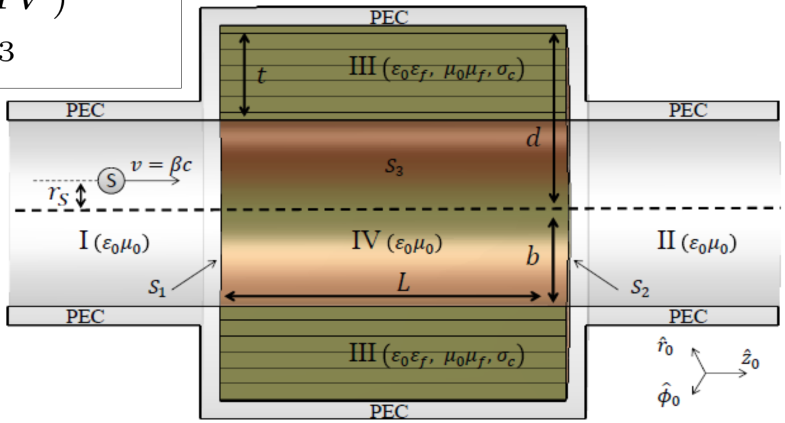
Splitting volumes:

We have 4 unknowns for the 4 volumes → We need 4 matching conditions!

Matching conditions:

3 Conditions: Field Matching of Magnetic fields at separation surfaces.

$$H_{S_1}^{(I)} = H_{S_1}^{(IV)} \quad H_{S_2}^{(II)} = H_{S_2}^{(IV)} \quad H_{S_3}^{(III)} = H_{S_3}^{(IV)}$$



Mode Matching method

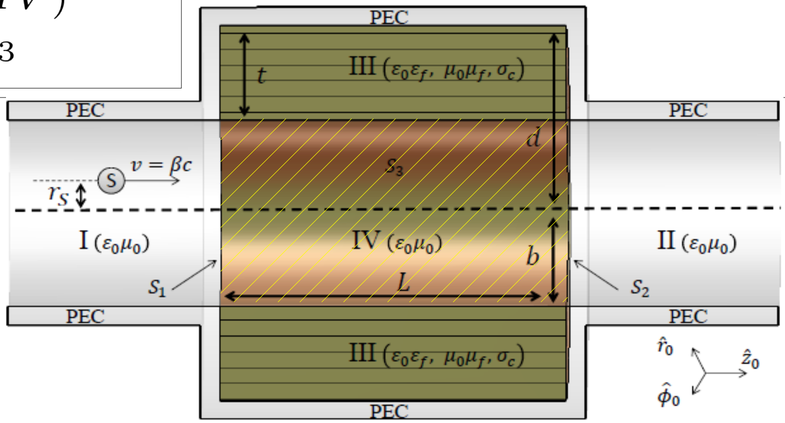
Splitting volumes:

We have 4 unknowns for the 4 volumes → We need 4 matching conditions!

Matching conditions:

3 Conditions: Field Matching of Magnetic fields at separation surfaces.

$$H_{S_1}^{(I)} = H_{S_1}^{(IV)} \quad H_{S_2}^{(II)} = H_{S_2}^{(IV)} \quad H_{S_3}^{(III)} = H_{S_3}^{(IV)}$$



1 Condition: Mode Matching in the cavity volume.

Modes in volume IV
solenoidal + irrotational
PEC boundary conditions[1]

$$E = \sum_n \mathbf{V}_n e_n + \sum_n \mathbf{F}_n f_n$$

Coefficients[2]
in function of external fields
(adiacent volumes)

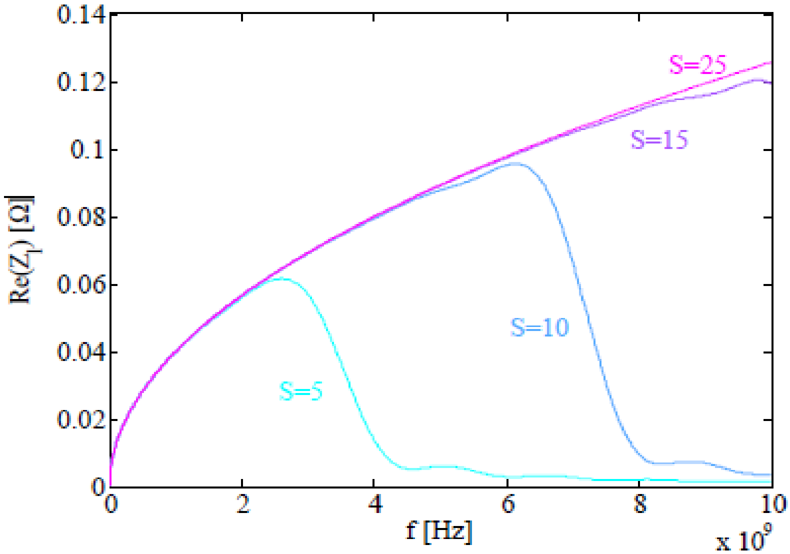
$$\begin{cases} \mathbf{V}_n = \dfrac{k_n}{k^2 - k_n^2} \displaystyle\int_S (E \times h_n^*) \cdot \hat{n}_0 \, \mathrm{d}S \\ \mathbf{F}_n = -j \dfrac{Z_o}{k} \displaystyle\int_S (H \times f_n^*) \cdot \hat{n}_0 \, \mathrm{d}S \end{cases}$$

N. B.:

1) The series converges non-uniformly at the boundaries.
2) Coefficients would be null with a standard electric Field Matching.

Method benchmark

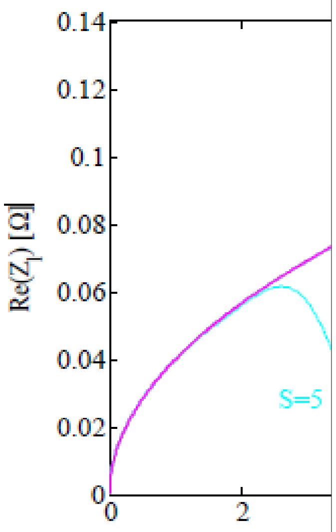
- Convergence tests.



Convergence demonstrated in function of the cavity modes.

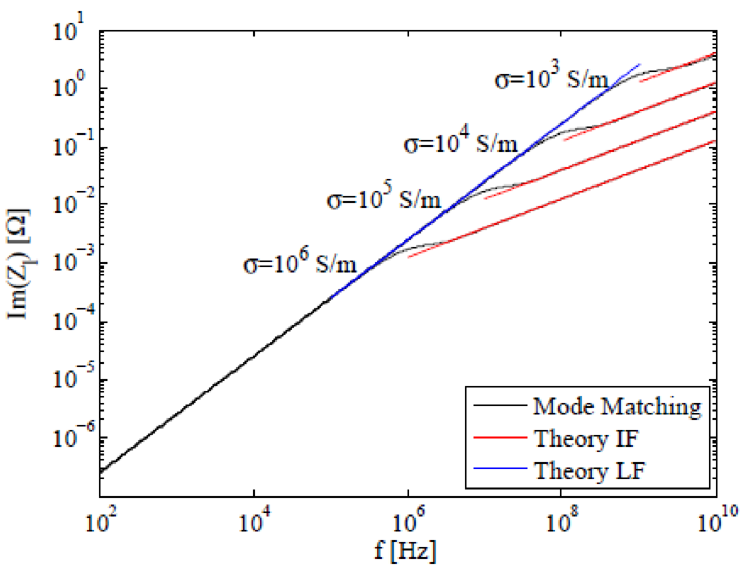
Method benchmark

✔ Convergence tests.



Convergence
function of th

✔ Classical “resistive wall”

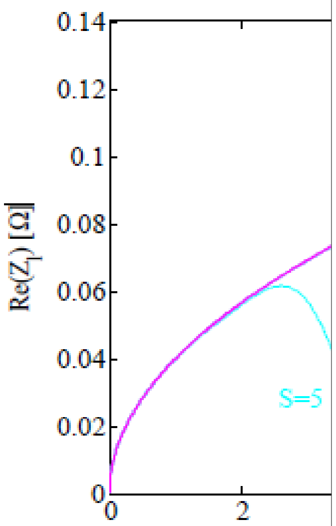


Benchmark for high conductivity materials (beam pipe model)

See also previous talk
“2D wall impedance theory”
by N.Mounet

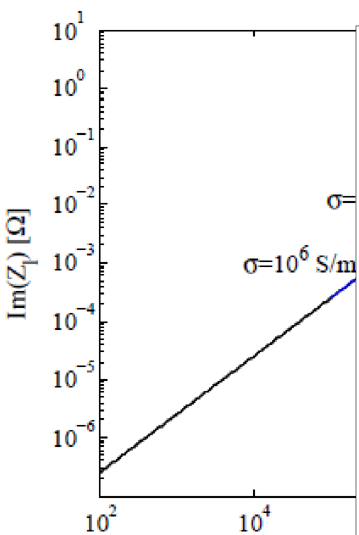
Method benchmark

✔ Convergence tests.



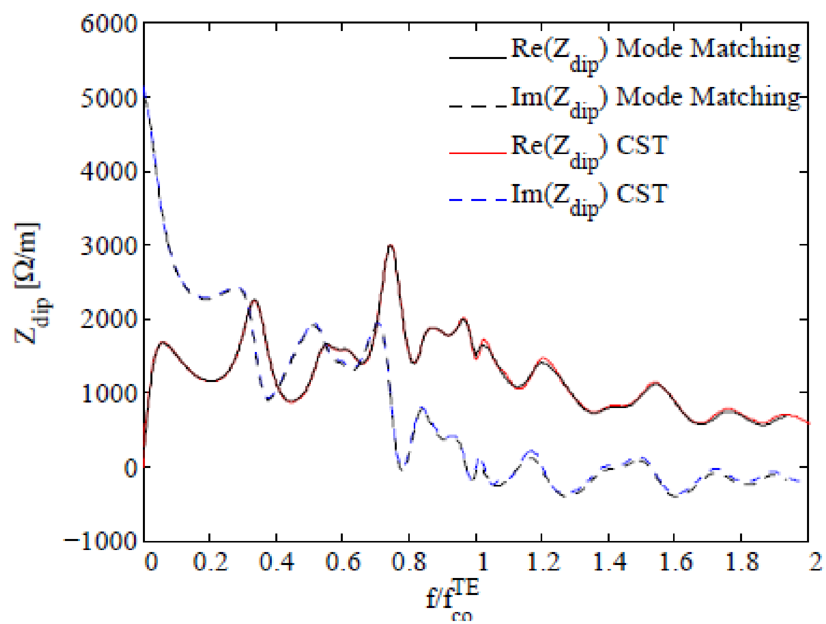
Convergence
function of th

✔ Classical "resistive wall"



Benchmark for
materials (be

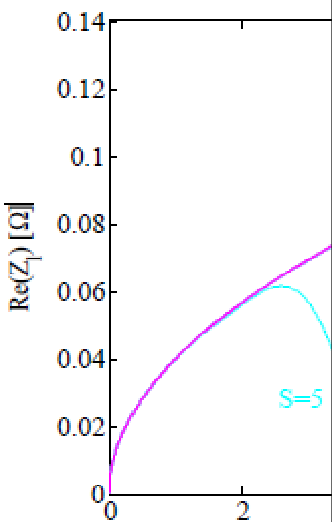
✔ Benchmark with CST.



Benchmark for low conductivity
materials (loaded cavities)

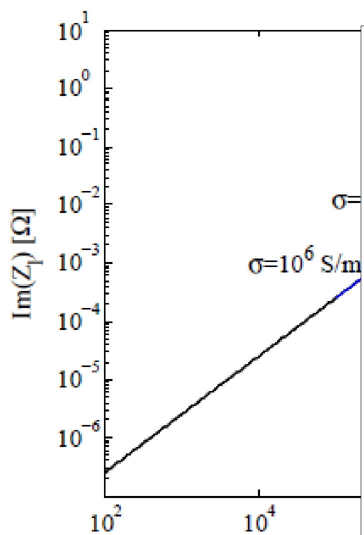
Method benchmark

✔ Convergence tests.



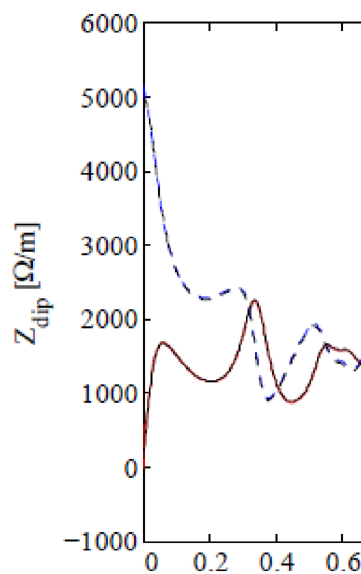
Convergence
function of th

✔ Classical "resistive wall"



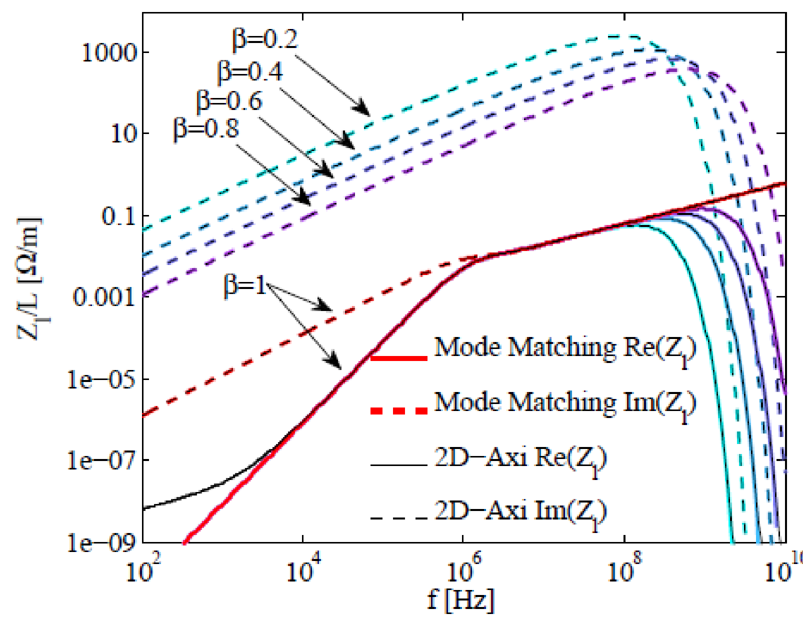
Benchmark for
materials (be

✔ Benchmark with CST.



Benchmark
materials

✔ Valid for every beam velocity.



Benchmark for low beam velocity
(low energy machines)

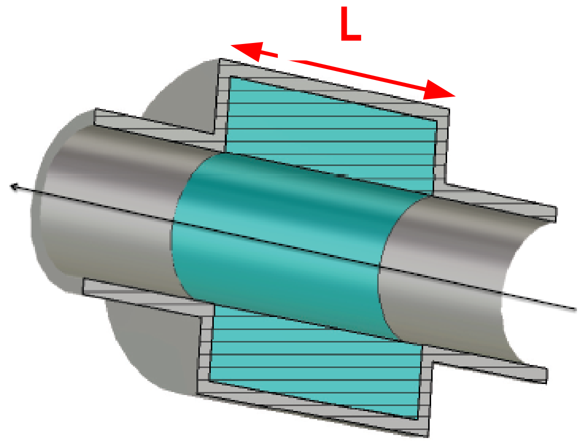
Applications

- ✔ Impedance dependence on length.
- ✔ Protrusion trapped modes below cut-off.
- ✔ Insert trapped modes at cut-off.

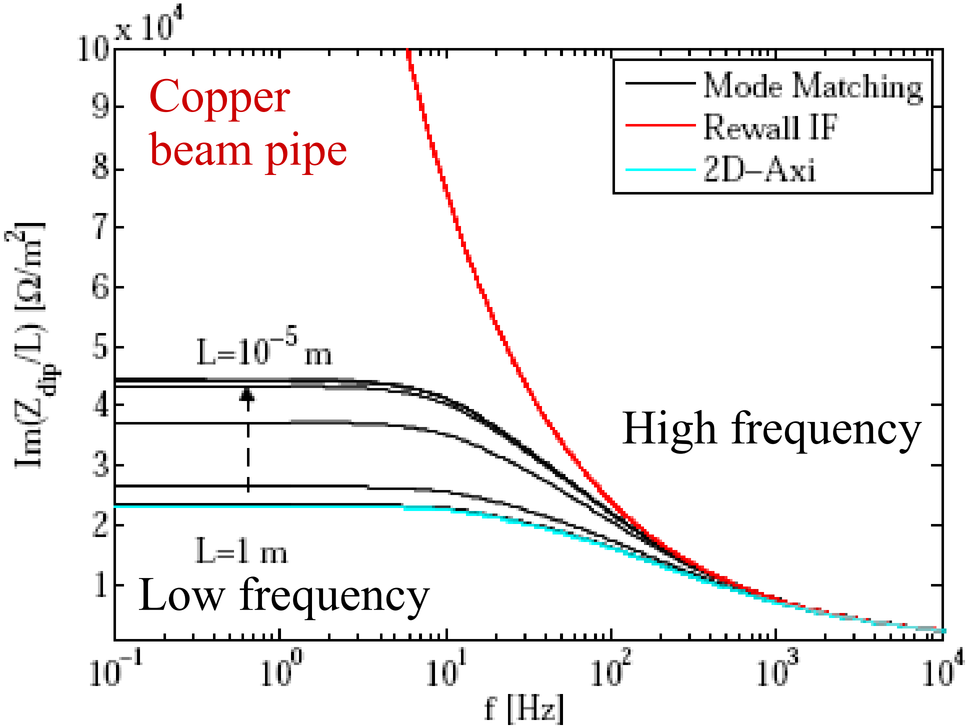
Applications

- ✔ Impedance dependence on length.
- ✔ Protrusion trapped modes below cut-off.
- ✔ Insert trapped modes at cut-off.

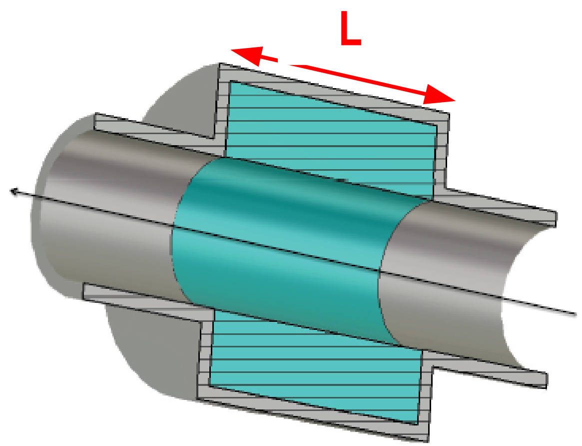
Applications



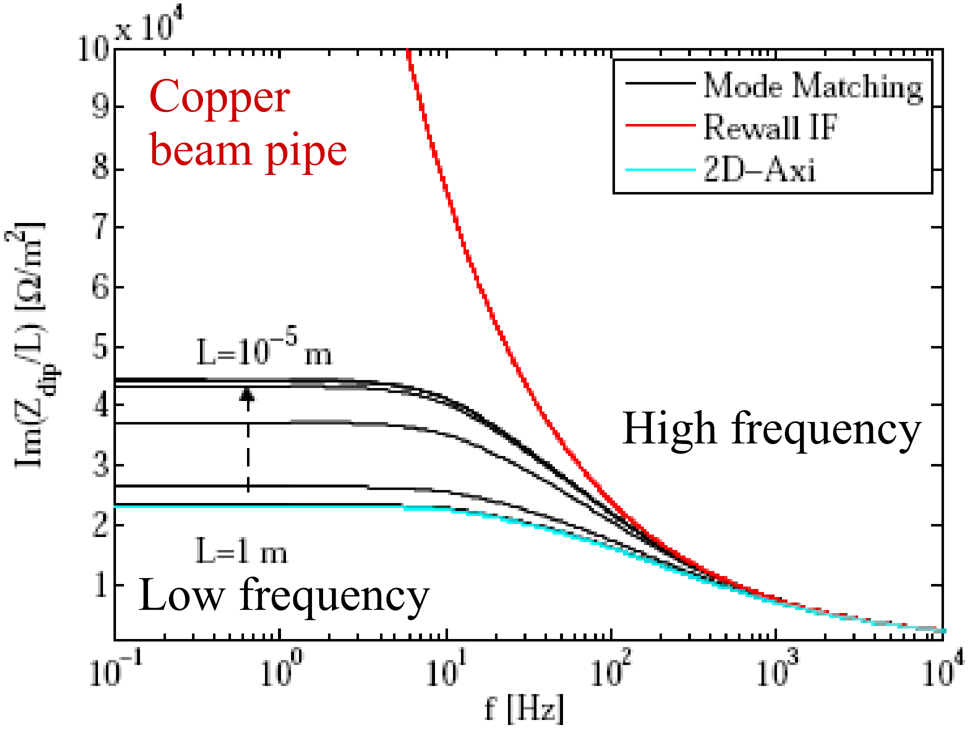
- Studied the impedance dependence on length L.
- Taking the ratio w.r.t. the 2D impedance model.

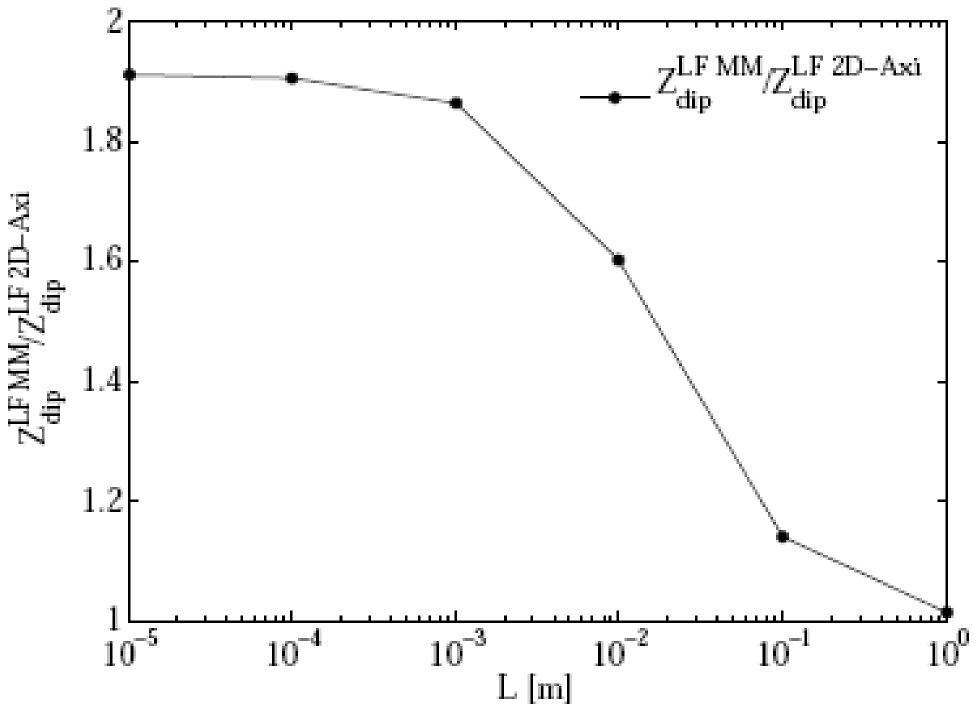


Applications



- Studied the impedance dependence on length L.
- Taking the ratio w.r.t. the 2D impedance model.
- Difference apparent only in the transverse impedance at low frequency, and very narrow gaps → for the moment an academical case.

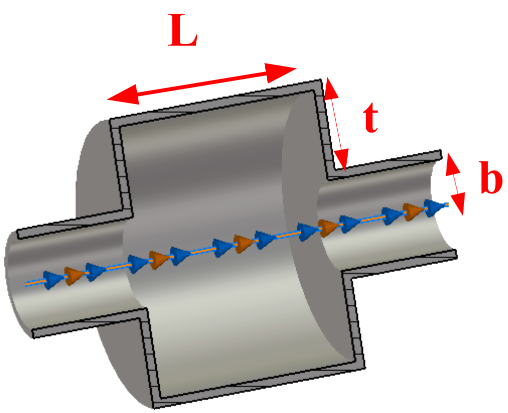




Applications

- ✔ Impedance dependence on length.
- ✔ Protrusion trapped modes below cut-off.
- ✔ Insert trapped modes at cut-off.

Trapped modes below cut-off



- Small empty cylindrical beam pipe protrusion.
- A trapped mode close to cut-off appears at frequency:

$$f_{trap} \simeq f_{\mathrm{TM}_{01}} + \Delta f_{\mathrm{TM}_{01}}$$

- Theory[1] predicts:

$$\Delta f_{\mathrm{TM}_{01}} = -f_{\mathrm{TM}_{01}} \frac{\alpha_{0,1}^2}{2} \left(\frac{A^2}{b} \right)^2$$

First Bessel zero

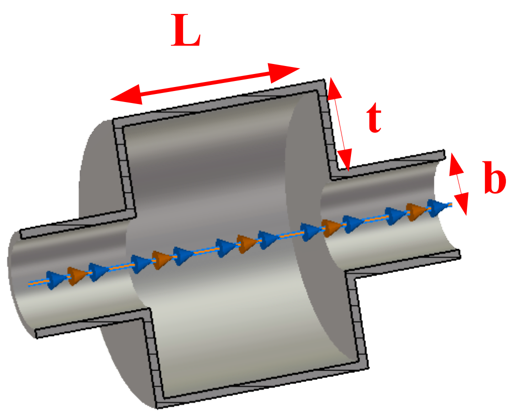
Protrusion Area $A = L \cdot t$

Pipe radius

Cut-off frequency

1) G. Stupakov and S. S. Kurennoy. Trapped electromagnetic modes in a waveguide with a small discontinuity. Phys. Rev. E , Jan 1994.

Trapped modes below cut-off



- Small empty cylindrical beam pipe protrusion.
- A trapped mode close to cut-off appears at frequency:

$$f_{trap} \simeq f_{\mathrm{TM}_{01}} + \Delta f_{\mathrm{TM}_{01}}$$

- Theory[1] predicts:

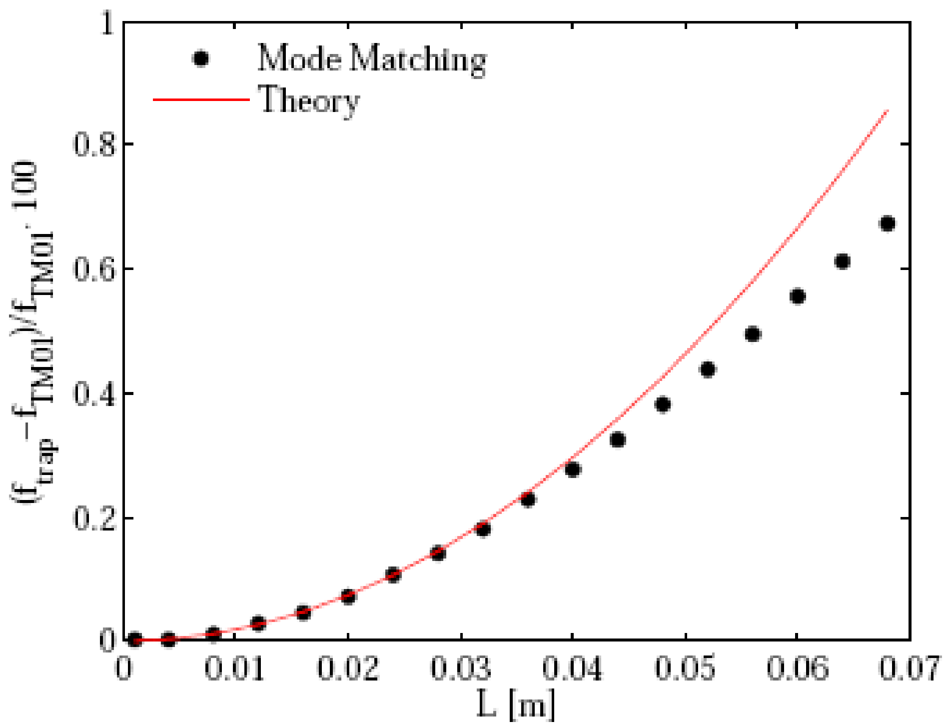
$$\Delta f_{\mathrm{TM}_{01}} = -f_{\mathrm{TM}_{01}} \frac{\alpha_{0,1}^2}{2} \left(\frac{A^2}{b} \right)^2$$

First Bessel zero

Protrusion Area $A = L \cdot t$

Pipe radius

Cut-off frequency

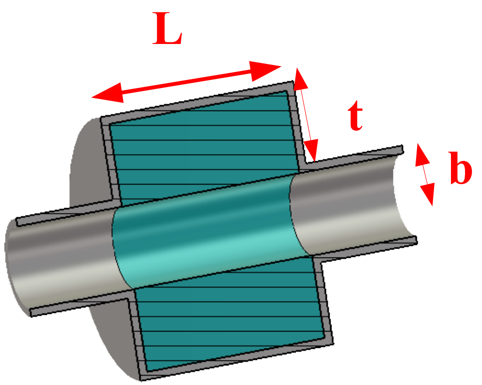


1) G. Stupakov and S. S. Kurennoy. Trapped electromagnetic modes in a waveguide with a small discontinuity. Phys. Rev. E , Jan 1994.

Applications

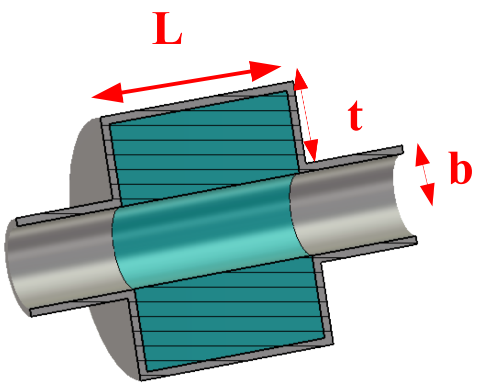
- ✔ Impedance dependence on length.
- ✔ Protrusion trapped modes below cut-off.
- ✔ Insert trapped modes at cut-off.

Trapped modes at cut-off

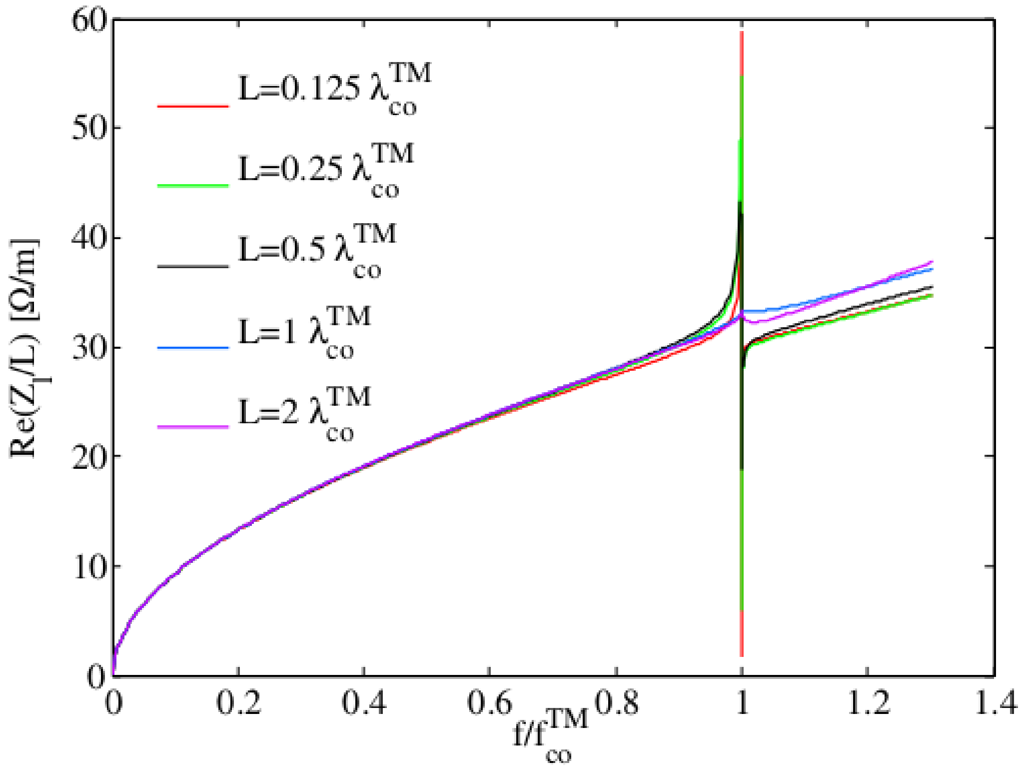


- Cylindrical cavity filled with material of $\sigma = 10^2\ \mathrm{S/m}$
- Longitudinal impedance case.

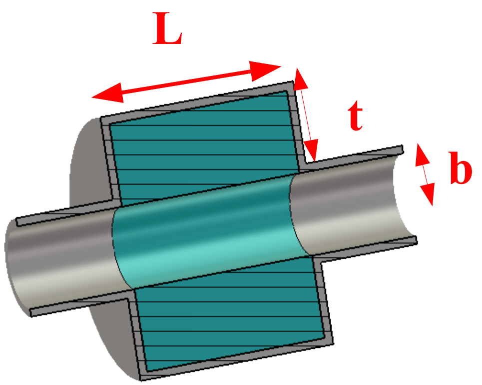
Trapped modes at cut-off



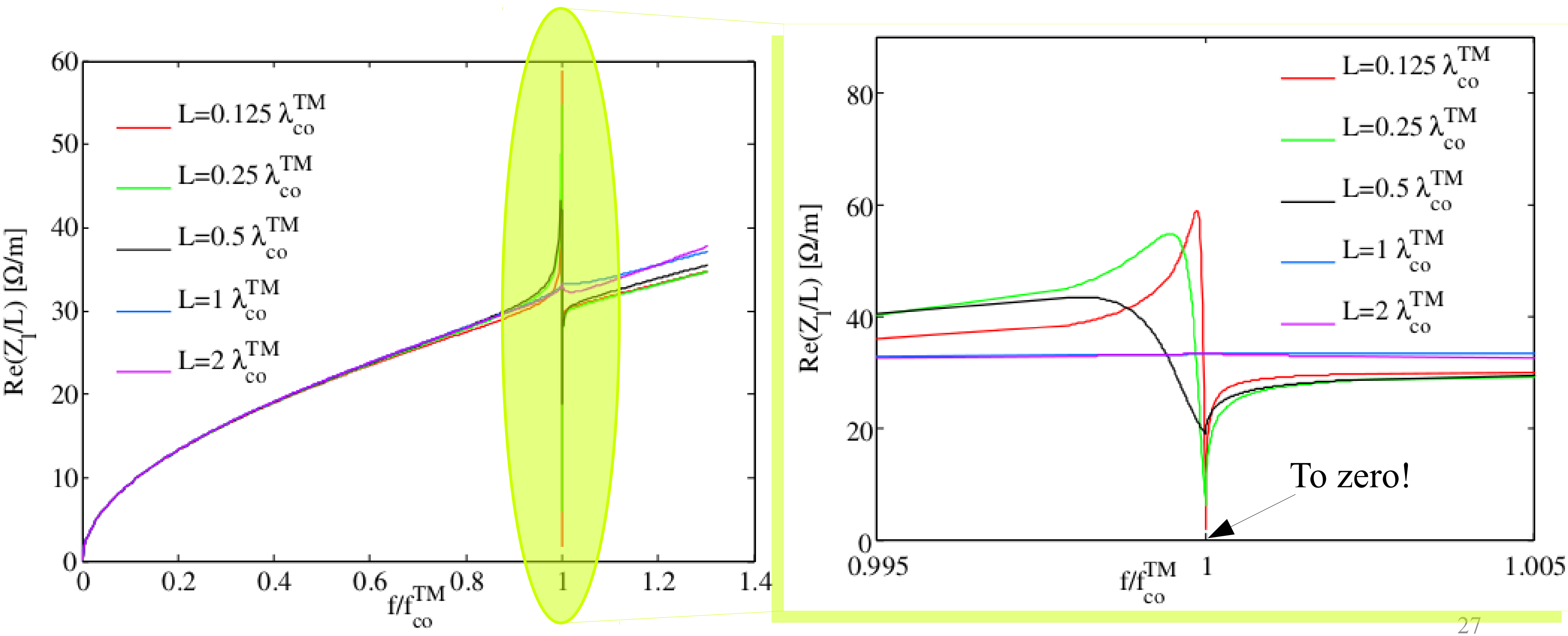
- Cylindrical cavity filled with material of $\sigma = 10^2\ \mathrm{S/m}$
- Longitudinal impedance case.
- A kink at cut-off appears for insert shorter than the cut-off wavelength $\lambda_{co}^{\mathrm{TM}} = c/f_{co}^{\mathrm{TM}}$



Trapped modes at cut-off

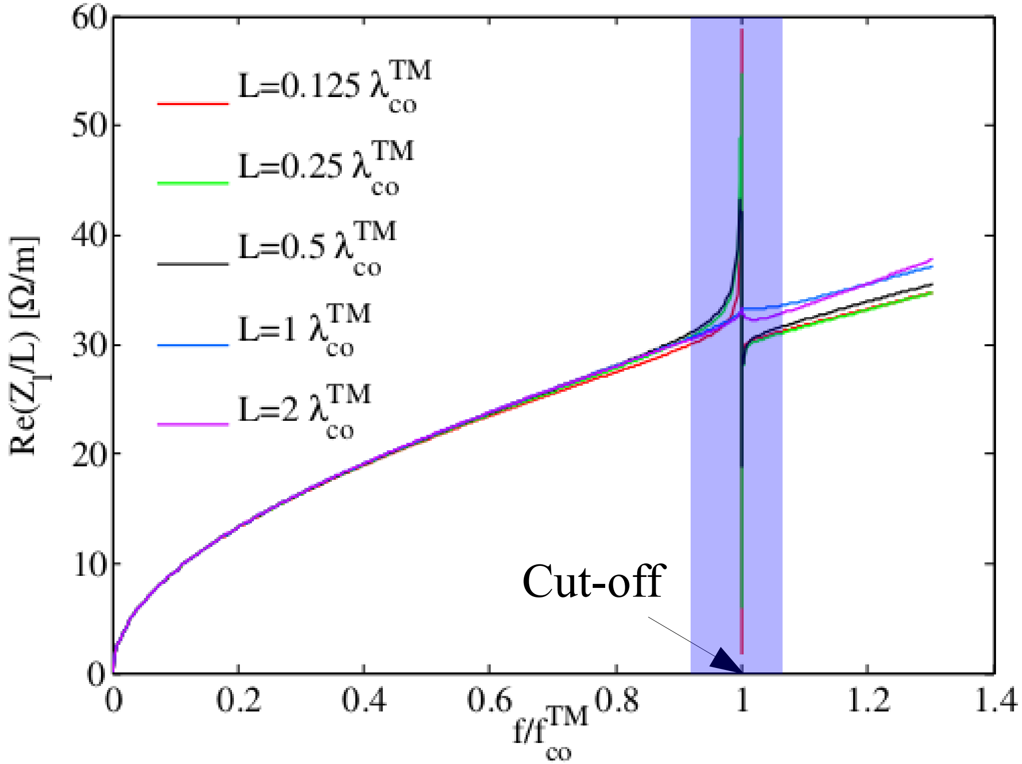


- Cylindrical cavity filled with material of $\sigma = 10^2$ S/m
- Longitudinal impedance case.
- A kink at cut-off appears for insert shorter than the cut-off wavelength $\lambda_{co}^{TM} = c/f_{co}^{TM}$



$R_i + j\omega L_i$

Trapped modes at cut-off

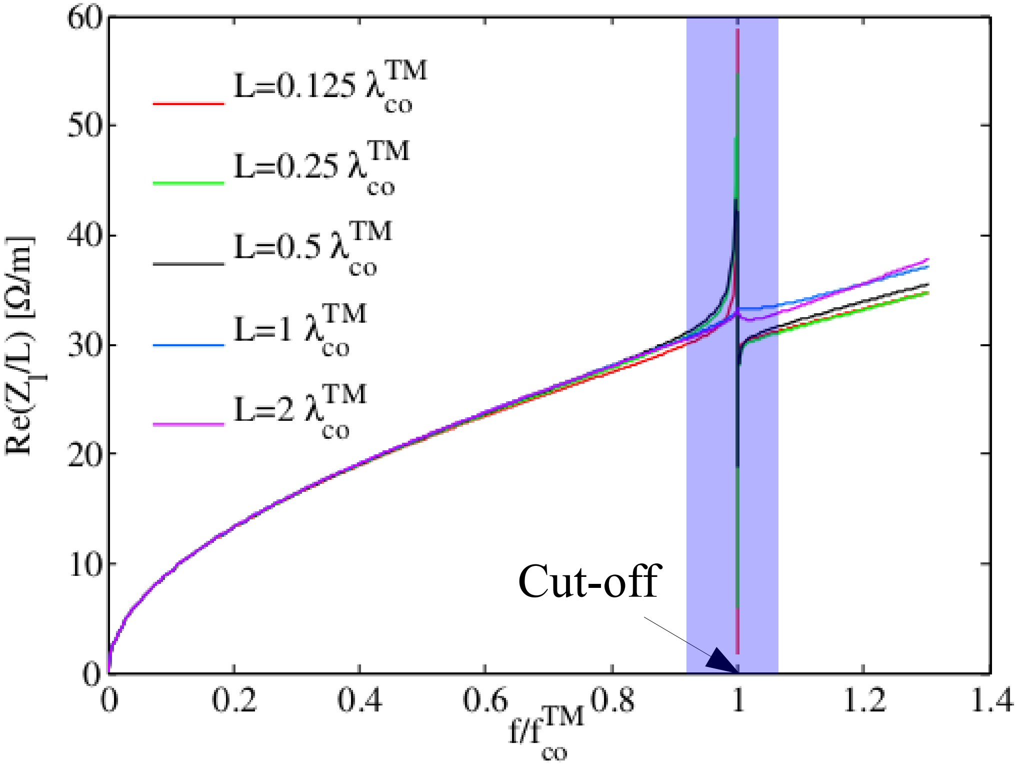


Longitudinal case: resistive wall

- **Insert impedance:** **inductive** $R_i + j\omega L_i$

$R_i + j\omega L_i$

Trapped modes at cut-off



Longitudinal case: resistive wall

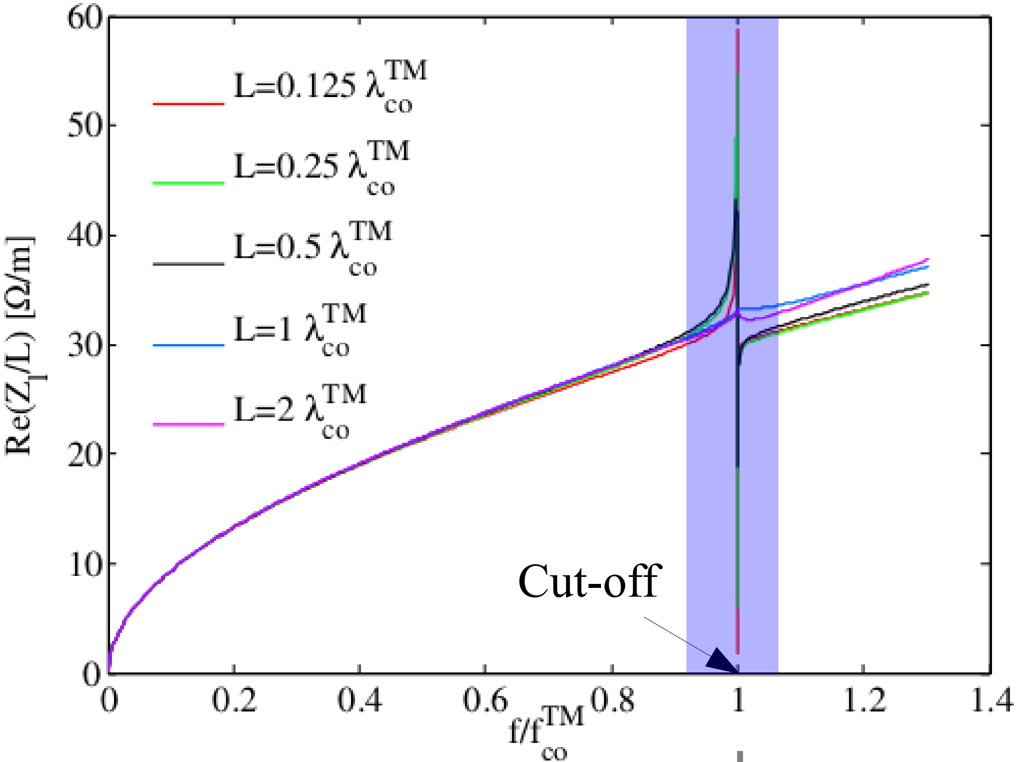
- **Insert impedance: inductive** $R_i + j\omega L_i$
- **Beam pipe impedance:**

Below cut off: **capacitive** $C_p = \dfrac{Y_o}{2\,\pi\sqrt{f_{co}^2 - f^2}}$

Above cut off: **resistive** $R_p = Z_o \dfrac{\sqrt{f^2 - f_{co}^2}}{f}$

$R_i + j\omega L_i$

Trapped modes at cut-off

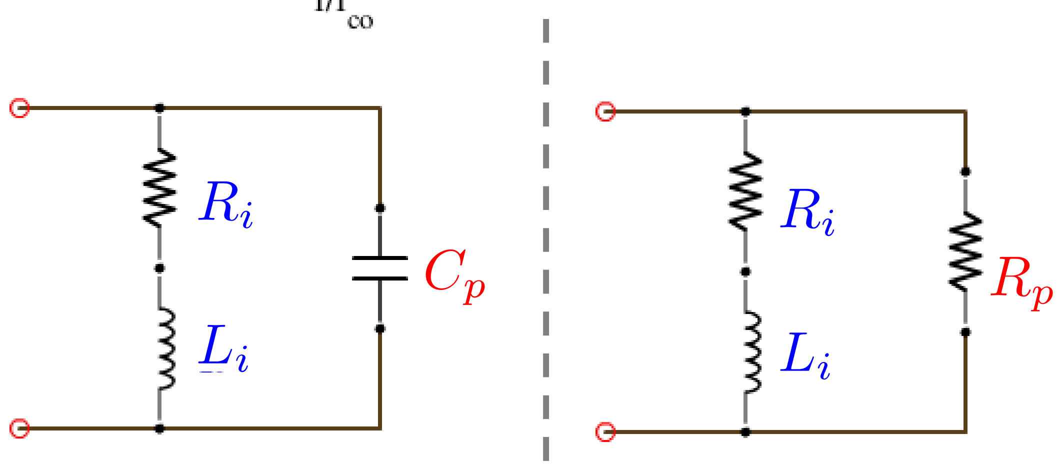


Longitudinal case: resistive wall

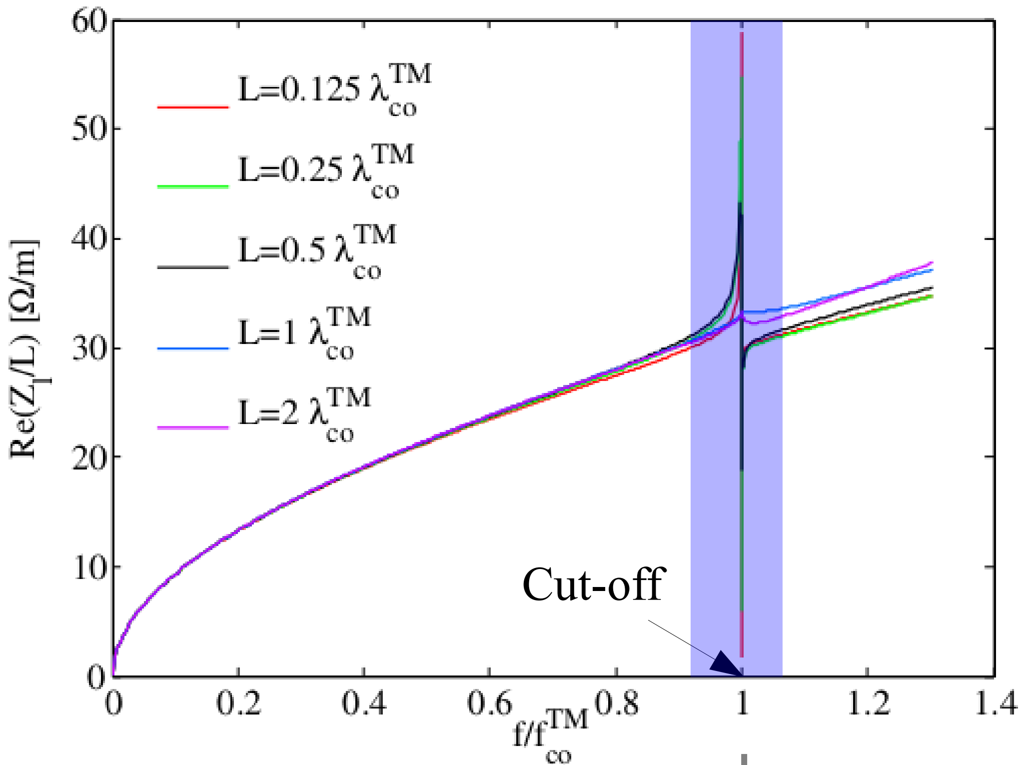
- **Insert impedance: inductive** $R_i + j\omega L_i$
- **Beam pipe impedance:**

Below cut off: **capacitive** $C_p = \dfrac{Y_o}{2\pi\sqrt{f_{co}^2 - f^2}}$

Above cut off: **resistive** $R_p = Z_o \dfrac{\sqrt{f^2 - f_{co}^2}}{f}$



Trapped modes at cut-off



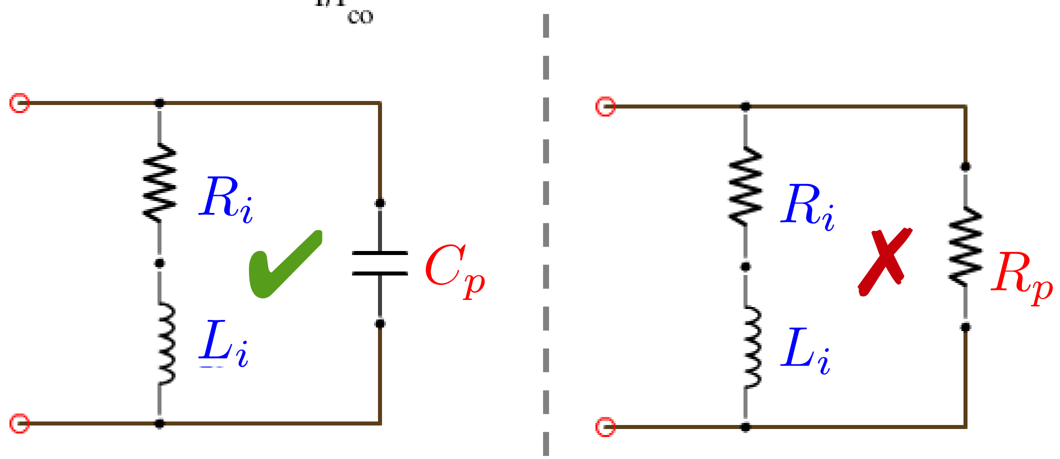
Longitudinal case: resistive wall

- **Insert impedance: inductive** $R_i + j\omega L_i$
- **Beam pipe impedance:**

Below cut off: **capacitive** $C_p = \dfrac{Y_o}{2\pi\sqrt{f_{co}^2 - f^2}}$

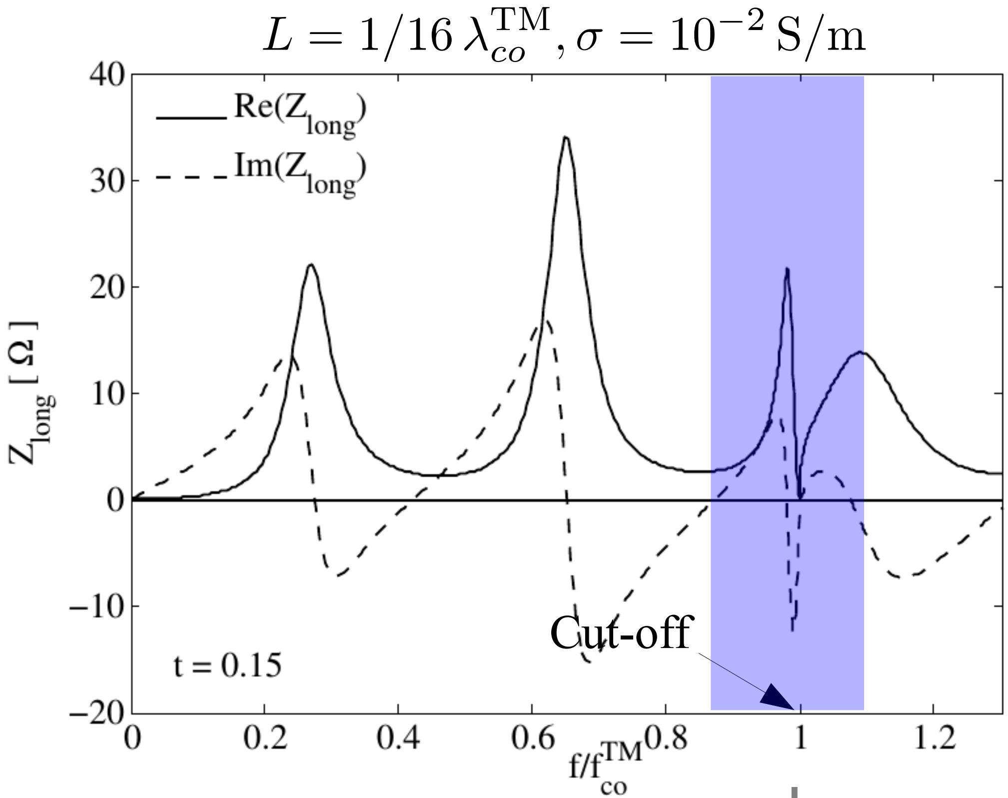
Above cut off: **resistive** $R_p = Z_o \dfrac{\sqrt{f^2 - f_{co}^2}}{f}$

✔ **Resonant behavior below cut-off!**



$R_i + j\omega L_i$

Trapped modes at cut-off

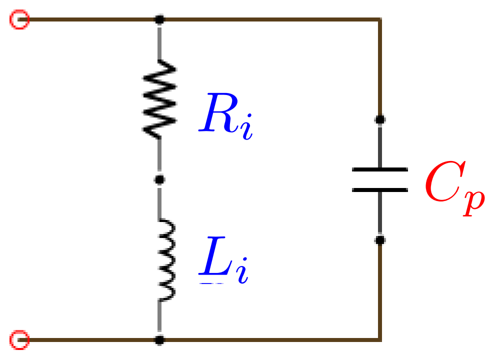


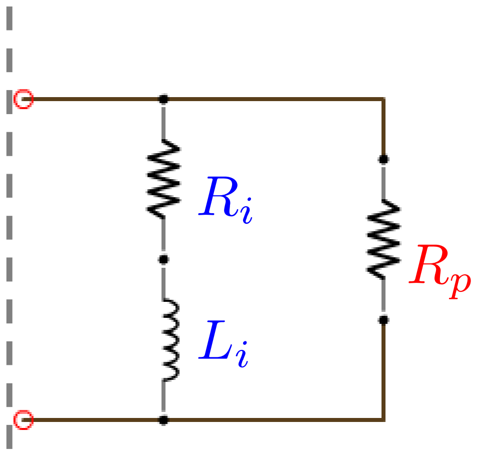
Longitudinal case: cavity #1

- **Insert impedance: inductive** side of the insert resonance $R_i + j\omega L_i$
- **Beam pipe impedance:**

Below cut off: **capacitive** $C_p = \dfrac{Y_o}{2\,\pi\sqrt{f_{co}^2 - f^2}}$

Above cut off: **resistive** $R_p = Z_o \dfrac{\sqrt{f^2 - f_{co}^2}}{f}$

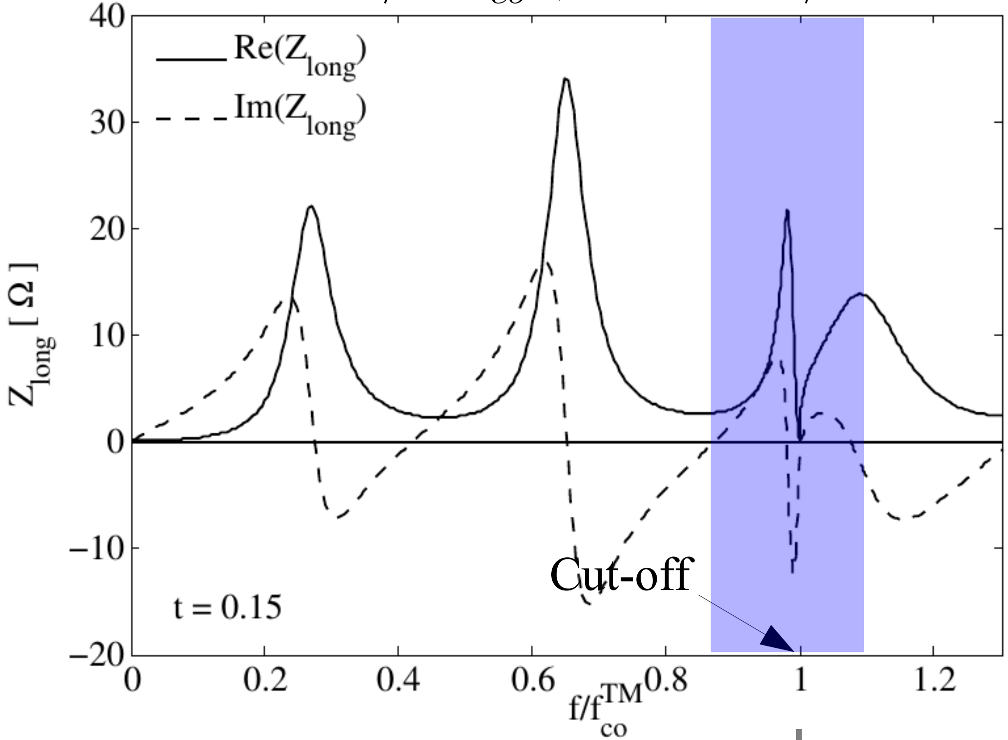




$R_i + j\omega L_i$

Trapped modes at cut-off

$L = 1/16\,\lambda_{co}^{\mathrm{TM}}, \sigma = 10^{-2}\,\mathrm{S/m}$



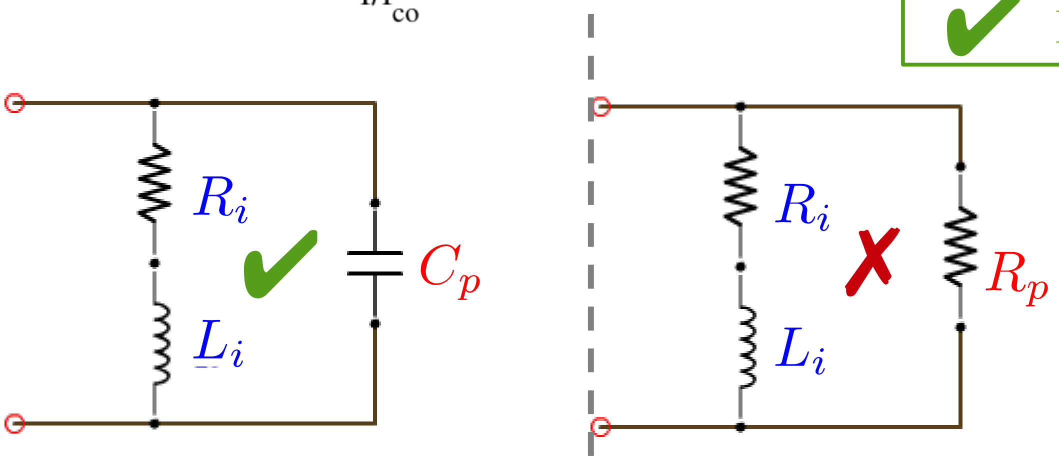
Longitudinal case: cavity #1

- **Insert impedance:** **inductive** side of the insert resonance $R_i + j\omega L_i$
- **Beam pipe impedance:**

Below cut off: **capacitive** $C_p = \dfrac{Y_o}{2\,\pi\sqrt{f_{co}^2 - f^2}}$

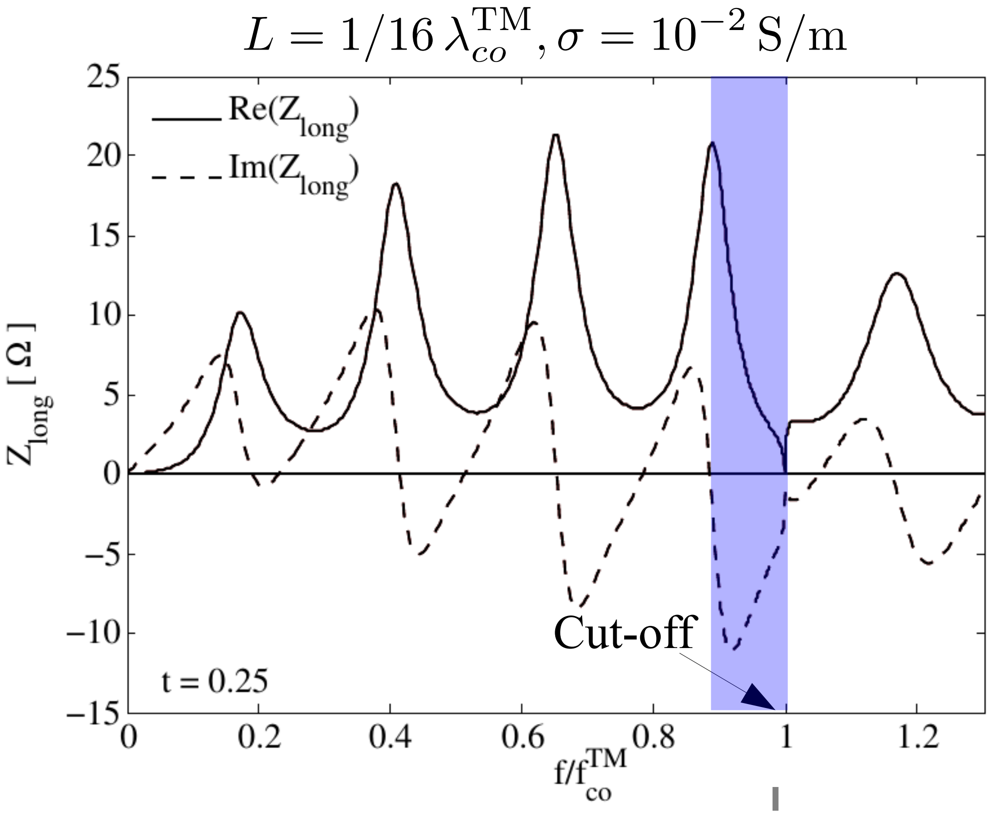
Above cut off: **resistive** $R_p = Z_o\dfrac{\sqrt{f^2 - f_{co}^2}}{f}$

✔ **Resonant behavior below cut-off!**



$R_i + j\omega L_i$

Trapped modes at cut-off



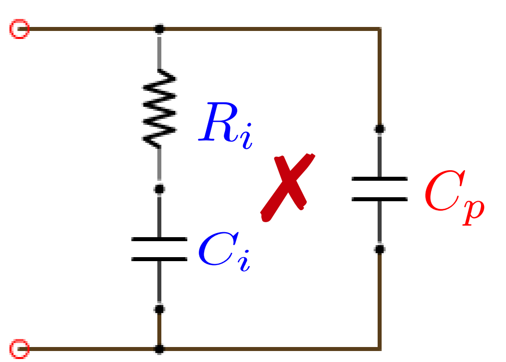
Longitudinal case: cavity #2

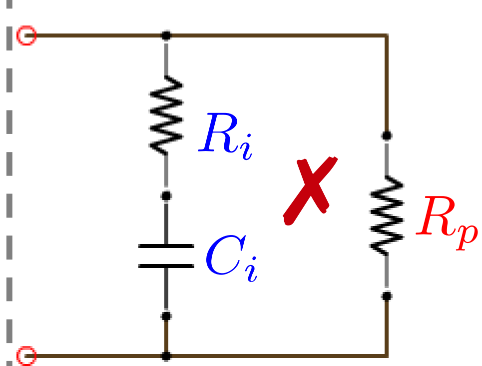
- **Insert impedance: capacitive** side of the insert resonance $R_i + 1/j\omega C_i$
- **Beam pipe impedance:**

Below cut off: **capacitive** $C_p = \dfrac{Y_o}{2\,\pi\sqrt{f_{co}^2 - f^2}}$

Above cut off: **resistive** $R_p = Z_o \dfrac{\sqrt{f^2 - f_{co}^2}}{f}$

✗ **No resonant behavior below cut-off!**



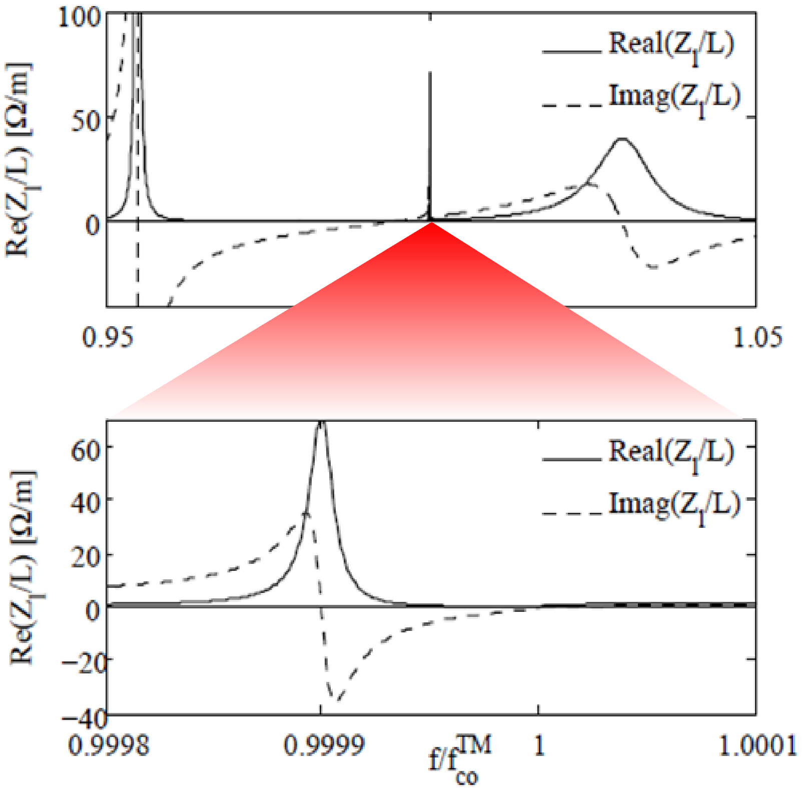


$R_i + j\omega L_i$

Flanges with Alumina

- **Trapped mode at cut off** → apparent for small conductivities $\sigma = 10^{-4} \ldots 10^{-2}\,\mathrm{S/m}$
- **Complex permittivity** → We can define an equivalent conductivity $\sigma_{eq} = \omega_0 \varepsilon_0 \varepsilon_r \tan\delta$
- **Beam pipe flanges:** very thin interconnections, filled with low losses materials such as Alumina $\varepsilon_r' = 9.4, \tan\delta = 4 \cdot 10^{-4} \rightarrow \sigma_{eq} = \omega_0 \varepsilon_0 \varepsilon_r \tan\delta = 5 \cdot 10^{-4}\,\mathrm{S/m}$ at cut-off.

$b = 5\,\mathrm{cm}, t = 30\,\mathrm{cm}, L = 8\,\mathrm{mm}$



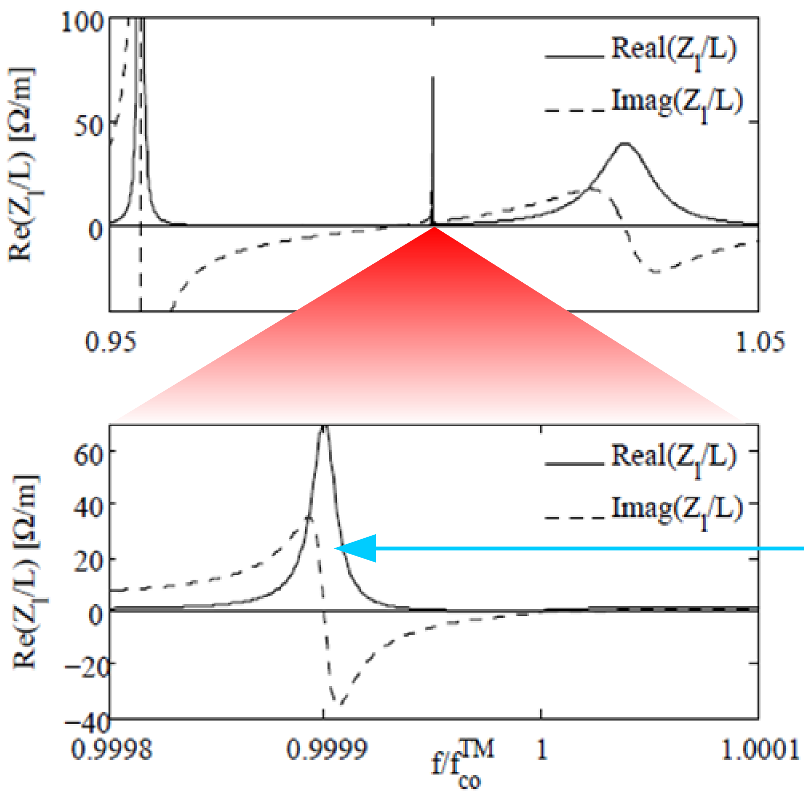
Demonstrated the presence of a **trapped mode** close to cut-off due to the reactive load of the pipes.

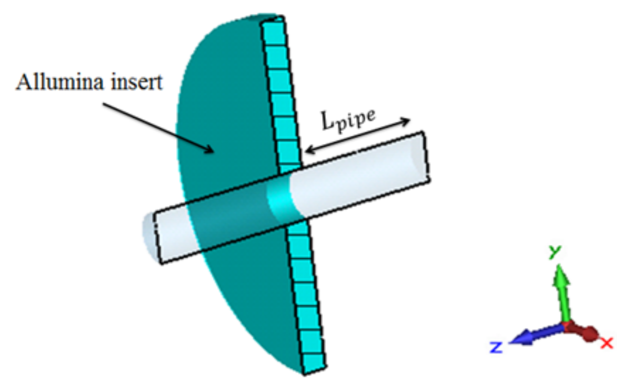
$R_i + j\omega L_i$

Flanges with Alumina

- **Trapped mode at cut off** → apparent for small conductivities $\sigma = 10^{-4} \ldots 10^{-2}\,\mathrm{S/m}$
- **Complex permittivity** → We can define an equivalent conductivity $\sigma_{eq} = \omega_0 \varepsilon_0 \varepsilon_r \tan\delta$
- **Beam pipe flanges:** very thin interconnections, filled with low losses materials such as Alumina $\varepsilon_r' = 9.4, \tan\delta = 4 \cdot 10^{-4} \rightarrow \sigma_{eq} = \omega_0 \varepsilon_0 \varepsilon_r \tan\delta = 5 \cdot 10^{-4}\,\mathrm{S/m}$ at cut-off.

$b = 5\,\mathrm{cm}, t = 30\,\mathrm{cm}, L = 8\,\mathrm{mm}$





We could perform CST Eigenmode simulations to prove the mode existence

$R_i + j\omega L_i$

Conclusions and outlook

- **Conclusions:**
  - ✓ Demonstrated Mode Matching capabilities and performance with extensive benchmarks.
  - ✓ Demonstrated a non relevant (for the moment) impact of segmentation on collimator impedance reduction.
  - ✓ Demonstrated potentially harmful existence of trapped mode at the beam pipe cut-off for low conductivity (or equivalent conductivity) materials.

- **Outlook:**
  - ✓ Extension of the Mode Matching to more complicated models.
  - ✓ Interface with CST Eigenmode solver for automatic eigenmode decomposition.
  - ✓ Including surface losses.
  - ✓ ….

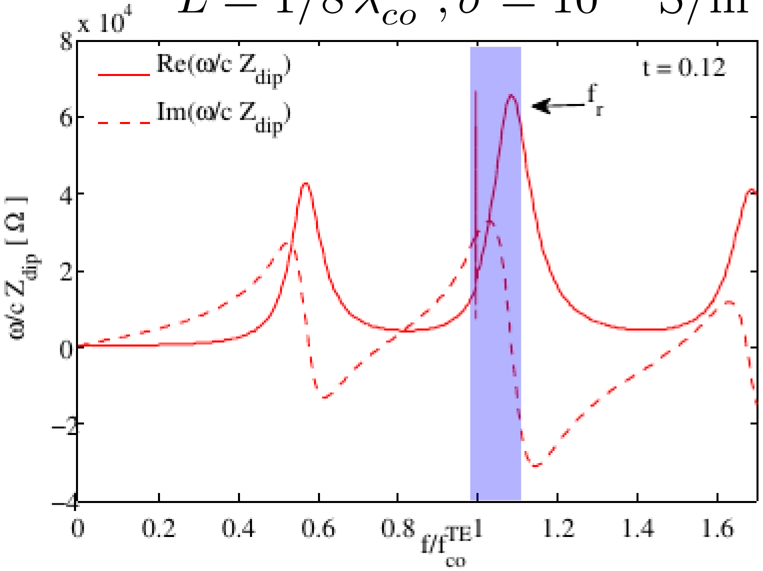
Many thanks!

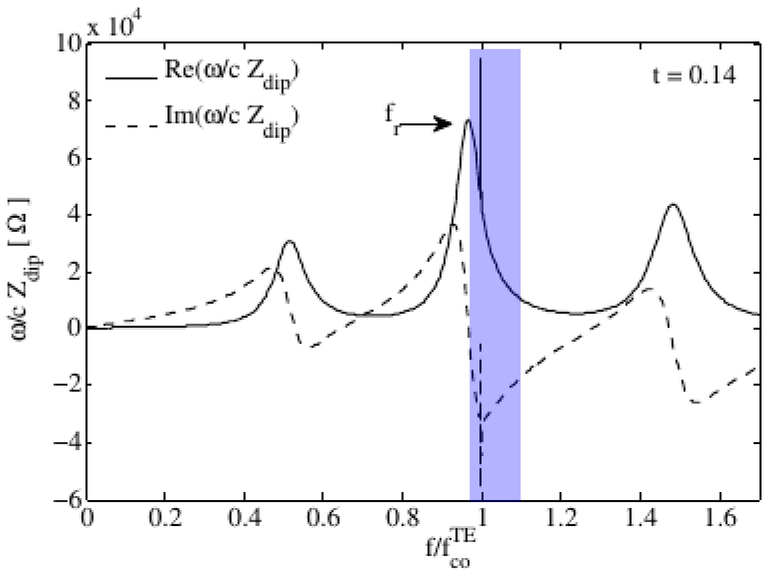
Backup slides

$R_i + j\omega L_i$

Trapped modes at cut-off

$L = 1/8\,\lambda_{co}^{\mathrm{TE}}, \sigma = 10^{-2}\,\mathrm{S/m}$





<u>Transverse</u> case: cavity #1-2

- **Insert impedance:** More complicated model, we can have resonances for **both** cases
- **Beam pipe impedance:**

Below cut off: **inductive** $L_p = \dfrac{Z_o}{2\,\pi\sqrt{f_{co}^2 - f^2}}$

Above cut off: **resistive** $R_p = Z_o \dfrac{\sqrt{f^2 - f_{co}^2}}{f}$

! The simple circuital approach is here **not sufficient**