

Impedance Computation in the Frequency Domain



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ICFA mini-Workshop on
“Electromagnetic wake fields and
impedances in particle accelerators”

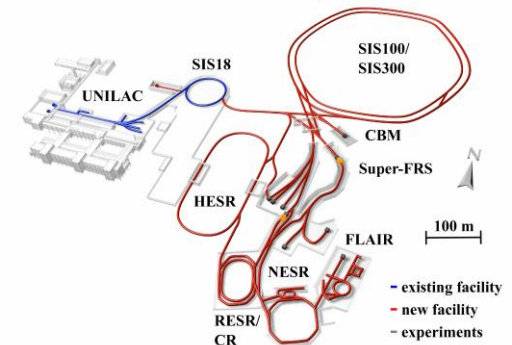


Contents

- Why Computation in Frequency Domain (FD) ?
- When do we need 3D?
→ Applications of 2D impedance computation
- The dispersion relation for the fields of a beam
- Different methods
 - Finite Integration Technique (FIT)
 - Finite Element Method (FEM)
- Details for FIT and FEM
→ Implementation of 2D impedance code
- Results for test cases
- Conclusion and Outlook

GSII

FAIR



Time domain vs. Frequency domain

- Time domain calculations e.g. by CST Particle Studio, GdfidL, Echo,... (Wake Potential)
- Impedances obtained by FFT
- Limitation by uncertainty relation $\Delta t \Delta f \geq 1$
- Long wake length for low frequency $\Delta z \geq \frac{\beta c}{\Delta f} \approx 300 \text{ m @ } 1 \text{ MHz}$
- Time step dt is limited by CFL criterion (for explicit time-stepping) → Resolving structure details makes dt small...
- Total computation time proportional to #timesteps
- High computational effort for low velocity (large extension of source fields)
- Time domain simulation requires fitting of dispersive material data (FD) on some impulse response model (TD)

Time domain vs. Frequency domain (2)



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- FD approach does not have those problems...
(but other ones...)
- FD is suitable for low and medium frequencies

When do we need 3D in FD? (1)

- 2D is sufficient for: Large length. What is large?
- Large longitudinal electrical length: “distributed impedance”

~~$$\Theta_{\parallel} = \frac{2\pi f l}{\beta c \sqrt{\mu_r \epsilon_r}} \gg 1$$~~

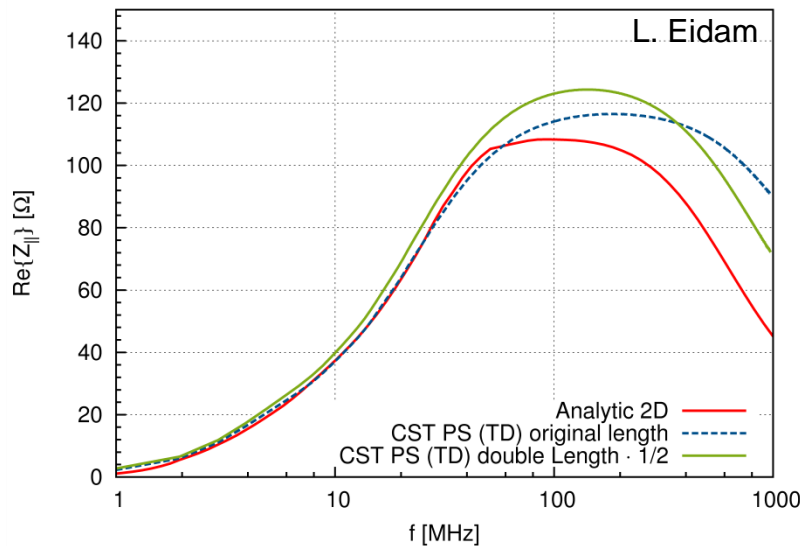
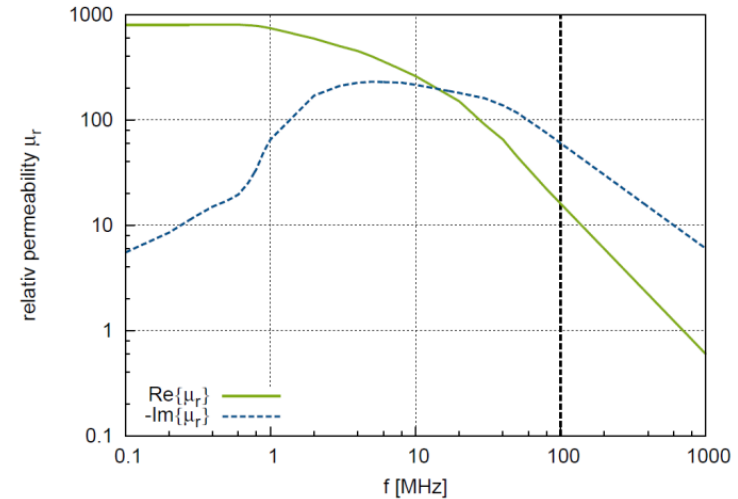
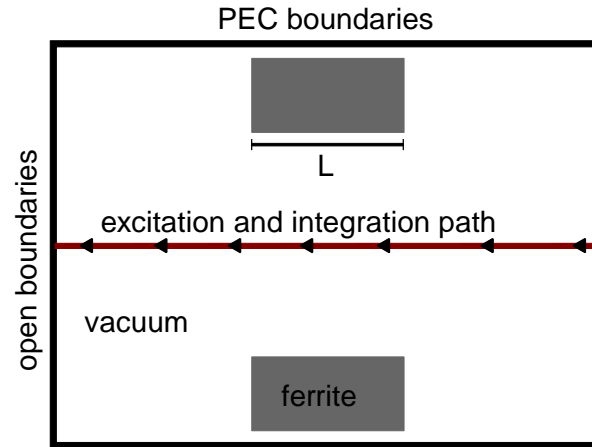
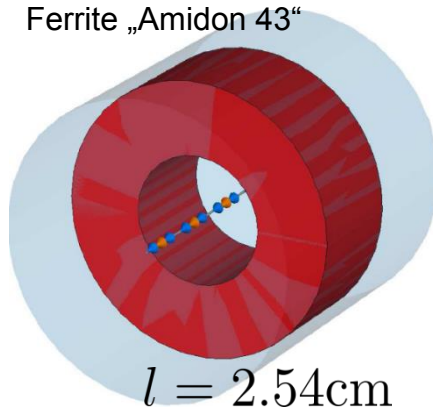
$$Z_{lumped} \ll Z_{dist}$$

$$\frac{\partial Z_{\parallel}(\omega, z)}{\partial z} = Z_{\parallel}^{total}(\omega) \delta(z - z_0)$$

$$\frac{\partial Z_{\parallel}(\omega, z)}{\partial z} = \frac{Z_{\parallel}^{total}(\omega)}{l}$$

A POSTERIORI JUSTIFICATION

When do we need 3D in FD? (2)



Time domain simulation requires fitting of material data on some impulse response model

Model Equation

→ Ferrite Material in FD

- From Maxwell's equations we have

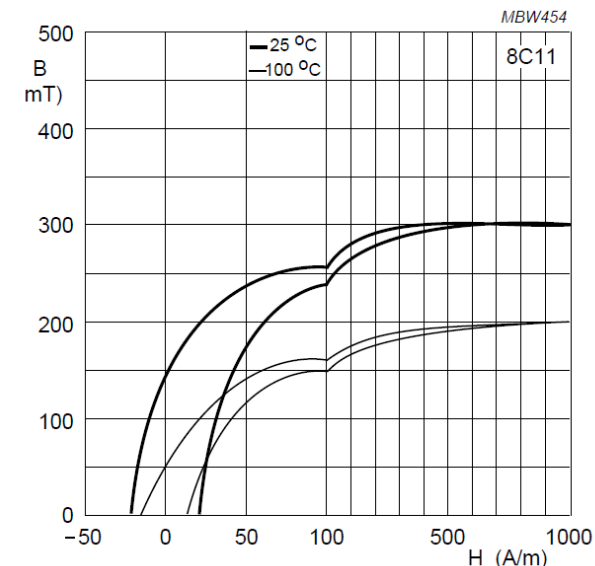
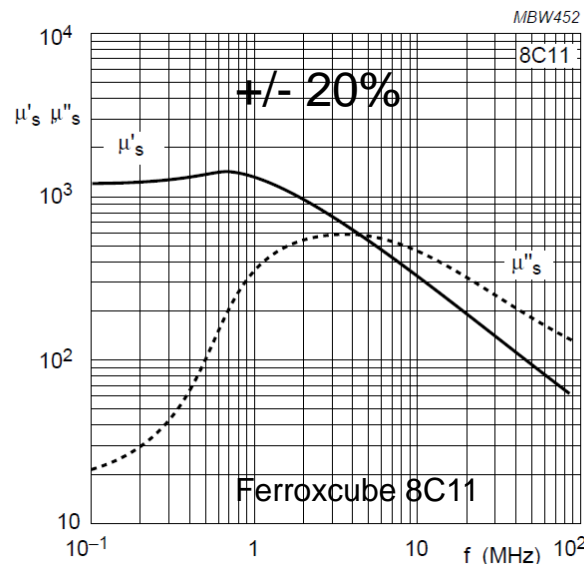
$$\nabla \times \underline{\mu}^{-1} \nabla \times \underline{\vec{E}} + i\omega \kappa \underline{\vec{E}} - \omega^2 \underline{\epsilon} \underline{\vec{E}} = -i\omega \underline{\vec{J}}_{ext}$$

- Charge implicitly included by continuity eq.

$$\underline{\mu} = \mu' - i\mu'' \quad \leftarrow \text{Magnetization / Polarization Losses}$$

$$\underline{\epsilon} = \epsilon' - i\epsilon''$$

- Unbiased small-signal complex permeability
- Linearized! Hysteresis loop approximated by ellipse in H-B space
- Remanence neglected
- Scalar!



Maxwell-Grid-Equations (MGE)

$$\nabla \times \underline{\underline{\mu}}^{-1} \nabla \times \underline{\underline{E}} + i\omega \kappa \underline{\underline{E}} - \omega^2 \epsilon \underline{\underline{E}} = -i\omega \underline{\underline{J}}_{ext}$$

$$\oint_{\partial A} \underline{\underline{E}} \cdot d\vec{s} = -\frac{d}{dt} \int_A \underline{\underline{B}} \cdot d\vec{A}$$

$$\oint_{\partial A} \underline{\underline{H}} \cdot d\vec{s} = \int_A \left(\frac{\partial \underline{\underline{D}}}{\partial t} + \underline{\underline{J}} \right) \cdot d\vec{A}$$

$$\oint_{\partial V} \underline{\underline{D}} \cdot d\vec{A} = \int_V \rho dV$$

$$\oint_{\partial V} \underline{\underline{B}} \cdot d\vec{A} = 0$$

FIT

$$\mathbf{C}\underline{\underline{e}} = -\frac{d}{dt} \widehat{\underline{\underline{b}}}$$

$$\widetilde{\mathbf{C}}\underline{\underline{h}} = \frac{d}{dt} \widehat{\underline{\underline{d}}} + \widehat{\underline{\underline{j}}}$$

$$\widetilde{\mathbf{S}}\widehat{\underline{\underline{d}}} = \underline{\underline{q}}$$

$$\mathbf{S}\widehat{\underline{\underline{b}}} = \underline{\underline{0}}$$

FIT is a mimetic discretization based on the INTEGRAL FORMULATION of Maxwell's equations (Weiland 1977)

$$\widetilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}} \mathbf{C}\underline{\underline{e}} + i\omega \mathbf{M}_{\kappa} \underline{\underline{e}} - \omega^2 \mathbf{M}_{\epsilon} \underline{\underline{e}} = -i\omega \widehat{\underline{\underline{j}}}_{ext}$$

Constitutive work of B.Doliwa on impedance computation with FIT has to be acknowledged

→ „Maxwell iteration“

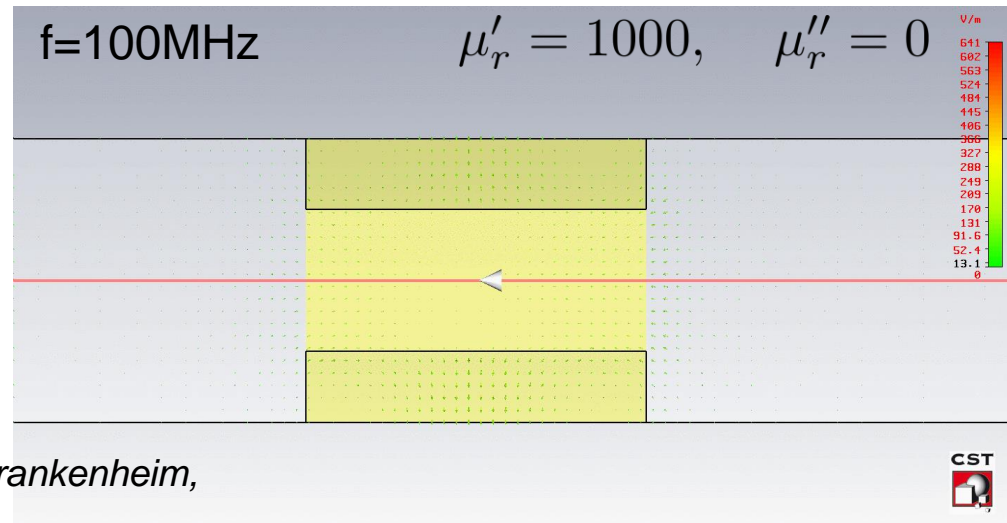
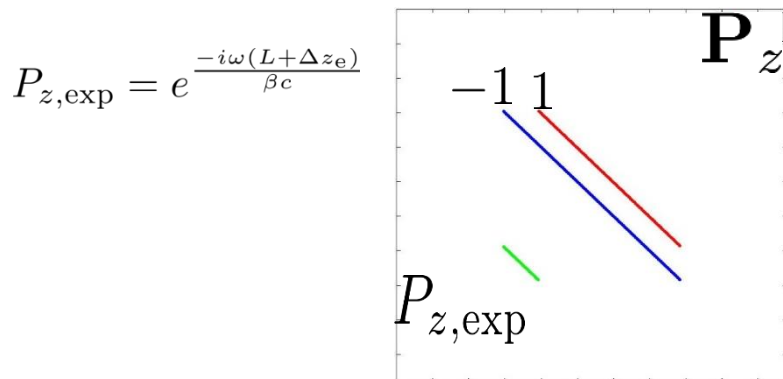
Finite Integration Technique (FIT) in FD

Mimetic Discretization by *Finite Integration Technique* (diagonal material matrices)

$$\tilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C}\underline{\hat{\mathbf{e}}} + i\omega\mathbf{M}_{\kappa}\underline{\hat{\mathbf{e}}} - \omega^2\mathbf{M}_{\epsilon}\underline{\hat{\mathbf{e}}} = -i\omega\hat{\mathbf{j}}_{ext}$$

Complex linear system of size $3N_p$, indefinite ill-conditioned matrix

- Matrix can be symmetrized but still Non-Hermitian in case of lossy material (complex Eigenvalues)
- Longitudinal Phaseshift given a priori
→ Periodic boundary conditions:



More details: U.Niedermayer and O. Boine Frankenheim,
Proc. of ICAP 2012 and references therein

Definition of coupling impedances in FD

→ Volume integral definition best applicable on mesh

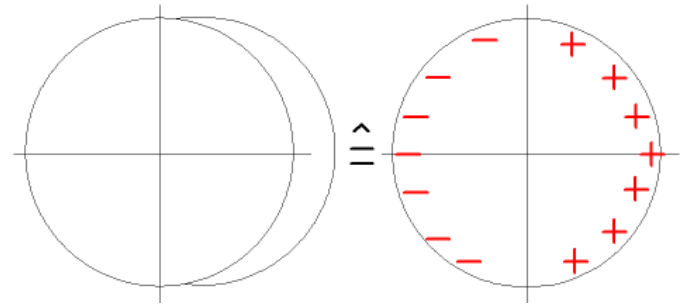


Uniform cylindrical beam: Radius of the beam Displacement of the beam

$$\sigma(\varrho, \varphi) \approx \frac{q}{\pi a^2} (\Theta(a - \varrho) + \delta(a - \varrho) d_x \cos \varphi)$$

$$\underline{J}_{s,z}(\varrho, \varphi, z, \omega) = \sigma e^{-i\omega z/v}$$

$$\underline{\rho}_s(\varrho, \varphi, z, \omega) = \frac{1}{v} \sigma e^{-i\omega z/v}$$



- Rigid beam
- Finite integration length due to decay of scattered fields

$$\underline{Z}_{\parallel}(\omega) = -\frac{1}{q^2} \int_{beam} \underline{\vec{E}} \cdot \underline{\vec{J}}_{\parallel}^* dV$$

$$\underline{Z}_{\perp,x}(\omega) = -\frac{v}{(q d_x)^2 \omega} \int_{beam} \underline{\vec{E}} \cdot \underline{\vec{J}}_{dip}^* dV$$

$$\underline{Z}_{\parallel}(\underline{\hat{e}}(\omega)) = \frac{1}{q^2} \underline{\hat{e}} \cdot \underline{\hat{j}}_s^*$$

$$\underline{Z}_{\perp}(\underline{\hat{e}}(\omega)) = \frac{\beta c}{\omega (q \cdot 2d_x)^2} \underline{\hat{e}} \cdot \underline{\hat{j}}_s^{dip*}$$

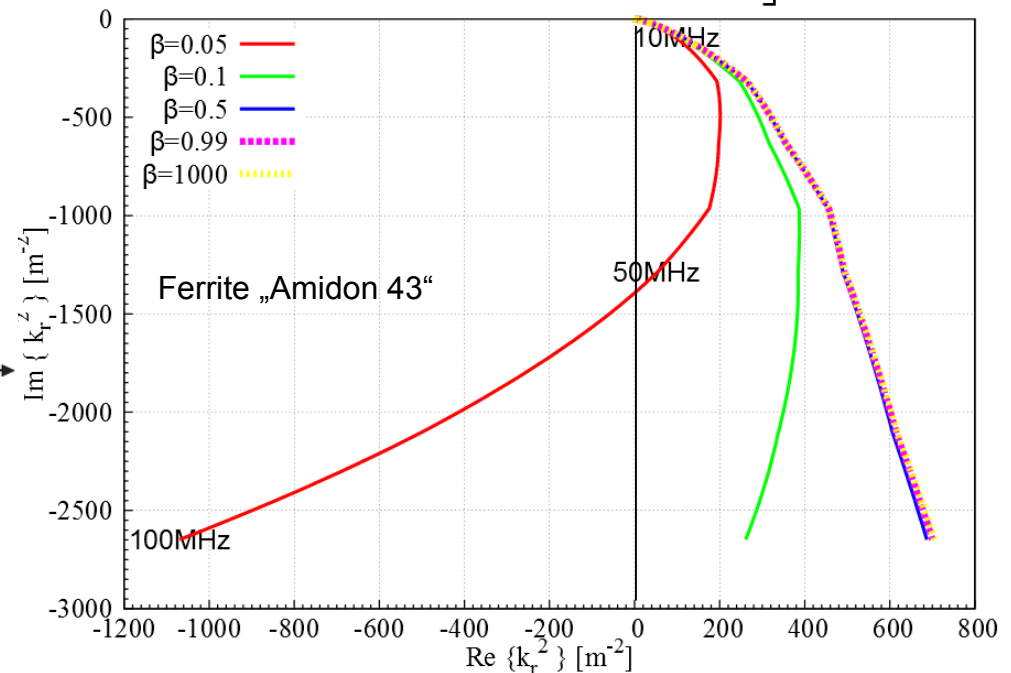
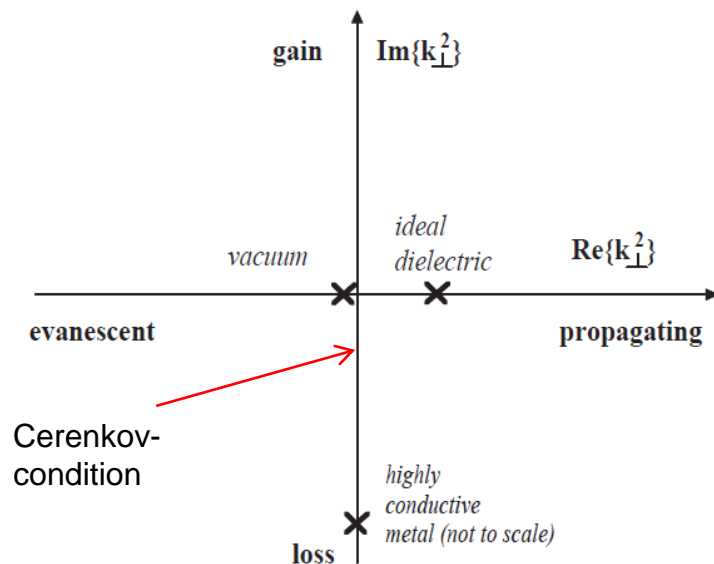
Imaginary part dominated by SPACE CHARGE for low beta!

A glance on the dispersion relation

$$(\Delta_{\perp} + k_{\perp}^2) E_z = r h s \quad \boxed{k_{\perp}^2 + k_z^2 = \omega^2 \underline{\mu} \underline{\epsilon}} \quad \partial_z \rightarrow -i \frac{\omega}{\beta c_0}$$

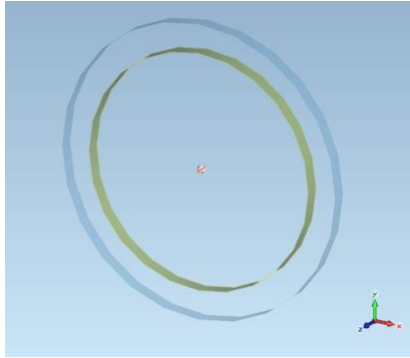
$$k_{\perp}^2 = \frac{\omega^2}{c_0^2} \left[n^2 (1 - \tan \delta_{\mu} \tan \delta_{\epsilon}) - \frac{1}{\beta^2} - i n^2 (\tan \delta_{\mu} + \tan \delta_{\epsilon}) \right]$$

n: lossless refraction index



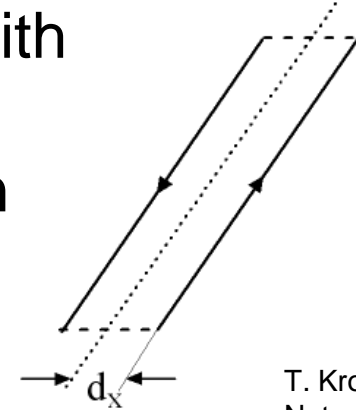
- Required transverse mesh resolution will always depend on β
- Radial model (approximation) $\beta \rightarrow \infty$, i.e. $\partial_z \rightarrow 0$

Simplified numerical method (according to “radial model“)

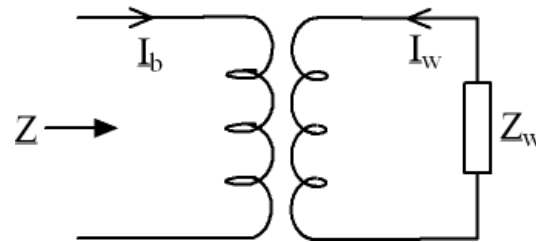
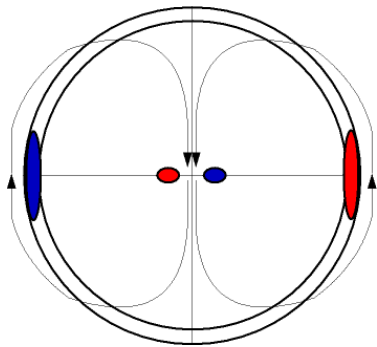


- Power loss calculation with CST EM Studio
- MQS Frequency-Domain

$$\delta P = \frac{1}{2} \int_{pipe} \underline{\vec{E}} \cdot \underline{\vec{J}}^* dV$$

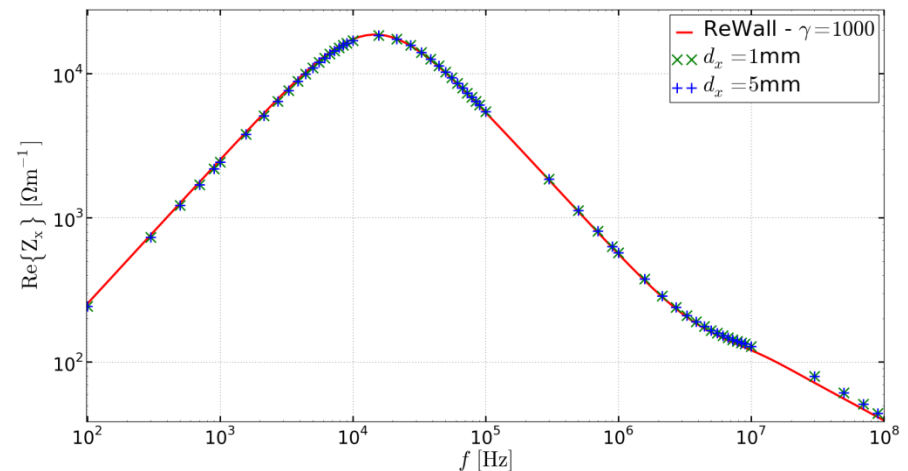


T. Kroyer, CERN-AB-Note-2008-017



Transformer between dipolar beam-current and wall-current

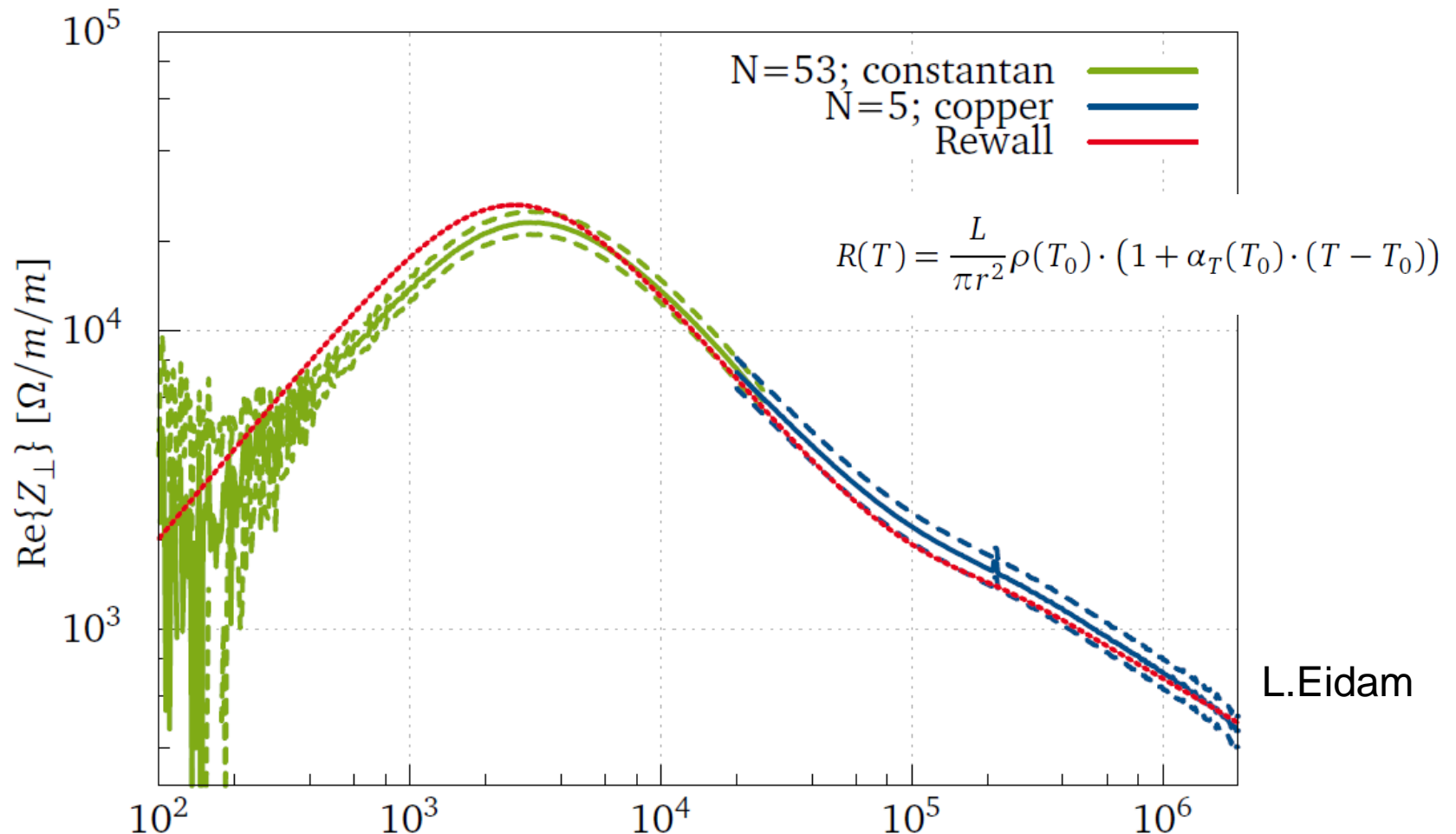
$$\frac{\text{Re} \{ Z_{\perp, x}(\omega) \}}{l} = \frac{c}{\omega d_x^2} \frac{1}{I^2} \frac{\delta P}{\delta z}$$



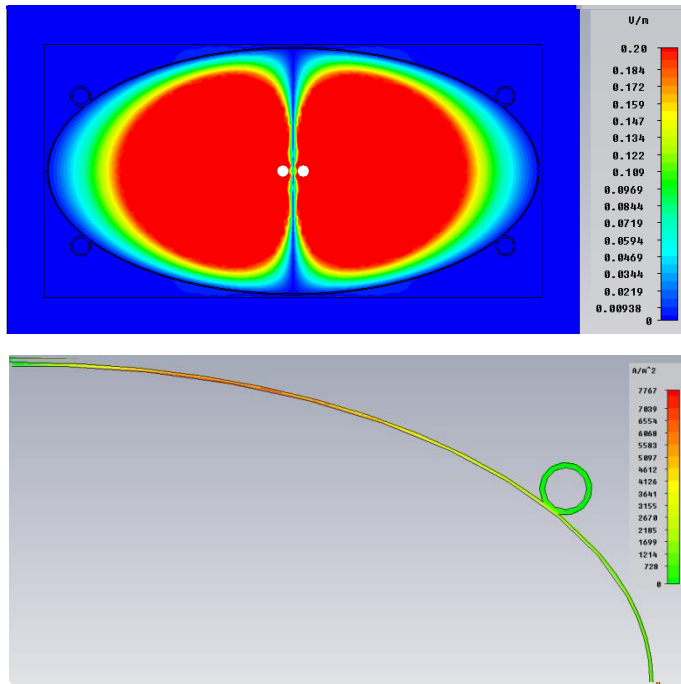
G. Nassibian and F. Sacherer, NIM A 1978

L. Vos, CERN-AB-2003-005 ABP

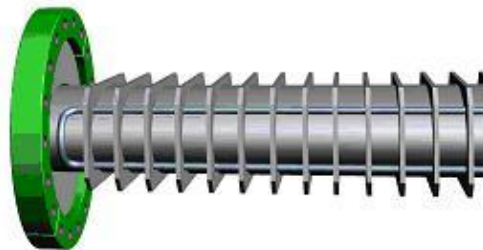
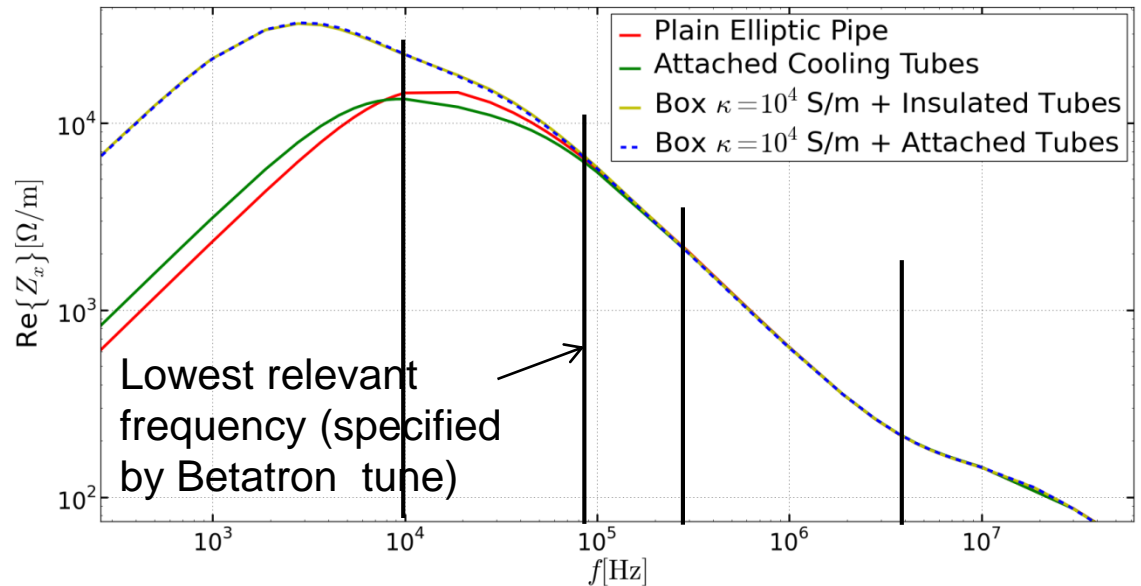
Coil measurement (according to “radial model“)



Numerical calculation for SIS100 dipole chamber (FD power loss calculation)



Longitudinal E-field and wall current for $f = 300 \text{ kHz}$

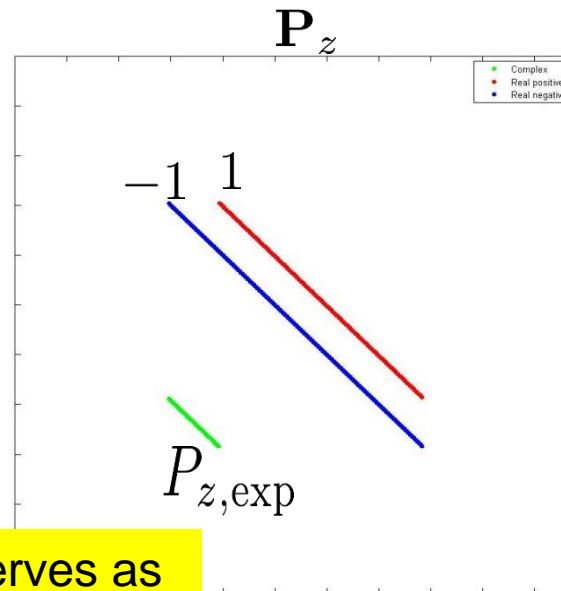


U.Niedermayer and O. Boine-Frankenheim,
Analytical and numerical calculations of resistive wall impedances for thin beam pipe structures at low frequencies, NIM A, 2012

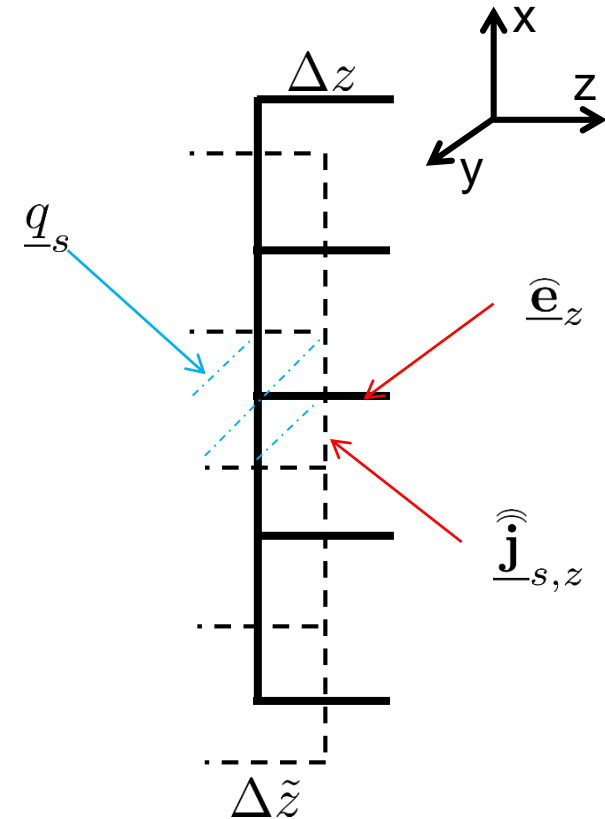
FIT in 2D (...back to full Maxwell)

Consider the a setup that is one cell long and has periodic boundary conditions

$$P_{z,\text{exp}} = e^{\frac{-i\omega(L+\Delta z_e)}{\beta c}}$$



The 2D solution also serves as boundary condition for 3D
 → Similar as waveguide port but enforced mode instead of Eigen-mode



$$P_z = -1 + e^{-i\omega \Delta z / v}$$

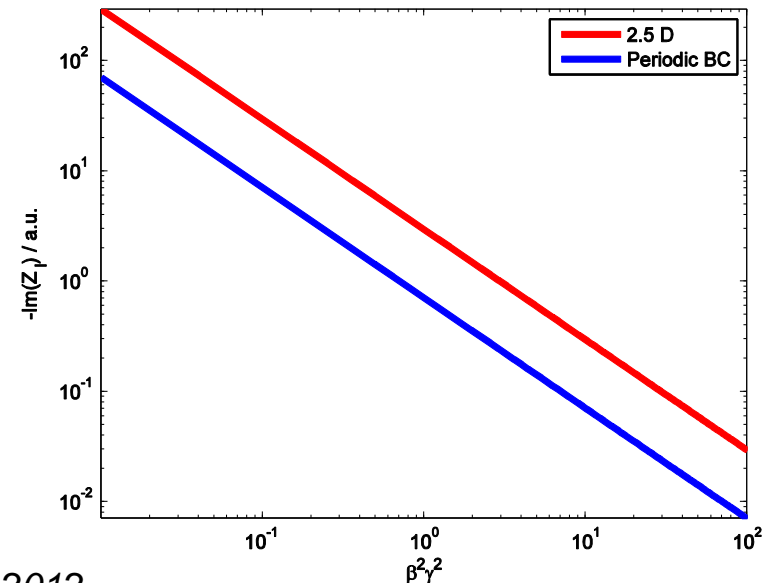
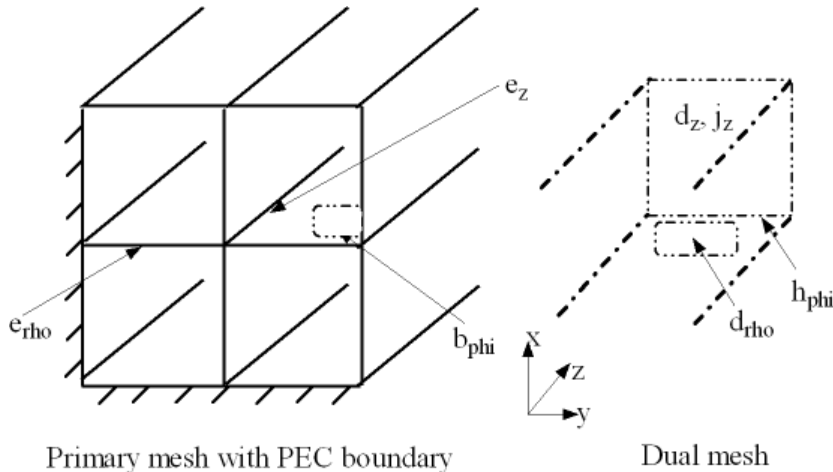
$$\tilde{P}_z = -P_z^H$$

Space charge impedance as test-case (plain perfectly conducting beam pipe)

$$1 - \frac{1}{\beta^2} = -\frac{1}{\beta^2 \gamma^2} \quad \left(I - \frac{\Delta z \Delta \tilde{z}}{v^2} \mathbf{M}_\mu^{-1} \mathbf{M}_\epsilon^{-1} \right) = -\frac{1}{\beta^2 \gamma^2} I$$

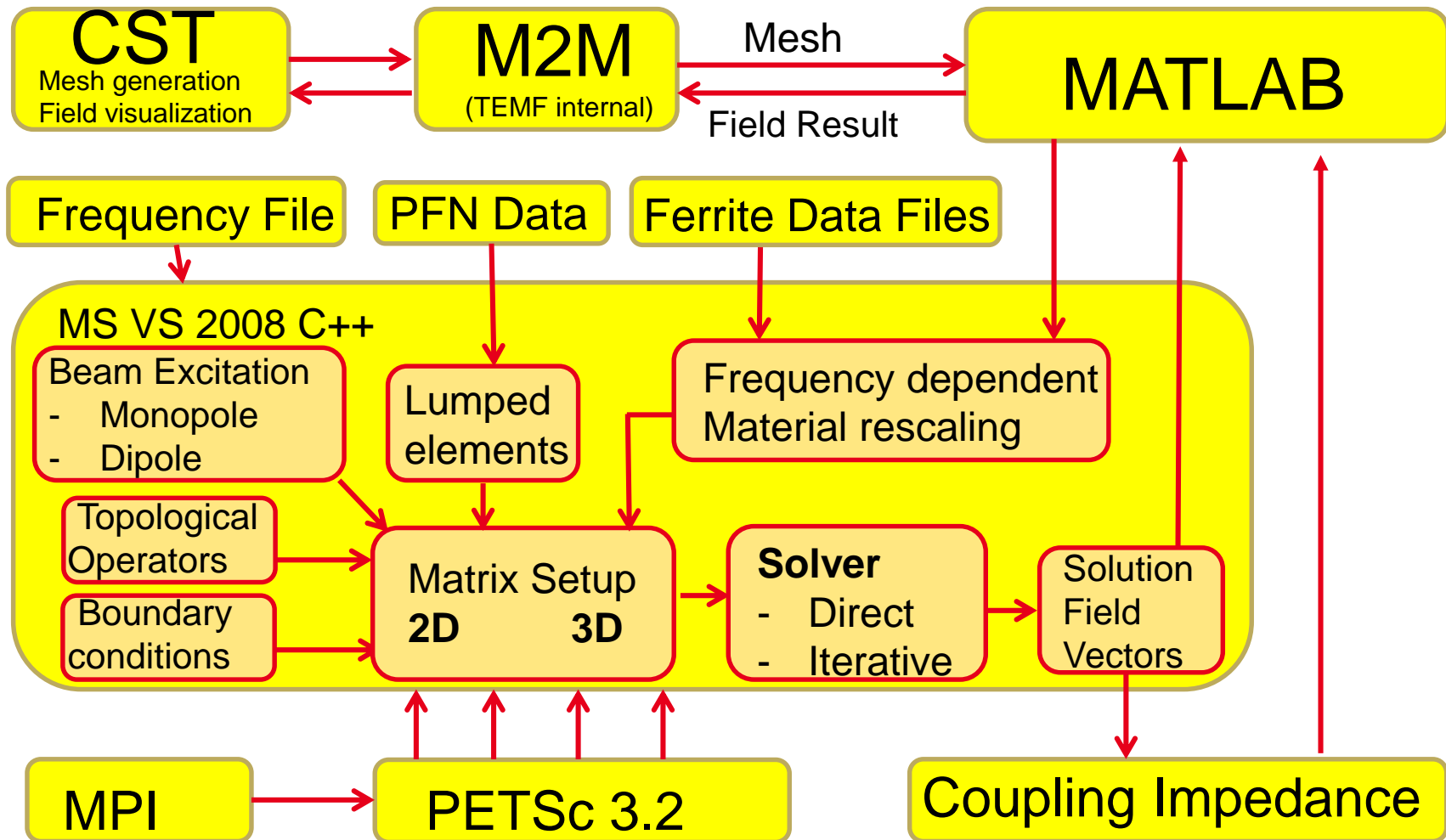
$$\left(\Delta_\perp - \frac{\omega^2}{\beta^2 \gamma^2 c^2} \right) \underline{E}_z = -\frac{i\omega \mu \sigma}{\beta^2 \gamma^2} e^{-i\omega z/v} \xrightarrow{\text{MQS}} \underline{Z}_\parallel^{SC} = -i\omega \frac{\mu_0 g l}{4\pi \beta^2 \gamma^2}$$

$$\underline{\mathbf{e}}_z = \frac{-i\omega}{4\beta^2 \gamma^2 - \omega^2 \delta_\perp^2 / c^2} \mathbf{M}_\mu \hat{\mathbf{j}}_{ext}$$



U.Niedermayer and O. Boine Frankenheim, Proc. of ICAP 2012

Implementation



Finite Element Method (FEM)

- Approximate a basis in a function space by a finite number of basis functions (“finite elements“)

$$u(x) \approx \bar{u}(x) = \sum_{j=1}^N u_j \varphi_j(x)$$

- The elements should have the following properties
 - compact support
 - continuous
 - linearly independent
- Usually on unstructured tetrahedral / triangular mesh
- Unstructured mesh approximates arbitrary geometry well
- Nondiagonal material matrices do not matter as much as in TD
- A large effort to implement from scratch!

FEniCS (www.fenicsproject.org)

(A. Logg, K. Mardal, A. Wells)



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- Use a compiler instead of a PhD student to go from weak formulation of PDE to running code! 😊
- Automated solution of PDE
→ compiler for weak formulations of PDE (UFL/FFC)
- Can be run by Python script

- Creates C++ code that runs the linear algebra backend

- Provides a Functional Analysis framework
→ work with “continuous“ functions on the mesh

- Mesh from GMSH (*C. Geuzaine, J. Remacle*)
<http://geuz.org/gmsh/>

FEM for the 2D impedance problem

$$\nabla \times \underline{\nu} \nabla \times \underline{\vec{E}} + i\omega\kappa \underline{\vec{E}} - \omega^2 \varepsilon \underline{\vec{E}} = -i\omega \underline{\vec{J}}_s$$

$$\underline{\nu} = \frac{1}{\underline{\mu}} = \frac{\mu' + i\mu''}{|\underline{\mu}|^2}$$

$$\underline{\vec{E}} = \begin{pmatrix} \vec{E}_{\perp}^r \\ E_z^r \end{pmatrix} + i \begin{pmatrix} \vec{E}_{\perp}^i \\ E_z^i \end{pmatrix}$$

$$\vec{E}_{\perp} \times \vec{n}|_{\partial\Omega} = 0$$

$$E_z|_{\partial\Omega} = 0$$

- Nodal functions only for scalar components
- Edge functions “Nedelec elements“ for vector components
→ Too much continuity for nodal elements
(normal component cannot jump on material edge)
- Divergence free Nedelec elements (lowest order edge functions)

→ Helmholtz split

Helmholtz Split



$$\nabla \times \frac{1}{\mu} \nabla \times \underline{\vec{E}} + i\omega\kappa \underline{\vec{E}} - \omega^2 \varepsilon \underline{\vec{E}} = -i\omega \underline{\vec{J}}_s$$

$$\underline{\vec{E}} = \underline{\vec{E}}_{curl} + \underline{\vec{E}}_{div} \quad \underline{\vec{E}}_{div} = -\nabla \underline{\Phi}$$

$$\nabla \cdot \varepsilon \nabla \underline{\Phi} = -\underline{\rho} = -\frac{1}{\beta c} \underline{\vec{J}}_{s,z}$$

$$\underline{\vec{R}} = \omega^2 \varepsilon \underline{\vec{E}}_{div} - i\omega \underline{\vec{J}}_s$$

$$\nabla \cdot \underline{\vec{R}} = 0$$

“Continuity Equation“

$$\nabla \times \frac{1}{\mu} \nabla \times \underline{\vec{E}}_{curl} + i\omega\kappa \underline{\vec{E}}_{curl} - \omega^2 \varepsilon \underline{\vec{E}}_{curl} = \underline{\vec{R}}$$

Statics solver

(combined E-static and stationary current)



$$\underline{\nabla} \cdot \underline{\varepsilon} \underline{\nabla} \underline{\Phi} = -\underline{\rho}$$

$$\underline{\varepsilon} = \varepsilon + \frac{\kappa}{i\omega}$$

$$\nabla \cdot \varepsilon \nabla \Phi^r + \nabla \cdot \frac{\kappa}{\omega} \nabla \Phi^i = -\rho^r$$

$$\nabla \cdot \varepsilon \nabla \Phi^i - \nabla \cdot \frac{\kappa}{\omega} \nabla \Phi^r = -\rho^i$$

Boundary Conditions

$$\int_{\Omega} \varepsilon \nabla \Phi^r \cdot \nabla v^r dV + \int_{\Omega} \nabla \frac{\kappa}{\omega} \Phi^i \cdot \nabla v^r dV + \int_{\partial\Omega} \dots = \int_{\Omega} \rho^r v^r dV$$

$$\int_{\Omega} \varepsilon \nabla \Phi^i \cdot \nabla v^i dV - \int_{\Omega} \nabla \frac{\kappa}{\omega} \Phi^r \cdot \nabla v^i dV + \int_{\partial\Omega} \dots = \int_{\Omega} \rho^i v^i dV$$

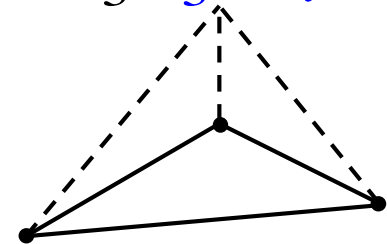
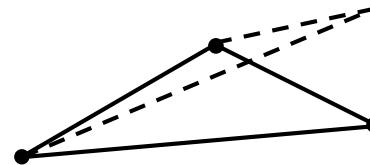
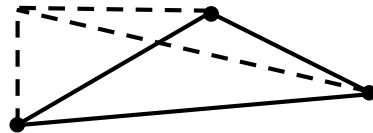
Solve with Galerkin approach and nodal-functions,
then project gradients on H(curl)

$$\underline{\vec{E}}_{div} = -\underline{\nabla} \underline{\Phi}$$

Nodal functions (2D)

Lowest order: "Hat functions"

$$\varphi(x, y) = u_1 N_1(x, y) + u_2 N_2(x, y) + u_3 N_3(x, y)$$



Pictures: H. De Gerssem

$$N_i = a_i + b_i x + c_i y$$

- Local Basis
- a, b, c such that $N_i(\vec{x}_j) = \delta_{ij}$
- Local to Global map required

Rotational part (1)

(Edge elements)

- Nedelec Elements

→ **Tangential continuity** (H(curl)-conforming)

$$\vec{E}_{\perp}^r(\vec{x}) = \sum_{i=1}^N e_i^r \vec{w}_i(\vec{x})$$

$$\vec{w}_i(\vec{x}) = N_k \nabla N_l - N_l \nabla N_k$$

$$N_i = a_i + b_i x + c_i y$$

$$\int_{l_j} \vec{w}_i \cdot \vec{t}_j ds = \delta_{ij}$$

Rotational part (2)

Rewriting the curlcurl Operator

$$\nabla \times \underline{\vec{E}} = \left(\begin{array}{cc|c} 0 & -\partial_z & \partial_y \\ \partial_z & 0 & -\partial_x \\ \hline -\partial_y & \partial_x & 0 \end{array} \right) \underline{\vec{E}} = \left(\begin{array}{c|c} iZ & A \\ \hline B & 0 \end{array} \right) \underline{\vec{E}} \quad \partial_z \rightarrow -i \frac{\omega}{\beta c}$$

$$Z = \begin{pmatrix} 0 & +\frac{\omega}{\beta c} \\ -\frac{\omega}{\beta c} & 0 \end{pmatrix} \quad Z^2 = -\frac{\omega^2}{\beta^2 c^2} \quad A = \begin{pmatrix} \partial_y \\ -\partial_x \end{pmatrix} = -B^T$$

$$\nabla \times \underline{\nu} \nabla \times \underline{\vec{E}} = \begin{pmatrix} A\nu_r B - \nu_r Z^2 & -Z\nu_i A \\ -B\nu_i Z & B\nu_r A \end{pmatrix} \begin{pmatrix} \vec{E}_\perp^r \\ E_z^r \end{pmatrix} - \begin{pmatrix} A\nu_i B - \nu_i Z^2 & Z\nu_r A \\ B\nu_r Z & B\nu_i A \end{pmatrix} \begin{pmatrix} \vec{E}_\perp^i \\ E_z^i \end{pmatrix} \\ + i \begin{pmatrix} A\nu_r B - \nu_r Z^2 & -Z\nu_i A \\ -B\nu_i Z & B\nu_r A \end{pmatrix} \begin{pmatrix} \vec{E}_\perp^i \\ E_z^i \end{pmatrix} + i \begin{pmatrix} A\nu_i B - \nu_i Z^2 & Z\nu_r A \\ B\nu_r Z & B\nu_i A \end{pmatrix} \begin{pmatrix} \vec{E}_\perp^r \\ E_z^r \end{pmatrix}$$

Only AB and BA are 2nd order operators

Boundary Conditions

$$\int_{\Omega} (A\nu_r B \vec{E}_\perp^r) \cdot \vec{v}^r dV = \int_{\Omega} (\nu_r B \vec{E}_\perp^r) (B\vec{v}^r) dV + \int_{\partial\Omega} \dots$$

$$\int_{\Omega} (B\nu_r A E_z^r) v^r dV = \int_{\Omega} (\nu_r A E_z^r) \cdot (A v^r) dV + \int_{\partial\Omega} \dots$$

Rotational part (3)

Weak Formulation of the whole CurlCurl Equation



$$\begin{aligned}(\nabla \times \nu \nabla \times \vec{E})^r - \omega \kappa \vec{E}^i - \omega^2 \varepsilon \vec{E}^r &= \vec{R}^r \\(\nabla \times \nu \nabla \times \vec{E})^i + \omega \kappa \vec{E}^r - \omega^2 \varepsilon \vec{E}^i &= \vec{R}^i\end{aligned}$$

Inner product

$$\int_{\Omega} dV$$

$$a_{\varepsilon} = -\omega^2 \left(\left(\vec{v}_{\perp}^r, \varepsilon \vec{E}^r \right) + \left(\vec{v}_{\perp}^i, \varepsilon \vec{E}^i \right) + \left(v_z^r, \varepsilon E_z^r \right) + \left(v_z^i, \varepsilon E_z^i \right) \right)$$

$$a_{RHS} = \left(\vec{v}_{\perp}^r, \vec{R}_{\perp}^r \right) + \left(\vec{v}_{\perp}^i, \vec{R}_{\perp}^i \right) + \left(v_z^r, R_z^r \right) + \left(v_z^i, R_z^i \right)$$

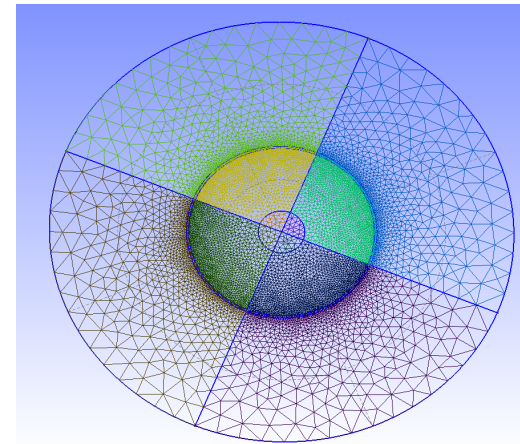
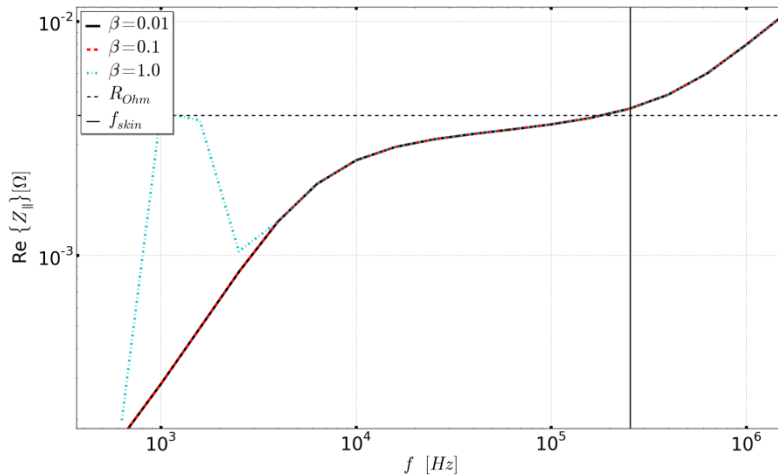
SOLVE($a_{\text{curlcurl}} + a_{\kappa} + a_{\varepsilon} == a_{RHS}$)

28 terms in the weak formulation

#DOFs (lowest order) = 2* #edges + 2* #vertices)

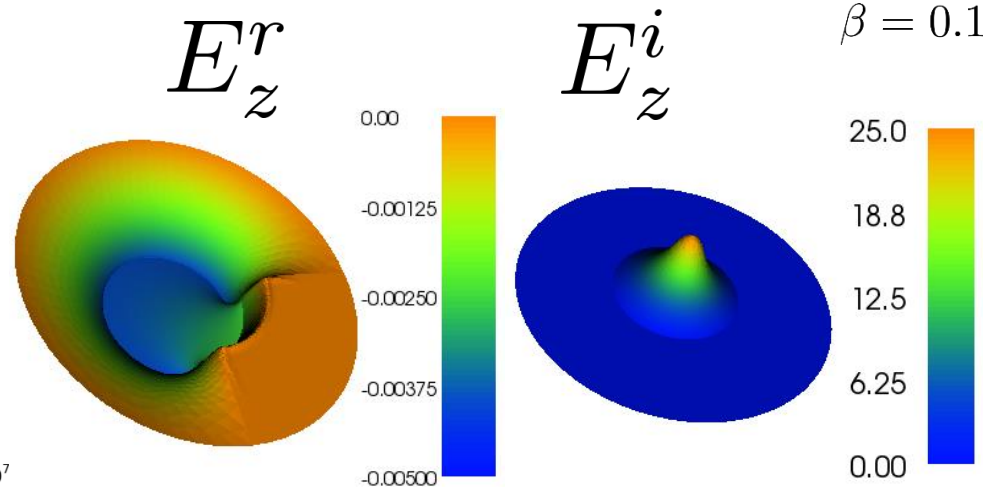
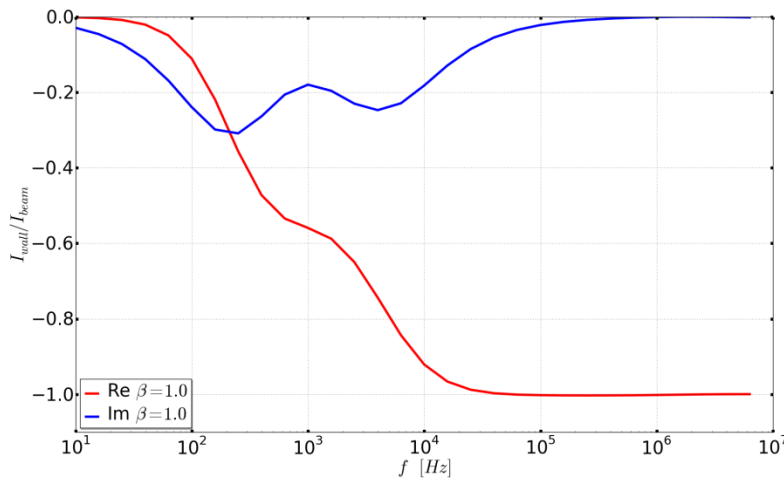
```
Z=assemble(inner(Ezr, Jzr)*dx)+i*assemble(inner(Ezi, Jzr)*dx)
```

Numerical Examples: Thin beam pipe (FEM)

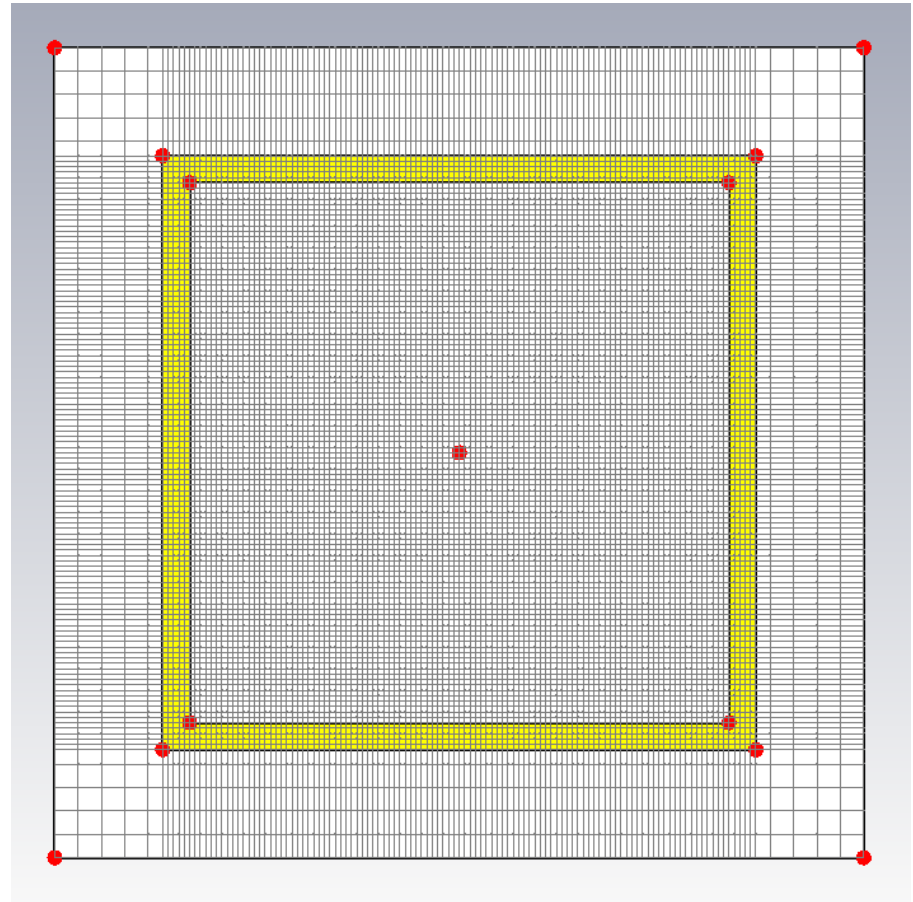
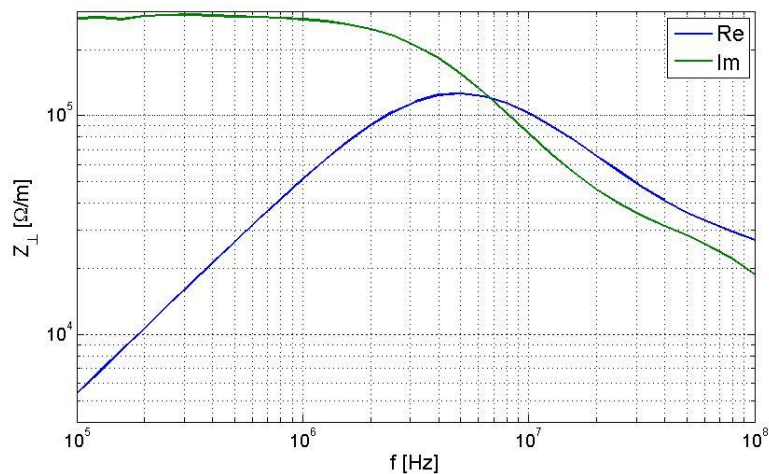
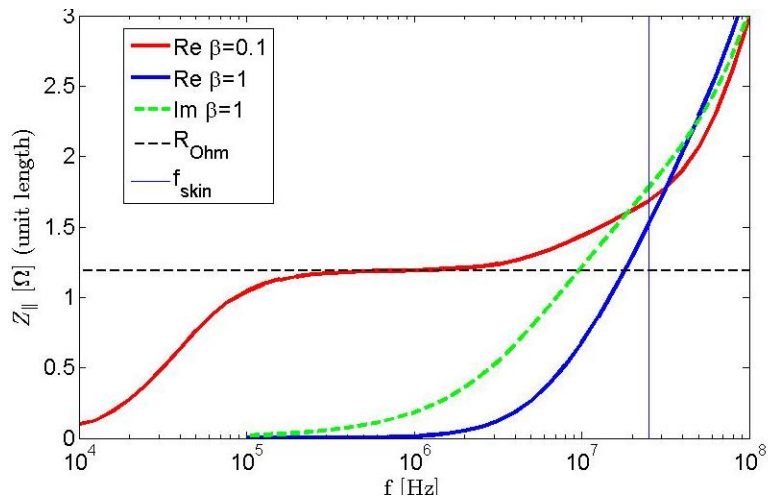


$f = 0.1$ MHz

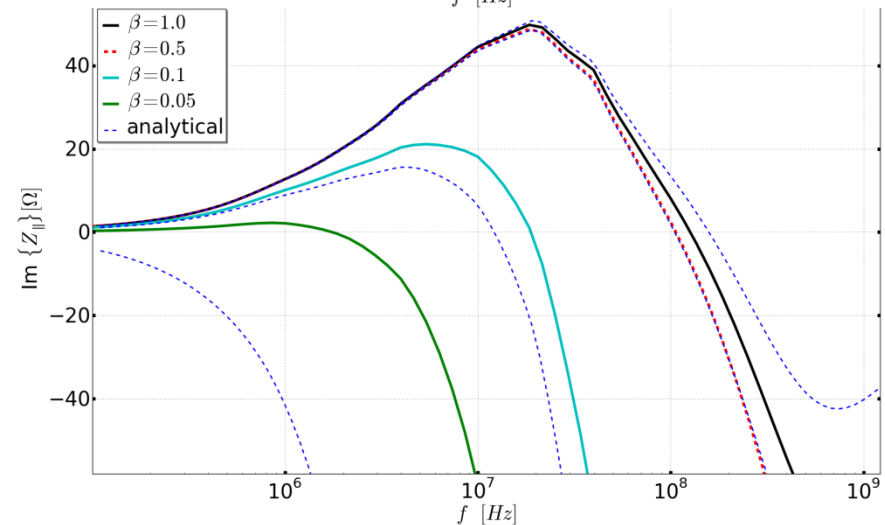
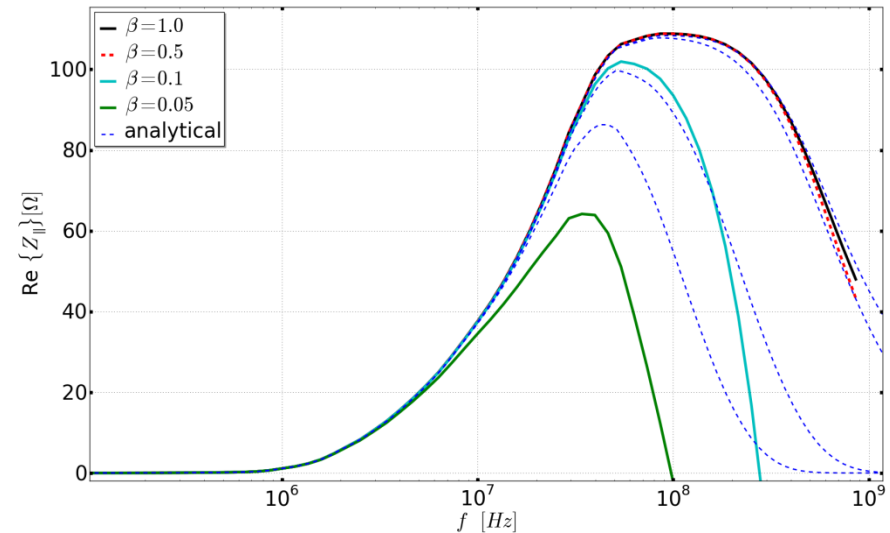
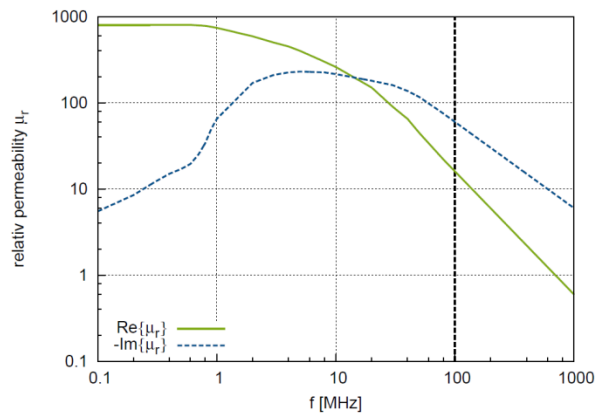
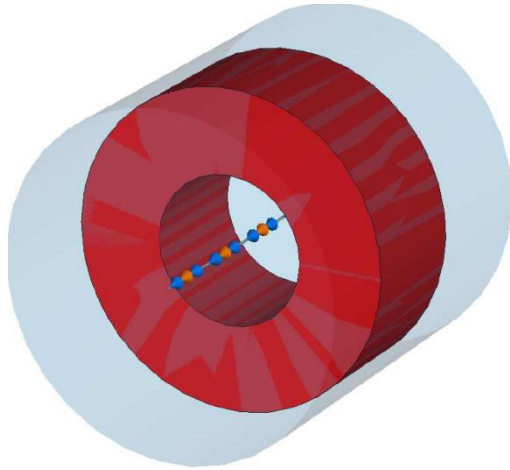
$\beta = 0.1$



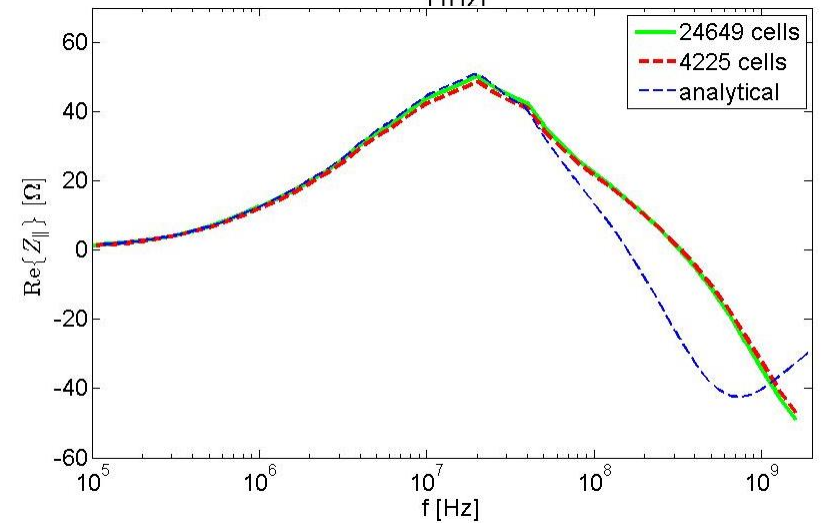
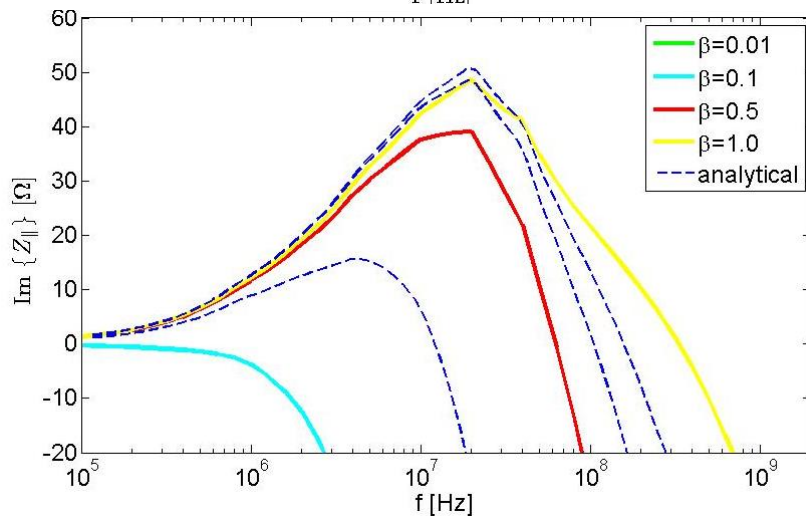
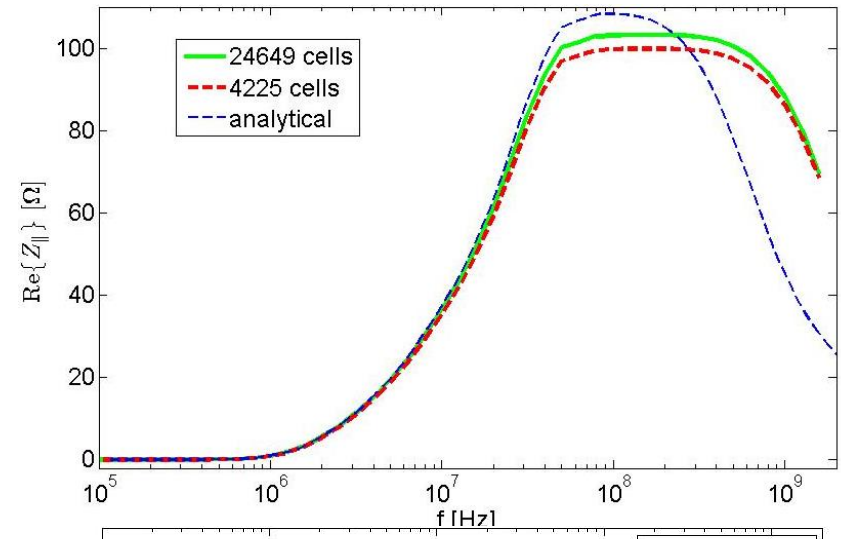
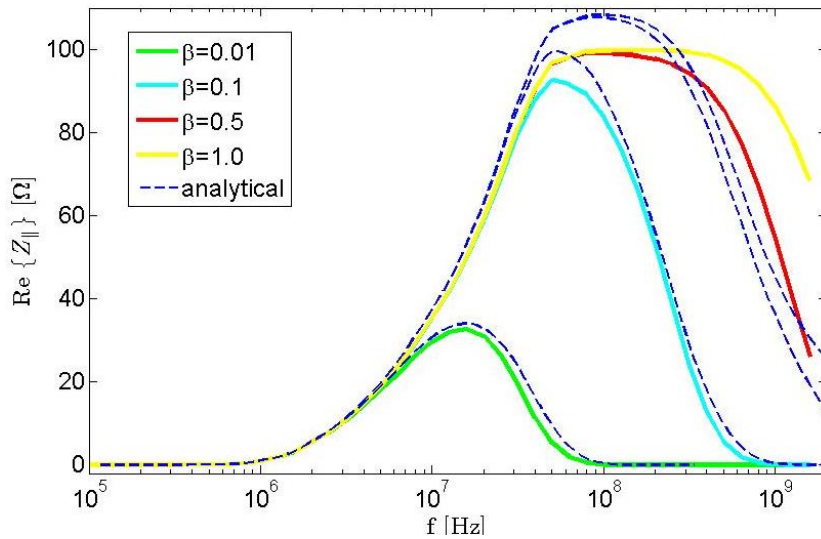
Numerical Examples: Rectangular beam pipe (staircase FIT) (no Helmholtz split)



Numerical Examples: Ferrite Ring (FEM)



Numerical Examples: Ferrite Ring (FIT)



Comparison of my FIT and FEM implementations

- Both solvers have recently been developed...
- Not been pushed to the limit yet!
- Not optimized, not parallelized, no nice interface
- Therefore, no performance comparison!

FIT (2D/3D)

- Windows based C++
- Linear algebra package PETSc
→ complex numbers available
- HEX mesh from CST
- Bad convergence on curved structures due to staircase mesh
- Extremely slow in 3D

FEM (2D)

- Linux based Python
- FEniCS framework (open source)
- Different linear algebra backends
- TET mesh from GMSH
- Helmholtz split necessary for lowest order
- No complex numbers but coupled systems can be treated easily

Conclusion and Outlook

- Both methods work well in 2D
- Convergence studies yet to be done
 - difficult since convergence depends on frequency, beta, material parameters, dimensions...
- Small skin depth hard to treat
 - Surface impedance boundary conditions (SIBC)
- Thin layers hard to treat
 - Thin sheet approaches
 - Impedance transmission conditions

Outlook: 3D simulation in Frequency Domain

- For both FIT and FEM possible
- Good parallel LES solvers necessary
 - They have to be able to deal with ill conditioned systems
- Requires the presented 2D solvers as beam entry and exit boundary conditions
 - Don't try to imprint analytical beampipe solution!
- For simulation above beam pipe cutoff also a waveguide port Eigenmode solver required
- **Large** development effort...
 - ...ongoing at TEMF

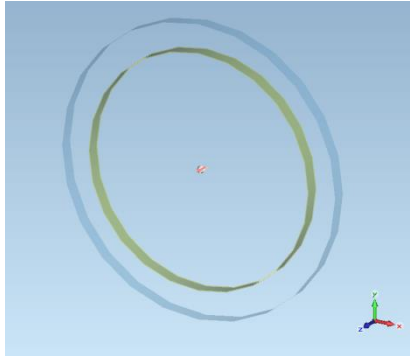
THE END

- Thank you for your kind attention
- Any questions?



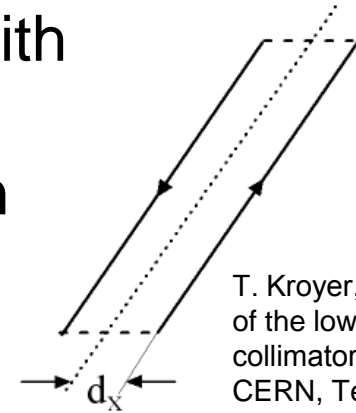
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Simplified numerical method (according to “radial model“)



- Power loss calculation with CST EM STUDIO®
- MQS Frequency-Domain

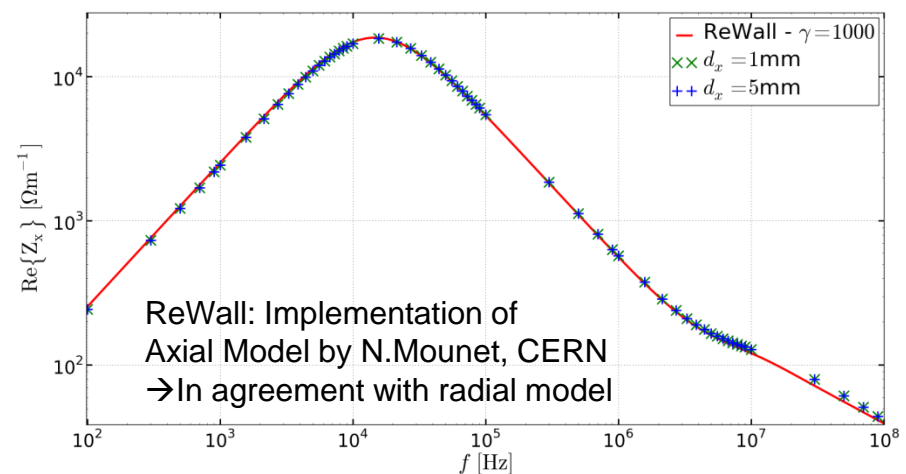
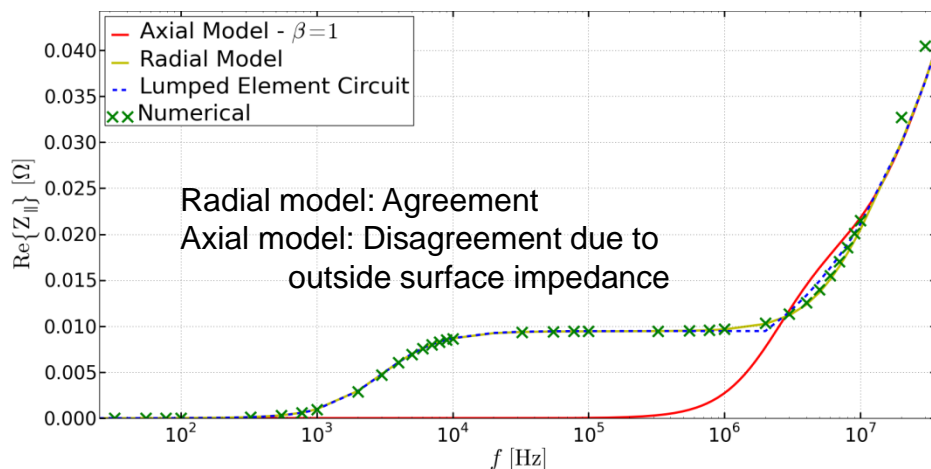
$$\delta P = \frac{1}{2} \int_{\text{pipe}} \underline{\vec{E}} \cdot \underline{\vec{J}}^* dV$$



T. Kroyer, “Simulation of the low frequency collimator impedance” CERN, Tech.Rep., 2008

$$\frac{\text{Re}\{Z_{\parallel}\}}{l} = \frac{1}{I^2} \frac{\delta P}{\delta z}$$

$$\frac{\text{Re}\{Z_{\perp,x}(\omega)\}}{l} = \frac{c}{\omega d_x^2} \frac{1}{I^2} \frac{\delta P}{\delta z}$$



Transformer Model

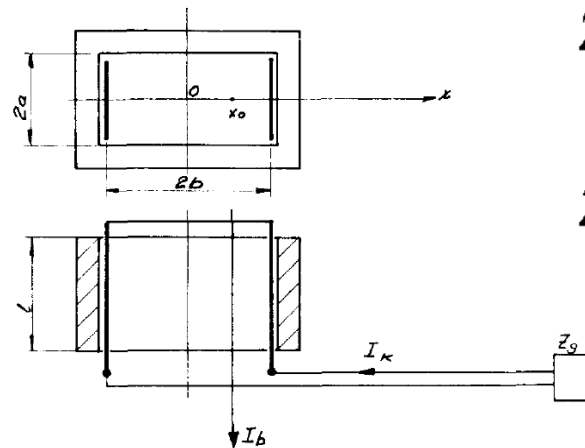
NUCLEAR INSTRUMENTS AND METHODS 159 (1979) 21-27 ; © NORTH-HOLLAND PUBLISHING CO.

METHODS FOR MEASURING TRANSVERSE COUPLING IMPEDANCES IN CIRCULAR ACCELERATORS

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CERN, CH-1211 Geneva, Switzerland

Received 26 June 1978

$$Z_k = j\omega L + Z_g,$$



$$Z_L = \frac{\omega^2 \mu_0^2 x_0^2 l^2}{4 a^2 Z_k} \Omega,$$

$$Z_T = \frac{c\omega\mu_0^2 l^2}{4 a^2 Z_k} \Omega/m$$

- Longitudinal impedance is zero at the center
- Both scale inversely with the Network impedance

The Kicker magnet

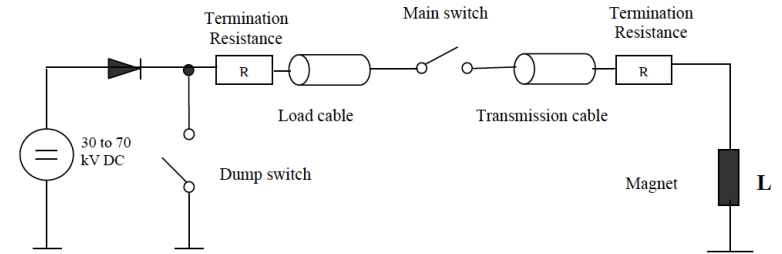
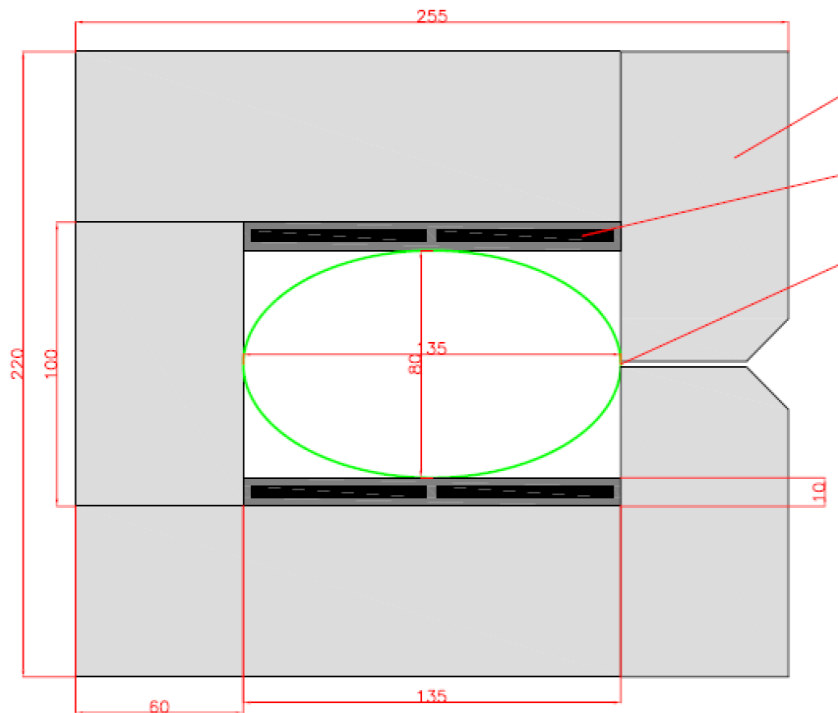


Figure 2.8-160: Schematic circuit diagram of a monopolar kicker

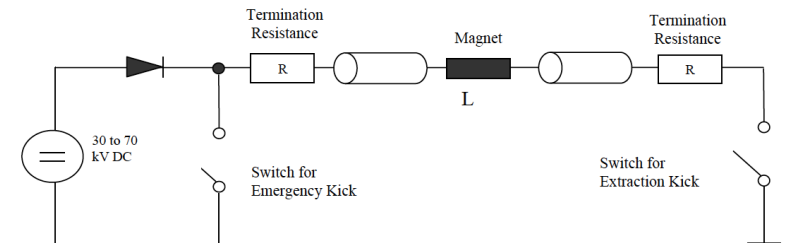
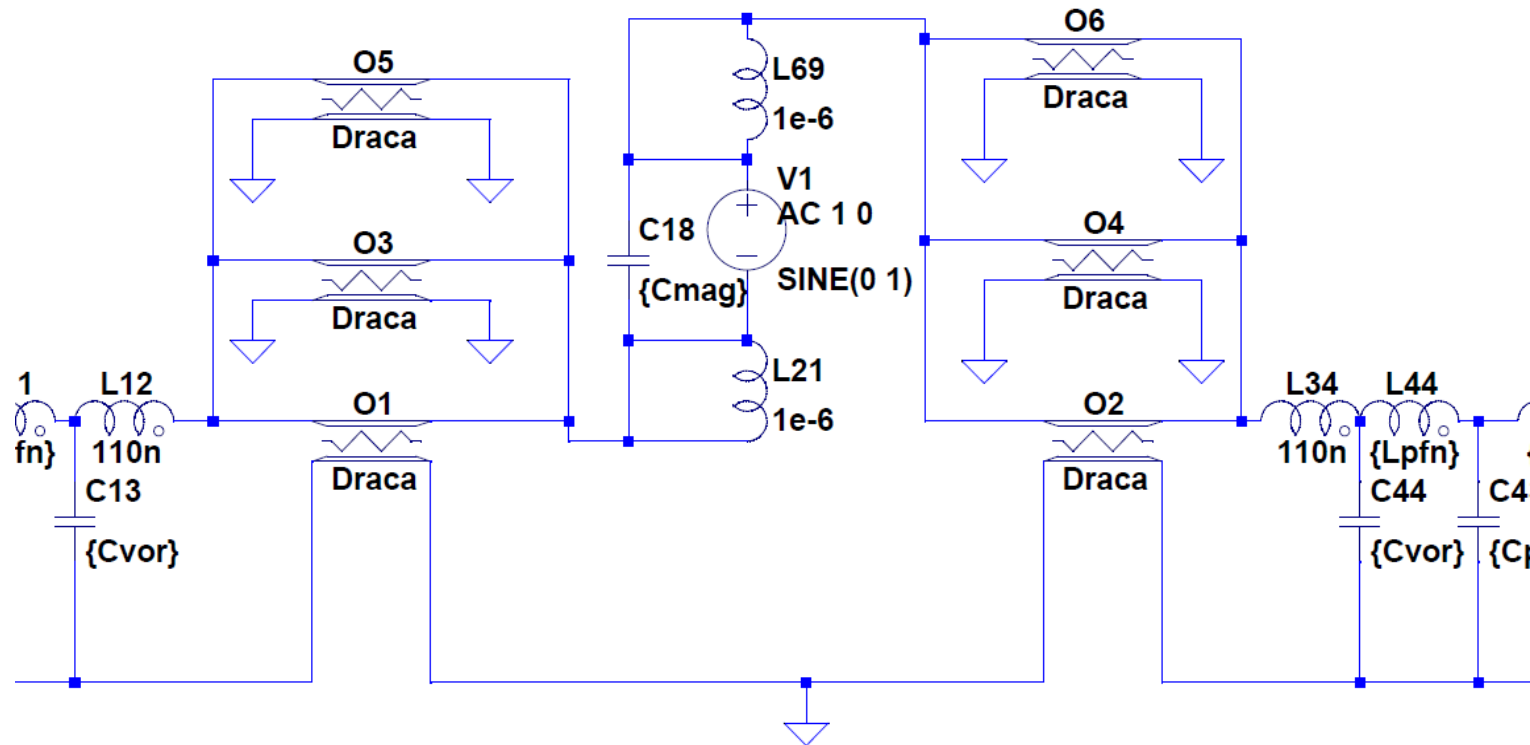


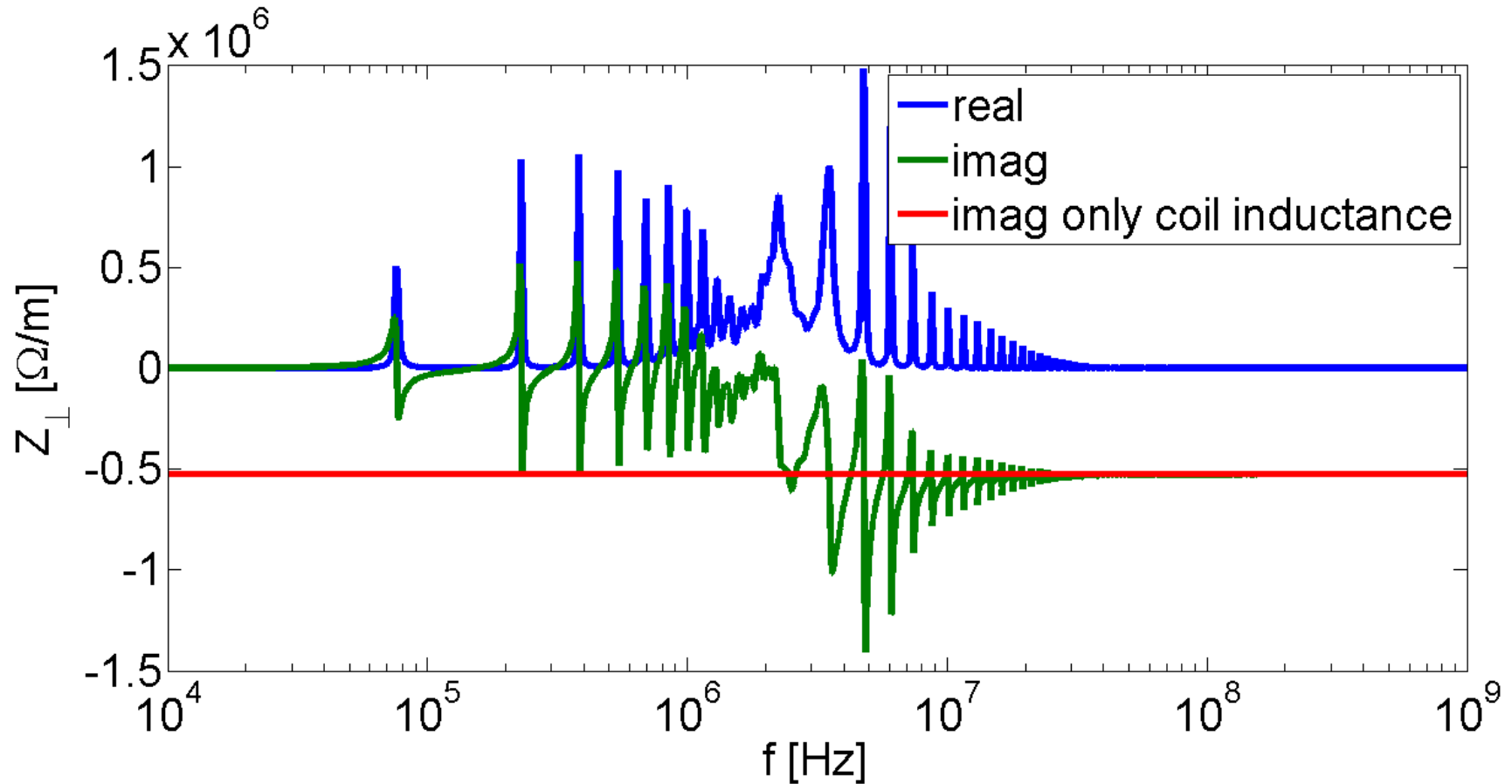
Figure 2.8-158: Schematic circuit – bipolar emergency / extraction kicker module

PFN Impedance determination

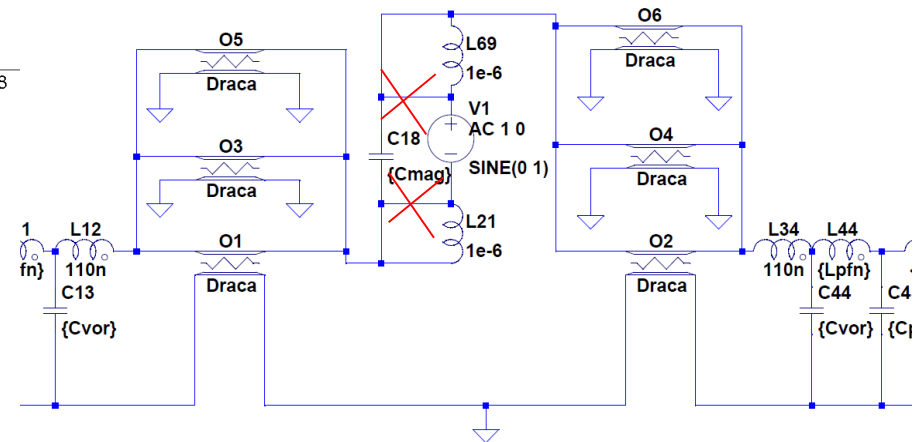
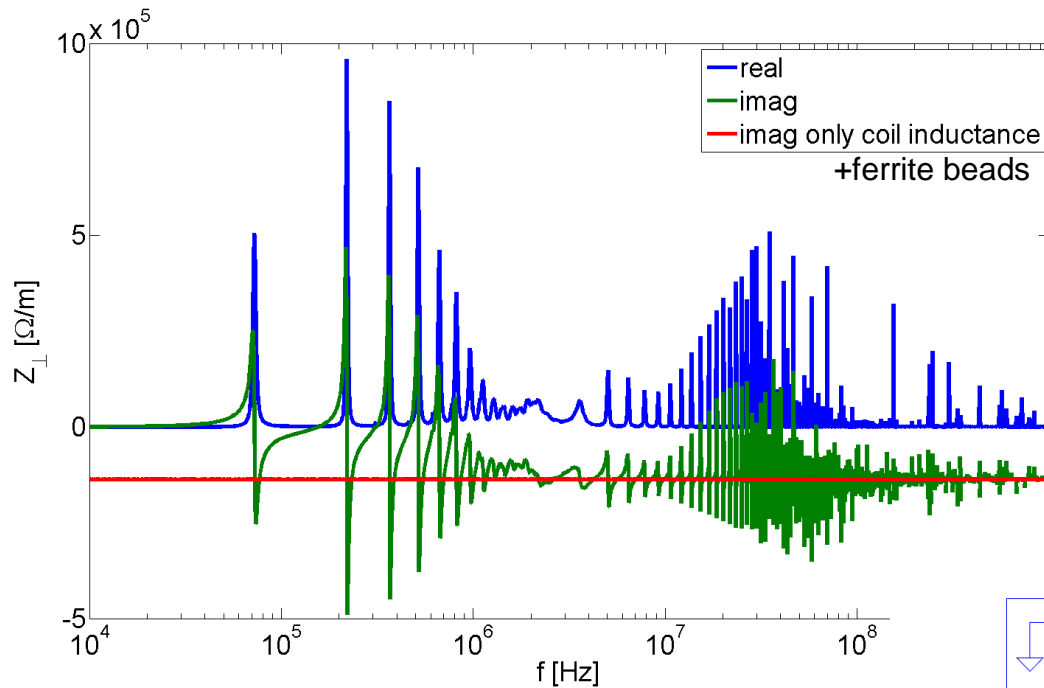


- Can also be measured like this with NWA

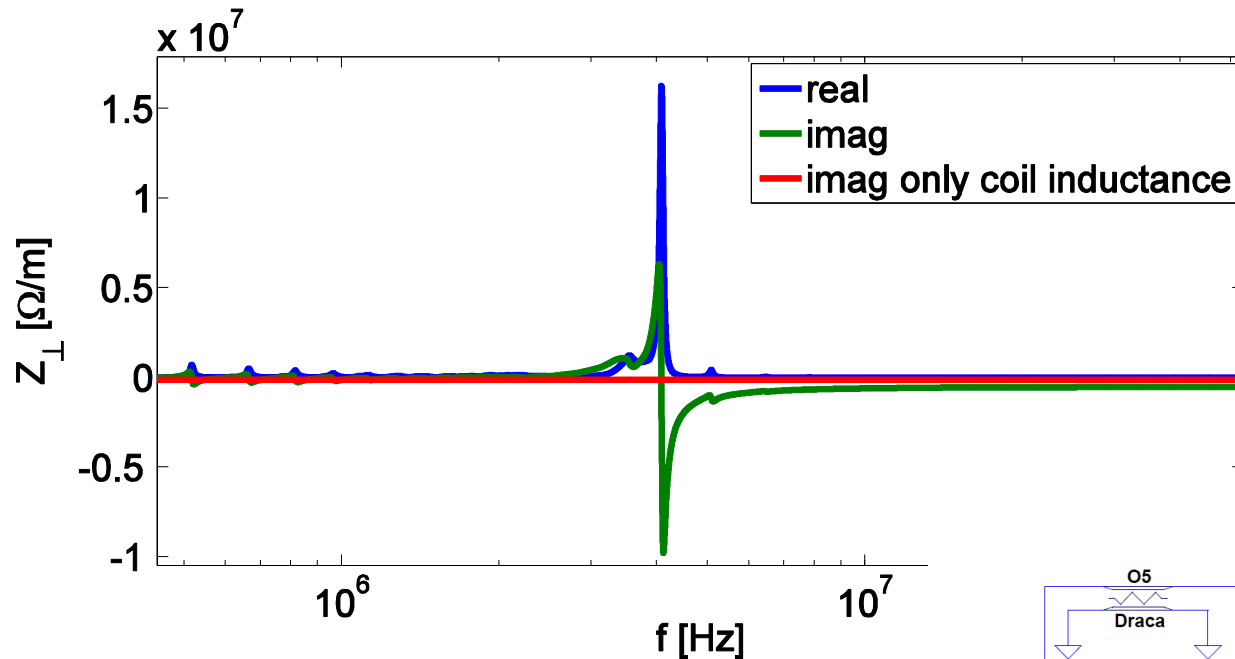
Transverse impedance



Optimization with ferrite bead (saturates at kicker puls current)



Cmag inside ferrites → Resonance!



$$f_r = 4 \text{ MHz}$$

