SUMMARY OF SESSION 1

Elias Métral

Impedance theory and related effects

- **V. Danilov: From the primary concept of coupling impedance to the most recent generalizations**

- **G. Rumolo: A review of impedance related collective effects**

V. Danilov (1/7)

- ("Known => For instance in Chao's book") 1st introduction of the impedance concept for particle accelerators => A. Sessler and V. Vaccaro (1967)

$$U = Z\, I$$

Ohm's law in electricity

~ Integrated force

$$Z = -\frac{E_z\, 2\, \pi\, R}{I}$$

=> Longitudinal impedance (frequency domain)

V. Danilov (2/7)

- …There was another paper in 1966…which could not be known…

Distribution: (closed) AR and ISR Scientific Staff.

ISR-RF/66-35
November 18, 1966

LONGITUDINAL INSTABILITY OF A COASTING BEAM ABOVE TRANSITION, DUE TO THE ACTION OF LUMPED DISCONTINUITIES.

by V.G. Vaccaro

1. Generalities

We assume that the electrical action on an ion beam, of a discontinuity in a tank is that of an impedance. We still consider the

where a is the beam radius. The passage of an ion beam induces a field in the discontinuity, which is given by:

$$E_d = -Z\ I/d, \qquad (4)$$

where d is the magnitude of the discontinuity, and Z is the impedance of the discontinuity.

REFERENCES

1) V.K. Neil and A.M. Sessler
Longitudinal Resistive Instabilities of Intense Coasting Beams in Particle Accelerator
Rev. Sci. Instr. 36, 429 (1965)

1) A.M. Sessler and V.G. Vaccaro
Longitudinal Instabilities of Azimuthally Uniform Beams in Circular Vacuum Chambers of Arbitrary Electrical Proprieties (in preparation).

V. Danilov (3/7)

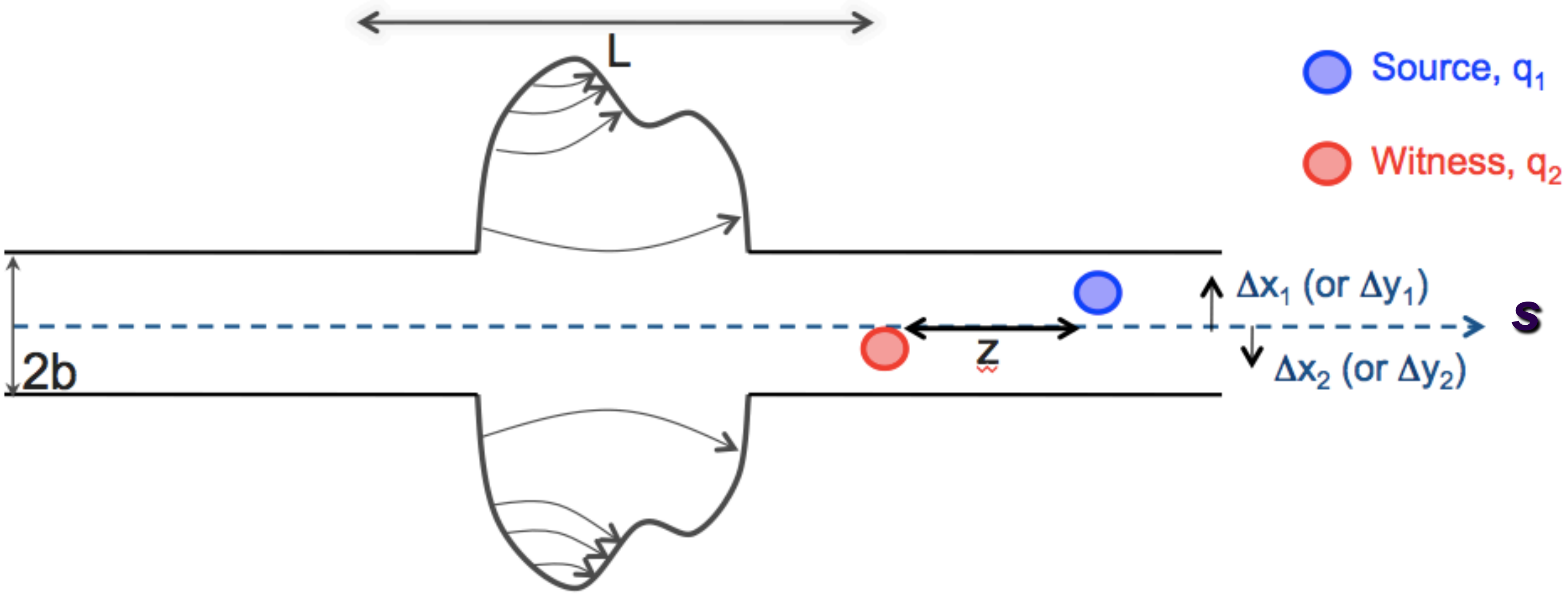
- Wake field (wake function) = concept in space / time domain (came few years later => 1969)

  => The 2 are linked by Fourier transforms

LNF - 69/80
23 Dicembre 1969

A. G. Ruggiero and V. G. Vaccaro : THE WAKE FIELD OF AN OSCILLATING PARTICLE IN THE PRESENCE OF CONDUCTIVE PLATES WITH RESISTIVE TERMINATIONS AT BOTH ENDS. -

V. Danilov (4/7)



G. Rumolo

V. Danilov (5/7)

- 2 fundamental approximations behind the "conventional impedances / wakes"
  - Rigid-beam approximation => $z = s_{witness} - s_{source} = \text{Constant}$
  - Impulse approximation => $\upsilon \, \Delta p = \int_0^L F \, ds$

Wake potential

- Several properties
  - No wake in front for $\beta = 1$
  - Etc.

Not true for CSR

V. Danilov (6/7)

- **Sessler-Vaccaro formalism works very well for longitudinal impedance, even for unusual wakes like CSR**

$$\int_0^L F_l \, ds = -e^2 \, W_l(z)$$

- **Transverse case is more complicated**
  - **Conventional definition**

$$\int_0^L F_r \, ds = -e^2 \, r_{source} \, W_r(z)$$

  - **… but several terms need to be added to correctly describe the beam dynamics**

$$\int_0^L F_r \, ds = -e^2 \, r_{source} \, W_r(z) - e^2 \, r'_{source} \, A_r(z) - e^2 \, r_{witness} \, D_r(z) + \ldots$$

Driving (or dipolar) wake

Angular wake => Fast damping in VEPP-2 and BEP

Detuning (or quadrupolar) wake => Effect on tune shift (see 2nd talk), TMCI, etc.

V. Danilov (7/7)

- With these additions, Sessler-Vaccaro formalism works well also in the transverse plane

- In the general case (e.g. case of a long element with large angle and coordinate changes), the force itself should be used instead of the integral (average) of a certain length
  - It is the force itself which enters into the linearized Vlasov equation (see 2nd talk)
  - The force has to be defined via the Green function (impulse response of the inhomogeneous differential equation)
  - Approach based on the division of the synchrotron phase space (by rings and sectors) was used by V. Danilov and E. Perevedentzev in 1992 and recently (2014) by A. Burov for his NHTVS (Nested Head-Tail Vlasov Solver)

- Extensions of the impedance concept: Space charge, e-cloud, CSR

G. Rumolo (1/5)

$$P = R\, I^2$$

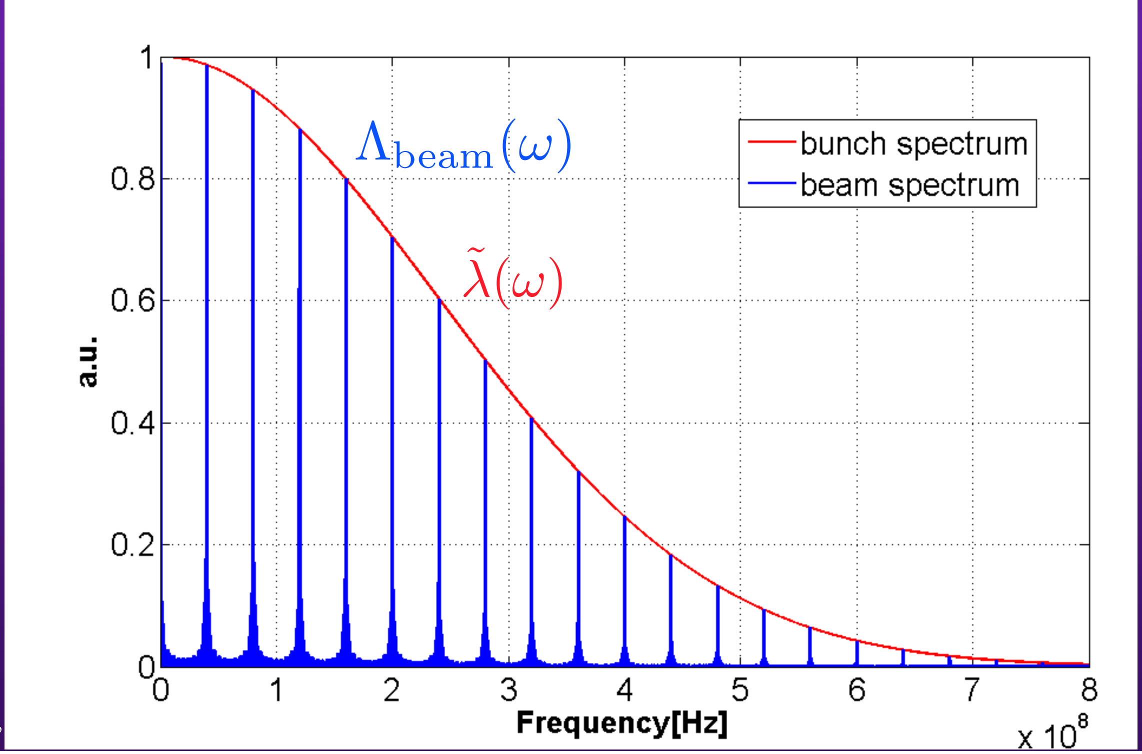
Joule's law in electricity

- **Longitudinal impedance / wake => Energy lost by the source:**
  - **Modes that remain trapped in the object**
    - **Partly dissipated on lossy walls or into purposely designed inserts or HOM absorbers**
    - **Partly transferred to following particles (or the same particle over successive turns), possibly feeding into an instability**
  - **Modes that propagate down the beam chamber (above cut-off), eventually lost on surrounding lossy materials**

G. Rumolo (2/5)

- **General formula for the energy loss of a bunch train (*M* identical equally spaced bunches)**

$$\Delta E_{\text{beam}} = \frac{e^2 \omega_0}{2\pi} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \text{Re}\left[Z_{||}(p\omega_0)\right] \cdot \left[\frac{1-\cos(\frac{2\pi M p}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$



G. Rumolo (3/5)

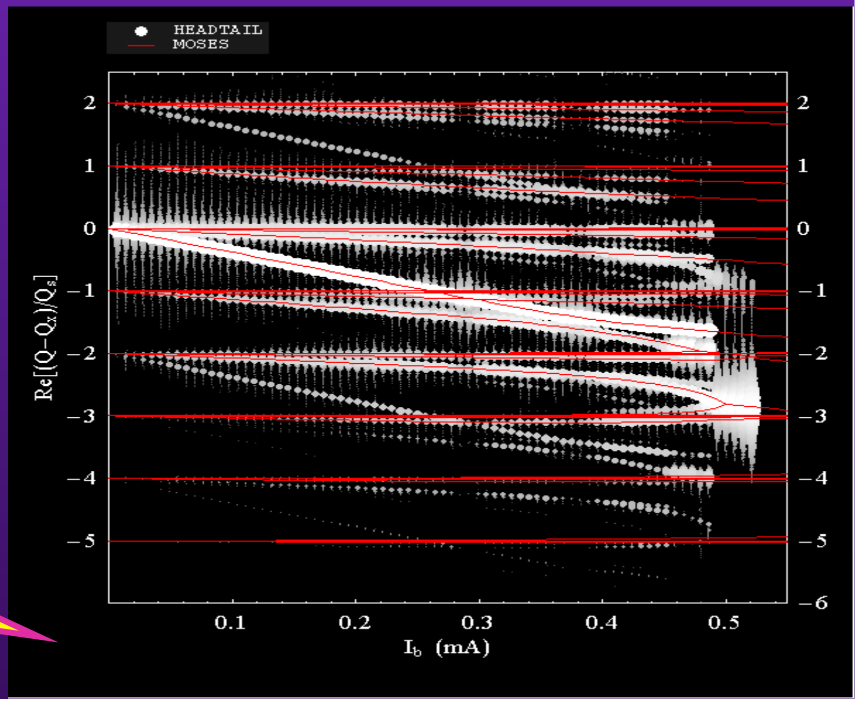
- **Motion in the LONGITUDINAL and TRANSVERSE planes**
  - Find the Hamiltonian *H* of the system from the equations of motion (2 for longitudinal and 4 for transverse – uncoupled)
  - Consider a distribution $\Psi = \Psi_0 \left(H \right) + \Delta\Psi$ (Stationary: $\Psi_0(H)$; Perturbation: $\Delta\Psi$)
  - Apply Vlasov equation $\frac{d\Psi}{dt} = 0$ and linearize it
  - Stable perturbation => e.g. PWD in longitudinal and shifts of transverse modes for purely imaginary impedance
  - Growing perturbation => Instabilities: SB, CB, low intensity (independent modes), high intensity (not independent modes)

G. Rumolo (4/5)

- **Several codes developed to solve Vlasov equation. Some can include multi-bunch, chromaticity, amplitude detuning and transverse damper, but they all consider linearised longitudinal motion and only dipole wake fields. E.g:**
  - **MOSES [Y. Chin, CERN/SPS/85-2 & CERN/LEP-TH/88-05]**
  - **NHTVS [A. Burov, Phys. Rev. ST AB 17, 021007 (2014)]**
  - **DELPHI [N. Mounet, HSC Meeting, 09/04/2014]**

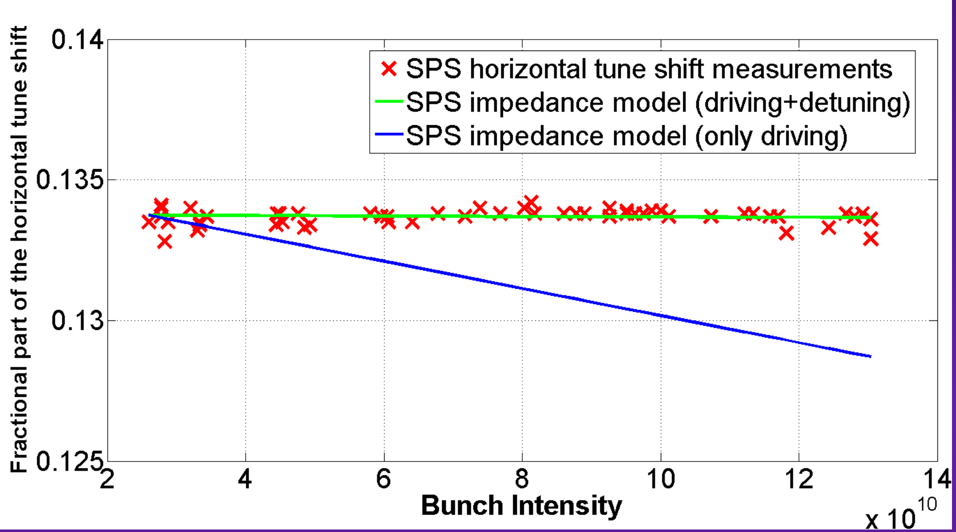
- **Numerical simulations**
  - **Discretization => Slicing of the bunch, with a slicing fine enough to sample the wake function**

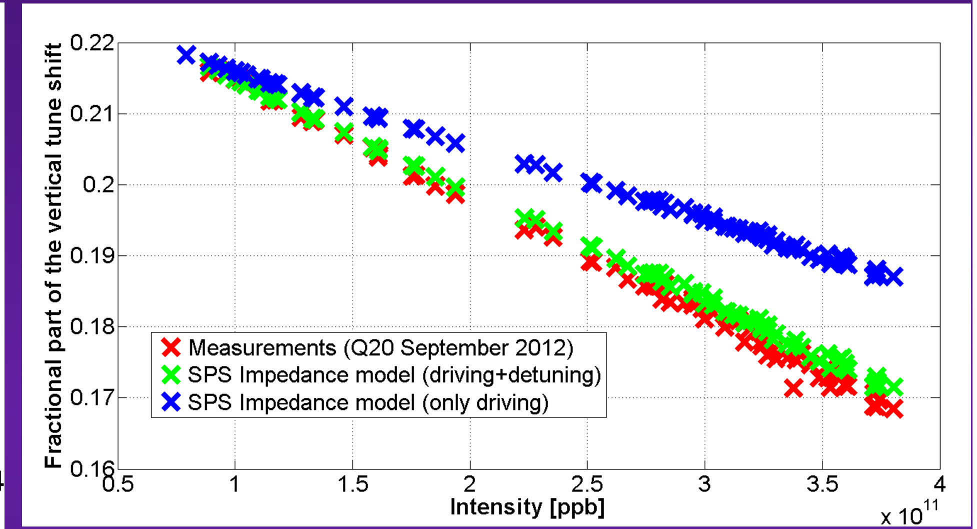
Benchmark between HEADTAIL tracking code and MOSES



G. Rumolo (5/5)

- **Importance of the detuning (quadrupolar) impedance**

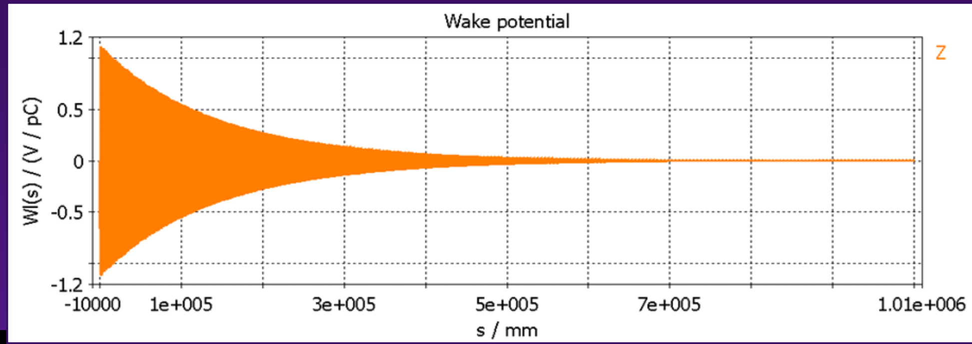




$$E_z = - \frac{Z\ I}{2\pi R}$$

A. Sessler and V. Vaccaro

February 6, 1967



The very 1st paper on impedance was written in November, 1966

Many thanks again for all Vittorio!
(and let's celebrate the ½ century in 2016-2017)