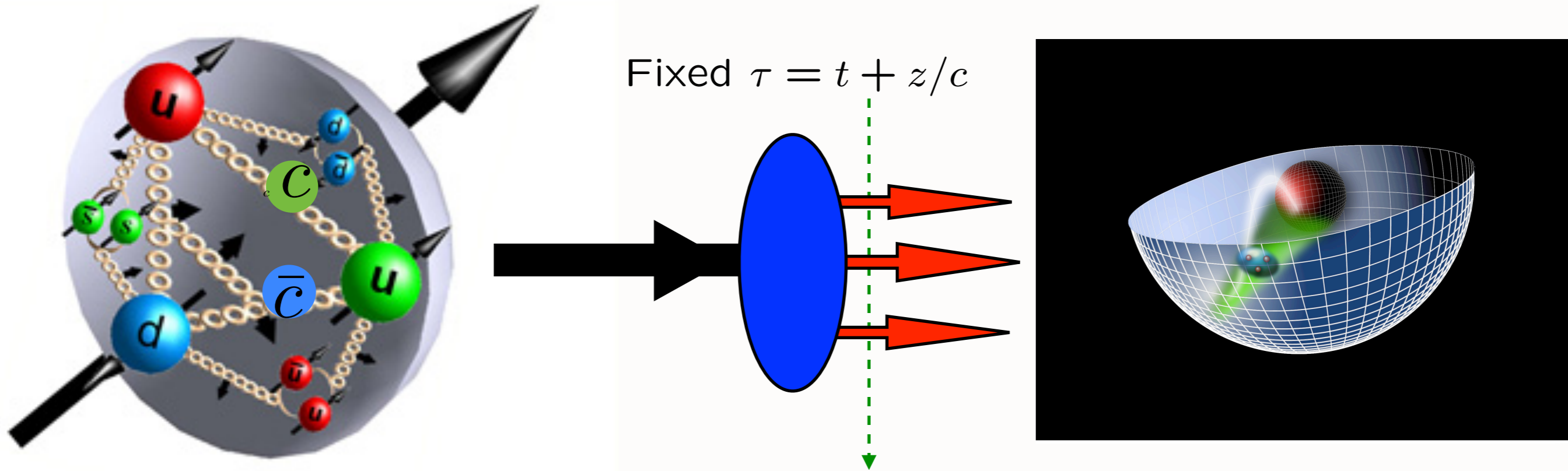
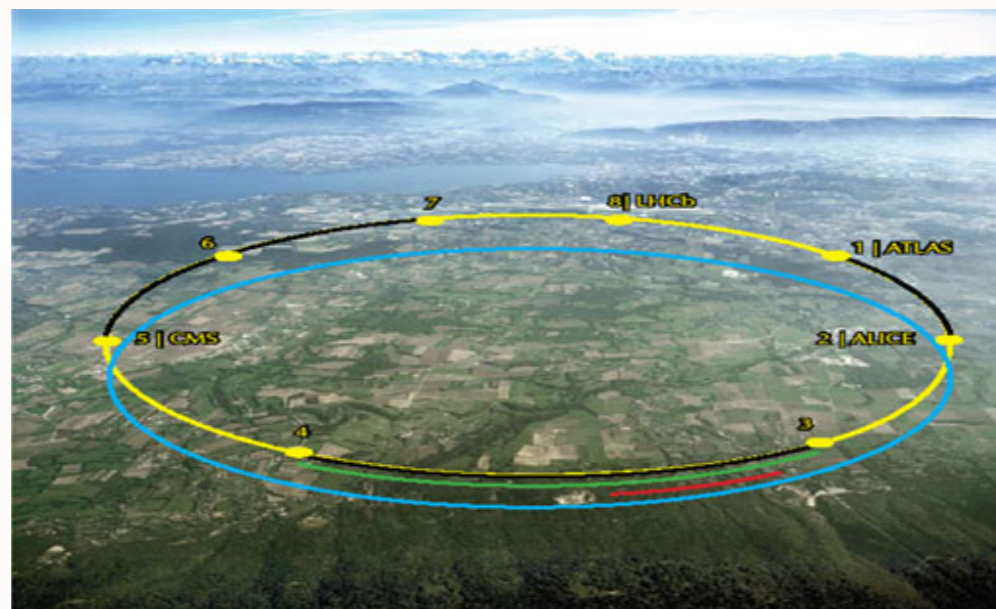


Novel QCD Phenomena and New Perspectives for Hadron Physics from Light-Front Holography



CERN TH Seminar

January 22, 2014



Stan Brodsky

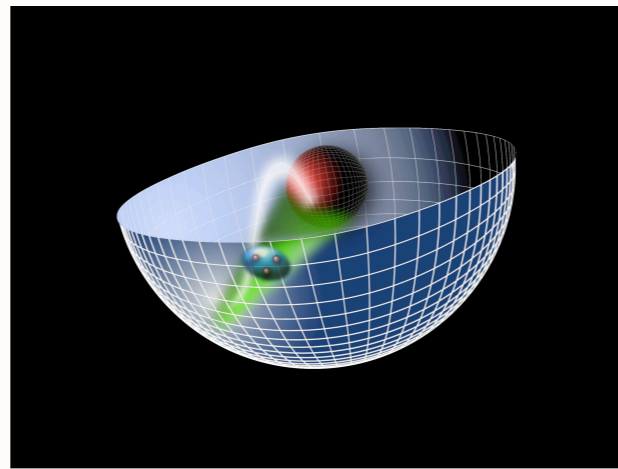


Stanford University, October 19-20, 2011

Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **Confinement in QCD -- What sets the QCD mass scale?**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Insights into QCD Condensates**
- **Chiral Symmetry**
- **Systematically improvable**
- **Eliminate scale ambiguities**





*AdS/QCD
Soft-Wall Model*

Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

Confinement scale:

$$1/\kappa \simeq 1/3 \text{ fm}$$

***Unique
Confinement Potential!
Conformal Symmetry
of the action***

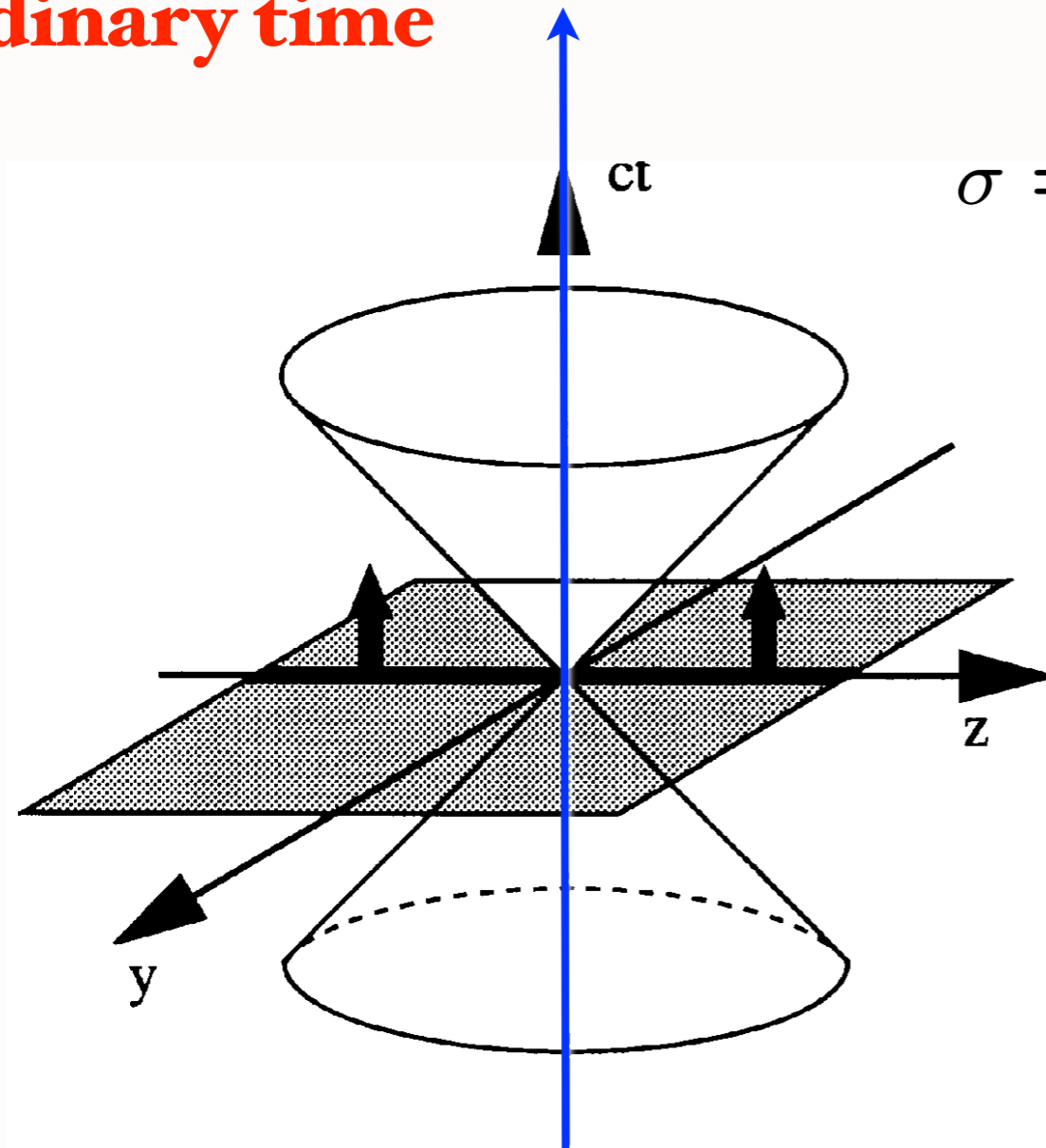
● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

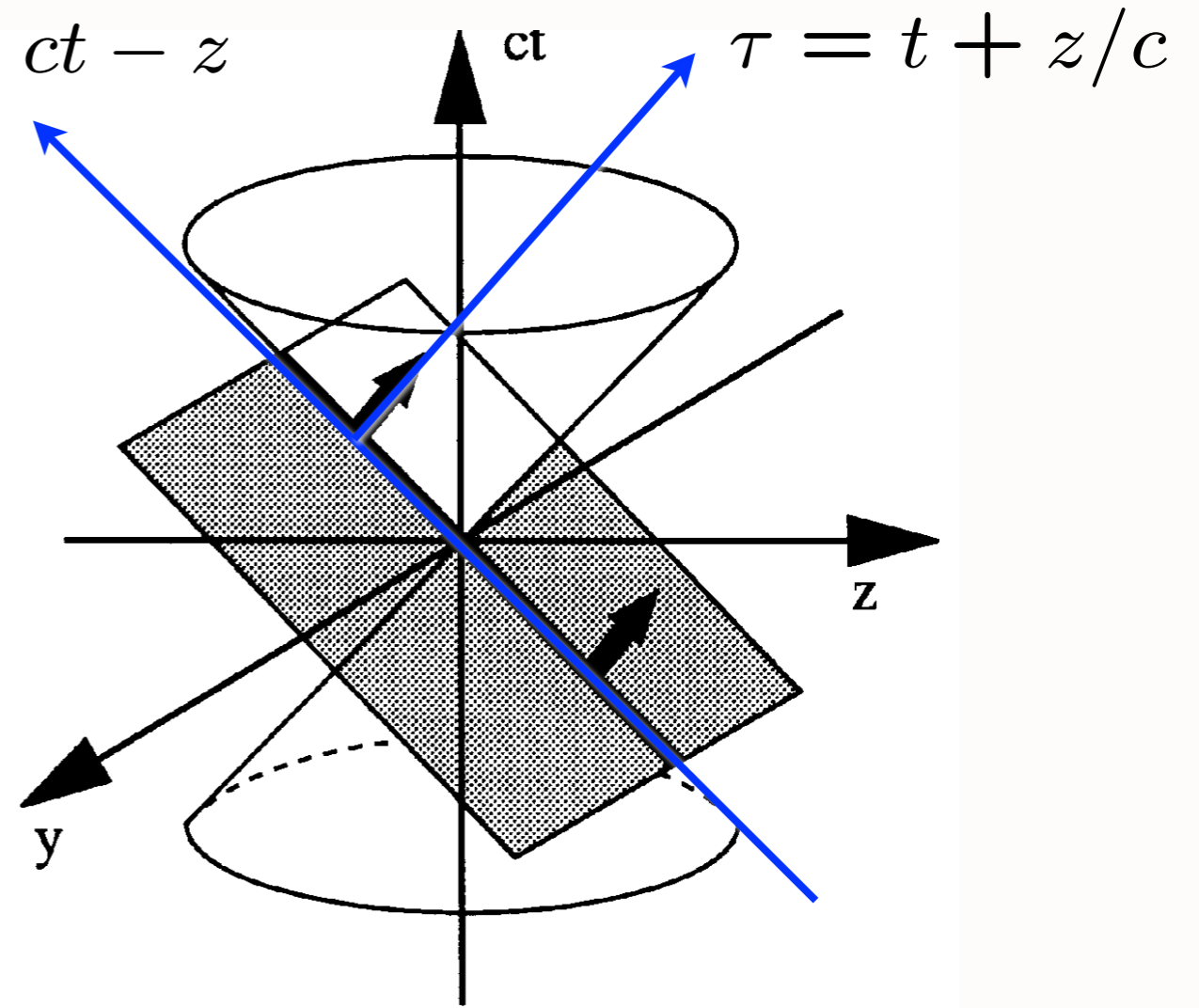
P.A.M Dirac, Rev. Mod. Phys.
21, 392 (1949)

Evolve in
ordinary time

Evolve in
light-front time!



$$\sigma = ct - z$$



$$\tau = t + z/c$$

Instant Form

Front Form

Each element of
flash photograph
illuminated
along the light front
at a fixed

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

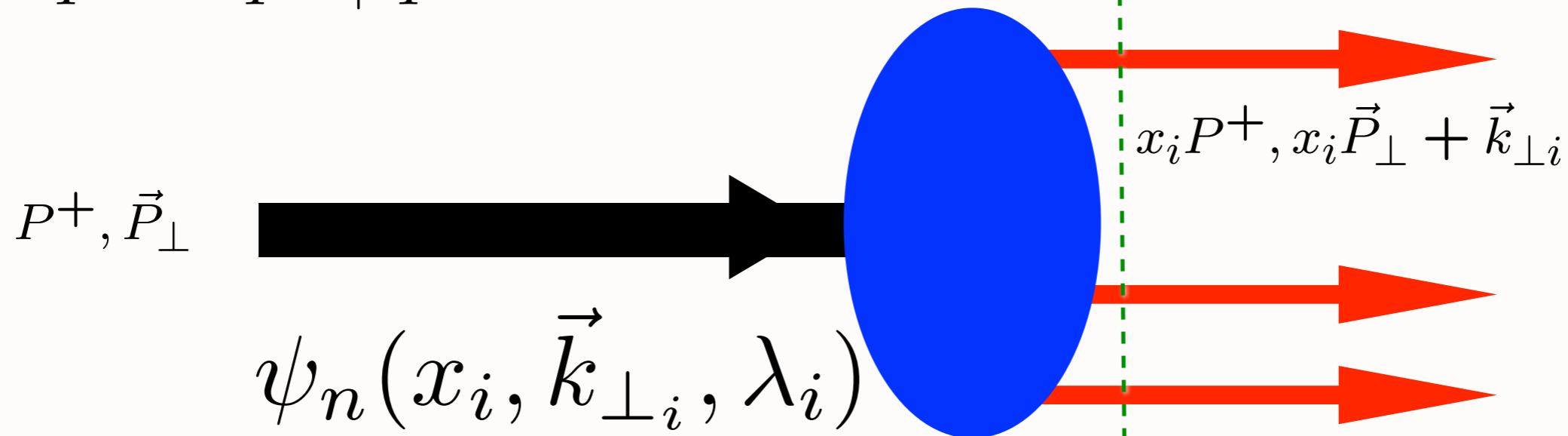


Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$$|p, J_z \rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i \rangle$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

P^+, \vec{P}_\perp

$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

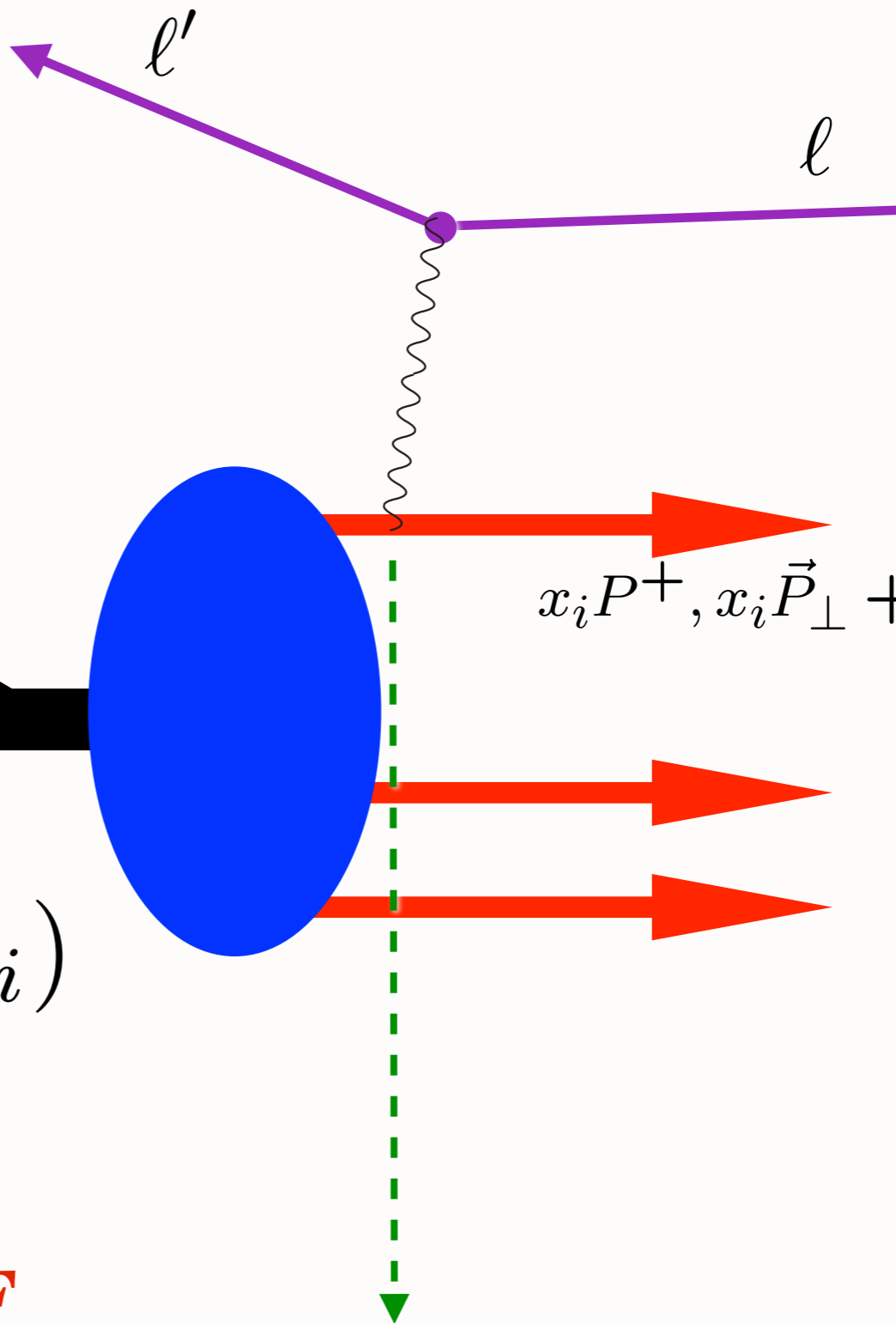
$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

Fixed $\tau = t + z/c$

$$x_{bj} = x = \frac{k^+}{P^+}$$

Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph



- LF coordinates

$$\begin{array}{llll} \tau = x^+ = x^0 + x^3 & \text{light-front time} & P^+ = P^0 + P^3 & \text{longitudinal momentum} \\ x^- = x^0 - x^3 & \text{longitudinal space variable} & P^- = P^0 - P^3 & \text{light-front Hamiltonian} \\ \mathbf{x}_\perp = (x^1, x^2) & \text{transverse space variable} & \mathbf{P}_\perp = (P^1, P^2) & \text{transverse momentum} \end{array}$$

- On shell relation $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 = \mathcal{M}^2$ leads to dispersion relation for LF Hamiltonian P^-

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \quad P^+ > 0$$

- Hamiltonian equation for the relativistic bound state

$$i \frac{\partial}{\partial x^+} |\psi(P)\rangle = P^- |\psi(P)\rangle = \frac{M^2 + \mathbf{P}_\perp^2}{P^+} |\psi(P)\rangle$$

where P^- is derived from the QCD Lagrangian: kinetic energy of partons plus confining interaction

- Construct LF Lorentz invariant Hamiltonian $P^2 = P^- P^+ - \mathbf{P}_\perp^2$

$$P_\mu P^\mu |\psi(P)\rangle = M^2 |\psi(P)\rangle$$



- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows unambiguous definition of partonic content of a hadron

$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

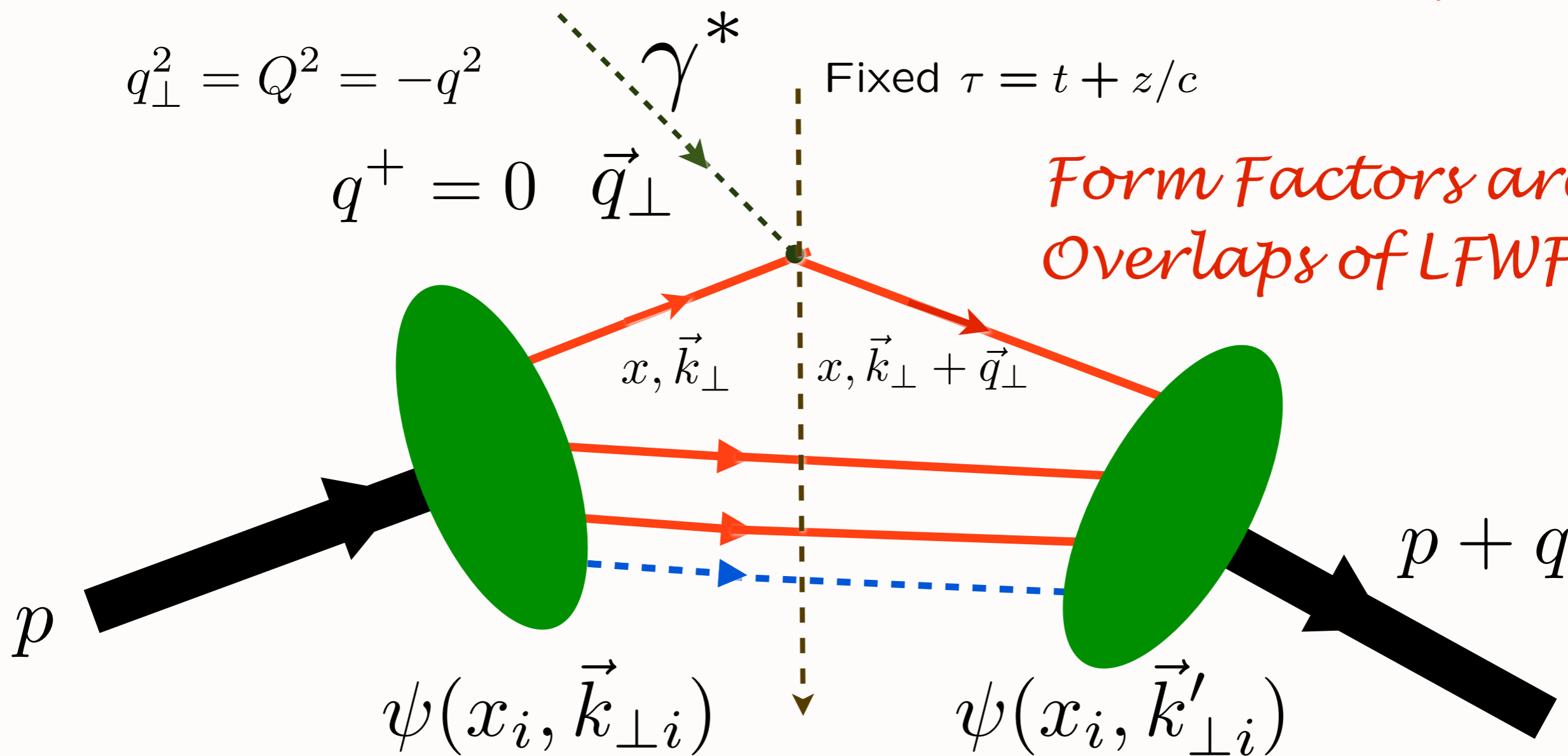
Interaction picture

$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

Fixed $\tau = t + z/c$

Form Factors are Overlaps of LFWFs



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

**Drell & Yan, West
Exact LF formula!**

Exact LF Formula for Pauli Form Factor

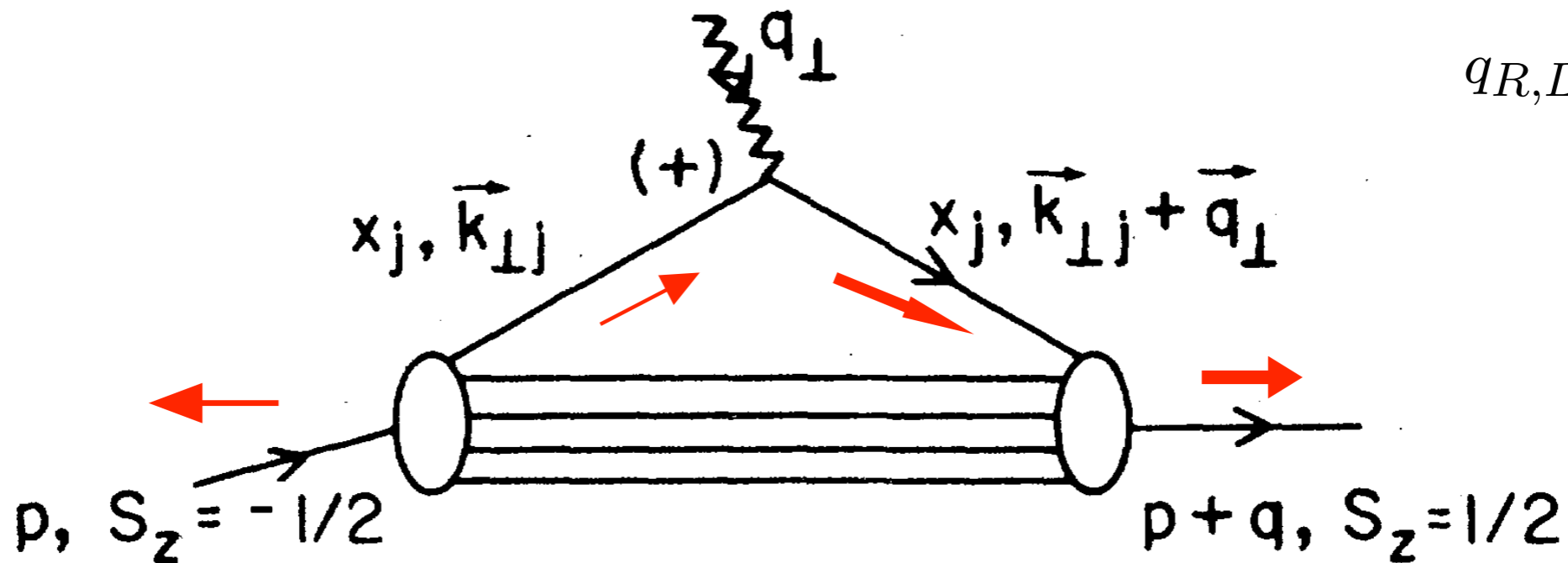
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx] [d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

Drell, sjb

$$q_{R,L} = q^x \pm iq^y$$

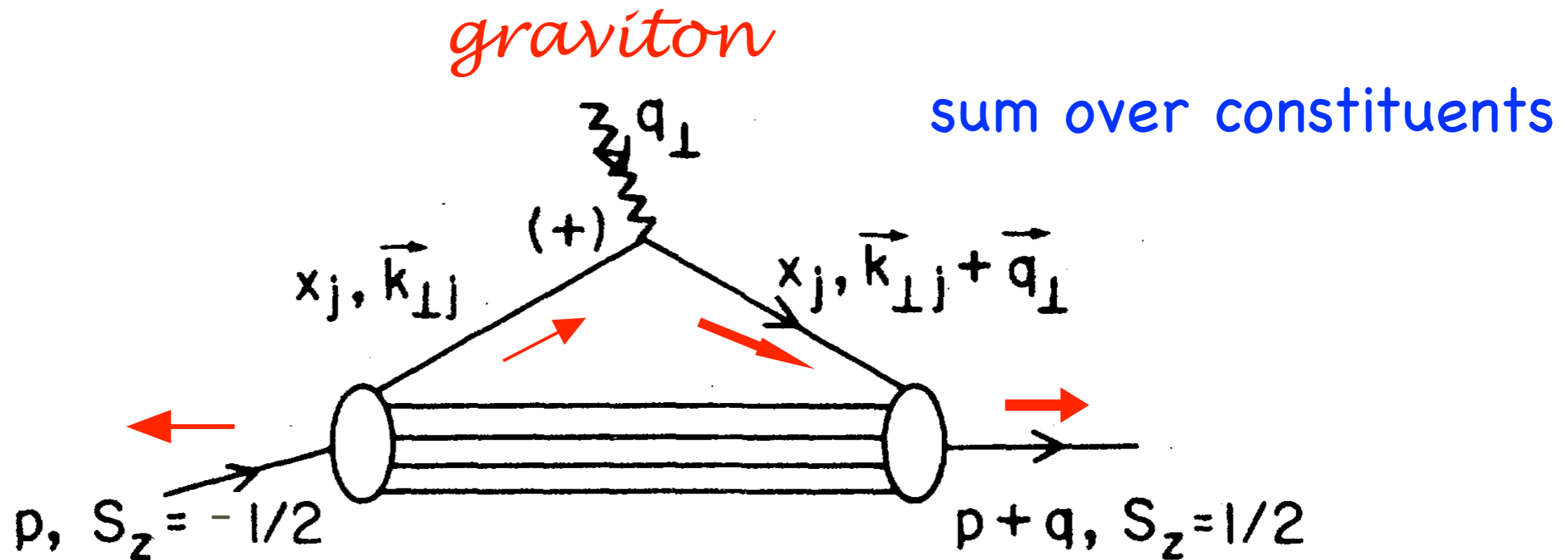


Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

*Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum*

Vanishing Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem



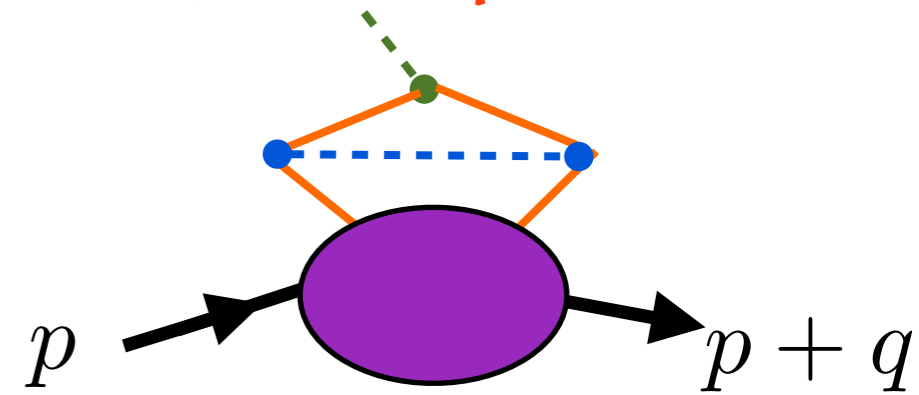
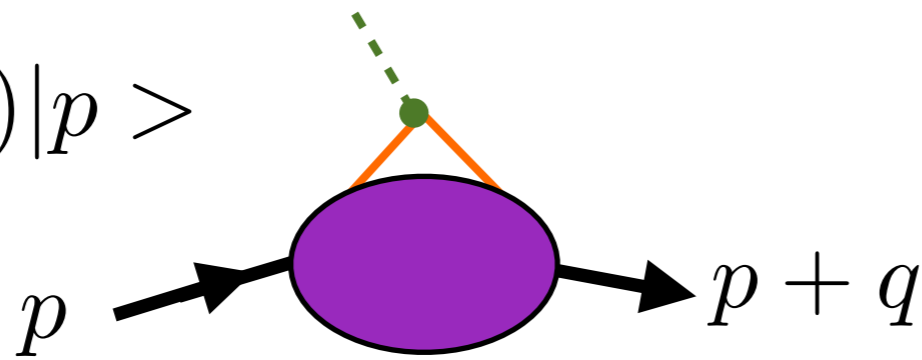
Hwang, Schmidt, sjb;
Holstein et al

$B(0) = 0$

Each Fock State

Calculation of proton form factor in Instant Form

$$\langle p + q | J^\mu(0) | p \rangle$$



- **Need to boost proton wavefunction from p to $p + q$: Extremely complicated dynamical problem; even the particle number changes**
- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**
- **Instant form: acausal boundary conditions**

$$|\psi_p(P^+, \vec{P}_\perp)\rangle = \sum_n \prod_{i=1}^n \frac{dx_i d^2\vec{k}_{\perp i}}{\sqrt{x_i} 16\pi^3} 16\pi^3 \delta\left(1 - \sum_{i=1}^n x_i\right) \delta^{(2)}\left(\sum_{i=1}^n \vec{k}_{\perp i}\right) \\ \times \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle.$$

$$q_{\lambda_q/\Lambda_p}(x, \Lambda) = \sum_{n, q_a} \int \prod_{j=1}^n dx_j d^2\vec{k}_{\perp j} \sum_{\lambda_i} |\psi_{n/H}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 \\ \times \delta\left(1 - \sum_i x_i\right) \delta^{(2)}\left(\sum_i \vec{k}_{\perp i}\right) \delta(x - x_q) \delta_{\lambda_a \lambda_q} \Theta(\Lambda^2 - \mathcal{M}_n^2),$$

Obeys DGLAP Evolution ***Defines quark distributions***

Connection to Bethe-Salpeter:

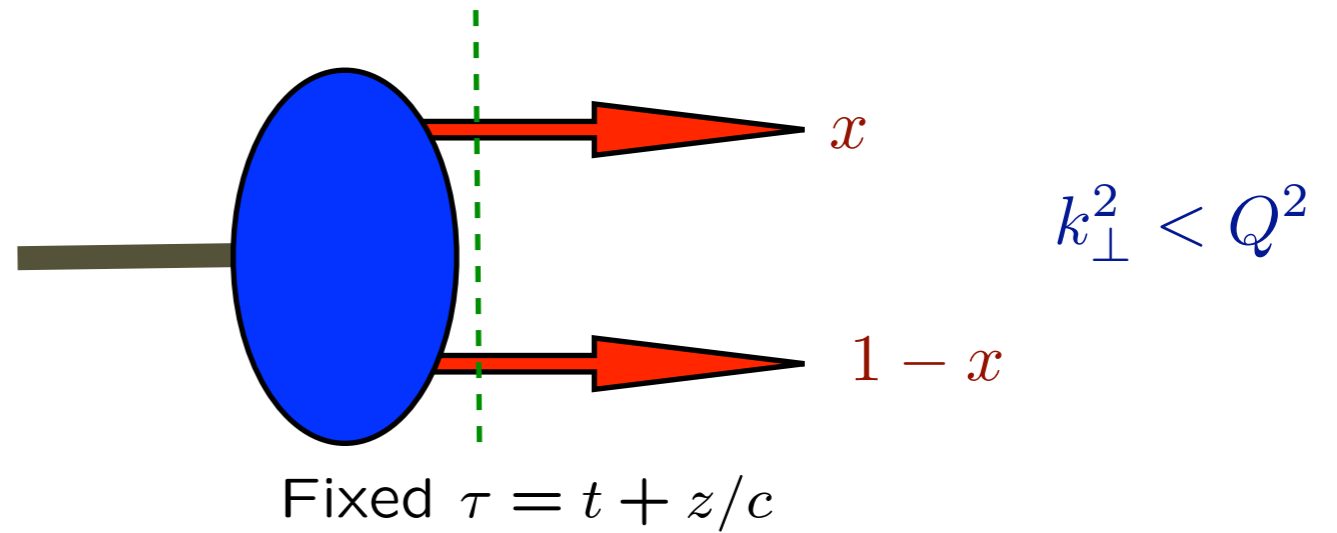
$$\int dk^- \Psi_{BS}(k, P) \rightarrow \psi_{LF}(x, \vec{k}_\perp) \quad \Psi_{BS}(x, P)|_{x^+=0}$$

Hadron Distribution Amplitudes

$$A^+ = 0$$

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

$$\sum_i x_i = 1$$



- Fundamental **gauge invariant** non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

**Conserved
LF Fock-State by Fock-State
Every Vertex**

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

**n-1 orbital angular
momenta**

Nonzero Anomalous Moment <--> Nonzero orbital angular momentum

Drell, sjb, Schmidt

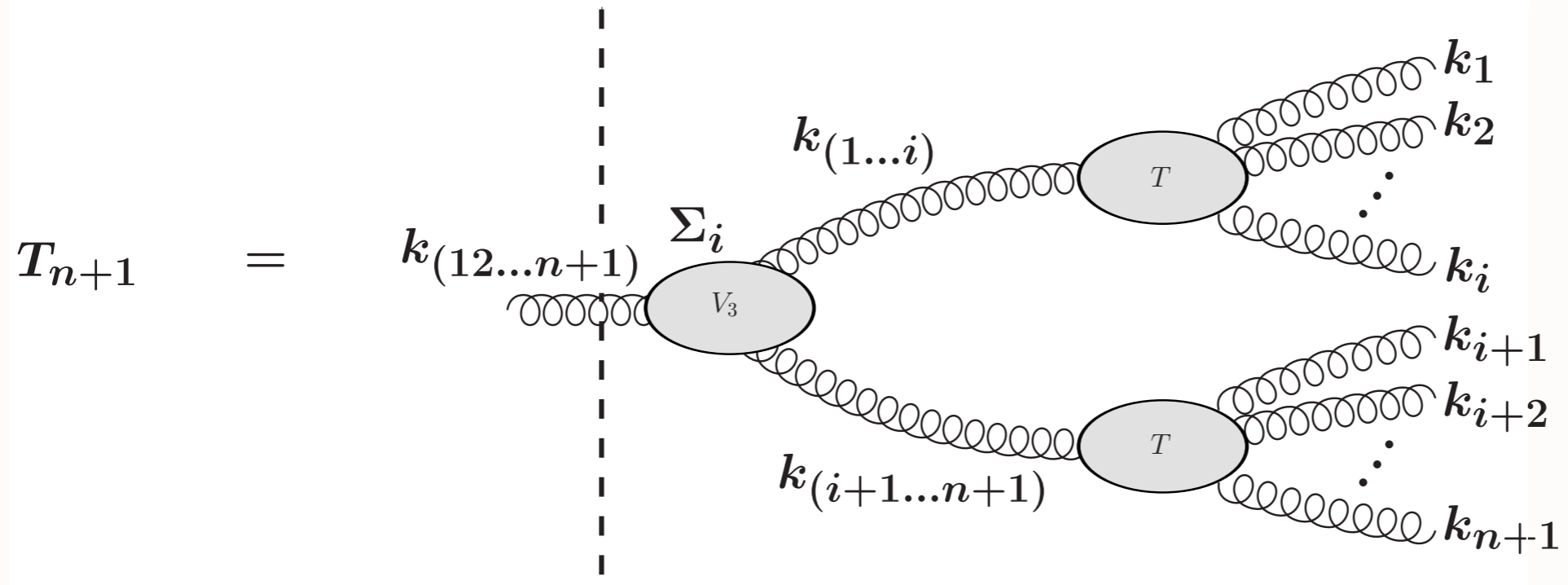
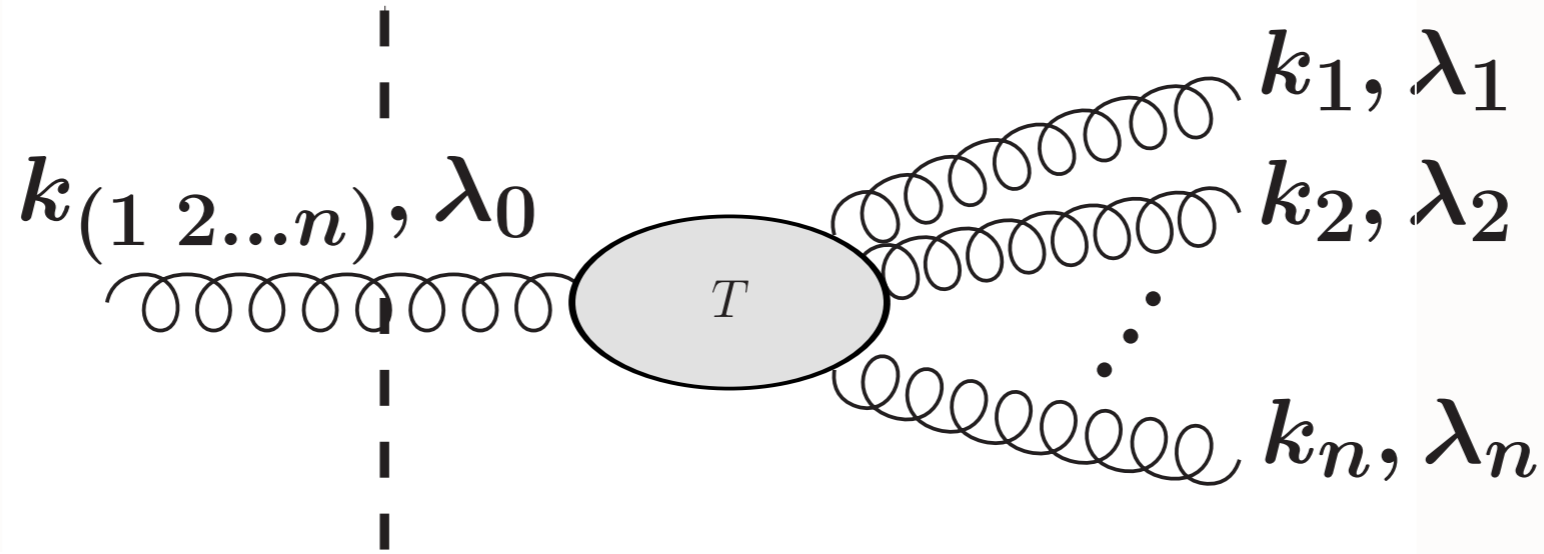
Parke-Taylor Amplitudes

Santiago-Cruz, Stasto

Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

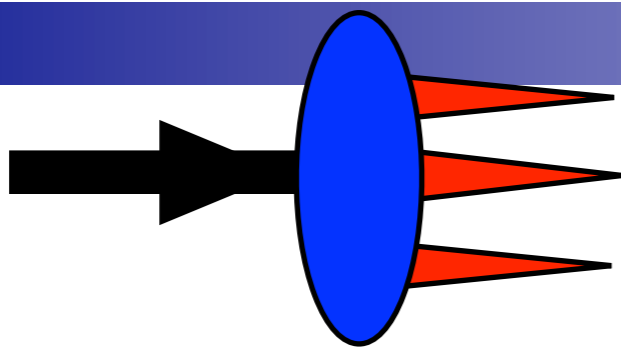
Cluster Decomposition Theorem for relativistic systems: **C. Ji & sjb**



Parke-Taylor amplitudes reflect LF angular momentum conservation

$$\langle ij \rangle = \sqrt{z_i z_j} \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) =$$

• *Light Front Wavefunctions:*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

GTMDs

$$x, \vec{k}_{\perp}, \vec{b}_{\perp}$$

Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$

Transverse density in position space

TMDs

$$x, \vec{k}_{\perp}$$

TMFFs

$$\vec{k}_{\perp}, \vec{b}_{\perp}$$

GPDs

$$x, \vec{b}_{\perp}$$

TMSDs

$$\vec{k}_{\perp}$$

PDFs

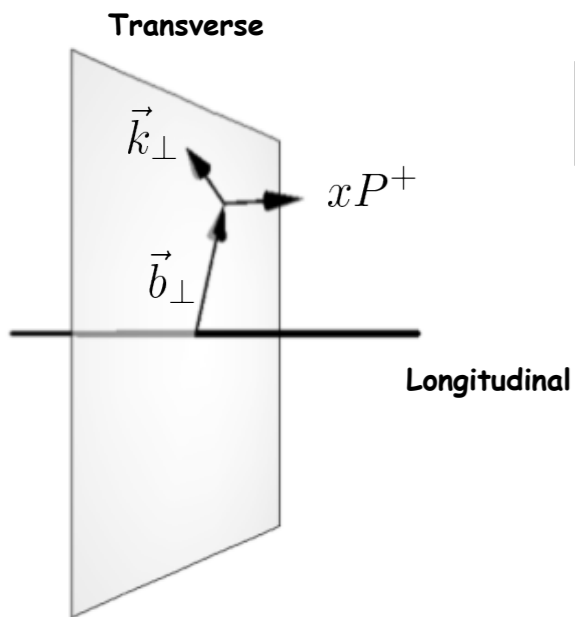
$$x,$$

FFs

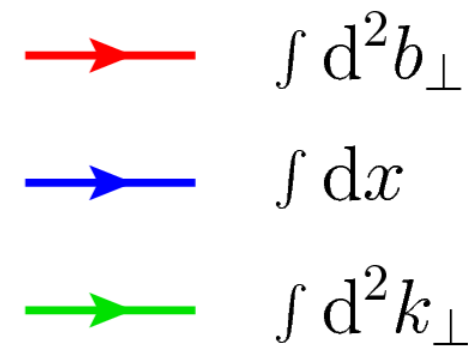
$$\vec{b}_{\perp}$$

Charges

*Lorce,
Pasquini*



Sivers, T-odd from lensing



Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

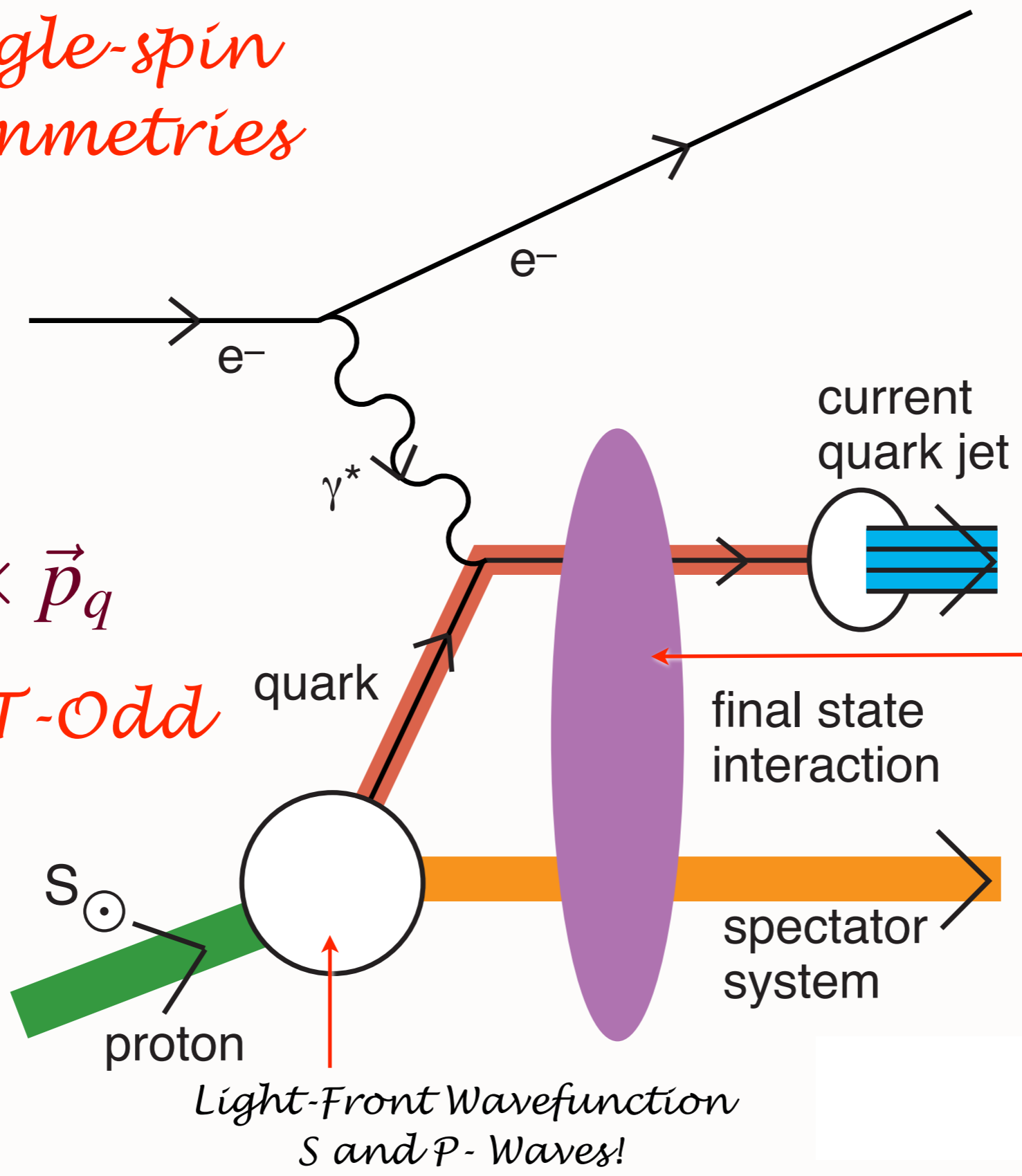
Collins, Burkardt, Ji, Yuan. Pasquini, ...

QCD S- and P-Coulomb Phases --Wilson Line

“Lensing Effect”

Leading-Twist Rescattering Violates pQCD Factorization!

Relation to confining interaction?



$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

**QED:
Lensing
involves soft
scales**

S_p
proton

*Light-Front Wavefunction
S and P-Waves!*

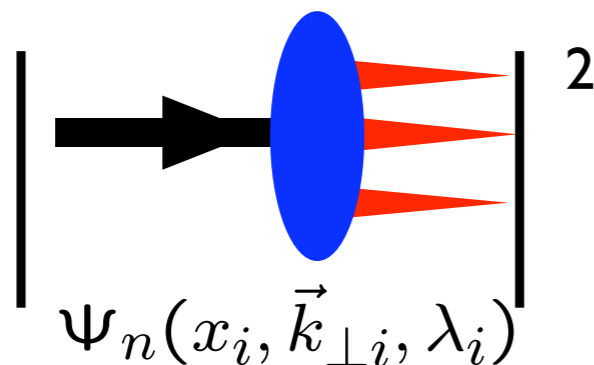
Sign reversal in DY!

Structure functions are not parton probabilities.

By Stanley J. Brodsky, Paul Hoyer,
Nils Marchal, Stephane Peigne, Francesco Sannino.
Phys.Rev. D65 (2002) 114025.

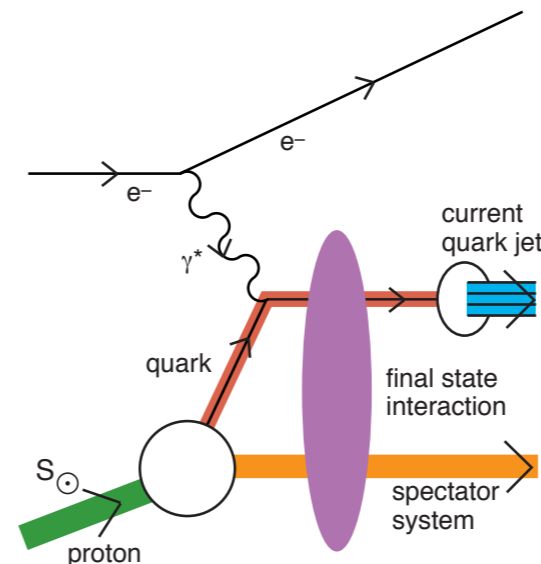
Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and J^z
- DGLAP Evolution; mod. at large x
- No Diffractive DIS



Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS



**Hwang,
Schmidt, sjb,**

Mulders, Boer

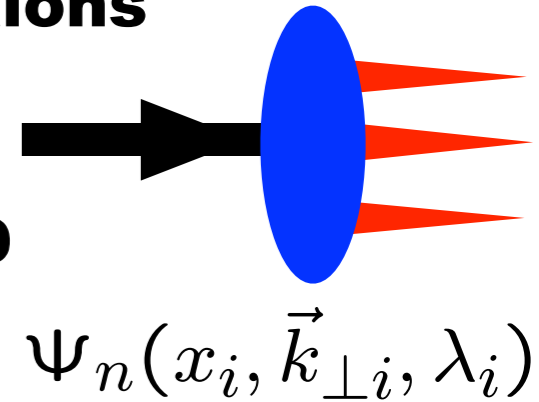
Qiu, Sterman

Collins, Qiu

**Pasquini, Xiao,
Yuan, sjb**

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**



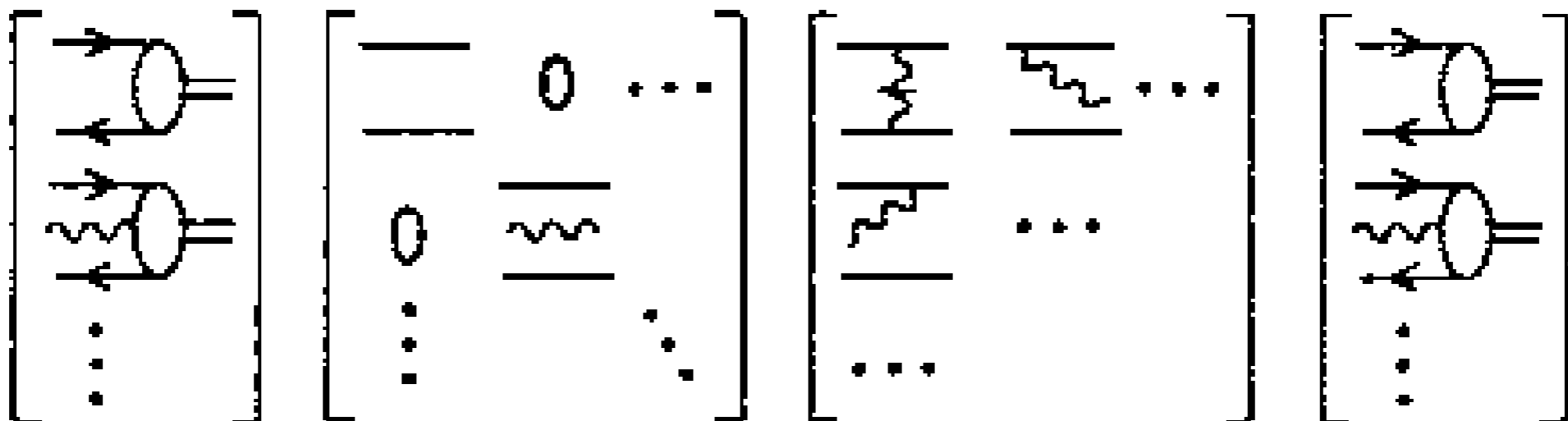
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, modulo 'lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q} | V | q\bar{q} \rangle & \langle q\bar{q} | V | q\bar{q}g \rangle & \cdots \\ \langle q\bar{q}g | V | q\bar{q} \rangle & \langle q\bar{q}g | V | q\bar{q}g \rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

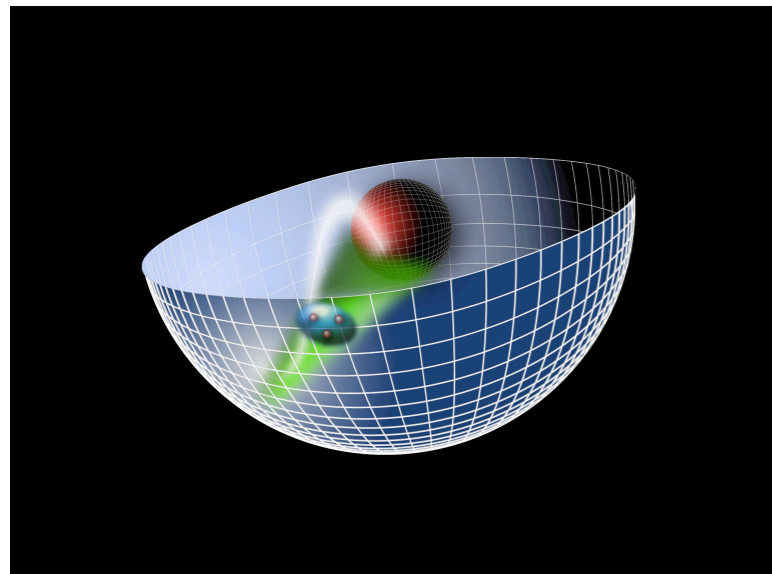
$$A^+ = 0$$



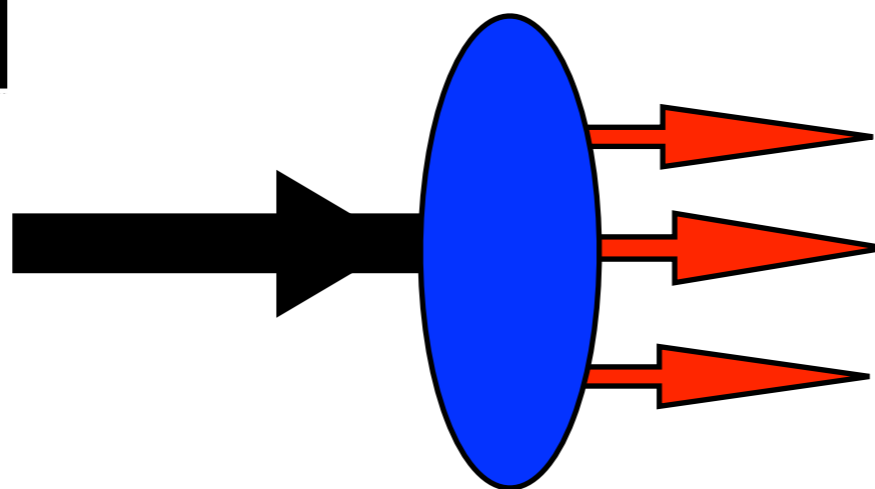
Minkowski space; frame-independent; no fermion doubling; no ghosts

- *Light-Front Vacuum = vacuum of free Hamiltonian!*

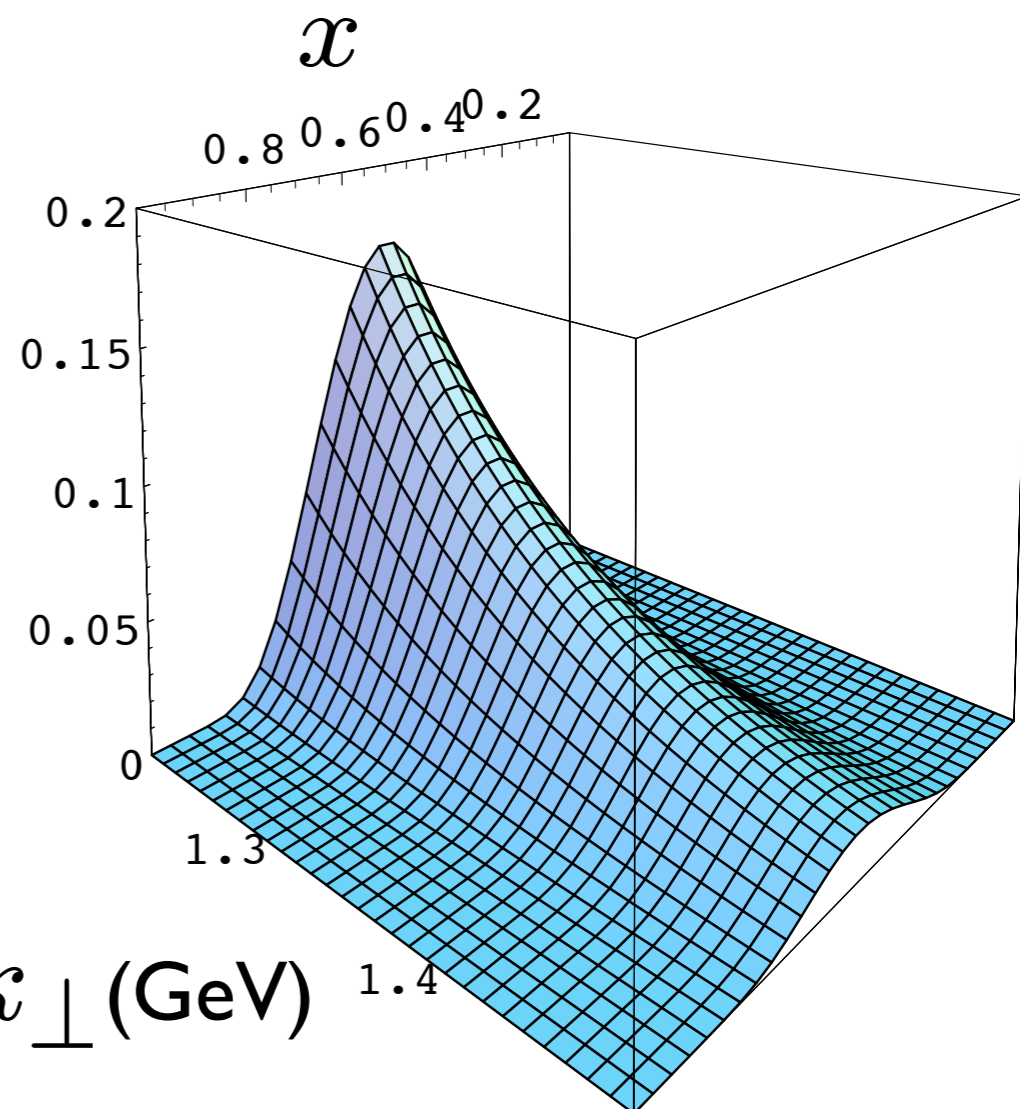
$$\phi(z)$$



- *Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

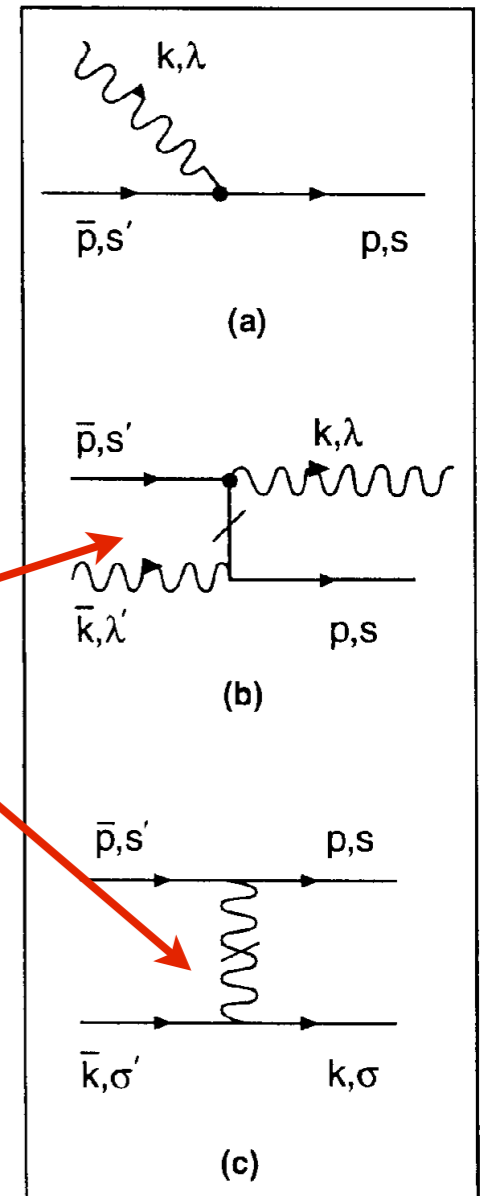
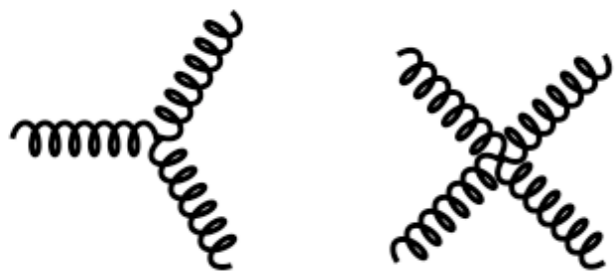


- *Light Front Wavefunctions:*
Schrödinger Wavefunctions
of Hadron Physics

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

H_{QCD}^{LF}

$$\begin{aligned} &= \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \tilde{\psi} - A_a^i (i\partial^\perp)^2 A_{ia} \\ &- \frac{1}{2} g^2 \int d^3x \text{Tr} [\tilde{A}^\mu, \tilde{A}^\nu] [\tilde{A}_\mu, \tilde{A}_\nu] \\ &+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \tilde{\psi} \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \tilde{\psi} \\ &- g^2 \int d^3x \bar{\psi} \gamma^+ \left(\frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \right) \tilde{\psi} \\ &+ g^2 \int d^3x \text{Tr} \left([i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa] \right) \\ &+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \tilde{A} \frac{\gamma^+}{i\partial^+} \tilde{A} \tilde{\psi} \\ &+ g \int d^3x \bar{\psi} \tilde{A} \tilde{\psi} \\ &+ 2g \int d^3x \text{Tr} (i\partial^\mu \tilde{A}^\nu [\tilde{A}_\mu, \tilde{A}_\nu]) \end{aligned}$$



Rigorous First-Principle Formulation of Non-Perturbative QCD

Exact frame-independent formulation of nonperturbative QCD!

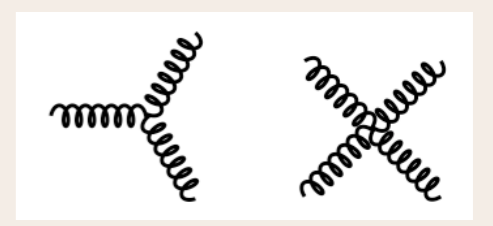
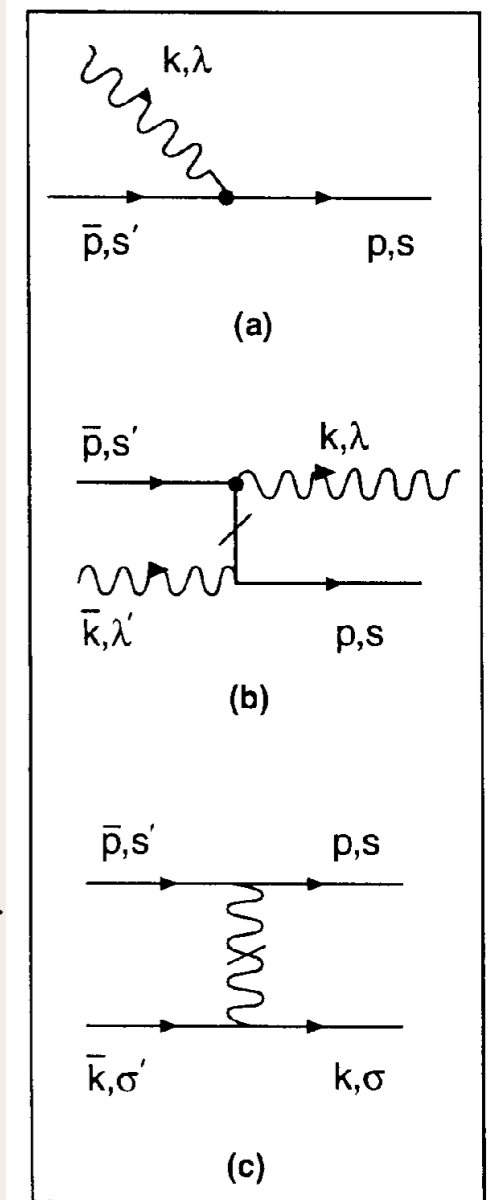
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

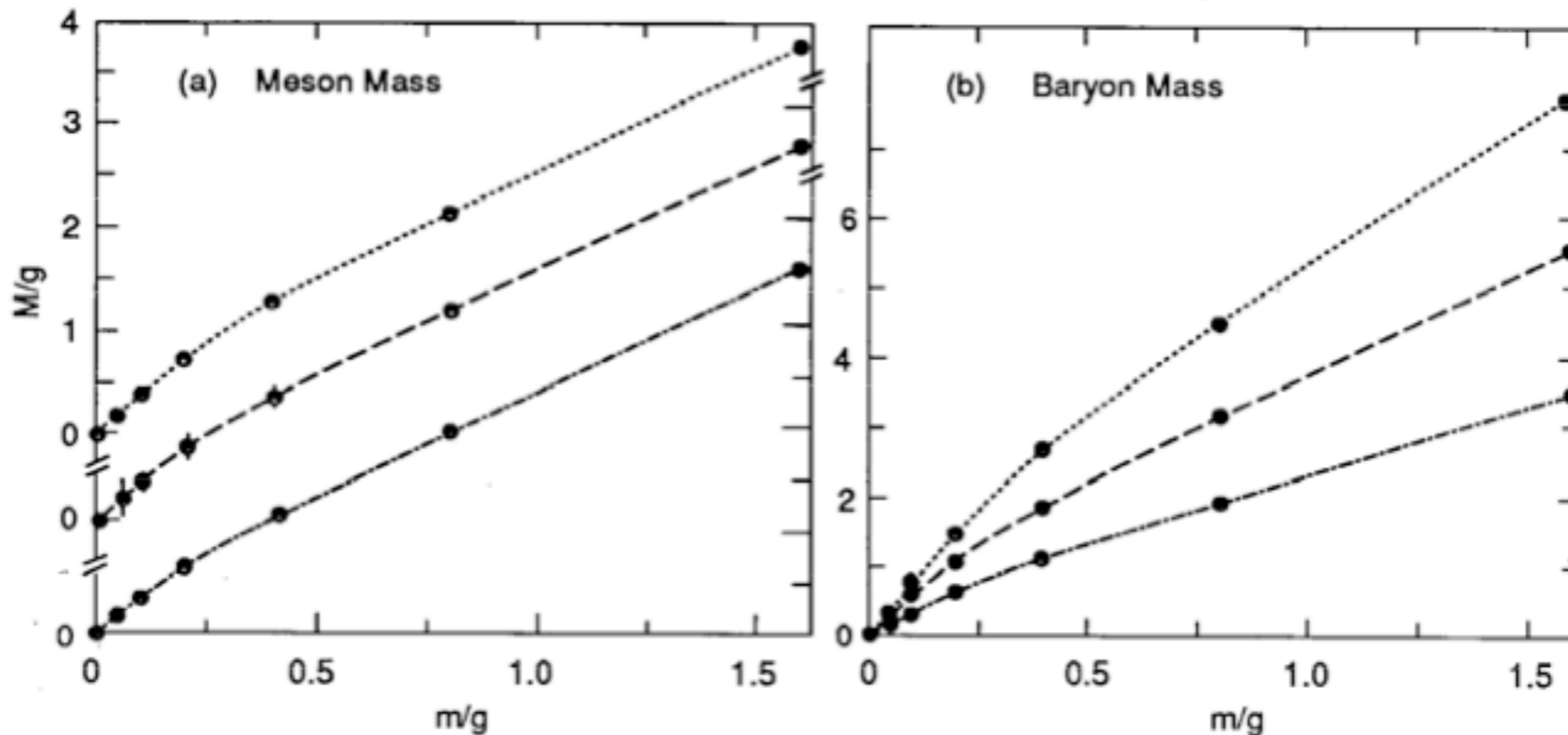


H_{LF}^{int}

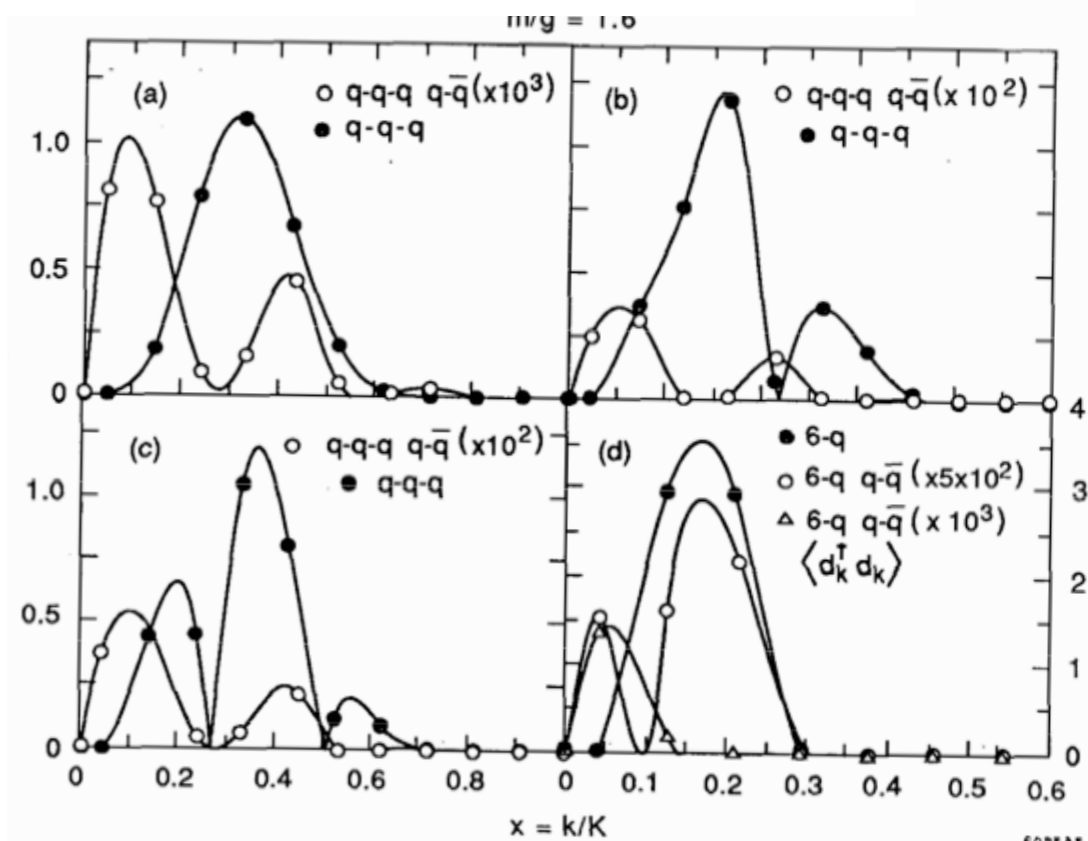
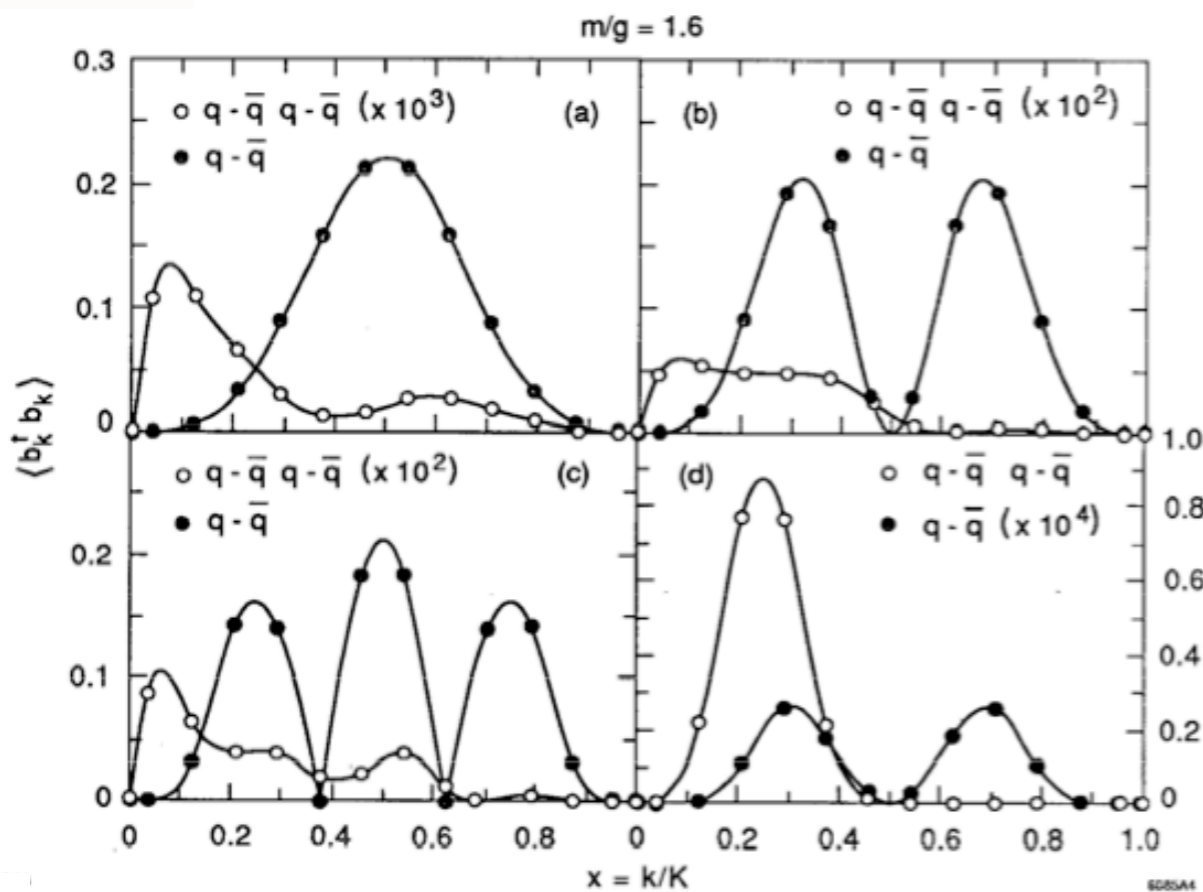
Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass

DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, $2K=24$. d) Eleventh

a-c) First three states in $N = 3$ baryon spectrum, $2K=21$. d) First $B = 2$ state.

state:

Hornbostel, Pauli, sjb

Remarkable Advantages of the Front Form

- **Light-Front Time-Ordered Perturbation Theory: Elegant, Physical**
- **Frame-Independent**
- **Few LF Time-Ordered Diagrams (not $n!$) -- all k^+ must be positive**
- **J^z conserved at each vertex**
- **Cluster Decomposition -- only proof for relativistic theory**
- **Automatically normal-ordered; LF Vacuum trivial up to zero modes**
- **Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED**
- **Reproduces Parke-Taylor Rules and Amplitudes (Stasto-Cruz)**
- **Hadronization at the Amplitude Level with Confinement**

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

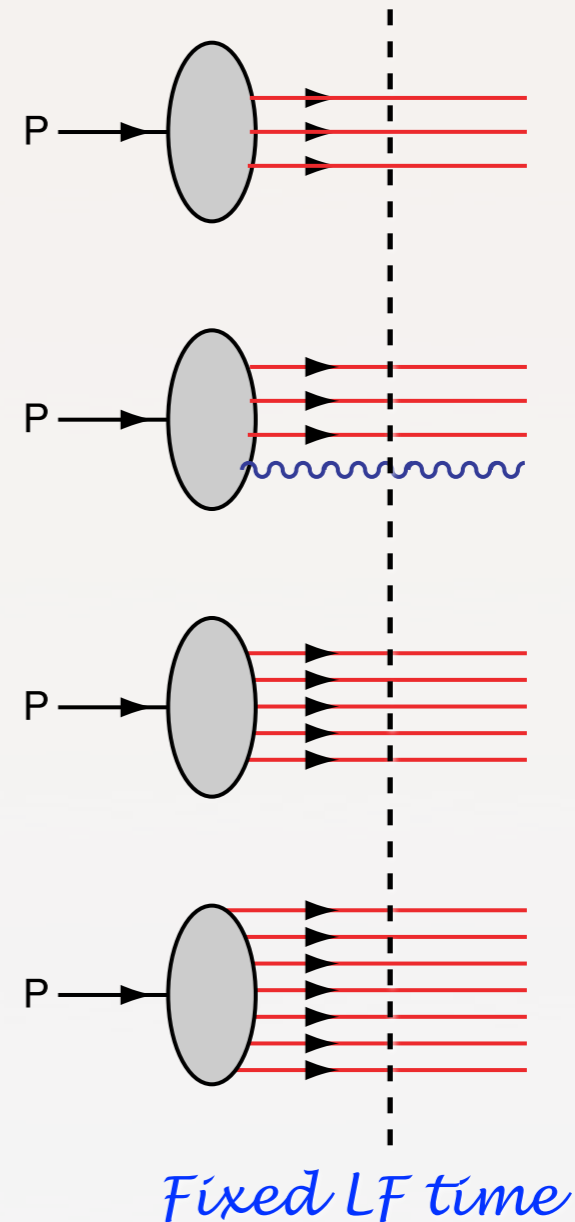
are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks
 $s(x), c(x), b(x)$ at high x !

$\bar{s}(x) \neq s(x)$
 $\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states

BFKL Pomeron

Hidden Color

Soft gluons in the infinite momentum wave function and the BFKL pomeron.

[Alfred H. Mueller](#) ([SLAC](#) & [Columbia U.](#)) . SLAC-PUB-10047, CU-TP-609, Aug 1993. 12pp.

Published in **Nucl.Phys.B415:373-385,1994.**

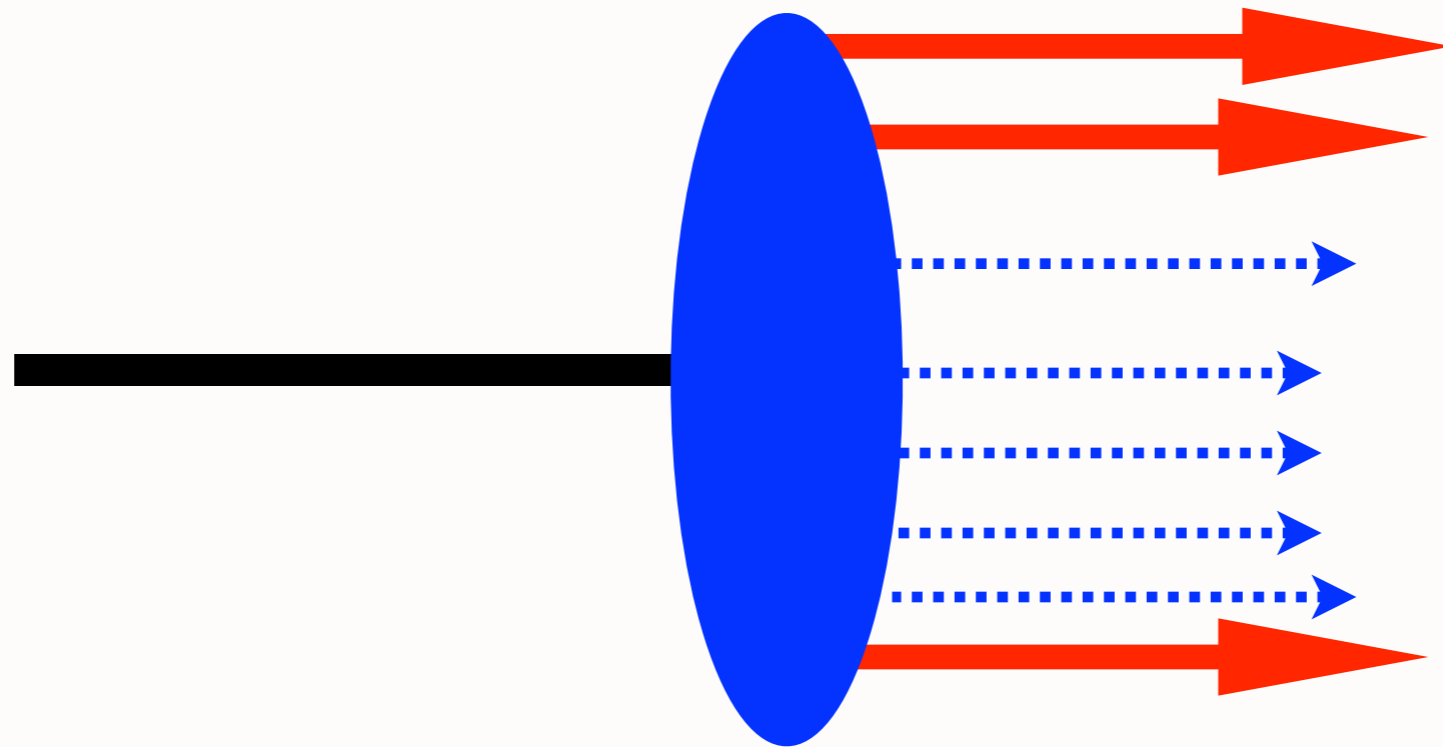
Light cone wave functions at small x.

[F. Antonuccio](#) ([Heidelberg, Max Planck Inst.](#) & [Heidelberg U.](#)) , [S.J. Brodsky](#) ([SLAC](#)) , [S. Dalley](#) ([CERN](#)) .

Phys.Lett.B412:104-110,1997.

e-Print: [hep-ph/9705413](#)

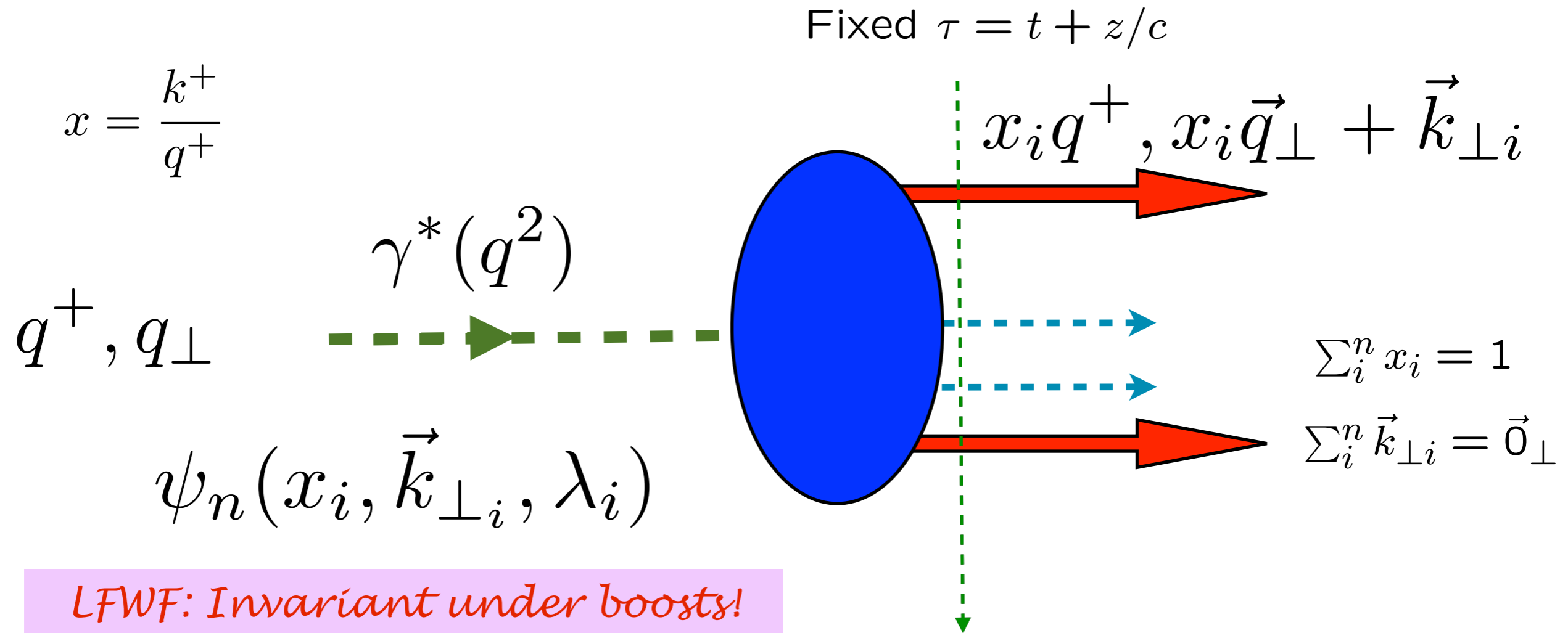
Mueller: BFKL derived from multi-gluon Fock State



Antonuccio, Dalley, sjb: Ladder Relations

Light-Front Wavefunctions of Virtual Photon

Virtual photon has space-like mass $q^2 = -Q^2 < 0$



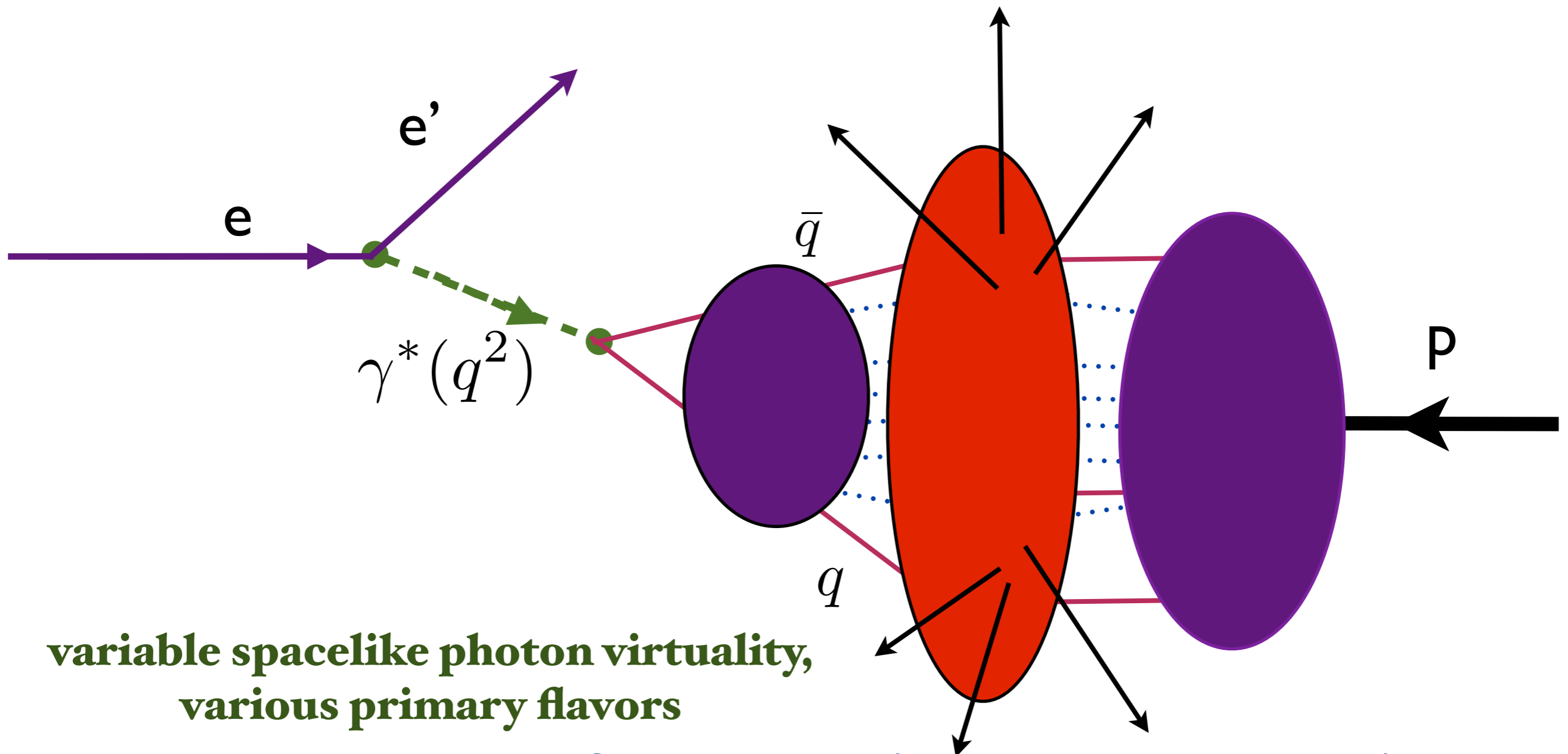
Witten, DGLAP, ERBL Evolution

Feynman virtuality from sum over all electron LF time-orderings

$$q^2 = q^+ q^- - \vec{q}_\perp^2$$

LHeC: Virtual Photon-Proton Collider

Perspective from the e-p collider frame



**variable spacelike photon virtuality,
various primary flavors**

photon and proton fragmentation vs. central regions

Saturation, nuclear shadowing, antishadowing

Advantages of the Dirac's Front Form for Hadron Physics



- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent -- no boosts!**
- **No dependence on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no condensates!**
- **Profound implications for Cosmological Constant**

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

*Eliminate higher Fock states
(retarded interactions)*

$$\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r})\right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{l(l+1)}{r^2} + V_{\text{eff}}(r, S, l)\right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

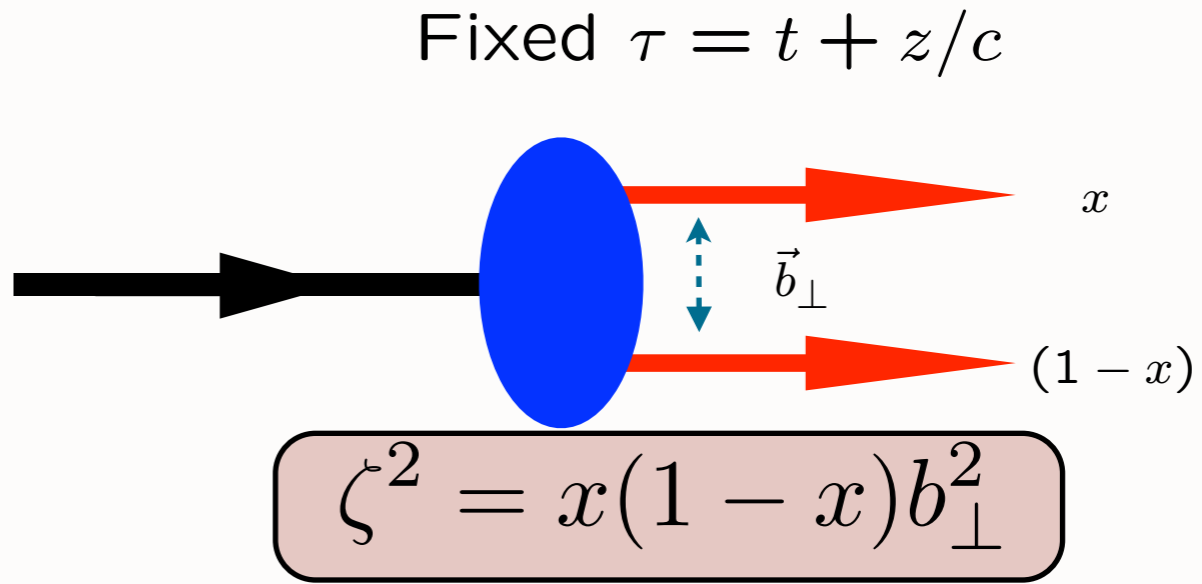
Coulomb potential

$$V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

*Semiclassical first approximation to QED --> **Bohr Spectrum***

Light-Front QCD

$$H_{QCD}^{LF}$$



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

*Eliminate higher Fock states
(retarded interactions)*

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{4\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

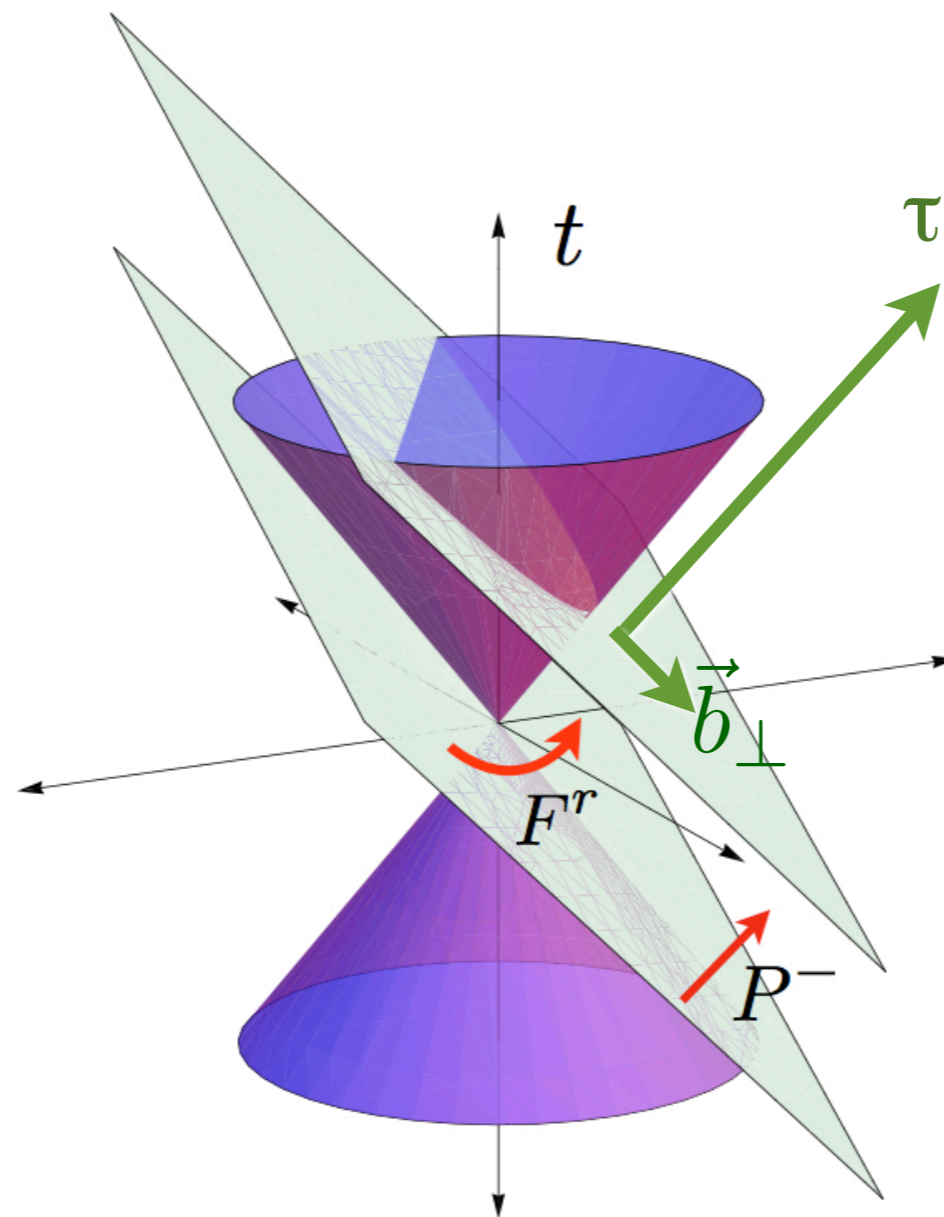
AdS/QCD:

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*Confining AdS/QCD
potential!*

Semiclassical first approximation to QCD

Sums an infinite # diagrams



$$\zeta_{\perp}^2 = b_{\perp}^2 x(1 - x)$$

$$\frac{d^2}{d\zeta_{\perp}^2} \rightarrow \frac{k_{\perp}^2}{x(1 - x)}$$

Null plane: a surface tangent to the light cone.

The null-plane Hamiltonians map the initial light-like surface onto some other surface, and therefore describe the dynamical evolution of the system.

The energy P^- translates the system in the null-plane time coordinate x^+ , whereas the spin Hamiltonians F_r rotate the initial surface about the surface of the light cone.

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\begin{aligned} \mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left(-\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions}. \end{aligned}$$

Change variables

$$(\vec{\zeta}, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left(\zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\begin{aligned} \mathcal{M}^2 &= \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta \phi^*(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \end{aligned}$$

Light-Front Schrödinger Equation

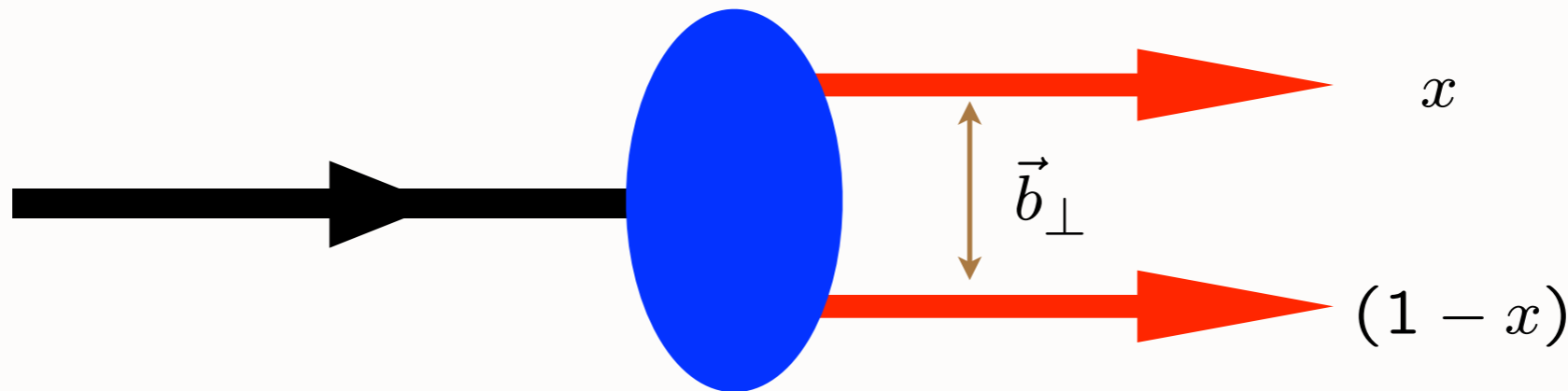
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

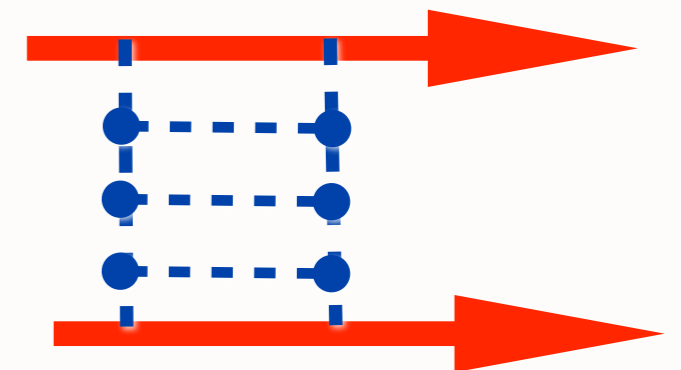
$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{4\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



U is the confining QCD potential
Conjecture: 'H'-diagrams generate

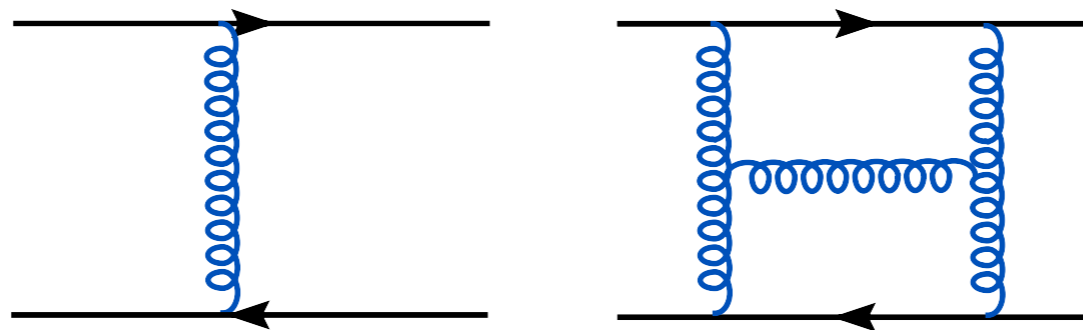
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$



Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \left[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2 + (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \right]$$

Smirnov, Smirnov, Steinhauser, 2010



$\log \kappa^2 \zeta^2$

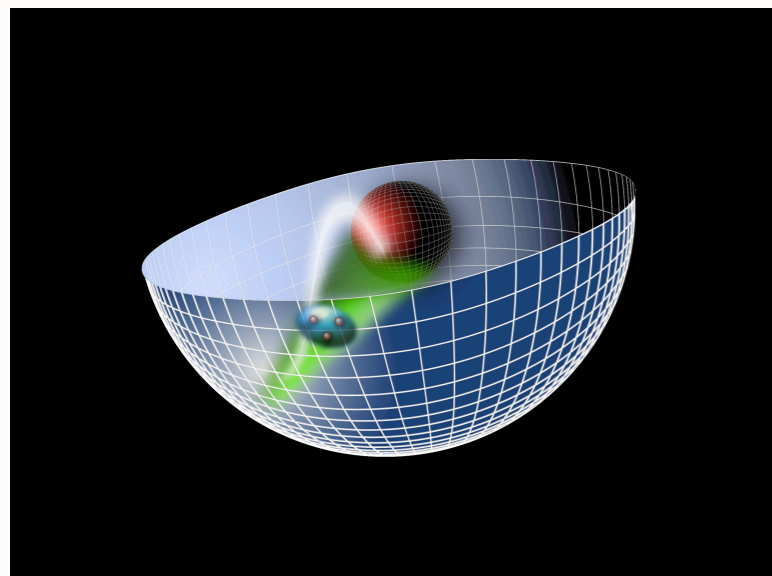
Summation of H graphs could yield confining potential

$$\phi(z)$$

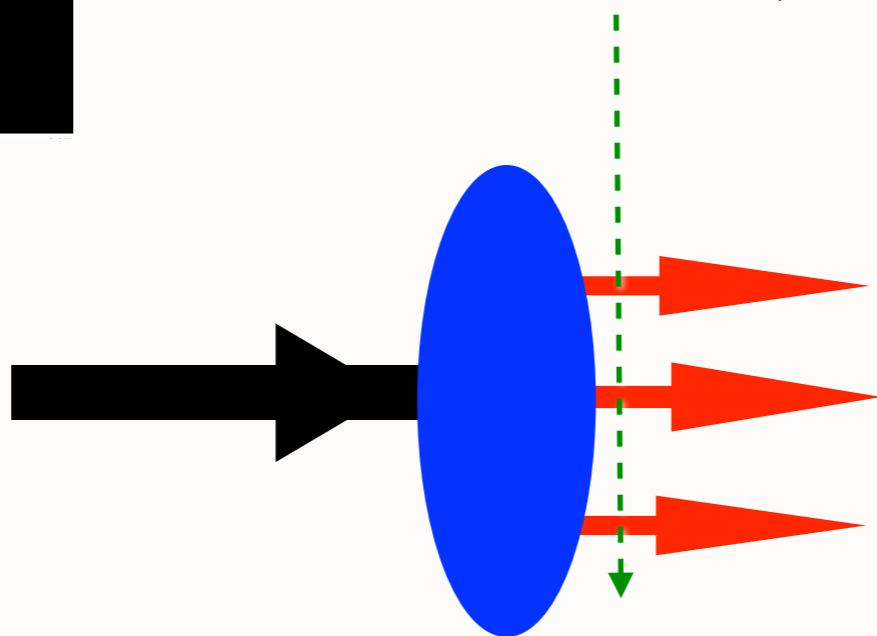
AdS₅: Conformal Template for QCD

• Light-Front Holography

Duality of AdS₅ with LF Hamiltonian Theory



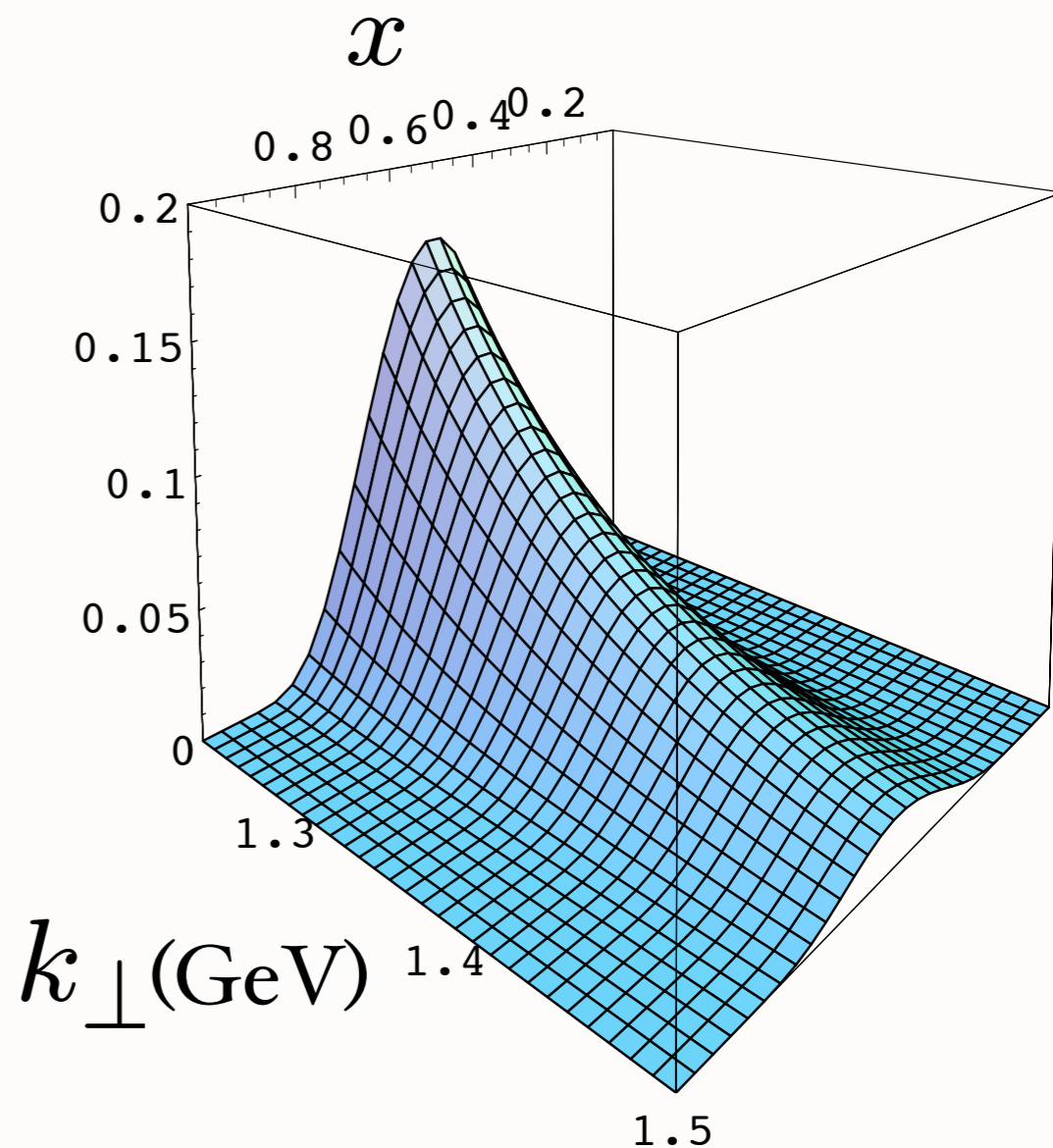
Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

• Light Front Wavefunctions:

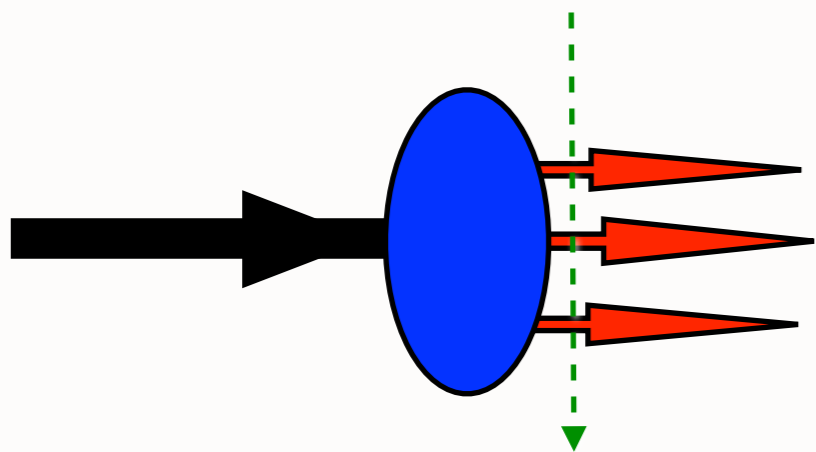
**Light-Front Schrödinger Equation
Spectroscopy and Dynamics**



Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

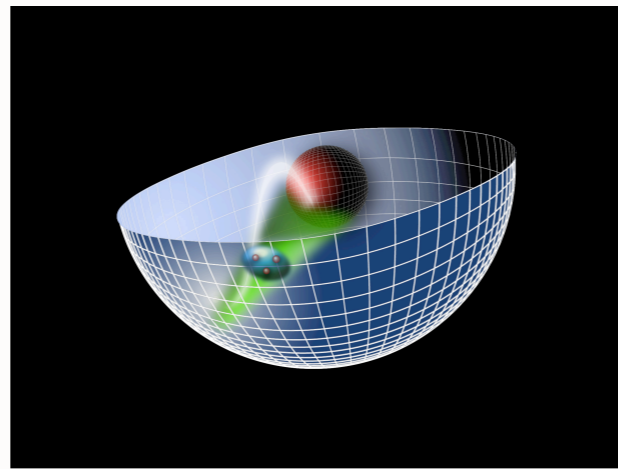
Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

*AdS/QCD
Soft-Wall Model*

*Single scheme-independent
fundamental mass scale*

κ



Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

$$1/\kappa \simeq 1/3 \text{ fm}$$

Confinement scale:

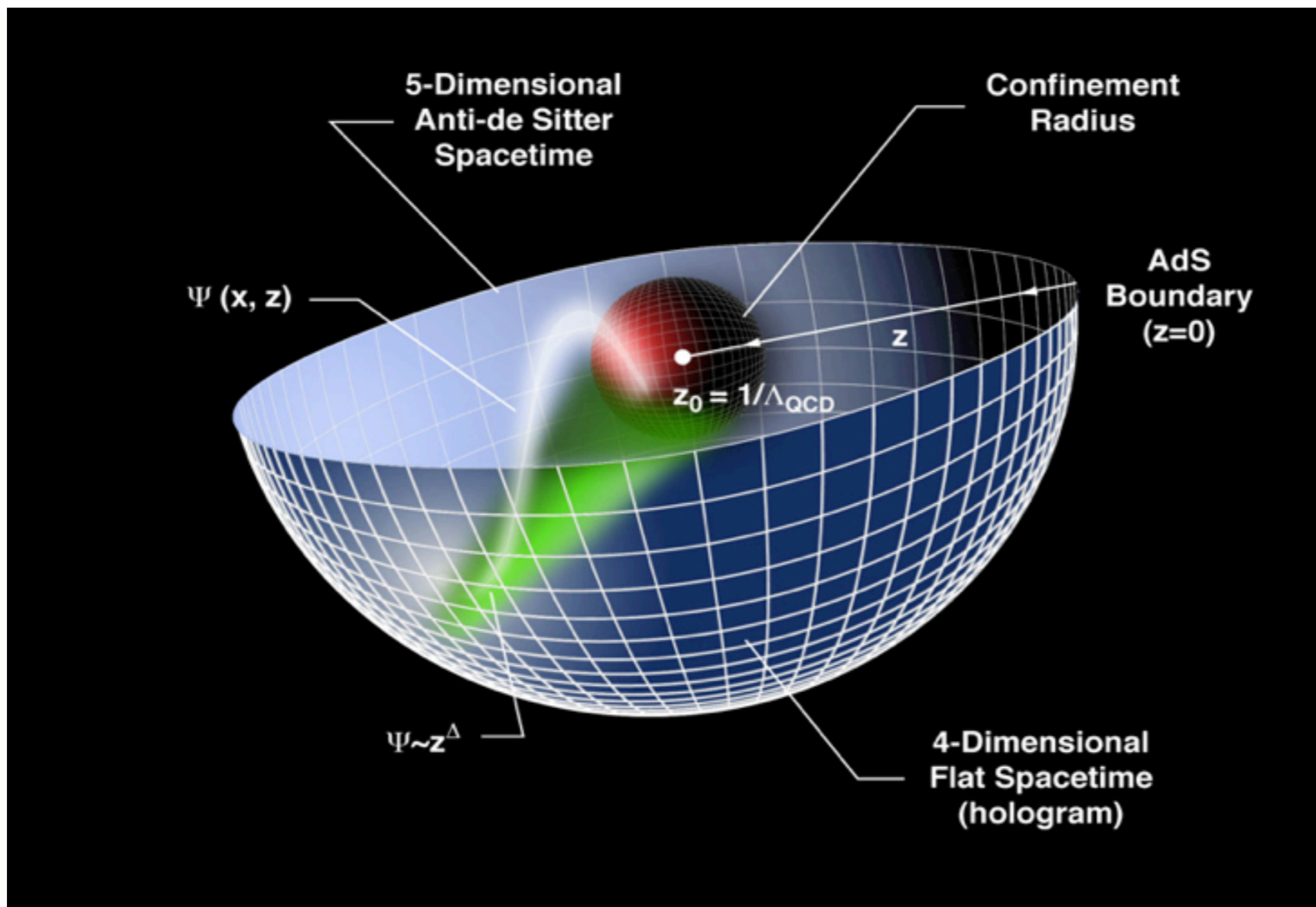
($\mathbf{m}_q=0$)

***Unique
Confinement Potential!***

*Conformal Symmetry
of the action*

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**




Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

AdS/CFT

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$.

- Action for massive scalar modes on AdS_{d+1} :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ -coordinates, $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

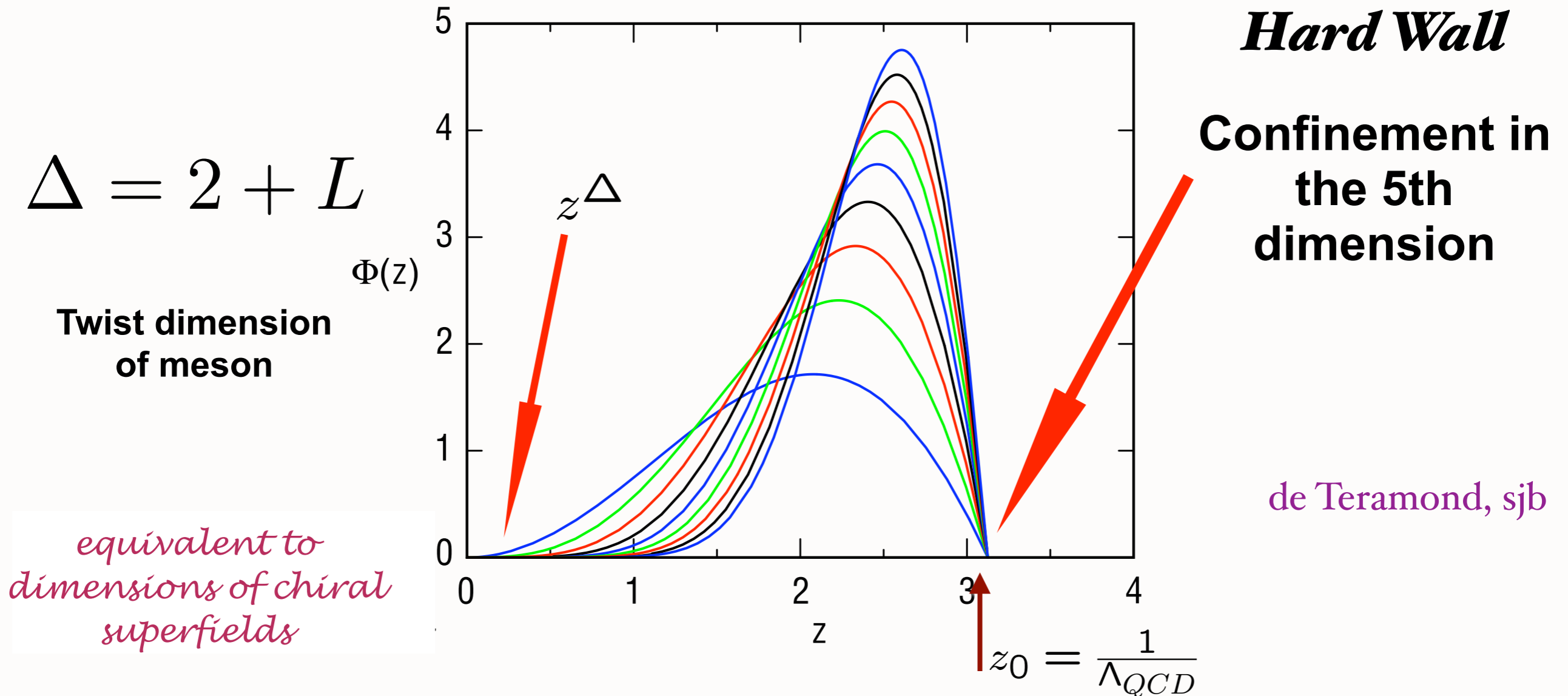
$$\left[z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \rightarrow z^\Delta$ as $z \rightarrow 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$

- Physical AdS modes $\Phi_P(x, z) \sim e^{-iP \cdot x} \Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small- z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator \mathcal{O} which creates a hadron out of the vacuum: $\langle P | \mathcal{O} | 0 \rangle \neq 0$.



Identify hadron by its interpolating operator at $z \rightarrow 0$

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale κ**
- **Uses AdS₅ as template for conformal theory**

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

General dilaton profile

- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE ($d = 4$)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \phi''(\zeta) + \frac{1}{4} \phi'(\zeta)^2 + \frac{2J-3}{2\zeta} \phi'(\zeta)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

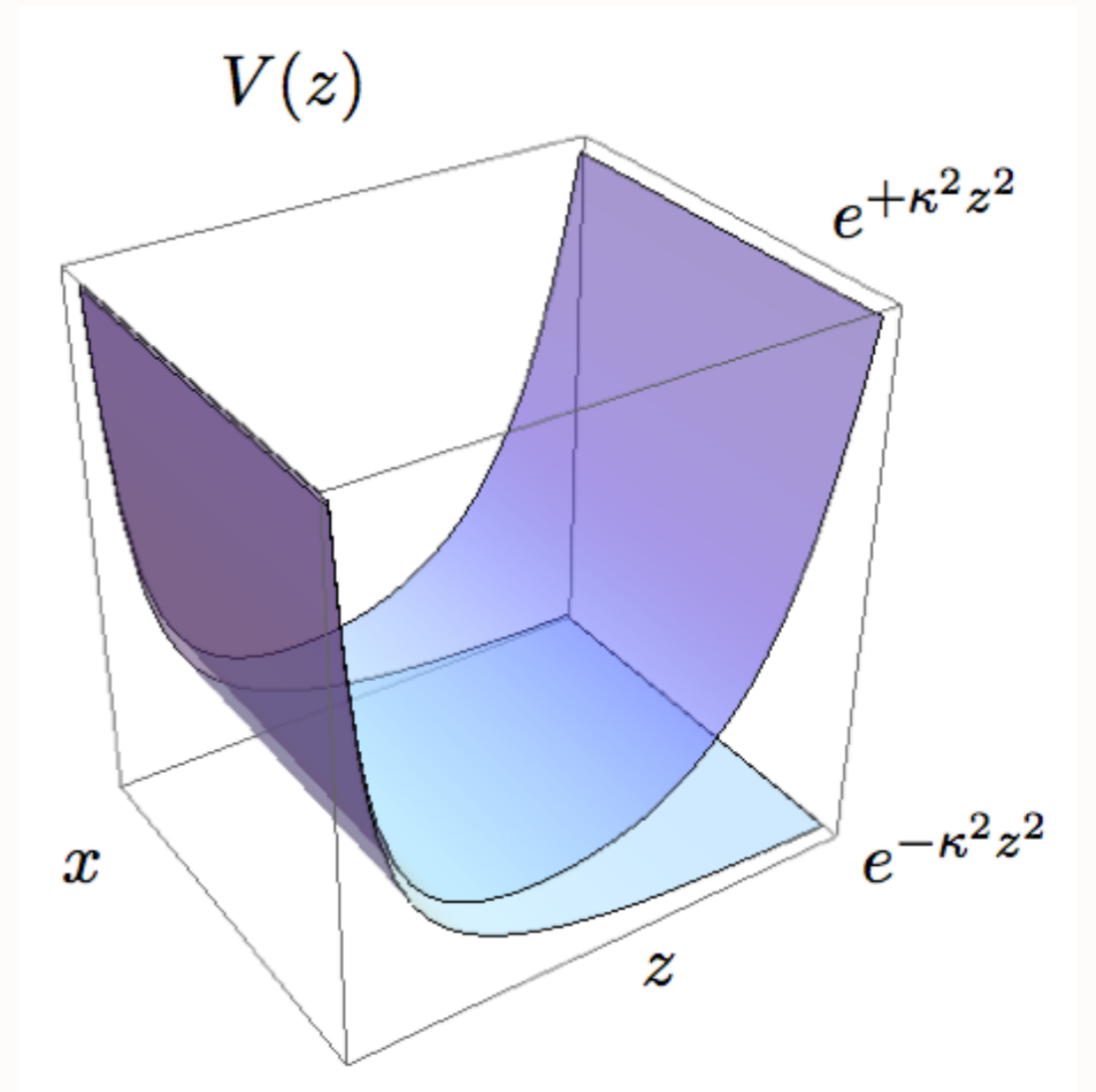
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

where $\varphi(z) \rightarrow 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

General-Spin Hadrons

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$



with $(\mu R)^2 = -(2 - J)^2 + L^2$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

• Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS₅

Identical to Light-Front Bound State Equation!

z

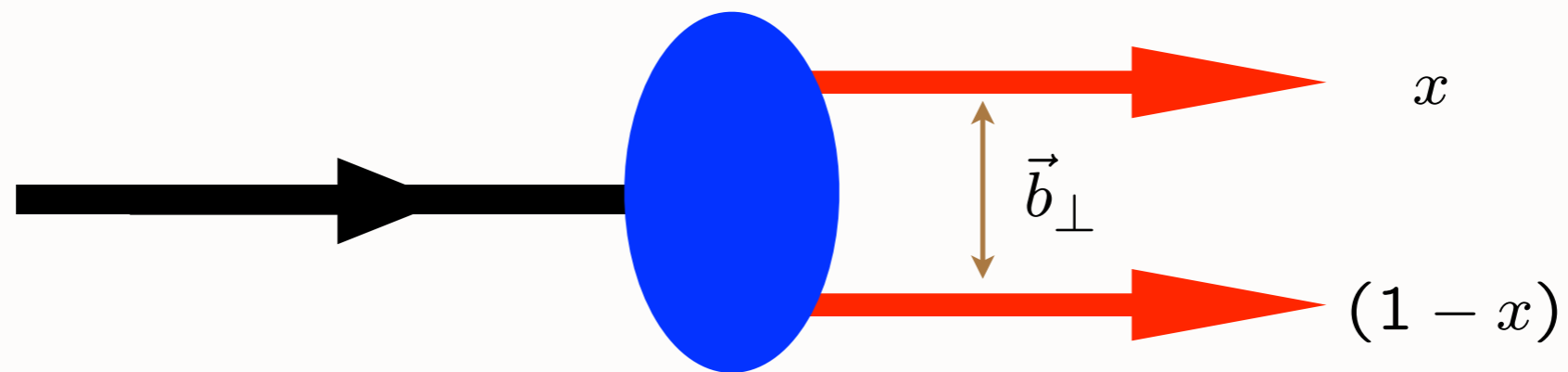


$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

$LF(3+1) \longleftrightarrow AdS_5$

$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$

$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

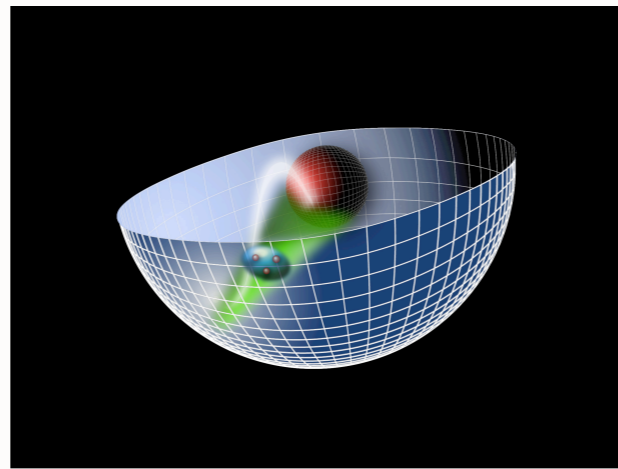
$$(\mu R)^2 = L^2 - (J - 2)^2$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

*AdS/QCD
Soft-Wall Model*

*Single scheme-independent
fundamental mass scale*

κ



Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

$$1/\kappa \simeq 1/3 \text{ fm}$$

Confinement scale:

($\mathbf{m}_q=0$)

***Unique
Confinement Potential!
Conformal Symmetry
of the action***

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

Quark separation increases with L

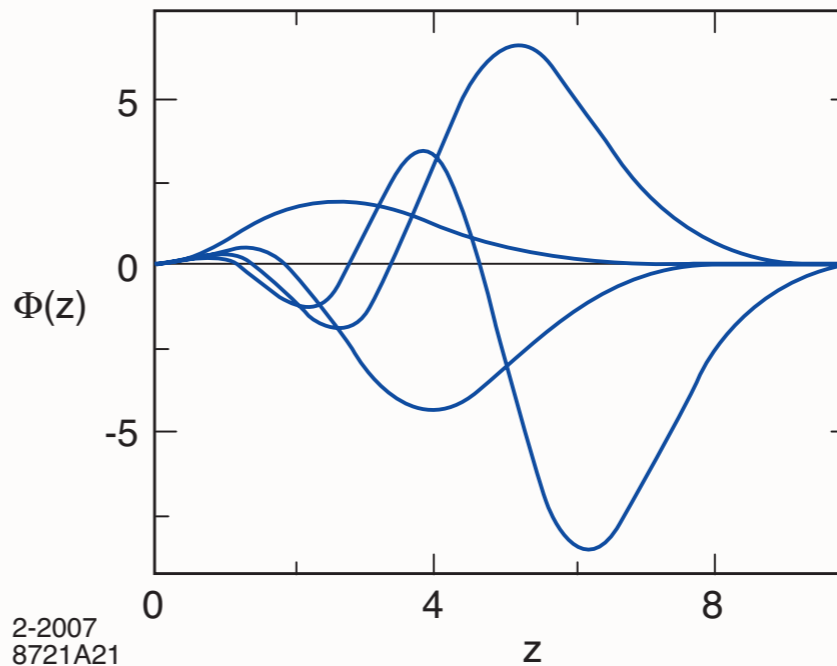
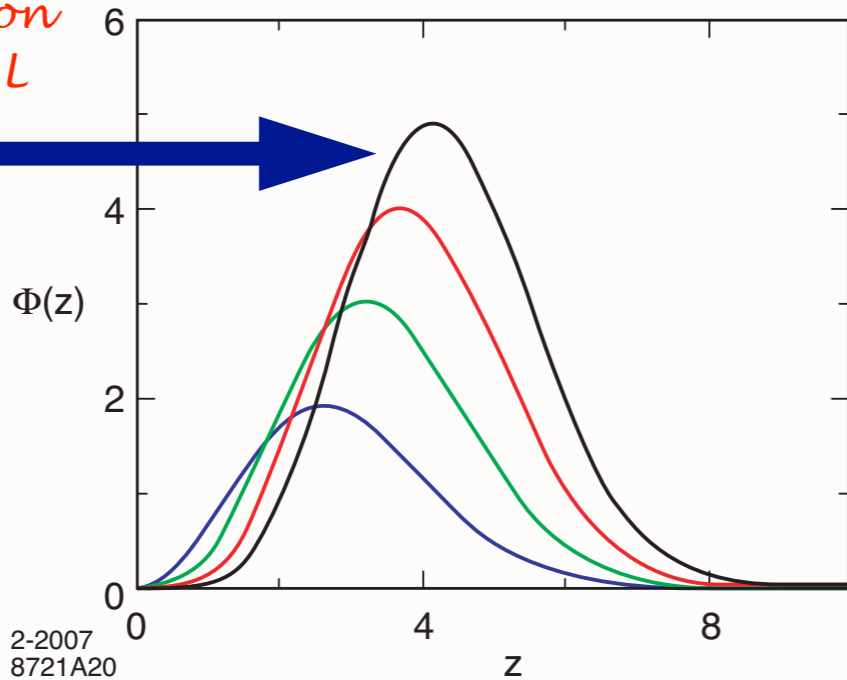
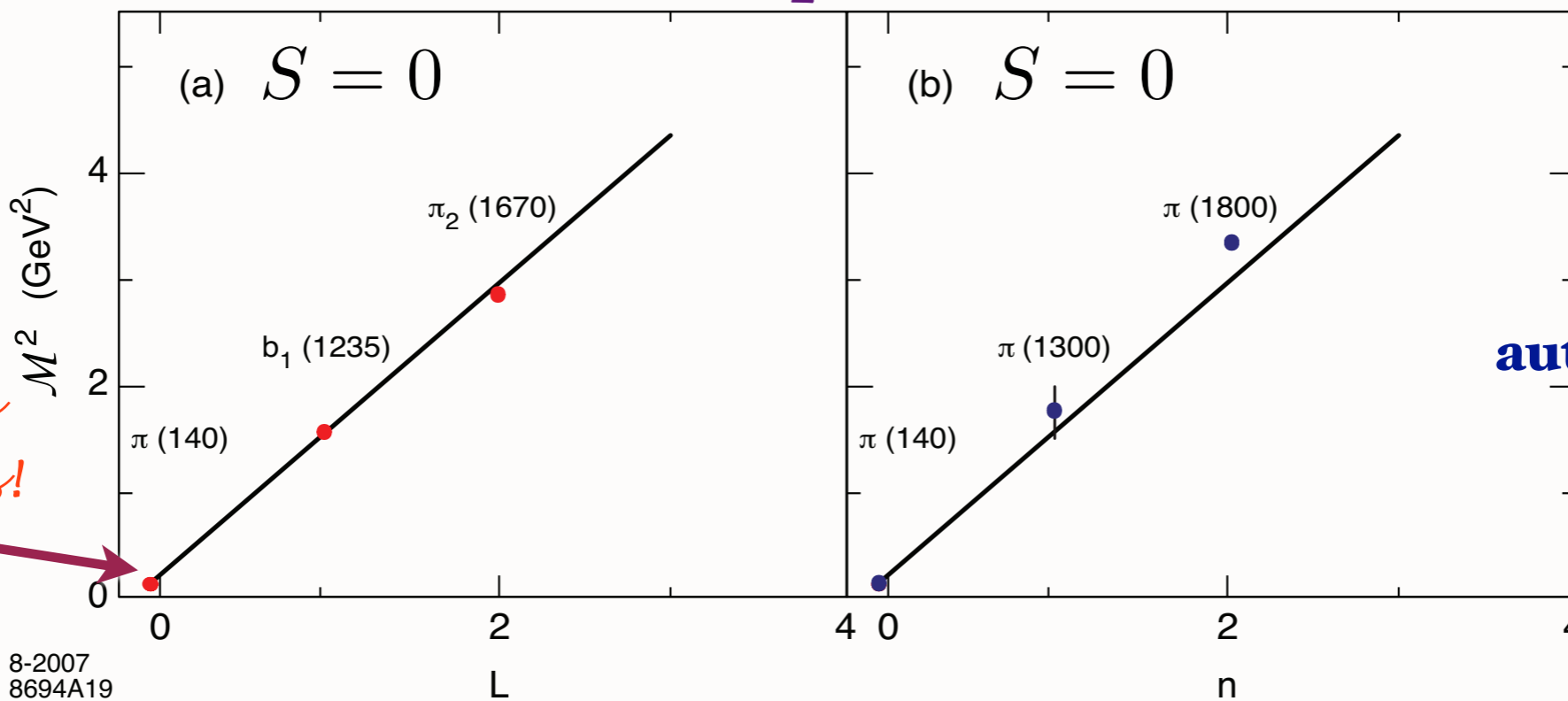


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Same slope in n and L !

Soft Wall Model



Pion has zero mass!



Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Meson Spectrum in Soft Wall Model

Pion: Negative term for $J=0$ cancels positive terms from LFKE and potential



- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

- $J = L + S, I = 1$ meson families

$$\mathcal{M}_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$$

$$4\kappa^2 \text{ for } \Delta n = 1$$

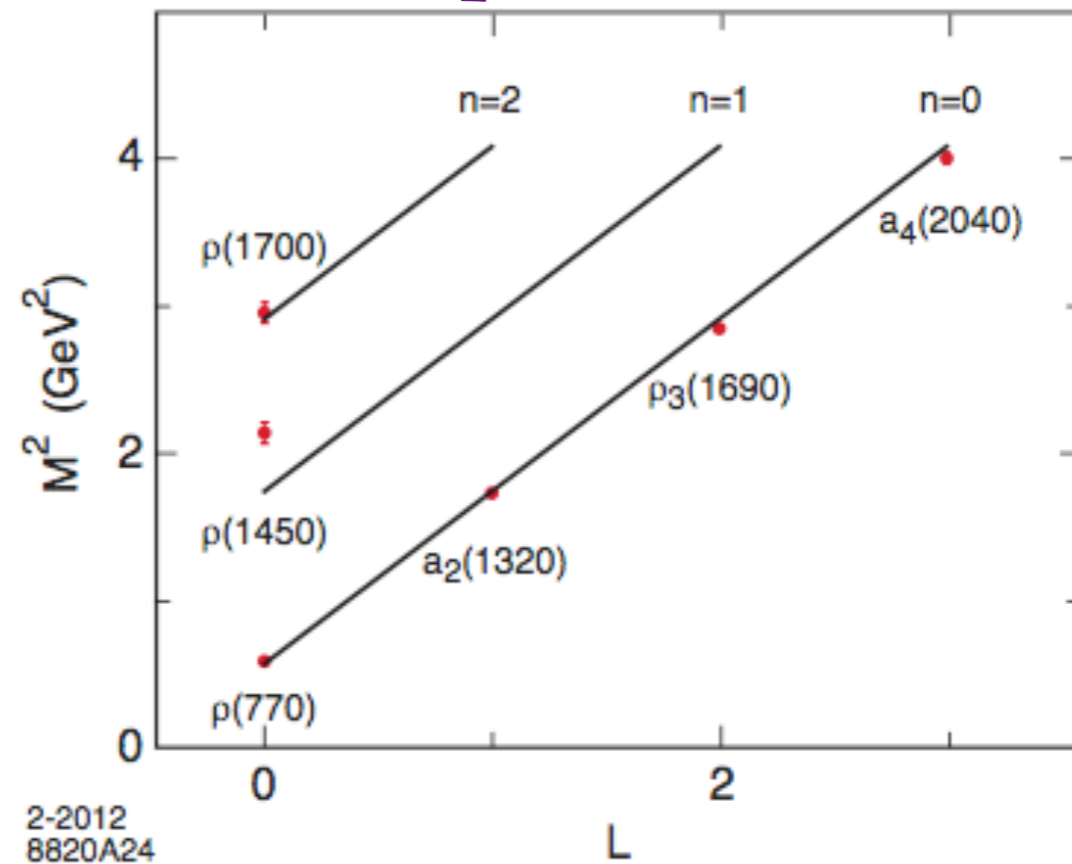
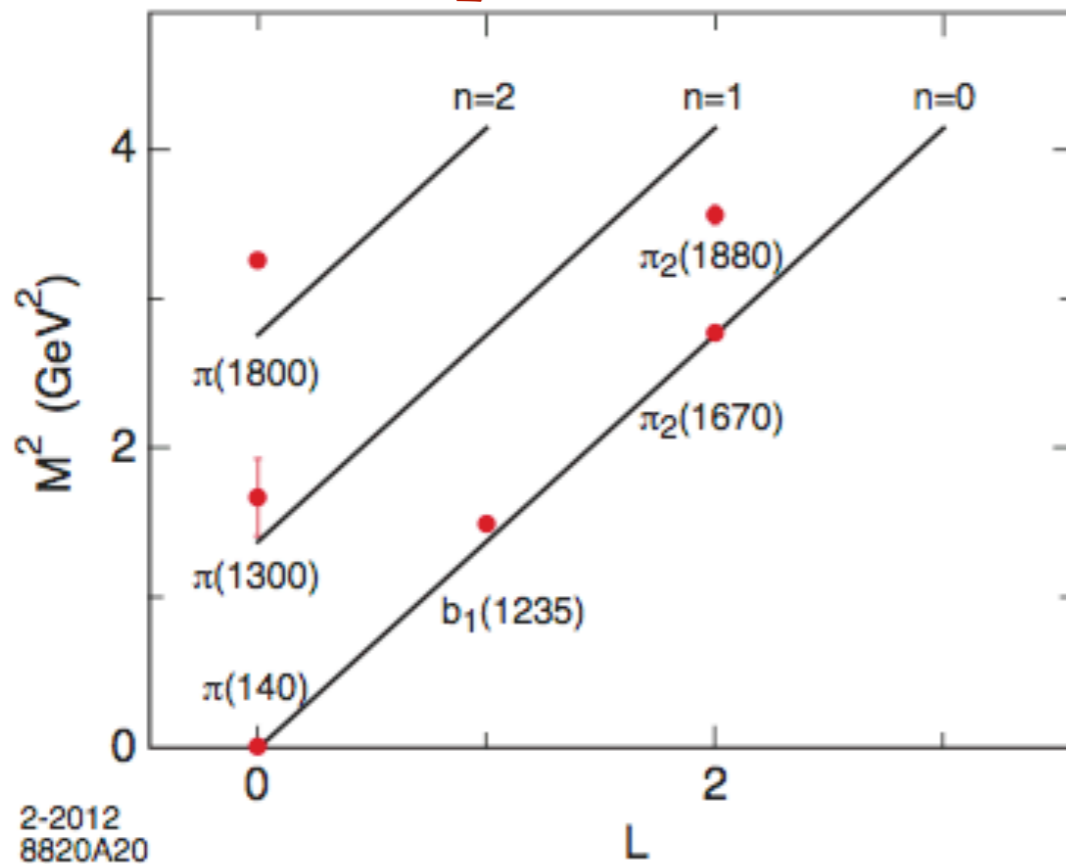
$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$

$$m_q = 0$$

Massless pion in Chiral Limit!

Same slope in n and L !

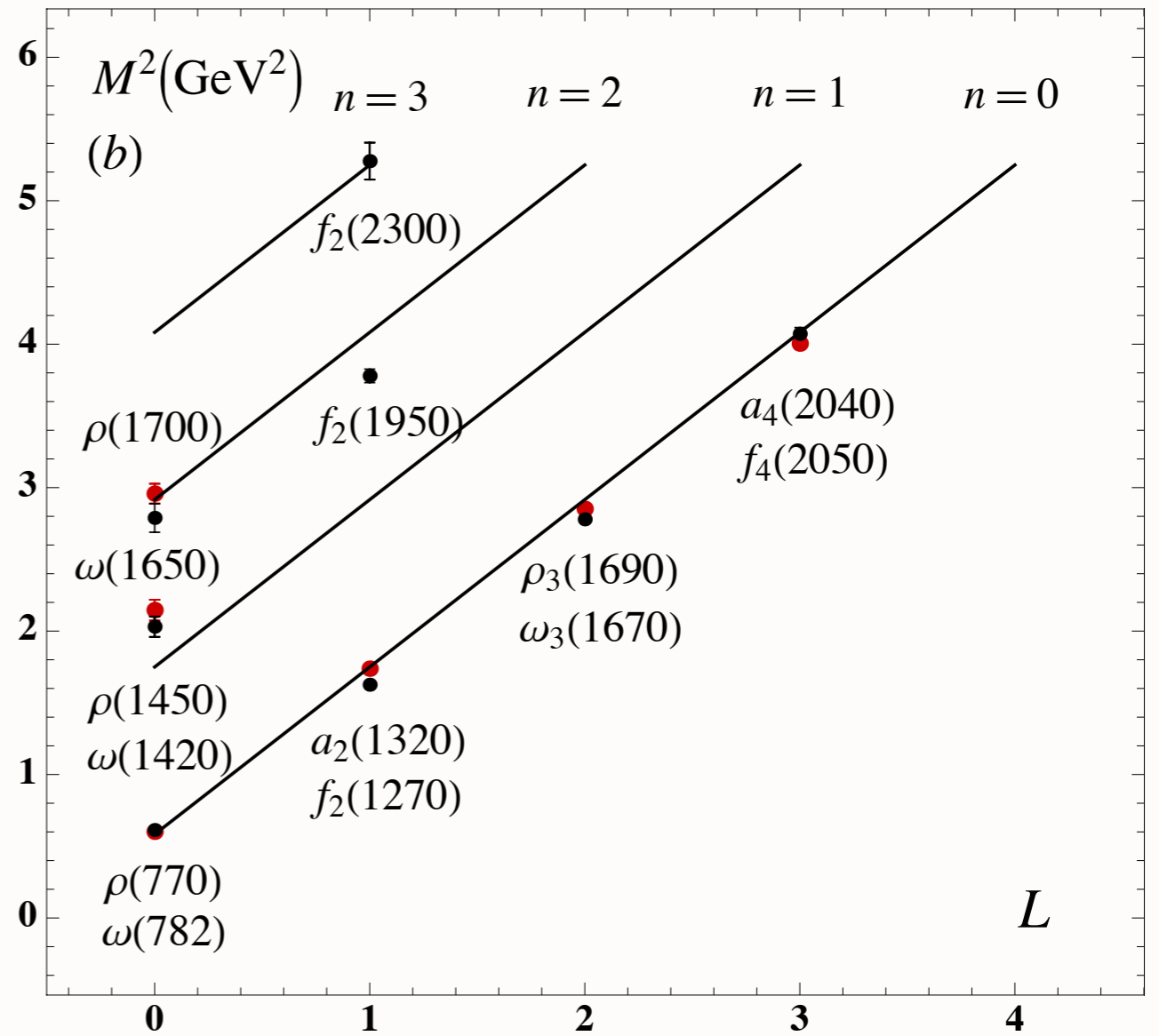
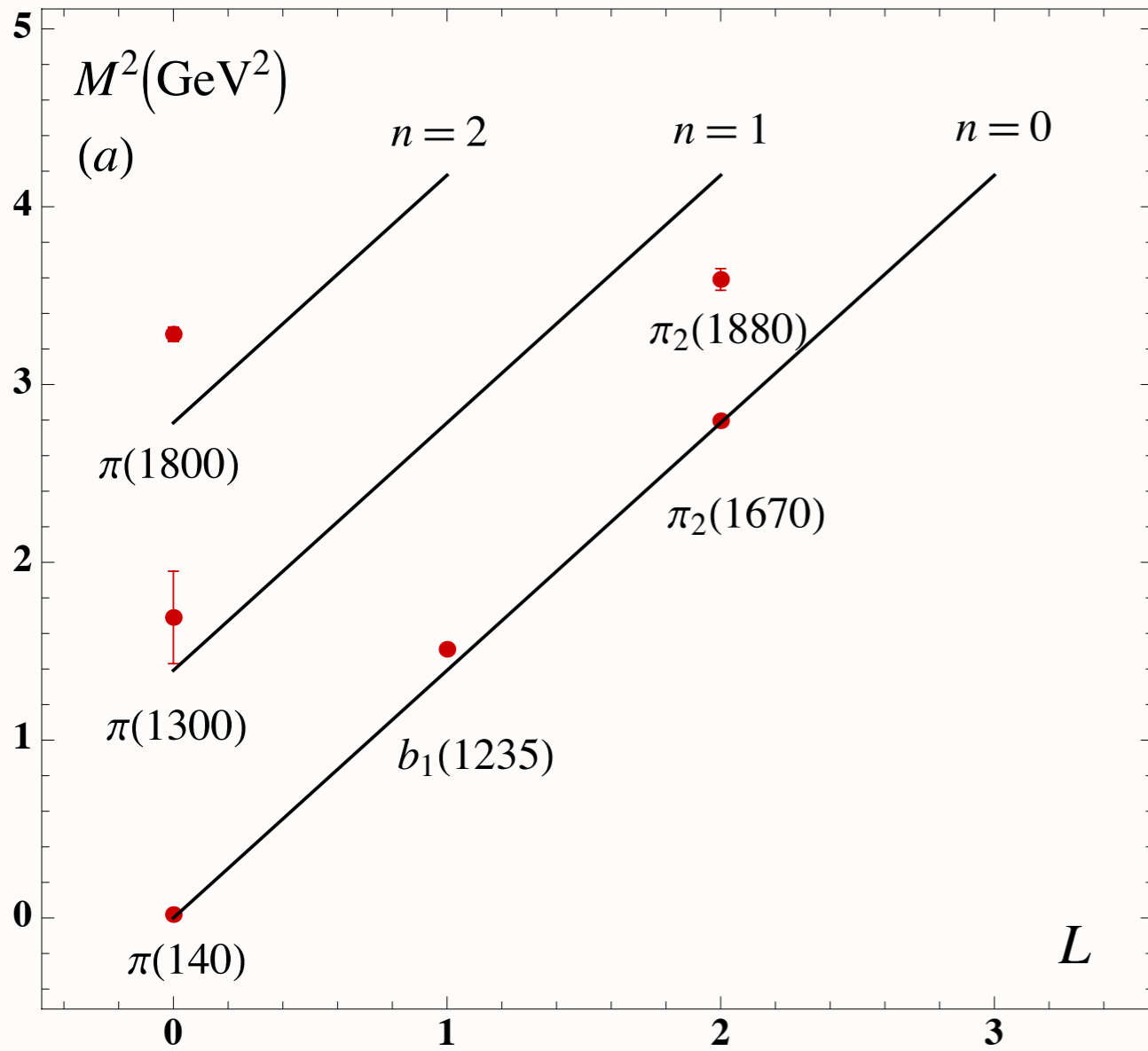


$I=1$ orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ -meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a -states

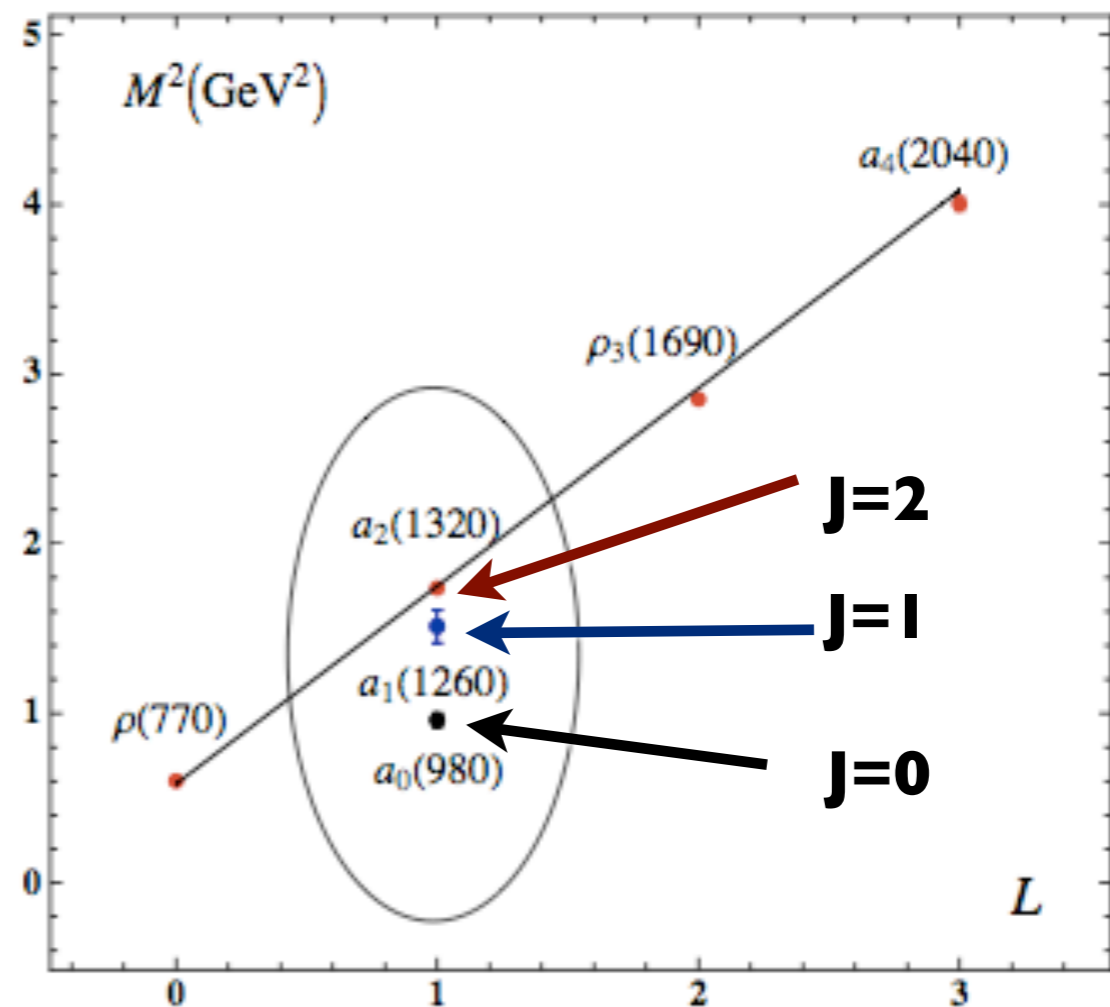
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules



$$\lambda = \kappa^2$$

$$\mathcal{M}_{n,J,L}^2 = 4\lambda \left(n + \frac{J+L}{2} \right)$$



- Triplet splitting for the $L = 1, J = 0, 1, 2$ vector meson a -states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

- Systematics of light meson spectra – orbital and radial excitations as well as important $J - L$ splitting, well described by light-front harmonic confinement model
- Linear Regge trajectories, a massless pion and relation between the ρ and a_1 mass $M_{a_1}/M_\rho = \sqrt{2}$ usually obtained from Weinberg sum rules [Weinberg (1967)]

Light-Front Holography

- **AdS₅/CFT₄ Duality between AdS₅ and Conformal Gauge Theory in 3+1 at fixed LF time** [G. de Téramond, H. G. Dosch, sjb](#)

Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche,
Ivan Schmidt, Alfredo Vega

- “**AdS₄/CFT₃ Construction from Collective Fields**” [Robert de Mello Koch](#), [Antal Jevicki](#), [Kewang Jin](#), [João P. Rodrigues](#)

- “**Exact holographic mapping and emergent space-time geometry**” [Xiao-Liang Qi](#)

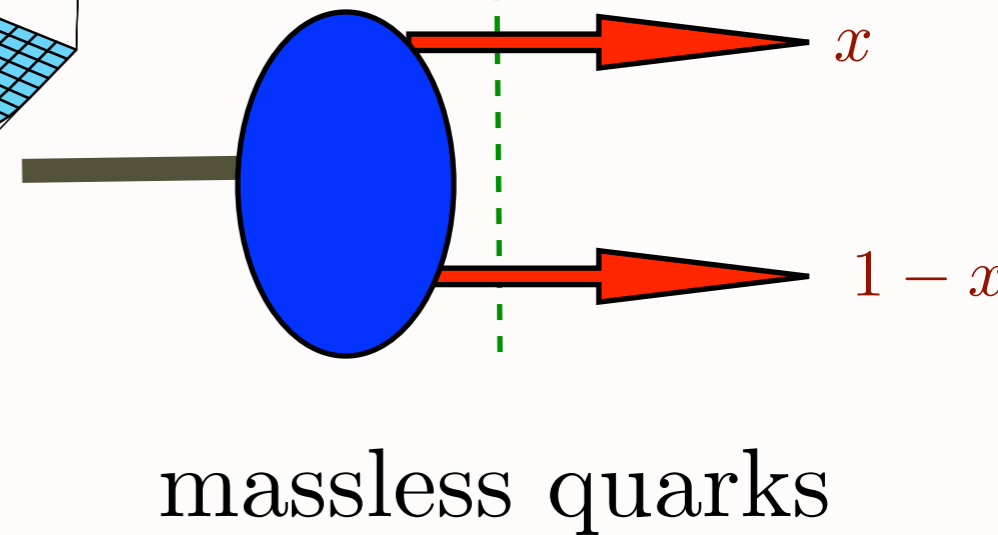
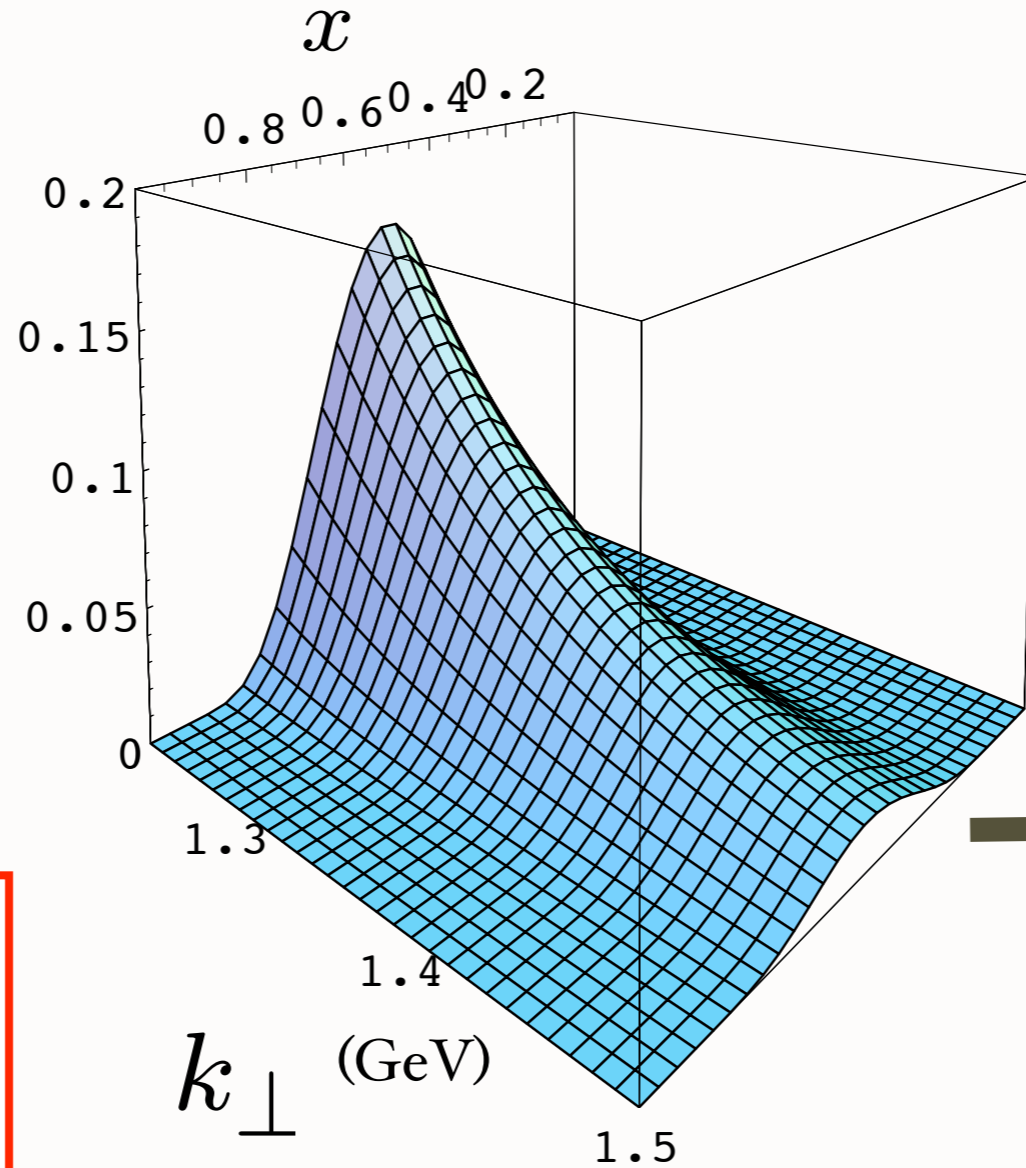
- **Ehrenfest arguments:** [Glazek and Trawinski](#)

Prediction from AdS/QCD: Meson LFWF

de Teramond,
Cao, sjb

“Soft Wall”
model

$$\psi_M(x, k_\perp^2)$$



Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Provides Connection of Confinement to Hadron Structure

Hadron Form Factors from AdS/QCD

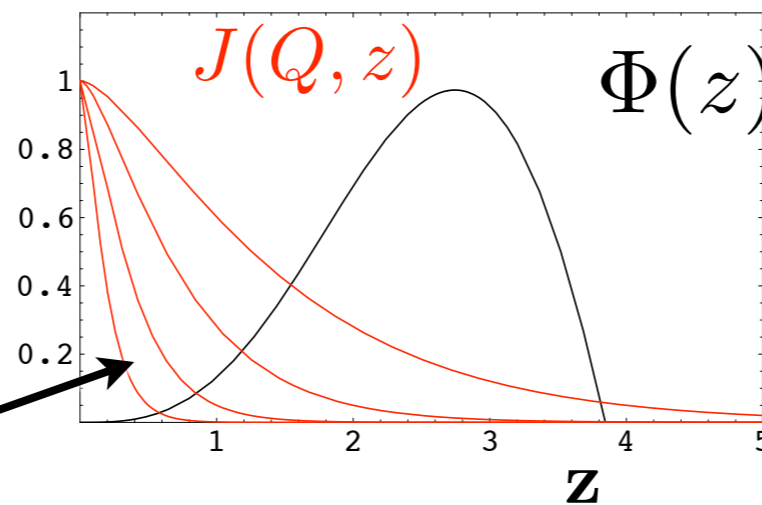
Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQ K_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$

high Q^2



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$.

Twist $\tau = n + L$

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

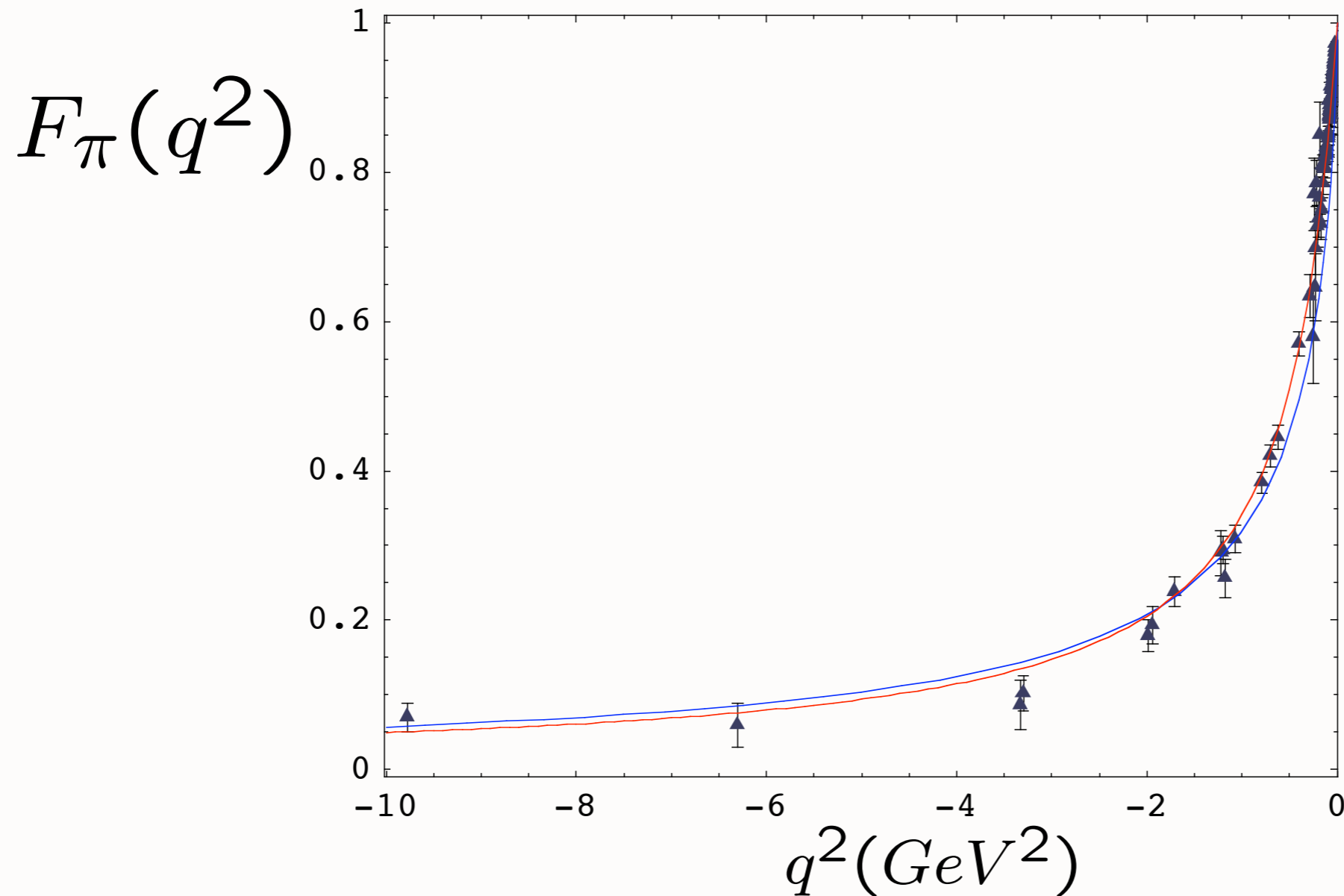
$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

— Soft Wall: Harmonic Oscillator Confinement

— Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

J. R. Forshaw*

*Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester,
Oxford Road, Manchester M13 9PL, United Kingdom*

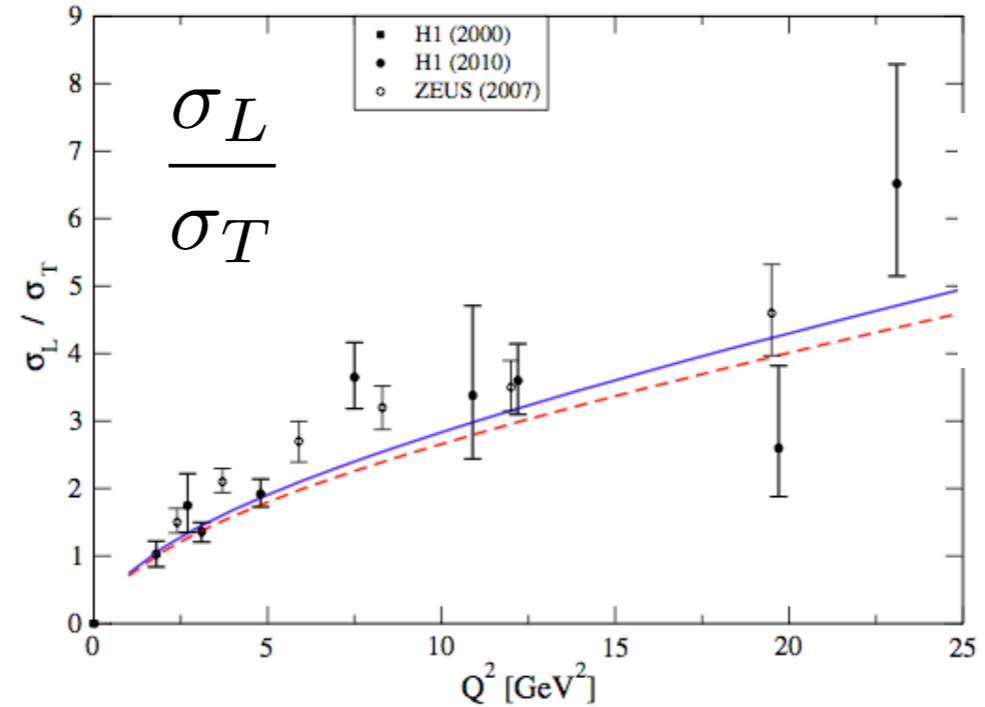
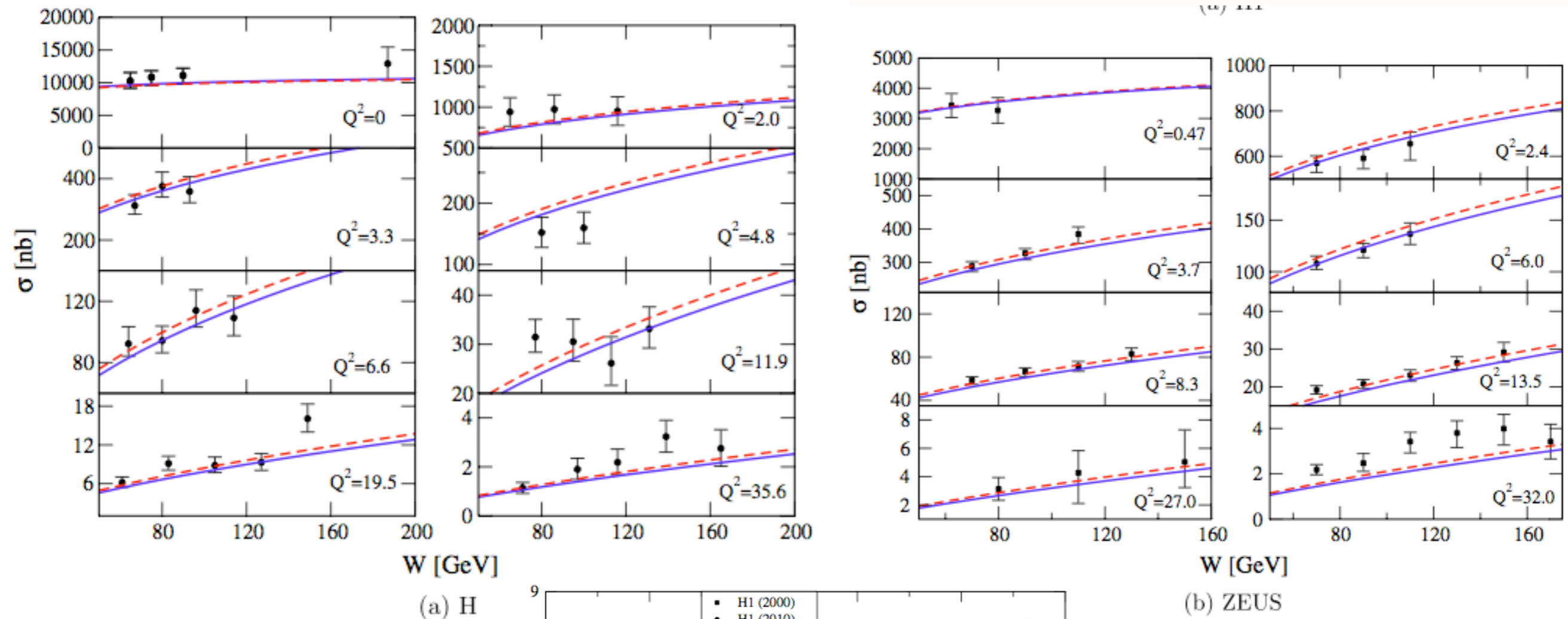
R. Sandapen†

Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada
(Received 5 April 2012; published 20 August 2012)

We show that anti-de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ -meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron-proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



**J. R. Forshaw,
R. Sandapen**

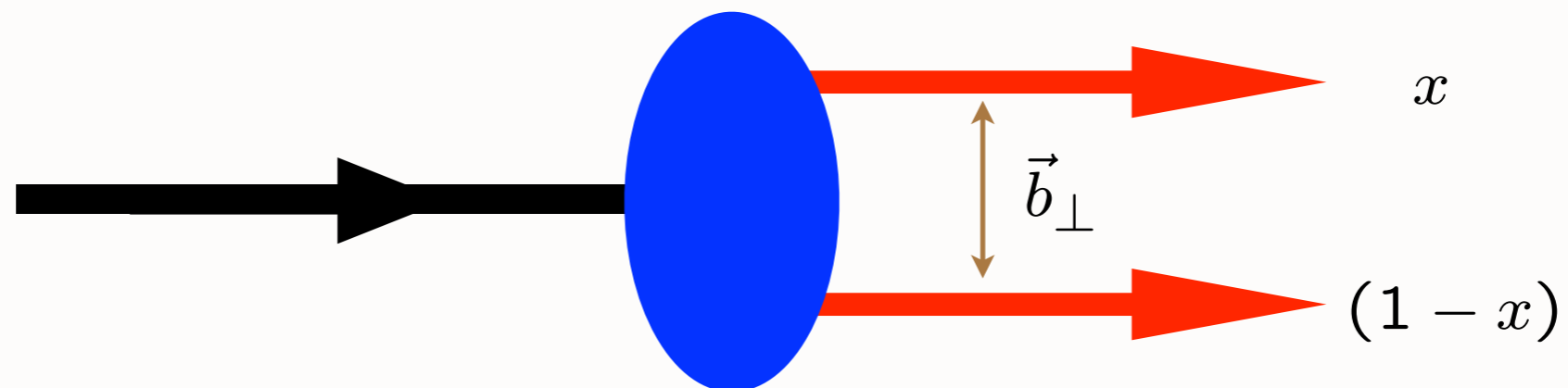
$$\gamma^* p \rightarrow \rho^0 p'$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

$LF(3+1)$ \longleftrightarrow AdS_5

$\psi(x, \vec{b}_\perp)$ \longleftrightarrow $\phi(z)$

$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$ \longleftrightarrow z



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x dx J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

de Teramond & sjb

Identical to LF Holography obtained from electromagnetic current

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

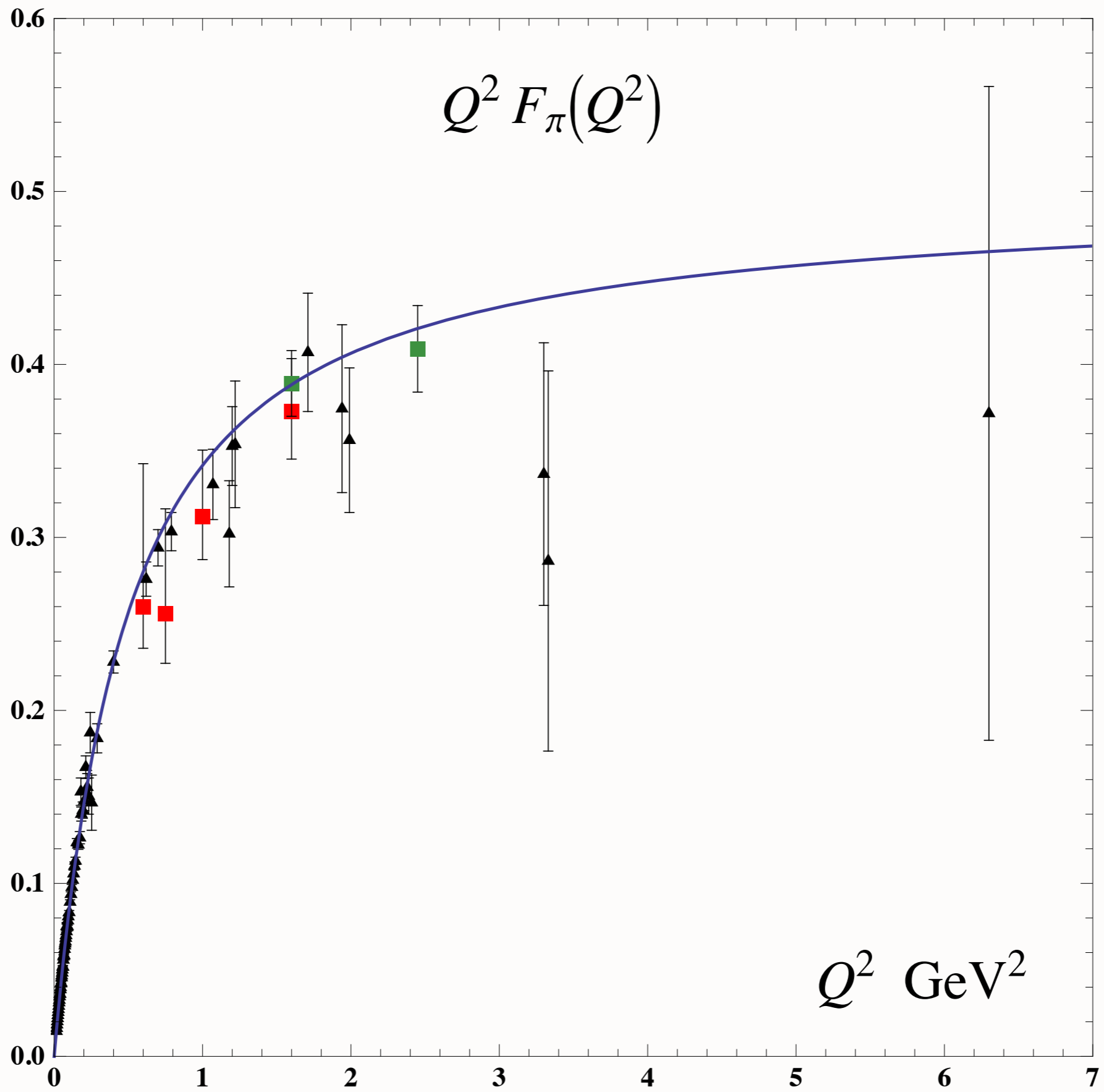
$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

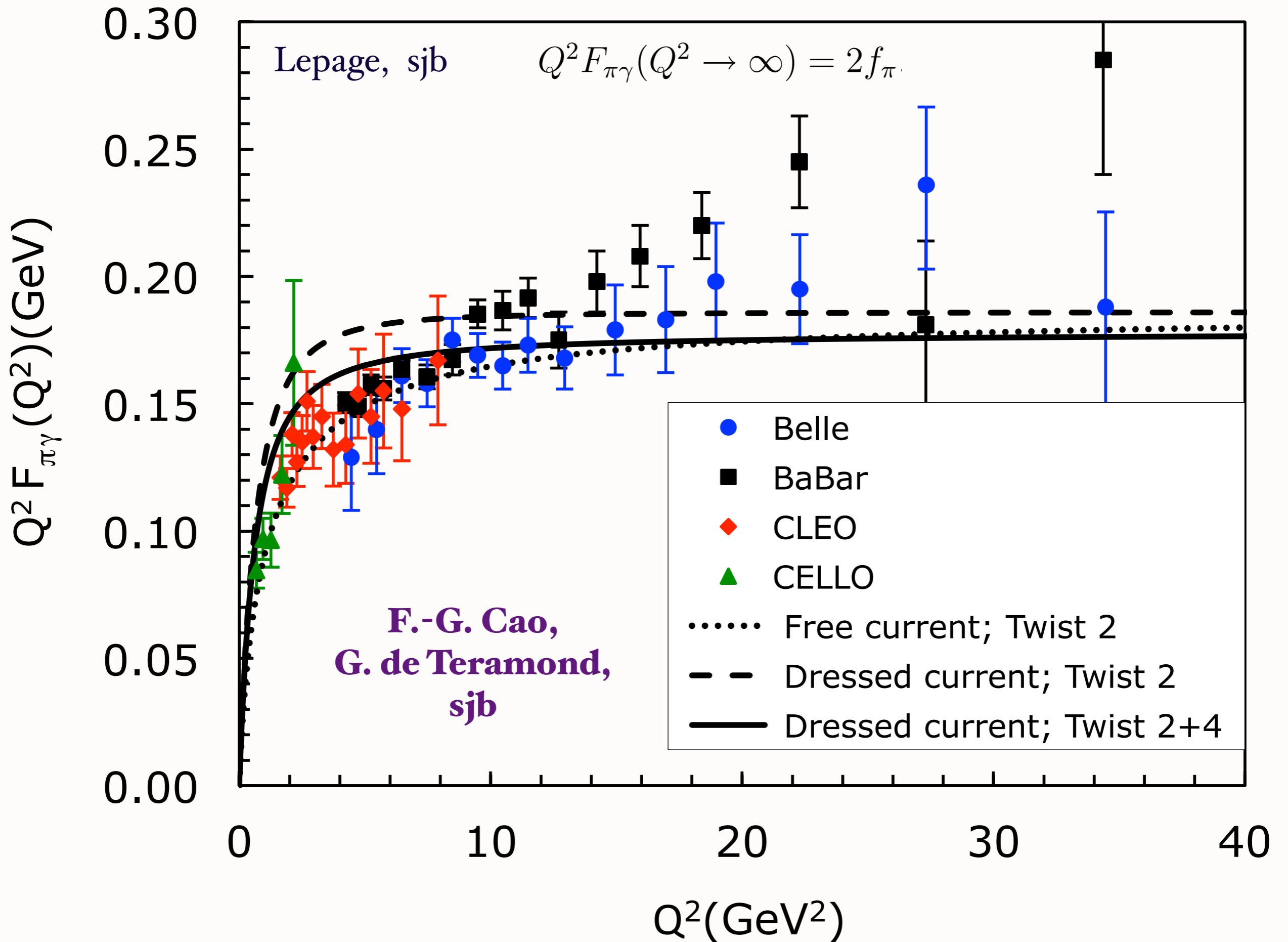
$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Dressed
Current
in Soft-Wall
Model*



Photon-to-pion transition form factor



Predicting the isospin asymmetry in $B \rightarrow K^* \gamma$ using holographic AdS/QCD distribution amplitudes for the K^*

M. Ahmady*

Department of Physics, Mount Allison University, Sackville, New Brunswick E4L 1E6, Canada

R. Sandapen†

*Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A 3E9, Canada
and Department of Physics, Mount Allison University, Sackville, New Brunswick E4L 1E6, Canada*

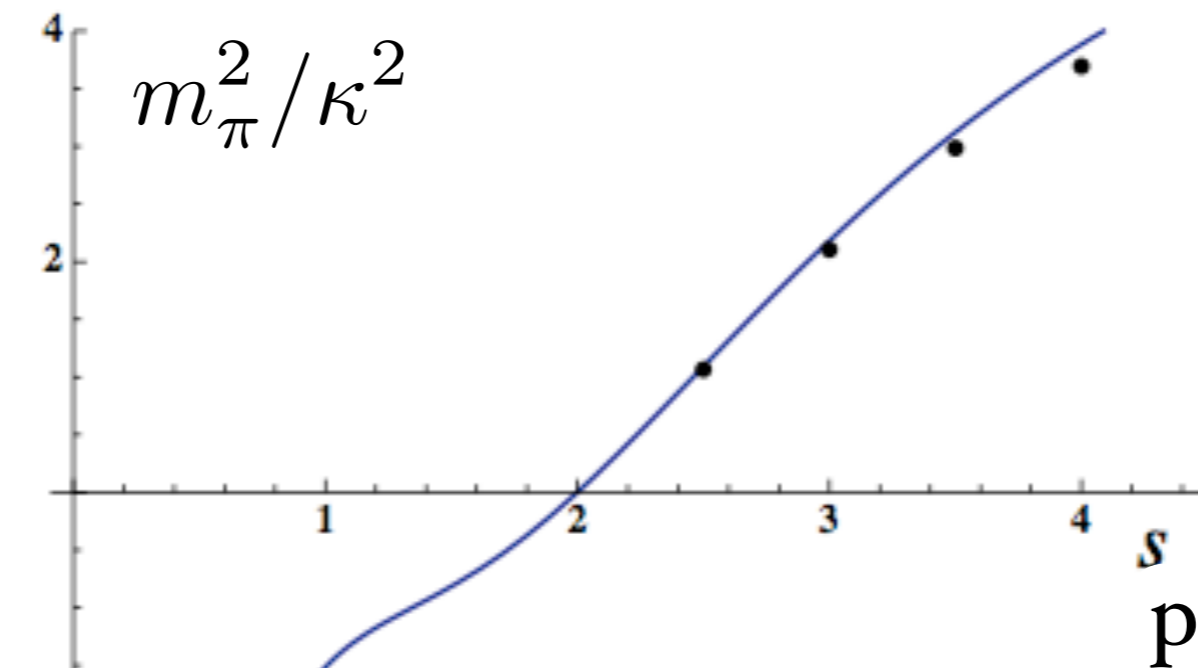
(Received 10 May 2013; published 26 July 2013)

$$\phi_\lambda(z, \zeta) = \mathcal{N}_\lambda \frac{\kappa}{\sqrt{\pi}} \sqrt{z(1-z)} \exp\left(-\frac{\kappa^2 \zeta^2}{2}\right) \\ \times \exp\left(-\frac{(1-z)m_q^2 + zm_{\bar{q}}^2}{2\kappa^2 z(1-z)}\right), \quad \text{where } \zeta = \sqrt{z(1-z)}r$$

We predict the isospin asymmetry well as the branching ratio for the decay $B \rightarrow K^* \gamma$ within QCD factorization using new anti-de Sitter/quantum chromodynamics (AdS/QCD) holographic distribution amplitudes (DAs) for the K^* meson. Our prediction for the branching ratio agrees with that obtained using standard QCD sum-rules (SR) DAs and with experiment. More interestingly, our prediction for the isospin asymmetry using the AdS/QCD DA does not suffer from the end-point divergence encountered when using the corresponding SR DA. We predict an isospin asymmetry of 3.2% in agreement with the most recent average measured value of $(5.2 \pm 2.6)\%$ quoted by the Particle Data Group.

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff
 $p=2!$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

● **Dosch, de Teramond, sjb**

Uniqueness

de Teramond, Dosch, sjb

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \quad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L : same slope!**
- **Massless pion in chiral limit! No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, FurlanFubini, Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569**

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics quark kinetic energy + quark-gluon dynamics quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$
$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if $m_q=0$

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

**Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement**

QCD Mass Scale from Confinement not Explicit

What determines the QCD mass scale Λ_{QCD} ?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK \quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

Conformal Invariance in Quantum Mechanics.

V. DE ALFARO

Istituto di Fisica Teorica dell'Università - Torino

Istituto Nazionale di Fisica Nucleare - Sezione di Torino

S. FUBINI and G. FURLAN (*)

CERN - Geneva

(ricevuto il 3 Maggio 1976)

Summary. — The properties of a field theory in one over-all time dimension, invariant under the full conformal group, are studied in detail. A compact operator, which is not the Hamiltonian, is diagonalized and used to solve the problem of motion, providing a discrete spectrum and normalizable eigenstates. The role of the physical parameters present in the model is discussed, mainly in connection with a semi-classical approximation.

$$G|\psi(\tau)\rangle = i\frac{\partial}{\partial\tau}|\psi(\tau)\rangle$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

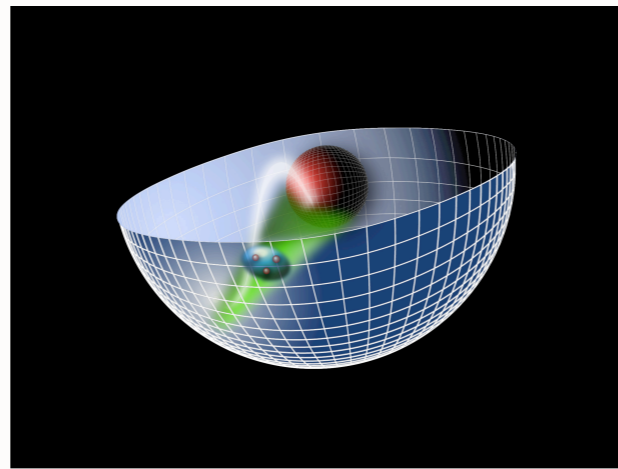
$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left(\frac{2tw + v}{\sqrt{4uw - v^2}} \right),$$

- **Identify with difference of LF time $\Delta x^+ / P^+$ between constituents**
- **Finite range**
- **Measure in Double Parton Processes**

Remarkable Features of Light-Front Schrödinger Equation

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$



*AdS/QCD
Soft-Wall Model*

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent

Unique color-confining potential

Zero mass pion for massless quarks

Regge trajectories with equal slopes in n and L

Light-Front Wavefunctions

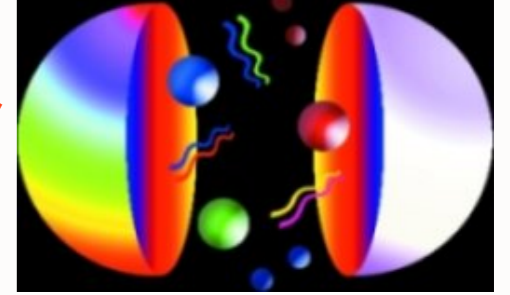
Conformal Symmetry

Light-Front Schrödinger Equation

Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

*Yukawa interaction
in 5 dimensions*



From Nick Evans

- Action for Dirac field in AdS_{d+1} in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g} e^{\varphi(z)} (i\bar{\Psi} e_A^M \Gamma^A D_M \Psi + h.c. + \varphi(z) \bar{\Psi} \Psi - \mu \bar{\Psi} \Psi)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP \cdot x} \Psi(z)$

$$\left[i \left(z \eta^{\ell m} \Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.$$

- Solution $(\nu = \mu R - \frac{1}{2}, \nu = L + 1)$

$$\Psi_+(z) \sim z^{\frac{5}{2} + \nu} e^{-\kappa^2 z^2 / 2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2} + \nu} e^{-\kappa^2 z^2 / 2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1) \quad \text{positive parity}$$

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

- Large N_C : $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C} \Lambda_{\text{QCD}}$

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint Π^\dagger , with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned} \psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2). \end{aligned} \quad \nu = L + 1$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

Baryon Spectrum in Soft-Wall Model

- Upon substitution $z \rightarrow \zeta$ and

$$\Psi_J(x, z) = e^{-iP \cdot x} z^2 \psi^J(z) u(P),$$

find LFWE for $d = 4$

$$\begin{aligned} \frac{d}{d\zeta} \psi_+^J + \frac{\nu + \frac{1}{2}}{\zeta} \psi_+^J + U(\zeta) \psi_+^J &= \mathcal{M} \psi_-^J, \\ -\frac{d}{d\zeta} \psi_-^J + \frac{\nu + \frac{1}{2}}{\zeta} \psi_-^J + U(\zeta) \psi_-^J &= \mathcal{M} \psi_+^J, \end{aligned}$$

where $U(\zeta) = \frac{R}{\zeta} V(\zeta)$

- Choose linear potential $U = \kappa^2 \zeta$
- Eigenfunctions

$$\psi_+^J(\zeta) \sim \zeta^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \quad \psi_-^J(\zeta) \sim \zeta^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2)$$

- Eigenvalues

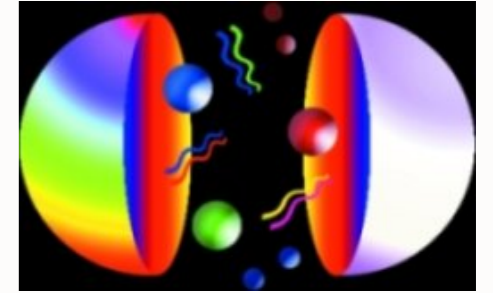
$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1), \quad \nu = L + 1 \quad (\tau = 3)$$

- Full $J - L$ degeneracy (different J for same L) for baryons along given trajectory !

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

*Chiral Symmetry
of Eigenstate!*

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

$$-\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - V(\zeta)\psi_- = M\psi_+,$$

$$\frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - V(\zeta)\psi_+ = M\psi_-,$$

$$M^2 = 4\kappa^2(n + \nu + 1)$$

Orbital assignment for baryon trajectories according to parity and internal spin.

$$\nu = |\mu R| - 1/2$$

	$S = \frac{1}{2}$	$S = \frac{3}{2}$
P = +	$\nu = L$	$\nu = L + \frac{1}{2}$
P = -	$\nu = L + \frac{1}{2}$	$\nu = L + 1$

$$M_{n,L,S=\frac{3}{2}}^{2(+)} = M_{n,L,S=\frac{1}{2}}^{2(-)}$$

No spin-orbit coupling

J=1/2 “Chiral partners”, e.g. N(1535) and N(1400), with different L, non-degenerate

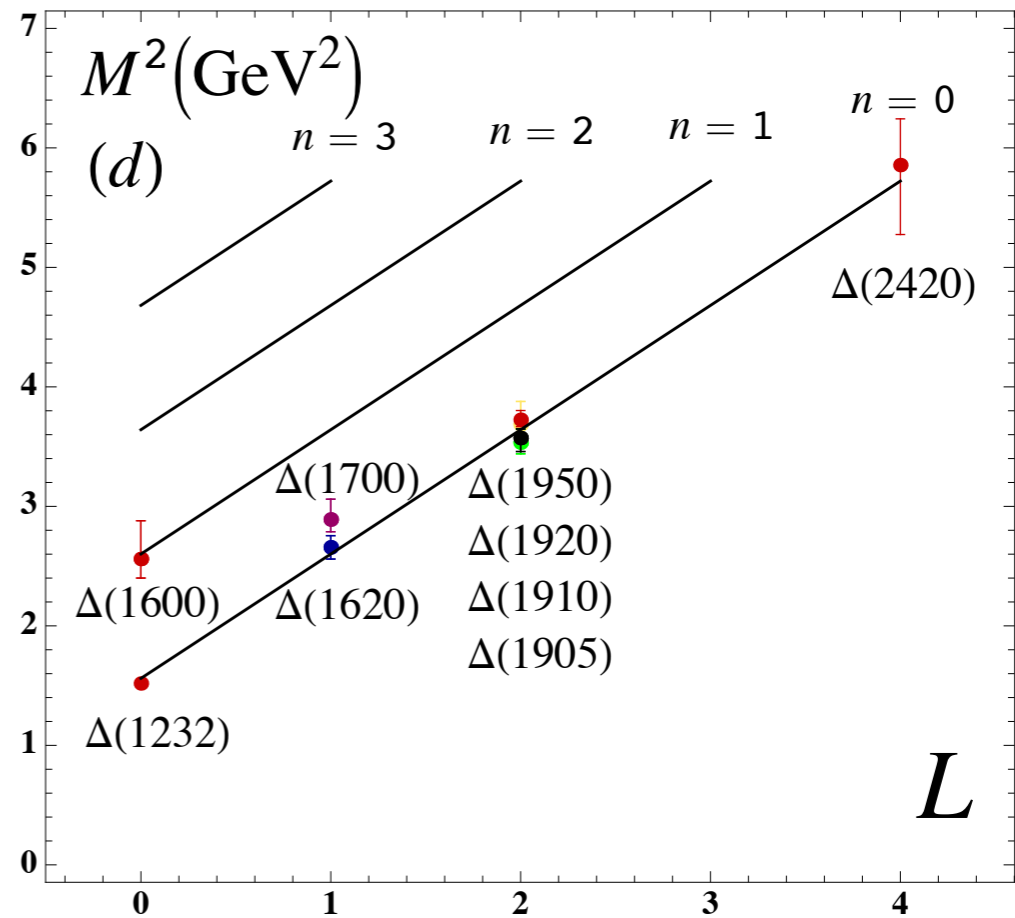
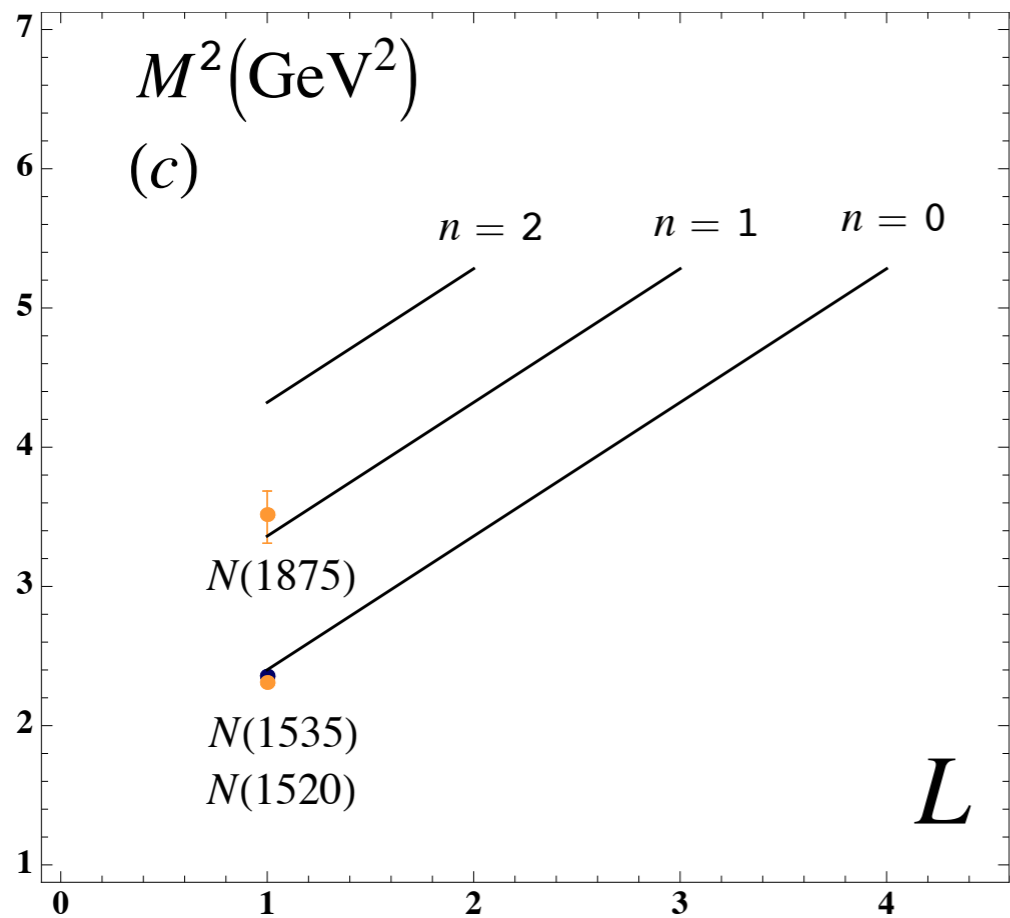
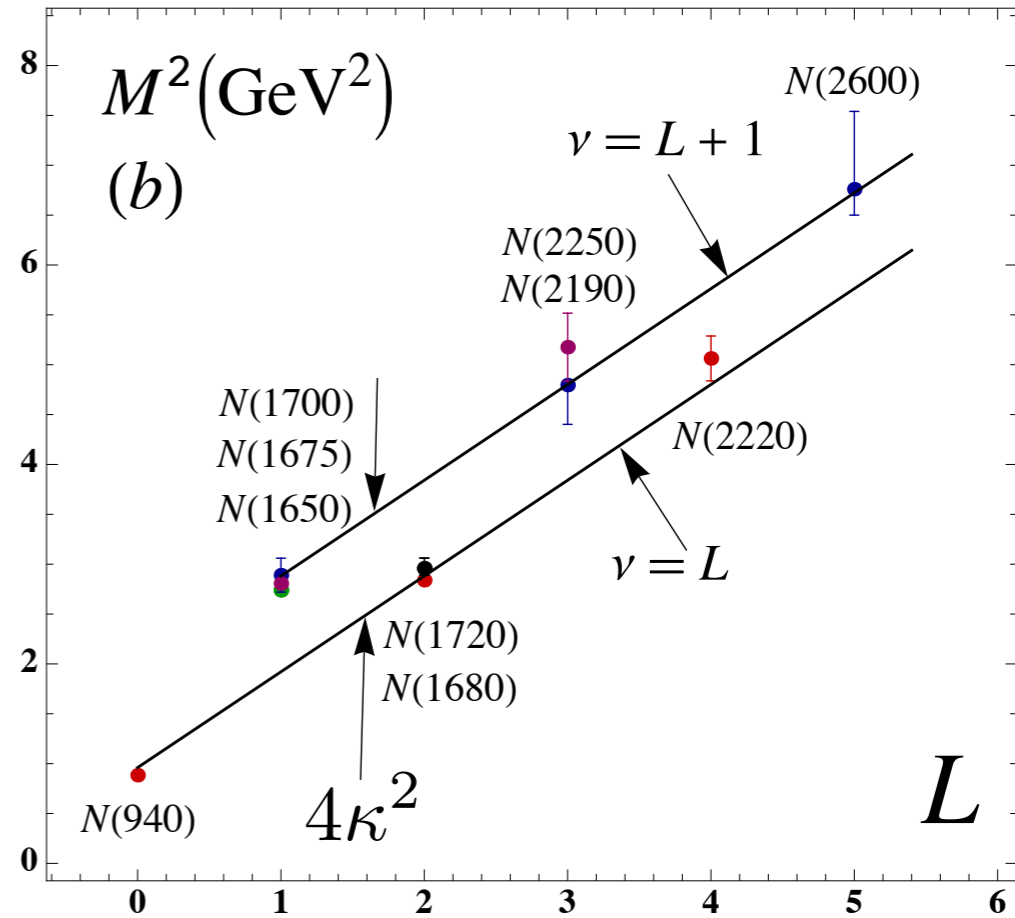
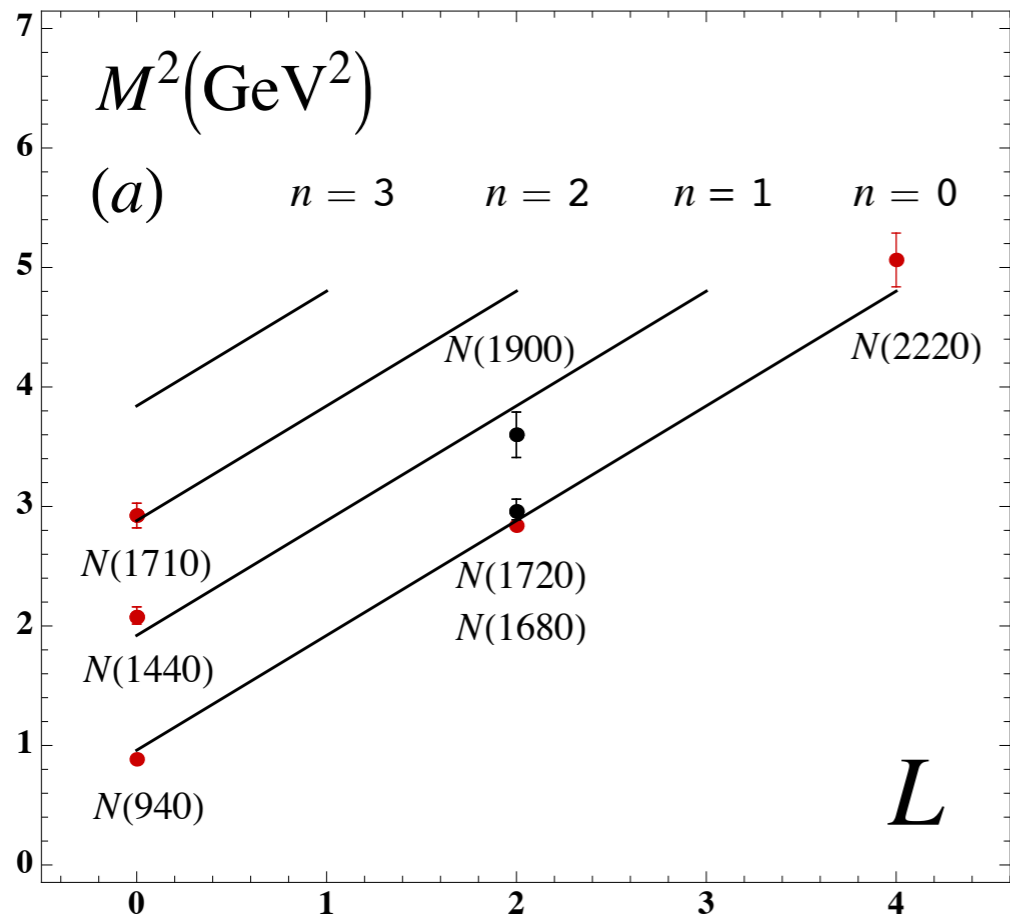
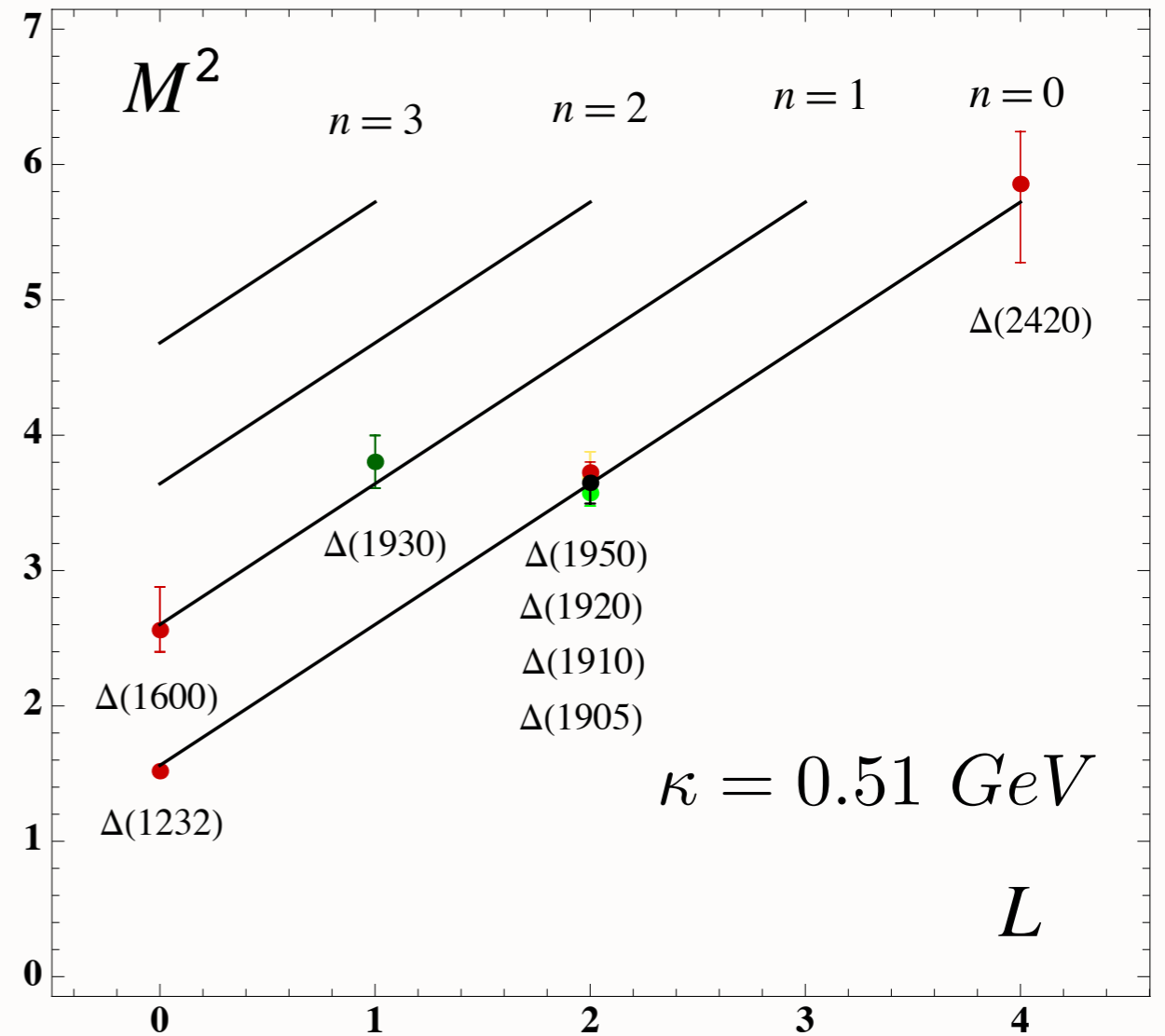
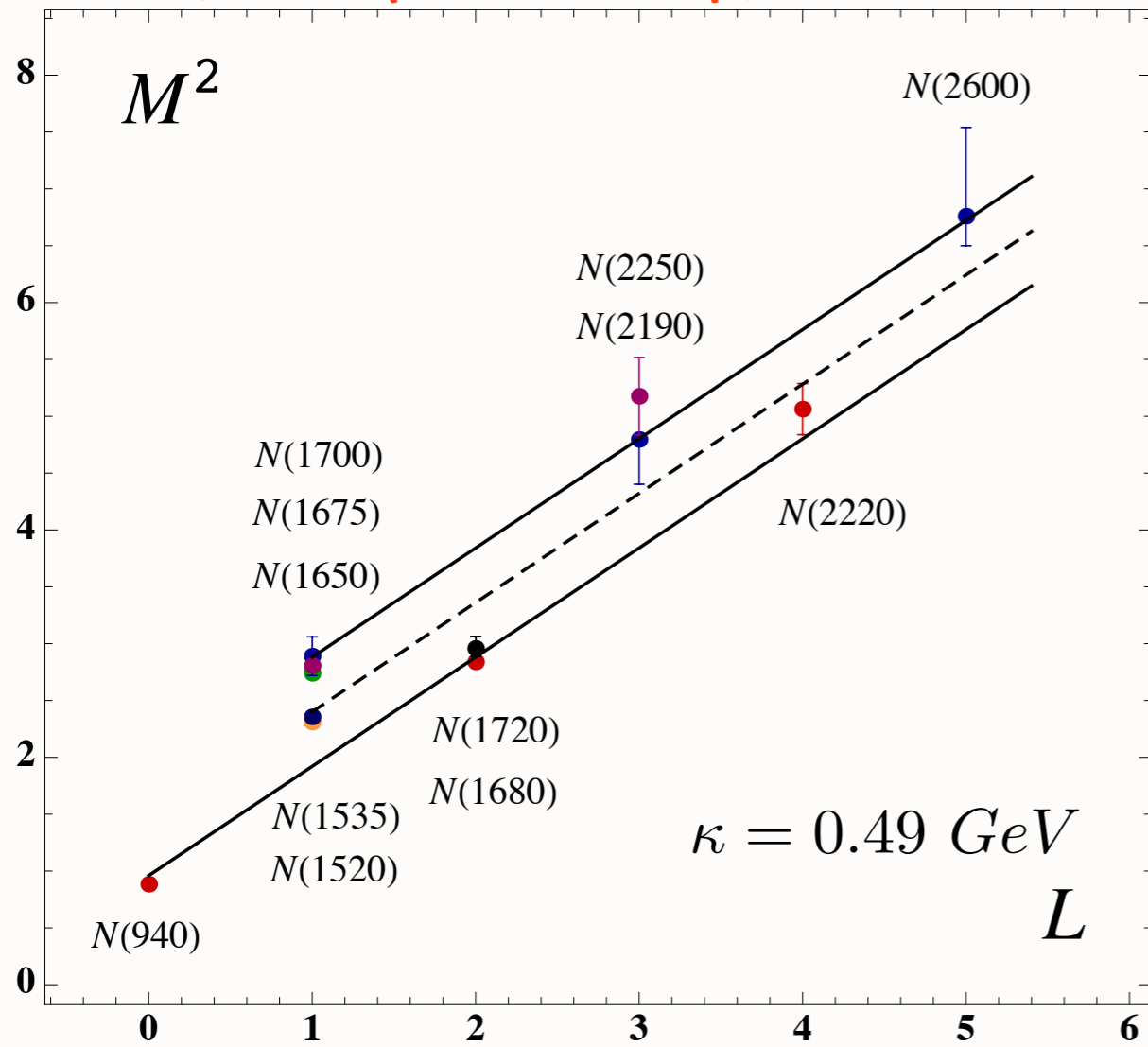


Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S , L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta_{\frac{5}{2}}^{-}(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

$SU(6)$	S	L	n	Baryon State			
56	$\frac{1}{2}$	0	0	$N_{\frac{1}{2}}^{1+}(940)$			
	$\frac{1}{2}$	0	1	$N_{\frac{1}{2}}^{1+}(1440)$			
	$\frac{1}{2}$	0	2	$N_{\frac{1}{2}}^{1+}(1710)$			
	$\frac{3}{2}$	0	0	$\Delta_{\frac{3}{2}}^{3+}(1232)$			
	$\frac{3}{2}$	0	1	$\Delta_{\frac{3}{2}}^{3+}(1600)$			
70	$\frac{1}{2}$	1	0	$N_{\frac{1}{2}}^{1-}(1535) \quad N_{\frac{3}{2}}^{3-}(1520)$			
	$\frac{3}{2}$	1	0	$N_{\frac{1}{2}}^{1-}(1650) \quad N_{\frac{3}{2}}^{3-}(1700) \quad N_{\frac{5}{2}}^{5-}(1675)$			
	$\frac{3}{2}$	1	1	$N_{\frac{1}{2}}^{1-} \quad N_{\frac{3}{2}}^{3-}(1875) \quad N_{\frac{5}{2}}^{5-}$			
	$\frac{1}{2}$	1	0	$\Delta_{\frac{1}{2}}^{1-}(1620) \quad \Delta_{\frac{3}{2}}^{3-}(1700)$			
56	$\frac{1}{2}$	2	0	$N_{\frac{3}{2}}^{3+}(1720) \quad N_{\frac{5}{2}}^{5+}(1680)$			
	$\frac{1}{2}$	2	1	$N_{\frac{3}{2}}^{3+}(1900) \quad N_{\frac{5}{2}}^{5+}$			
	$\frac{3}{2}$	2	0	$\Delta_{\frac{1}{2}}^{1+}(1910) \quad \Delta_{\frac{3}{2}}^{3+}(1920) \quad \Delta_{\frac{5}{2}}^{5+}(1905) \quad \Delta_{\frac{7}{2}}^{7+}(1950)$			
70	$\frac{1}{2}$	3	0	$N_{\frac{5}{2}}^{5-} \quad N_{\frac{7}{2}}^{7-}$			
	$\frac{3}{2}$	3	0	$N_{\frac{3}{2}}^{3-} \quad N_{\frac{5}{2}}^{5-} \quad N_{\frac{7}{2}}^{7-}(2190) \quad N_{\frac{9}{2}}^{9-}(2250)$			
	$\frac{1}{2}$	3	0	$\Delta_{\frac{5}{2}}^{5-} \quad \Delta_{\frac{7}{2}}^{7-}$			
56	$\frac{1}{2}$	4	0	$N_{\frac{7}{2}}^{7+} \quad N_{\frac{9}{2}}^{9+}(2220)$			
	$\frac{3}{2}$	4	0	$\Delta_{\frac{5}{2}}^{5+} \quad \Delta_{\frac{7}{2}}^{7+} \quad \Delta_{\frac{9}{2}}^{9+} \quad \Delta_{\frac{11}{2}}^{11+}(2420)$			
70	$\frac{1}{2}$	5	0	$N_{\frac{9}{2}}^{9-} \quad N_{\frac{11}{2}}^{11-}$			
	$\frac{3}{2}$	5	0	$N_{\frac{7}{2}}^{7-} \quad N_{\frac{9}{2}}^{9-} \quad N_{\frac{11}{2}}^{11-}(2600) \quad N_{\frac{13}{2}}^{13-}$			

PDG 2012

Baryon Spectroscopy from AdS/QCD and Light-Front Holography



de Teramond, sjb

$$\mathcal{M}_{n,L,S}^{2(+)} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{3}{4} \right), \quad \text{positive parity}$$

$$\mathcal{M}_{n,L,S}^{2(-)} = 4\kappa^2 \left(n + L + \frac{S}{2} + \frac{5}{4} \right), \quad \text{negative parity}$$

**All confirmed
resonances
from PDG
2012**

See also Forkel, Beyer, Federico, Klempt

CERN TH
January 22, 2014

New Perspectives for Hadron Physics

Stan Brodsky
SLAC
STANFORD LINEAR ACCELERATOR

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$**
$$J^z = +1/2 : \langle L^z \rangle = 1/2, \langle S_q^z \rangle = 0$$
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$.**

No mass-degenerate parity partners!

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}(\kappa^2 z^2) e^{-\kappa^2 z^2/2}$$

- Normalization ($F_1^p(0) = 1$, $V(Q=0, z) = 1$)

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

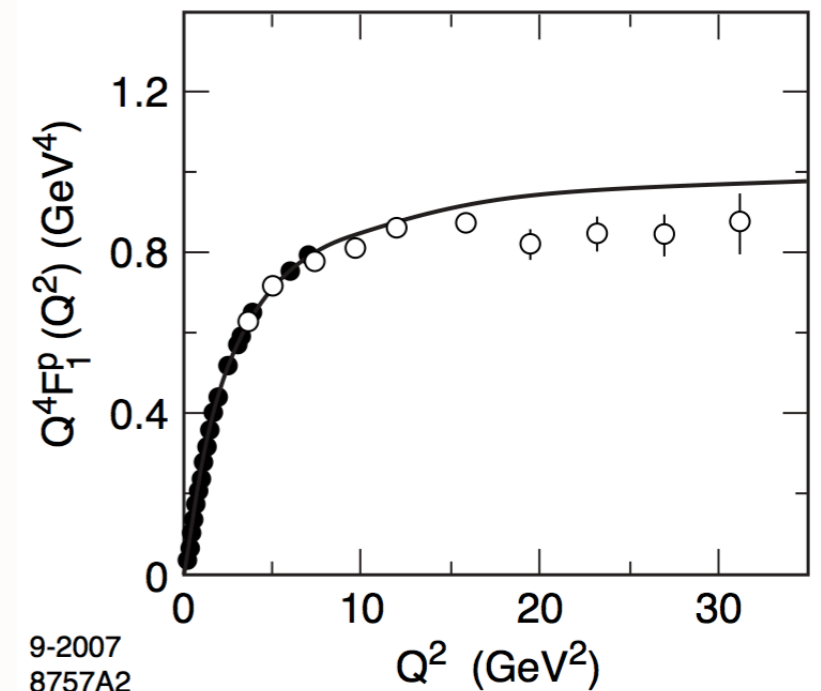
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

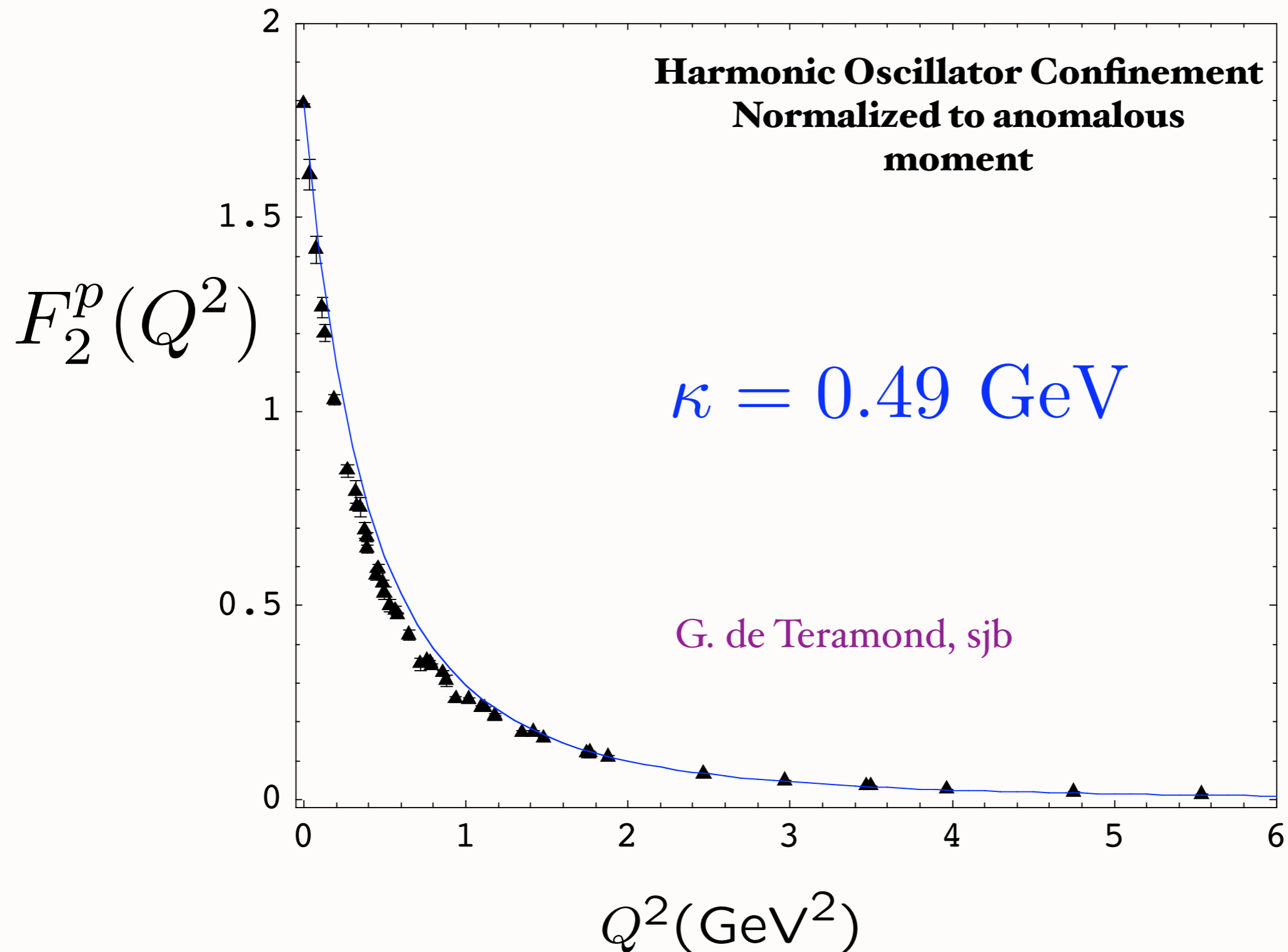
$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

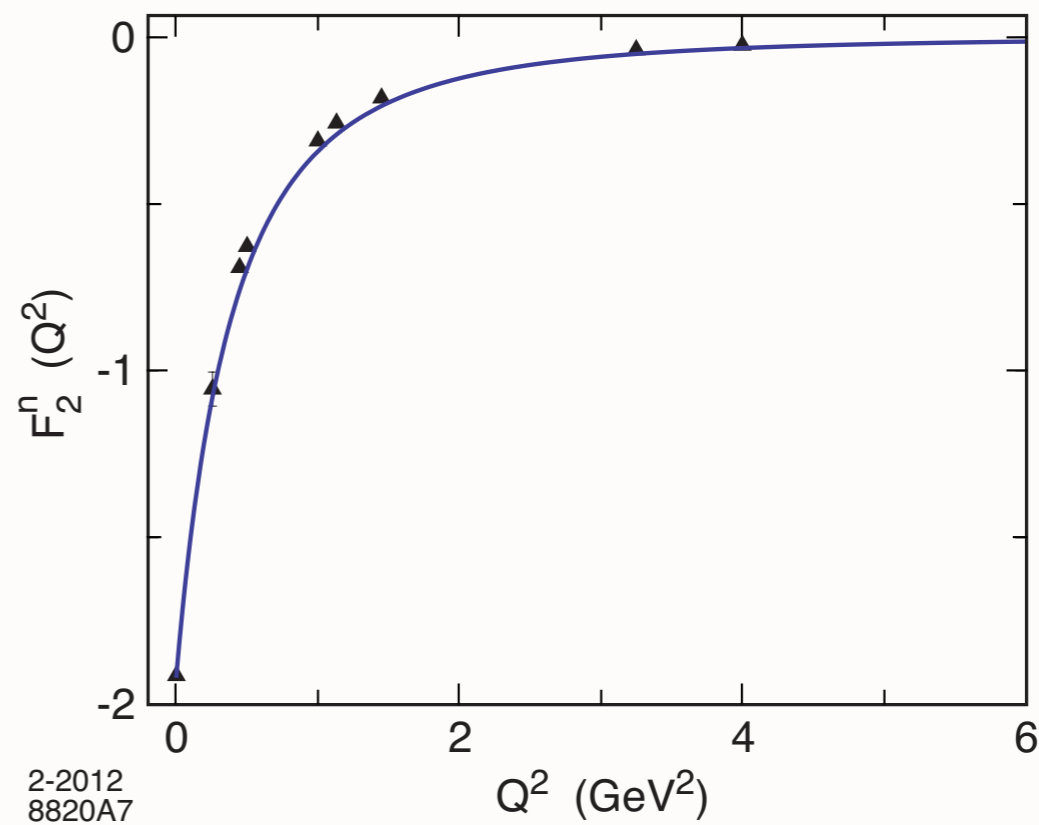
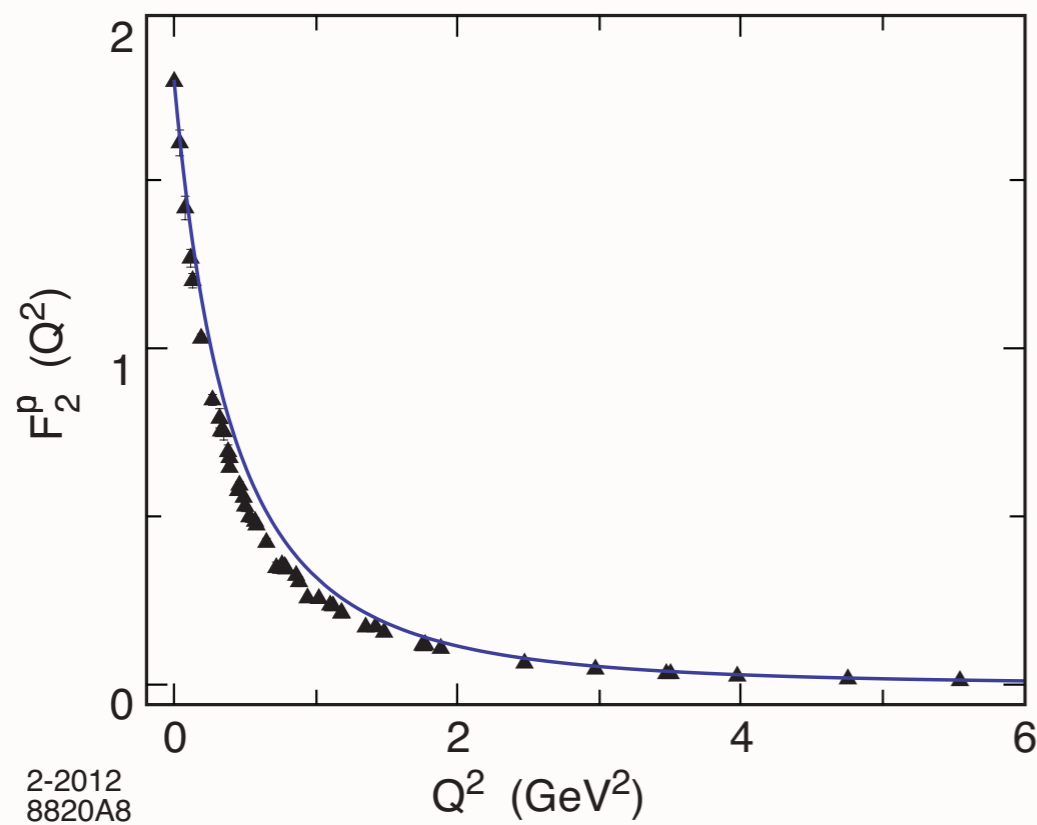
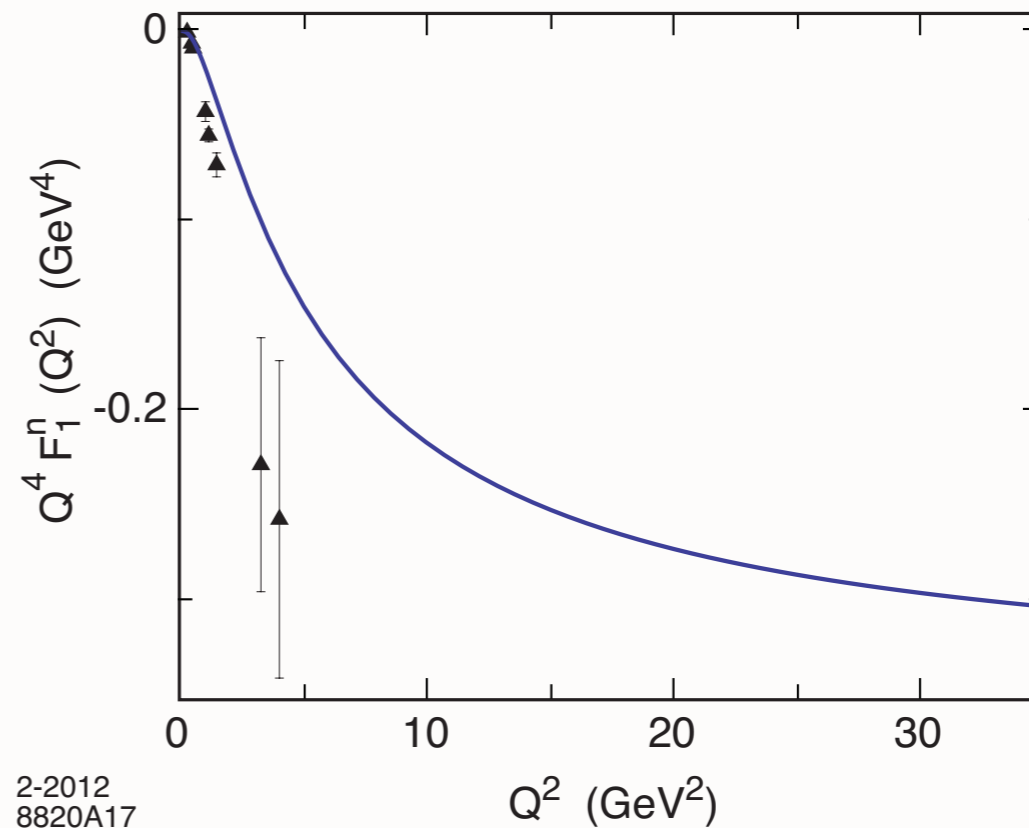
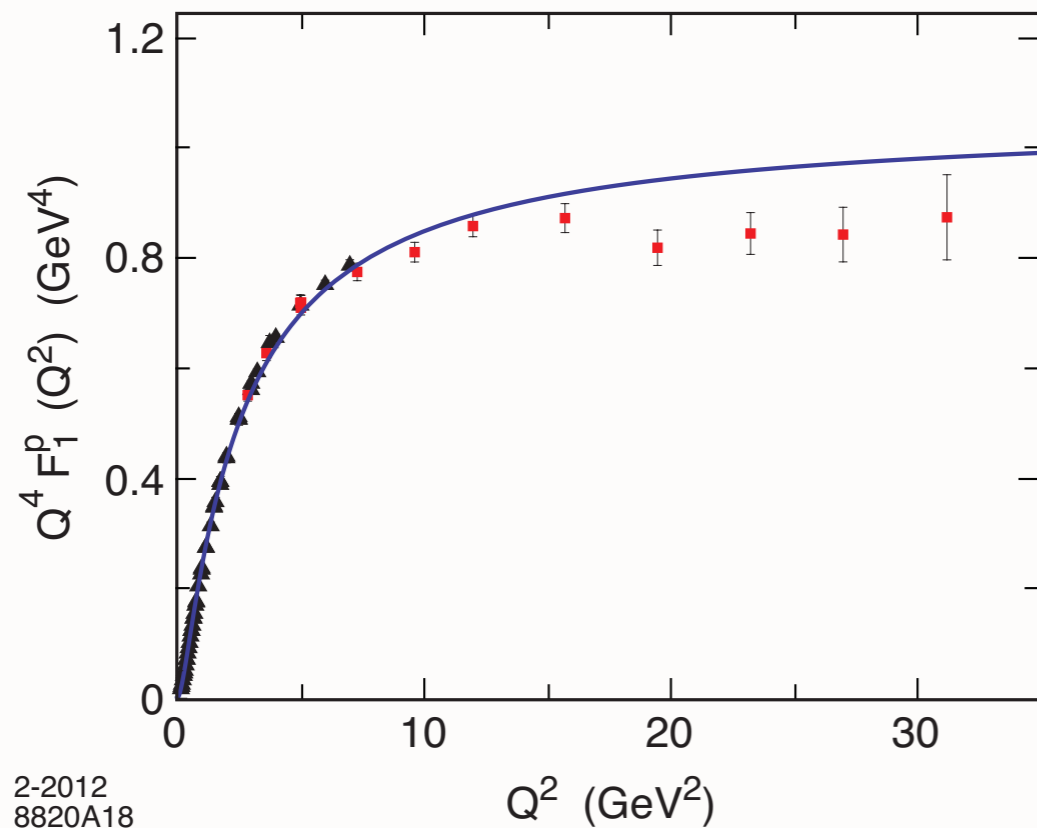


Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



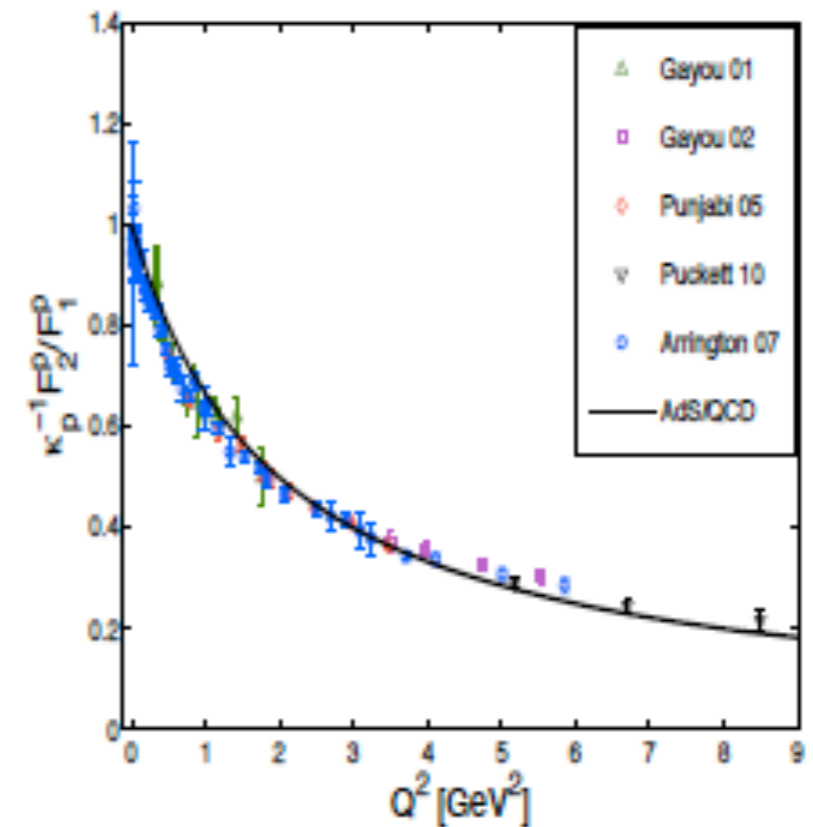
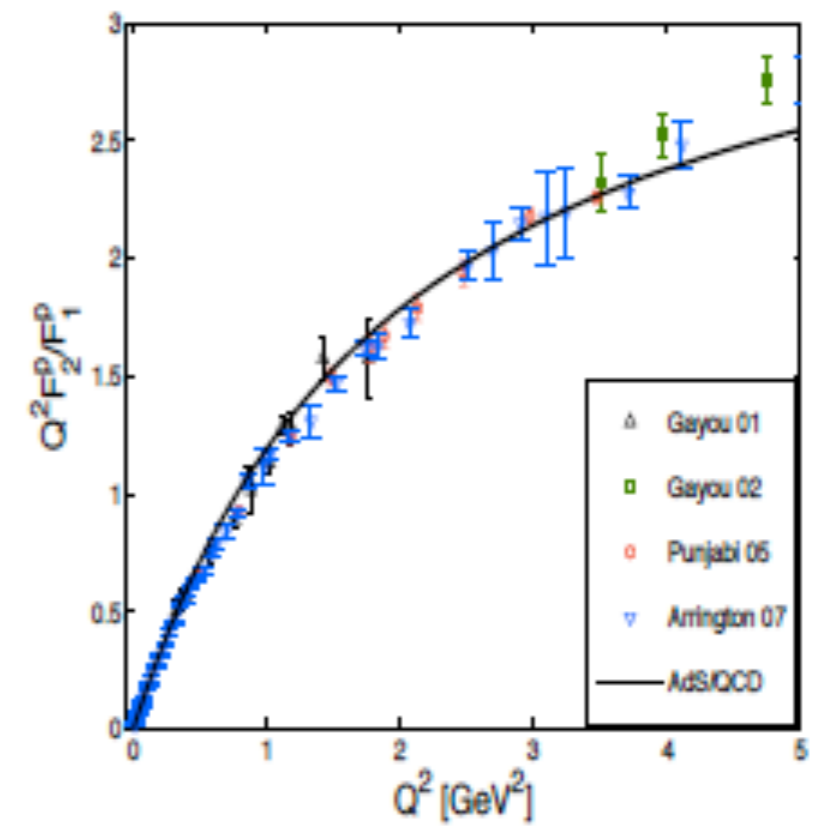
Using $SU(6)$ flavor symmetry and normalization to static quantities



Nucleon and flavor form factors in a light front quark model in AdS/QCD

Dipankar Chakrabarti, Chandan Mondal

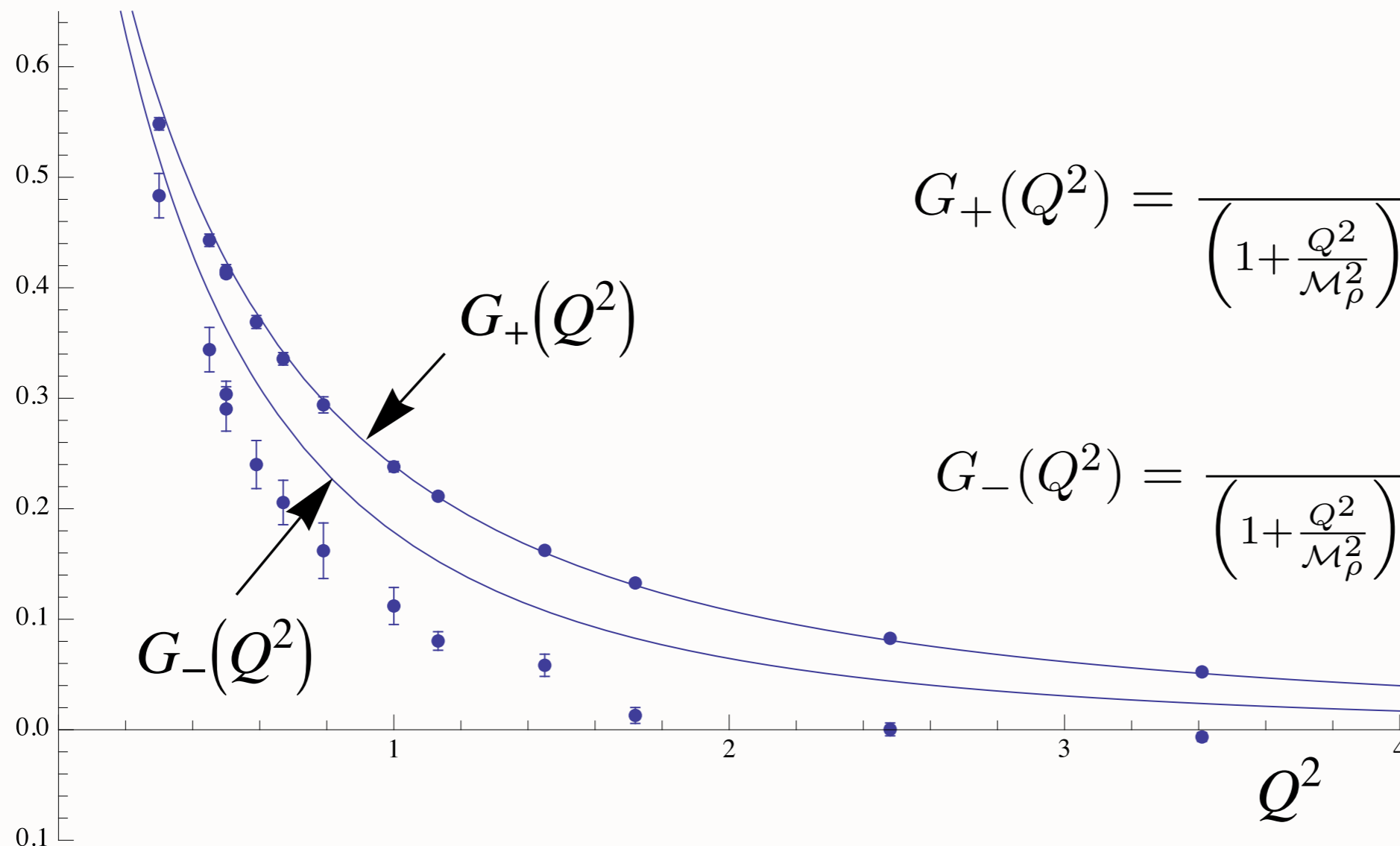
¹Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India.



Flavor Decomposition of Elastic Nucleon Form Factors

G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F_{u,1}^p = \frac{5}{3}G_+ + \frac{1}{3}G_-$, $F_{d,1}^p = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F_{u,1}^n = \frac{1}{3}G_+ + \frac{2}{3}G_-$, $F_{d,1}^n = \frac{5}{3}G_+ + \frac{1}{3}G_-$



$$G_+(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right)}$$

$$G_-(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \left(1 + \frac{Q^2}{M_{\rho''}^2}\right)}$$

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \rightarrow N^*(1440)$: $\Psi_+^{n=0,L=0} \rightarrow \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1N \rightarrow N^*}^p(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q, z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $(F_{1N \rightarrow N^*}^p(0) = 0, \quad V(Q=0, z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

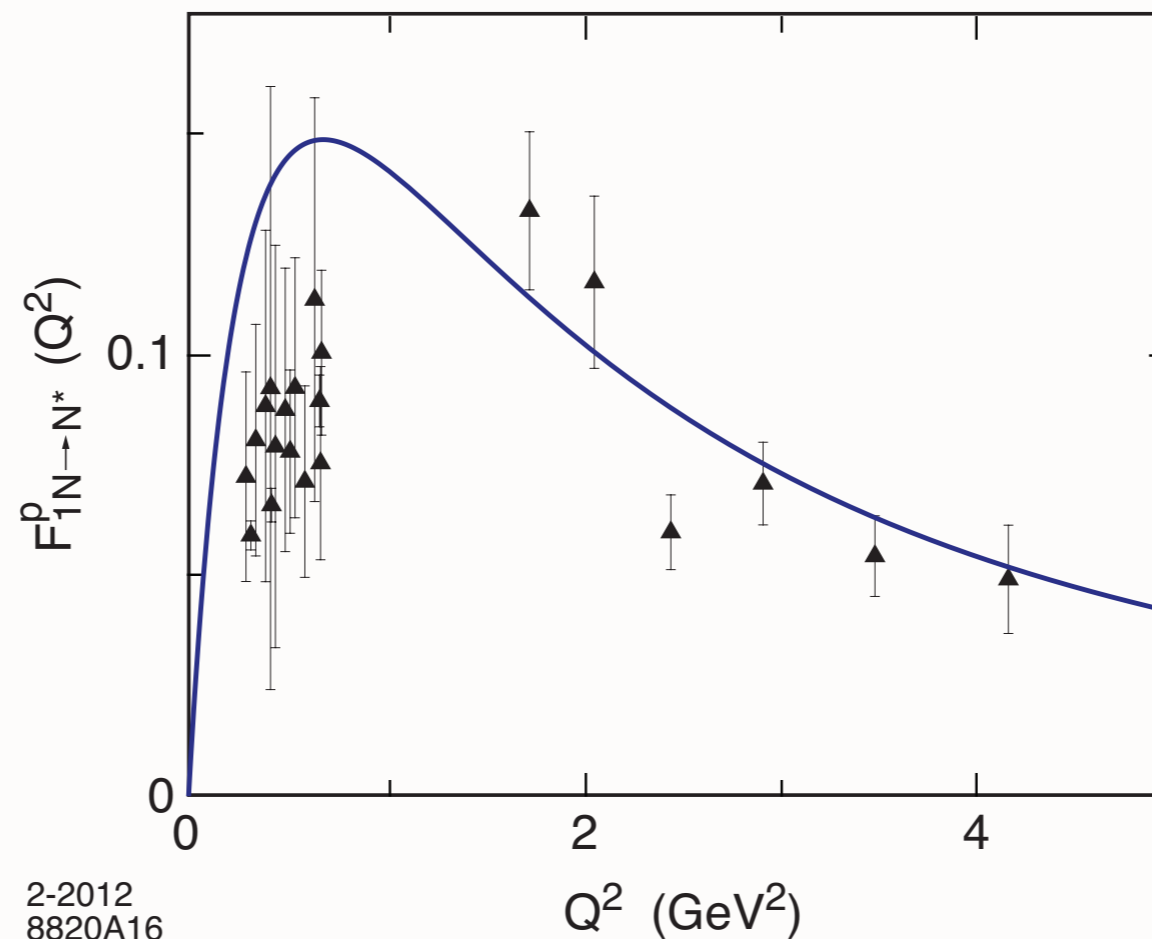
with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

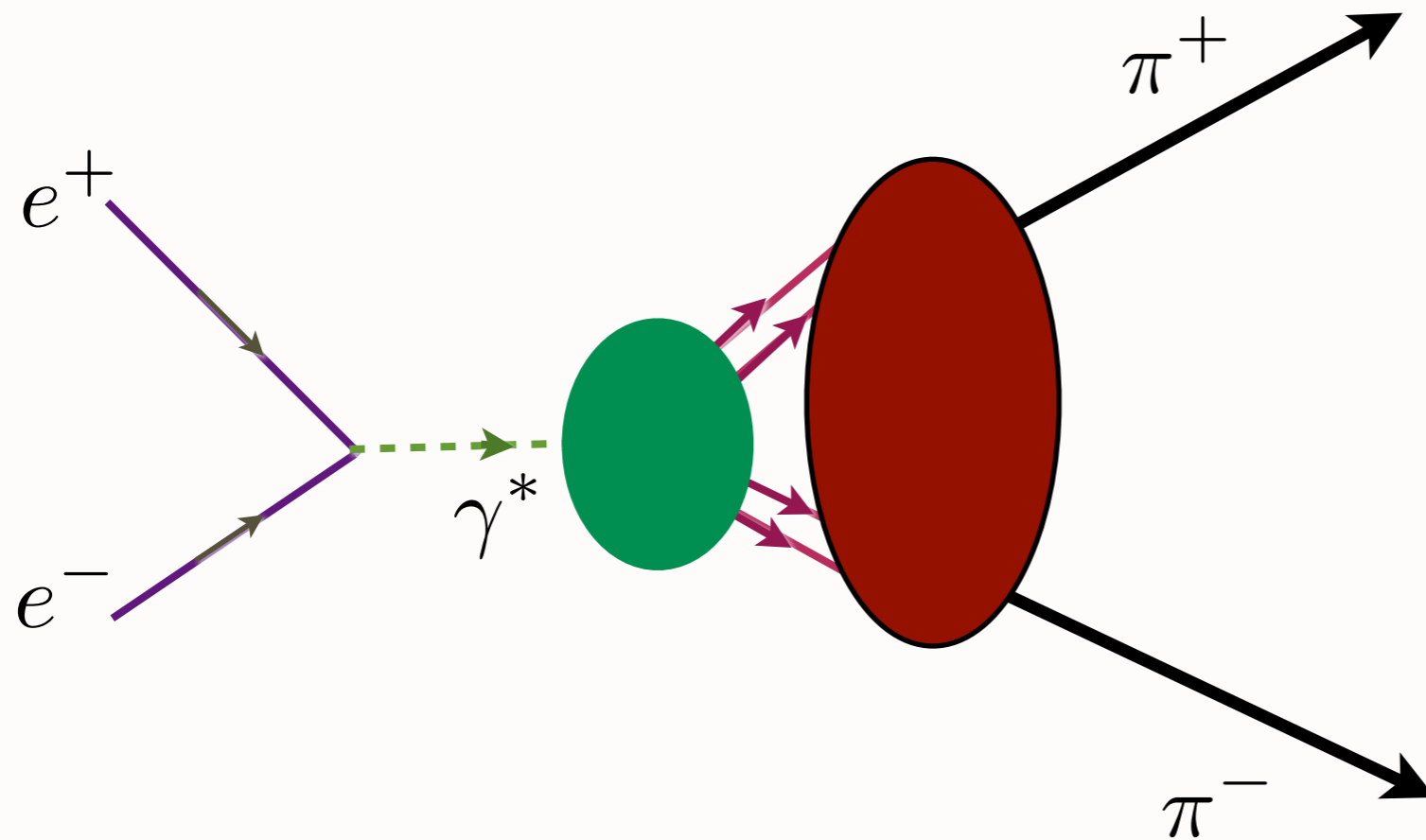
Nucleon Transition Form Factors

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

Dressed soft-wall current brings in higher Fock states and more vector meson poles

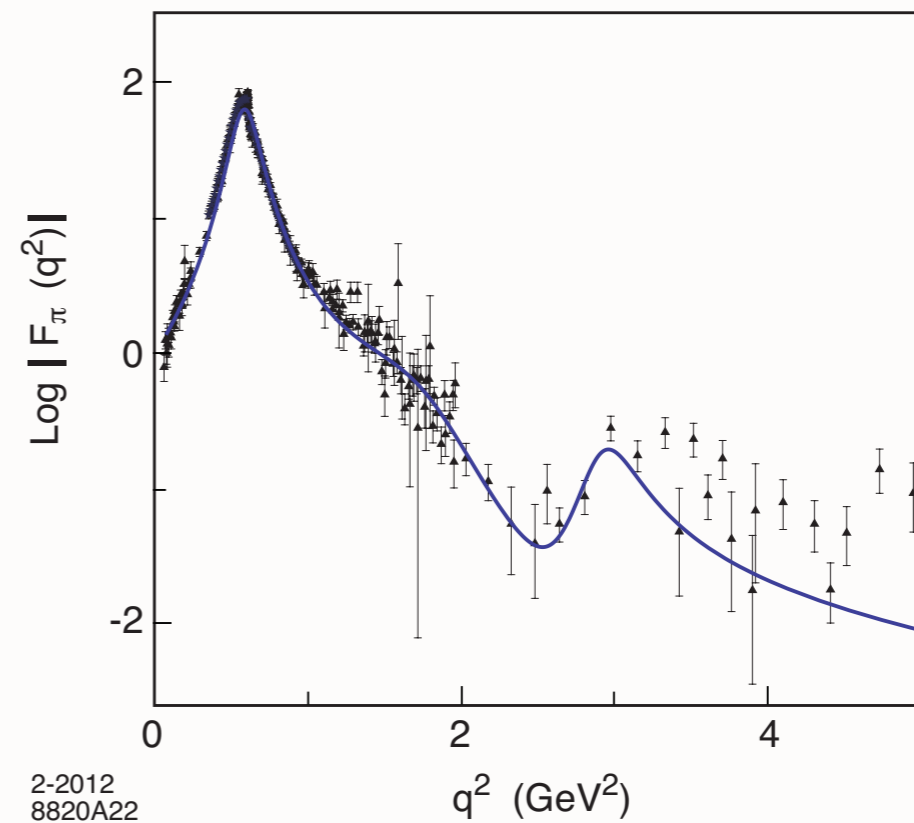
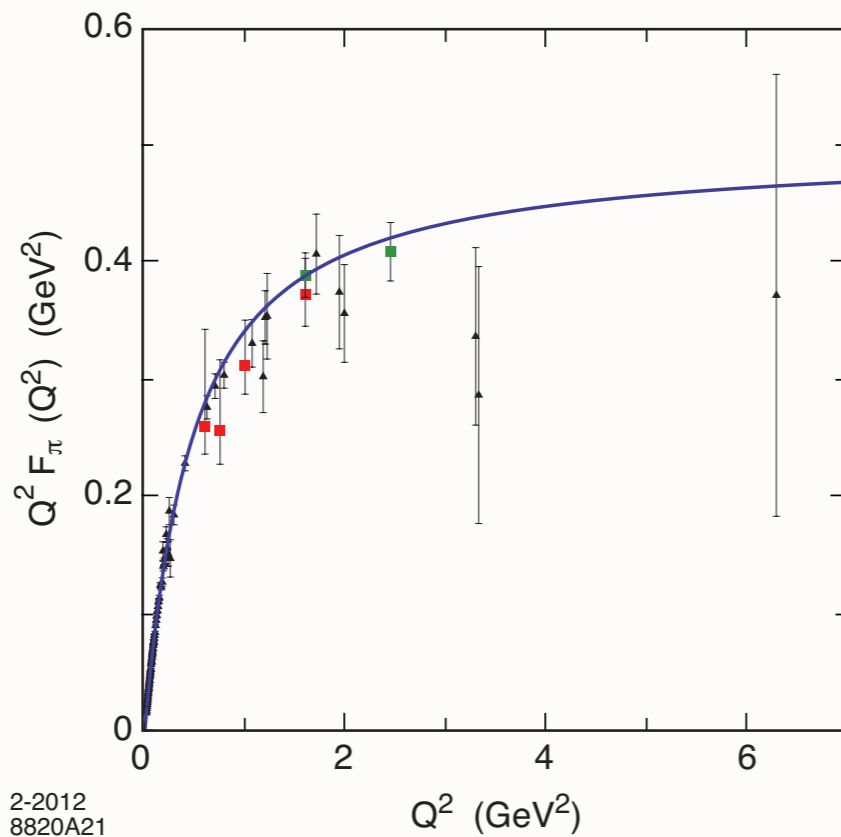


Higher Fock Components in LF Holographic QCD

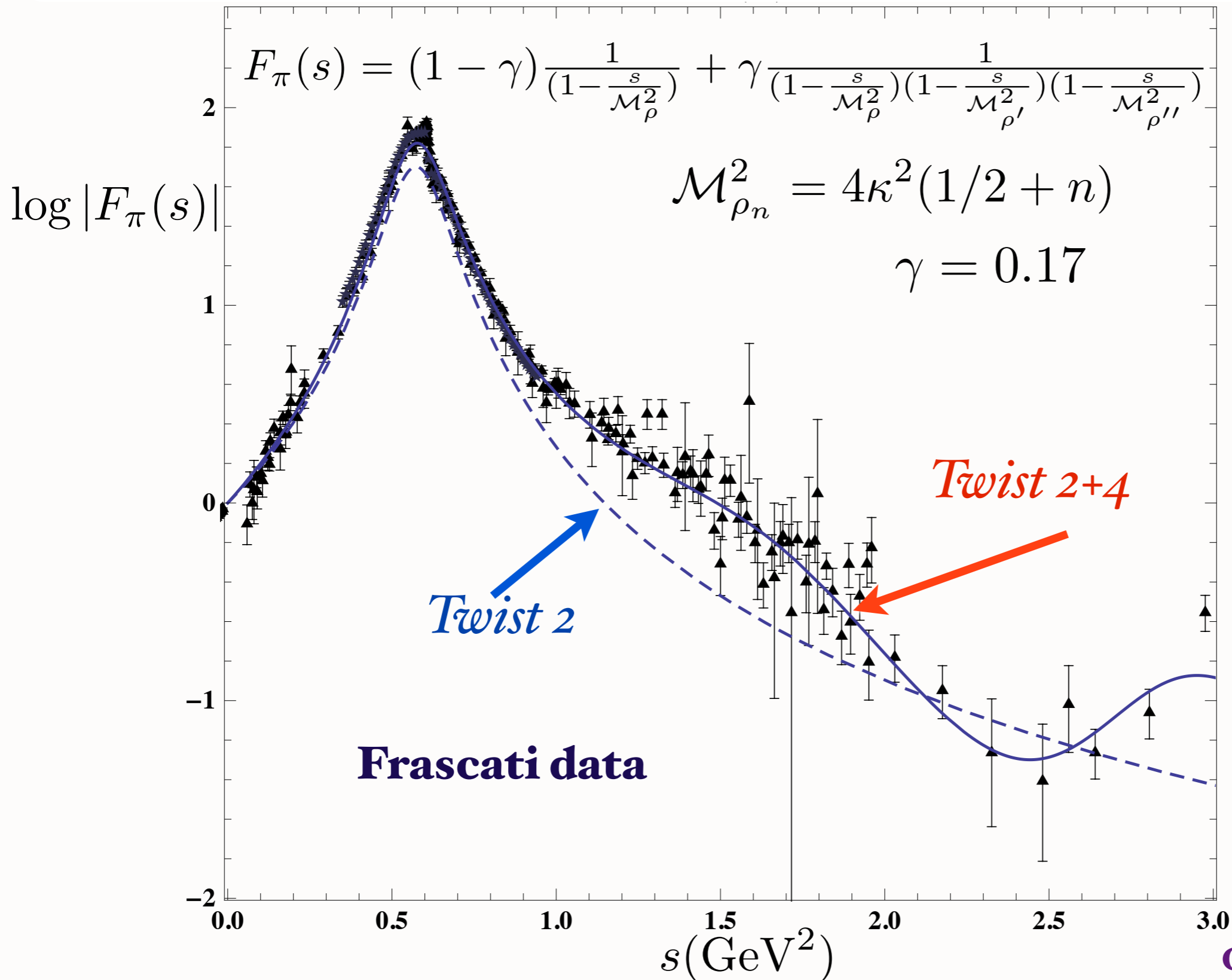
- Effective interaction leads to $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$ but also to $q \rightarrow qq\bar{q}$ and $\bar{q} \rightarrow \bar{q}q\bar{q}$
- Higher Fock states can have any number of extra $q\bar{q}$ pairs, but surprisingly no dynamical gluons
- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \dots$$

- Modify form factor formula introducing finite width: $q^2 \rightarrow q^2 + \sqrt{2}i\mathcal{M}\Gamma$ ($P_{q\bar{q}q\bar{q}} = 13\%$)



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

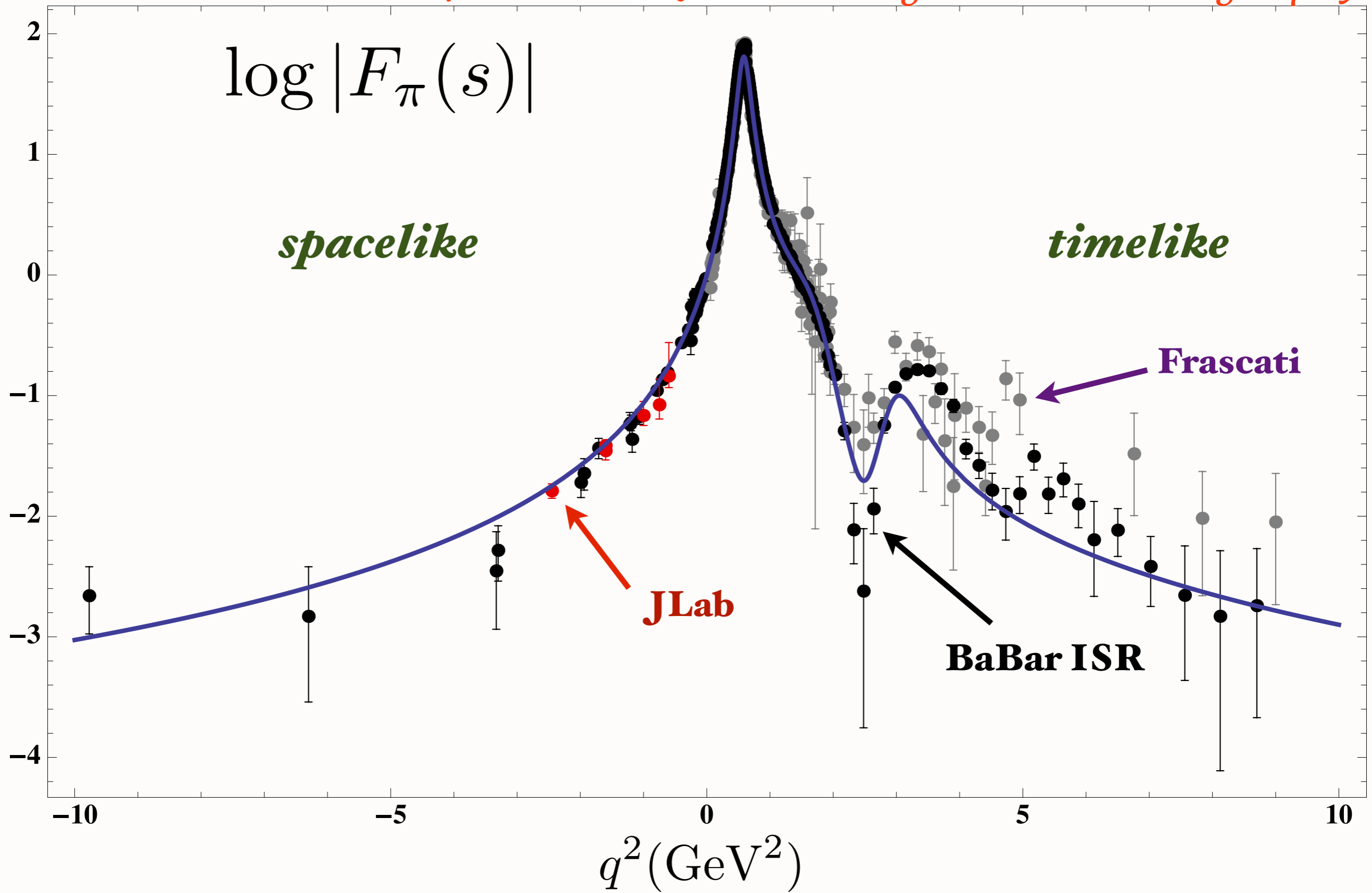


**Prescription for
Timelike poles :**

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

**14% four-quark
probability**

Pion Form Factor from AdS/QCD and Light-Front Holography



Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)}(p_\pi + q - k) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take $A_z \propto \Phi_\pi(z)/z$, $\Phi_\pi(z) = \sqrt{2P_{q\bar{q}}} \kappa z^2 e^{-\kappa^2 z^2/2}$, $\langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar{q}}$

- Find $(\phi(x) = \sqrt{3} f_\pi x(1-x), f_\pi = \sqrt{P_{q\bar{q}}} \kappa / \sqrt{2\pi})$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\bar{q}} Q^2 (1-x) / 4\pi^2 f_\pi^2 x} \right]$$

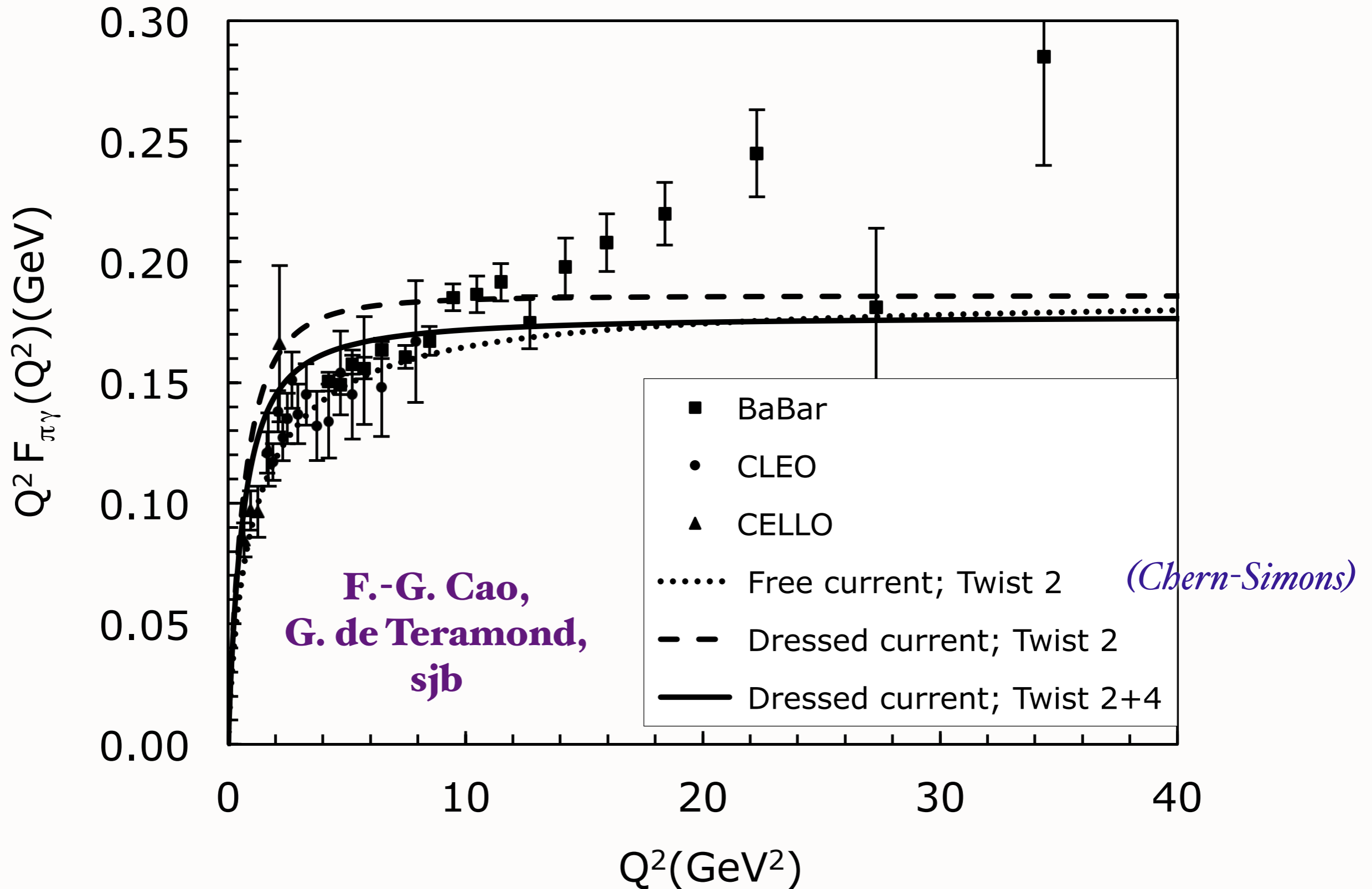
normalized to the asymptotic DA [$P_{q\bar{q}} = 1 \rightarrow$ Musatov and Radyushkin (1997)]

- Large Q^2 TFF is identical to first principles asymptotic QCD result $Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

Photon-to-pion transition form factor

Lepage, sjb

$$Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi.$$



Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

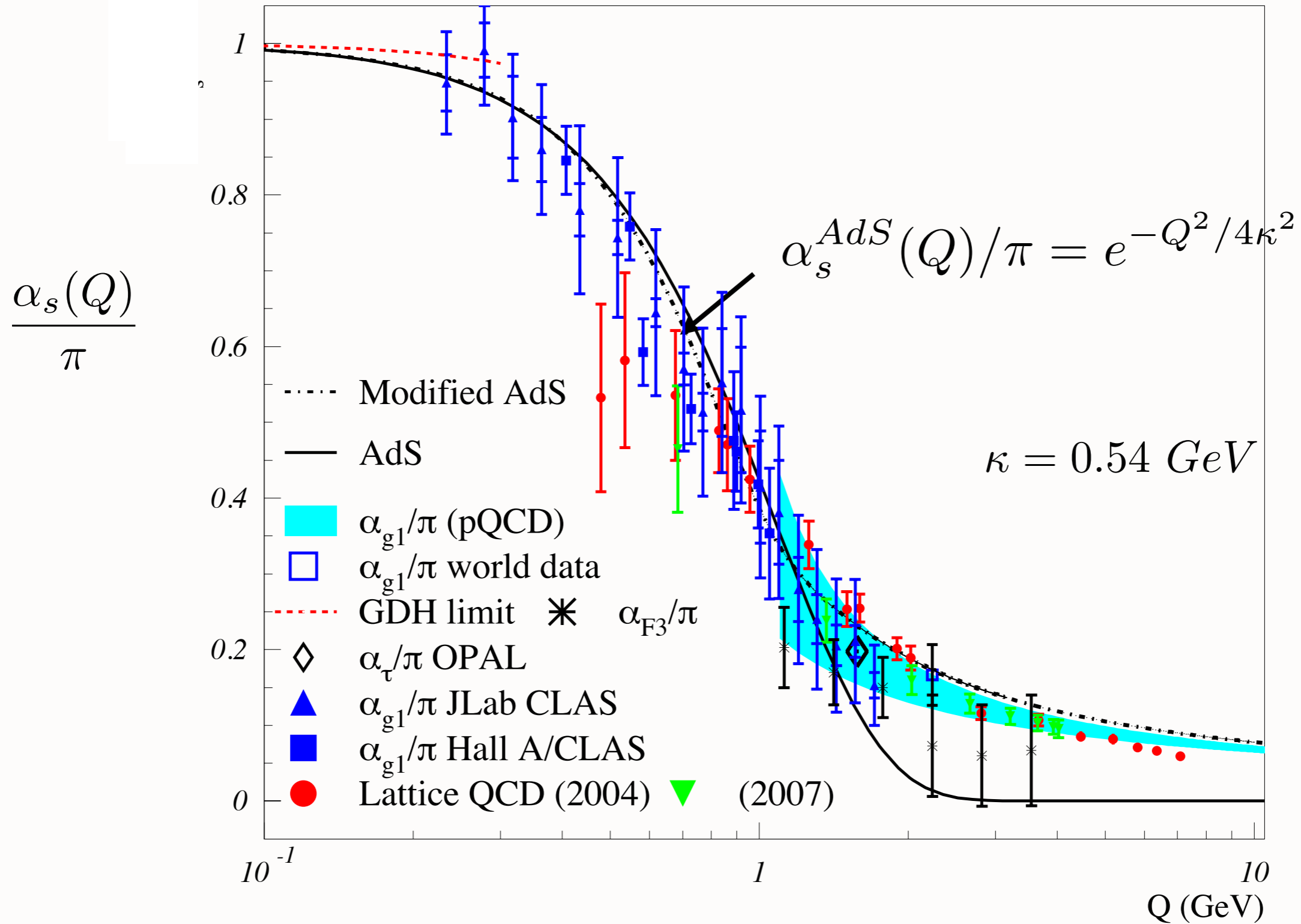
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for $Q < 1 \text{ GeV}$

$$e^\varphi = e^{+\kappa^2 z^2}$$

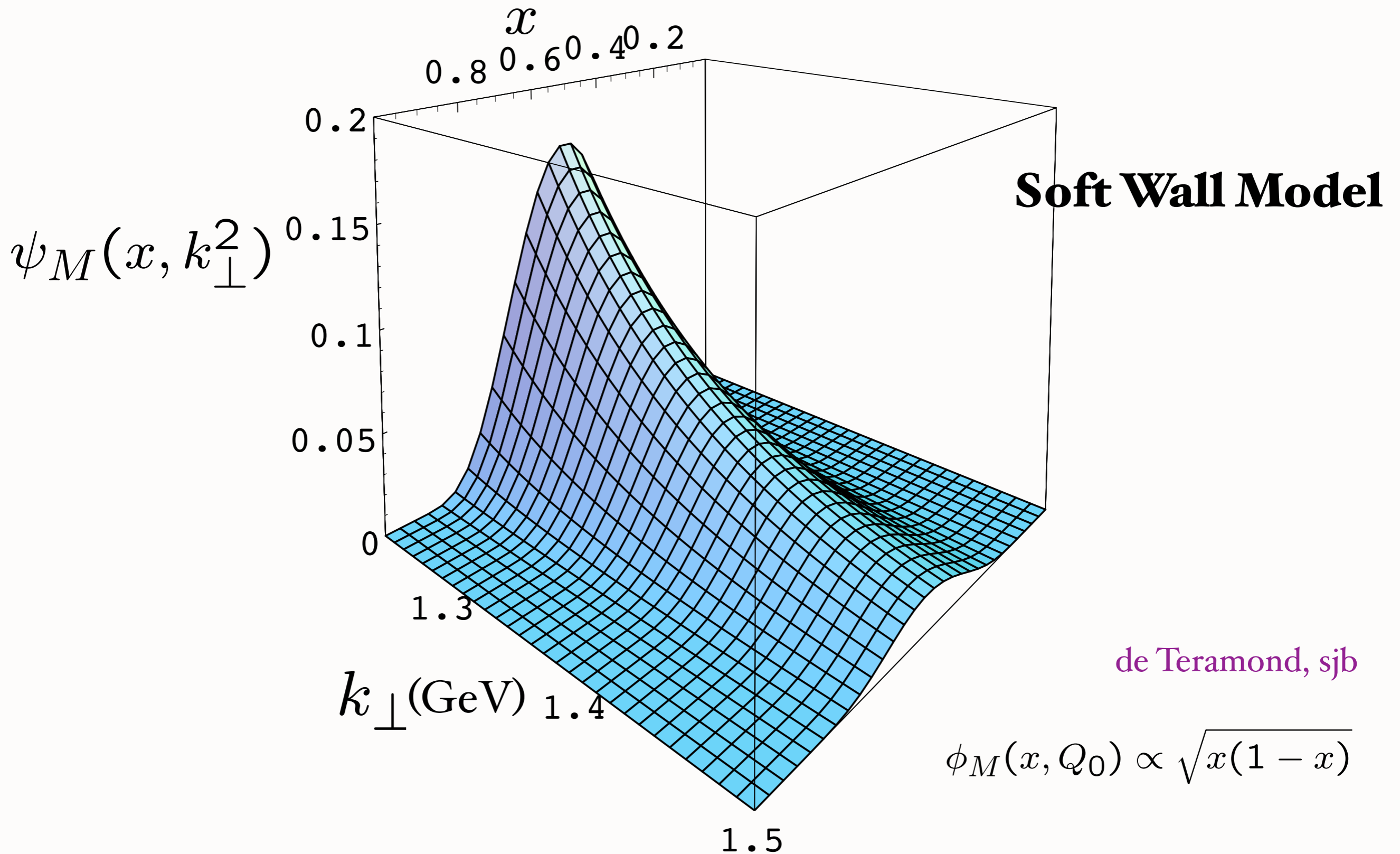
Deur, de Teramond, sjb

Extensions of AdS/QCD LF Holography

- *Massive quarks*
- *Broken Chiral Symmetry*
- *Structure Functions*
- *Counting Rules at $x \sim 1$, Duality*
- *Nucleon GPDs*

Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche, Ivan Schmidt, Alfredo Vega
Ian Cloet, C. D. Roberts
Ruben Sandapen, Jeff Forshaw
Burkardt, Schmidt, Lepage, sjb

Prediction from AdS/CFT: Meson LFWF

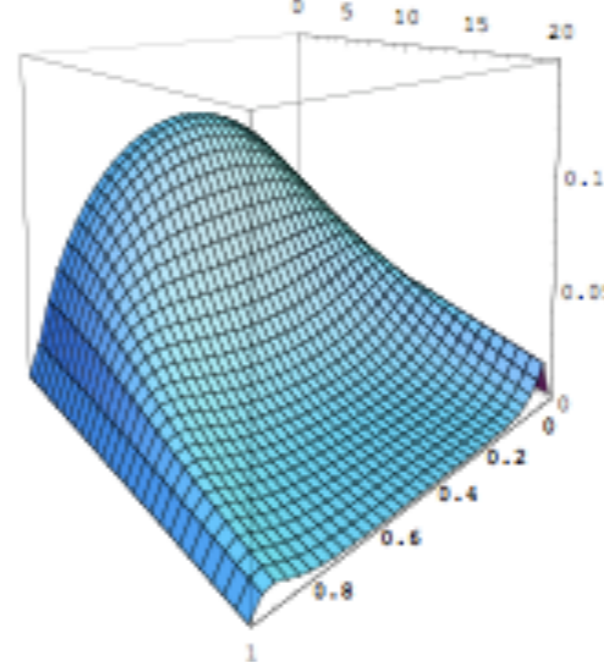
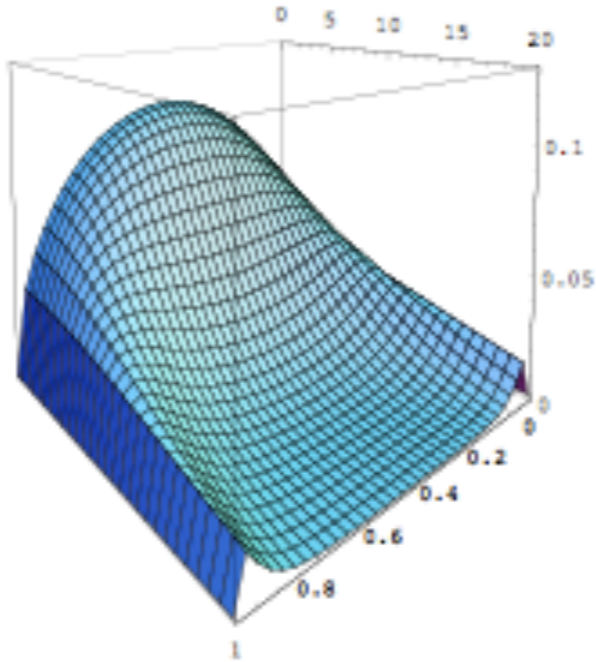


Increases PQCD prediction for $F_{\pi}(Q^2)$ by 16/9

$$|\pi^+\rangle = |u\bar{d}\rangle$$

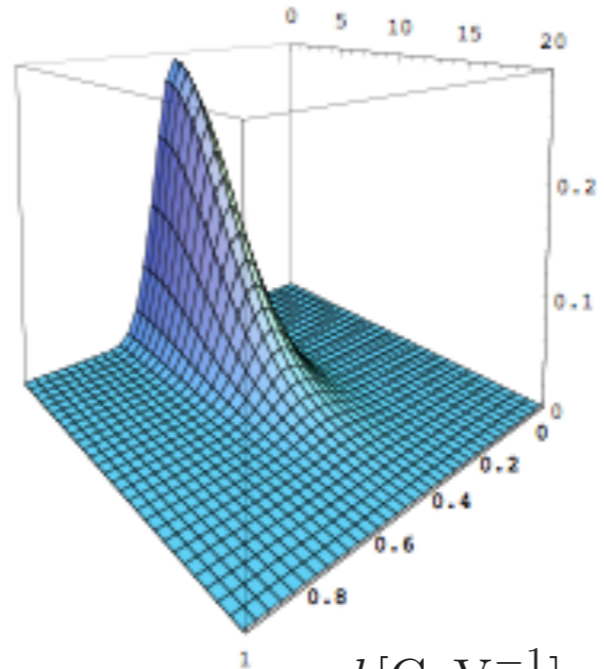
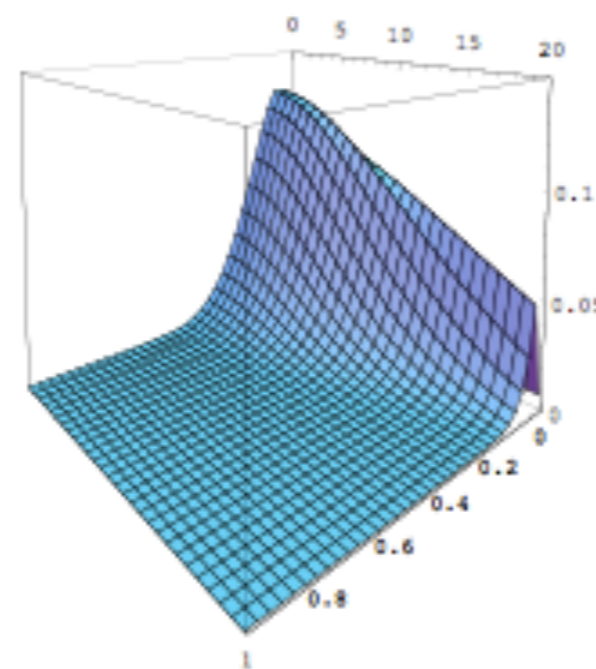
$$m_u = 2 \text{ MeV}$$

$$m_d = 5 \text{ MeV}$$



$$|K^+\rangle = |u\bar{s}\rangle$$

$$m_s = 95 \text{ MeV}$$



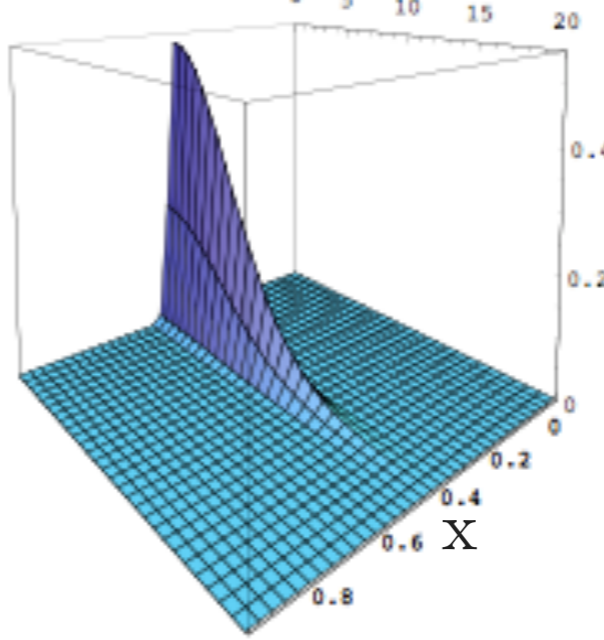
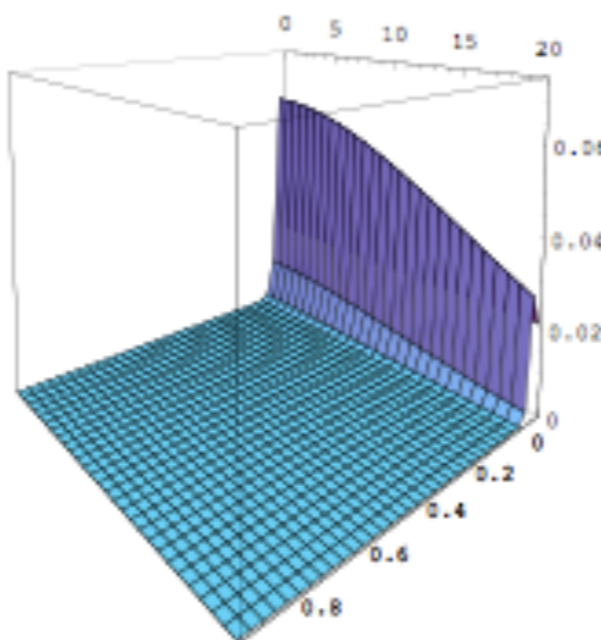
$$|D^+\rangle = |c\bar{d}\rangle$$

$$m_c = 1.25 \text{ GeV}$$

$$|\eta_c\rangle = |c\bar{c}\rangle$$

$$|B^+\rangle = |u\bar{b}\rangle$$

$$m_b = 4.2 \text{ GeV}$$



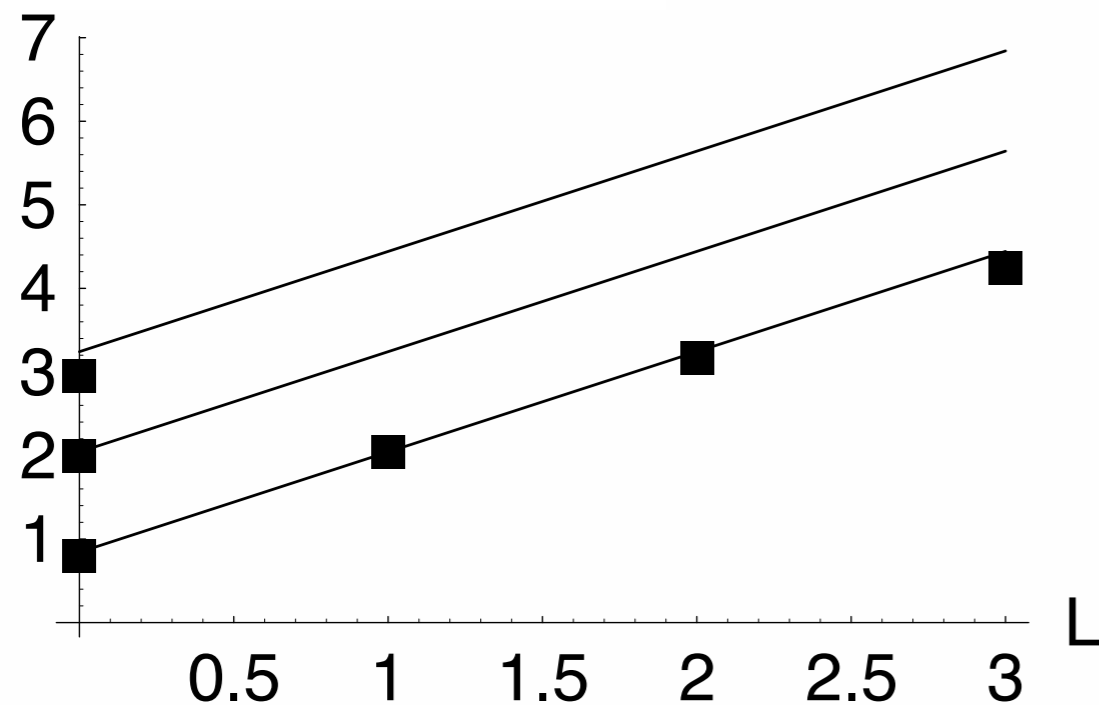
$$|\eta_b\rangle = |b\bar{b}\rangle$$

$$\kappa = 375 \text{ MeV}$$

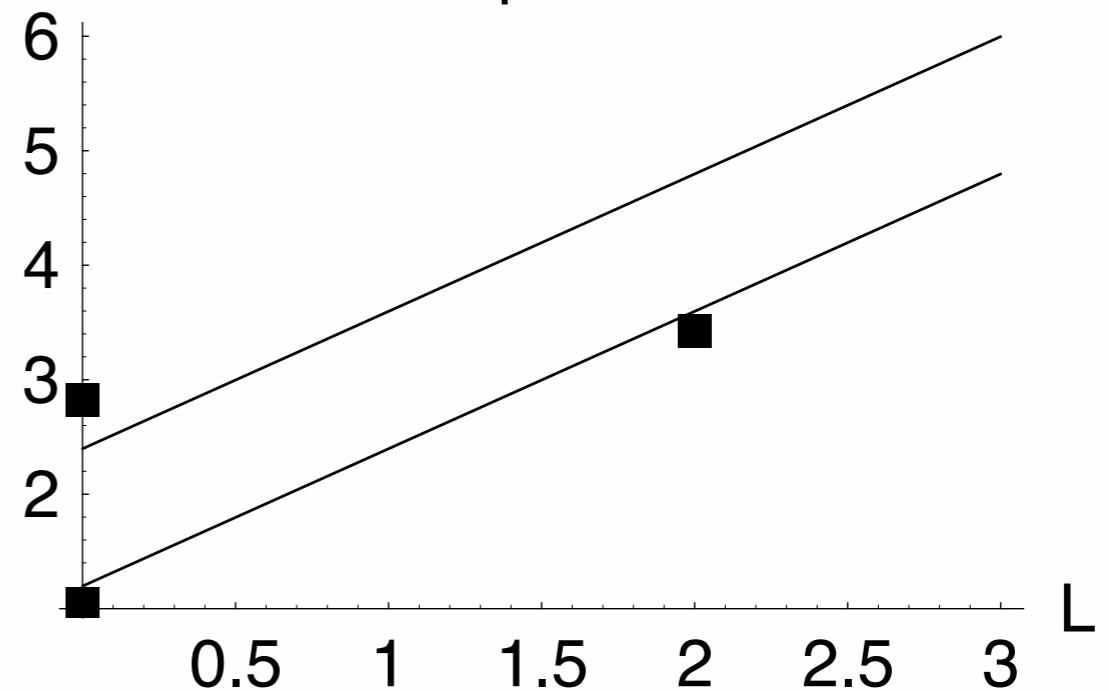
Application to Strange Hadrons

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

$\mathcal{M}^2(\text{GeV}^2)$



$\mathcal{M}^2(\text{GeV}^2)$



$n = 0 : K^*(892), K^*(1410), K^*(1680), K^*(2045)$

$n = 1 : K^*(1430)$

$n = 2 : K^*(1789)$

$n = 0 : \phi(1012), \phi(1850)$

$n = 1 : \phi(1680)$

G. de Teramond, H. G. Dosch, sjb

Preliminary

Decay constants f_P in MeV of pseudoscalar mesons

Meson	Data [13]	Our
π^-	$130.4 \pm 0.03 \pm 0.2$	131
K^-	$156.1 \pm 0.2 \pm 0.8$	155
D^+	206.7 ± 8.9	167
D_s^+	257.5 ± 6.1	170
B^-	193 ± 11	139
B_s^0	$253 \pm 8 \pm 7$	144
B_c	$489 \pm 5 \pm 3$ [14]	159

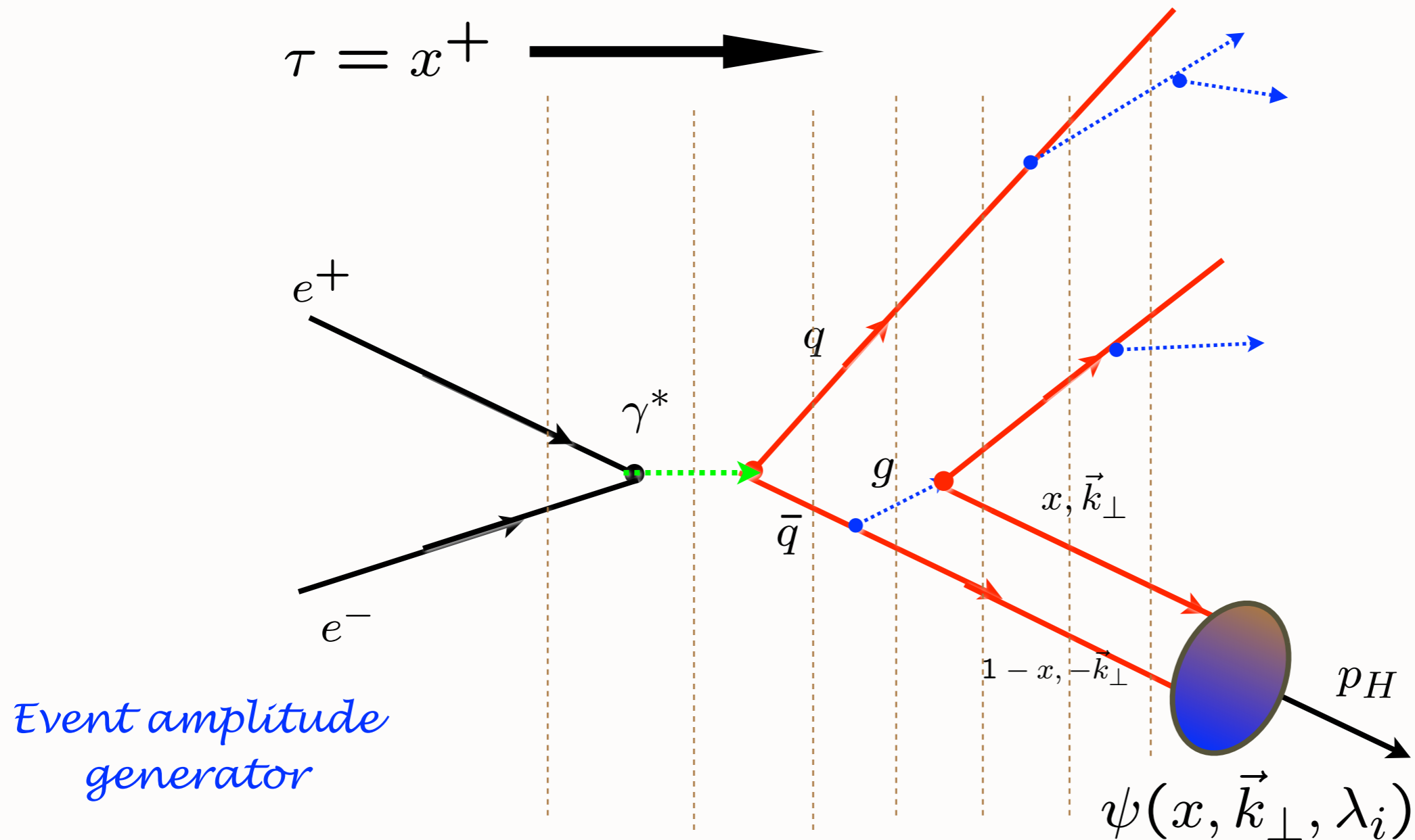
Decay constants f_V in MeV of vector mesons

Meson	Data [13]	Our	Meson	Data [13]	Our
ρ^+	210.5 ± 0.6	170	ρ^0	154.7 ± 0.7	120
D^*	$245 \pm 20_{-2}^{+3}$ [15]	167	ω	45.8 ± 0.8	40
D_s^*	$272 \pm 16_{-20}^{+3}$ [16]	170	ϕ	76 ± 1.2	58
B^*	$196 \pm 24_{-2}^{+39}$ [15]	139	J/ψ	277.6 ± 4	116
B_s^*	$229 \pm 20_{-16}^{+41}$ [15]	144	$\Upsilon(1s)$	238.5 ± 5.5	56

Applications to Collider Physics

- **Non-Perturbative Structure Functions**
- **Fundamental understanding of angular momentum**
- **Higher Fock States: Intrinsic Heavy Quarks**
- **Higgs at High x_F**
- **Hadronization at the Amplitude Level**
- **Direct Higher-Twist Processes: Violation of leading twist scaling**
- **Collisions of Flux-Tubes: Ridge effect in p-p scattering**
- **Multiparton amplitudes: Cluster decomposition, J_z conservation, Parke-Taylor**
- **Multi-gluon initiated processes: Novel nuclear effects**
- **Non-Universal Anti-shadowing**
- **Hadronization from first principles -- at the Amplitude Level**
- **Principle of Maximum Conformality**
- **Connection to Pomeron**

Hadronization at the Amplitude Level

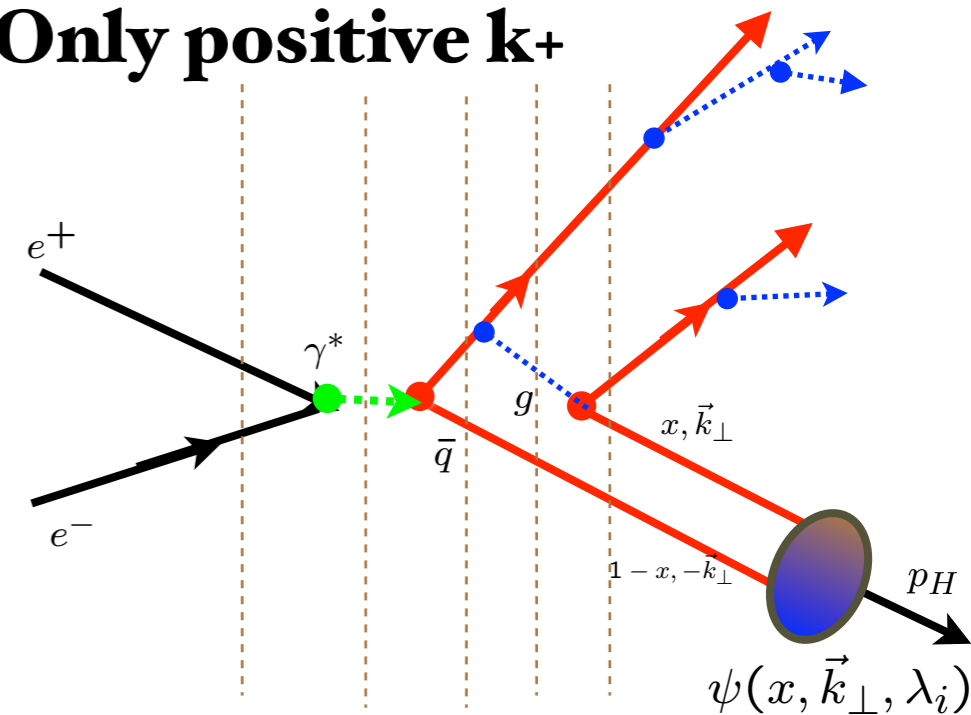


Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Off-Shell T-Matrix

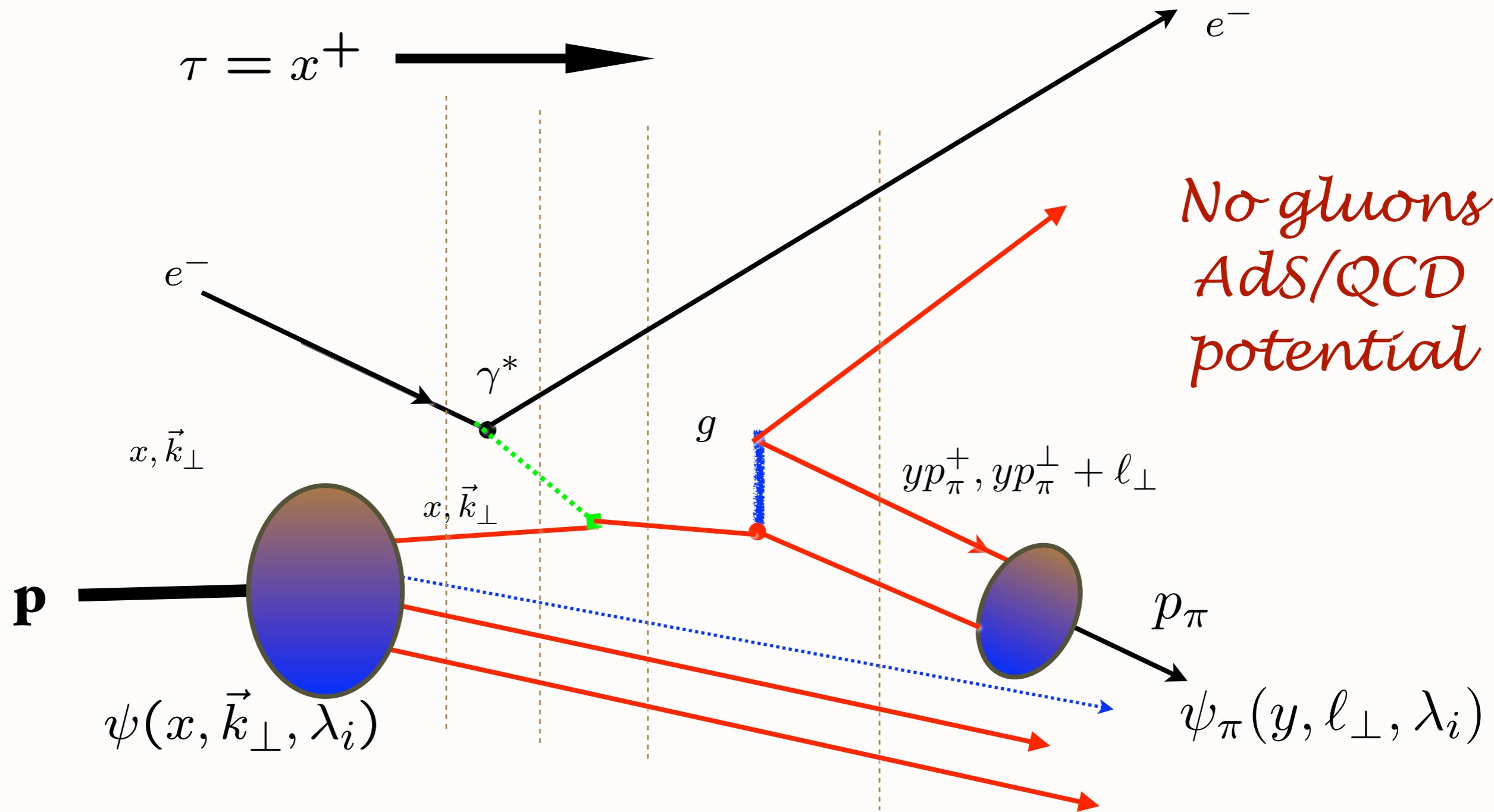
Event amplitude generator

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k_+**
- **J^z Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** Chueng Ji, sjb
- **“History”-Numerator structure universal**
- **Renormalization- alternate denominators**
- **LFWF takes Off-shell to On-shell**
- **Tested in QED: $g-2$ to three loops**



Roskies, Suaya, sjb

Hadronization at the Amplitude Level



**Construct helicity amplitude using Light-Front Perturbation theory;
coalesce quarks via LFWFs**

Only Hadrons can Appear!

BLFQ

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- Hamiltonian light-front field theory within an AdS/QCD basis.
J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
G.F. de Teramond, P. Sternberg, X. Zhao, E.G. Ng, C. Yang, sjb

AdS/QCD and Light-Front Holography

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

- **Zero mass pion for $m_q = 0$ ($n=J=L=0$)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2 : Dimensional counting**
 $[Q^2]^{n-1} F(Q^2) \rightarrow \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

String Theory

- Conformal template:
- Use isometries of AdS₅

Goal: First Approximant to QCD

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS₅ space

Counting rules for Hard Exclusive Scattering
Regge Trajectories

AdS/QCD

Conformal behavior at short distances

QCD at the Amplitude Level

Confinement at large distance
Unique!

Semi-Classical QCD / Wave Equations

Holography

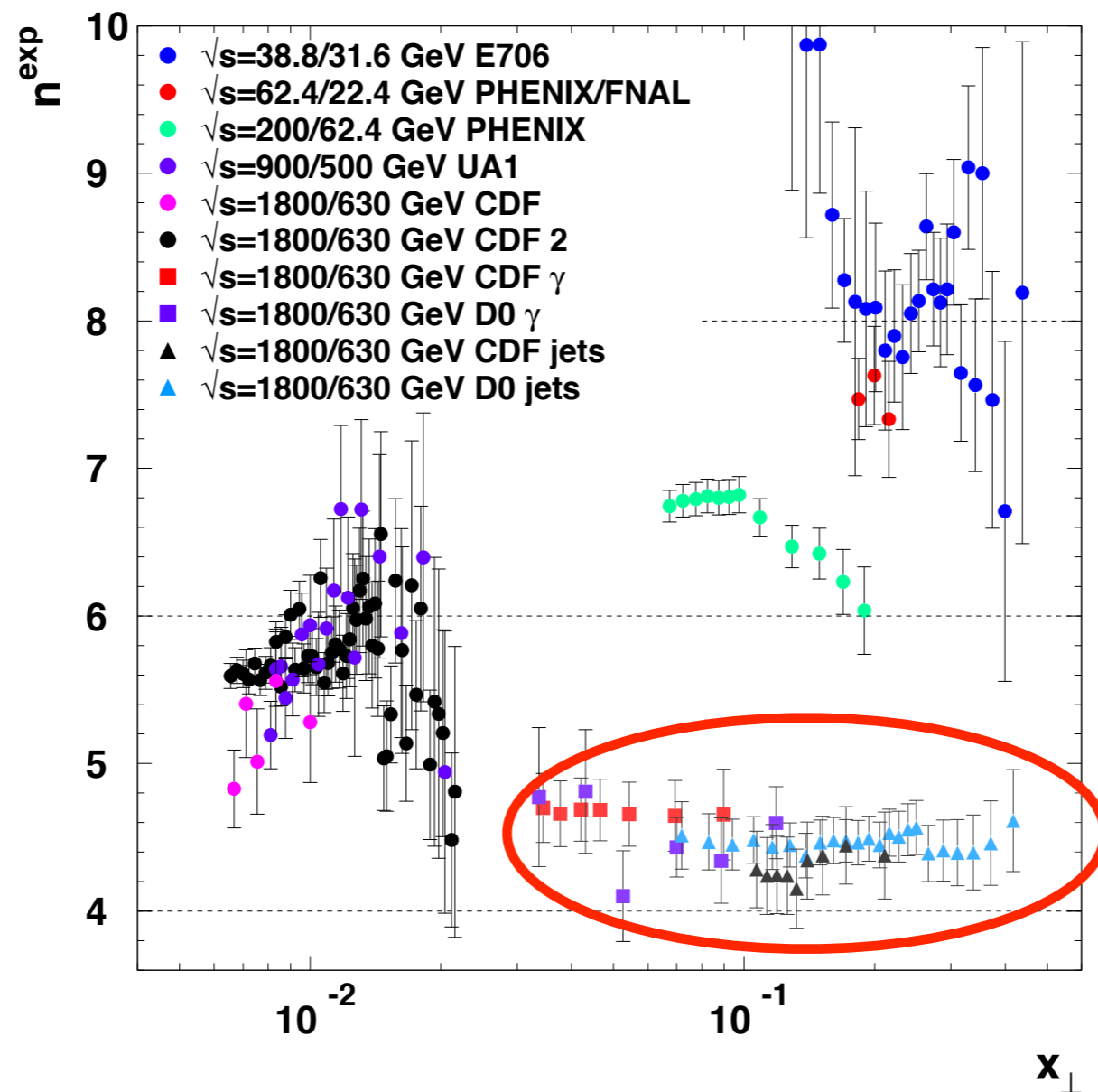
Boost Invariant 3+1 Light-Front Wave Equations

$J=0, 1, 1/2, 3/2$ plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

$$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$



Photons and Jets
agree with PQCD
 x_T scaling
Hadrons do not!

Arleo, Hwang, Sickles, sjb

- Significant increase of the hadron n^{exp} with x_{\perp}
 - $n^{\text{exp}} \simeq 8$ at large x_{\perp}
- Huge contrast with photons and jets !
 - n^{exp} constant and slight above 4 at all x_{\perp}

Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles; sjb

*Small color-singlet
Color Transparent
Minimal same-side energy*

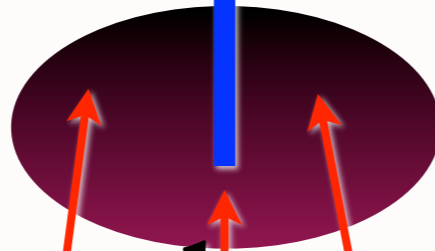
*Explains
Baryon
anomaly*

$qq \rightarrow B\bar{q}$

p

$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$



u

g

d

g

u

$$n_{\text{active}} = 6$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

d

*Collision can produce 3
collinear quarks*

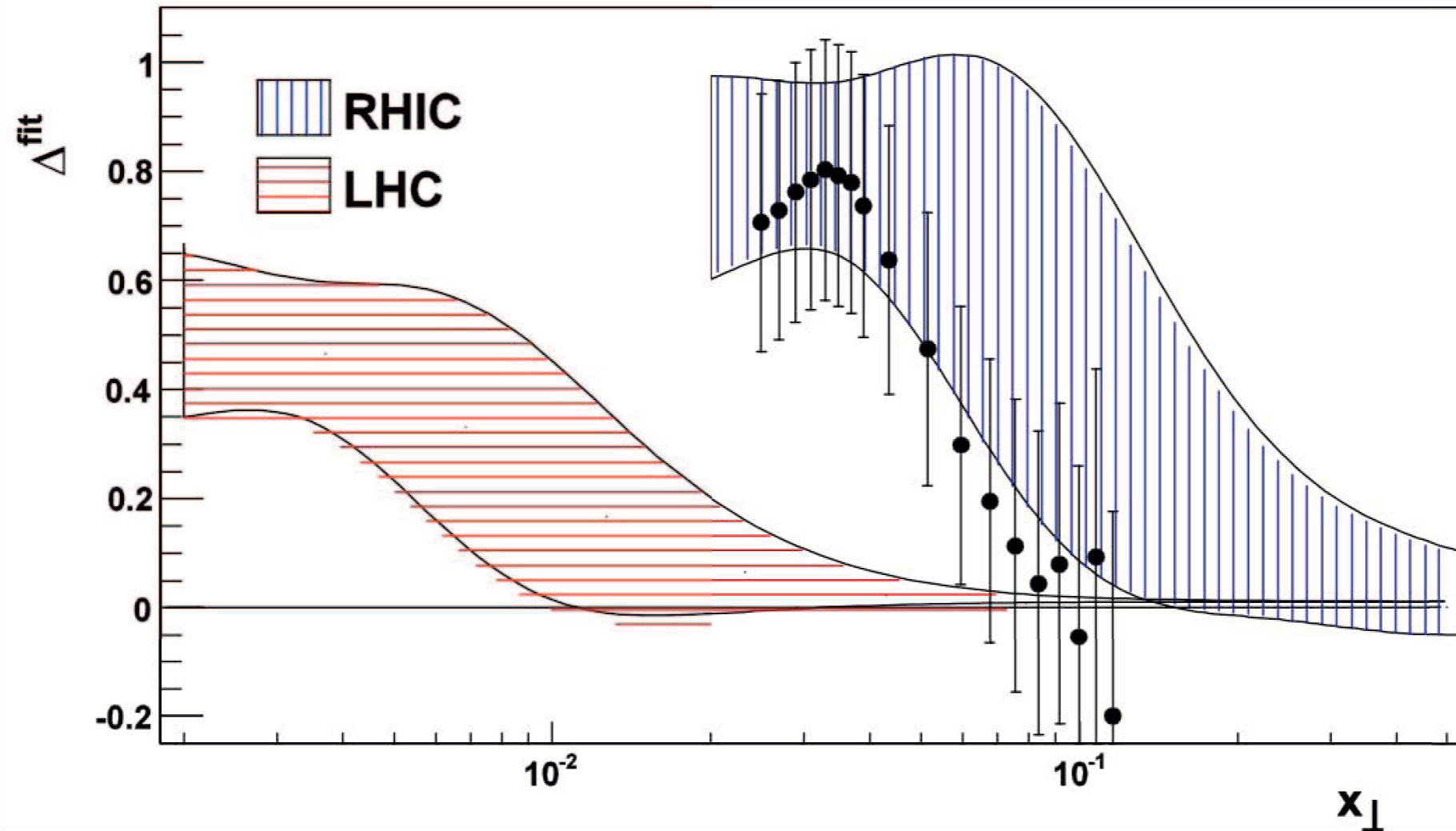
[The Nucleus as a Color Filter in {QCD} Decays:
Hadroproduction in Nuclei.](#)

By Stanley J. Brodsky, Paul Hoyer.
Phys.Rev.Lett. 63 (1989) 1566.

PHENIX results

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

[A. Bezilevsky, APS Meeting

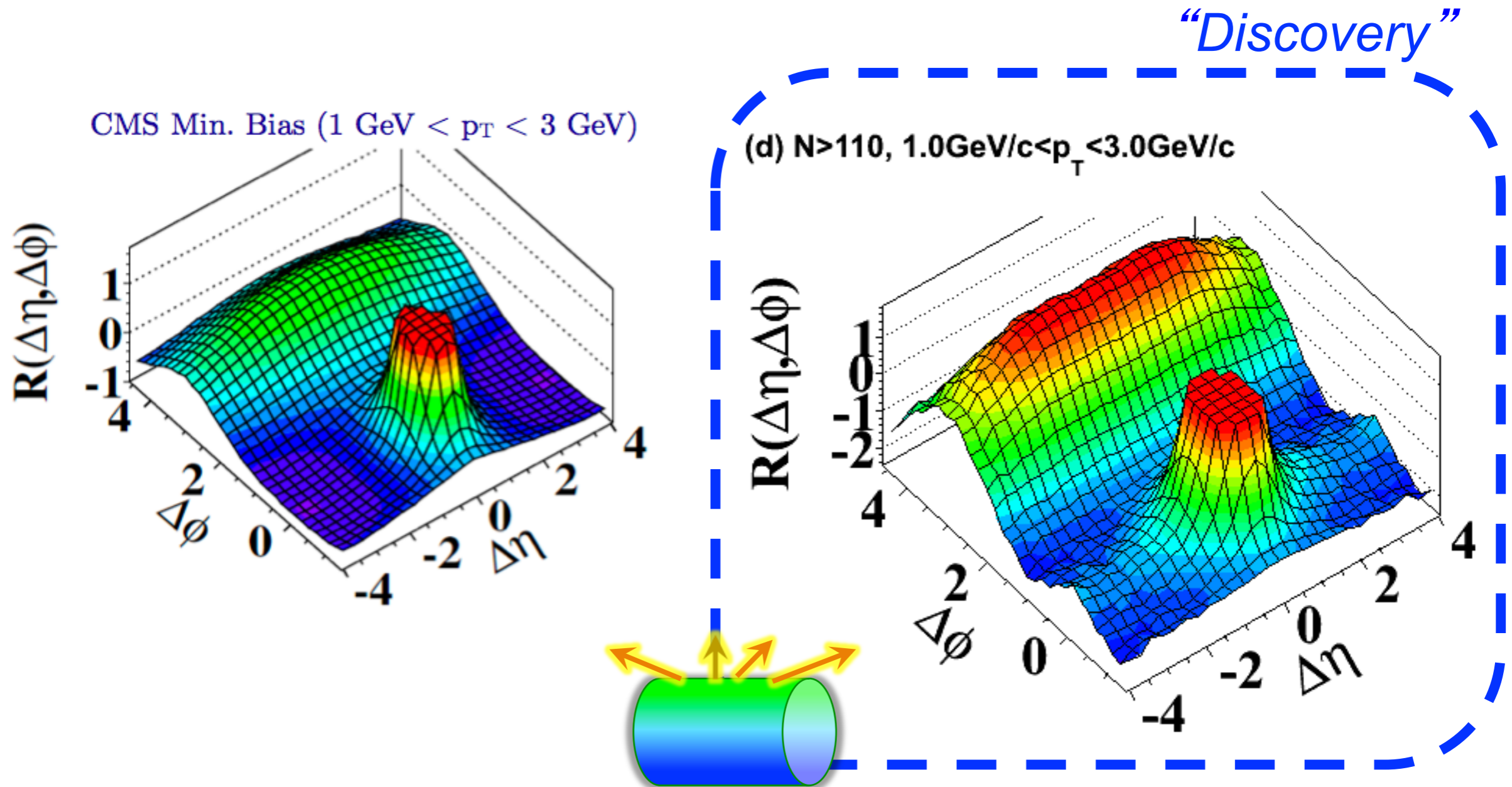


- Magnitude of Δ and its x_{\perp} -dependence consistent with predictions

$$\Delta = n_{\text{expt}} - n_{PQCD}$$

Arleo, Hwang, Sickles, sjb

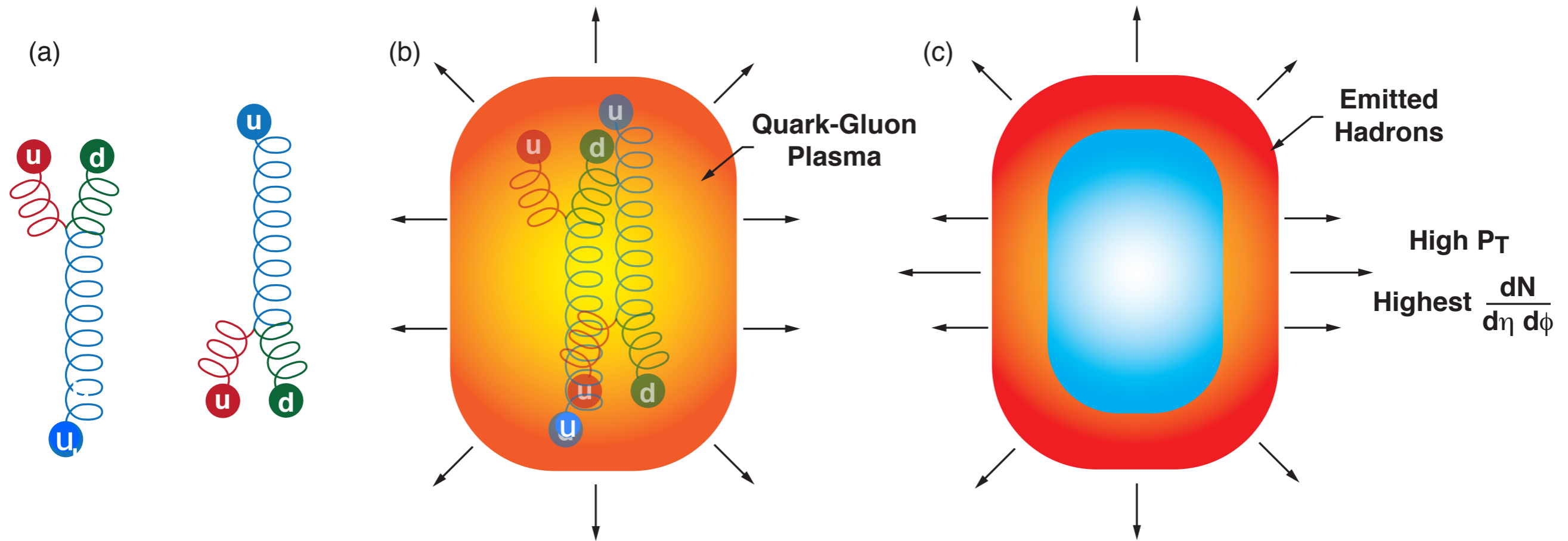
Two particle correlations: CMS results



- ◆ Ridge: Distinct long range correlation in η collimated around $\Delta\phi \approx 0$ for two hadrons in the intermediate $1 < p_T, q_T < 3 \text{ GeV}$

Possible origin of same-side CMS ridge in p p Collisions

Bjorken, Goldhaber, sjb



$$\vec{V} = \sum_{i=1}^N [\cos 2\phi_i \hat{x} + \sin 2\phi_i \hat{y}]$$

Possible multiparticle ridge-like correlations in very high multiplicity proton-proton collisions

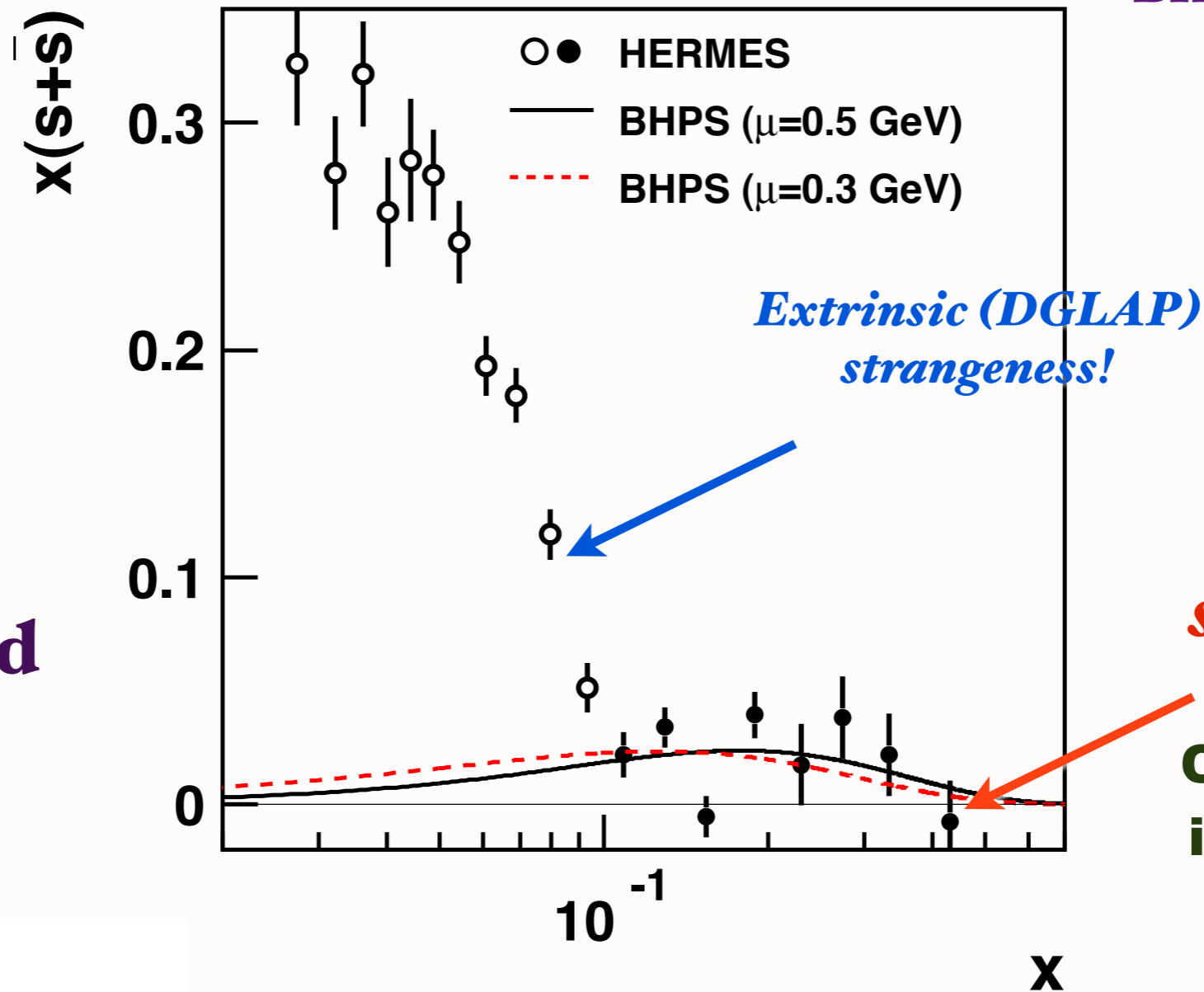
Bjorken, Goldhaber, sjb

We suggest that this “ridge”-like correlation may be a reflection of the rare events generated by the collision of aligned flux tubes connecting the valence quarks in the wave functions of the colliding protons.

The “spray” of particles resulting from the approximate line source produced in such inelastic collisions then gives rise to events with a strong correlation between particles produced over a large range of both positive and negative rapidity.

HERMES: Two components to $s(x, Q^2)$!

BHPS: Hoyer, Sakai,
Peterson, sjb



*Intrinsic
strangeness!*

**Consistent with
intrinsic charm
data**

QCD: $\frac{1}{M_Q^2}$ scaling

Comparison of the HERMES $x(s(x) + \bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5 \text{ GeV}^2$ using $\mu = 0.5 \text{ GeV}$ and $\mu = 0.3 \text{ GeV}$, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

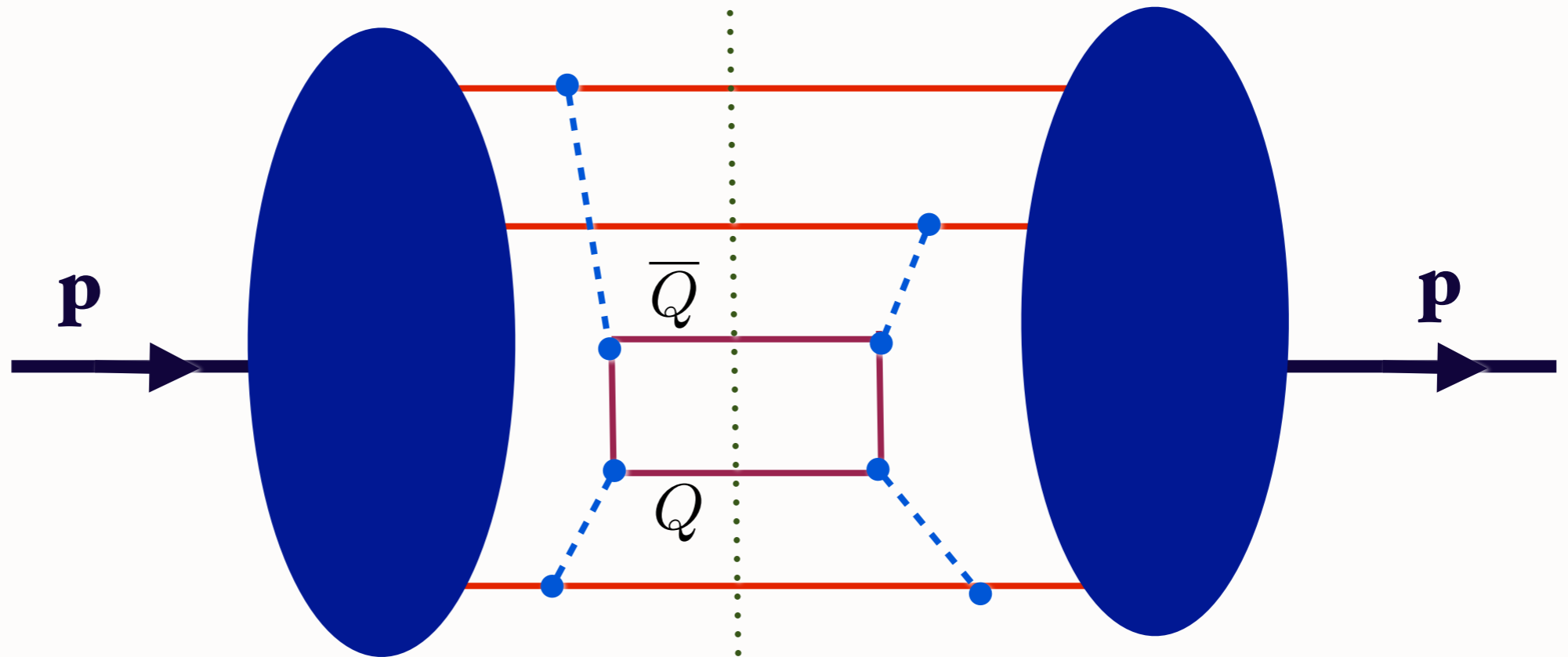
$$s(x, Q^2) = s(x, Q^2)_{\text{extrinsic}} + s(x, Q^2)_{\text{intrinsic}}$$

W. C. Chang and
J.-C. Peng
arXiv:1105.2381

Fixed LF time

*Proton Self Energy
Intrinsic Heavy Quarks*

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

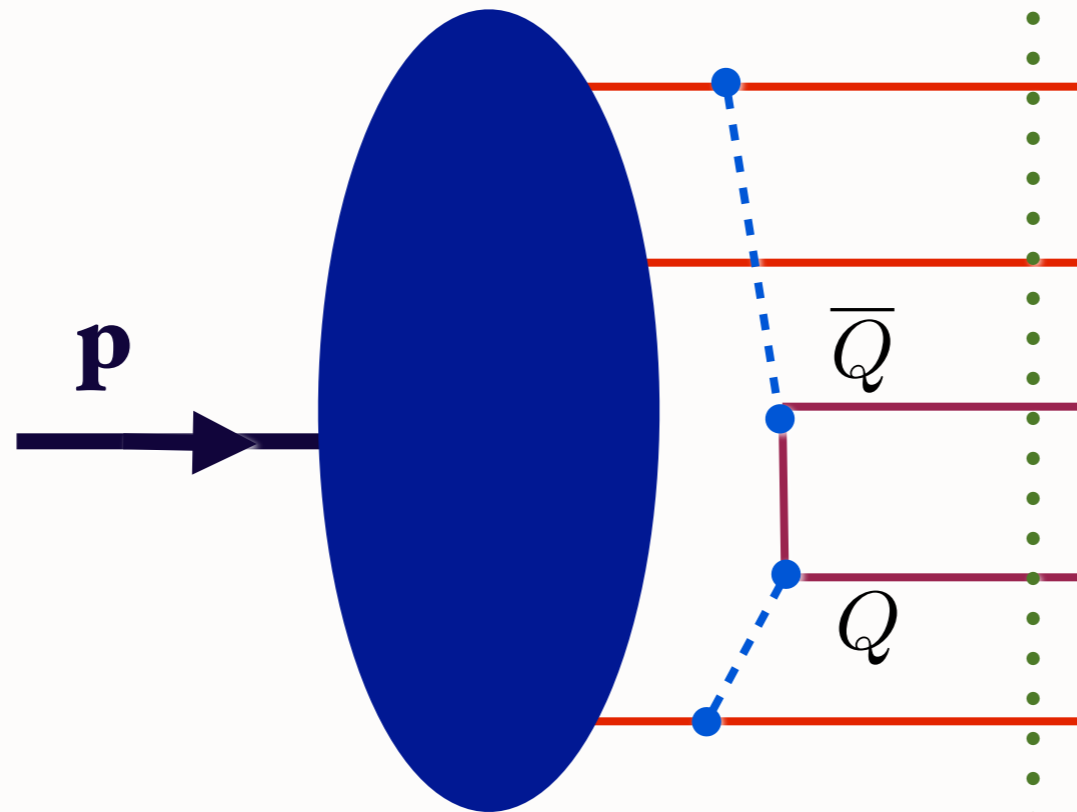


$$\text{Probability (QED)} \propto \frac{1}{M_{\ell}^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

**Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.**

*Proton 5-quark Fock State:
Intrinsic Heavy Quarks*



*QCD predicts
Intrinsic Heavy
Quarks at high x !*

Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

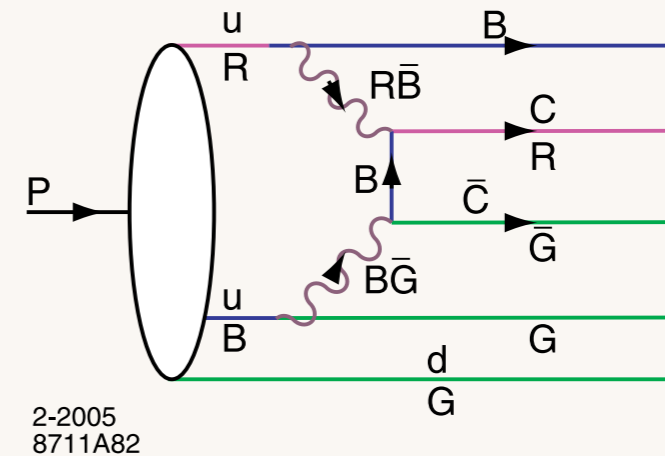
Probability (QED) $\propto \frac{1}{M_{\ell}^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

**Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.**

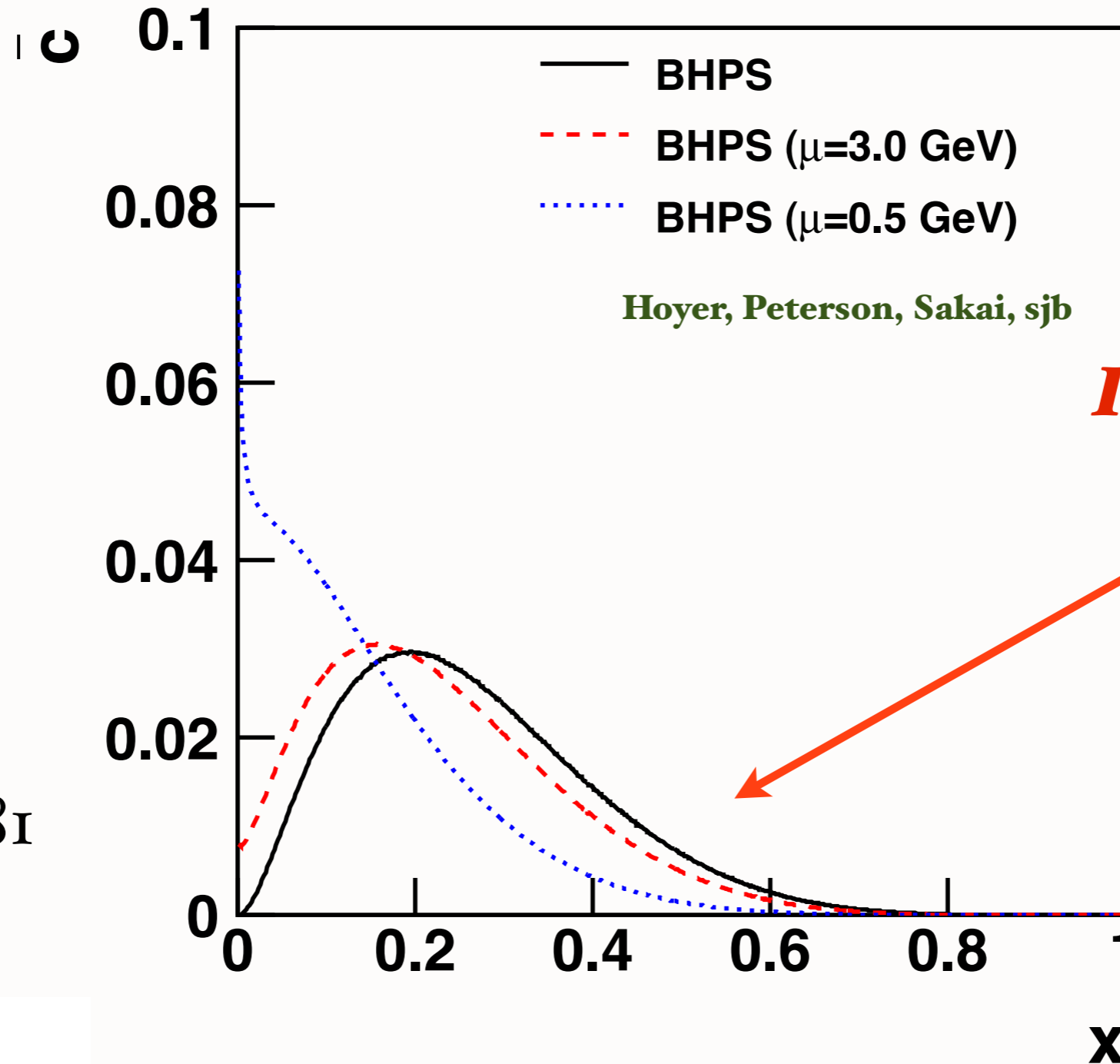
Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!



- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Cannot use $c(x, Q^2)$ to determine $g(x, Q^2)$

QCD ($1/m_Q^2$) scaling: predict IC !



W. C. Chang and
J.-C. Peng

arXiv:1105.2381

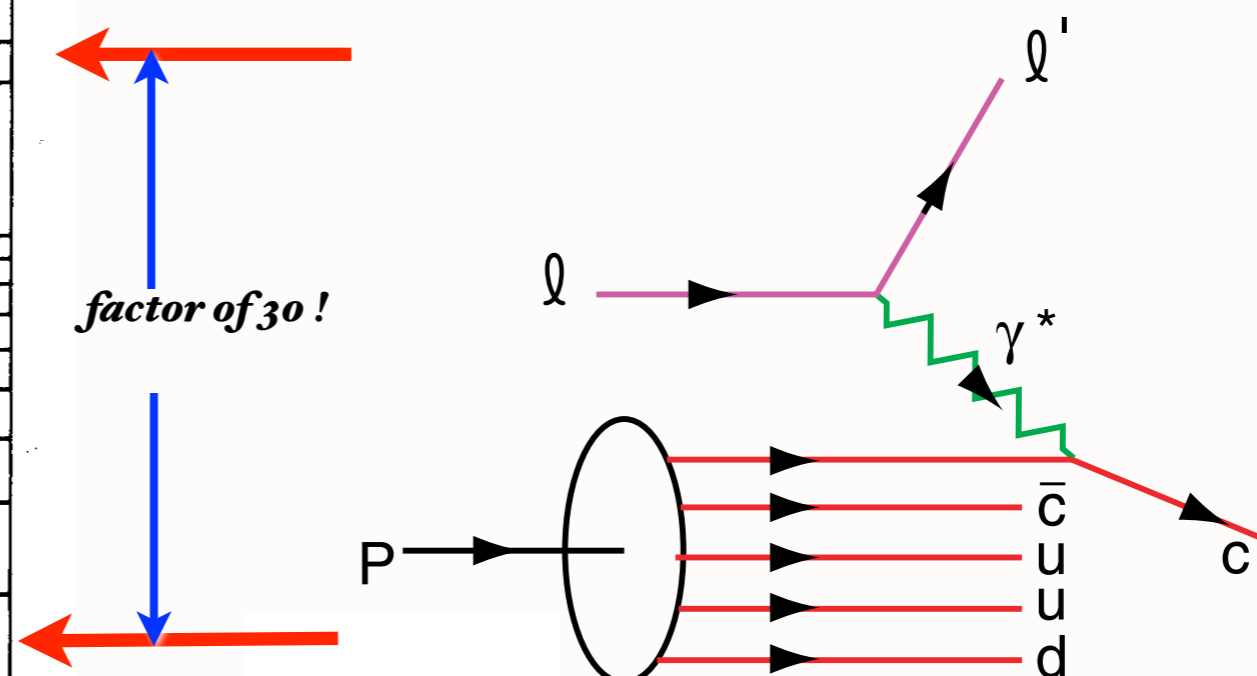
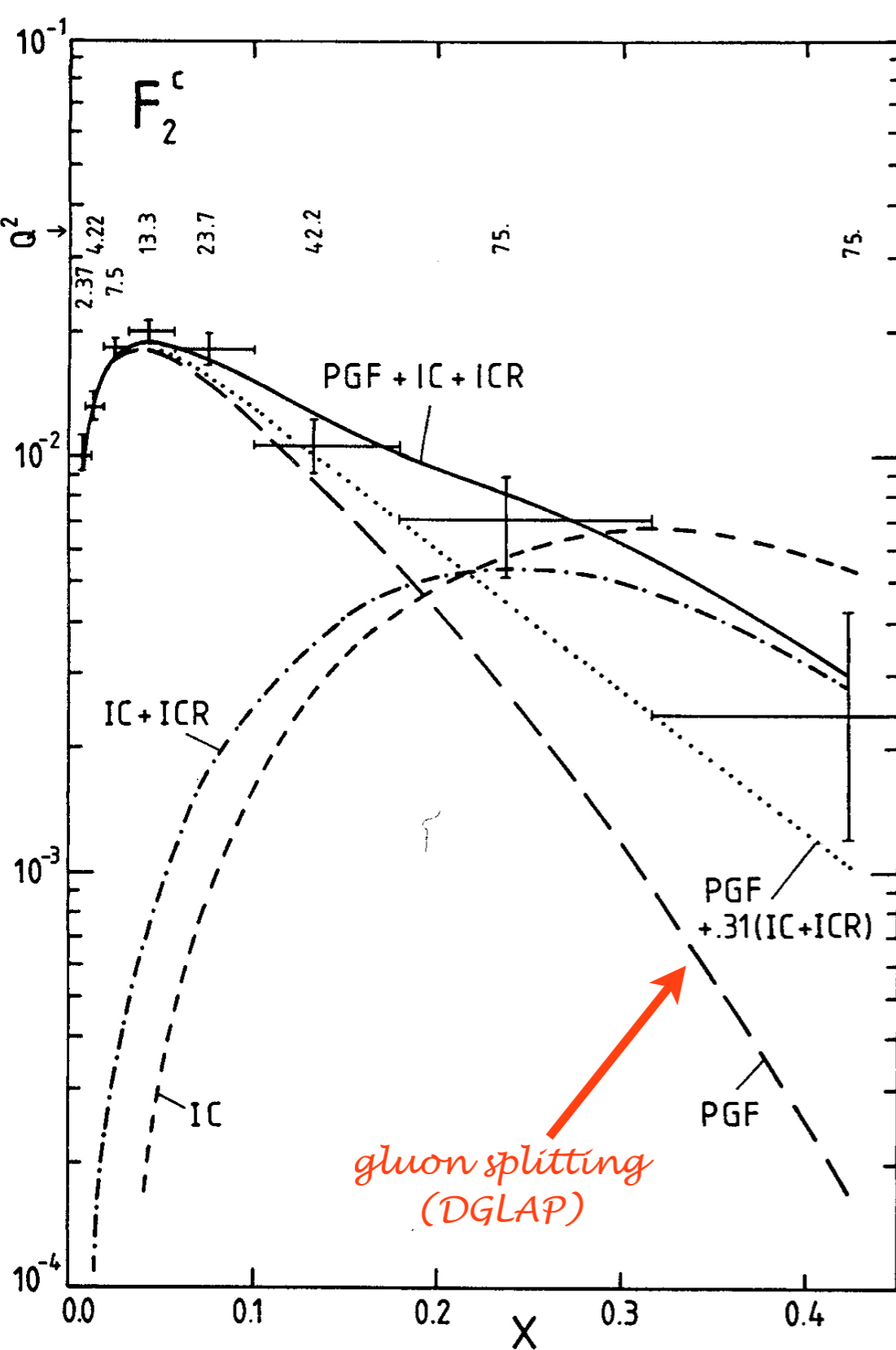
Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75 \text{ GeV}^2$ using $\mu = 3.0 \text{ GeV}$, and $\mu = 0.5 \text{ GeV}$, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

Consistent with EMC

Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm Hoyer, Peterson, Sakai, sjb



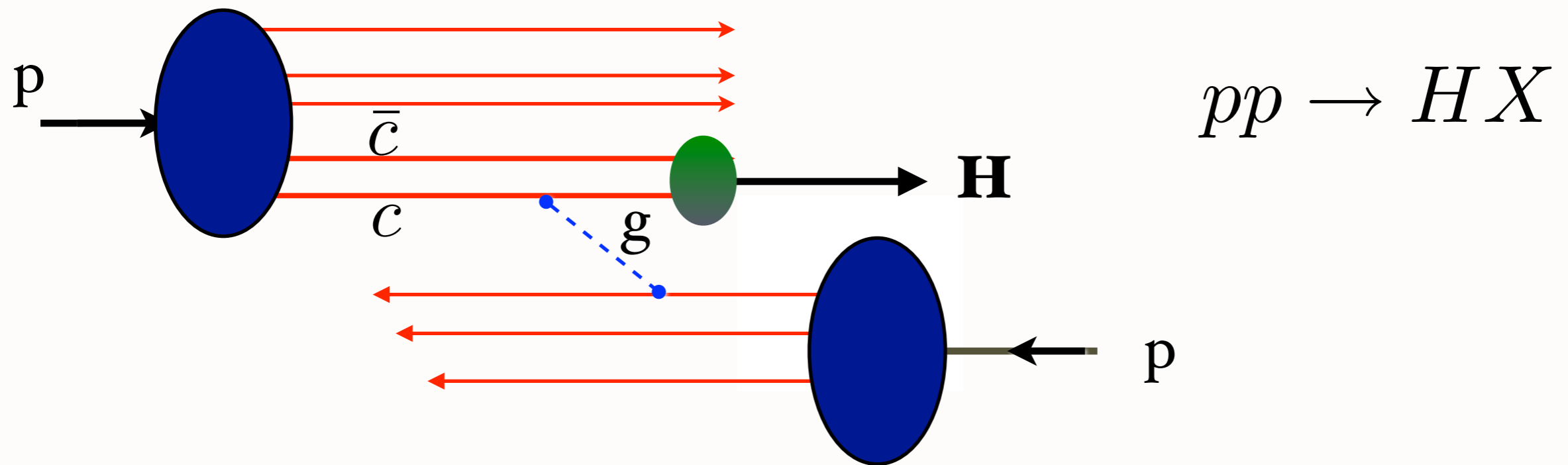
factor of 30!

DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

*Intrinsic Charm Mechanism for Inclusive
High- x_F Higgs Production*



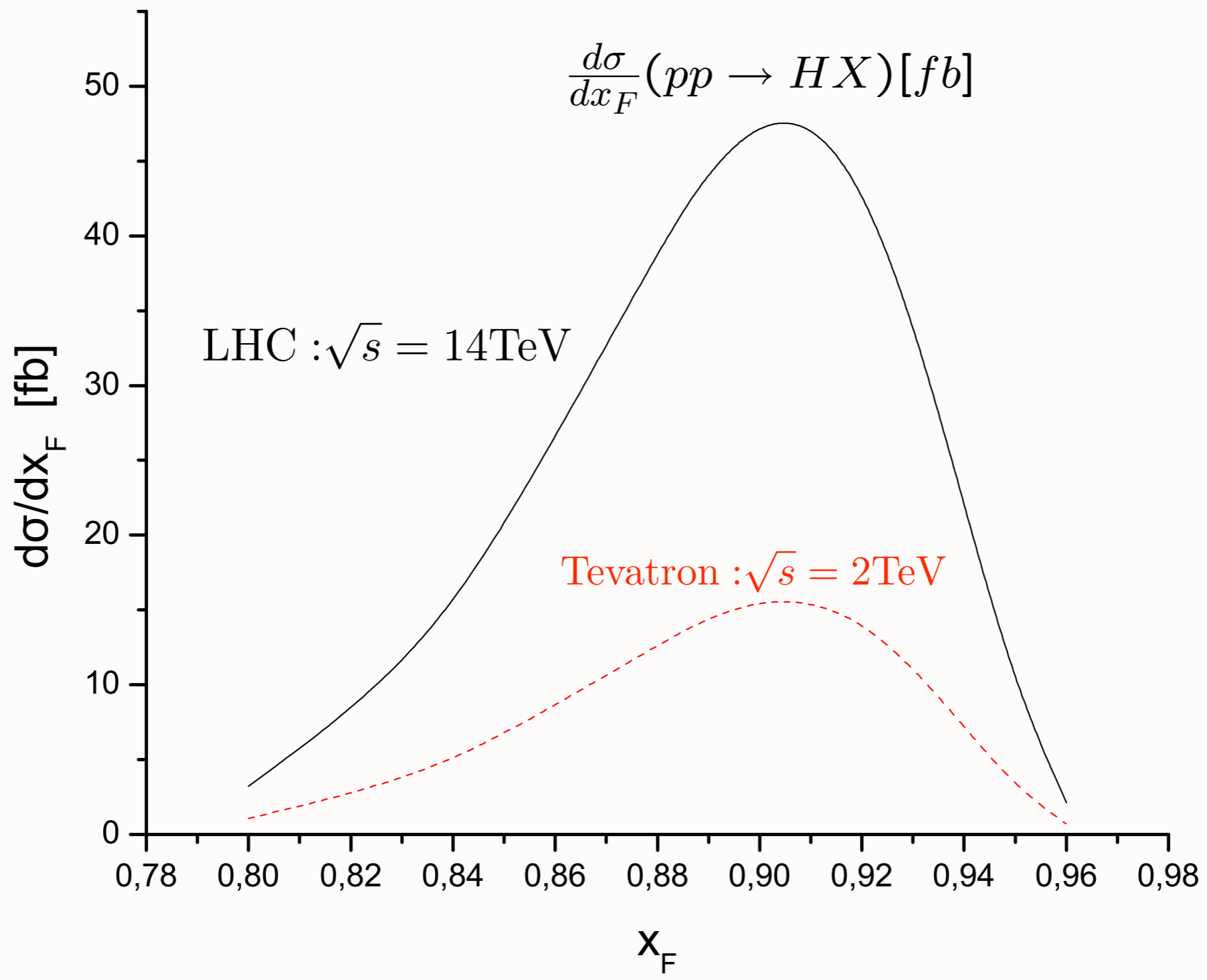
Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

AFTER: Higgs production at threshold!

Intrinsic Heavy Quark Contribution to Inclusive Higgs Production

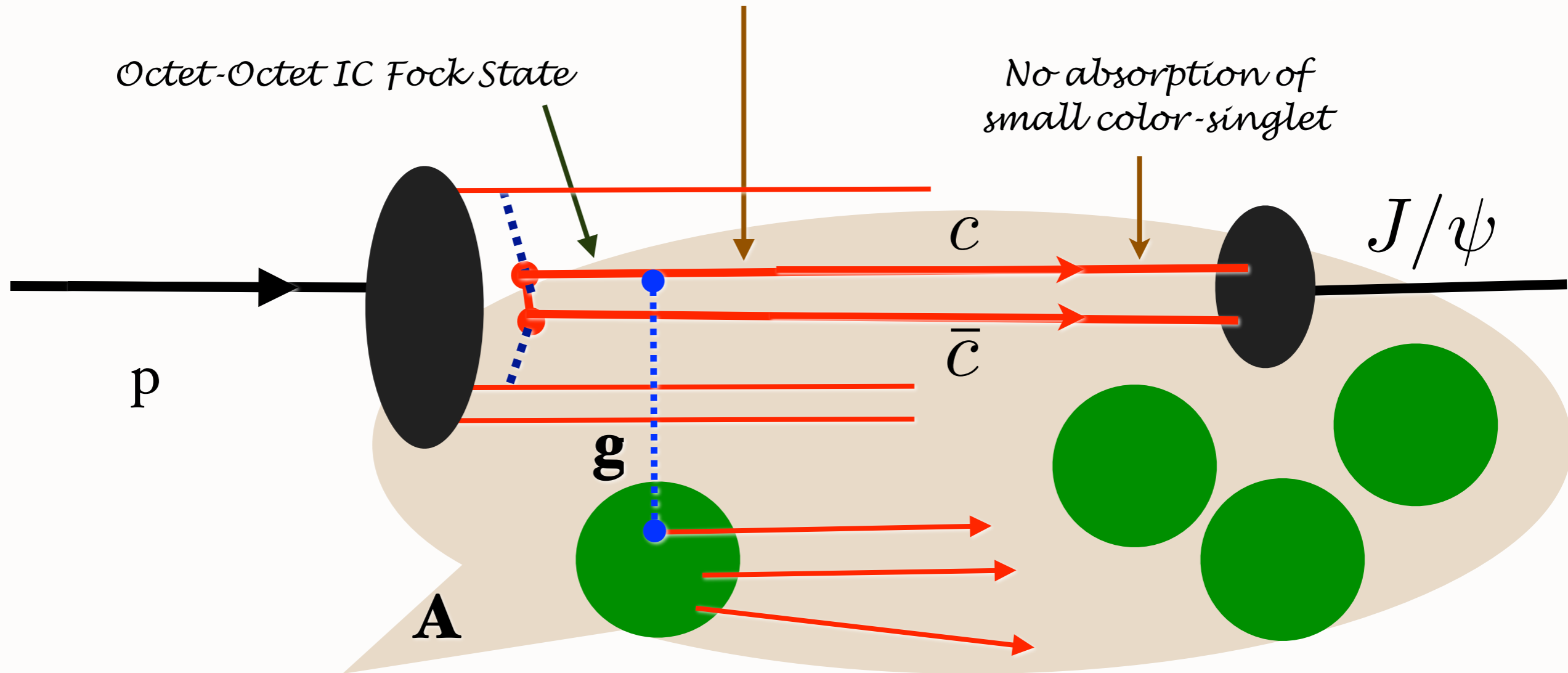


High x_F

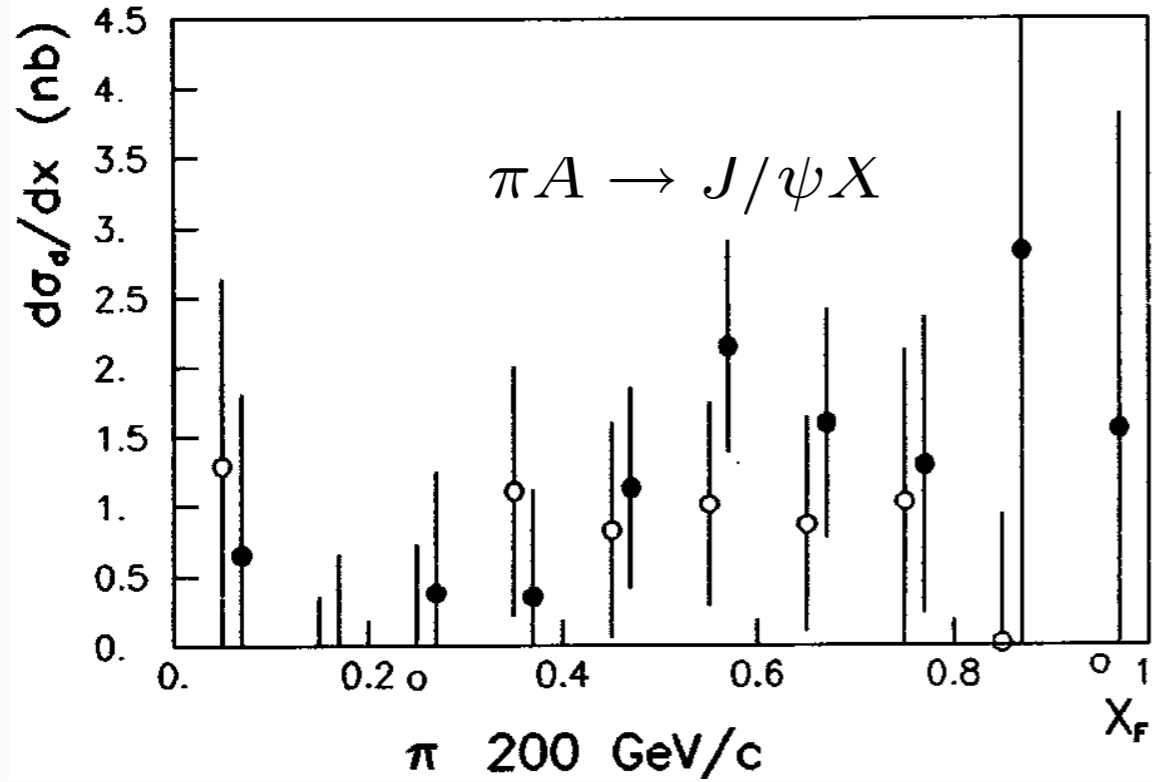
*Color-Opaque IC Fock state
interacts on nuclear front surface*

**Kopeliovich,
Schmidt, Soffer, sjb**

Scattering on front-face nucleon produces color-singlet $c\bar{c}$ pair

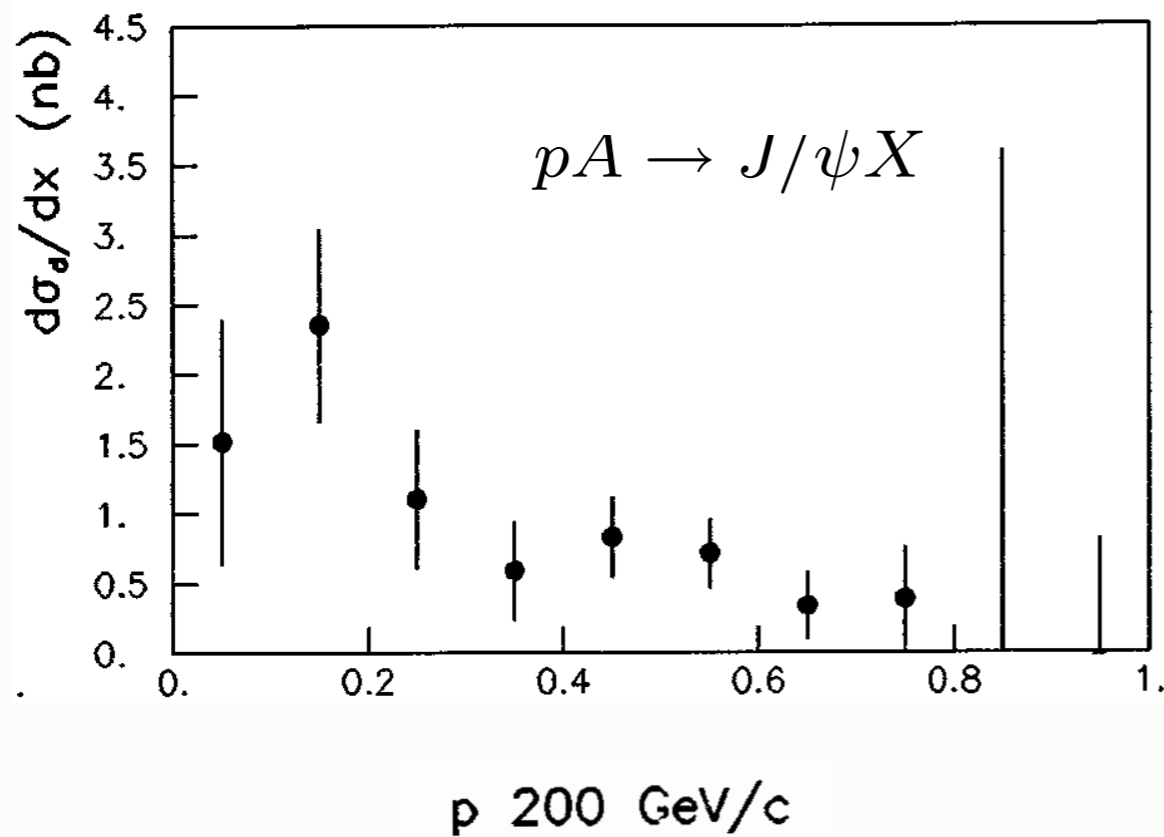


$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$

$A^{2/3}$ component



J. Badier et al, NA3

Excess beyond conventional PQCD subprocesses

- IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \rightarrow J/\psi X$
(Mueller, Gunion, Tang, SJB)
- Color Octet IC Explains $A^{2/3}$ behavior at high x_F (NA3, Fermilab) *Color Opacity*
(Kopeliovitch, Schmidt, Soffer, SJB)
- IC Explains $J/\psi \rightarrow \rho\pi$ puzzle
(Karliner, SJB)
- IC leads to new effects in B decay
(Gardner, SJB)

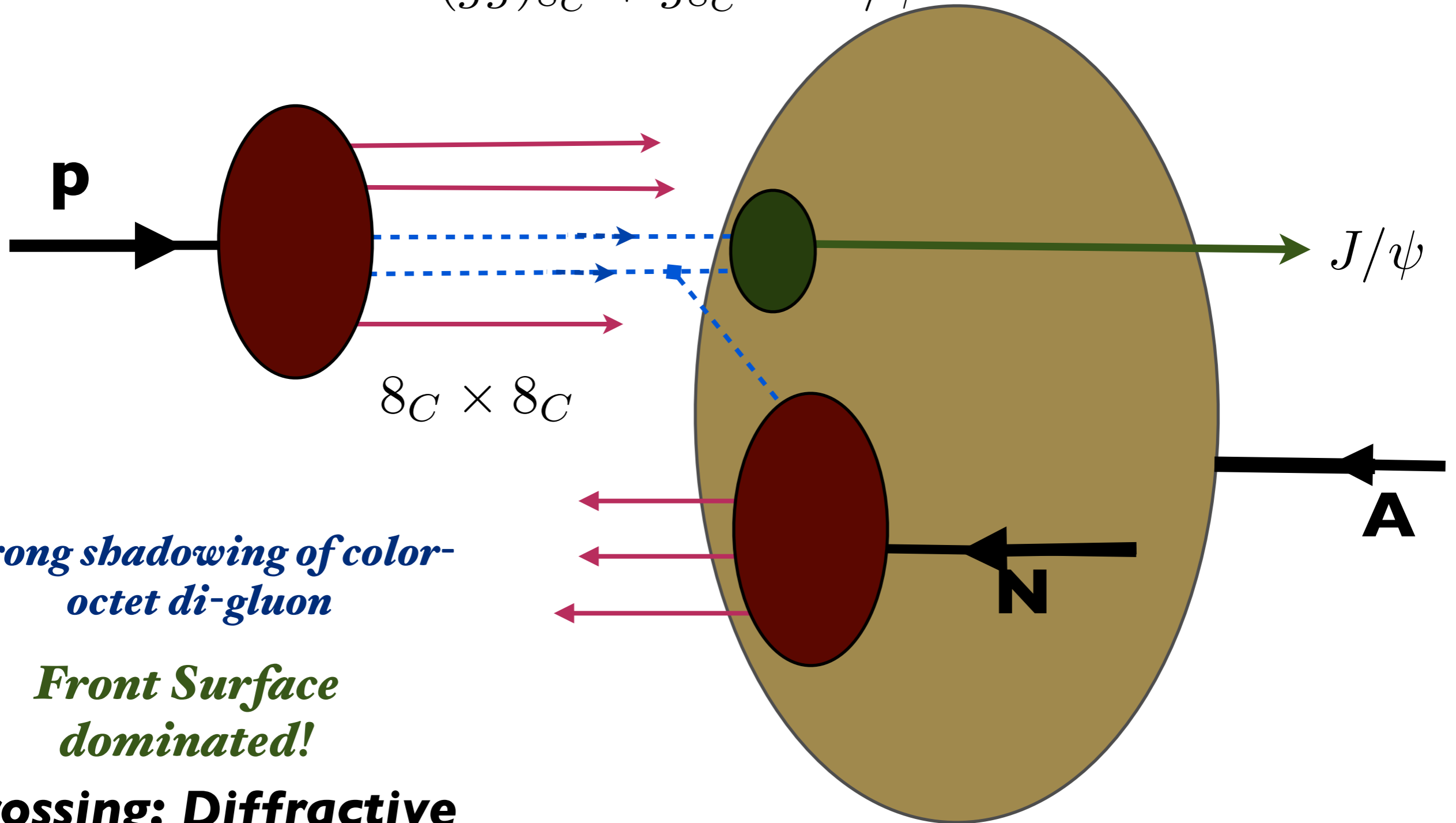
Higgs production at $x_F = 0.8$

Forward rapidity $y \sim 4$

$$pA \rightarrow J/\psi X$$

Zhu, sjb

$$(gg)_{8_C} + g_{8_C} \rightarrow J/\psi$$



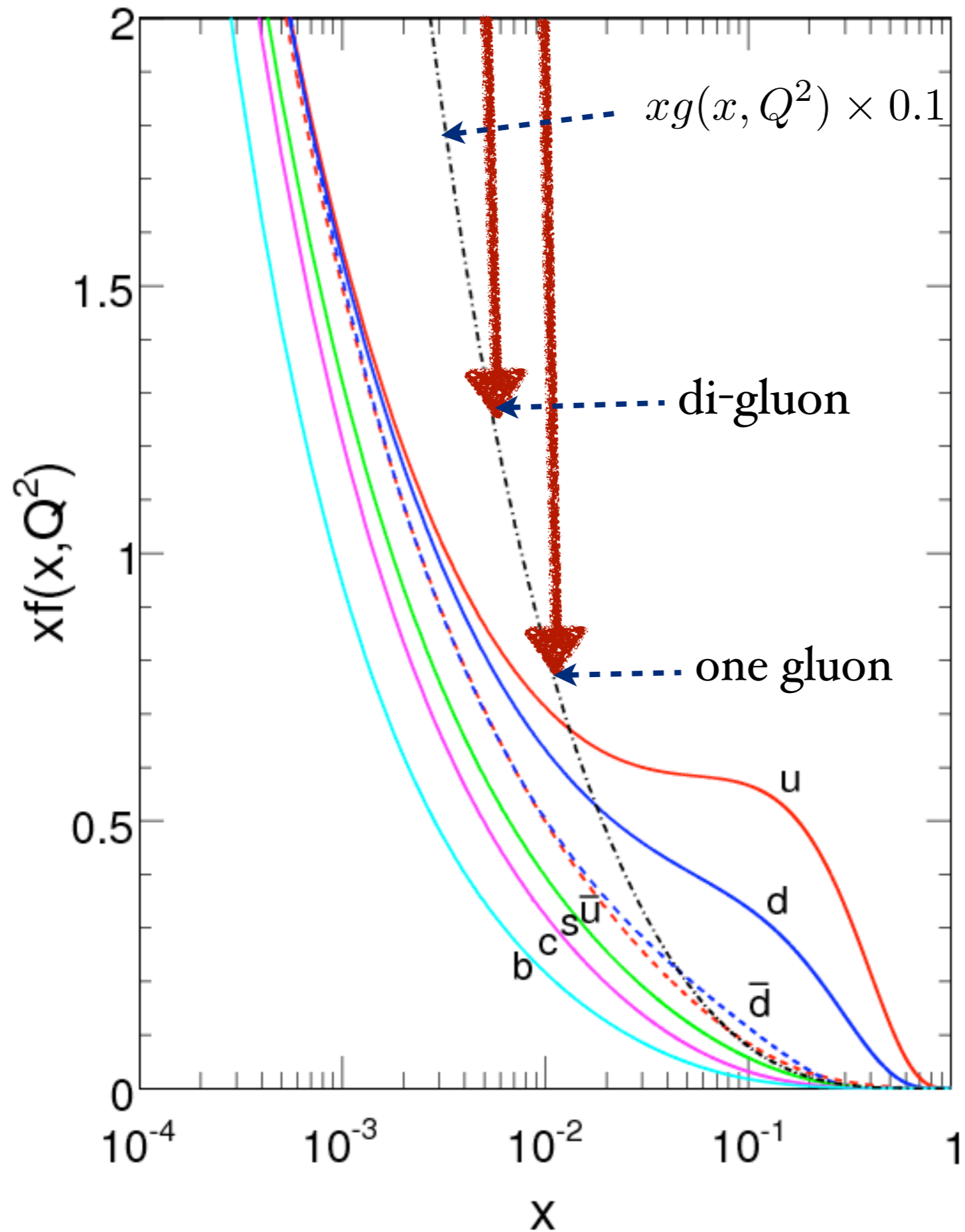
Strong shadowing of color-octet di-gluon

Front Surface dominated!

Crossing: Diffractive & pomeron exchange

Double-gluon subprocess

Two gluons at $g(0.005) \sim \frac{13}{0.005} = 2600$ vs. one gluon at $g(0.01) \sim \frac{8}{0.01} = 800$



QCD and the Standard-Model Vacuum on the Light Front

- **Light Front Quantization**
- **The LF Vacuum and the Physical Universe**
- **QCD Condensates and the Cosmological Constant**
- **Higgs Model on The LF and the Cosmological Constant**
- **Light-Front Holography and AdS/QCD**

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$

$$\Omega_{\Lambda} = 0.76(\text{expt})$$

$$(\Omega_{\Lambda})_{QCD} \propto \langle 0 | q\bar{q} | 0 \rangle^4$$

QCD Problem Solved if quark and gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb Proc.Nat.Acad.Sci. 108 (2011) 45-50 “Condensates in Quantum Chromodynamics and the Cosmological Constant”

C. Roberts, R. Shrock, P. Tandy, sjb Phys.Rev. C82 (2010) 022201 “New Perspectives on the Quark Condensate”

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0\rangle = E_0|\psi_0\rangle, E_0 = \min\{E_i\}$$

*Eigenstate defined at one time t over all space;
Acausal! Frame-Dependent*

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0\rangle_{LF} = M_0^2|\psi_0\rangle_{LF}, M_0^2 = 0.$$

*Frame-independent eigenstate at fixed LF time $\tau = t+z/c$
within causal horizon*

Frame-independent description of the causal physical universe!

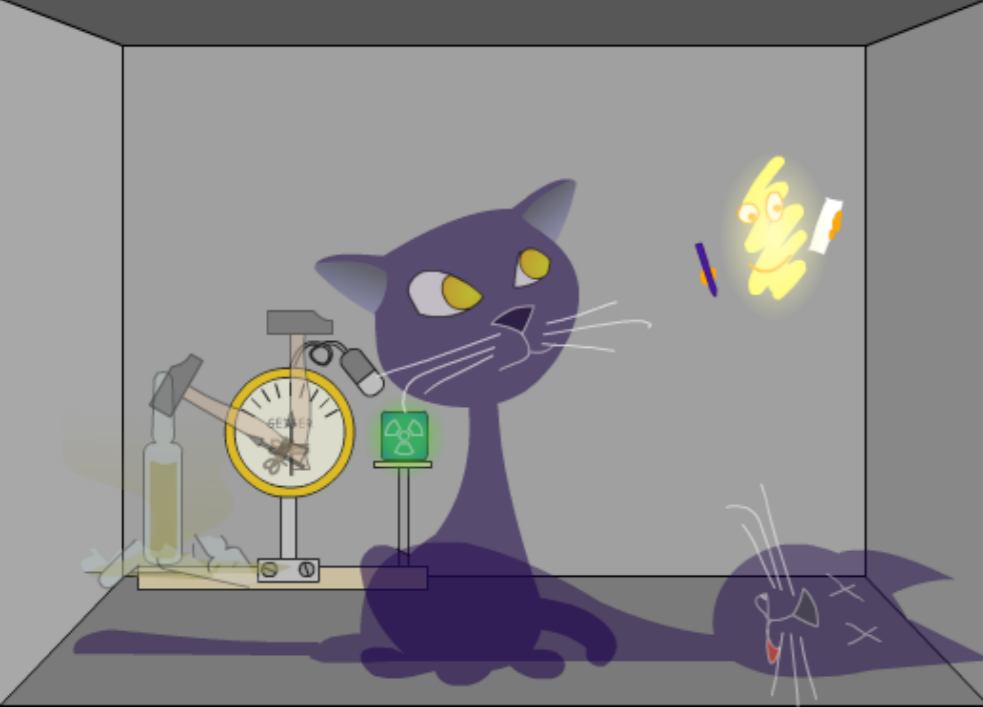
Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind

Maris, Roberts, Tandy

Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant --
Eliminates 45 orders of magnitude
conflict**



Original Note of Warning

Chiral Magnetism (or Magnetohydrochironics)

A. Casher and L. Susskind, Phys. Rev. D9 (1974) 436

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.¹ Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.² A number of investigations

➤ *These authors argue that dynamical chiral-symmetry breaking can be realized as a property of hadrons, instead of via a nontrivial vacuum exterior to the measurable degrees of freedom*

The essential ingredient required for a spontaneous symmetry breakdown in a composite system is the existence of a divergent number of constituents

– DIS provides evidence for divergent sea of low-momentum partons – parton model.

PHYSICAL REVIEW C **82**, 022201(R) (2010)

New perspectives on the quark condensate

Stanley J. Brodsky,^{1,2} Craig D. Roberts,^{3,4} Robert Shrock,⁵ and Peter C. Tandy⁶

¹*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

²*Centre for Particle Physics Phenomenology: CP³-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

³*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

⁴*Department of Physics, Peking University, Beijing 100871, China*

⁵*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

⁶*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 25 May 2010; published 18 August 2010)

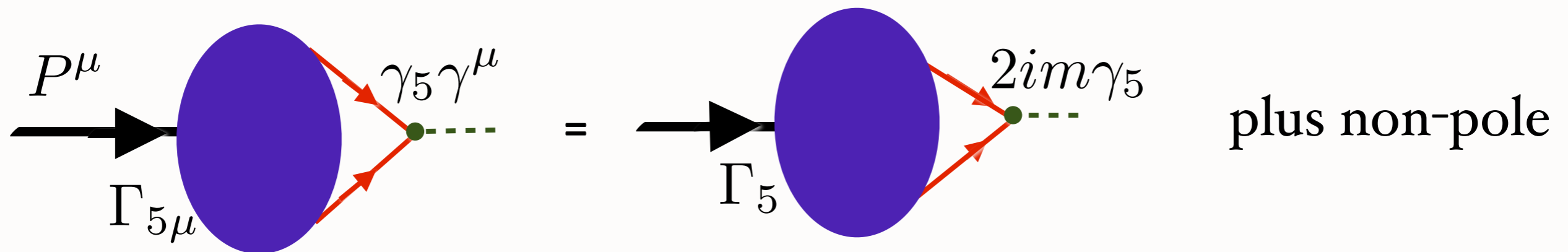
We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

Ward-Takahashi Identity for axial current

GMOR satisfied, no VEV

$$P^\mu \Gamma_{5\mu}(k, P) + 2im\Gamma_5(k, P) = S^{-1}(k + P/2)i\gamma_5 + i\gamma_5 S^{-1}(k - P/2)$$

$$S^{-1}(\ell) = i\gamma \cdot \ell A(\ell^2) + B(\ell^2) \quad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



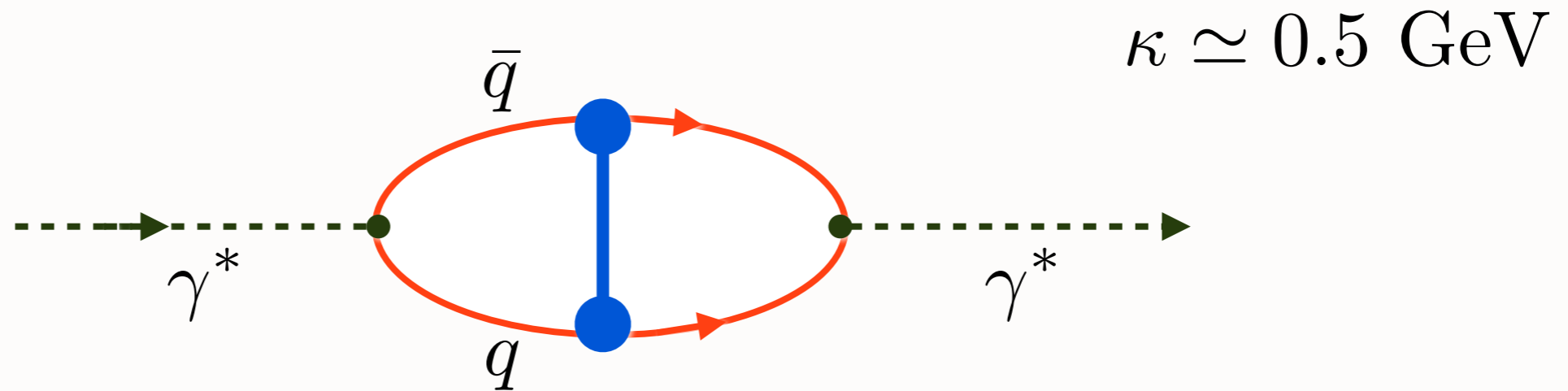
Identify pion pole at $P^2 = m_\pi^2$

$$P^\mu \langle 0 | \bar{q} \gamma_5 \gamma^\mu q | \pi \rangle = 2m \langle 0 | \bar{q} i \gamma_5 q | \pi \rangle$$

$$f_\pi m_\pi^2 = -(m_u + m_d) \rho_\pi$$

Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

$$M^2 = 4\kappa^2(n + L + S/2) \quad \text{light-quark meson spectra}$$



$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 \left(1 + \mathcal{O}\left(\frac{\kappa^4}{s^2}\right) + \dots \right)$$

mimics dimension-4 gluon condensate $\langle 0 | \frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0) | 0 \rangle$ in

$e^+e^- \rightarrow X, \tau$ decay, $Q\bar{Q}$ phenomenology

Standard Model on the Light-Front

- Same phenomenological predictions
- Higgs field has three components
- Real part creates Higgs particle
- Imaginary part (Goldstone) become longitudinal components of W, Z
- *Higgs VEV of instant form becomes $k^+=0$ LF zero mode!*
- Analogous to a background static classical Zeeman or Stark Fields
- Zero contribution to T^{μ}_{μ} ; zero coupling to gravity

Determinations of the vacuum Gluon Condensate

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle [\text{GeV}^4]$$

-0.005 ± 0.003 from τ decay.

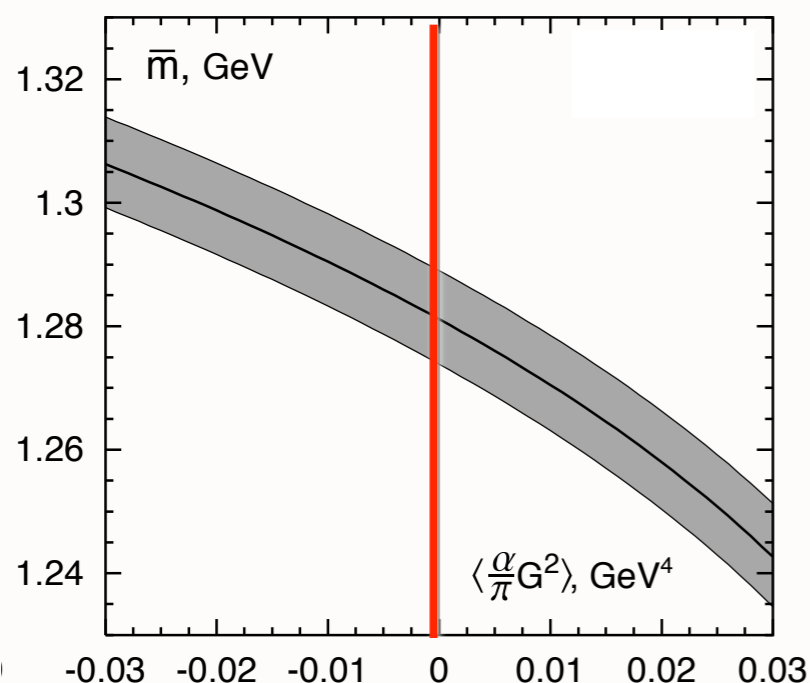
Davier et al.

$+0.006 \pm 0.012$ from τ decay.

Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk



*Consistent with zero
vacuum condensate*

Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state $M=0$.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: “In-hadron” condensates (Maris, Tandy Roberts)
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$\begin{aligned} (\Omega_\Lambda)_{QCD} &\sim 10^{45} \\ (\Omega_\Lambda)_{EW} &\sim 10^{56} \end{aligned} \quad \Omega_\Lambda = 0.76(\text{expt})$$

QCD gives $\Lambda=\text{zero}$ if Quark and Gluon condensates reside within hadrons, not vacuum!

Electroweak contribution gives $\Lambda=\text{zero}$ from Zero Mode solution to Higgs Potential

*Electroweak Problem also could be solved in **technicolor**-- condensates within technihadrons*

$$(\Omega_\Lambda)_{QCD} = 0 \quad (\Omega_\Lambda)_{EW} = 0$$

Central Question: What is the source of Dark Energy?

$\Omega_\Lambda = 0.76(\text{expt})$ *Higgs Zero-Mode Curvature?*

QCD Myths

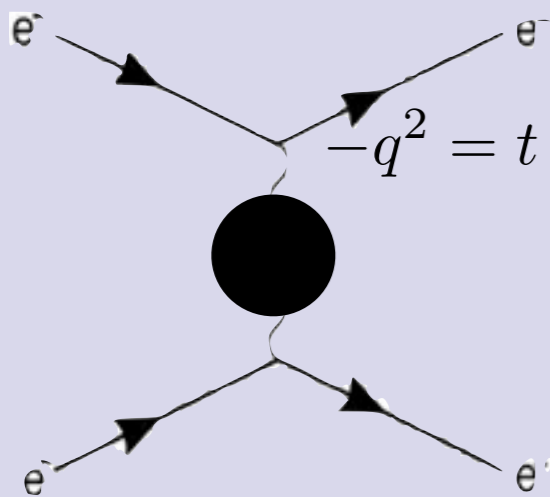
- **Anti-Shadowing is Universal**
- **ISI and FSI are higher twist effects and universal**
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **heavy quarks only from gluon splitting**
- **renormalization scale cannot be fixed**
- **QCD condensates are vacuum effects**
- **Infrared Slavery**
- **Nuclei are composites of nucleons only**
- **Real part of DVCS arbitrary**

Lessons from QED

In the (physical) Gell Mann-Low scheme, the momentum scale of the running coupling is the virtuality of the exchanged photon; independent of initial scale.

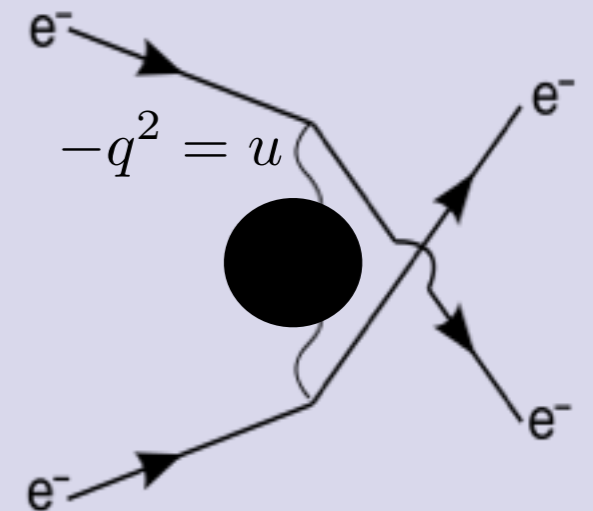
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \quad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Example: ee-scattering



$$\mathcal{M}_{ee \rightarrow ee} = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$

Two separate scales;
one for each skeleton graph.



For any other scale choice an infinite set of diagrams must be taken into account to obtain the correct result!

In any other scheme, the correct scale displacement must be used

$$\log \frac{\mu_{MS}^2}{m_\ell^2} = 6 \int_0^1 dx x(1-x) \log \frac{m_\ell^2 + Q^2 x(1-x)}{m_\ell^2}, \quad Q^2 \gg m_\ell^2 \rightarrow \log \frac{Q^2}{m_\ell^2} - \frac{5}{3}$$

$$\alpha_{MS}(e^{-5/3} q^2) = \alpha_{GM-L}(q^2).$$

Principle of Maximum Conformality (PMC)

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Scale-Free
Conformal Series

Running Coupling
Effects

Higher Twist from
Hadron Dynamics

Intrinsic Heavy
Quarks

Light by Light
Loops

BLM/PMC: Absorb β -terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Principle of Maximum Conformality (PMC)

- **Sets pQCD renormalization scale correctly at every finite order**
- **Predictions are scheme-independent**
- **Satisfies all principles of the renormalization group**
- **Agrees with Gell Mann-Low procedure for pQED in Abelian limit**
- **Shifts all β terms into α_s , leaving conformal series**
- **Automatic procedure: R_δ scheme**
- **Number of flavors n_f set**
- **Eliminates $n!$ renormalon growth**
- **Choice of initial scale irrelevant**
- **Eliminates unnecessary systematic error -- conventional guess is scheme-dependent, disagrees with QED**
- **Reduces disagreement with pQCD for top/anti-top asymmetry at Tevatron from 3σ to 1σ**

**Xing-Gang Wu, Martin Mojaza
Leonardo di Giustino, SJB**



Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza*

*CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark
and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

Stanley J. Brodsky†

SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Xing-Gang Wu‡

*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China
(Received 13 January 2013; published 10 May 2013)*

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

δ -Renormalization Scheme (\mathcal{R}_δ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the **modified minimal subtraction** scheme ($\overline{\text{MS}}$) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu_{\overline{\text{MS}}}^2 = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: \mathcal{R}_δ -scheme

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu_\delta^2 = \mu_{\overline{\text{MS}}}^2 \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

Exposing the Renormalization Scheme Dependence

Observable in the \mathcal{R}_δ -scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \dots$$

$$\mathcal{R}_0 = \overline{\text{MS}}, \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS} \quad \mu^2 = \mu_{\overline{\text{MS}}}^2 \exp(\ln 4\pi - \gamma_E), \quad \mu_{\delta_2}^2 = \mu_{\delta_1}^2 \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \text{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$ -term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p . Grouping the different δ_k -terms, one recovers in the $N_c \rightarrow 0$ Abelian limit the dressed skeleton expansion.

Special Degeneracy in PQCD

There is nothing special about a particular value for δ , thus for any δ

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3 \\ + [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4$$

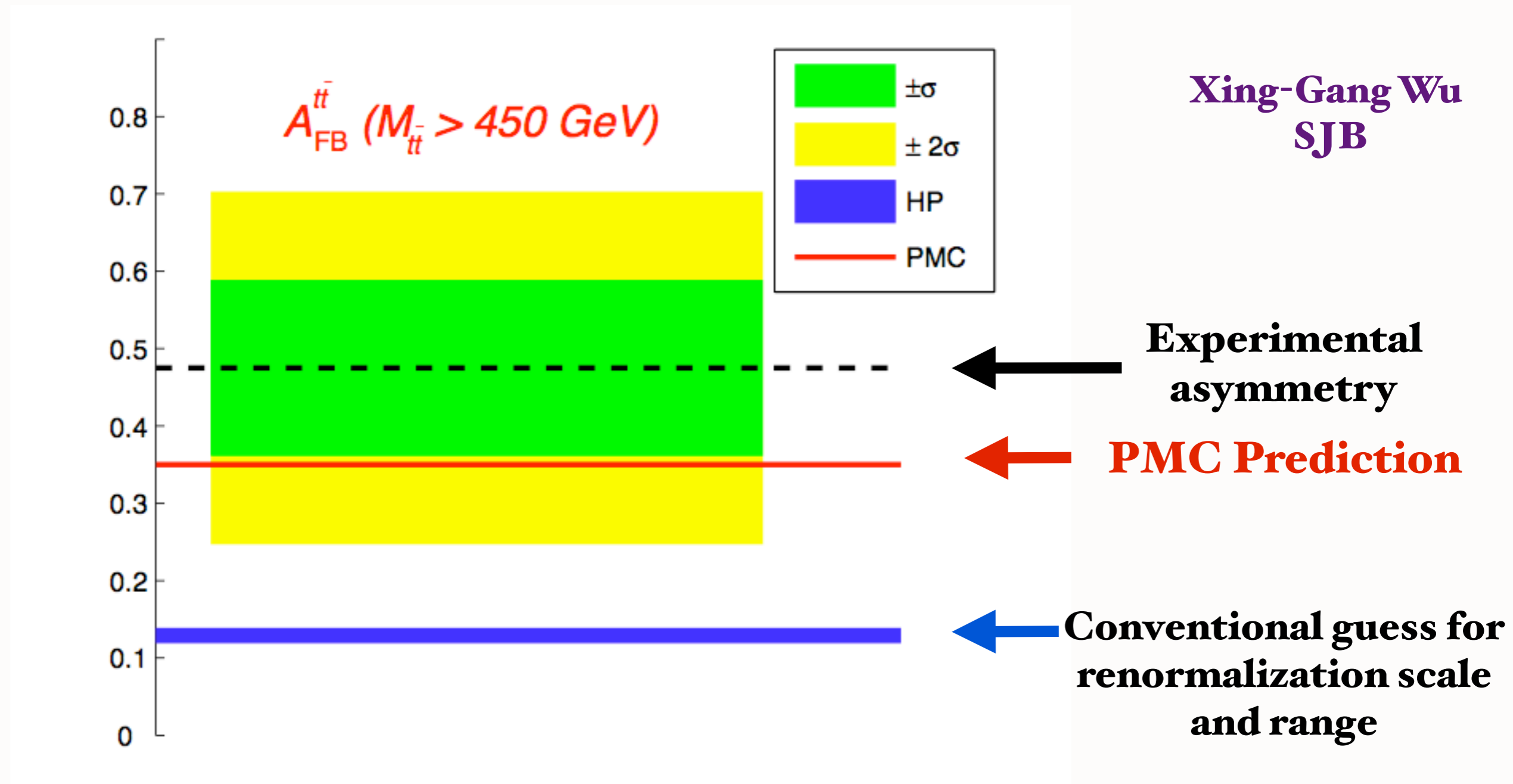
According to the **principal of maximum conformality** we must set the scales such to absorb all 'renormalon-terms', i.e. **non-conformal terms**

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \dots) \underline{r_{2,1}} \\ + (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \dots) \underline{r_{3,2}} + (\beta_0^3 + \dots) r_{4,3} \\ + r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \dots) \underline{r_{3,1}} \\ + \dots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \dots + \frac{(-1)^n}{n!} \frac{d^{n-1}\beta}{(d \ln \mu^2)^{n-1}} r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \dots$$

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)

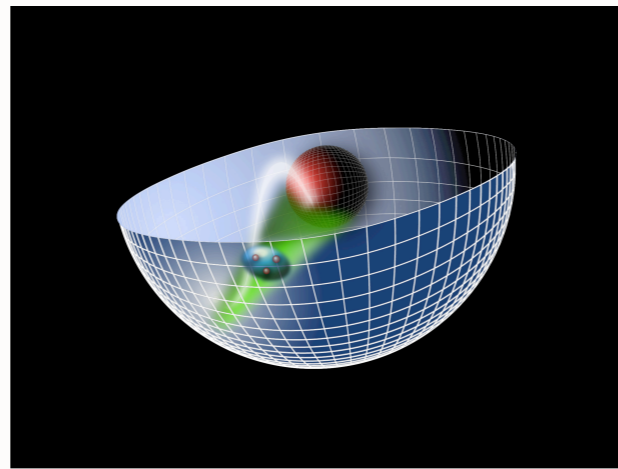


Top quark forward-backward asymmetry predicted by pQCD NNLO within 1σ of CDF/D0 measurements using PMC/BLM scale setting

Conformal Template

- **Self-Consistent breaking of scale invariance--Unique Confining Potential and Dilaton**
- **Non-Perturbative QCD Running Coupling**
- **Principle of Maximum Conformality -- sets renormalization scale in PQCD -- result is scheme independent!**
- **ERBL evolution and eigensolutions**

Frishman, Sachrajda, Lepage, sjb; Braun



*AdS/QCD
Soft-Wall Model*

Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

$$\kappa \simeq 0.6 \text{ GeV}$$

Confinement scale:

$$1/\kappa \simeq 1/3 \text{ fm}$$

***Unique
Confinement Potential!
Conformal Symmetry
of the action***

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

An analytic first approximation to QCD

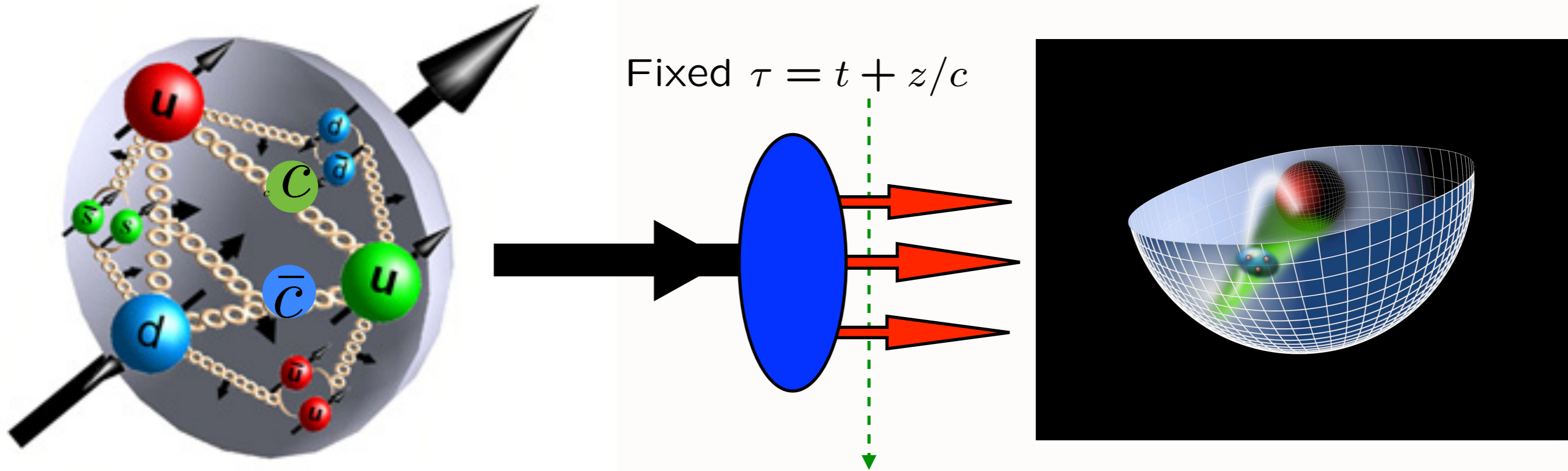
AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

New Perspectives for QCD

- **Light-Front QCD and Holography**
- **Unique Color Confinement Potential**
- **Principle of Maximal Conformality**
- **Non-Universal Anti-Shadowing and other Novel Nuclear Effects**
- **Lensing effects and Factorization Breaking**
- **Direct and Multiparton Processes**
- **Heavy Quark Distributions and Novel Higgs Production Mechanisms**
- **Ridge Correlations at the LHC**
- **The QCD Vacuum and the Cosmological Constant**

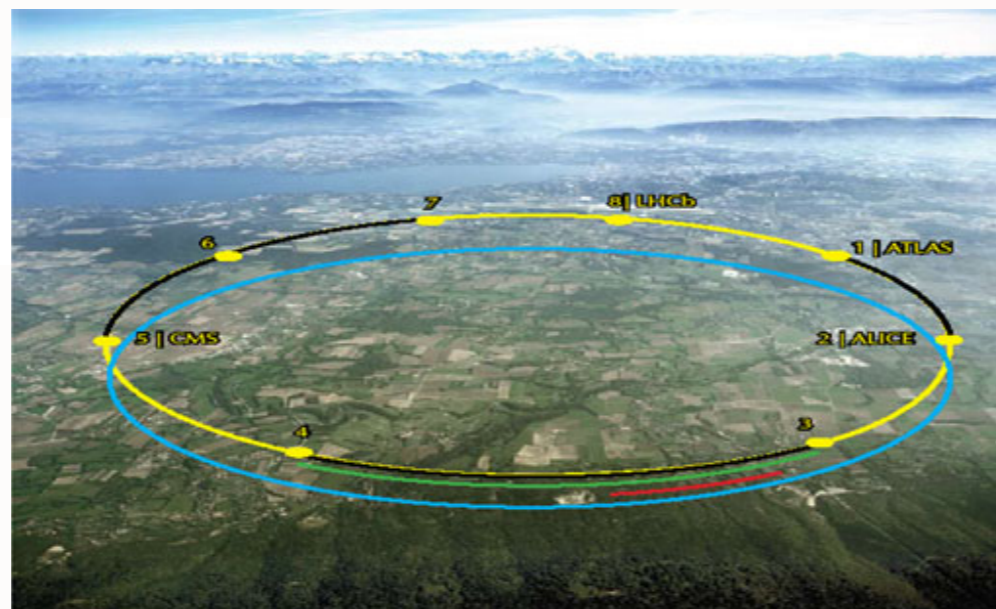
Novel QCD Phenomena and New Perspectives for Hadron Physics from Light-Front Holography



CERN TH Seminar

January 22, 2014

Stan Brodsky



Stanford University, October 19-20, 2011