SEMI-ANALYTIC APPROACHES TO MAGNET DESIGN

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FOREWORD

- In the design of accelerator systems, a parametric analysis is needed in the phase of conceptual design
  - Having approximated handy equations for a fast exploration and to have an insight on the multidimensional phase space is a plus
  - The added value (analytical insight, handy equations) is huge

- Methods
  - Do not solve the whole problem, but a simplified version
  - When analytical methods do not work, try a numerical method and then make empirical fits
  - A 5%-10% approximation is more than acceptable in this phase

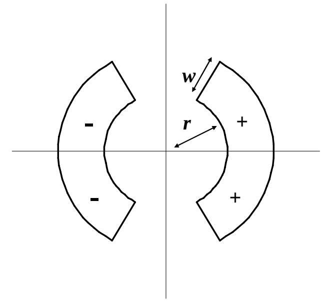
- We summarize several works carried out for the magnet design in the last 3 years:
  - Equations for short sample field/gradient → electromagnetic design
  - Equations for forces and stresses → mechanical design
  - Equations for the stored energy → quench protection

CONTENTS

- A few key ideas
- Equations for dipoles and some examples
- Equations for quadrupoles and some examples
- Conclusions

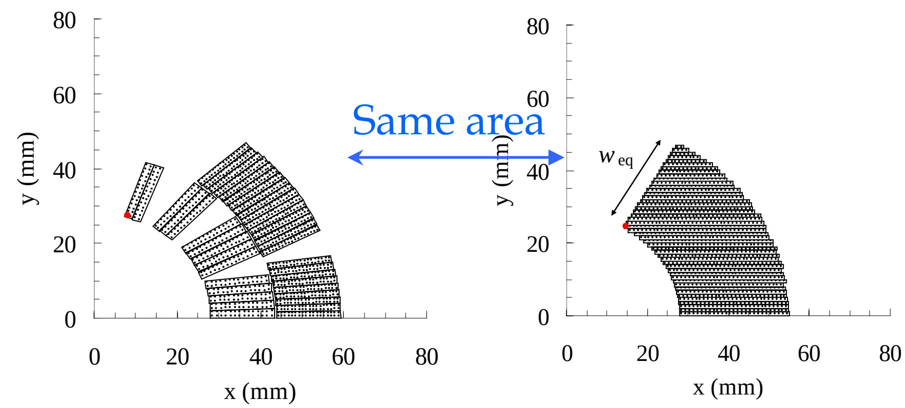
Key ideas (1): Coil equivalent width

- Our simplified model relies on simple sector geometry
- Main parameters
  - Magnet aperture (radius) r
  - Coil width w



- How to apply the approach to take into account of multilayer coils, copper wedges …
  - Let A be the coil surface
  - Equivalent coil width

$$w_{eq} \equiv \left(\sqrt{1 + \frac{3A}{2\pi r^2}} - 1 \right) r$$



The LHC Main dipole cross-section

A 60° sector coil with same aperture and equivalent width

The issue of ratio peak field – central field

- The central field (gradient) produced by an ironless sector coil is easily computed using Biot-Savart
  - Dipoles: proportional to coil width and current density

$$B = j\gamma_0 w$$

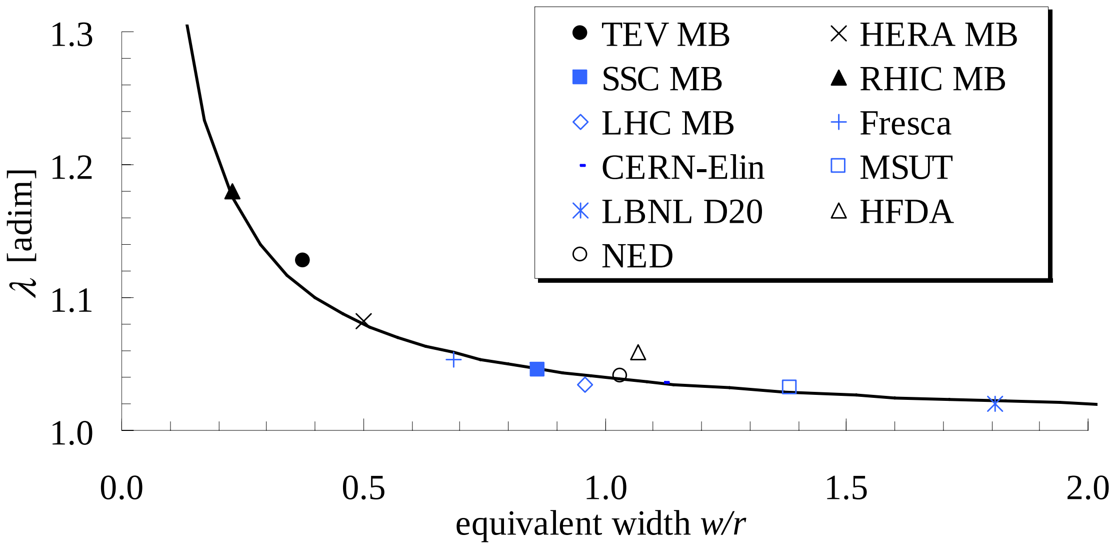
  - Quadrupoles: logarithmic function of *w/r*

$$G = j\gamma_0 \log\left(1 + \frac{w}{r}\right)$$

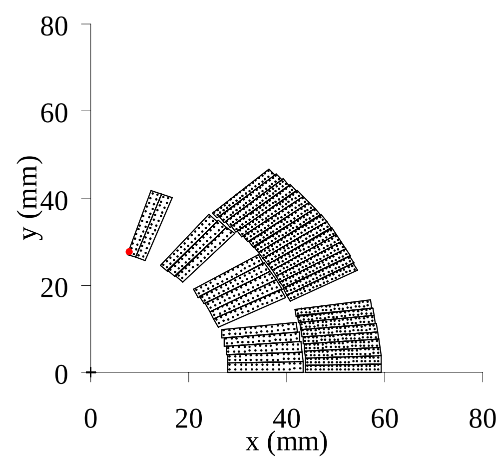
- A superconducting magnet is limited by the peak field in the coil

KEY IDEAS (2): ESTIMATING THE RATIO PEAK FIELD – CENTRAL FIELD IN DIPOLES

- The ratio peak field central field λ is a function of the ratio between the coil width and the aperture *w/r*
  - Empirical fit $$\lambda(w,r)=1+\frac{ar}{w}$$ $a\sim0.04$
  - It tends to 1 for large ratios *w/r*



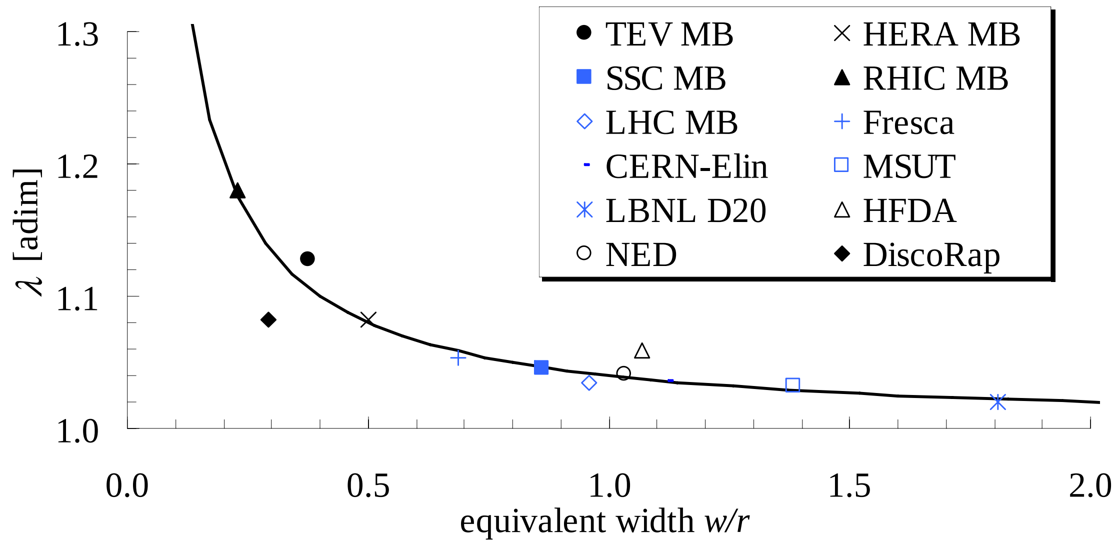
Ratio peak field / central field for some dipoles (markers) and empirical fit found for sector coils (solid line)



Peak field location in the cross-section of the main LHC dipole

Key ideas (2): estimating the ratio peak field – central field in dipoles

- The ratio peak field central field λ is a function of the ratio between the coil width and the aperture *w/r*
  - Empirical fit $\lambda(w,r) = 1 + \frac{ar}{w}$ a~0.04
  - You can make better …



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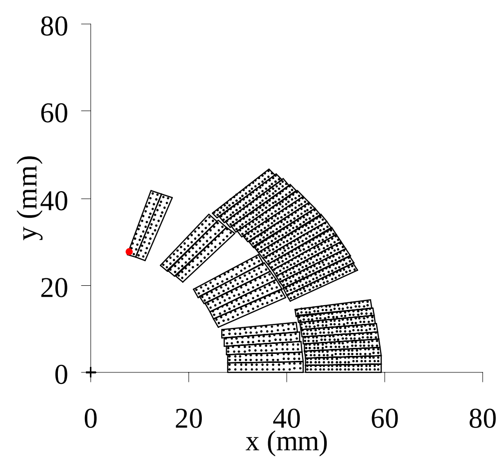
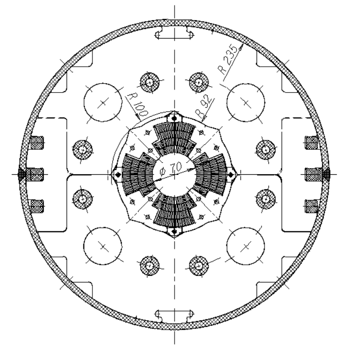
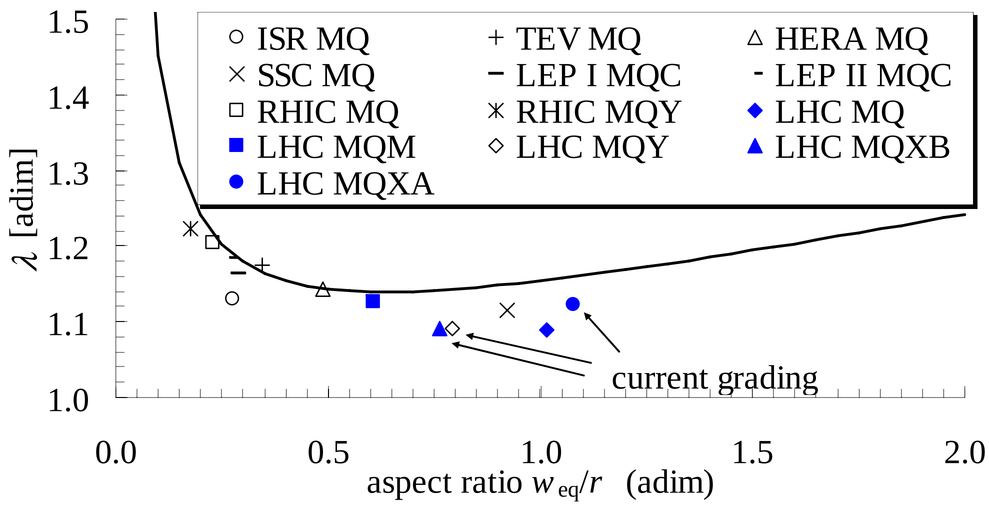


Peak field location in the cross-section of the main LHC dipole

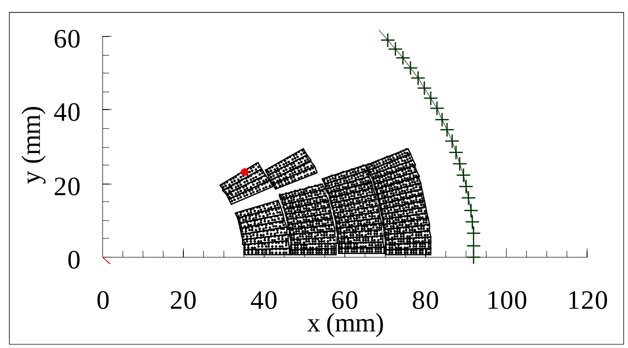
KEY IDEAS (3): ESTIMATING THE RATIO PEAK FIELD – CENTRAL FIELD IN QUADRUPOLES

- The ratio peak field central field λ is a function of the ratio between the coil width and the aperture *w/r*
  - Empirical fit $$\lambda(r,w) \sim \lambda_f(r,w) = a_{-1}\frac{r}{w} + 1 + a_1\frac{w}{r}$$
  - It diverges for large ratios *w/r*





Ratio peak field / central field for some quadrupoles (markers) and empirical fit found for sector coils (solid line)



Peak field location in the cross-section of the LHC MQXA (IR quadrupole)

CRITICAL SURFACE FIT

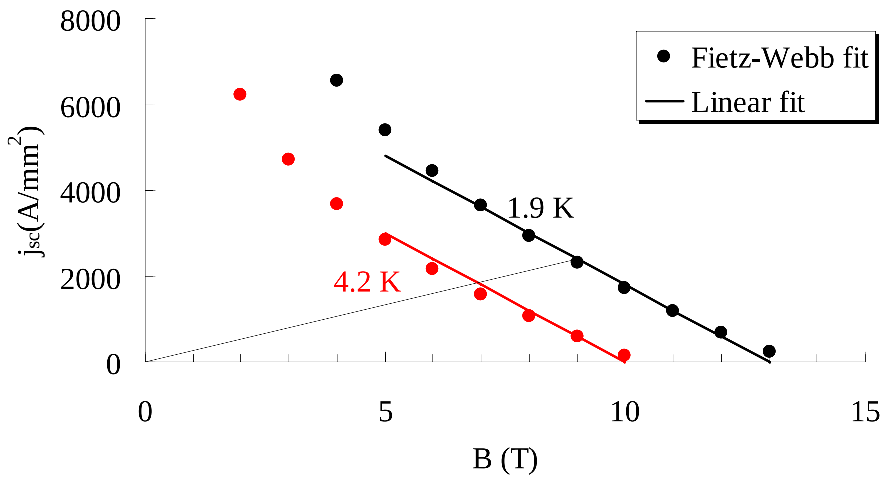
- Nb-Ti critical surface for a given temperature can be written according to the Fietz-Webb fit

$$j_{sc,c} = C(T)B^{\alpha-1}\left(1-\frac{B}{B^*_{c2}(T)}\right)^{\beta}$$

  - A linear fit is rather good in the high field domain of usual interest

$$j_c(B) = \kappa c(b - B)$$

  - Advantage: the loadline equation can be solved explicitly



Critical surface of Nb-Ti, linear fit, and loadline

Key ideas (4): Critical surface fit for Nb3Sn

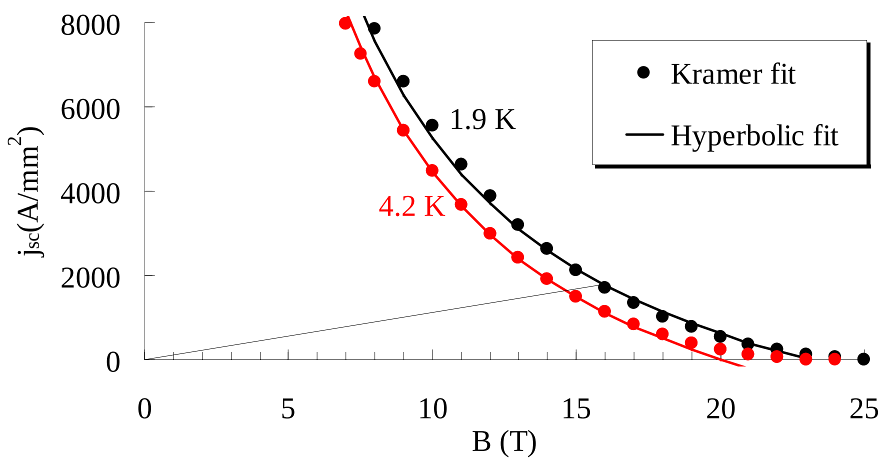
- Nb-Ti critical surface for a given temperature can be written according to Kramer fit

$$j_{sc}(B) = \frac{C(T,\varepsilon)}{\sqrt{B}}\left(1 + \frac{B}{B^*_{c2}(T,\varepsilon)}\right)^2$$

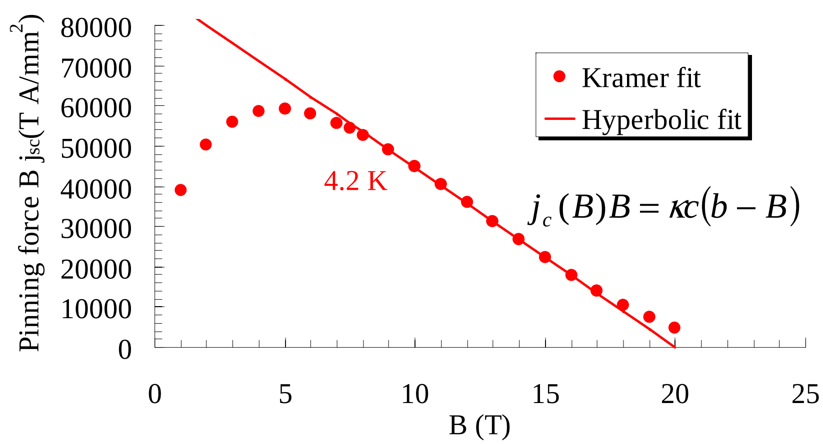
  - An shifted hyperbolic fit is rather good in the high field domain of usual interest

$$j_c(B) = \kappa c\left(\frac{b}{B} - 1\right)$$

  - Advantage: the loadline equation can be solved explicitly



Critical surface of Nb_3Sn, hyperbolic fit, and loadline



The hyperbolic fit corresponds to a linear pinning force

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Equations for dipoles

- Short sample field

$$B_{ss} \sim \frac{\kappa c b \gamma_0 w_{eq}}{1+\kappa c \gamma_0 \left(w_{eq}+ar\right)} \qquad B_{ss} \sim \frac{\kappa c \gamma_0 w_{eq}}{2}\left(\sqrt{\frac{4b}{\kappa c \gamma_0 (w_{eq}+ar)}+1}-1\right)$$

- Short sample current density

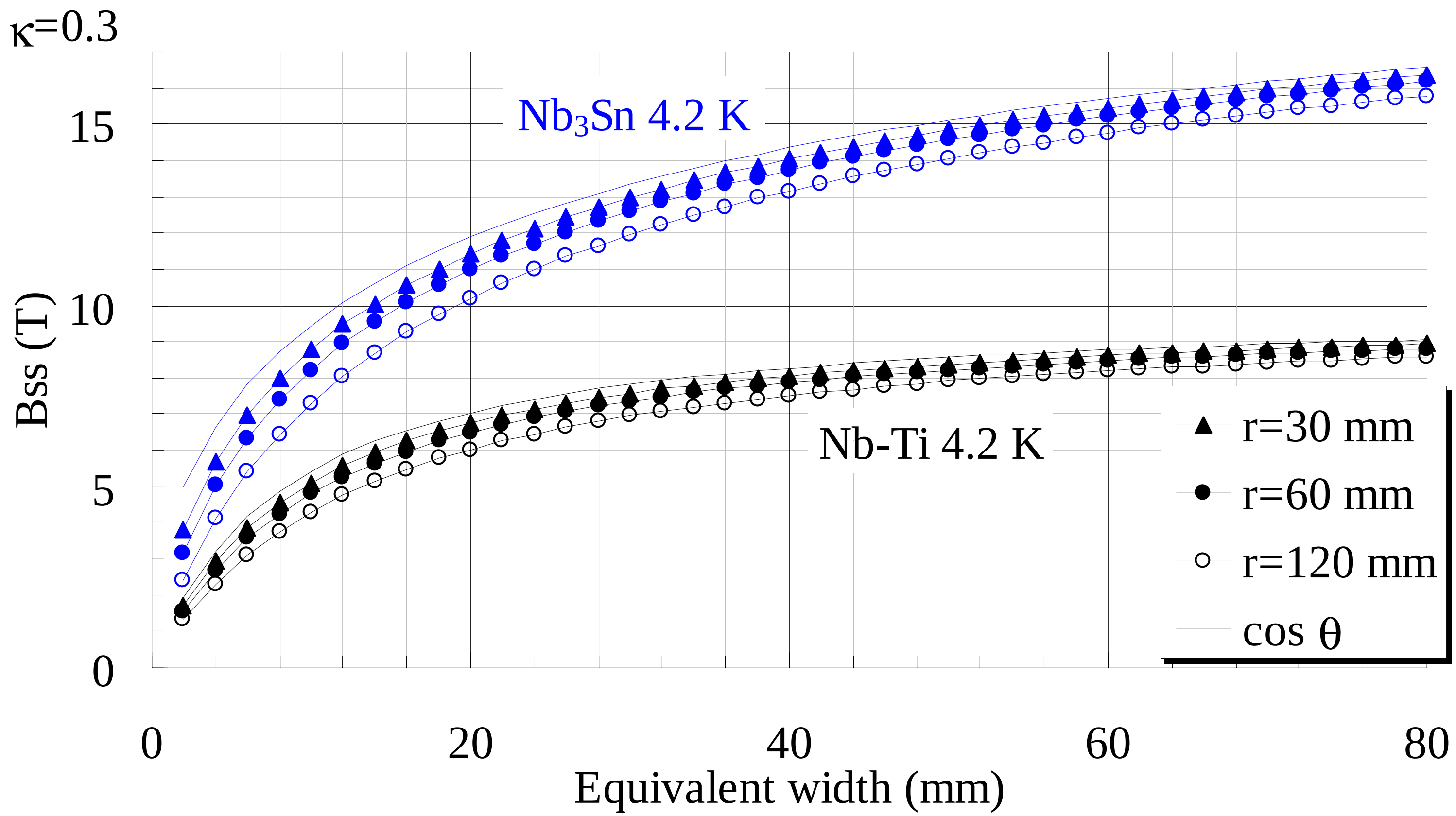
$$j_{ss} \sim \frac{\kappa c b}{1+\kappa c \gamma_0 \left(w_{eq}+ar\right)} \qquad j_{ss} \sim \frac{\kappa c}{2}\left(\sqrt{\frac{4b}{\kappa c \gamma_0 (w_{eq}+ar)}+1}-1\right)$$

- Pressure in the midplane at current density *j*

$$\sigma_{\varphi} \sim \frac{\sqrt{3}}{6\pi} j^2 \mu_0 \mathrm{Max}_{\rho \in [r, r+w_{eq}]}\left[\frac{3\rho^2 (r+w_{eq}) - r^3 - 2\rho^3}{\rho}\right]$$

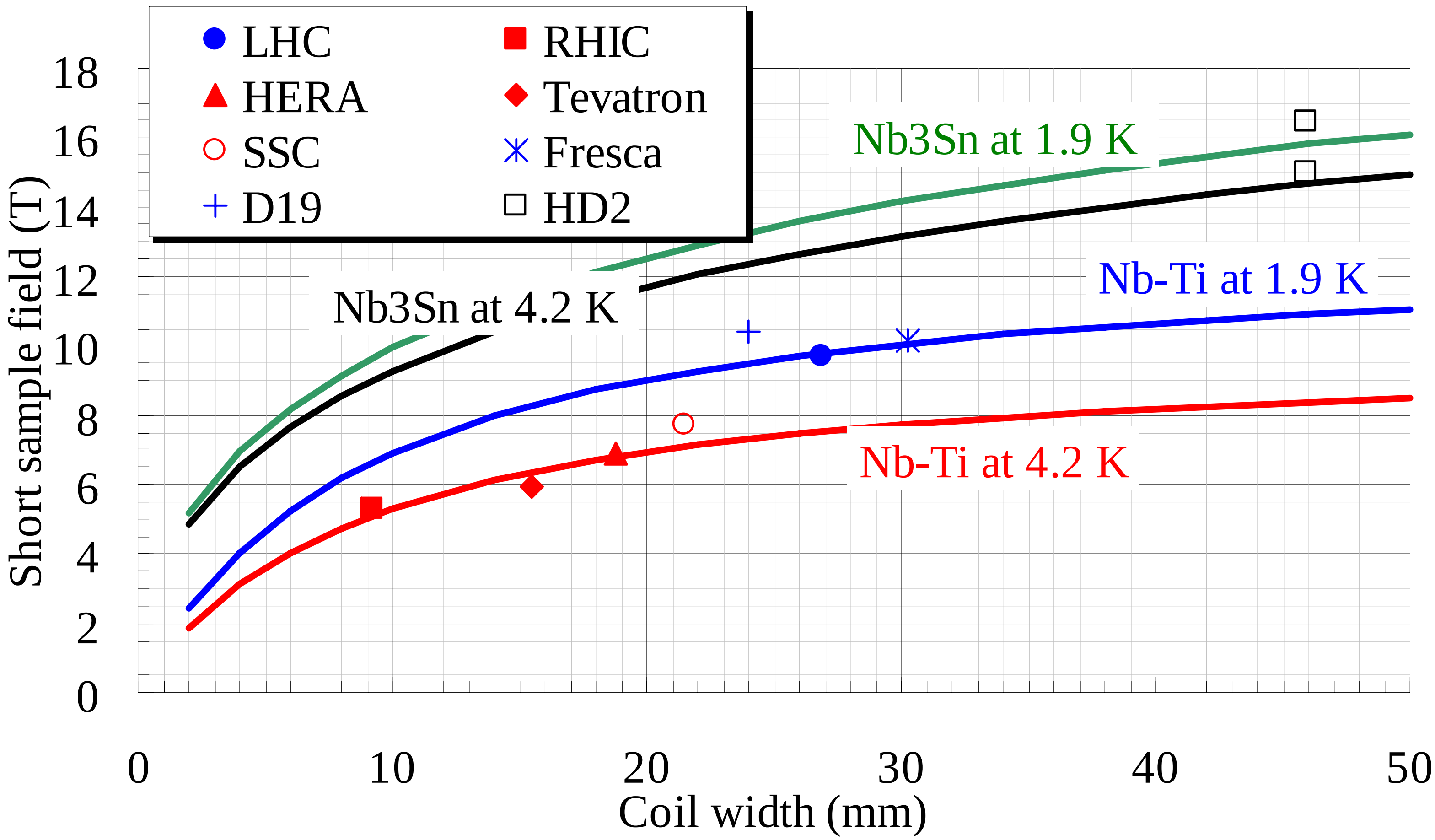
(Plus corrective terms for iron)

SHORT SAMPLE FIELD VERSUS APERTURE



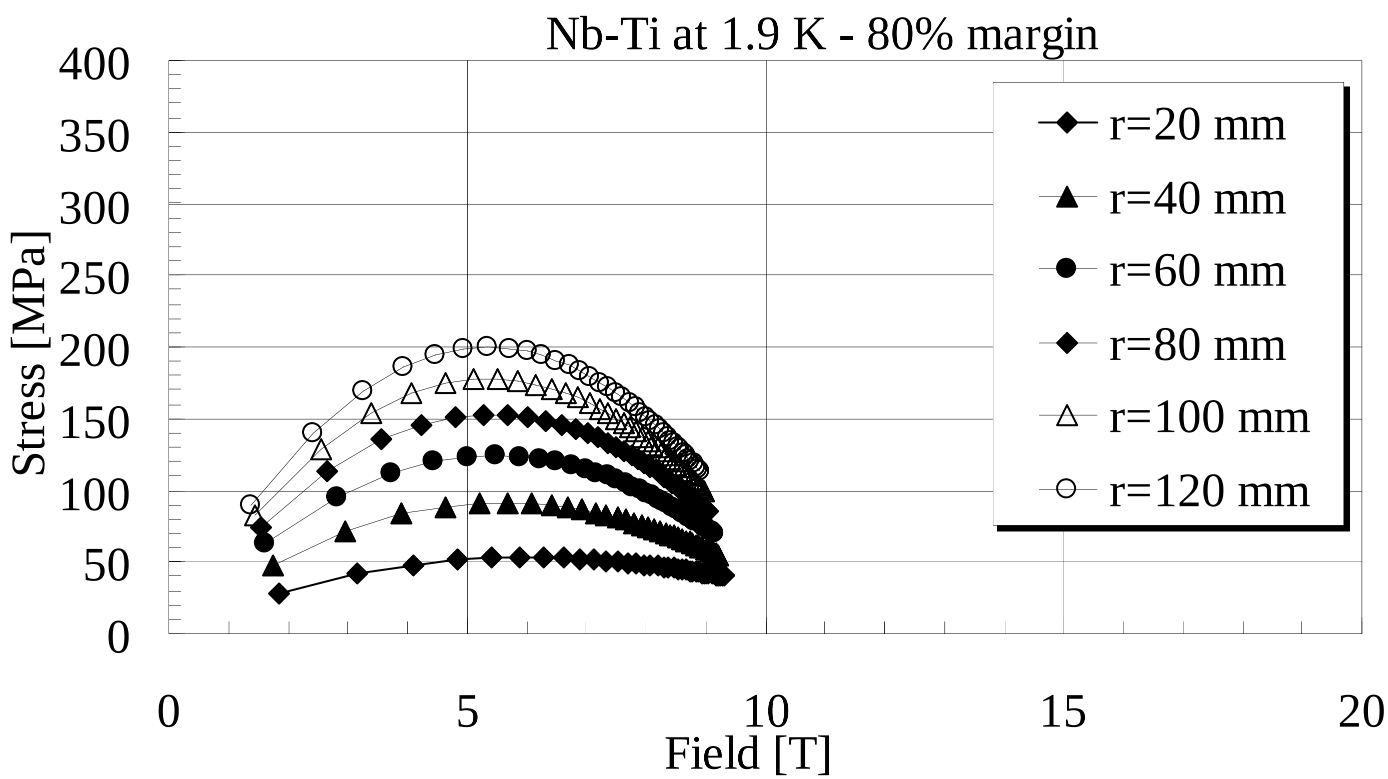
Short sample field versus coil equivalent width, for different apertures. Nb-Ti and Nb_3Sn at 4.2 K, both with filling ratio of 0.30

Short sample field versus coil width



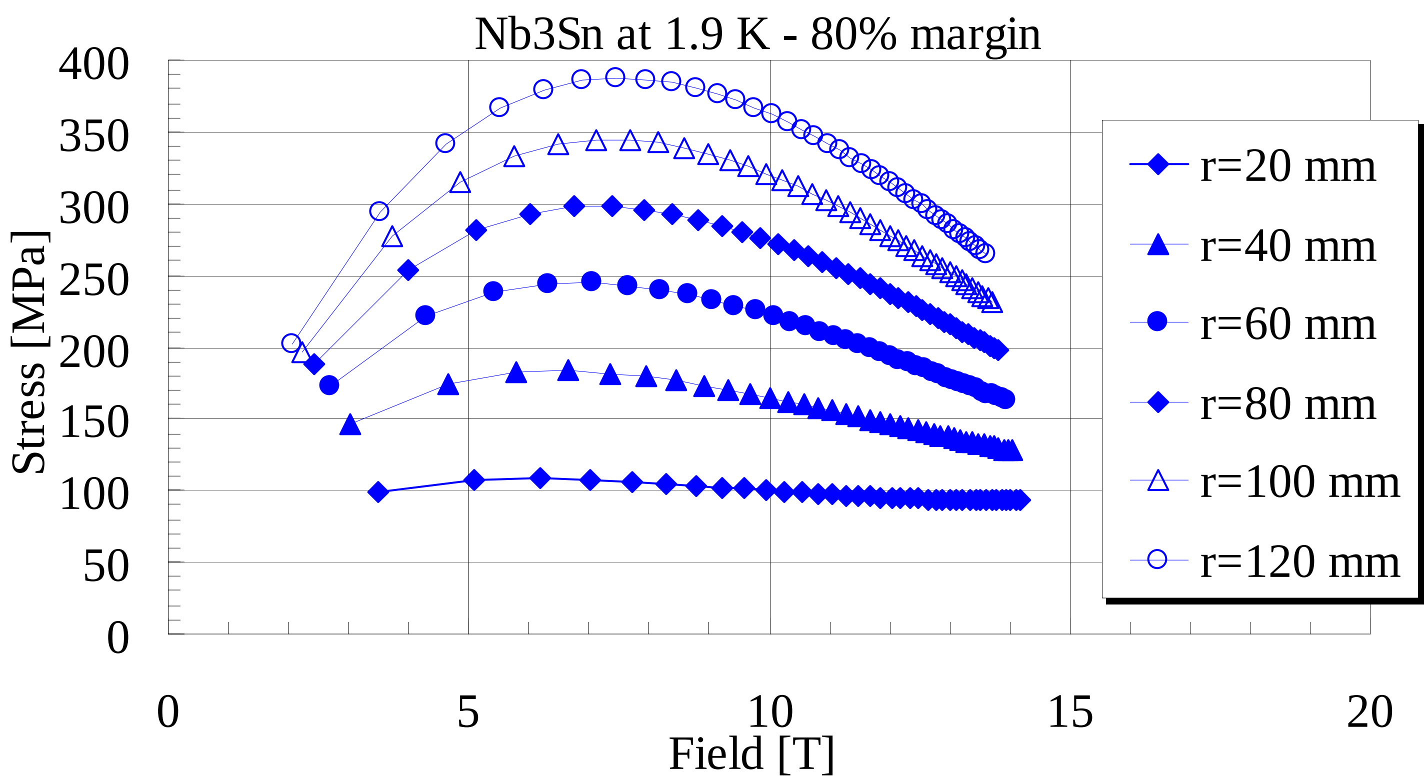
Short sample field versus coil equivalent width, for cos theta and a filing factor of 0.3,
and numerical evaluation of short sample field in some dipole lay-outs

Pressure versus short sample field, aperture and coil width



Stress in the midplane versus operational field (80% of short sample) for different aperture radii, and increasing coil widths (from 2 to 80 mm), Nb-Ti at 1.9 K with 0.3 filling factor

Pressure versus short sample field, aperture and coil width



Stress in the midplane versus operational field (80% of short sample) for different aperture radii, and increasing coil widths (from 2 to 80 mm), Nb_3Sn at 1.9 K with 0.3 filling factor

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Equations for quadrupoles

- Short sample gradient

$$G_{ss} \sim \frac{\kappa c b \gamma_0 \log\left(1+\frac{w_{eq}}{r}\right)}{1+\kappa c r\left(a_{-1}\frac{r}{w_{eq}}+1+a_1\frac{w_{eq}}{r}\right)\gamma_0 \log\left(1+\frac{w_{eq}}{r}\right)}$$

$$G_{ss} \sim \frac{\kappa c \gamma_0 \log\left(1+\frac{w_{eq}}{r}\right)}{2}\left(\sqrt{\frac{4b}{\left(a_{-1}\frac{r}{w_{eq}}+1+a_1\frac{w_{eq}}{r}\right)\gamma_0 \log\left(1+\frac{w_{eq}}{r}\right) r\kappa c}+1}-1\right)$$

- Short sample current density

$$j_{ss} = \frac{G_{ss}}{\gamma_0 \log\left(1+\frac{w_{eq}}{r}\right)}$$

- Pressure in the midplane at current *j*

$$\sigma_\varphi \sim \frac{\sqrt{3}}{16\pi} j^2 \mu_0 \mathrm{Max}_{\rho\in[r,r+w_{eq}]} \frac{1}{\rho^2}\left[\rho^4 - r^4 + 4\rho^4 \ln\left(\frac{r+w_{eq}}{\rho}\right)\right]$$

(Plus corrective terms for iron)

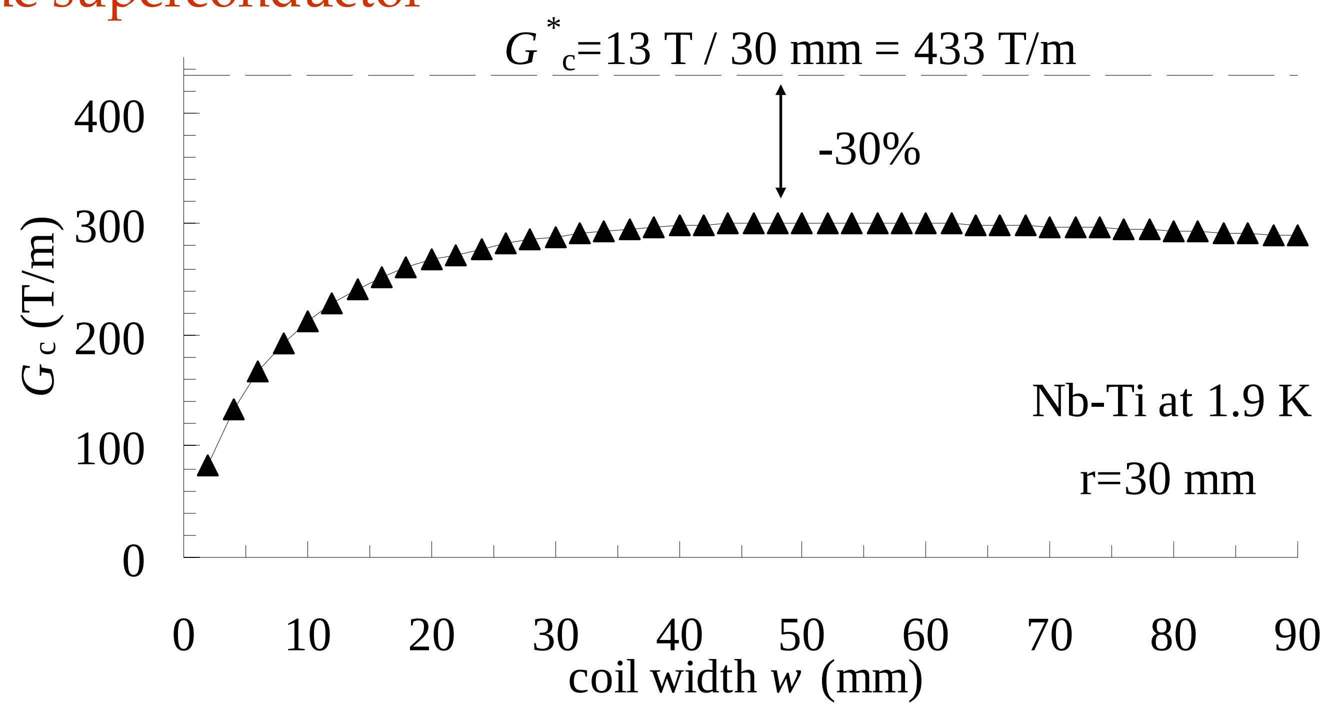
- Stored energy at current *j*

$$U \sim \frac{3\mu_0 j^2}{16} r^4 \left(\left[\left(1+\frac{w_{eq}}{r}\right)^4 - 1\right] - 4\ln\left(1+\frac{w_{eq}}{r}\right)\right)$$

(Plus corrective terms for grading and iron)

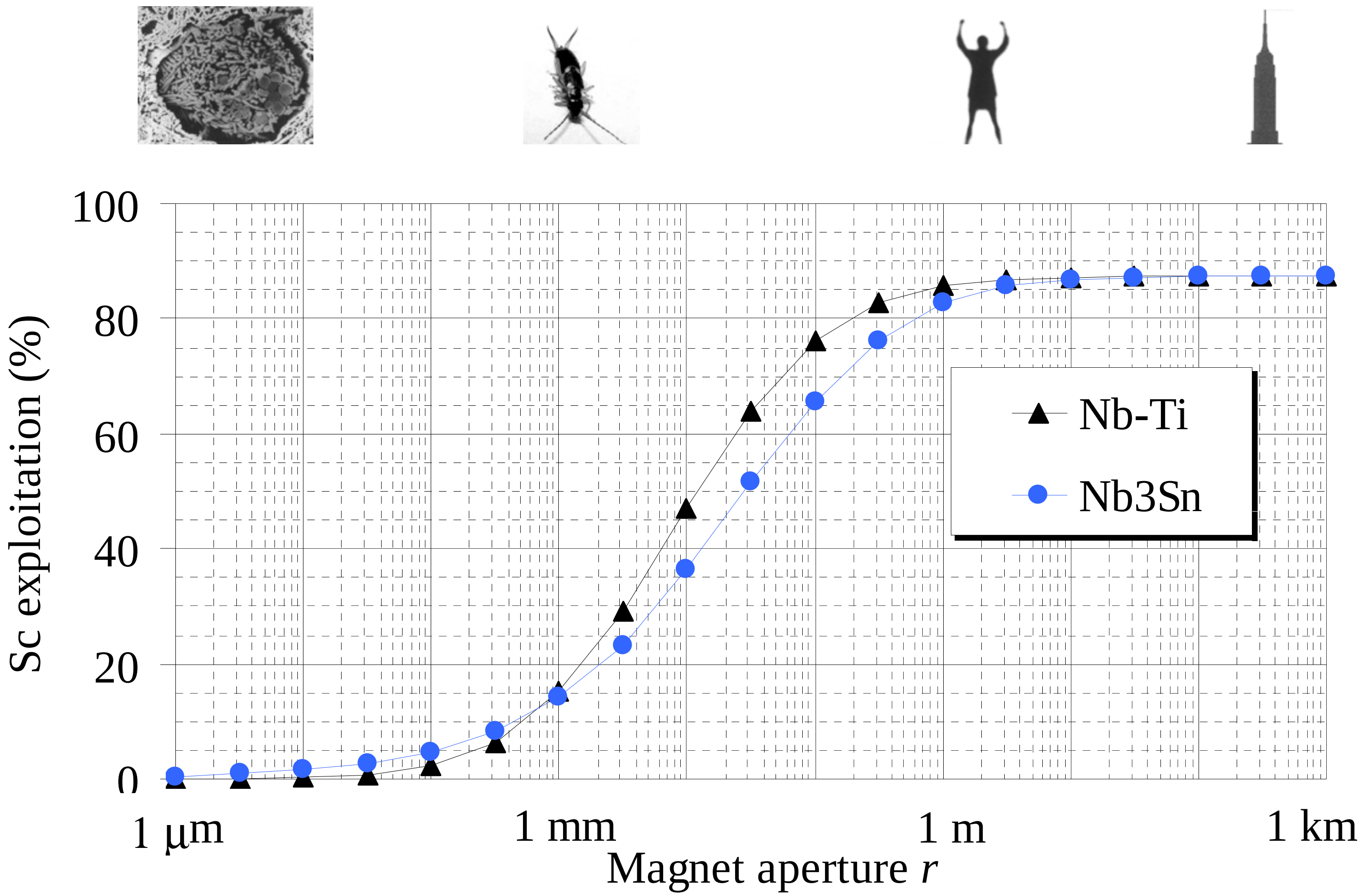
Short sample gradient vs coil width

- Adding more and more coil does not help
- Contrary to the dipole case, one cannot reach the full potential of the superconductor



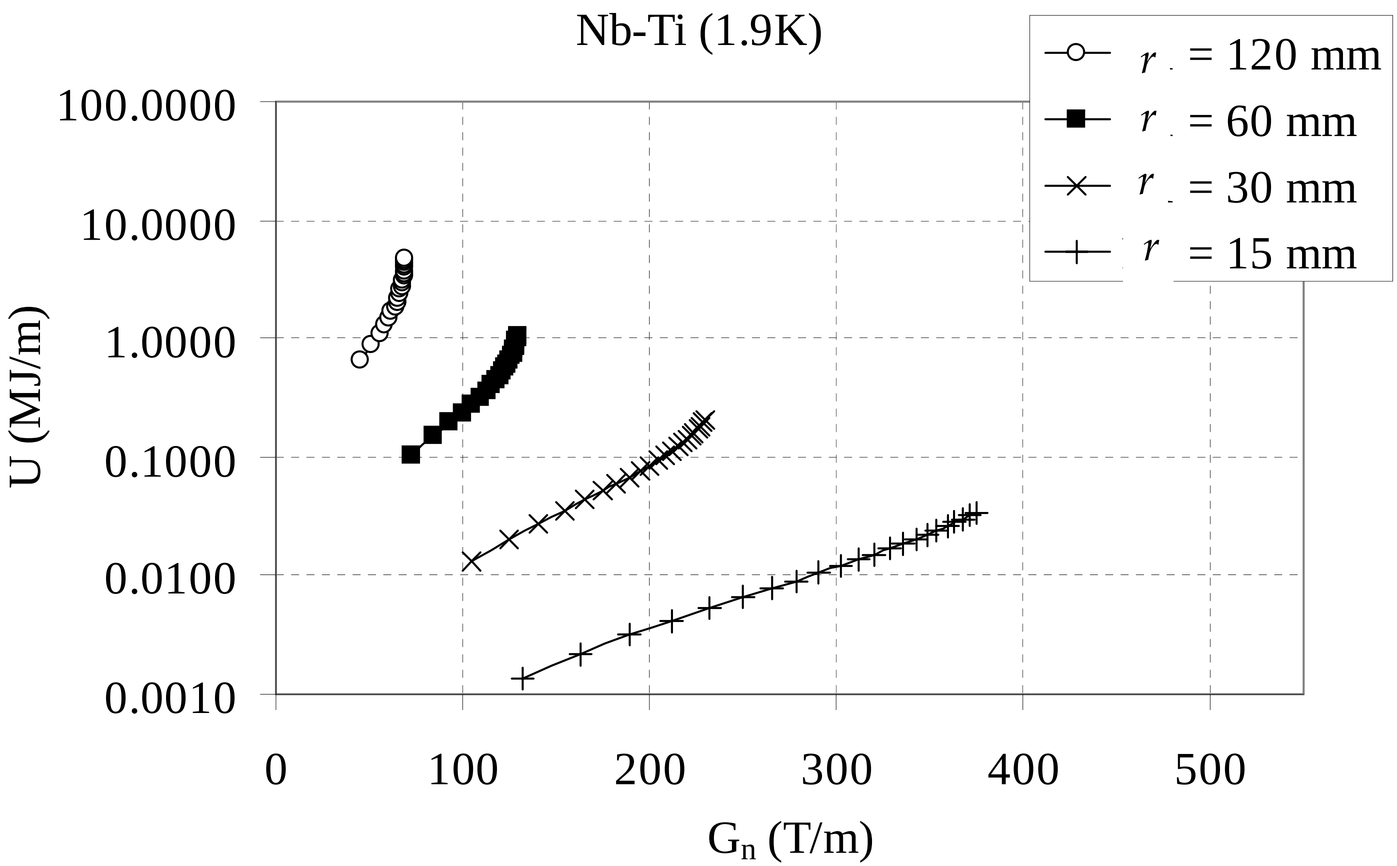
Short sample gradient versus coil width for a 30 mm radius aperture quadrupole, sector coil, Nb-Ti at 1.9 K

AN EXTREME EXTRAPOLATION OF OUR EQUATIONS …



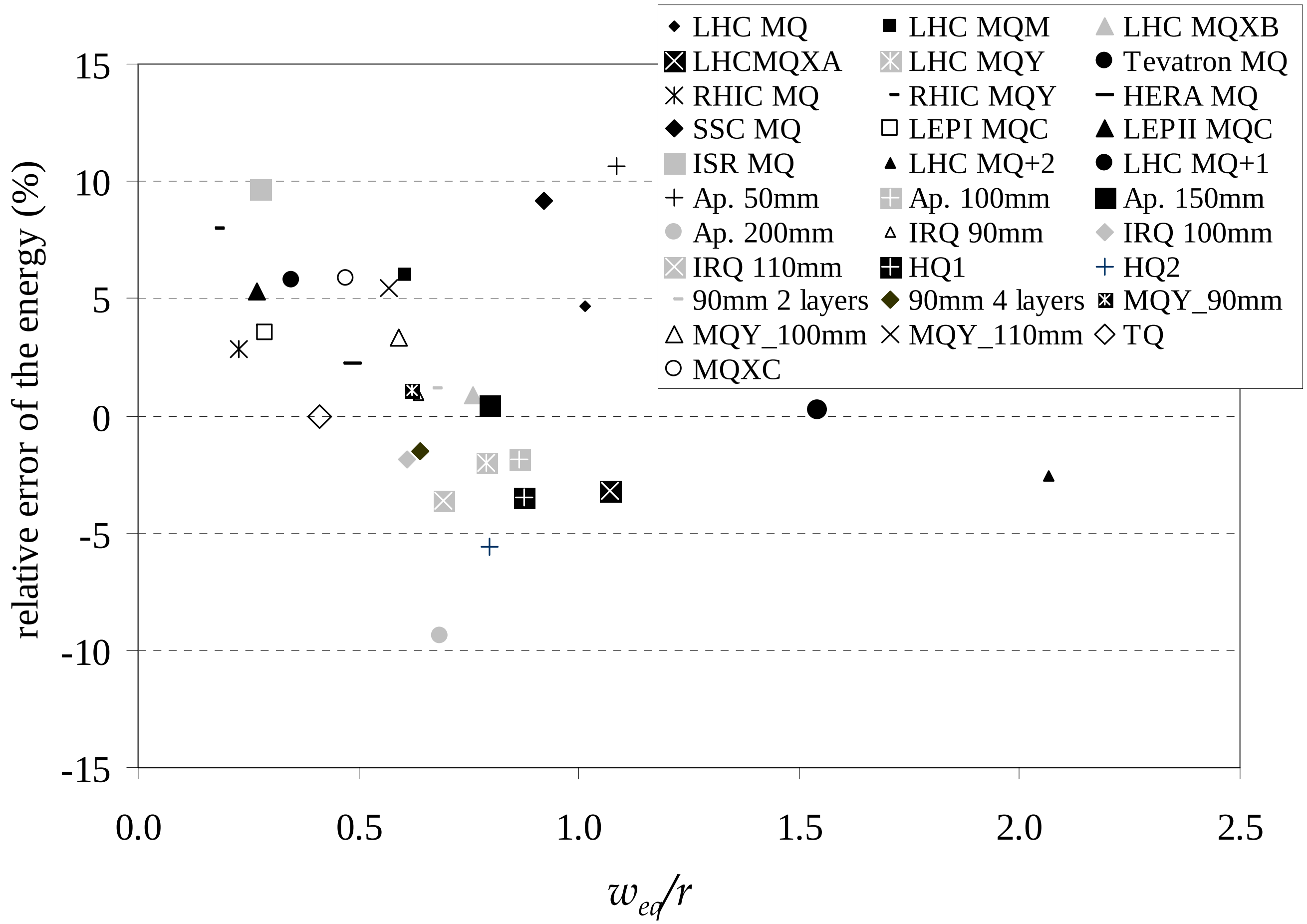
Percentage of superconductor exploitation for a quadrupole based on sector coils, versus aperture width, for Nb-Ti and Nb3Sn at 1.9 K

Stored energy in Nb-Ti



Stored energy per length in a superconducting quadrupole, Nb-Ti at 1.9 K, versus nominal gradient (80% of short sample) for different apertures and increasing coil widths

STORED ENERGY IN QUADRUPOLES



Agreement between semi-analytical approximation and numerical results of stored energy in superconducting quadrupoles

CONCLUSIONS

- Equations for estimating relevant quantities in magnet design are available
- Short sample field and current
  - In quadrupoles [L. Rossi and E. Todesco, PR-STAB **9** 102401 (2006)]
  - In sector dipoles [L. Rossi and E. Todesco, PR-STAB **10** 112401 (2007)]
  - In dipoles, other designs [L. Rossi and E. Todesco, to be presented at ASC 2007]
- Forces and pressure
  - In quadrupoles [P. Fessia, F. Regis and E. Todesco, IEEE Trans. Appl. Supercond. **17** (2007)]
  - In dipoles [P. Fessia, F. Regis and E. Todesco, to be presented at ASC 2007]
- Stored energy
  - In quadrupoles [F. Borgnolutti, A. Mailfert and E. Todesco, IEEE Trans. Appl. Supercond. in press, to be presented in EPAC07]
  - In dipoles [F. Borgnolutti, A. Mailfert and E. Todesco, in preparation]

CONCLUSIONS

- This effort of parametric studies involves not only magnet design …
  - Parametric analysis of the optics for the LHC upgrade [J. P. Koutchouk, EPAC06, J. P. Koutchouk and E. Todesco CARE HHH workshop, Valencia]
  - Parametric analysis of the energy deposition for different IR apertures and lengths [F. Broggi, F. Cerutti, C. Hoa, J.P. Koutchouk, G. Sterbini, E Wildner, PAC07 and F. Cerutti, M. Mauri, A. Mereghetti, E. Todesco, E. Wildner to be presented in EPAC08]
  - Expected field errors in superconducting quadrupoles [B. Bellesia, J. P. Koutchouk and E. Todesco, PR-STAB **10** 062401 (2007)]

- Previous related work, in the same spirit
  - Quadrupoles [G. Ambrosio, F. Ametrano, G. Bellomo, F. Broggi, L. Rossi, G. Volpini INFN-TC **95-25** (1995)]
  - Dipoles [S. Caspi, P. Ferracin, PAC05 and P. Ferracin, S. Caspi, S. Gourlay, IEEE Trans. Appl. Supercond. **16** (2006) 354]