An Overview of Neutrino Physics

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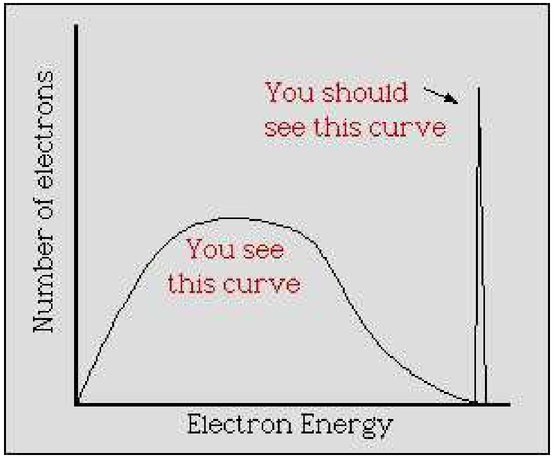
The Outline

- History and Introduction
- Neutrinos in the Standard Model
- Neutrino mass and Mixing
- Neutrino Oscillation

A brief history of the neutrino

- In 1914 Chadwick discovered that the electron energy spectrum in Nuclear β decay is continuous : ${}^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + e^-$



- But conservation of energy momentum $\Longrightarrow$ electron energy $\approx m_X c^2 - m_Y c^2 \rightarrow$ a fixed value
- Energy conservation may be violated ?
- 1930, Wolfgang Pauli proposed the existence of an electrically neutral particle, which he called neutron, as a desparate remedy to save the energy conservation principle

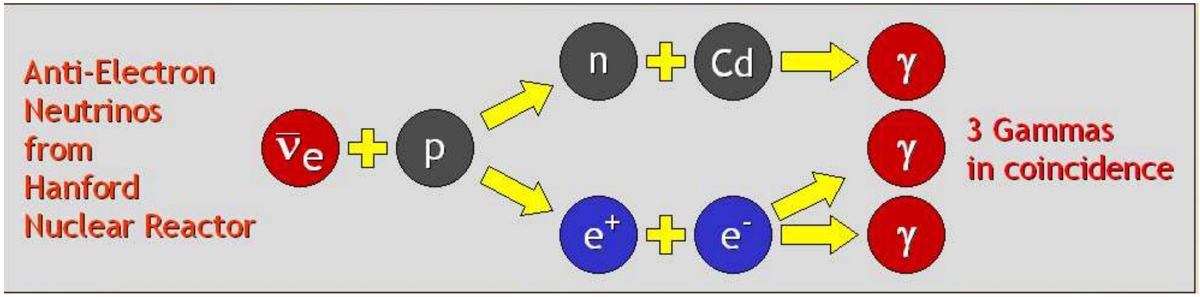
A brief history of the neutrino

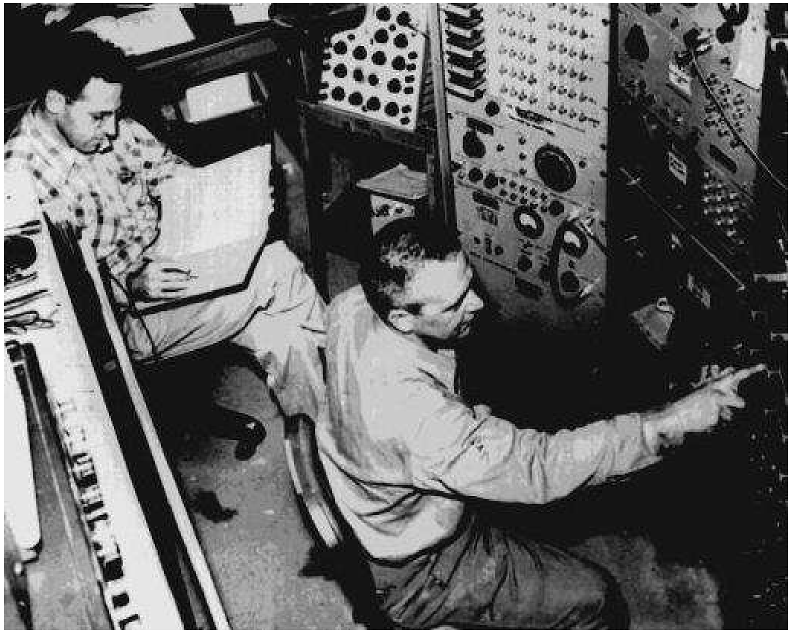
- Neutron was discovered in 1932 by Chadwick
- The name Neutrino (the little neutral one) was given by Fermi in 1933
- In 1934 Fermi gave a theory for explain β decay — weak interactions
- Neutrinos are weakly interacting
- Their mass is much smaller than the electron mass – may be massless
- They do not carry any electric charge
- They are fermions with spin 1/2.
- Extremely difficult to detect them experimentally

" I have postulated something which can never be detected — Pauli"

The Ghost Riders

- Experimental Discovery of ν_e by Reines and Cowans in 1956
- 25 years after it was postulated !!





- Clyde Cowan (1919-1974)
- Fred Reines (1918 - 1998)
- Nobel Prize to F. Reines in 1995

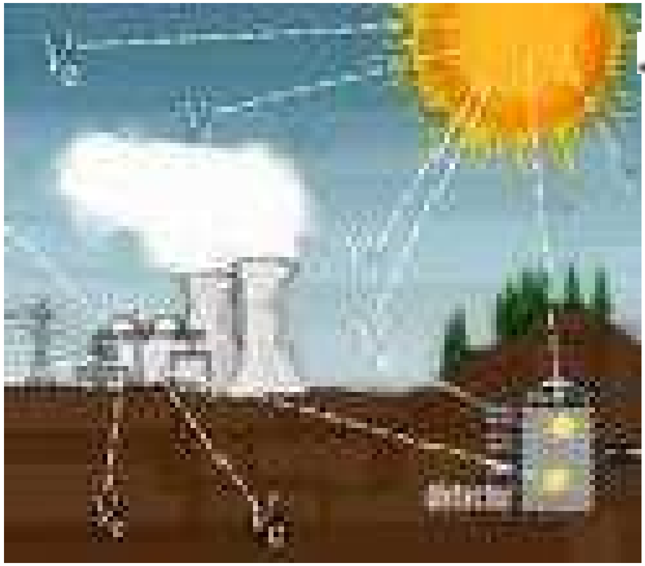
More than one type of Neutrinos

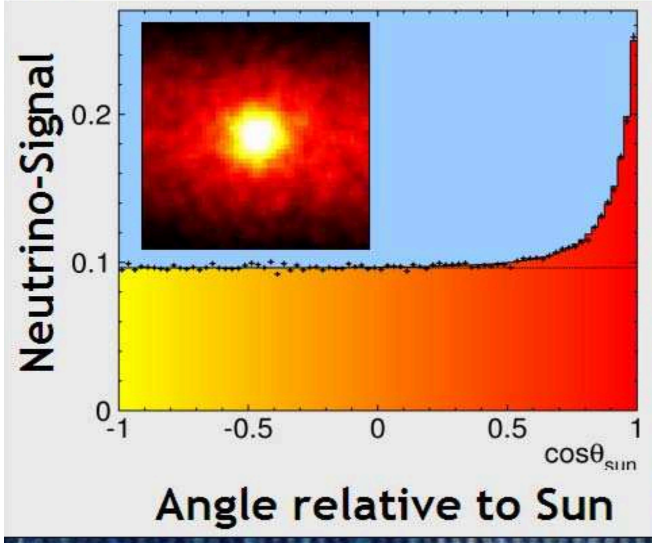
- In 1960 Bruon Pontecorvo suggested that the neutrino produced in $\pi^+ \to \mu^+ + \nu$ may be different from the neutrino produced in β-decay
- Can the neutrinos produced in pion decays, can subsequently be converted into electrons ?
- The experiment was carried out in Brookhaven in 1962 using a beam of 15 GeV protons from AGS accelerator and the muon neutrino (ν_μ) was discovered.
- Nobel prize to Lederman. Schwartz and Steinberger in 1988
- The third type of neutrino ν_τ was discovered in 2000 by the DONUT experiment
- From $Z \to \nu\bar{\nu}$, LEP experiment gives $N_\nu = 2.994 \pm 0.012$
- Big Bang Nucleosynthesis give $N_\nu = 3.14 \pm 0.7$
- There exist atleast three types of neutrinos
- Are there more ?

Neutrinos are Unique

- The second most abundant particle in the Universe
  - Cosmic microwave background photons: 400 / cc
  - Cosmic microwave neutrinos: 330 /cc
- The lightest massive particle
  - A million times lighter than electron
  - No direct mass measurement yet
  - Can change from one type to another $\rightarrow$ neutrino oscillation $\rightarrow$ very small mass and flavour mixing
- Interacts weakly
  - This allows them to carry information directly from astrophysical objects
  - But this also makes their detection extremely difficult
- Can be its own antiparticle (?)

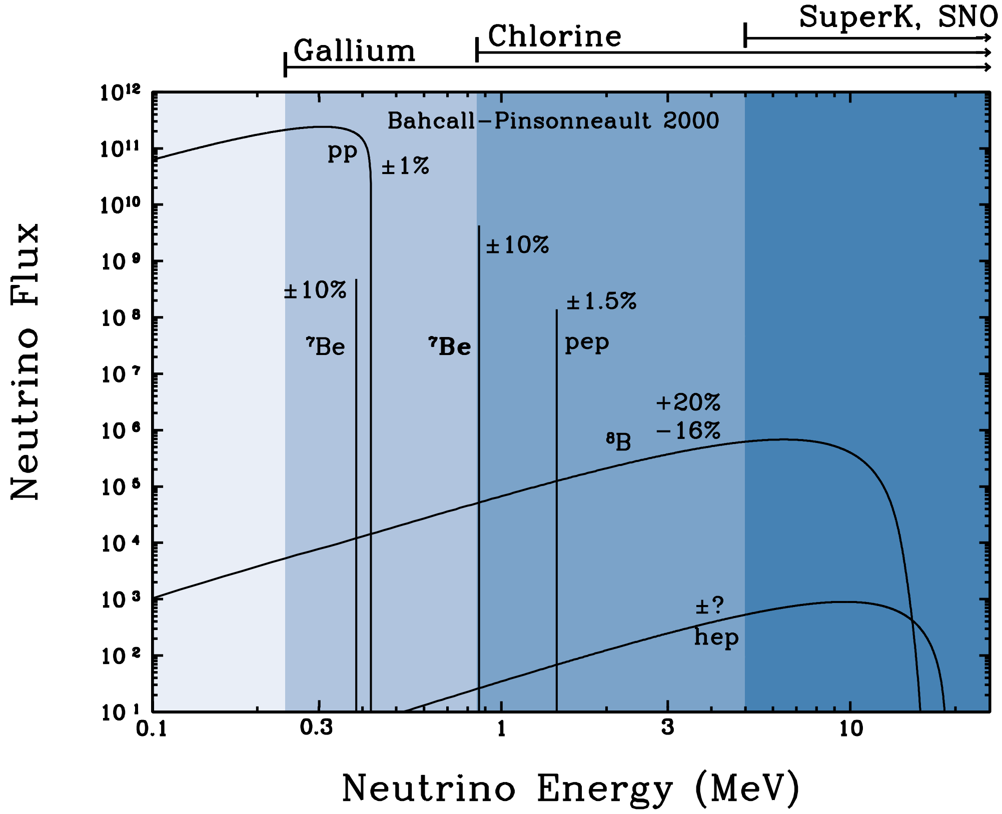
Neutrinos are everywhere





- The Sun is an intense source of MeV neutrinos

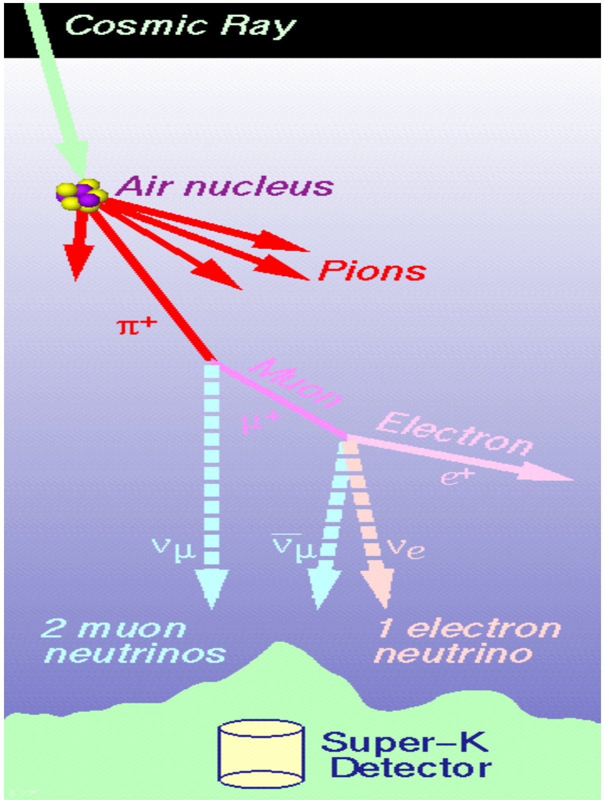
$$4p \rightarrow^4 He + 2e^+ + 2\nu_e + 28\text{MeV}$$



- Solar Neutrino Spectrum

Neutrinos are everywhere

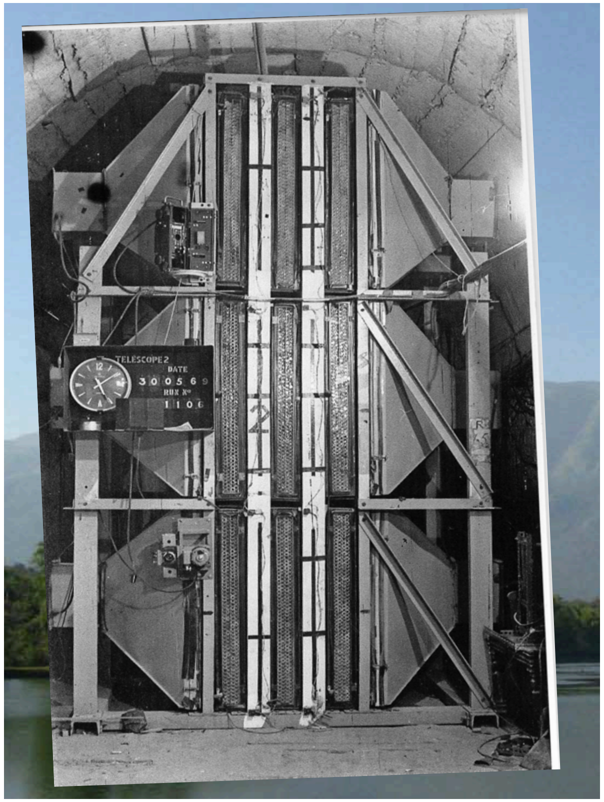
- Neutrinos are generated in our Atmosphere



Cosmic Ray + $A_{air} \rightarrow \pi^{+} + \ldots$

$\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$

$\mu^{+} \rightarrow e^{+} + \bar{\nu}_{\mu} + \nu_{e}$



One of the first detections Kolar Gold Mine, India, 1965

Neutrinos are everywhere

- They come from inside our Earth

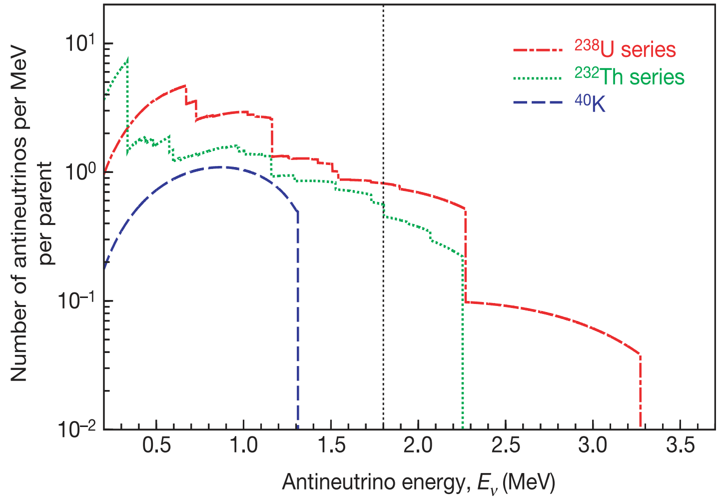
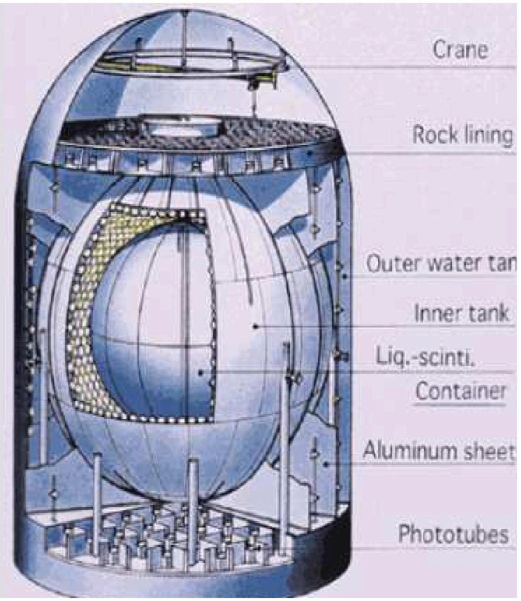
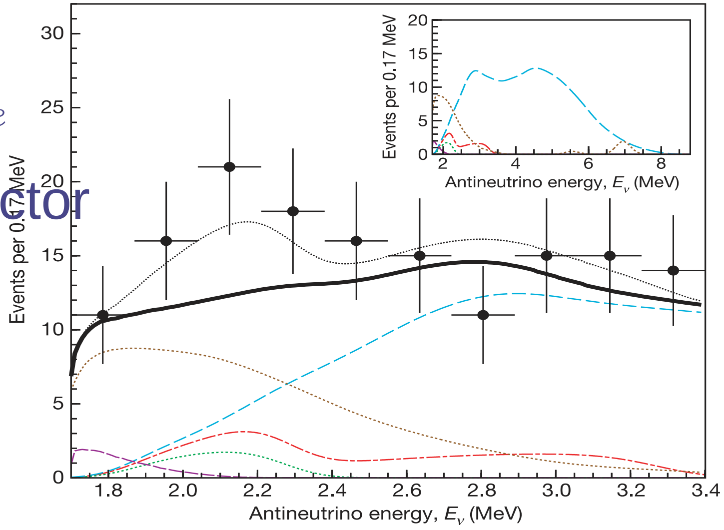


Figure 1 | The expected ^{238}U, ^{232}Th and ^{40}K decay chain electron antineutrino energy distributions. KamLAND can only detect electron antineutrinos to the right of the vertical dotted black line; hence it is insensitive to ^{40}K electron antineutrinos.

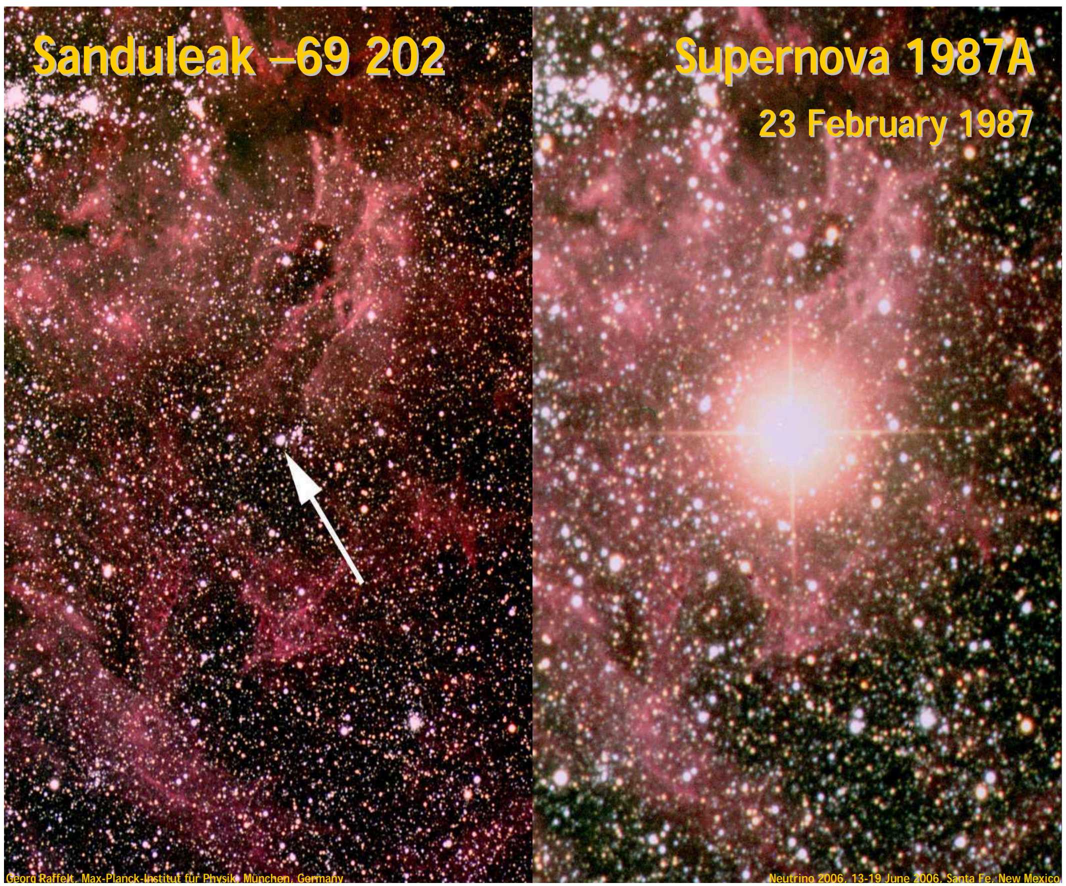


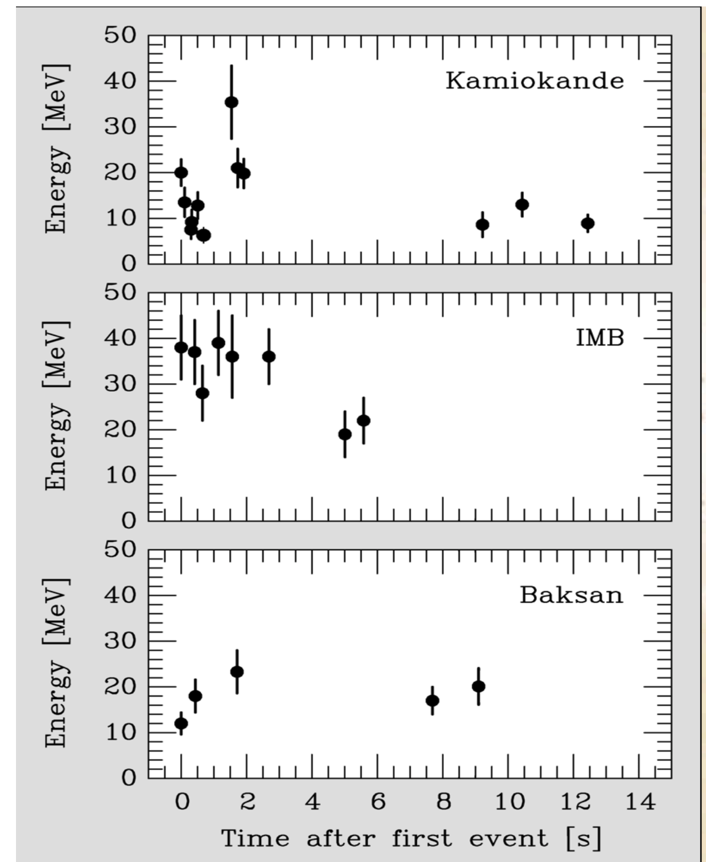
- The U, Th and K inside the earth produce $\bar{\nu}_e$
- Geoneutrinos detected by KamLAND detector in Japan



Neutrinos are everywhere

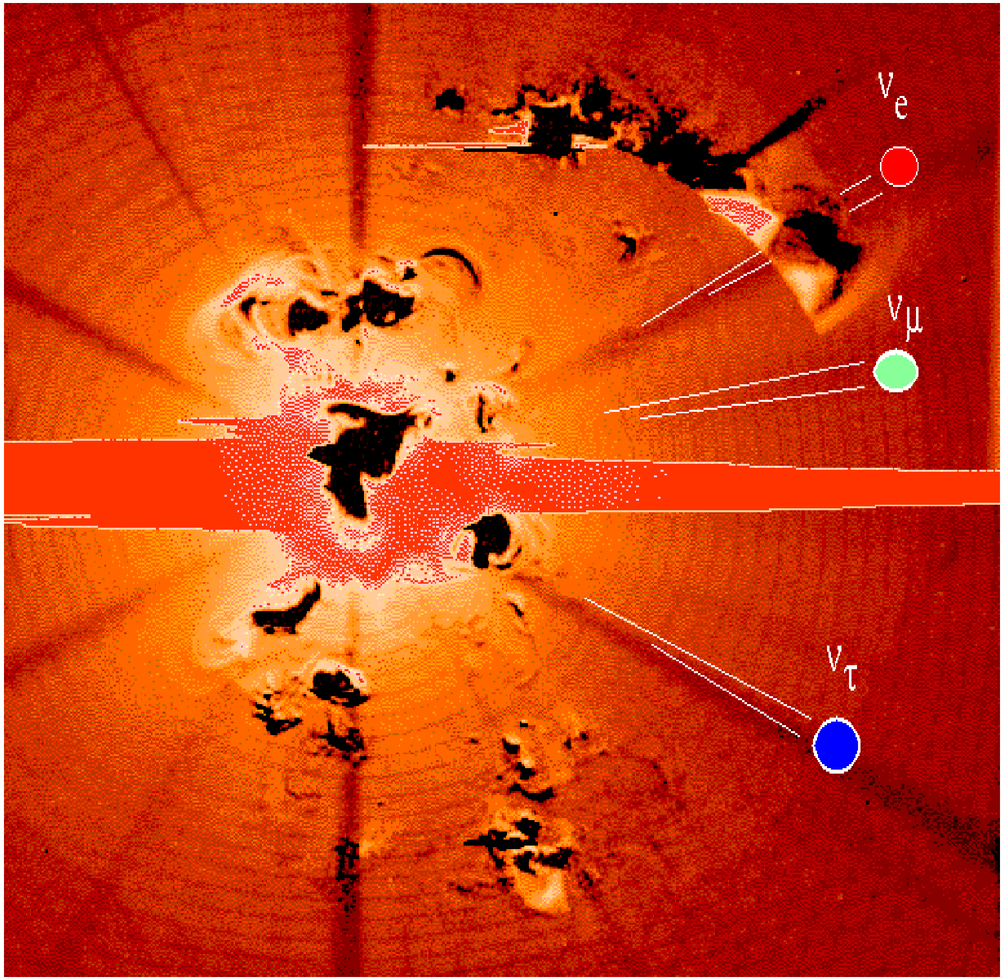
- They come from SuperNovae explosion
- Only about 1% of SN energy is released in photons
- Remaining 99% comes out as neutrinos

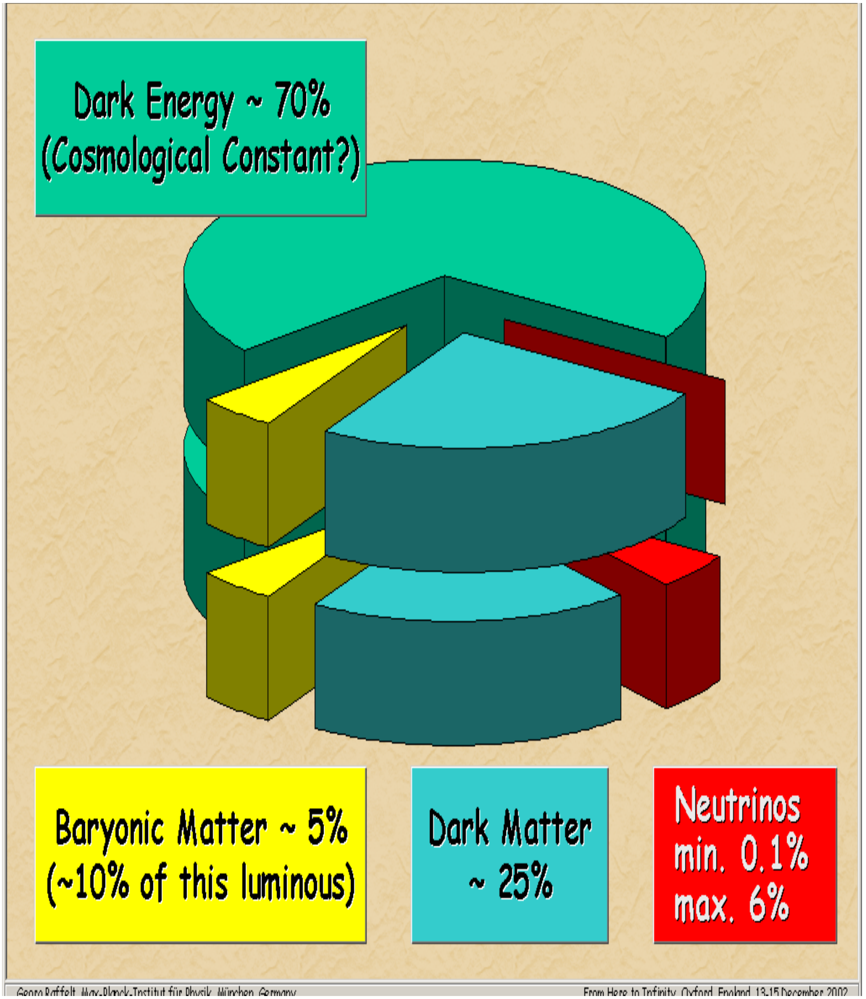




Neutrinos are everywhere

- They are generated during BigBang





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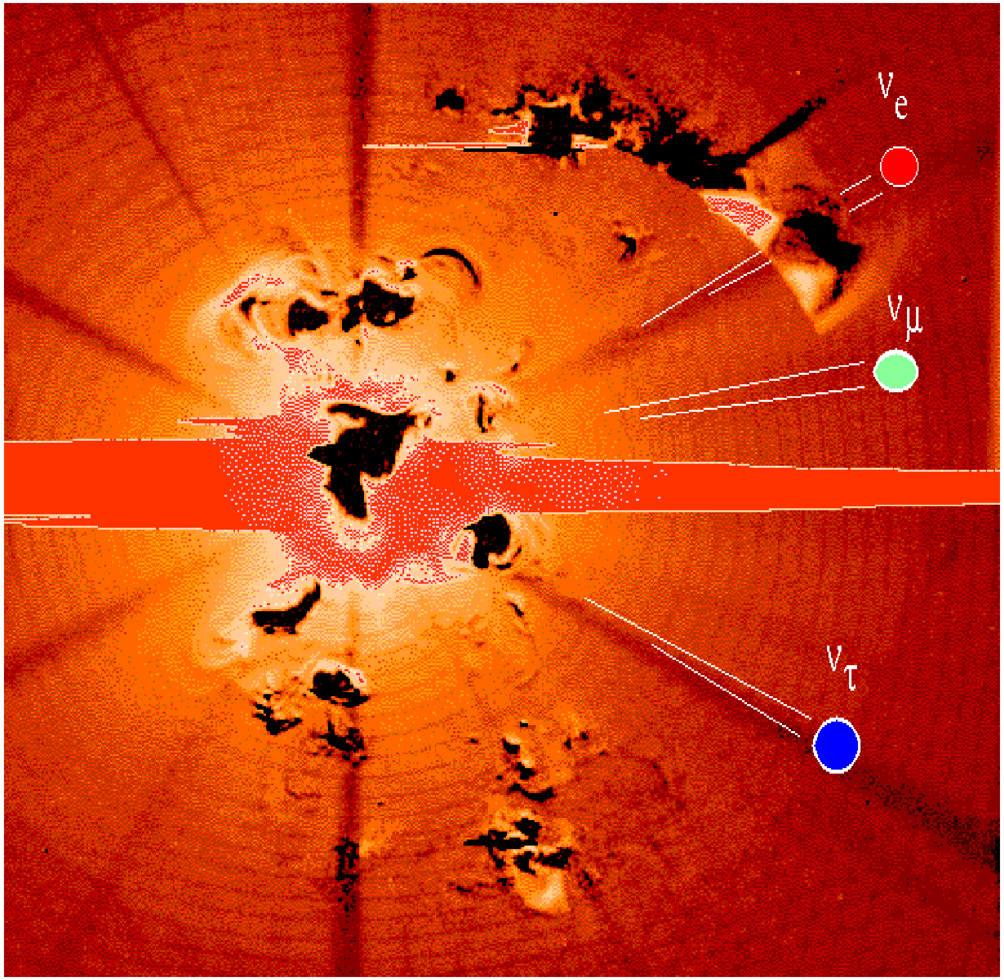
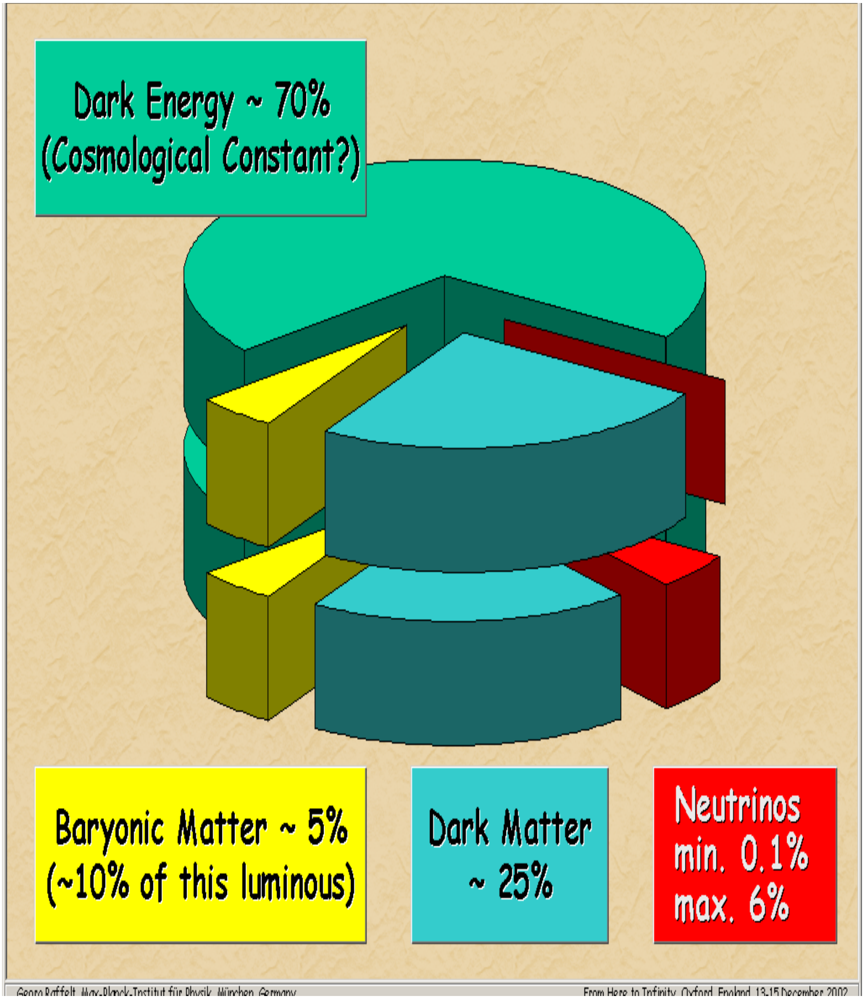
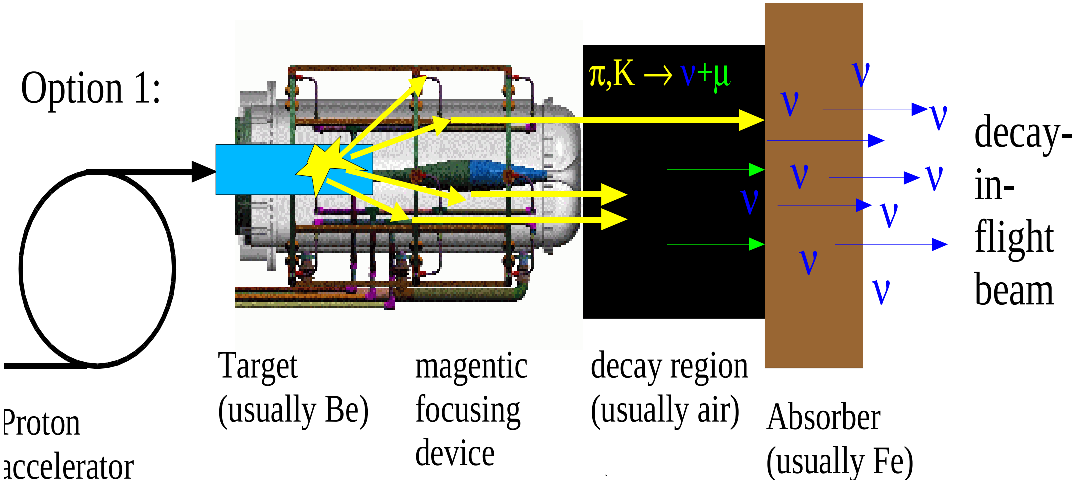
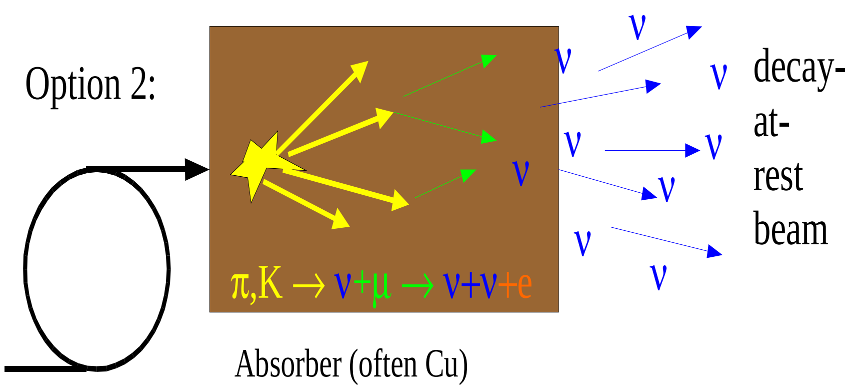




Neutrinos are everywhere

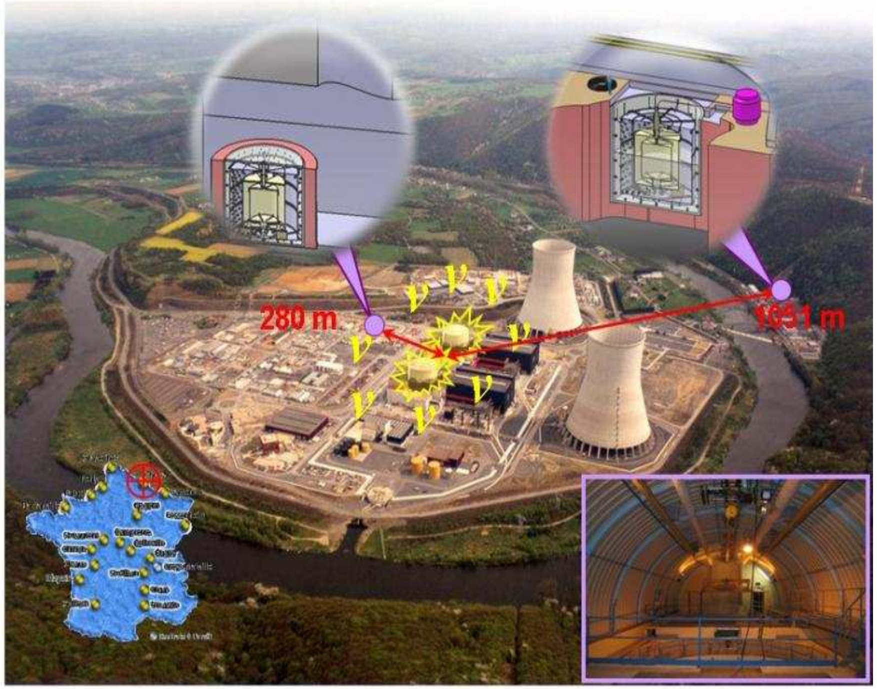
- They can be produced in particle accelerators



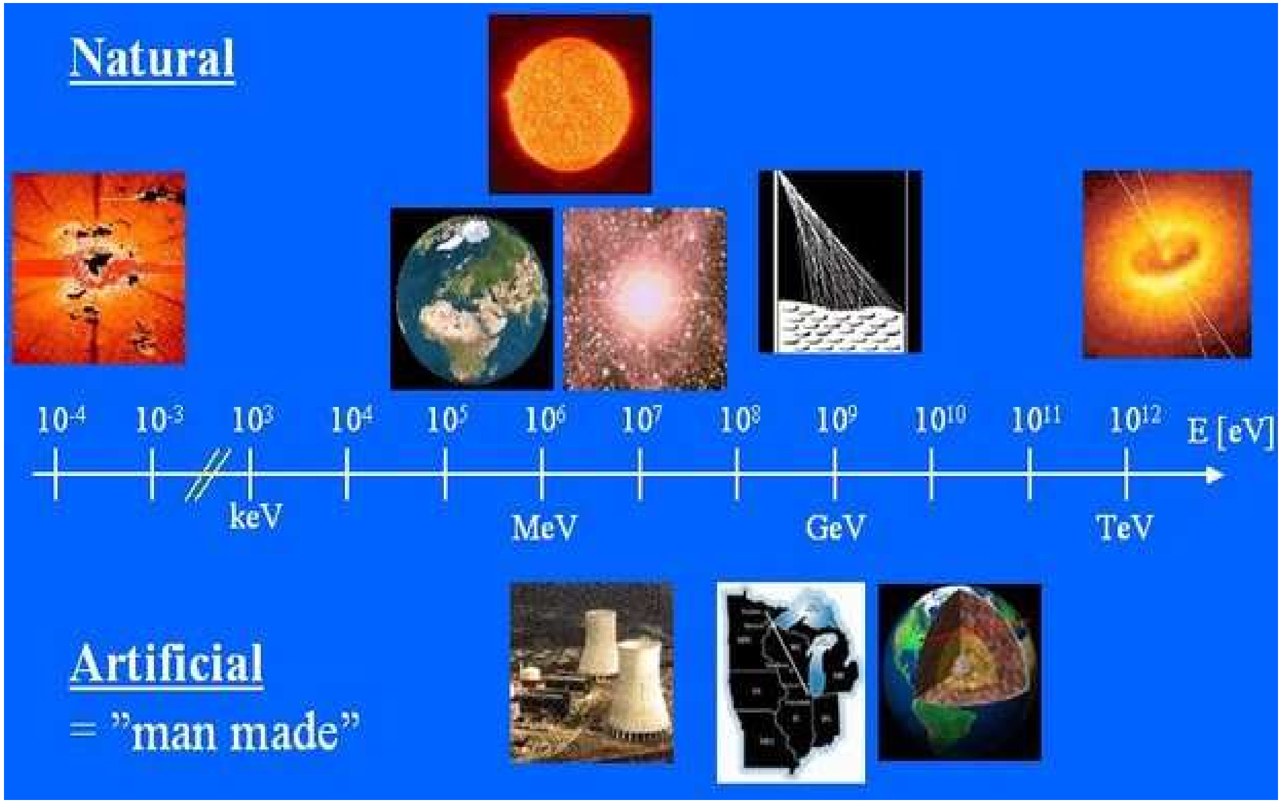


Neutrinos are everywhere

- They are also produced in nuclear reactors



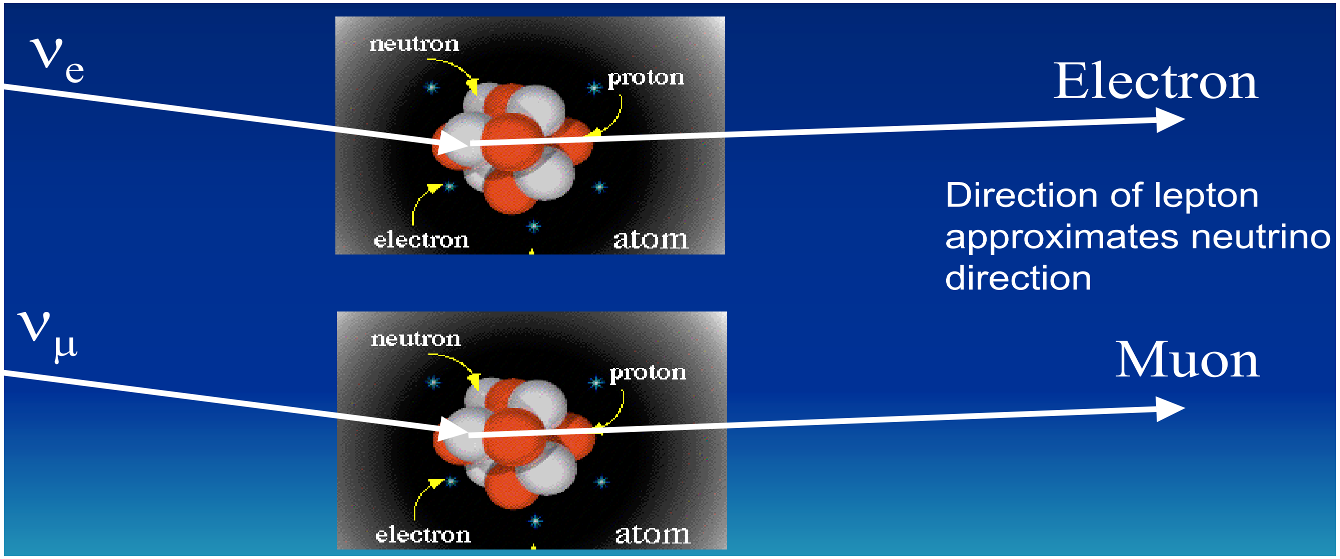
Neutrinos are everywhere



- 1 eV = 1.6 $\times 10^{-19}$ Joule   1 MeV = 10^6 eV   1 GeV = 10^9 eV
- Span an energy range from KeV - TeV

Neutrino Detection Principle

- A neutrino is detected by observing the product of its interaction with matter



- Interaction strength – very weak
- Huge detectors

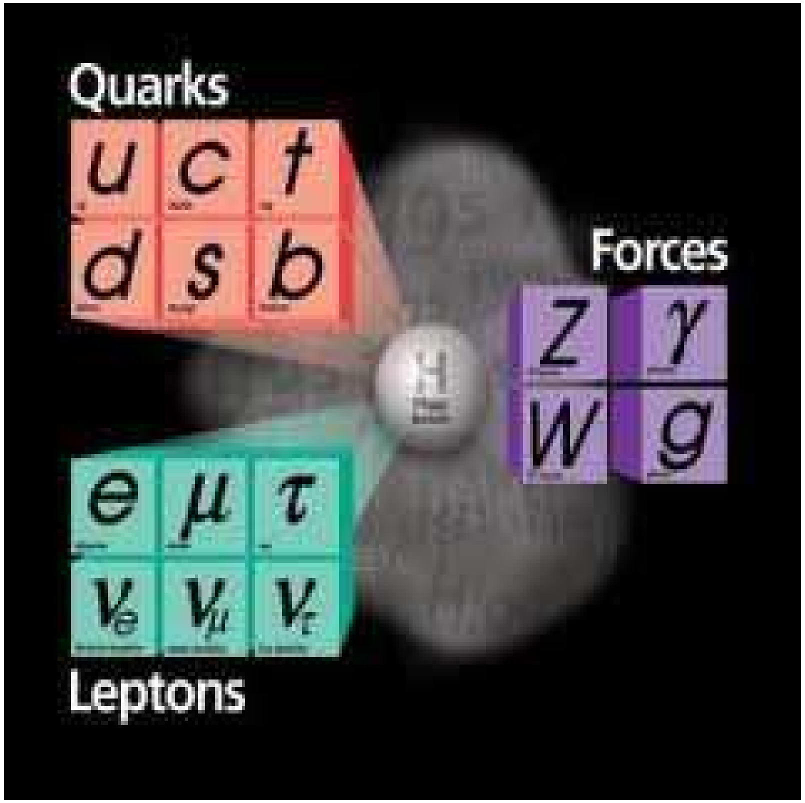
The elusive Neutrino

- Since neutrinos are weakly interacting the detectors has to be
  - BIG : to have enough statistics
  - DEEP : to reduce cosmic radiation
  - CLEAN : to reduce radioactive background

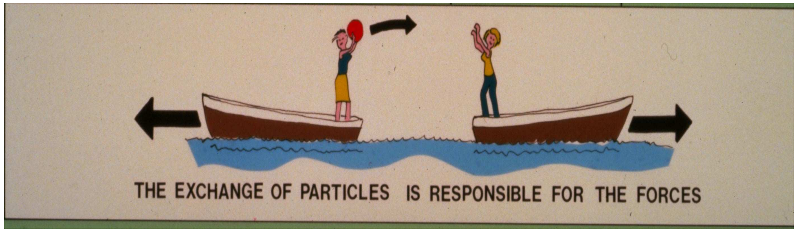
- SuperKamiokande
- 50,000 litres of water
- 10,000 photomultiplier tube
- 10^{25} neutrinos passing through/day
- Neutrino events /day 5-10

The Standard Model of Particle Physics

- Aim: To describe the Fundamental Particles and their Interaction
- Guided by symmetry principles governing strong, weak and electromagnetic interaction : $SU(3)_C \times SU(2)_L \times U(1)_Y$



Force	Strength	Carrier
Strong nuclear	1	Gluons
Electromagnetic	.001	Photon
Weak nuclear	.00001	Z^0, W^+, W^-



- Contains the antiparticles also

Neutrinos in the Standard Model

- Neutrinos are weakly interacting : $SU(2)_L \times U(1)_Y$
- Neutrinos are part of lepton doublets l_l in 3 flavours

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$:(2:-1)

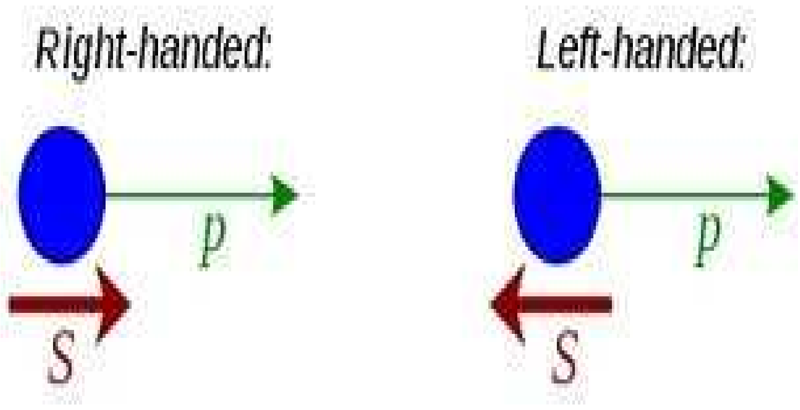
- Right handed leptons occur as singlets : $e_R{}^-, \mu_R{}^-, \tau_R{}^-$:(1,-2)
- The quantum numbers are related as Q = T_3 + $Y/2$
- The corresponding antiparticles are also there

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} e^+ \\ \nu_e{}^c \end{pmatrix}_R$$

- Neutrinos are left-handed and antineutrinos are righthanded
- No right handed neutrino $\rightarrow$ Parity Violation

Parity Violation in Weak Interactions

- Parity refers to the symmetry of space inversion
- In 1956 Lee and Yang proposed violation of parity in weak interactions to solve the $\tau - \theta$ puzzle.
- 1957 Wu et al. discovered Parity violation in β-decay of ^{60}Co
- 1957 : Landau, Lee and Yang, Salam proposed that neutrinos are massless and only left-handed



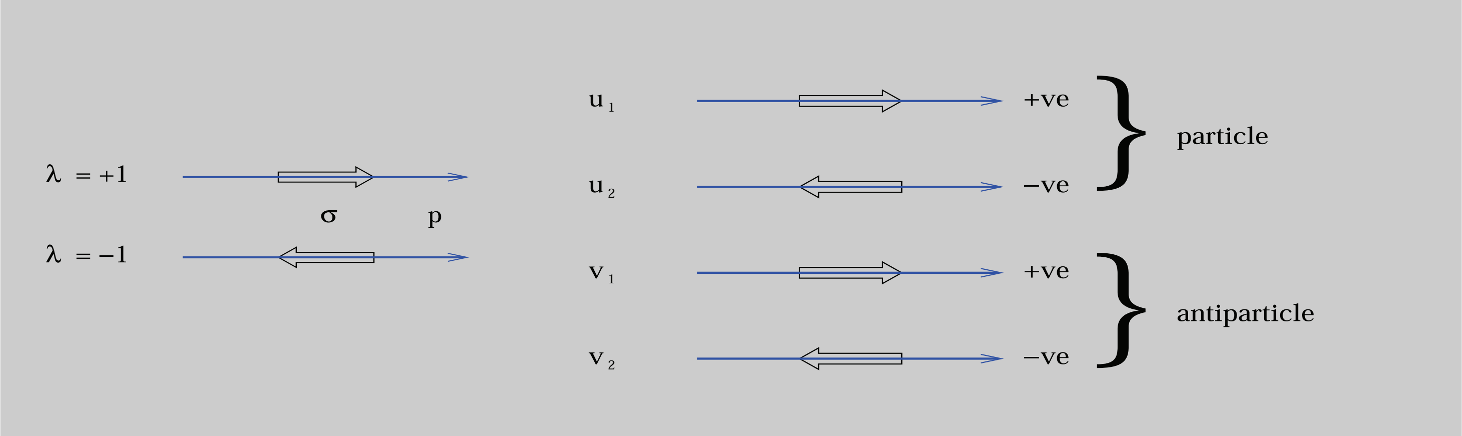
- In 1958 Goldhaber et al. measured the neutrino helicity to be left-handed
- V-A form of weak interaction

Helicity

- Helicity is the projection of spin along the direction of motion

$$\vec{\Sigma}.\hat{p} = \begin{pmatrix} \vec{\sigma}.\hat{p} & 0 \\ 0 & \sigma.\hat{p} \end{pmatrix}$$

- Helicity operator commutes with the Dirac Hamiltonian $H = \alpha.\mathbf{p} + \beta m$
- $\Longrightarrow$ Energy eigenstates of Dirac Hamiltonian are also eigenstates of helicity,



Chirality or Handedness

- The chirality ooperator is γ_5

Consider now the chirality/handedness operator in the Pauli-Dirac representation:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

which satisfies:

$$\{\gamma_5, \gamma^\mu\} = 0$$

This anti-commutation relationship is true in any Dirac matrix representation. Lets evaluate the commutator of the chirality operator with the Dirac hamiltonian:

$$[\gamma_5, H] = [\gamma_5, \vec{\alpha}\cdot\vec{p} + m\beta] = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix} m & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m \end{pmatrix} - \begin{pmatrix} m & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m \end{pmatrix}\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \Rightarrow$$

$$[\gamma_5, H] = \begin{pmatrix} \vec{\sigma}\cdot\vec{p} & -m \\ m & \vec{\sigma}\cdot\vec{p} \end{pmatrix} - \begin{pmatrix} \vec{\sigma}\cdot\vec{p} & m \\ -m & \vec{\sigma}\cdot\vec{p} \end{pmatrix} \Rightarrow$$

$$[\gamma_5, H] = 2m\begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

- Thus chirality does not commute with the Dirac Hamiltonian if mass is non-zero

Physical meaning of Chirality

Lets now investigate the physical meaning of the chirality. Consider the massless Dirac equation:

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = 0$$

Let $\Psi(x) = u(\vec{p})e^{-ip^{\mu}x_{\mu}}$ be a solution of the Dirac equation. By substituting we get that:

$$(\gamma^0 p_0 - \vec{\gamma}\cdot\vec{p})u(\vec{p}) = 0 \Rightarrow$$

$$\gamma^0 p_0 u(\vec{p}) = \vec{\gamma}\cdot\vec{p}\,u(\vec{p}) \Rightarrow$$

$$\gamma_5\gamma^0\gamma^0 p_0 u(\vec{p}) = \gamma_5\gamma^0\vec{\gamma}\cdot\vec{p}\,u(\vec{p}) \Rightarrow$$

$$p_0\gamma_5 u(\vec{p}) = \gamma_5\gamma^0\vec{\gamma}\cdot\vec{p}\,u(\vec{p}) \qquad (3)$$

Physical meaning of Chirality

If this is a positive energy solution then we have that $p^0 > 0$ and (3) becomes:

$$\gamma_5 u(\vec{p}) = \gamma_5 \gamma^0 \vec{\gamma} \cdot \hat{p}\, u(\vec{p}) \qquad (4)$$

If this is a negative solution then, $p^0 < 0$ and

$$\gamma_5 u(\vec{p}) = -\gamma_5 \gamma^0 \vec{\gamma} \cdot \hat{p}\, u(\vec{p}) \qquad (5)$$

Lets compute the matrix product on the right side:

$$\gamma_5 \gamma^0 \vec{\gamma} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} = \vec{\Sigma} \Rightarrow$$

$$\vec{\Sigma} = \gamma_5 \gamma^0 \vec{\gamma} \qquad (6)$$

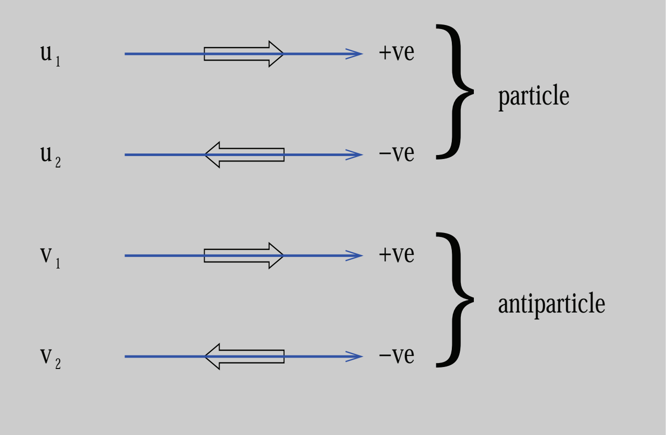
$$p^0 > 0 \quad \Rightarrow \quad \gamma_5 u(\vec{p}) = \begin{pmatrix} \vec{\sigma} \cdot \hat{p} & 0 \\ 0 & \vec{\sigma} \cdot \hat{p} \end{pmatrix} u(\vec{p}) = \vec{\Sigma} \cdot \hat{p}\, u(\vec{p}) \quad (7)$$

and

$$p^0 < 0 \quad \Rightarrow \quad \gamma_5 u(\vec{p}) = -\begin{pmatrix} \vec{\sigma} \cdot \hat{p} & 0 \\ 0 & \vec{\sigma} \cdot \hat{p} \end{pmatrix} u(\vec{p}) = -\vec{\Sigma} \cdot \hat{p}\, u(\vec{p}) \quad (8)$$

Chirality and Helicity

- For m=0 eigenstates of chirality are eigenstates of helicity



- Let us define the projection operators $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$ for left and right handed states
- $p_0 > 0$

$$\gamma_5 u_1(p) = \vec{\Sigma}.\hat{p} u_1(p) = u_1(p) \Longrightarrow P_L u_1(p) = 0, P_R u_1(p) = u_1(p)$$
$$\gamma_5 u_2(p) = \vec{\Sigma}.\hat{p} u_2(p) = -u_2(p) \Longrightarrow P_L u_2(p) = u_2(p), P_R u_2(p) = 0$$

- $p_0 < 0$

$$\gamma_5 v_1(p) = -\vec{\Sigma}.\hat{p} v_1(p) = -v_1(p) \Longrightarrow P_L v_1(p) = v_1(p), P_R v_1(p) = 0$$
$$\gamma_5 v_2(p) = -\vec{\Sigma}.\hat{p}_2 v(p) = v_2(p) \Longrightarrow P_L v_2(p) = 0, P_R v_2(p) = v_2(p)$$

- P_L projects -ve (+ve) helicity states for particles (antiparitcles)
- P_R projects +ve (-ve) helicity states for particles (antiparticles)

Chirality and Helicity

- $\Longrightarrow$ Left handed particles and right-handed antiparticles have -ve helicity
- $\Longrightarrow$ Right-handed particles and left-handed antiparticles have +ve helicity

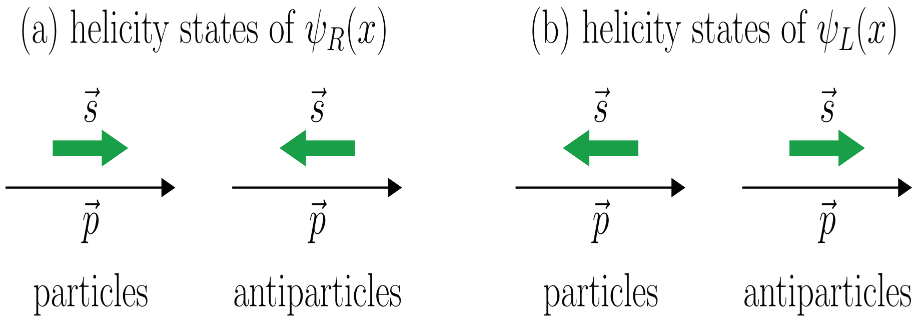


FIG. 2.1. Helicity states of the massless chiral fields $\psi_R(x)$ and $\psi_L(x)$.

Giunti and Kim, Fundamentals of Neutrino Physics and Astrophysics

- For massive fields left handed particles are predominantly -ve helicity +ve helicity component $\sim m/E$
- For massive fields right handed particles are predominantly +ve helicity with a -ve helicity component $\sim m/E$

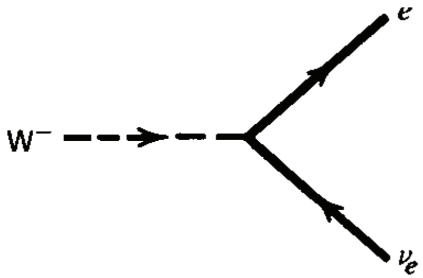
Neutrino Interaction in SM

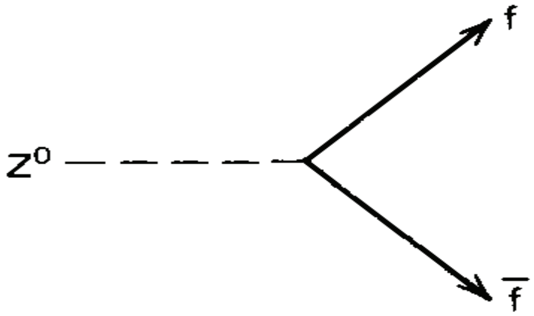
- Neutrinos interact with W boson through the "charged current" interaction that gives rise to the term

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} \bar{\ell} \gamma^{\mu} (1 - \gamma_5) \nu W_{\mu}^{-} + h.c. \tag{1}$$

- The interaction with Z boson is the "neutral current" interaction that gives rise to the term

$$\mathcal{L}_{NC} = \frac{g}{(2 \cos \theta_W)} \bar{\nu} \gamma_{\mu} (1 - \gamma_5) \nu Z^{\mu} \tag{2}$$





Lepton Number

- Neutrinos are electrically neutral
- Neutrinos carry Lepton Number +1, antineutrinos carry lepton number -1 (Global $U(1)$ symmetry, $\psi \to \psi' = e^{i\alpha}\psi$)
- Introduced in 1953 by Konopinski and Mahmoud to explain certain missing decay modes
- Known reactions conserve generational lepton number L_e, L_μ and L_τ as well as total Lepton number $L_e + L_\mu + L_\tau$
- Allowed Decays

  $\pi \to \mu^- \bar{\nu}_\mu$

  $\mu^- \to e^- \bar{\nu}_e \nu_\mu$
- Forbidden Decays

  $\mu \to e\gamma$

  (L_e and L_μ not conserved separately)

Fermion Mass in Standard Model

- $L_{Dirac} = \bar{\psi}(i\gamma^{\mu}\delta_{\mu} - m)\psi$
- Mass term : $m\bar{\psi}\psi$
- Chiral decomposition : $\psi = \psi_L + \psi_R$
- $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$, the left and right handed states
- $M_{Dirac} = -m_D(\bar{\psi}_L\psi_L + \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_R)$
- $\bar{\psi}_L\psi_L = \bar{\psi}_R\psi_R$ =0 (Using property of γ matrices) (Check!)
- Thus, $L_{Dirac} = -m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$
- In SM ψ_L belongs to SU(2) doublet and ψ_R is SU(2) singlet e.g $\psi_L = (\nu_e, e^-)^T, \psi_R = e_R$
- Y is also not conserved ($e_L : Y = -1, e_R : Y = -2$)
- Mass terms break gauge symmetry

Fermion Mass in Standard Model

- Fermion masses can be generated through Higgs : $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} (2, +1)$
- $L_Y = y_e \bar{\psi}_L \phi e_R$, (the fermions and Higgs doublets : $2 \times 2 = 3 \oplus 1$)
- $\bar{\psi}_L : Y = 1, \phi : Y = 1, e_R : Y = -2$
- $[L_Y] = 4 \ \rightarrow$ y -> dimension less
- When Higgs develops Vaccum expectation value : $\phi = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$
- Mass term : $\frac{y_e}{\sqrt{2}} \overline{e_L} v e_R$, $m_e = \frac{y_e v}{\sqrt{2}}$
- y_e is arbitrary $\longrightarrow m_e$ is not a prediction of SM
- When we include other leptons and quarks there are different Yukawa couplings $\rightarrow$ free parameters of SM
- No right handed neutrinos and hence no Dirac mass term in SM

Dirac Neutrino Mass beyond SM

- Simplets extentsion of SM : add N_R : (1,0)
- Mass term $\rightarrow \mathcal{L}_{Dirac} = m_D \bar{\nu}\nu = m_D(\bar{\nu_L} N_R + \bar{N_R}\nu_L)$: $\nu \equiv \nu_L + N_R$
- Lepton number $L_\nu + L_N$ is conserved
- Mass is generated by Higgs mechanism through Yukawa coupling
- $-\mathcal{L} = \overline{N}_R Y_\nu \tilde{\phi}^\dagger l_L$ + h.c
- $\tilde{\phi} = i\sigma_2 \phi^*$
- Same ϕ cannot give mass to upper and lower fermions of the doublet $\rightarrow$ Y cannot be conserved
- $i\sigma_2\phi^*$ same transformation properties as ϕ under SU(2)

Dirac Mass Matrix

- When there are several fields with the same quantum numbers, we define the Dirac mass matrix, $(m_D)_{ij}$

$$\mathscr{L}_{\rm D} = - \sum_{\alpha,\beta=e,\mu,\tau} M^{\rm D}_{\alpha\beta}\, \overline{\nu_{\alpha L}}\, \nu_{\beta R} + \text{H.c.} \qquad M^{\rm D}_{\alpha\beta} = \frac{v}{\sqrt{2}}\, Y_{\alpha\beta}$$

$$v = \left(\sqrt{2}\, G_{\rm F}\right)^{-1/2} = 246\,\text{GeV}$$

complex 3×3 Dirac mass matrix

$$\mathscr{L}_{\rm D} = - \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

L_e, L_μ, L_τ are not conserved

- In the SM, the fermion fields are present in three copies, and the Dirac mass matrices are 3×3 matrices. In general, however, m_D does not have to be a square matrix.

Majorana Mass and cgarge conjugation

- Majorana mass term uses the charge conjugated states

need charge conjugation:

electron e^-: $[\gamma_\mu (i\partial^\mu + e\,A^\mu) - m]\,\psi = 0 \quad (1)$

positron e^+: $[\gamma_\mu (i\partial^\mu - e\,A^\mu) - m]\,\psi^c = 0 \quad (2)$

Try $\psi^c = S\,\psi^*$, evaluate $(S^*)^{-1}\,(2)^*$ and compare with (1):

$$S = i\gamma_2$$

and thus

$$\psi^c = i\gamma_2\,\psi^* = i\gamma_2\,\gamma_0\overline{\psi}^T \equiv C\,\overline{\psi}^T$$

slide courtsey: W. Rodejohann

Properties of C

Properties of C:

$$C^\dagger = C^T = C^{-1} = -C$$

$$C\,\gamma_\mu\,C^{-1} = -\gamma_\mu^T$$

$$C\,\gamma_5\,C^{-1} = \gamma_5^T$$

$$C\,\gamma_\mu\gamma_5\,C^{-1} = (\gamma_\mu\gamma_5)^T$$

properties of charged conjugate spinors:

$$(\psi^c)^c = \psi$$

$$\overline{\psi^c} = \psi^T\,C$$

$$\overline{\psi_1}\,\psi_2^c = \overline{\psi_2^c}\,\psi_1$$

$$\boxed{(\psi_L)^c = (\psi^c)_R}$$

$$\boxed{(\psi_R)^c = (\psi^c)_L}$$

slide courtsey: W. Rodejohann

Majorana Mass

- Let us consider the Dirac mass term Thus, $\mathcal{L}_{Dirac} = -m(\overline{N}_R \nu_L + \overline{\nu}_L N_R)$
- In order to get Majorana mass one does not need an extra N_R but use the right-handed state $(\nu_L)^c \equiv (\nu^c)_R$
- where, $\nu^C = C\bar{\nu}^T = i\gamma_2\gamma_0\gamma_0\nu^* = i\gamma_2\nu^*$
- Then, $\mathcal{L}^M_{mass} == \frac{1}{2} m_M (\overline{\nu_L{}^c}\nu_L + \overline{\nu_L}\nu_L{}^c)$
- The full Majorana Lagrangian
  $\mathcal{L}^M = \frac{1}{2}\left[\bar{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_L{}^c} i\partial\!\!\!/ \nu_L{}^c - m(\overline{\nu_L}^c \nu_L + \overline{\nu_L}\nu_L{}^c)\right]$
- The overall factor of 1/2 to avoid doublecounting as ν_L and $\nu_L{}^c$ are not independent
- The Majorana field $\nu \equiv \nu_L + \nu^c{}_R$
  $\nu^c = (\nu_L)^c + (\nu^c{}_R)^c = (\nu^c)_R + \nu_L = \nu$
  $\rightarrow$ A Majorana Neutrino is its own Antineutrino
- In terms of ν $\mathcal{L}^M = \frac{1}{2}\left[\overline{\nu} i\partial\!\!\!/ \nu - m\overline{\nu}\nu\right]$

Can particles be their own antiparticles ?

- A few years before he mysteriously disappeared at sea, Italian physicist Ettore Majorana posed a puzzle for future researchers. In 1937 he varied Dirac equation to predict a class of particles that are its own antiparticles: the Majorana fermions.
- Is the Dirac equation a real equation like the Klein-Gordon equation "
- For Majorana representation all non-zero elements of all the four γ_μ's are purely imaginary. This leads to $\psi = \psi^c$

Palsh. B. Pal, arXiv 1006.1718

Majorana Mass Matrix

- The Majorana mass term in matrix form can be written as,
  $\mathcal{L}^{M}_{mass} = -\frac{1}{2}(\overline{\nu^c_L} M \nu_L) + h.c. = \frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau}(\nu^T_{\alpha L} C^\dagger M_{\alpha\beta}\nu_{\beta L}) + h.c.$
  $(\overline{\nu^c_L} = -\nu^T_L C^\dagger)$
-

$$\sum_{\alpha,\beta=e,\mu,\tau} \nu^T_{\alpha L} C^\dagger M_{\alpha\beta}\nu_{\beta L} = \sum_{\alpha,\beta=e,\mu,\tau} \nu^T_{\beta L} C^\dagger M_{\alpha\beta}\nu_{\alpha L}$$

using $C^T = -C$ and '-' sign due anti-commutation of fermion fields

- Interchanging α, β in the last term,
  $\sum_{\alpha,\beta=e,\mu,\tau} \nu^T_{\alpha L} C^\dagger M_{\alpha\beta}\nu_{\beta L} = \sum_{\alpha,\beta=e,\mu,\tau} \nu^T_{\alpha L} C^\dagger M_{\beta\alpha}\nu_{\beta L}$
- $\Longrightarrow$ $M_{\beta\alpha} = M_{\alpha\beta}$

Lepton Number

- $\mathcal{L}_{mass} == \frac{1}{2} m_M (\overline{\nu_L{}^c} \nu_L + \bar{\nu_L} \nu_L{}^c)$
- Under global U(1) gauge transformation, $\nu_L \rightarrow e^{i\varphi} \nu_L$ and $\nu_L^T \rightarrow e^{i\varphi} \nu_L^T$ Majorana Mass Term becomes,

$$\mathcal{L}_{mass}^{M} = -m \frac{1}{2} (-e^{2i\varphi} \nu_L^T C^{\dagger} \nu_L + e^{-2i\varphi} \nu_L^{\dagger} C \nu_L^{*}) \quad (3)$$

- Not invariant under the global U(1) gauge transformation
- Violation of Lepton Number by 2 units
- Can lead to neutrinoless double β decay $(A, Z) \rightarrow (A, Z+2) + 2e^-$
- Charged particle cannot be Majorana

How to generate Majorana Mass ?

- $\mathcal{L}_{mass} == \frac{1}{2} m_M (\overline{\nu_L{}^c} \nu_L + \overline{\nu_L} \nu_L{}^c)$
- Can we have a Majorana mass term in SM ?
- The $SU(2)_L \times U(1)_Y$ quantum numbers :
  $\nu_L : \quad (2, -1), \quad \nu_L{}^c = i\gamma_2 \nu_L^* : \quad (2, 1)$
  $\overline{\nu_L{}^c} \nu_L = (2, -1) \times (2, -1) = (1, -2) + (3, -2)$
  $\overline{\nu_L} \nu_L{}^c = (1, 2) + (3, 2)$
- Thus Y is not conserved
- No Majorana mass term in SM
- Majorana mass term can be generated at tree-level through a term $\overline{\nu_L}^c \nu_L X$
- To ensure gauge invariance X needs to have Q=0
- Scalar triplet with (Y = 2)
  $(3, -2) \times (3, 2) = (1, 0) + (3, 0) + (5, 0)$

Neutrino Mixing: Dirac case

- -$\mathcal{L}_{Dirac} = \overline{\nu}_{L\alpha} M_{\alpha\beta} N_{R\beta} + h.c.\ \alpha, \beta = e, \mu, \tau$
- $(\nu_e, \nu_\mu \nu_\tau)$ are the weak interaction eigenstates
- The mass matrix M is in general non-diagonal and non-hermitian and can be diagonalized by a Bi-Unitary transformation

$$M_{diag} = UMV^\dagger$$

- - $\mathcal{L}_{Dirac} = (\overline{\nu}_{L\alpha} U^\dagger)(UM_{\alpha\beta}V^\dagger)(VN_{R\beta}) + h.c. = \overline{\nu'}_{L\alpha} M_{dia} N'_{R\beta} + h.c.$
- The weak eigenstates are related to the mass eigenstates as,

$$\nu_L\alpha = U_{\alpha i}\nu'_{Li}, \quad N_R^\alpha = V_{\alpha i} N'_{Ri} \text{ (i =1,2,3)}$$

Neutrino Mixing: Dirac case

- The charged current Lagrangian
- Using the primed basis ${\nu'}_i$ the Charged Current Lagrangian is

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} (\overline{\nu'}_1, \overline{\nu'}_2, \overline{\nu'}_3) U^\dagger \gamma^\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L W^+_\mu + h.c.$$

$U^\dagger \to$ PMNS or MNSP mixing matrix for leptons in the basis where the charged lepton mass matrix is diagonal analogous to CKM matrix in the quark sector

- $2N^2$ parameters for a $N \times N$ complex matrix
- Unitarity $\Longrightarrow N^2$ independent elements
- ${}^N C_2 = N(N-1)/2$ angles, $N^2 - N(N-1)/2 = N(N+1)/2$ phases
  $\Longrightarrow$ 2 generation: 1 angle, 3 phases, 3 generation: 3 angles, 6 phases
- Not all phases are physically observable

Neutrino Mixing: Dirac case

- The weak charged current : $j_w{}^\mu = \sum_{k=1,3} \sum_{\alpha=e,\mu\tau} \bar{\nu_k}_L U^*_{\alpha k} \gamma^\mu l_{\alpha l}$
- Making phase transformations : $\nu_k \to e^{i\phi_k} \nu_k$ and $l_\alpha = e^{i\phi_\alpha} l_\alpha$
- $j_w{}^\mu = \sum_{k=1,3} \sum_{\alpha=e,\mu\tau} \bar{\nu_k}_L e^{-i\phi_k} U^*_{\alpha k} e^i \phi_\alpha \gamma^\mu l_{\alpha l}$
  $j_w{}^\mu = e^{-i(\phi_1 - \phi_e)} \sum_{k=1,3} \sum_{\alpha=e,\mu\tau} \bar{\nu_k}_L e^{-i(\phi_k - \phi_1)} U^*_{\alpha k} e^i \phi_\alpha - \phi_e \gamma^\mu l_{\alpha l}$
- $\Longrightarrow$ 5 arbitrary phases which can be used to absorb 5 out of 6 phases in U
- A common rephasing of all the fields leaves the charged current invariant $\Longrightarrow$ conservation of total lepton number
- $\Longrightarrow$ the mixing matrix contains 3 angles and 1 physical phase

Parametrizing the Neutrino Mixing Matrix

- $U_{PMNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13},\delta) R_{12}(\theta_{12})$
- $U_{PMNS} =$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- 3 mixing angles and 1 phase
- This phase is responsible for CP violation in lepton sector

Neutrino Mixing: Majorana case

- The Majorana mass matrix,

$$\begin{aligned}\mathcal{L}^{M}_{mass} &= -\frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau} \nu^{T}_{\alpha L} C^{\dagger} M_{\alpha\beta}\nu_{\beta L}) + h.c. \\ &= -\frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau} \nu^{T}_{\alpha L} C^{\dagger} U^{T}(U^{*} M_{\alpha\beta} U^{\dagger}) U \nu_{\beta L} + h.c. \\ &= -\frac{1}{2}\nu^{T}_{kL} C^{\dagger} M_{dia}\nu_{kL} + h.c.\end{aligned}$$

- $M_{diag} = U^{*} M U^{\dagger}$ or $M_{diag} = U^{T} M U$
- Can be shown starting from a bi-unitary transformation and assuming the mass matrix to be symmetric

Phases of the Majorana Mixing

- The weak charged current : $j_w{}^\mu = \sum_{k=1,3} \sum_{\alpha=e,\mu\tau} \bar{\nu_k}_L U^*_{\alpha k} \gamma^\mu l_{\alpha l}$
- U in general contains 6 phases
- The Majorana mass term $-\frac{1}{2}\nu^T_{kL} C^\dagger M_{\alpha\beta} \nu_{\beta L}$ not invariant under the phase transformation $\nu_k \to e^{i\phi_k}\nu_k$
- $\Longrightarrow$ Phases of the mixing matrix can not be absorbed in the neutrino field. If absorbed then again reappear in the mass matrix.
- $\Longrightarrow$ Only 3 phases can be absorbed by rephasing the charged lepton fields
- The Majorana Mixing Matrix $U = U^{Dirac} P$ where P is the phase matrix $P = Dia(0, e^{i\alpha}, e^{i\beta})$

PART-II : Neutrino Oscillation

Neutrino Oscillations

- Do neutrinos change flavour after passing through a distance ?
- Proposed by Pontecorvo in 1957 in analogy with $K_0 - \bar{K}_0$ oscillations
- Quantum Mechanical Interference phenomena
- ν_e, ν_μ, ν_τ produced in weak interactions – flavour states
- ν_1, ν_2, ν_3 are the mass eigenstates that propagate
- If neutrinos have mass then,
  $\nu_\alpha = U_{\alpha i} \nu_i$ (In terms of Fields) $|\nu_\alpha >= U^*_{\alpha i} |\nu_i >$ (In terms of States)
  U is the neutrino mixing matrix 3×3 for three flavours
- This leads to Neutrino Oscillations

Neutrino Oscilltions in Vaccum: Two Flavours

- If neutrinos have mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

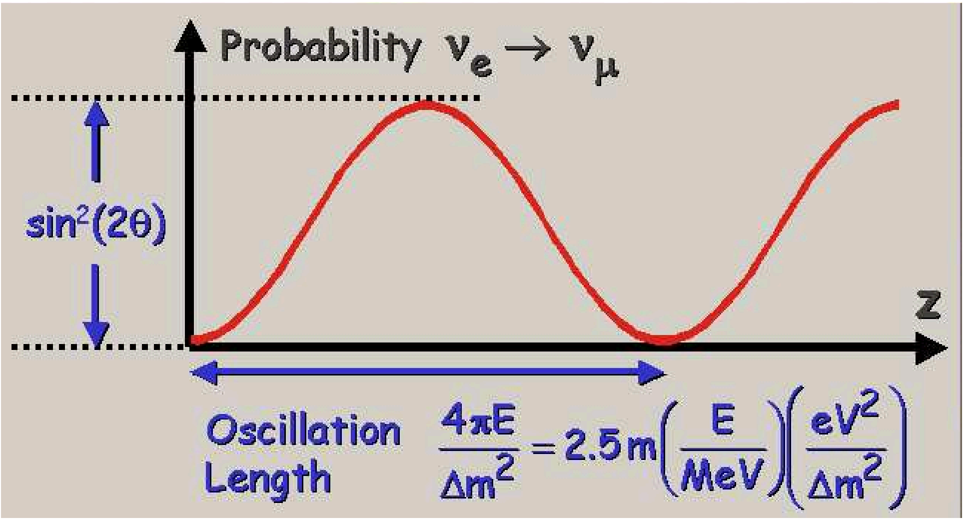
- Neutrinos acquire different phases as they propagate

$$|\nu_j(t)\rangle = \exp(-iE_j t)|\nu_j(0)\rangle \qquad E_j = p^2 + m_i^2/2p$$

- A phase difference develop between the terms since $m_1 \neq m_2$
- At some later time
$|\nu_e(t)\rangle = \cos\theta \exp(-iE_1 t)|\nu_1(0)\rangle + \sin\theta \exp(-iE_2 t)|\nu_2(0)\rangle \neq |\nu_e\rangle$
- Survival Probability (in vacuum)
$|\langle\nu_e(t)|\nu_e\rangle|^2 = P_{\nu_e\nu_e} = 1 - \sin^2 2\theta \sin^2(1.27\Delta m^2 L/E)$;
- Oscillation Probability (in vacuum)
$P_{\nu_e\nu_\mu} = 1 - P_{\nu_e\nu_e} = \sin^2 2\theta \sin^2(1.27\Delta m^2 L/E)$

Oscilltion Probability in Vacuum

$$P_{\nu_e \nu_\mu} = \sin^2 2\theta \sin^2(1.27 \Delta m^2 L/E) = \sin^2 2\theta \sin^2(\pi L/\lambda)$$



$\Delta m^2 = m_2^2 - m_1^2$

$\theta \rightarrow$ mixing angle

$L \rightarrow$ Distance travelled (in m/Km)

$E \rightarrow \nu$ Energy (in MeV/GeV)

- Neutrino Oscillation requires
  Non-zero neurino mass
  Non-zero mixing angles
  Oscillation effect $\Delta m^2 \sim E/L$

- Oscillation Wavelength
  $\lambda = 2.5m(E/MeV)(eV^2/\Delta m^2)$
- $\lambda >> L, \sin^2(\pi L/\lambda) \rightarrow 0$
- $\lambda << L, \sin^2(\pi L/\lambda) \rightarrow 1/2$
- $\lambda \sim 2L, \sin^2(\pi L/\lambda) \sim 1 \rightarrow \Delta m^2 \sim E/L$

Neutrino oscillations in vacuum

Neutrino oscillations in vacuum

- Generalisation to more than 2 flavours

$$\mathcal{P}(\nu_\alpha \longrightarrow \nu_\beta) = | < \nu_\beta|\nu_\alpha(t) > |^2 = \Sigma_{i,j} U_{\alpha i} U^\star_{\beta i} U^\star_{\alpha j} U_{\underline{\text{e}}taj}\, e^{-i2\Delta_{ij}}$$

$$= \delta_{\alpha\beta} - 4\,\Sigma_{i>j}\, Re(U_{\alpha i} U^\star_{\beta i} U^\star_{\alpha j} U_{\beta j})\, \sin^2(\Delta_{ij})$$

$$+2\,\Sigma_{i>j}\, Im(U_{\alpha i} U^\star_{\beta i} U^\star_{phaj} U_{\beta j})\, \sin(2\Delta_{ij}) U^\star_{\beta i} U^\star_{\alpha j} U_{\beta j})\, \sin(2\Delta_{ij})$$

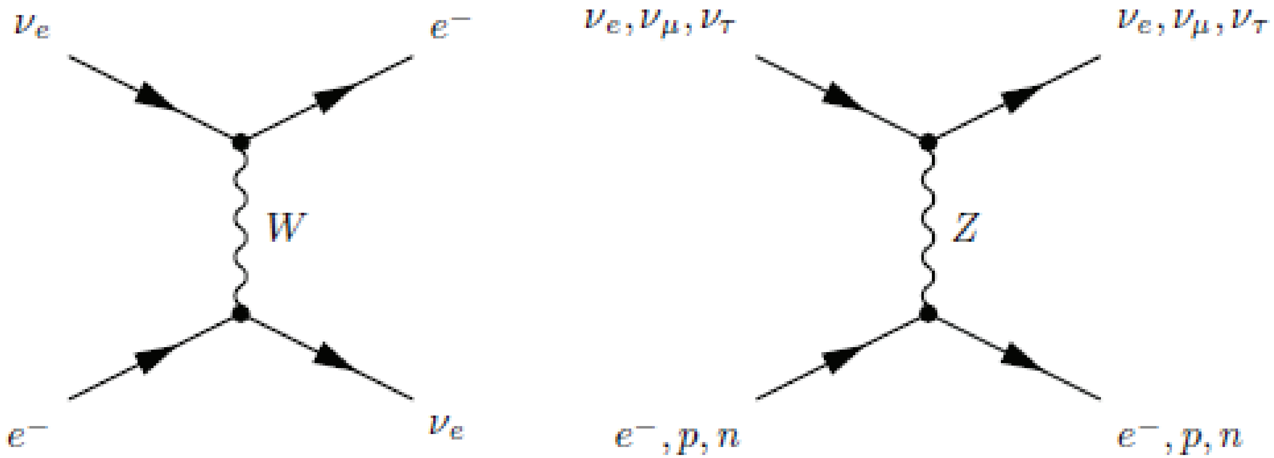
$$\Delta_{ij} = \frac{1.267\Delta m^2_{ij}(eV^2)L(Km)}{E(GeV)}.$$

For real U

$$\mathcal{P}(\nu_\alpha \longrightarrow \nu_\beta) == \delta_{\alpha\beta} - 4\,\Sigma_{i>j}\, Re(U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j})\, \sin^2(\Delta_{ij})$$

Matter Effects

- Neutrinos travelling through matter will interact with it.
- Normal matter only has e^-, p and n (CP, CPT violating).
- Only ν_e and $\bar{\nu}_e$ have charge current interactions.
- All the neutrino types can have neutral current interactions



- This interaction modifies the mixing and masses in matter
- The probabilities are also different than vacuum oscillation case
- The neutrino flavour conversion in matter violate CP, CPT.

Matter Potential

- Neutrinos undergo forward elastic scattering (no momentum change)
- Energies are much below the electroweak scale,
  $\mathcal{H}_{eff}^{CC}(x) = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_e(x)\gamma^\mu(1-\gamma^5)\nu_e(x)\right] \left[\bar{e}(x)\gamma_\mu(1-\gamma^5)e\right]$
- To calculate the effective potential one needs to average over the electron background i,e compute $\langle(\bar{e}_L\gamma_\mu e_L)\rangle$:
  $< \bar{e}_L\gamma_0 e_L >= N_e, < \bar{e}_L\vec{\gamma}e_L >=< \vec{v_e} >, \bar{e}_L\gamma_0\gamma_5 e_L >=< \frac{\vec{\sigma_e}\vec{p_e}}{E_e} >, \bar{e}_L\vec{\gamma}\gamma_5 e_L >=< \vec{\sigma_e} >,$
- For unpolarized electrons at rest only the first one contributes
- $\langle H_{eff}^{CC}(x)\rangle = \frac{G_F N_e}{\sqrt{2}}\bar{\nu}_e(x)\gamma^o\nu_e(x)$

$V_{CC} = \sqrt{2}G_F N_e$

For antineutrinos, $V_{CC} = -\sqrt{2}G_F N_e$

(The current for charge conjugate field is $-\overline{\psi}\gamma^\mu\psi$)

For detailed derivation e.g. Giunti and Kim Book

Matter Potential : Neutral Currents

- One can similarly get for the neutral current

$$V_{NC} = \sqrt{2} G_F \sum_f N_f [I_{3L}^{(f)} - 2\sin^2\theta_W Q^{(f)}]$$

f	$I_{3L}^{(f)}$	$Q^{(f)}$
e	$-1/2$	-1
$--INSERT--657,329p$	$1/2$	1
n	$-1/2$	0

- Normal matter has $N_e = N_p$ and so their contribution cancels out and only the neutron contribution stays
- $V_{NC} = -\sqrt{2} G_F \frac{N_n}{2}$

Matter effects: Two Flavours

- Mass eigenvalues in matter :
$$\frac{m^2_{1m,2m}}{2E} = \frac{1}{2E}\left(\frac{A}{2} \mp \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}\right)$$

- The mass squared difference in matter:
$$\Delta {m_m}^2 = \sqrt{A - \Delta m^2 \cos 2\theta)^2 + \sin^2 2\theta}$$

- Effective mixing angle θ_M in matter :

$$\tan 2\theta_M = \frac{\Delta m^2_{21} \sin 2\theta}{\Delta m^2_{21} \cos 2\theta - A}$$

$\Delta m^2 \cos 2\theta = A = 2\sqrt{2} G_F n_e$,

$\theta_M \to \pi/4$ MSW Resonance

- Mixing angle Maximal

L. Wolfenstein, PRD **17**, 1978

S.P. Mikheyev, A.Yu. Smirnov, SJNP **42**, 1985

- Mass squared difference in Matter Minimal

Intrinsic Neutrino Properties Δm^2, θ

- Survival probability in constant density matter is
$$P^m_{ee} = 1 - \sin^2 2\theta_m \sin^2\left(\frac{\Delta m^2_m L}{4E}\right)$$