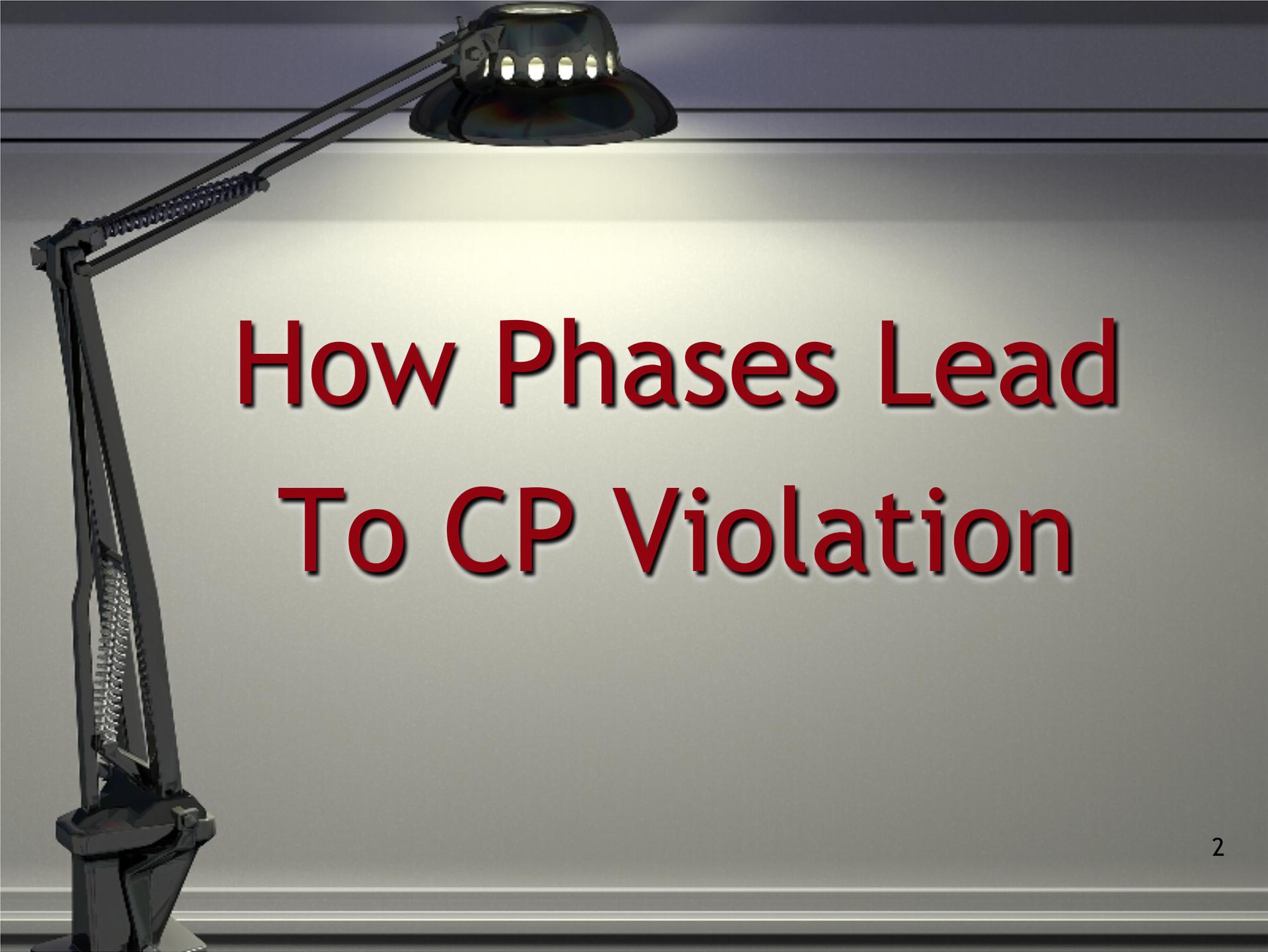


CP Violation

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NASA Hubble Photo

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A desk lamp with a silver-colored base and a black adjustable arm is positioned on the left side of the frame. The lamp's head is tilted downwards, casting a bright, circular glow on the surface below. The background is a plain, light-colored wall. The text "How Phases Lead To CP Violation" is centered on the wall, illuminated by the lamp's light.

How Phases Lead To CP Violation

\cancel{CP} always comes from *phases*.

Therefore, \cancel{CP} always requires an *interference* between (at least) two amplitudes.

For example, an interference between two Feynman diagrams.

Let us consider how a CP-violating rate difference between two CP-mirror-image processes, such as $B^+ \rightarrow D^0 K^+$ and $B^- \rightarrow \bar{D}^0 K^-$, arises.

Suppose some process P has the amplitude —

$$A = M_1 e^{i\theta_1} e^{i\delta_1} + M_2 e^{i\theta_2} e^{i\delta_2}$$

CP-invariant magnitude }
 CP-odd “weak” phase from constants }
 CP-even “strong” phase

The diagram shows the amplitude equation $A = M_1 e^{i\theta_1} e^{i\delta_1} + M_2 e^{i\theta_2} e^{i\delta_2}$. Red arrows and brackets indicate the following groupings:

- A bracket on the left groups M_1 and M_2 under the label "CP-invariant magnitude".
- A bracket on the right groups $e^{i\delta_1}$ and $e^{i\delta_2}$ under the label "CP-odd 'weak' phase from constants".
- A bracket at the bottom groups $e^{i\theta_1}$ and $e^{i\theta_2}$ under the label "CP-even 'strong' phase".

Then the CP-mirror-image process \bar{P} has the amplitude —

$$\bar{A} = M_1 e^{i\theta_1} e^{-i\delta_1} + M_2 e^{i\theta_2} e^{-i\delta_2}$$

Then the rates for \bar{P} and P differ by —

$$\bar{\Gamma} - \Gamma = |\bar{A}|^2 - |A|^2 = 4M_1M_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)$$

Assumes equal amounts of
the initial states of \bar{P} and P .

$$\bar{\Gamma} - \Gamma = |\bar{A}|^2 - |A|^2 = 4M_1M_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)$$

A CP-violating rate difference
requires 3 ingredients:

- Two interfering amplitudes
- These two amplitudes must have different CP-even phases
- These two amplitudes must have different CP-odd phases

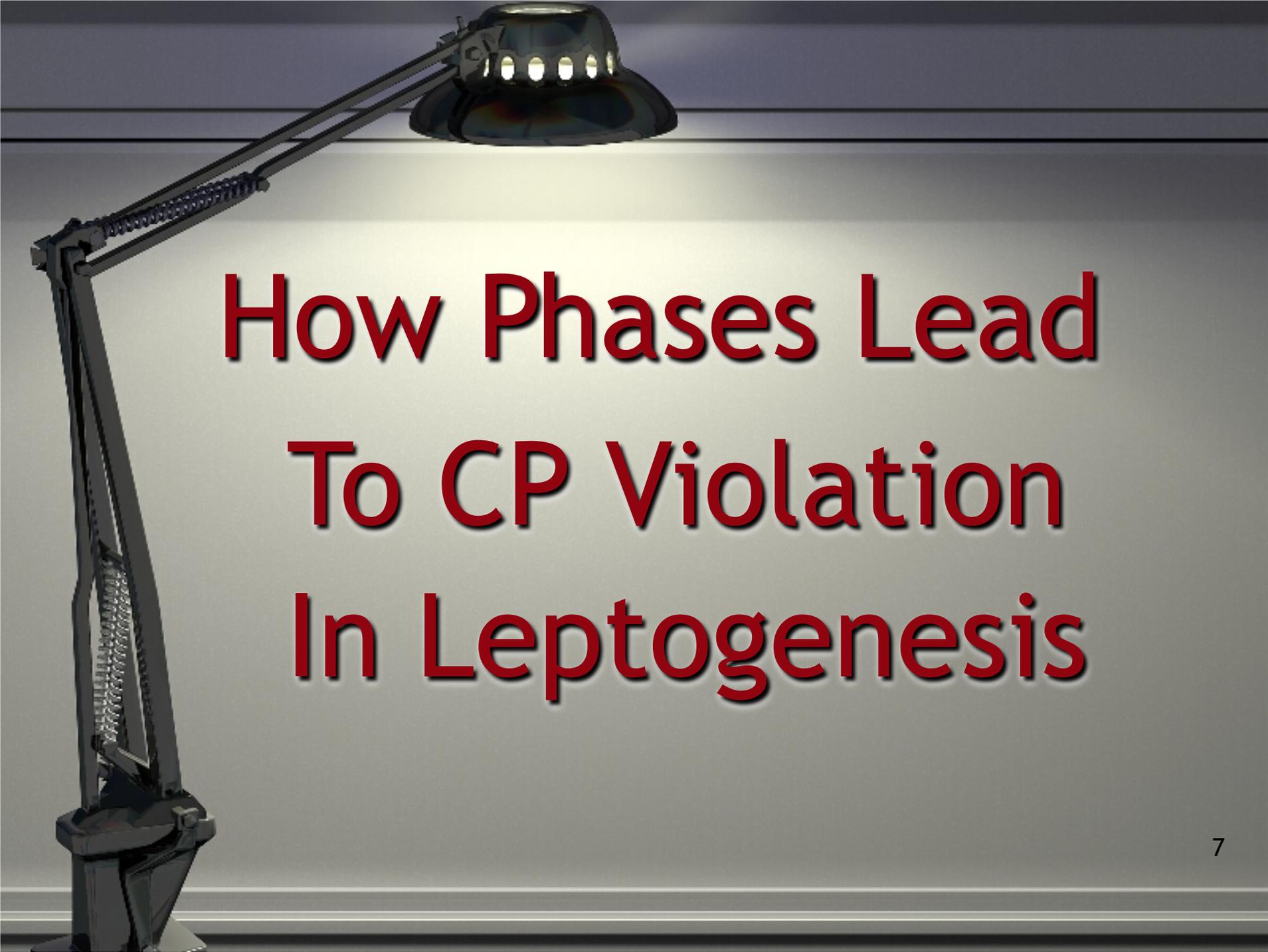
Mixing Can Lead to Unequal Amounts of the CP-Mirror-Image Initial States

$$\begin{array}{ccc} \text{From } K^0 \text{ only} & \swarrow & \text{From } \bar{K}^0 \text{ only} \\ \Gamma(K_L \rightarrow \pi^- \ell^+ \nu) & - & \Gamma(K_L \rightarrow \pi^+ \ell^- \bar{\nu}) \neq 0 \text{ violates CP.} \end{array}$$

$$\text{But } K_L \propto (1 + \varepsilon)K^0 - (1 - \varepsilon)\bar{K}^0,$$

where ε is the usual CP-violating parameter arising from $K^0 - \bar{K}^0$ mixing, and $|1 + \varepsilon|^2 \neq |1 - \varepsilon|^2$.

*This rate difference comes from ε ,
rather than the 3 ingredients just listed.*



How Phases Lead To CP Violation In Leptogenesis

Let us look at an example.

This example illustrates that ~~CP~~ in *any decay* always involves amplitudes *beyond* those of lowest order in the Hamiltonian.

The Yukawa interaction leading to heavy neutrino decay is —

$$-\mathcal{L}_Y = \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(y_{\alpha i} \left[\bar{\nu}_{\alpha L} \bar{H}^0 - \bar{\ell}_{\alpha L} H^- \right] N_{iR} + y_{\alpha i}^* \bar{N}_{iR} \left[H^0 \nu_{\alpha L} - H^+ \ell_{\alpha L} \right] \right)$$

Now consider two diagrams contributing to N_1 decay:

$$\Gamma\left(N_1 \rightarrow e^- + H^+\right) = \left| y_{e1} K_{\text{Tree}} + y_{\mu 1}^* y_{\mu 2} y_{e2} K_{\text{Loop}} \right|^2$$

When we go to the CP-mirror-image decay, $N_1 \rightarrow e^+ + H^-$, all the coupling constants get complex conjugated, but the kinematical factors do not change.

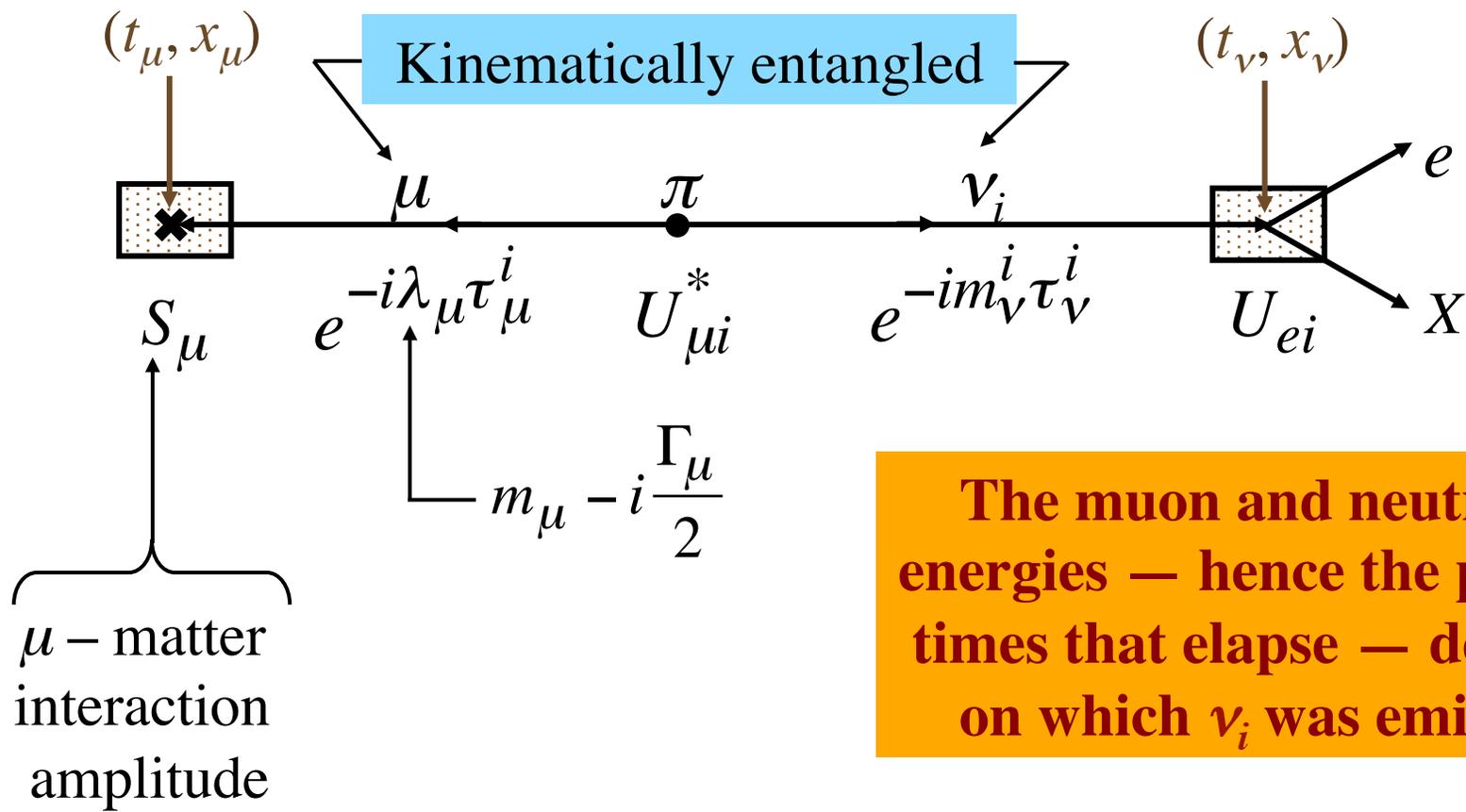
$$\Gamma\left(N_1 \rightarrow e^+ + H^-\right) = \left| y_{e1}^* K_{\text{Tree}} + y_{\mu 1} y_{\mu 2}^* y_{e2}^* K_{\text{Loop}} \right|^2$$

All three ingredients needed for ~~CP~~ are present.

$$\begin{aligned} & \Gamma\left(N_1 \rightarrow e^- + H^+\right) - \Gamma\left(N_1 \rightarrow e^+ + H^-\right) \\ &= 4 \operatorname{Im}\left(y_{e1}^* y_{\mu 1}^* y_{e2} y_{\mu 2}\right) \operatorname{Im}\left(K_{\text{Tree}} K_{\text{Loop}}^*\right) \end{aligned}$$



Neutrino Oscillation



$$\text{Amp} = \sum_{i=1,2,3} S_\mu e^{-i\left(m_\mu - i\frac{\Gamma_\mu}{2}\right)\tau_\mu^i} U_{\mu i}^* e^{-im_\nu^i \tau_\nu^i} U_{ei}$$

(Lorentz invariant)

$$\text{Amp} = \sum_{i=1,2,3} S_{\mu} e^{-i\left(m_{\mu} - i\frac{\Gamma_{\mu}}{2}\right)\tau_{\mu}^i} U_{\mu i}^* e^{-im_{\nu}^i\tau_{\nu}^i} U_{ei}$$

How do the kinematical phase factors depend on i ?

In the phase factor for the recoiling *muon*,

$$\tau_{\mu}^i = \frac{1}{m_{\mu}} \left(E_{\mu}^i t_{\mu} - p_{\mu}^i x_{\mu} \right)$$

Energy and momentum
of muon in π rest frame

Choosing $x_{\mu} = v_{\mu} t_{\mu} = \frac{p_{\mu}}{E_{\mu}}$ to avoid (Event rate) = 0,

Velocity, momentum, and energy of muon
in π rest frame for massless neutrinos

And using —

$$E_{\mu}^i = \frac{m_{\pi}^2 + m_{\mu}^2 - (m_{\nu}^i)^2}{2m_{\pi}} \quad \text{and} \quad (p_{\mu}^i)^2 = (E_{\mu}^i)^2 - m_{\mu}^2,$$

we find that to lowest (first) order in $\Delta m_{ij}^2 \equiv (m_{\nu}^i)^2 - (m_{\nu}^j)^2$,

$$\tau_{\mu}^i - \tau_{\mu}^j = 0.$$

That is, to lowest order, the muon phase factor

$$e^{-i \left(m_{\mu} - i \frac{\Gamma_{\mu}}{2} \right) \tau_{\mu}^i}$$

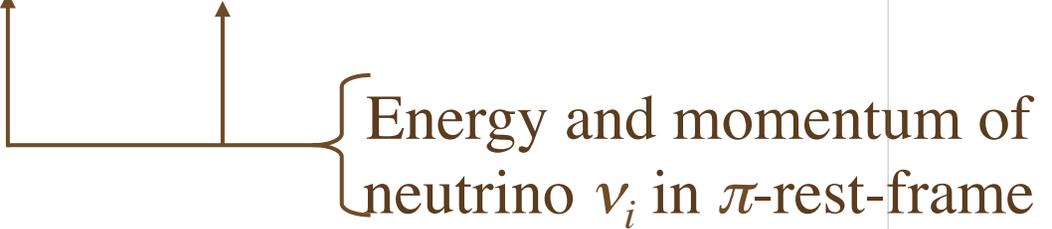
actually does not depend on i , so it will not influence the $|\text{Ampl}|^2$, and can be dropped.

(First noticed neglecting muon decay by Akhmedov and Smirnov)

In the phase factor for the *neutrino*, $e^{-im_{\nu}^i \tau_{\nu}^i}$,

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$$m_{\nu}^i \tau_{\nu}^i = E_{\nu}^i t_{\nu} - p_{\nu}^i x_{\nu}.$$


 Energy and momentum of
neutrino ν_i in π -rest-frame

Since in practice neutrinos are ultra relativistic,
we choose $t_{\nu} = x_{\nu} \equiv L^0$ to avoid (Event rate) = 0.

Using —

$$E_{\nu}^i = \frac{m_{\pi}^2 + (m_{\nu}^i)^2 - m_{\mu}^2}{2m_{\pi}} \quad \text{and} \quad (p_{\nu}^i)^2 = (E_{\nu}^i)^2 - (m_{\nu}^i)^2,$$

we find that to lowest (first) order in $\Delta m_{ij}^2 \equiv (m_\nu^i)^2 - (m_\nu^j)^2$,

$$m_\nu^i \tau_\nu^i - m_\nu^j \tau_\nu^j = \Delta m_{ij}^2 \frac{L^0}{2E^0}$$

} Distance ν travels
in the π rest frame

↑

} Energy ν would have in the π
rest frame if it were massless

Thus, we may take the neutrino phase factor, $e^{-im_\nu^i \tau_\nu^i}$,
to be —

$$e^{-i(m_\nu^i)^2 \frac{L^0}{2E^0}}$$

Using this result, and dropping the i -independent muon interaction and propagation amplitudes, we have —

$$\text{Amp} = \sum_{i=1,2,3} U_{\mu i}^* e^{-i(m_{\nu}^i)^2 \frac{L^0}{2E^0}} U_{ei}$$

[Recall that L^0 and E^0 are the neutrino travel distance and energy (neglecting its mass) *in the π rest frame.*]

From $\Delta p \Delta x \geq \hbar$, we cannot observe ν oscillation vs. travel distance in the lab unless there is a spread in lab-frame π momenta, so that the π is somewhat localized.

Because neutrinos are ultra-relativistic, when the parent π is moving in the lab, the ν travel distance and energy in the lab frame, L and E , are related to their π -rest-frame counterparts, L^0 and E^0 , by —

$$\frac{L}{E} = \frac{L^0}{E^0}$$

Thus, in terms of lab-frame variables,

$$\text{Amp} = \sum_{i=1,2,3} U_{\mu i}^* e^{-i \left(m_{\nu}^i \right)^2 \frac{L}{2E}} U_{ei}$$

This leads to —

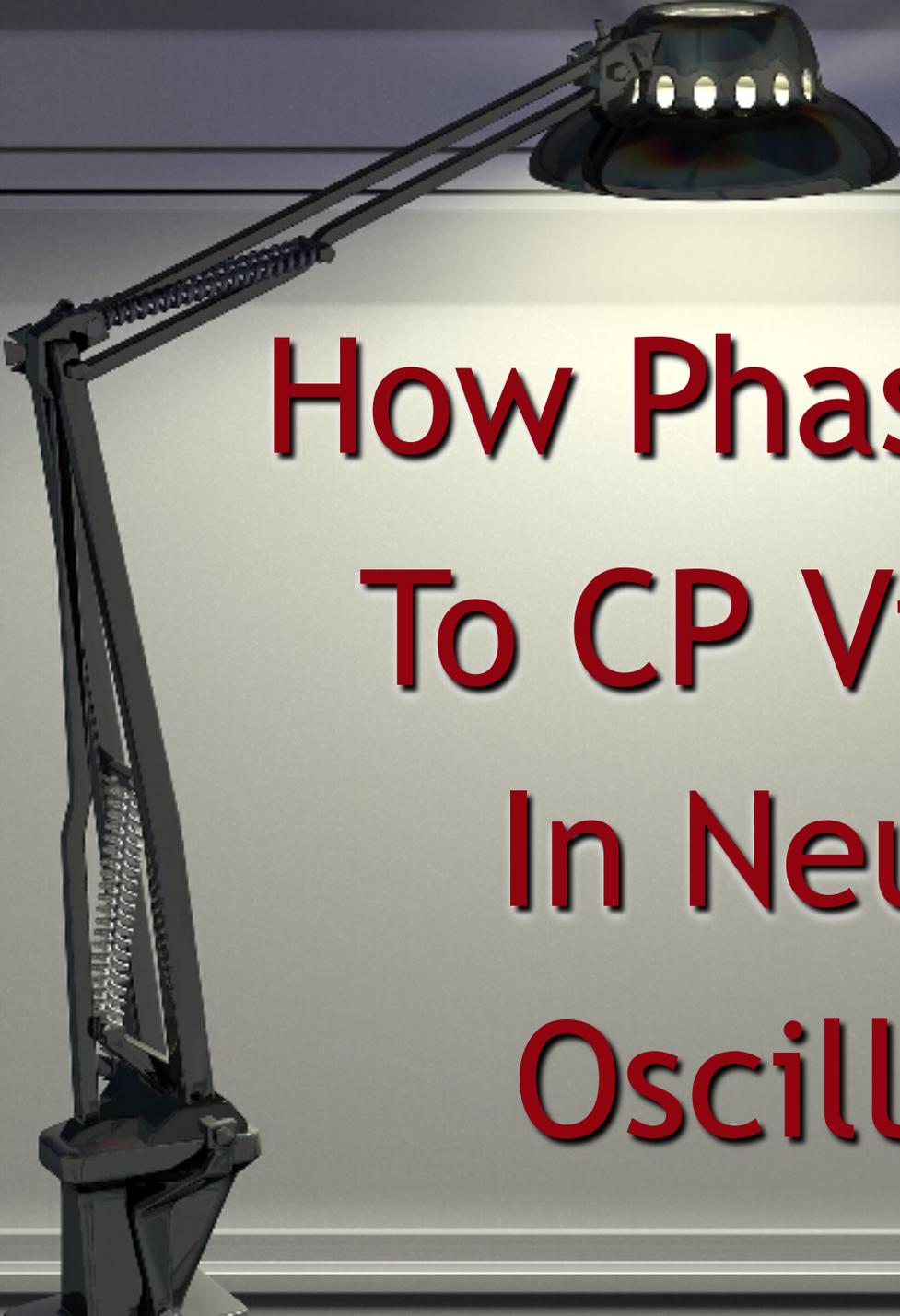
$$P(\nu_\mu \rightarrow \nu_e) = |\text{Amp}|^2 = -4 \sum_{i>j} \text{Re}\left(U_{\mu i}^* U_{ei} U_{\mu j} U_{ej}^*\right) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) + 2 \sum_{i>j} \text{Im}\left(U_{\mu i}^* U_{ei} U_{\mu j} U_{ej}^*\right) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)$$

This is the usual result.

We derived it now in the same way as we treat B-factory experiments.

We allowed for the $\nu - \mu$ kinematical entanglement, which proved to be irrelevant.

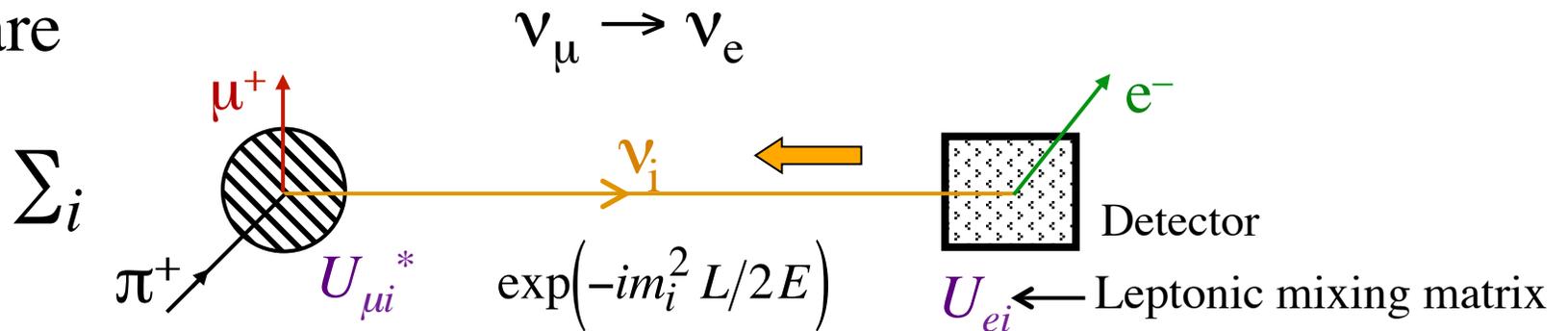
We didn't need to make any assumption about how the energies of the different neutrino mass eigenstates are related.



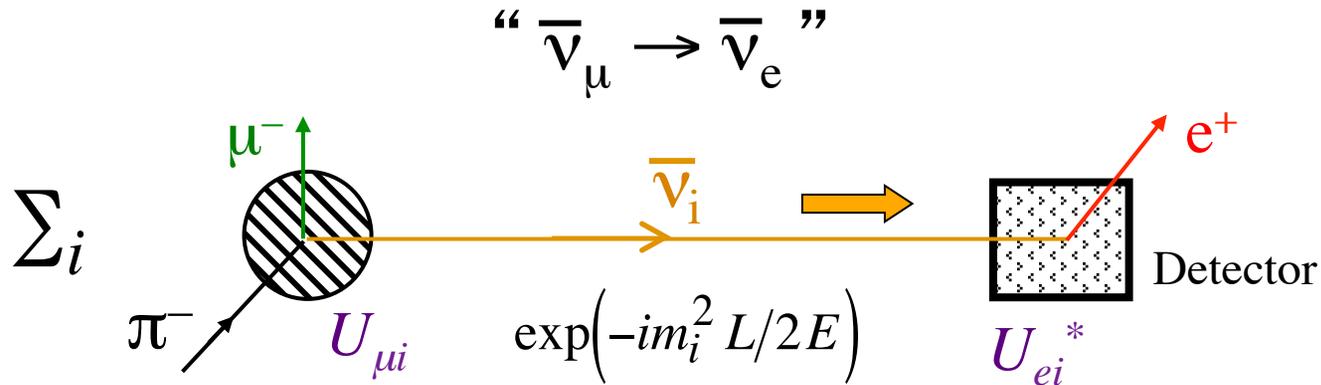
How Phases Lead To CP Violation In Neutrino Oscillation

The ingredients for ~~CP~~ in neutrino oscillation, even if $\bar{\nu} = \nu$

Compare

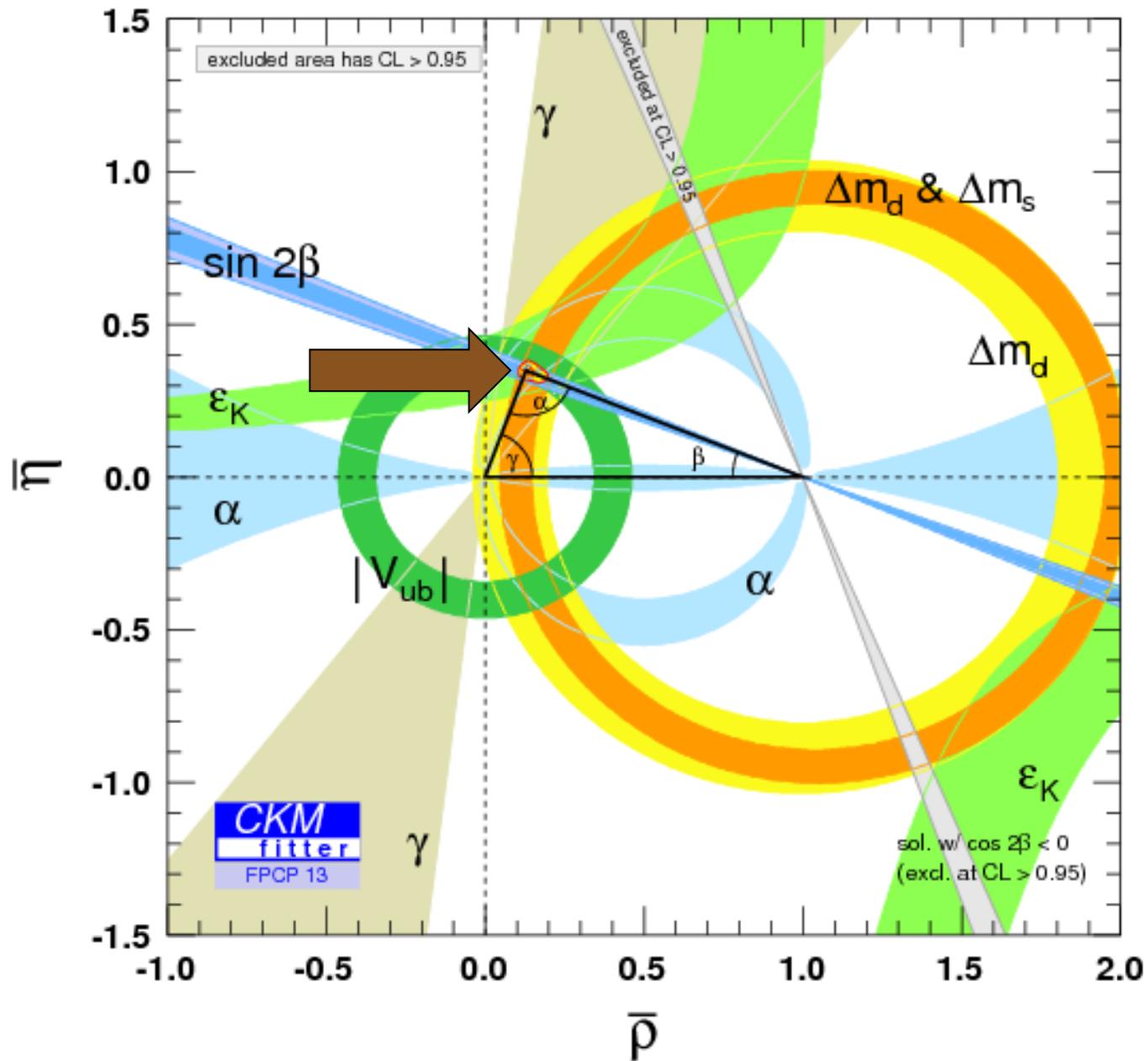


with





CKM Constraints and the Unitarity Triangle



Experimental
 constraints on
 the CKM
 matrix