CP Violation in the Higgs Sector

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Workshop “The Flavor of Higgs”

Weizmann Institut of Science, Tel Aviv, June 26, 2014

Partially based on work by

Brod, Haisch, Zupan – JHEP 1311 (2013) 180

Harnik, Martin, Okui, Primulando, Yu – Phys.Rev. D88 (2013) 7, 076009

Bishara, Grossman, Harnik, Robinson, Shu, Zupan – JHEP 1404 (2014) 084

Delaunay, Perez, de Sandes, Skiba – Phys.Rev. D89 (2014) 035004

Chen, Falkowski, Low, Vega-Morales – arXiv:1405.6723

Dolan, Harris, Jankowiak, Spannowski – arXiv:1406.3322

Motivation

“Es ist schon alles gesagt, nur noch nicht von allen”

“Everything has been said already, though not yet by everybody”

[Karl Valentin]

Motivation – I

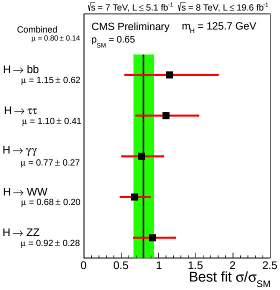
- We have found New Physics (NP) at the LHC! ⇒ The Higgs
- Yet, we still need to find NP beyond the Standard Model (BSM)
- The discovery of the/a Higgs boson opens a new window to NP BSM
- CP violation
  - In the quark sector consistent with SM
  - Already probe scales of up to $\mathcal{O}(10^4)$ TeV
  - CP violation in the Higgs sector?
- What do we know already / what can we learn in the future?

Motivation – II

- Higgs couplings completely determined in the SM
- That is why we need to measure them!
- For instance, in the SM

$$\mathcal{L}_Y = -\sum_i \frac{y_i}{\sqrt{2}} \bar{f}_i f_i \, h$$

- SM Yukawas are
  - flavor-diagonal
  - real (CP-conserving)



[CMS-PAS-HIG-13-005]

SM EFT

- No BSM particles at LHC ⇒ use EFT with only SM fields
  [See, e.g., Buchmüller et al. 1986, Grzadkowski et al. 2010]

$$\boxed{\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{SM}} + \mathcal{L}^{\text{dim.6}} + \dots}$$

For instance,

$$\mathcal{L}^{\text{eff}} \supset -\left(\alpha + \beta \frac{H^\dagger H}{\Lambda^2}\right) (\bar{Q}_L t_R H) + \text{h.c.}$$

Inserting $H = (0, (v+h)/\sqrt{2})^T$ yields

$$\mathcal{L}^{\text{eff}} \supset -\underbrace{\left(\alpha + \beta \frac{v^2}{2\Lambda^2}\right)}_{\equiv y_t^{\text{SM}}} \frac{v}{\sqrt{2}} \bar{t}_L t_R - \underbrace{\left(\alpha + 3\beta \frac{v^2}{2\Lambda^2}\right)}_{\equiv y_t^{\text{SM}} + 2\beta \frac{v^2}{2\Lambda^2}} \frac{h}{\sqrt{2}} \bar{t}_L t_R + \text{h.c.}$$

- α, β can be complex

- Test with collider and low-energy experiments

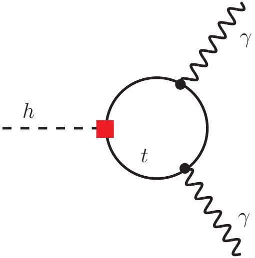
CP violation – reminder

- In the SM, only CP violation comes from electroweak sector (CKM phase)
- Switch off weak interactions:
  $K_1 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$, $K_2 = (K^0 - \bar{K}^0)/\sqrt{2}$
  are CP-even / CP-odd eigenstates
- Weak interactions lead to a superposition via box diagrams – K_L and K_S
- They are not CP eigenstates
- Analogy would be scalar h^0 and pseudoscalar A^0 Higgs in 2HDM
- If Higgs potential is not CP symmetric, lightest mass eigenstate is superposition $p\, h^0 + q\, A^0$

Outline

- CP violation in *htt*
- CP violation in *hbb*
- CP violation in $h\tau\tau$
- CP violation in *hVV* couplings
- Summary

From $h \to \gamma\gamma$ ...



- In the SM, Yukawa coupling to fermion f is

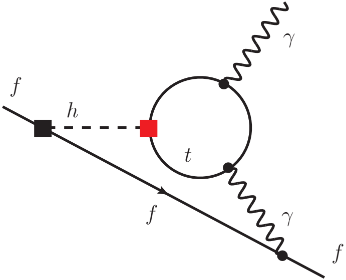
$$\mathcal{L}_Y = -\frac{y_f}{\sqrt{2}} \bar{f} f \, h$$

- We will look at modification

$$\mathcal{L}'_Y = -\frac{y_f}{\sqrt{2}} \left(\kappa_f \, \bar{f} f + i \tilde{\kappa}_f \, \bar{f} \gamma_5 f \right) h$$

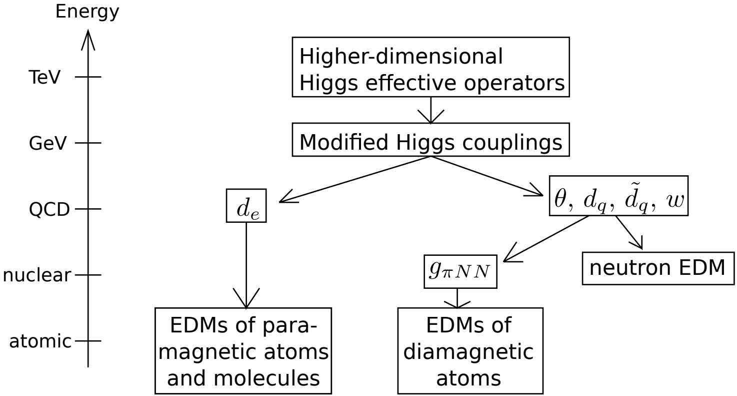
- New contributions will modify Higgs production cross section and decay rates

. . . to electric dipole moments



- Attaching a light fermion line leads to EDM
- Indirect constraint on *CP*-violating Higgs coupling
- SM “background” enters at three- and four-loop level
- Complementary to collider measurements
- Constraints depend on additional assumptions

Electric Dipole Moments (EDMs) – Generalities



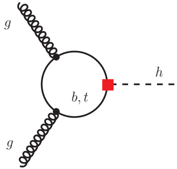
[Adapted from Pospelov et al., 2005]

Anomalous *htt* couplings

Constraints from $gg \to h$

- $gg \to h$ generated at one loop
- Have effective potential

$$V_{\text{eff}} = -c_g \frac{\alpha_s}{12\pi} \frac{h}{v} G^a_{\mu\nu} G^{\mu\nu,a} - \tilde{c}_g \frac{\alpha_s}{8\pi} \frac{h}{v} G^a_{\mu\nu} \widetilde{G}^{\mu\nu,a}$$



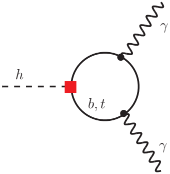
- c_g, $\tilde{c}_g$ given in terms of loop functions
- $\kappa_g \equiv c_g / c_{g,\text{SM}}$, $\tilde{\kappa}_g \equiv 3\tilde{c}_g / 2c_{g,\text{SM}}$

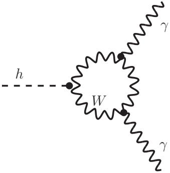
$$\frac{\sigma(gg \to h)}{\sigma(gg \to h)_{\text{SM}}} = |\kappa_g|^2 + |\tilde{\kappa}_g|^2 = \kappa_t^2 + 2.6\,\tilde{\kappa}_t^2 + 0.11\,\kappa_t\,(\kappa_t - 1)$$

Constraints from $h \to \gamma\gamma$

- $h \to \gamma\gamma$ generated at one loop
- Have effective potential

$$V_{\text{eff}} = -c_\gamma \frac{\alpha}{\pi} \frac{h}{v} F_{\mu\nu} F^{\mu\nu} - \tilde{c}_\gamma \frac{3\alpha}{2\pi} \frac{h}{v} F_{\mu\nu} \widetilde{F}^{\mu\nu}$$





- c_γ, $\tilde{c}_\gamma$ given in terms of loop functions
- $\kappa_\gamma \equiv c_\gamma / c_{\gamma,\text{SM}}$, $\tilde{\kappa}_\gamma \equiv 3\tilde{c}_\gamma / 2c_{\gamma,\text{SM}}$

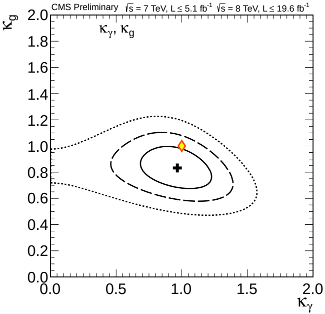
$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{\text{SM}}} = |\kappa_\gamma|^2 + |\tilde{\kappa}_\gamma|^2 = (1.28 - 0.28\,\kappa_t)^2 + (0.43\,\tilde{\kappa}_t)^2$$

LHC input

- Naive weighted average of ATLAS, CMS

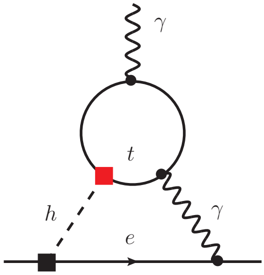
$$\kappa_{g,\text{WA}} = 0.91 \pm 0.08\,, \quad \kappa_{\gamma,\text{WA}} = 1.10 \pm 0.11$$

- We set $\kappa^2_{g/\gamma,\text{WA}} = |\kappa_{g/\gamma}|^2 + |\tilde\kappa_{g/\gamma}|^2$



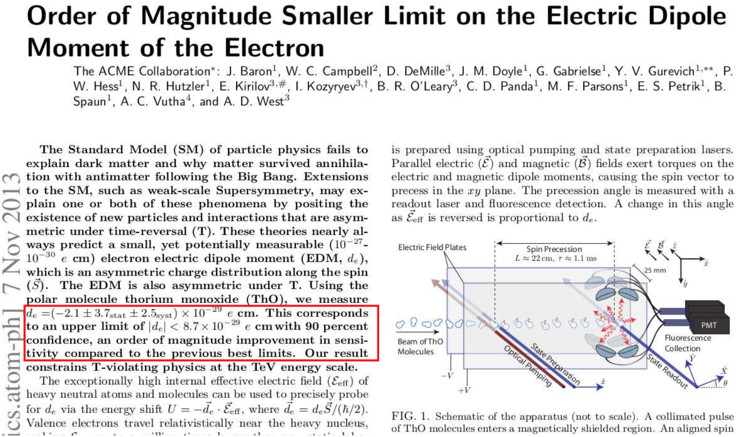
[CMS-PAS-HIG-13-005]

Electron EDM



- EDM induced via "Barr-Zee" diagrams [Weinberg 1989, Barr & Zee 1990]
- $|d_e/e| < 8.7 \times 10^{-29}$ cm (90% CL) [ACME 2013] with ThO molecules
- Constraint on $\tilde{\kappa}_t$ vanishes if Higgs does not couple to electron

ACME result on electron EDM



Order of Magnitude Smaller Limit on the Electric Dipole Moment of the Electron

The ACME Collaboration*: J. Baron[1], W. C. Campbell[2], D. DeMille[3], J. M. Doyle[1], G. Gabrielse[1], Y. V. Gurevich[1,**], P. W. Hess[1], N. R. Hutzler[1], E. Kirilov[3,#], I. Kozyryev[3,†], B. R. O'Leary[3], C. D. Panda[1], M. F. Parsons[1], E. S. Petrik[1], B. Spaun[1], A. C. Vutha[4], and A. D. West[3]

The Standard Model (SM) of particle physics fails to explain dark matter and why matter survived annihilation with antimatter following the Big Bang. Extensions to the SM, such as weak-scale Supersymmetry, may explain one or both of these phenomena by positing the existence of new particles and interactions that are asymmetric under time-reversal (T). These theories nearly always predict a small, yet potentially measurable (10^{-27}-10^{-30} e cm) electron electric dipole moment (EDM, d_e), which is an asymmetric charge distribution along the spin ($\vec{S}$). The EDM is also asymmetric under T. Using the polar molecule thorium monoxide (ThO), we measure $d_e = (-2.1 \pm 3.7_{\mathrm{stat}} \pm 2.5_{\mathrm{syst}}) \times 10^{-29}$ e cm. This corresponds to an upper limit of $|d_e| < 8.7 \times 10^{-29}$ e cm with 90 percent confidence, an order of magnitude improvement in sensitivity compared to the previous best limits. Our result constrains T-violating physics at the TeV energy scale.

The exceptionally high internal effective electric field ($\mathcal{E}_{\mathrm{eff}}$) of heavy neutral atoms and molecules can be used to precisely probe for d_e via the energy shift $U = -\vec{d}_e \cdot \vec{\mathcal{E}}_{\mathrm{eff}}$, where $\vec{d}_e = d_e \vec{S}/(\hbar/2)$. Valence electrons travel relativistically near the heavy nucleus,

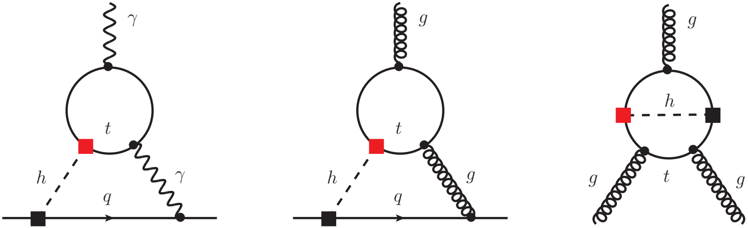
is prepared using optical pumping and state preparation lasers. Parallel electric ($\vec{\mathcal{E}}$) and magnetic ($\vec{\mathcal{B}}$) fields exert torques on the electric and magnetic dipole moments, causing the spin vector to precess in the xy plane. The precession angle is measured with a readout laser and fluorescence detection. A change in this angle as $\vec{\mathcal{E}}_{\mathrm{eff}}$ is reversed is proportional to d_e.

FIG. 1. Schematic of the apparatus (not to scale). A collimated pulse of ThO molecules enters a magnetically shielded region. An aligned spin

- Expect order-of-magnitude improvements!

Workshop "Fundamental Physics at the Intensity Frontier" [arXiv:1205.2671 [hep-ex]]

Neutron EDM



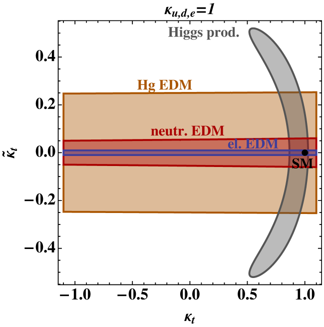
- Three operators; will mix, need to perform RGE analysis

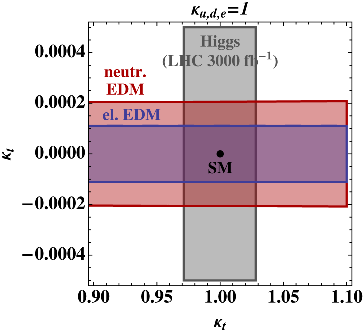
$$\frac{d_n}{e} = \Big\{(1.0 \pm 0.5)\left[-5.3\,\kappa_q\tilde{\kappa}_t + 5.1\cdot 10^{-2}\,\kappa_t\tilde{\kappa}_t\right]$$

$$+ (22 \pm 10)\,1.8\cdot 10^{-2}\,\kappa_t\tilde{\kappa}_t\Big\}\cdot 10^{-25}\,\mathrm{cm}\,.$$

- $w \propto \kappa_t\tilde{\kappa}_t$ subdominant
- $|d_n/e| < 2.9 \times 10^{-26}$ cm (90% CL) [Baker et al., 2006]

Combined constraints on top coupling





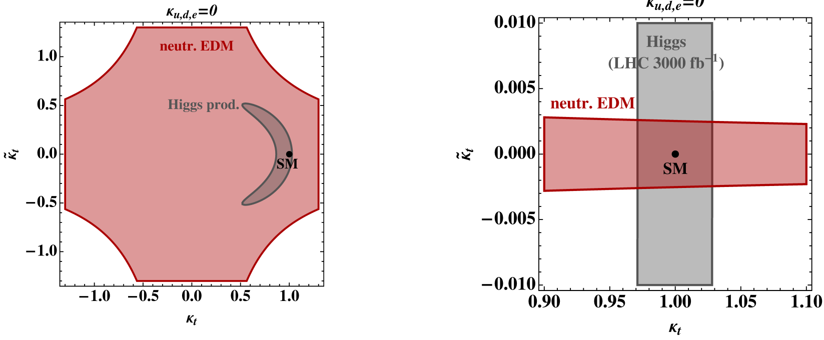
- Assume SM couplings to electron and light quarks
- Future projection for 3000fb^{-1} @ high-luminosity LHC

  [J. Olsen, talk at Snowmass Energy Frontier workshop]
- Factor 90 (300) improvement on electron (neutron) EDM

  [Fundamental Physics at the Energy Frontier, arXiv:1205.2671]

Combined constraints on top couplings

- Set couplings to electron and light quarks to zero
- Contribution of Weinberg operator will lead to strong constraints in the future scenario



Anomalous *hbb* couplings

Collider constraints

- Modifications of $gg \to h$, $h \to \gamma\gamma$ due to $\kappa_b \neq 1$, $\tilde{\kappa}_b \neq 0$ are subleading
- $\Rightarrow$ Main effect: modifications of branching ratios / total decay rate

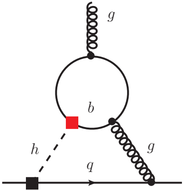
$$\mathrm{Br}(h \to b\bar{b}) = \frac{\big(\kappa_b^2 + \tilde{\kappa}_b^2\big)\mathrm{Br}(h \to b\bar{b})_{\mathrm{SM}}}{1 + \big(\kappa_b^2 + \tilde{\kappa}_b^2 - 1\big)\mathrm{Br}(h \to b\bar{b})_{\mathrm{SM}}}$$

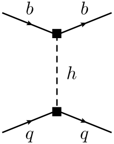
$$\mathrm{Br}(h \to X) = \frac{\mathrm{Br}(h \to X)_{\mathrm{SM}}}{1 + \big(\kappa_b^2 + \tilde{\kappa}_b^2 - 1\big)\mathrm{Br}(h \to b\bar{b})_{\mathrm{SM}}}$$

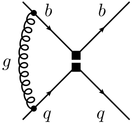
- Use naive averages of ATLAS / CMS signal strengths $\hat{\mu}_X$ for $X = b\bar{b}$, $\tau^+\tau^-$, $\gamma\gamma$, WW, ZZ
- $\hat{\mu}_X = \mathrm{Br}(h \to X)/\mathrm{Br}(h \to X)_{\mathrm{SM}}$ up to subleading corrections of production cross section

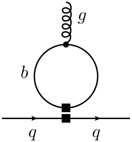
RGE analysis of the *b*-quark contribution to EDMs

- EDMs suppressed by small bottom Yukawa
- ≈ 3 scale uncertainty in CEDM Wilson coefficient
- Two-step matching at M_h and m_b:







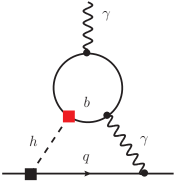


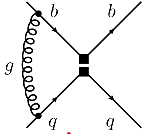
- Integrate out Higgs
- $\mathcal{O}_1^q = \bar{q}q\,\bar{b}i\gamma_5 b$

- Mixing into
- $\mathcal{O}_4^q = \bar{q}\sigma_{\mu\nu}T^a q\,\bar{b}i\sigma^{\mu\nu}\gamma_5 T^a b$

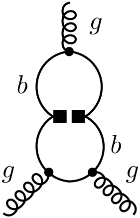
- Matching onto
- $\mathcal{O}_6^q = -\frac{i}{2}\frac{m_b}{g_s}\bar{q}\sigma^{\mu\nu}T^a\gamma_5 q G^a_{\mu\nu}$

RGE analysis of the *b*-quark contribution to EDMs

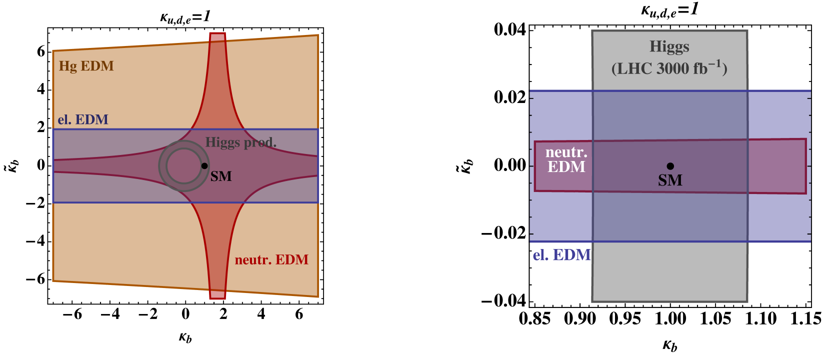




- $\mathcal{C}_5^q(\mu_b) = -4\,\frac{\alpha\,\alpha_s}{(4\pi)^2}\,Q_q\log^2\frac{m_b^2}{M_h^2} + \left(\frac{\alpha_s}{4\pi}\right)^3\frac{\gamma_{14}^{(0)}\gamma_{48}^{(0)}\gamma_{87}^{(0)}}{48}\,\log^3\frac{m_b^2}{M_h^2} + \mathcal{O}(\alpha_s^4)\,,$
- $\mathcal{C}_6^q(\mu_b) = \left(\frac{\alpha_s}{4\pi}\right)^2\frac{\gamma_{14}^{(0)}\gamma_{48}^{(0)}}{8}\,\log^2\frac{m_b^2}{M_h^2} + \mathcal{O}(\alpha_s^3)\,,$
- $\mathcal{C}_7(\mu_b) = \left(\frac{\alpha_s}{4\pi}\right)^2\frac{\gamma_{5,11}^{(1)}}{2}\,\log\frac{m_b^2}{M_h^2} + \mathcal{O}(\alpha_s^3)\,.$



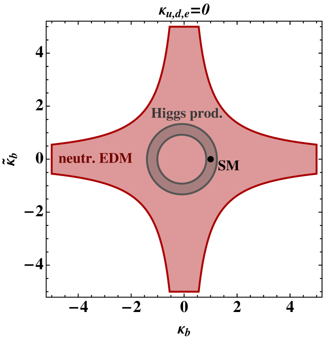
Combined constraints on bottom couplings

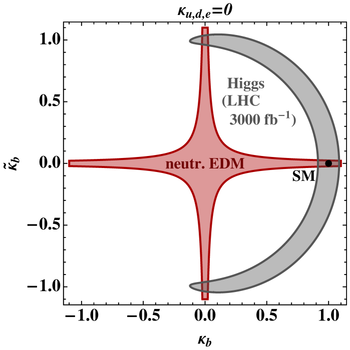


- Assume SM couplings to electron and light quarks
- Future projection for 3000fb^{-1} @ high-luminosity LHC
- Factor 90 (300) improvement on electron (neutron) EDM

Combined constraints on bottom couplings

- Set couplings to electron and light quarks to zero
- Contribution of Weinberg operator will lead to competitive constraints in the future scenario





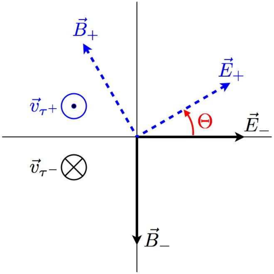
Anomalous $h\tau\tau$ couplings

CP violation in $h \to \tau^+\tau^-$

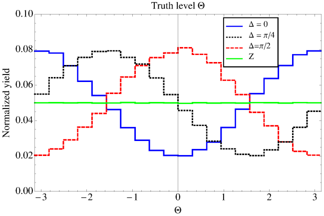
[Harnik et al., Phys.Rev. D88 (2013) 7, 076009 [arXiv:1308.1094[hep-ph]]]

- $\mathcal{L}'_Y \supset -\frac{y_\tau}{\sqrt{2}} h\bar{\tau}(\cos\Delta + i\gamma_5 \sin\Delta)\tau$
- Consider the decay $h \to \tau^+\tau^-$ where $\tau \to \rho\nu$ and $\rho^\pm \to \pi^\pm\pi^0$
- $\tau^+ - \tau^-$ spin correlation sensitive to the CP phase Δ
- τ spin information encoded in momentum distribution of its decay products

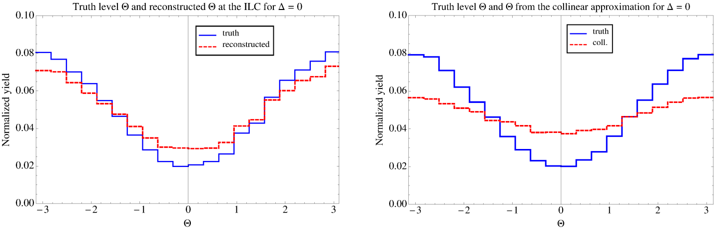
- Using some well-justified approximations, write differential cross section as $c - A\cos(\Theta - 2\Delta)$
- Here, Θ depends on the final-state momenta
- Find Δ as minimum in the Θ distribution



CP violation in $h \to \tau^+\tau^-$

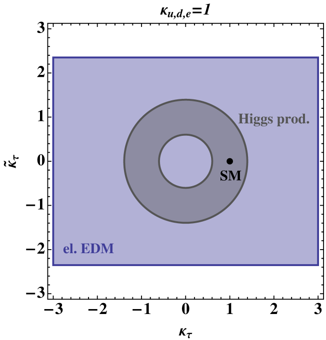


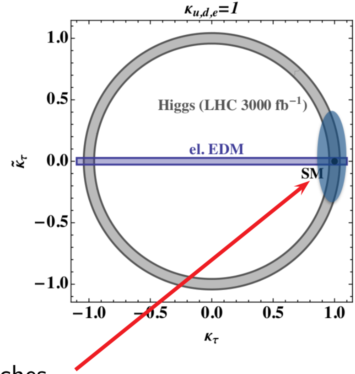
- At ILC can reconstruct both neutrino momenta
- At LHC use "collinear approximation" [Ellis et al., Nucl. Phys. B (297) 221 (1988)]
- Accuracy of 4.4° (ILC), 11.5° (high-lumi LHC)



Combined constraints on τ couplings

- Effect of modified $h\tau\tau$ coupling on κ_γ, $\tilde{\kappa}_\gamma$ again subleading
- Get simple constraint from modification of branching ratios





- Shaded region shows reach for direct searches

  [Harnik et al., Phys.Rev. D88 (2013) 7, 076009 [arXiv:1308.1094[hep-ph]]]

Radiative Higgs decays

More EFT

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{SM}} + \mathcal{L}^{\text{dim.6}} + \ldots$$

$$Q_{HD} \equiv (H^\dagger D_\mu H)^* (H^\dagger D_\mu H)\,,$$

$$Q_{HWB} \equiv (H^\dagger \sigma^a H) W^a_{\mu\nu} B^{\mu\nu}\,,$$

$$Q_{HW} \equiv (H^\dagger H) W^a_{\mu\nu} W^{a\mu\nu}\,,$$

$$Q_{H\widetilde{W}} \equiv (H^\dagger H) \widetilde{W}^a_{\mu\nu} W^{a\mu\nu}\,,$$

$$Q_{HB} \equiv (H^\dagger H) B_{\mu\nu} B^{\mu\nu}\,,$$

$$Q_{H\widetilde{B}} \equiv (H^\dagger H) \widetilde{B}_{\mu\nu} B^{\mu\nu}\,,$$

$$Q_{H\widetilde{W}B} \equiv (H^\dagger \sigma^a H) \widetilde{W}^a_{\mu\nu} B^{\mu\nu}\,.$$

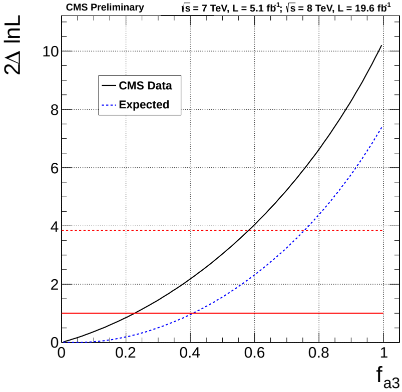
- $Q_{HD} \propto S$, $Q_{HWB} \propto T$
- Test with collider and low-energy experiments

$h \to \gamma\gamma$ vs. $h \to ZZ$

$$\begin{aligned}\mathcal{L}^{\text{eff}} \supset \quad & c_V \frac{M_Z^2}{v} h Z_\mu Z^\mu + \hat{c} \frac{\alpha}{\pi v} h F_{\mu\nu} F^{\mu\nu} + \hat{c}_{ZZ} \frac{\alpha}{\pi v} h Z_{\mu\nu} Z^{\mu\nu} \\ & + \tilde{c}_{ZZ} \frac{\alpha}{2\pi v} h Z_{\mu\nu} \widetilde{Z}^{\mu\nu} + \tilde{c} \frac{\alpha}{2\pi v} h F_{\mu\nu} \widetilde{F}^{\mu\nu}\end{aligned}$$

- $\tilde{c}$ and $\tilde{c}_{ZZ}$ couplings are CP odd
- CP violation only in dim.-5 operators, generated at one loop
- $h \to ZZ$: CP-conserving contribution generated at tree level
  ⇒ Need $\mathcal{O}(10^{-2})$ – $\mathcal{O}(10^{-3})$ measurement to see CP violation
- $h \to \gamma\gamma$: CP-conserving contribution generated at one loop
  ⇒ Large $\mathcal{O}(1)$ CP-violating effects are possible

$h \to ZZ^* \to 4\ell$

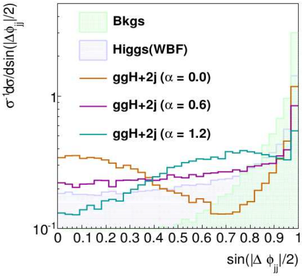


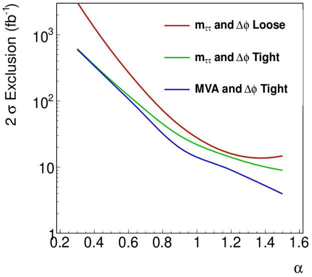
[CMS-HIG-13-002]

- f_{a3} related to $Z_{\mu\nu}\widetilde{Z}^{\mu\nu}$ interaction
- $f_{a3} < 0.47$ @ 95% CL

$pp \to h + 2j$ in gluon fusion

- In VBF $hZ_\mu Z^\mu$ vs. $hZ_{\mu\nu}\widetilde{Z}^{\mu\nu}$
- In gluon fusion $hG_{\mu\nu}\widetilde{G}^{\mu\nu}$ vs. $hG_{\mu\nu}\widetilde{G}^{\mu\nu}$
- In both cases main sensitivity from angular correlations of tagging jets
- Use $\sin(|\Delta\phi_{jj}|/2)$ [Del Duca et al., 2006; Klamke et al., 2007]
- The model: $\mathcal{L} = \cos(\alpha)\, y_f\, \bar{\psi}_f \psi_f\, h + \sin(\alpha)\, \tilde{y}_f\, \bar{\psi}_f\, i\gamma_5 \psi_f\, h$





More recent ideas

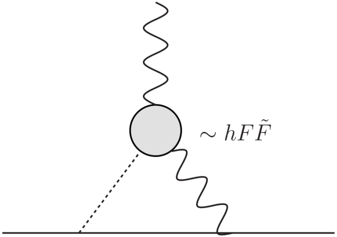
$h \to \gamma\gamma$ with converted photons

[Bishara et al., JHEP 1404 (2014) 084 [arXiv:1312.2955[hep-ph]]]

- Total rate $\Gamma_{h\to\gamma\gamma}$ always quadratic in the CP-violating parameter
  - Rate is always enhanced by CPV contribution
- Construct an observable in $h\gamma\gamma$ linear in the CP-violating parameter $\Rightarrow$ differential rate
- Effects can be $\mathcal{O}(1)$
- The measurement is very challenging

Limits from electron EDM

- Constraint on $y_e \cdot \tilde{c}$ from electron EDM



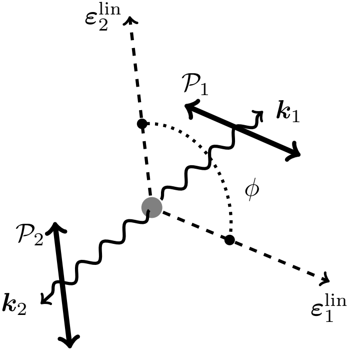
- $\tilde{c} \lesssim 10^{-3}$ for SM electron Yukawa [McKeen et al., Phys.Rev. D86 (2012) 113004 [arXiv:1208.4597[hep-ph]] – updated with new ACME result]
- Vanishes if Higgs does not couple to electron, or if there are cancellations

$h \to \gamma\gamma$ – how it works (in principle)

- Higgs is a scalar – no information on $\tilde{c}$ from angular distribution of photons
- Need to measure photon polarization
- For perfect linear polarization analyzers

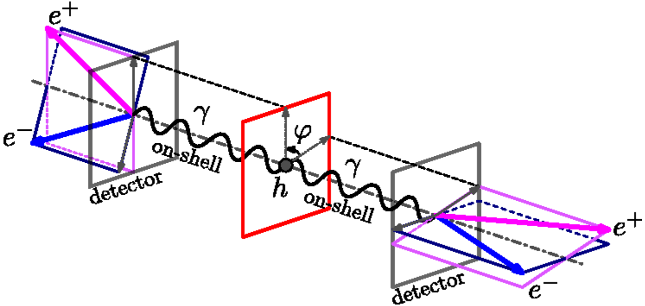
$$\frac{d\Gamma}{d\phi} = \frac{2}{\pi}\Gamma_{h\to\gamma\gamma}\underbrace{\cos^2(\phi+\xi)}_{\supset \hat{c}\tilde{c}\sin 2\phi}$$

- Here $\xi \equiv \tan^{-1}(\tilde{c}/\hat{c})$
- Shift in the modulation of the rate, linear in ξ



$h \to \gamma\gamma$ – how it works (in practice)

- Need to measure opening angles of $\mathcal{O}(10^{-4})$ to $\mathcal{O}(10^{-3})$
- At the limit of ATLAS / CMS pixel detectors
- Single angle carries information about CPV, can take φ

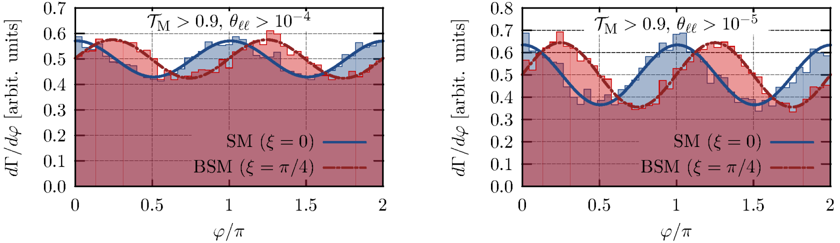


$h \to \gamma\gamma$ – how it works (maybe in the future)

- Differential spectrum has following form

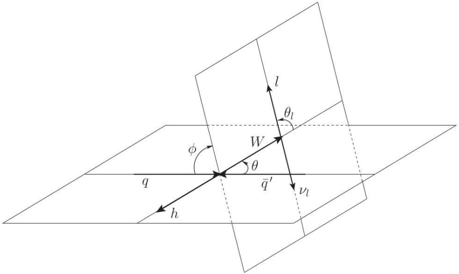
$$\frac{d\Gamma_{\mathrm{HBH}}}{d\varphi} = \mathcal{A} + \mathcal{B}\cos(2\xi + 2\varphi)$$

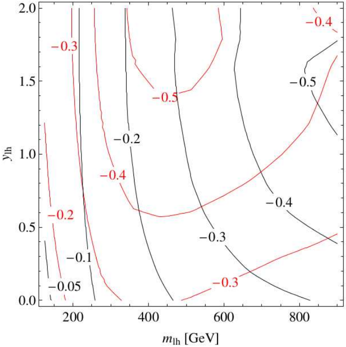
- Possible to find large effects in parts of phase space
- Choose cuts that select phase space regions with large effects
- Unrealistic for LHC



Other Observables – $q\bar{q} \to Wh$

[Delaunay et al., Phys.Rev. D89 (2014) 035004 [arXiv:1308.4930[hep-ph]]]





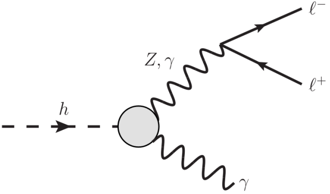
- Define up-down asymmetry $A_{CP} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$
- At Tevatron $A_{CP}^{p\bar{p}} \simeq -23\% \ldots - 6.3\%$
- Initial state symmetric at LHC – need additional cuts

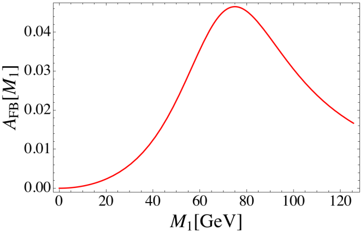
Other Observables – $h \to \ell^+\ell^-\gamma$

[Chen et al., arXiv:1405.6723[hep-ph]]

- Analoguous to "direct CP violation" in flavor physics
- Instead of triple products use "strong" phases of off-shell Z, γ
- Weak phases from coefficients in

  $\mathcal{L} \supset h/v(A_1 F^{\mu\nu} Z_{\mu\nu} + A_2 F^{\mu\nu} \tilde{Z}_{\mu\nu} + A_3 F^{\mu\nu} F_{\mu\nu} + A_4 F^{\mu\nu} \tilde{F}_{\mu\nu})$
- Lepton A_{FB} is CPV observable





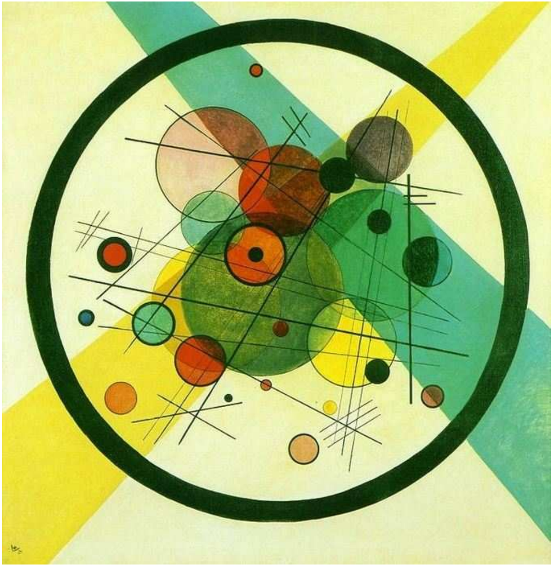
Other Observables – $h \to \ell^+\ell^-\gamma$

- …may be seen at high-luminosity LHC
- Can consider Z final state
- Or cross diagram to get $A_{FB}(e^+e^- \to hZ)$
- Or look at $A_{FB}(q\bar{q} \to hZ)$ – 100 TeV collider?

Summary

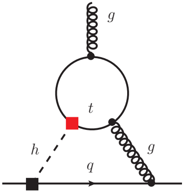
- CP violation in the Higgs sector is not so easy to see
- EDMs give strong bounds on CP violation
    - . . . but depend on additional assumptions
- Will have huge experimental progress in the future

Outlook



Appendix

Mercury EDM

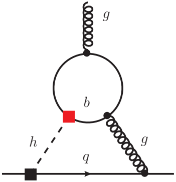


- Diamagnetic atoms also provide constraints
- $|d_{\rm Hg}/e| < 3.1 \times 10^{-29}\,$cm (95% CL) [Griffith et al., 2009]
- Dominant contribution from CP-odd isovector pion-nucleon interaction

$$\frac{d_{\rm Hg}}{e} = -\left(4^{+8}_{-2}\right)\,\left[3.1\,\tilde\kappa_t - 3.2\cdot 10^{-2}\,\kappa_t\tilde\kappa_t\right]\cdot 10^{-29}\,{\rm cm}$$

- Again, $w \propto \kappa_t\tilde\kappa_t$ subdominant, but does not vanish if Higgs does not couple to light quarks

Constraints from EDMs



- Contributions to EDMs suppressed by small Yukawas; still get meaningful constraints in future scenario
- For electron EDM, simply replace charges and couplings
- Have extra scale $m_b \ll M_h \Rightarrow \log m_b^2/M_h^2$

$$d_q(\mu_W) \simeq -4\, e\, Q_q\, N_c\, Q_b^2 \frac{\alpha}{(4\pi)^3} \sqrt{2} G_F\, m_q\, \kappa_q \tilde{\kappa}_b \frac{m_b^2}{M_h^2} \left(\log^2 \frac{m_b^2}{M_h^2} + \frac{\pi^2}{3} \right) ,$$

$$\tilde{d}_q(\mu_W) \simeq -2 \frac{\alpha_s}{(4\pi)^3} \sqrt{2} G_F\, m_q\, \kappa_q \tilde{\kappa}_b \frac{m_b^2}{M_h^2} \left(\log^2 \frac{m_b^2}{M_h^2} + \frac{\pi^2}{3} \right) ,$$

$$w(\mu_W) \simeq -g_s \frac{\alpha_s}{(4\pi)^3} \sqrt{2} G_F\, \kappa_b \tilde{\kappa}_b \frac{m_b^2}{M_h^2} \left(\log \frac{m_b^2}{M_h^2} + \frac{3}{2} \right) .$$