Systematic studies of texture zeros in lepton mass matrices

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Brighton, June 20th, 2014

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FWF Der Wissenschaftsfonds.

FWF Project P 24161-N16

Flavour symmetries in the lepton sector

One of the most interesting questions in flavour physics:

$\rightarrow$ Question for flavour symmetries.

$\Rightarrow$ Two possibilities:

- Abelian symmetries $\rightarrow$ texture zeros in the mass matrices.
- Non-Abelian symmetries.

Non-Abelian symmetries

Approach intensively studied recently: residual symmetries in the mass matrices:

→ Lam; Hernandez, Smirnov; Toorop, Feruglio, Hagedorn; Holthausen, Lim, Lindner; Hagedorn, Meroni, Vitale; King, Neder, Stuart; Lavoura, POL; ...

R.M. Fonseca and W. Grimus,
Classification of lepton mixing matrices from residual symmetries
[arXiv:1405.3678 [hep-ph]].

Result of this paper:

- Assuming Majorana neutrinos → Classification of **all** possible mixing matrices fully determined by residual symmetries.
- 16 sporadic mixing matrices, 1 infinite series of mass matrices.
- Only members of the infinite series of mixing matrices are compatible with the data at 3σ.

$$|U|^2 = \frac{1}{3}\begin{pmatrix} 1 & 1+\mathrm{Re}\,\sigma & 1-\mathrm{Re}\,\sigma \\ 1 & 1+\mathrm{Re}(\omega\sigma) & 1-\mathrm{Re}(\omega\sigma) \\ 1 & 1+\mathrm{Re}(\omega^2\sigma) & 1-\mathrm{Re}(\omega^2\sigma) \end{pmatrix}; \quad \omega = e^{2\pi i/3}; \quad \sigma = e^{2\pi i p/n}.$$

Abelian symmetries

Abelian symmetries impose texture zeros (TZ) in the mass matrices.

In this talk we focus on texture zeros in the *lepton* mass matrices.

Mostly studied situation:

- Diagonal charged-lepton mass matrix M_ℓ + TZ in the neutrino mass matrix M_ν:
  - *Dirac neutrinos:* Hagedorn, Rodejohann[1]
  - *Majorana neutrinos:* Frampton, Glashow, Marfatia; Xing[2],...
- TZ in M_ℓ and M_ν, both non-diagonal + additional symmetries (e.g. Hermitian mass matrices, Fritzsch textures).

→ Our aim: Study **all types of texture zeros (without other imposed symmetries)** in M_ℓ and M_ν.

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[1] Hagedorn, Rodejohann, JHEP **0507** (2005) 034.

[2] Frampton, Glashow, Marfatia, Phys. Lett. **B 536** (2002) 79; Xing, Phys. Lett. **B 530** (2002) 159.

Systematic studies of texture zeros in the lepton mass matrices

POL and Walter Grimus:

A complete survey of texture zeros in the lepton mass matrices.

[arXiv:1406.3546 [hep-ph]].

Three parts:

1. Classification of texture zeros in the lepton mass matrices.
2. Which sets of texture zeros are compatible with the experimental observations?
3. What is the predictive power of texture zeros?

I. Classification of texture zeros in the lepton mass matrices

Classification of texture zeros in the lepton mass matrices

Setting:

	fields	mass term	mass matrix
charged leptons	$3\ \ell_{iL}$, $3\ \ell_{iR}$	$-\overline{\ell_L} M_\ell \ell_R + \text{H.c.}$	M_ℓ complex 3×3
Dirac ν**s**	$3\ \nu_{iL}$, $3\ \nu_{iR}$	$-\overline{\nu_R} M_D \nu_L + \text{H.c.}$	M_D complex 3×3
Majorana ν**s**	$3\ \nu_{iL}$	$\frac{1}{2}\nu_L^T C^{-1} M_L \nu_L + \text{H.c.}$	M_L c. symm. 3×3

How many different types of texture zeros (TTZ) in the lepton mass matrices?

$\rightarrow$ number is **huge**:

- Dirac neutrinos: (M_ℓ, M_D): $2^9 \times 2^9 = 262144$ different TTZ.
- Majorana neutrinos: (M_ℓ, M_L): $2^9 \times 2^6 = 32768$ different TTZ.

Texture zeros and weak-basis transformations

Lepton masses and mixing matrix U_{PMNS} invariant under weak-basis transformations.

$\Rightarrow$ Use weak-basis transformations to **divide texture zeros into equivalence classes** making the same physical predictions.[3]

Field redefinition: $\ell_{L,R} = V_{L,R}^{(\ell)} \ell'_{L,R}, \quad \nu_{L,R} = V_{L,R}^{(\nu)} \nu'_{L,R}$

matrices $V_{L,R}^{(\ell,\nu)}$ *unitary* (kinetic terms invariant); $V_L^{(\ell)} = V_L^{(\nu)}$ (gauge invariance).

Transformation of mass matrices under weak-basis transformations

Dirac neutrinos: $M'_\ell = V_L^\dagger M_\ell V_R^{(\ell)} \quad$ and $\quad M'_D = V_R^{(\nu)\dagger} M_D V_L$.

Majorana neutrinos: $M'_\ell = V_L^\dagger M_\ell V_R^{(\ell)} \quad$ and $\quad M'_L = V_L^T M_L V_L$.

[3] Hagedorn, Rodejohann, JHEP **0507** (2005) 034.

Texture zeros and weak-basis transformations

Weak-basis transformations which leave the number of texture zeros in a mass matrix invariant:

V_L and $V_R^{(\ell,\nu)}$ are (up to phases) permutation matrices

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

$\rightarrow$ “weak-basis permutations”

$\Rightarrow$ Can divide TTZ into equivalence classes.

Equivalence classes of texture zeros

Take all TTZ and

- Remove textures which do not make any predictions at all. (Some TTZ may always be created through weak-basis transformations.[4])
- Charged-lepton masses non-zero and non-degenerate $\rightarrow$ Only allow textures with $\mathrm{rank}(M_\ell) = 3$.
- Neutrino masses non-degenerate $\rightarrow$ Only allow textures with $\mathrm{rank}(M_\nu) = 2$ or 3.
- Arrange remaining textures into equivalence classes using weak-basis permutations.

$\Rightarrow$

- Dirac neutrinos 2^{18} TTZ $\rightarrow$ **570 classes.**
- Majorana neutrinos 2^{15} TTZ $\rightarrow$ **298 classes.**

[4] Branco, Emmanuel-Costa, Gonzalez Felipe, Phys. Lett. **B 477** (2000) 147; Branco, Emmanuel-Costa, Gonzalez Felipe, Serodio, Phys. Lett. **B 670** (2009) 340.

II. Which sets of texture zeros are compatible with the experimental observations?

χ^2-analysis

Tested which TTZ are compatible with the experimental observations by means of a χ^2-analysis.

About 75% of all TTZ are compatible with the experimental observations!

For further investigation: Go through list of compatible textures and keep only maximally restrictive TTZ.

Maximally restrictive types of texture zeros

A set of texture zeros compatible with experiment is **maximally restrictive**, if imposing an additional texture zero it becomes incompatible with the physical observations.

Results of the χ^2-analysis

Neutrino nature	Dirac		Majorana	
Neutrino mass spectrum	normal	inverted	normal	inverted
Number of textures	570	570	298	298
Compatible with experiment	434	433	218	228
Compatible and maximally restrictive	**30**	**29**	**33**	**27**

$\rightarrow$ Out of all possibilities: Only about **30 maximally restrictive viable classes of texture zeros** for each Dirac and Majorana neutrinos and normal and inverted spectrum.

III. What is the predictive power of texture zeros?

Predictivity measures for models with texture zeros

Lepton physics observables:

- Already measured:

$$m_e,\ m_\mu,\ m_\tau,\ \Delta m^2_{21},\ \Delta m^2_{31},\ \sin^2\theta_{12},\ \sin^2\theta_{23},\ \sin^2\theta_{13}.$$

- Probably measured in the (near?) future:

$$m_0,\ \delta,\ m_{\beta\beta},\ \text{two Majorana phases.}$$

Idea behind our predictivity measures

Given a viable set of texture zeros and **fixing all but one** of the (already measured) observables to their experimentally observed values, how much can the **remaining observable at most deviate** from its experimental or best-fit value?

Predictivity measures for models with texture zeros

Predictivity criteria:

- Charged-lepton masses: A set of texture zeros correctly predicts $m_\ell \in \{m_e, m_\mu, m_\tau\}$ if (fixing all other parameters to their experimental values):

$$\frac{1}{2} m_\ell^{\mathrm{exp}} \leq m_\ell \leq 2 m_\ell^{\mathrm{exp}}.$$

- Oscillation parameters: A set of texture zeros correctly predicts $\mathcal{O} \in \{\Delta m^2_{21}, \Delta m^2_{31}, s^2_{12}, s^2_{23}, s^2_{13}\}$ if (fixing all other parameters to their experimental values):

  $\mathcal{O}$ can deviate from its best-fit value by at most 10σ.

- Not yet measured observables $\mathcal{O} \in \{m_0, \delta, m_{\beta\beta}, \text{Majorana phases}\}$: A set of texture zeros correctly predicts $\mathcal{O}$ if (fixing all measured observables to their experimental values):

$$\frac{\mathcal{O}^{\mathrm{max}} - \mathcal{O}^{\mathrm{min}}}{\mathrm{range}(\mathcal{O})} \leq 0.2.$$

Results of the predictivity analysis

We investigated only the **maximally restrictive TTZ** compatible with the data.

- None of the investigated textures can predict any of the charged-lepton masses.
- No set of texture zeros can predict any of the five neutrino oscillation parameters.
- Most investigated textures can predict m_0. (Mostly $m_0 = 0$.)
- Dirac neutrinos: Maximally predictive and compatible textures: $\delta \in \{0, \pi\}$. ($\rightarrow$ no CP violation in neutrino oscillations).
- Majorana neutrinos: Many textures can predict δ (also $\delta \neq 0, \pi$.)
- None of the textures for Majorana neutrinos can predict the Majorana phases.
- Almost all maximally restrictive and compatible sets of textures in (M_ℓ, M_L) predict $m_{\beta\beta}$.
- $m_{\beta\beta}$ can be big (larger than $0.1\,\mathrm{eV}$) or small (normal neutrino mass spectrum). Inverted spectrum: $m_{\beta\beta} < 0.1\,\mathrm{eV}$.

Conclusions

- Three families of leptons.
- Discussed Dirac and Majorana neutrinos.
- Required tree-level mass matrices to be compatible with exp. data.
- Same gauge structure as standard model: ℓ_R, ν_R gauge singlets.
- **Classified all possible texture zeros** in (M_ℓ, M_D) or (M_ℓ, M_L), also **non-diagonal** M_ℓ.
- Through weak-basis permutations: **570** classes of TZ for Dirac neutrinos, **298** classes for Majorana neutrinos.
- Maximally restrictive classes: about 30 for each Dirac/Majorana and normal/inverted neutrino mass spectrum.
- For maximally restrictive textures: **predictivity analysis.**
- None of these textures can predict an already measured observable.

In summary:

Texture zeros are astonishingly weak in their predictions. Predictive mass matrices need also **relations among the non-zero matrix elements**.

Thank you for your attention!