Lepton Masses and Mixing Angles from Point Interactions

Yukihiro Fujimoto
(Osaka Univ.)

Based on arXiv:1405.5872

Collaborating with

Kenji Nishiwaki (Harish-Chandra Research Inst.)
Makoto Sakamoto (Kobe Univ.)
Ryo Takahashi (Simane Univ.)

Mysteries of the Standard Model

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❒ Generations

Who ordered the same packages in this world… ?

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  What determine the flavor mixing structure …?

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What determine the flavor mixing structure …?

❐ CP phase

What determine the value of CP phase …?

Purpose

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- ❐ Flavor mixing
- ❐ Mass Hierarchy
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- ❐ Flavor mixing
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in the context of higher-dimensional gauge theories.

Idea

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- Extra dimension
- Point interactions (Extra boundary points)
- y-dependent scalar VEV

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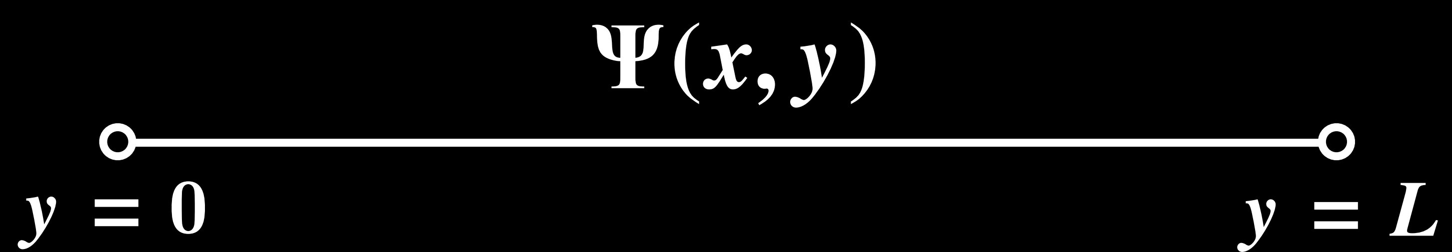

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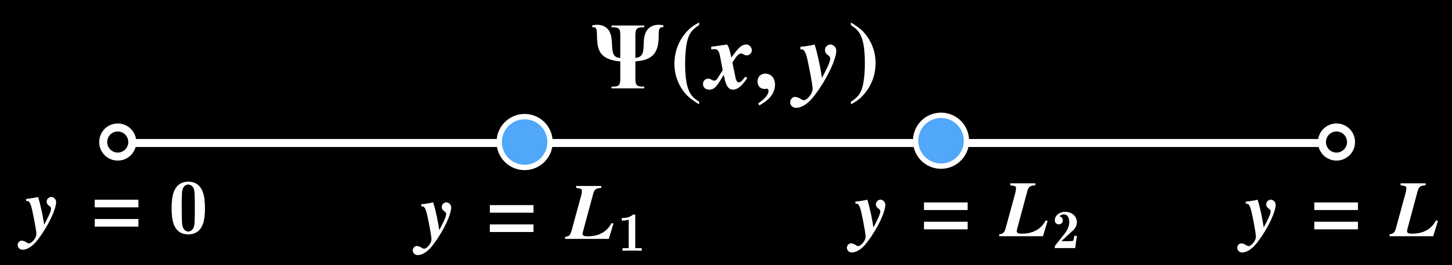

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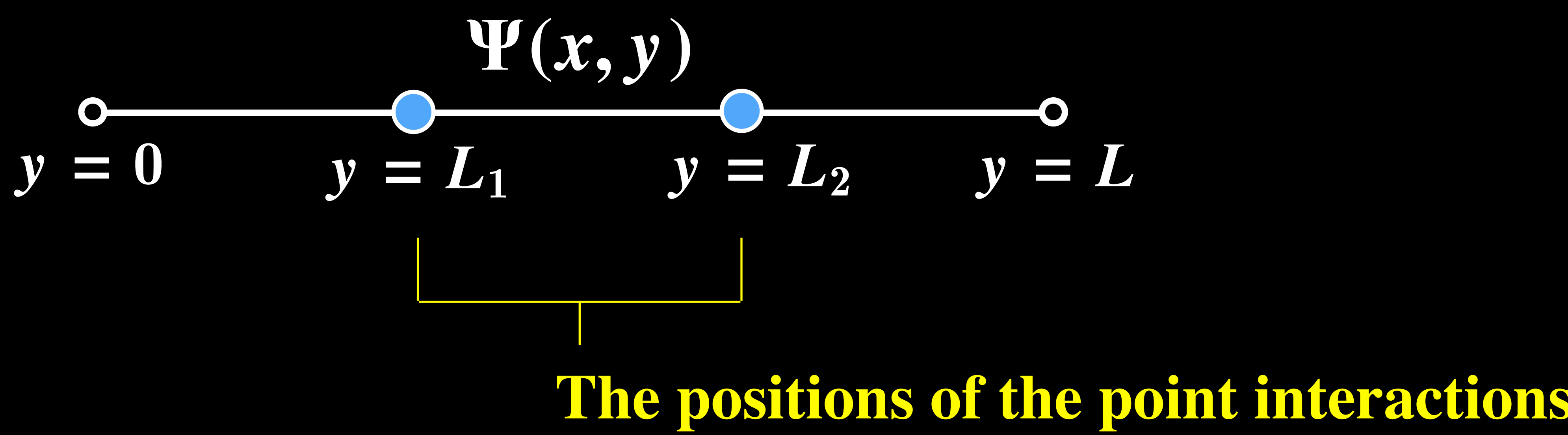

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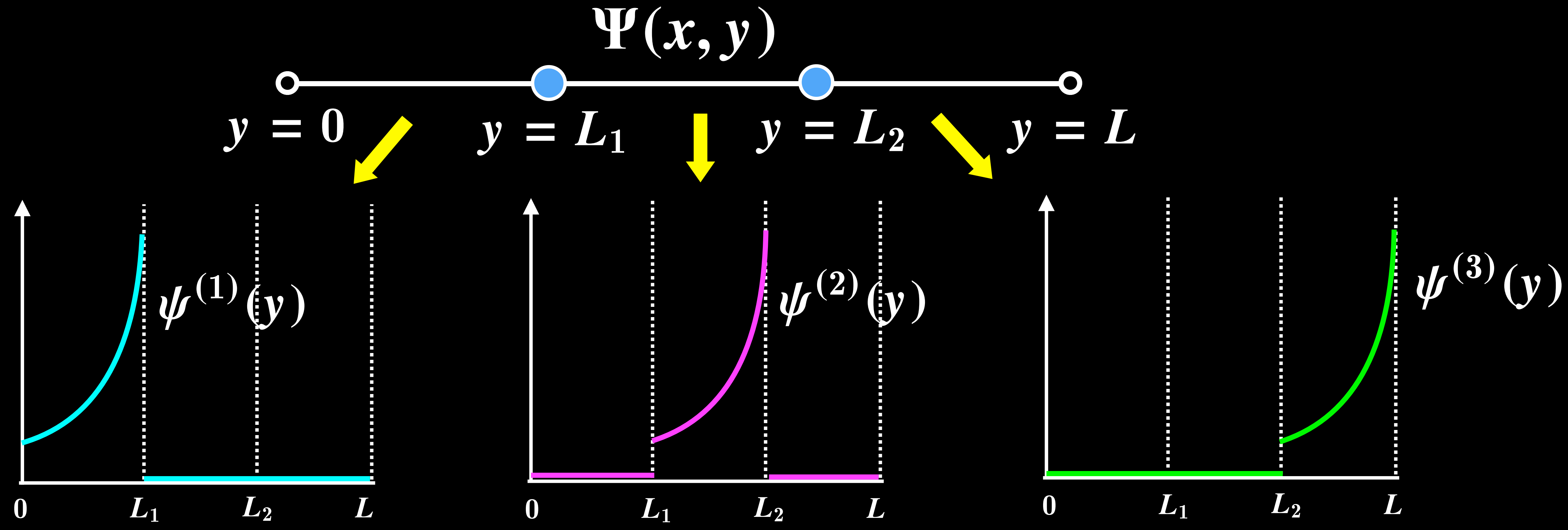

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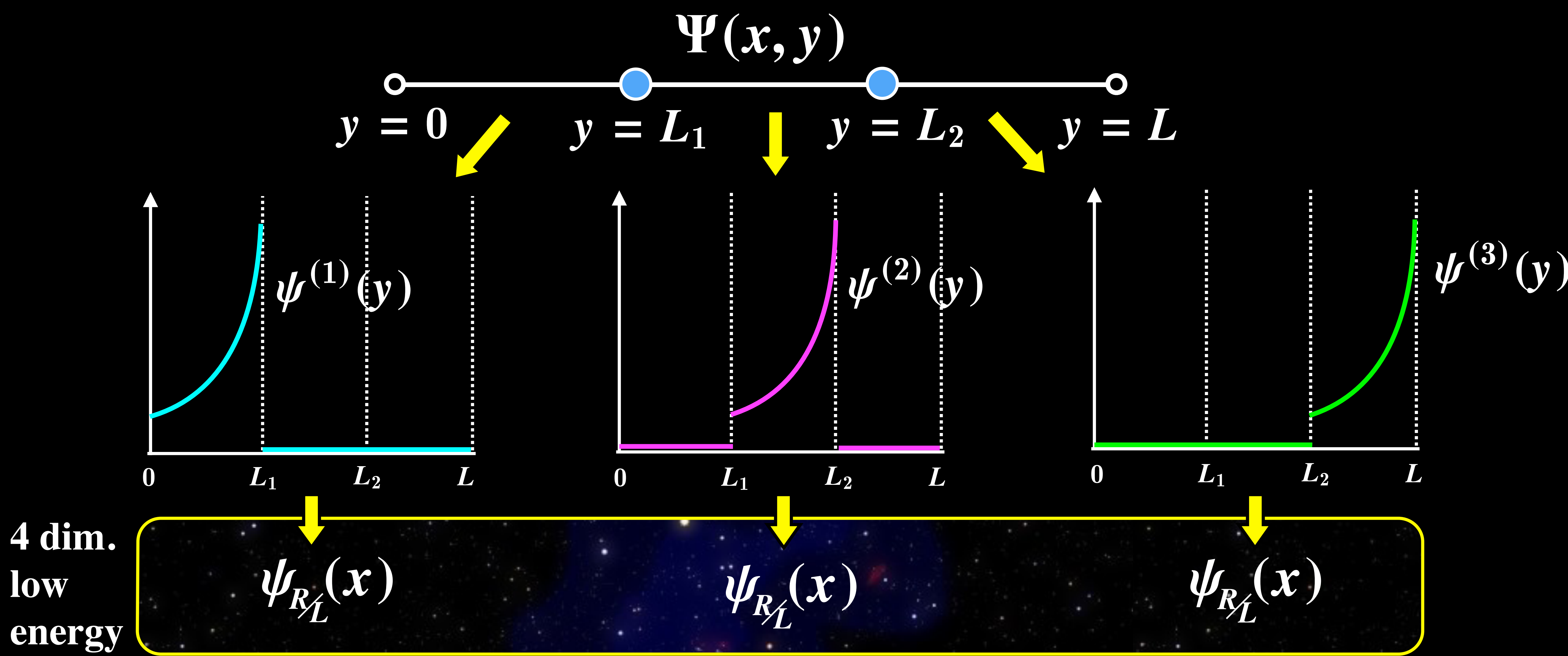

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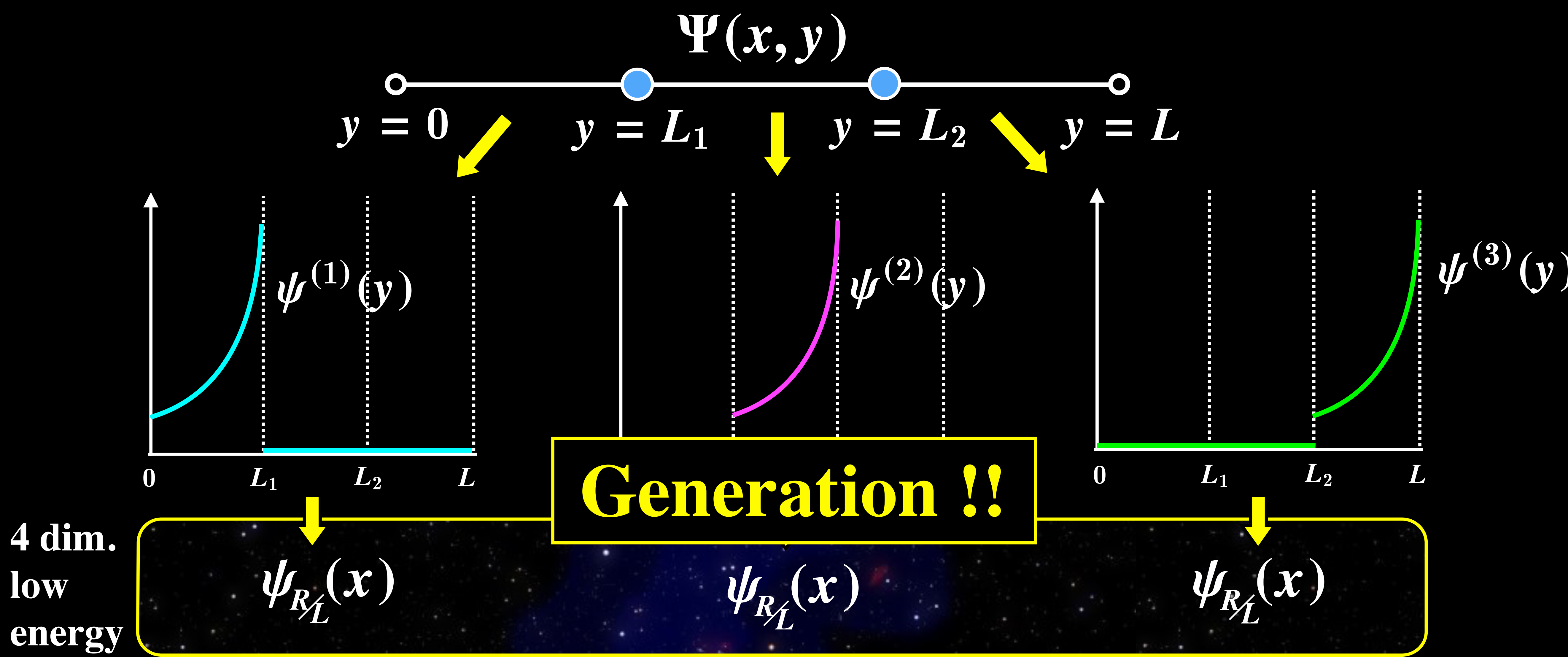

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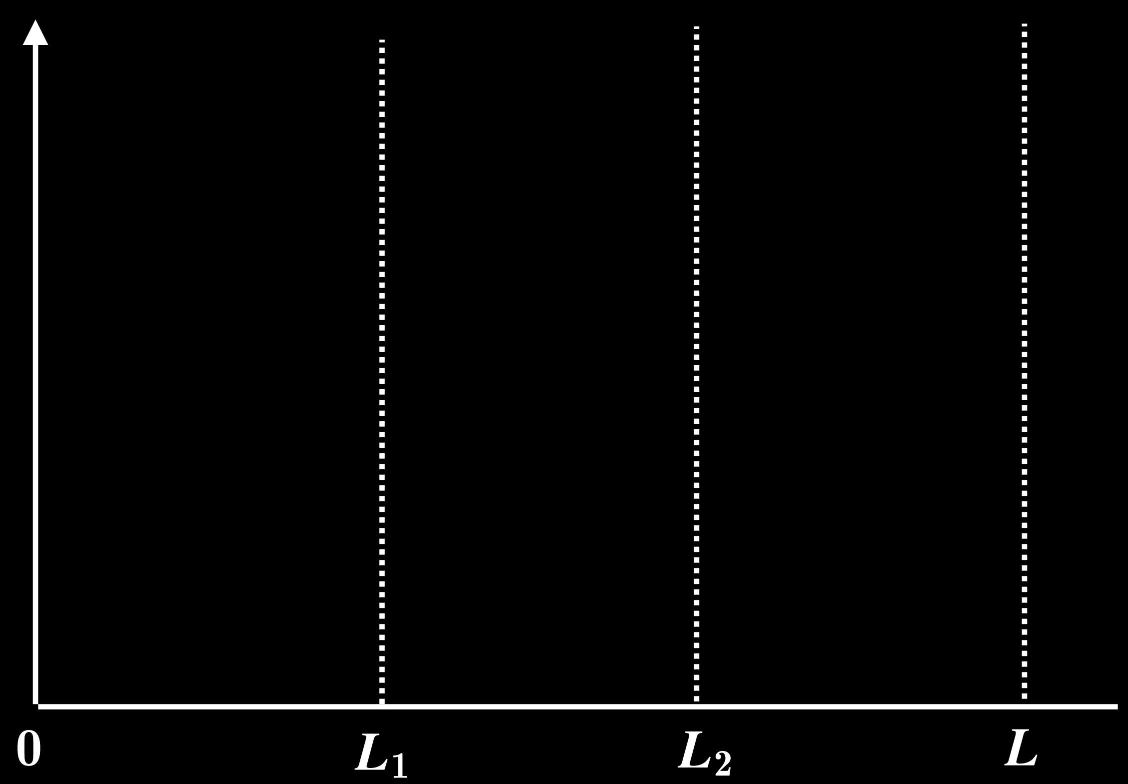

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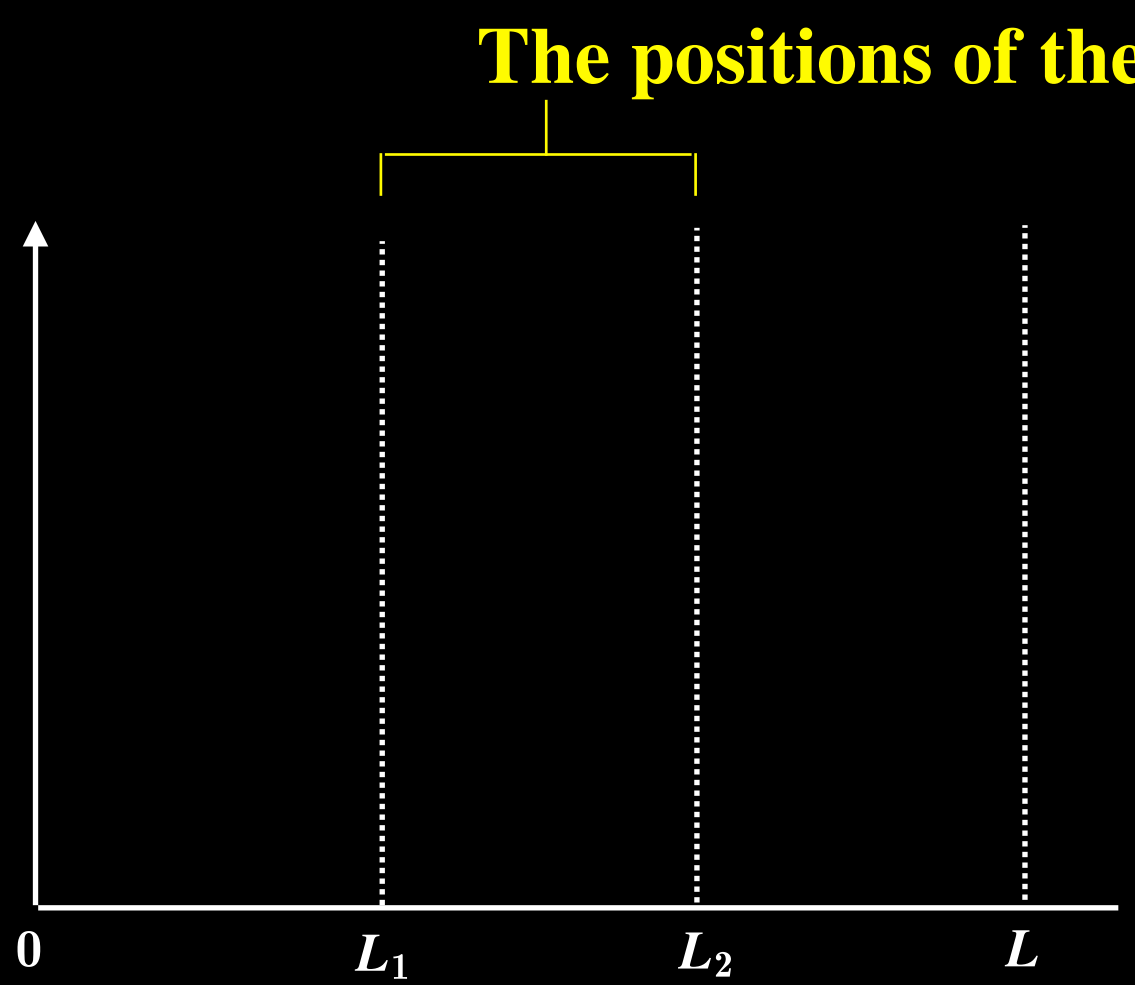

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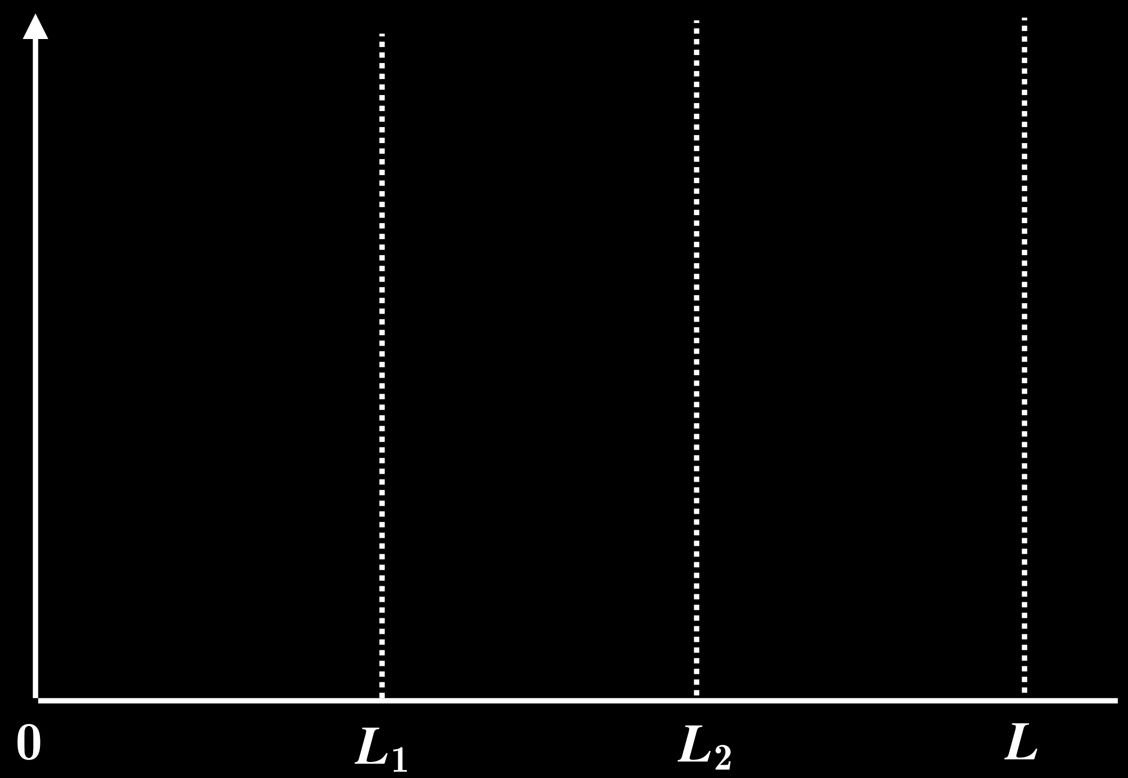
$L(x, y)$

SU(2) doublet



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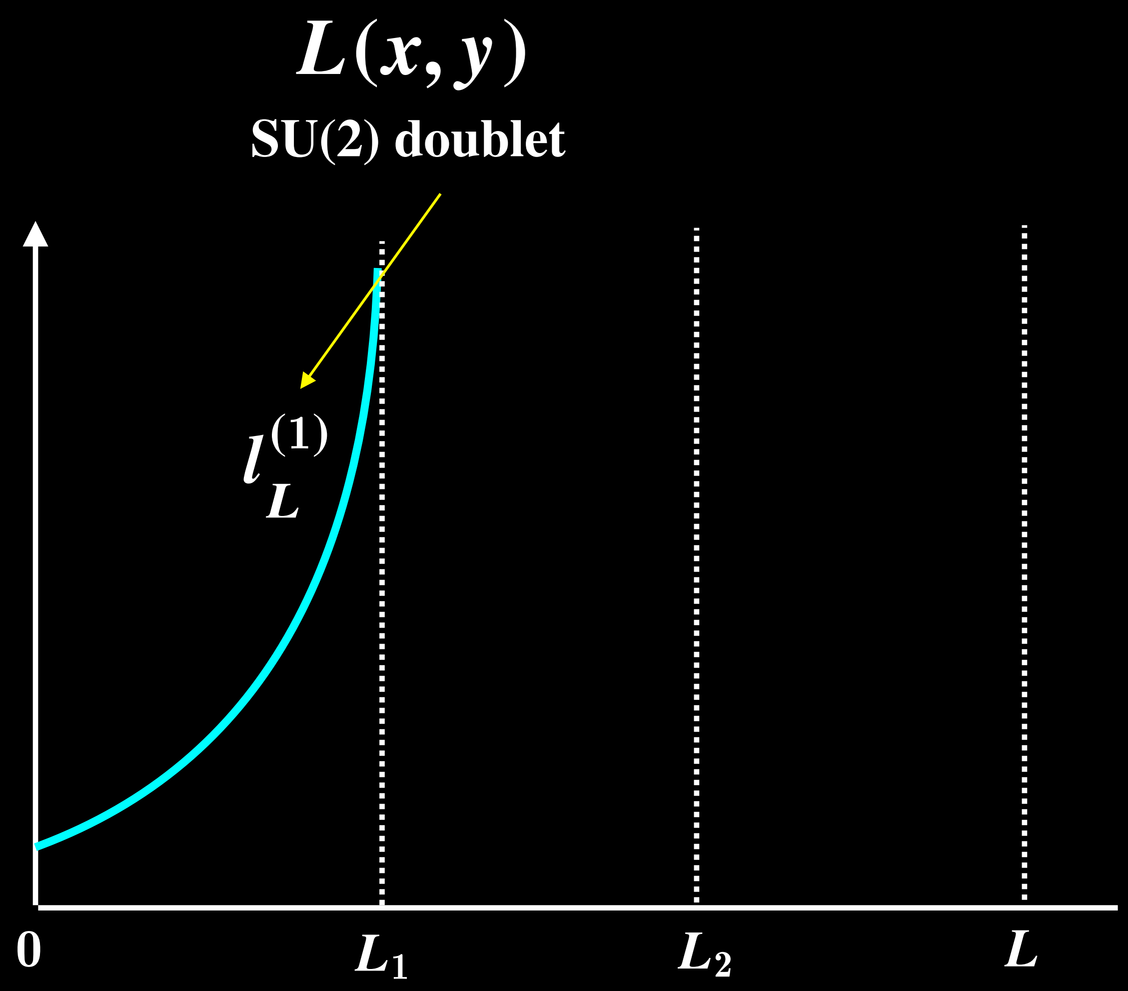

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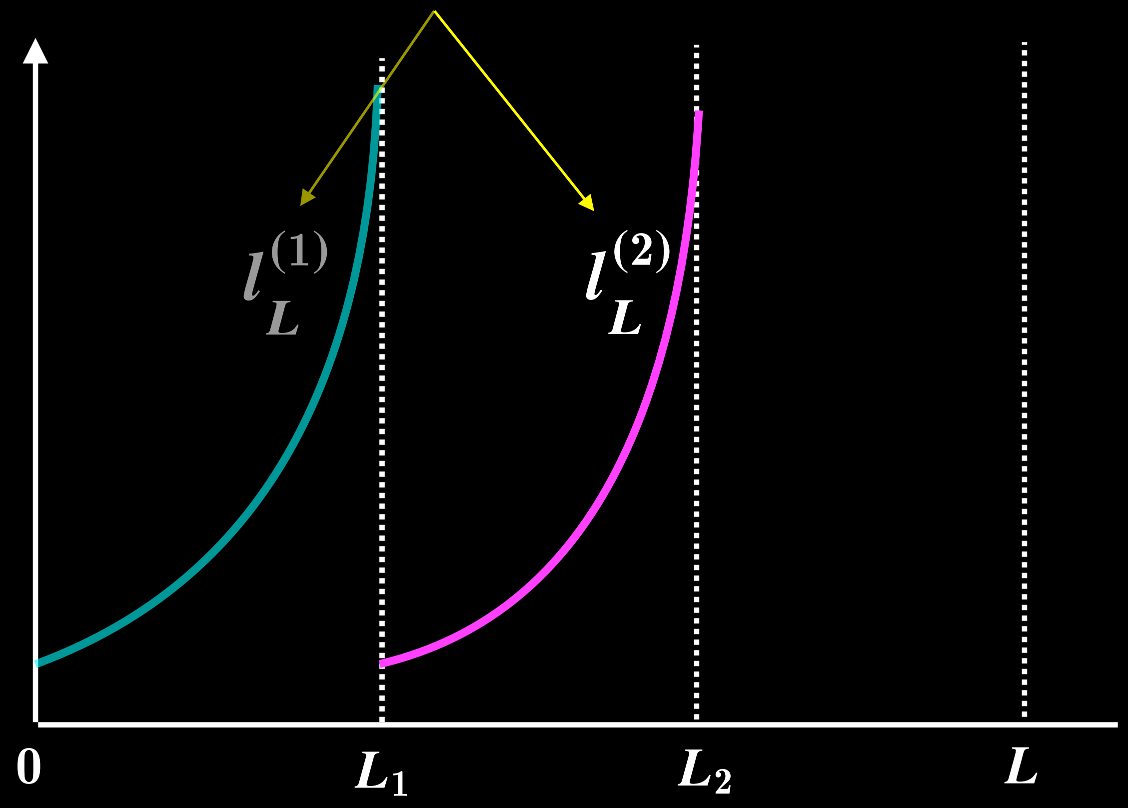

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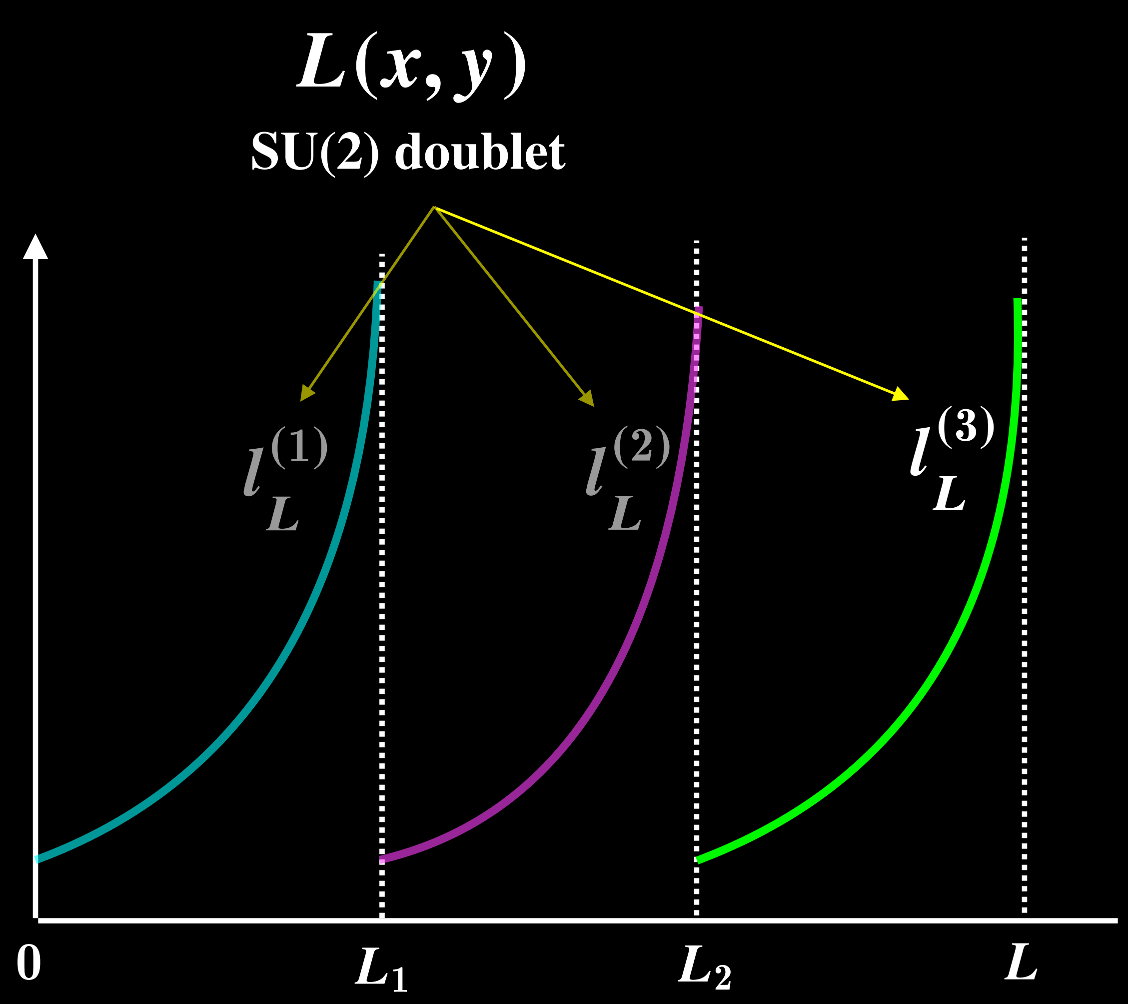

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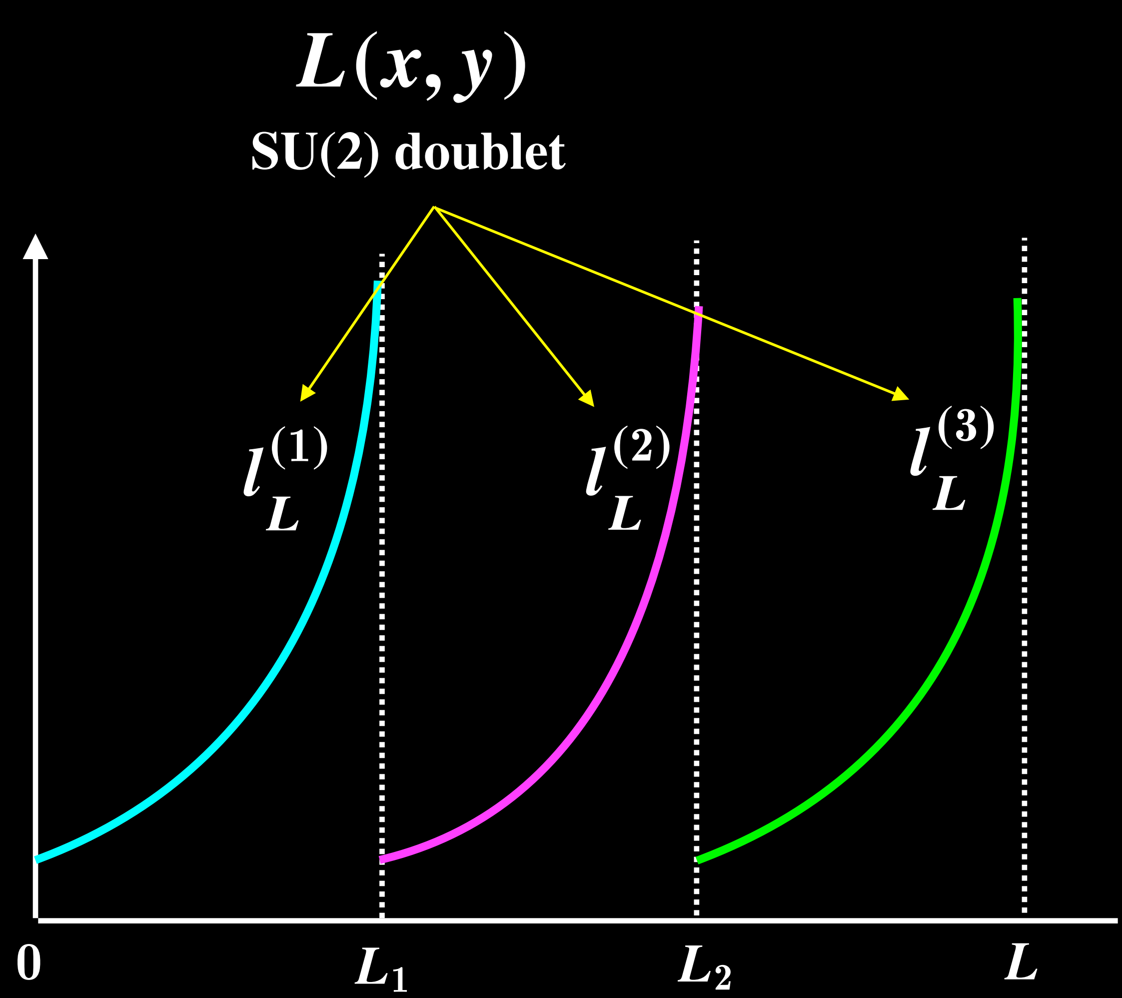

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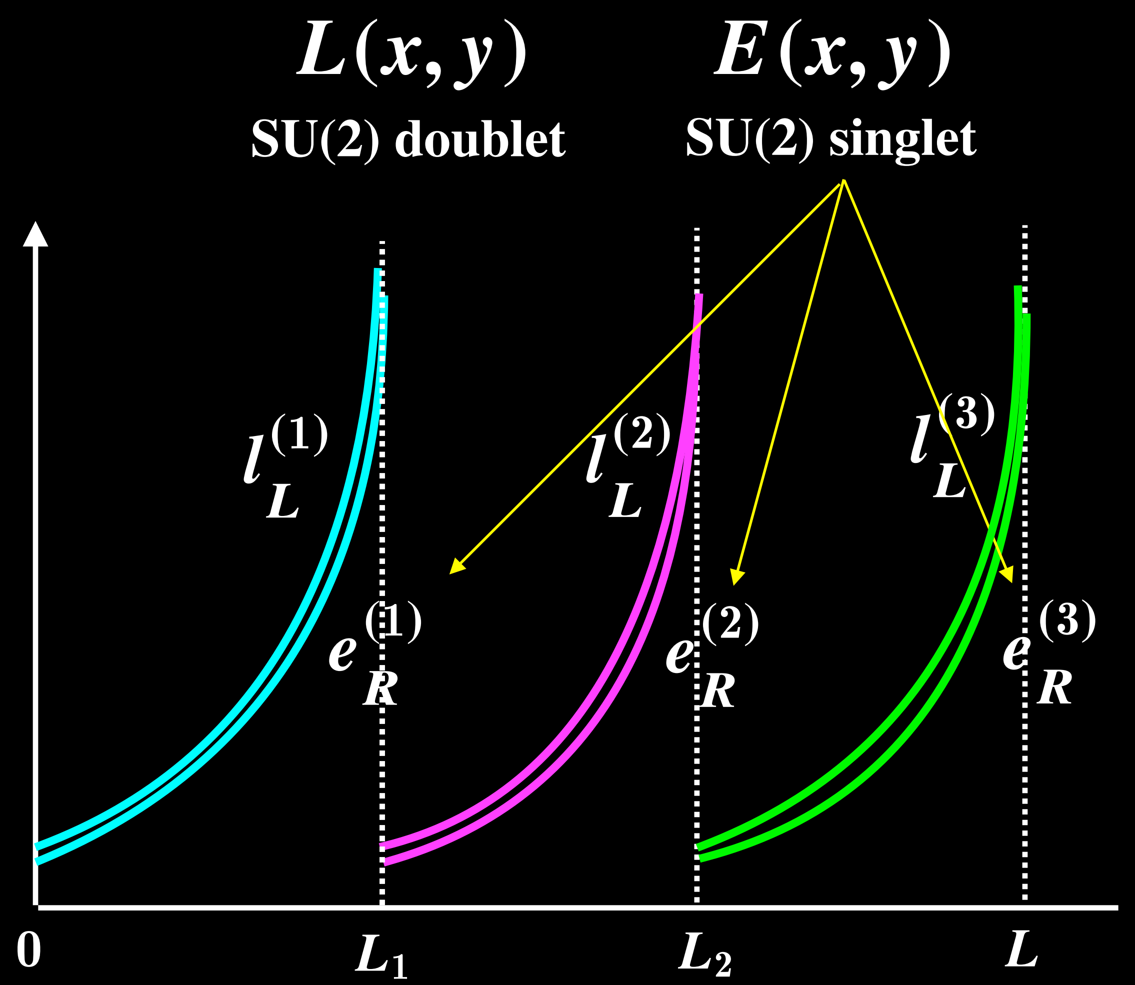

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- **y-dependent scalar VEV : $\langle \Phi(y) \rangle \sim e^{M_\Phi y}$**

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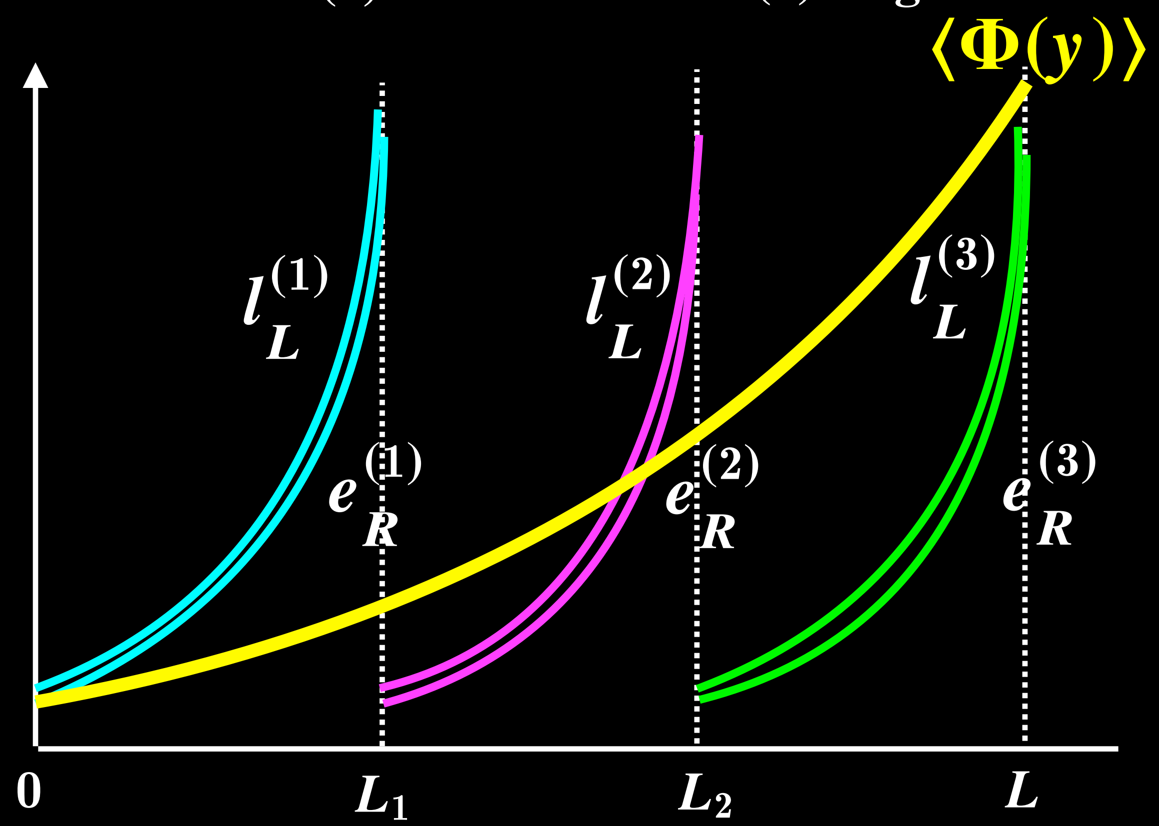

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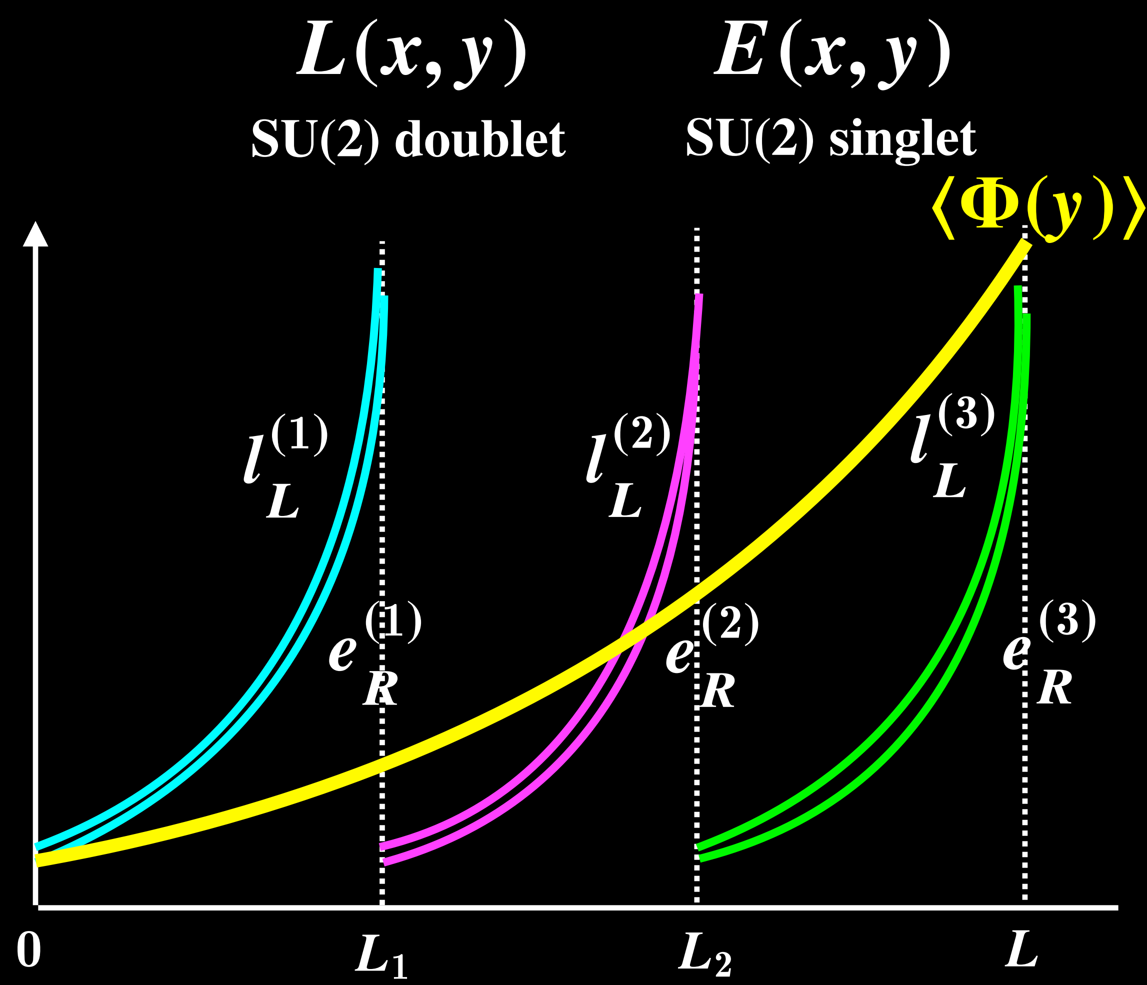

$$m_i = \lambda \int_0^L \langle \Phi(y) \rangle \langle H(y) \rangle f_R^{(i)}(y) g_L^{(i)}(y)$$

Small overlap → Small mass
Large overlap → Large mass

Mass hierarchy !!

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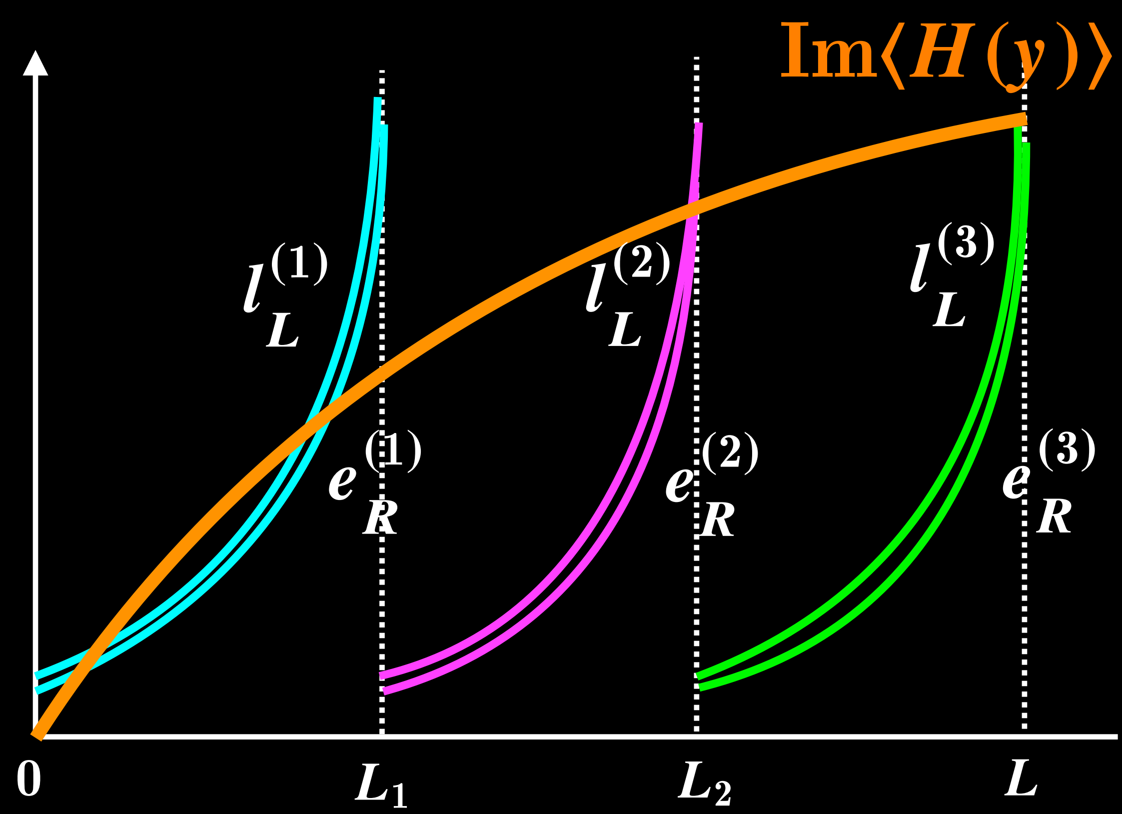

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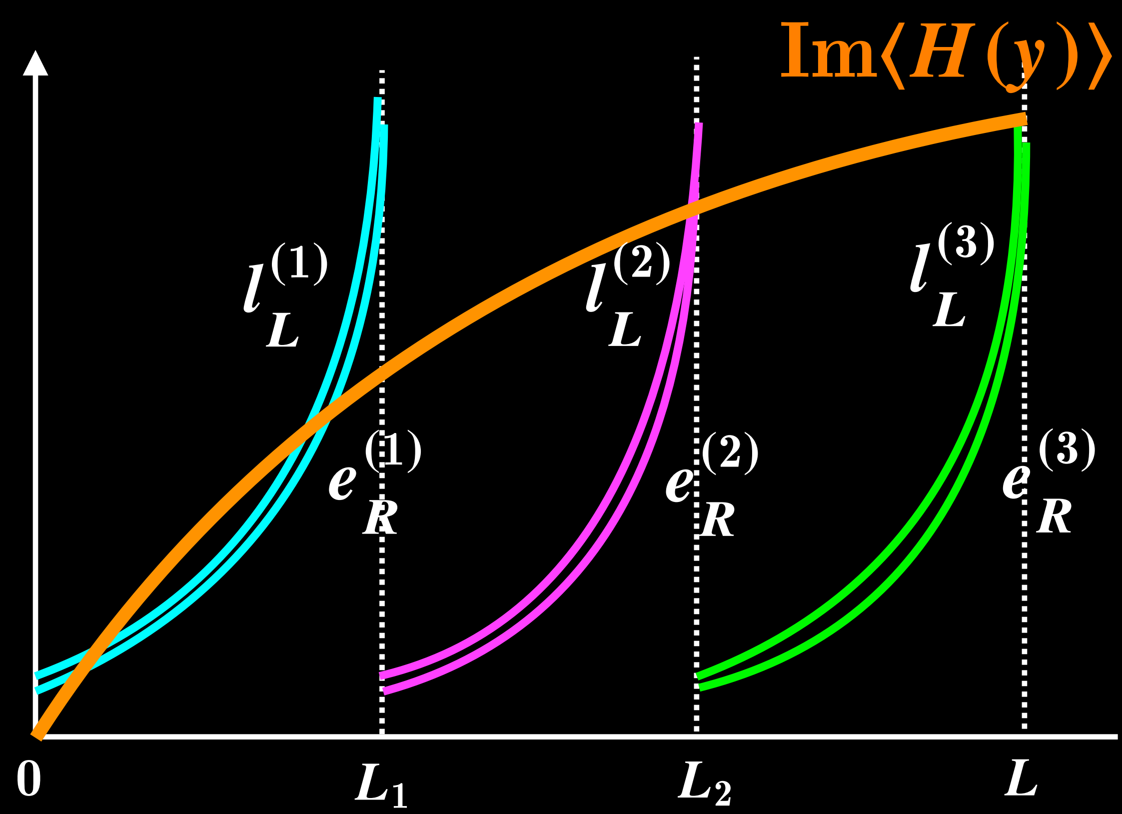

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Different phases to the masses through the overlap integrals.

CP phase !!

Features

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❒ Flavor mixing

The Flavor mixing was controlled by the configuration of the extra dimension with restricted form.

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▶ **Mass matrices**

$$M^{(\nu)} = \begin{pmatrix} m_{11}^{(\nu)} & m_{12}^{(\nu)} & 0 \\ 0 & m_{22}^{(\nu)} & m_{23}^{(\nu)} \\ m_{31}^{(\nu)} & 0 & m_{33}^{(\nu)} \end{pmatrix} \qquad M^{(e)} = \begin{pmatrix} m_{11}^{(e)} & m_{12}^{(e)} & m_{13}^{(e)} \\ 0 & m_{22}^{(e)} & m_{23}^{(e)} \\ 0 & 0 & m_{33}^{(e)} \end{pmatrix}$$

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Always three 0 entry in the mass matrices !!

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The origin of the CP phase for the leptons and the quarks is the same: Higgs VEV.

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$$\langle H(y)\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e^{i\frac{\theta}{L}y}$$

$\rightarrow$ $\boxed{\textbf{Quarks}}$ $\longrightarrow$ δ_{CP}

$\rightarrow$ $\boxed{\textbf{Leptons}}$ $\longrightarrow$ δ'_{CP}

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The origin of the CP phase for the leptons and the quarks is the same: Higgs VEV.

—> CP phase of the leptons can be a prediction after fitting the CP phase of the quarks !!

Setup

Setup

- **SU(2)×U(1) gauge theory on a circle**

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  with $\left\{\begin{array}{l}\textbf{One generation fermions}\\ \textbf{Higgs doublet \& gauge singlet scalar}\end{array}\right.$

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with { One generation fermions / Higgs doublet & gauge singlet scalar }

Gauge fields

$$W^{(a)}_{MN}(x,y)$$
$$B_{MN}(x,y)$$

scalars

$$H(x,y)$$
$$\Phi(x,y)$$

Fermions

$$\begin{pmatrix} \nu(x,y) \\ e(x,y) \end{pmatrix}$$
$$\nu'(x,y)$$
$$e'(x,y)$$

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Gauge fields

$$W^{(a)}_{MN}(x,y)$$
$$B_{MN}(x,y)$$

Fermions

$$\begin{pmatrix} \nu(x,y) \\ e(x,y) \end{pmatrix} \leftarrow \sim \text{4d Left handed}$$

$$\nu'(x,y),\quad e'(x,y) \leftarrow \sim \text{4d Right handed}$$

scalars

$$H(x,y) \leftarrow \text{Symmetry Breaking \& CP source}$$

$$\Phi(x,y) \leftarrow \text{Mass hierarchy}$$

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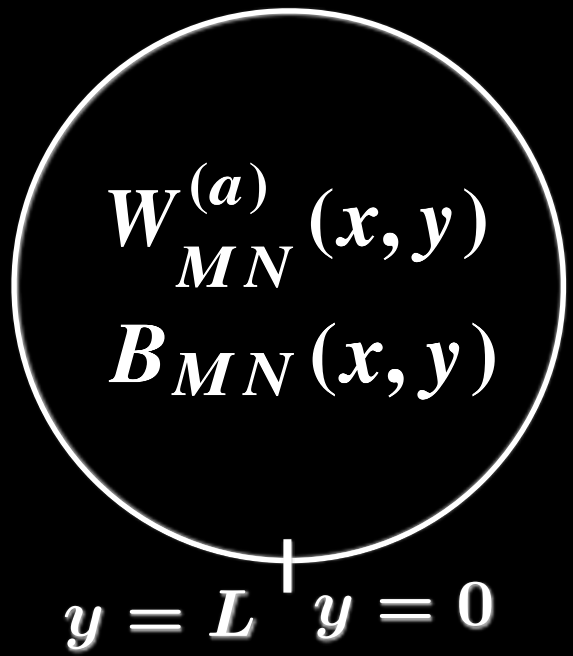
  ★ **Gauge fields do not feel point interactions.**



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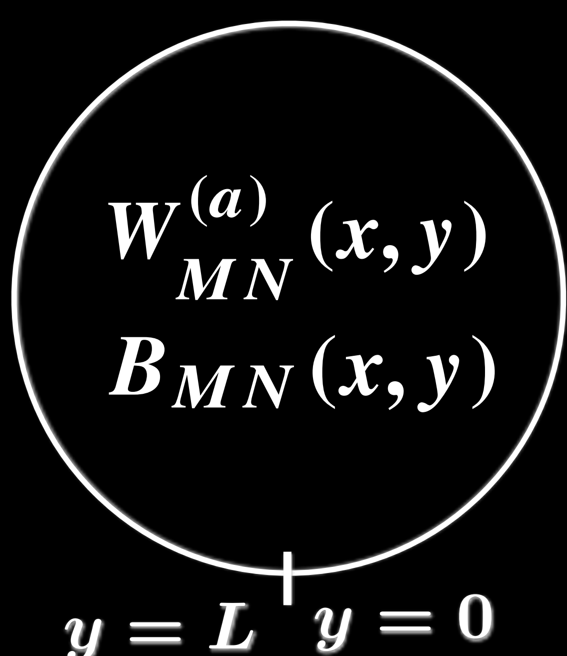

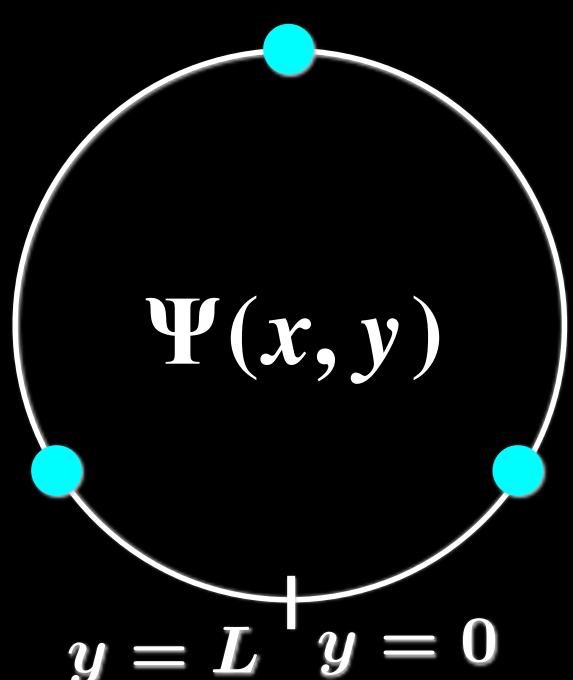

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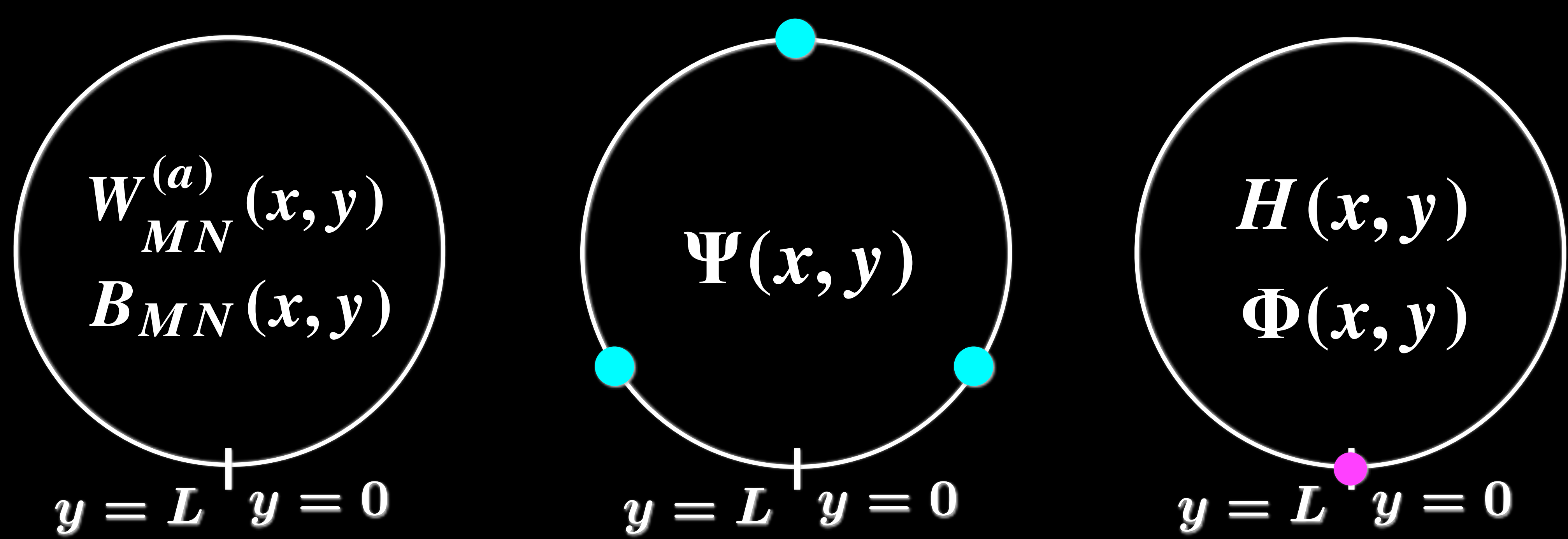

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  compatible with { ★ 5d gauge invariance / ★ action principle } etc.

Boundary Conditions (BC's)

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❐ Gauge fields

★ Periodic BC

$$\begin{cases} G(0) = G(L) \\ \partial_y G(0) = \partial_y G(L) \end{cases}$$

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❒ Higgs doublet

★ Twisted BC

$$\begin{cases} H(L) = e^{i\theta} H(0) \\ \partial_y H(L) = e^{i\theta} \partial_y H(0) \end{cases}$$

Boundary Conditions (BC's)

❒ Gauge singlet scalar

★ Robin BC

$$\begin{cases} \Phi(0) + L_+ \partial_y \Phi(0) = 0 \\ \Phi(L) - L_- \partial_y \Phi(L) = 0 \end{cases} \qquad (-\infty \leq L_\pm \leq +\infty)$$

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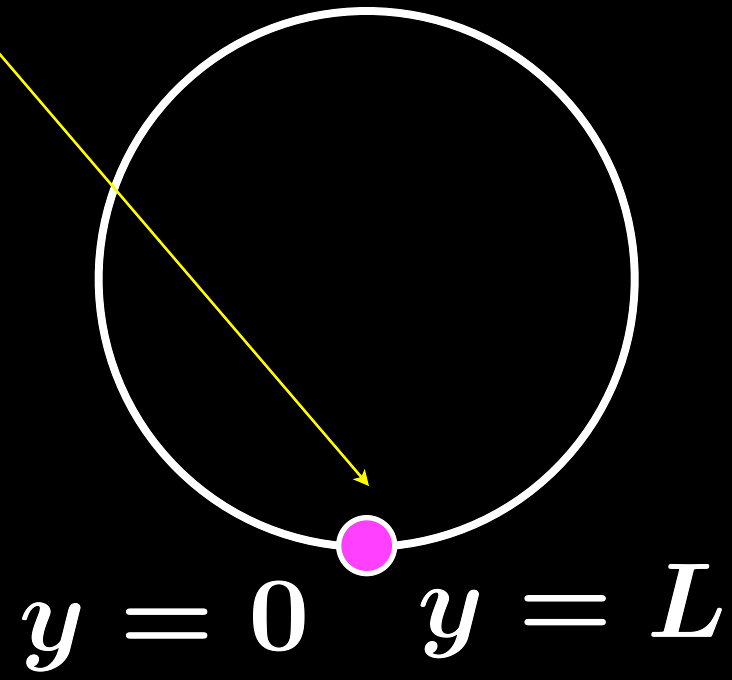

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★ Dirichlet BC

$$\Psi_R(y) = 0 \quad \text{@ point interactions}$$

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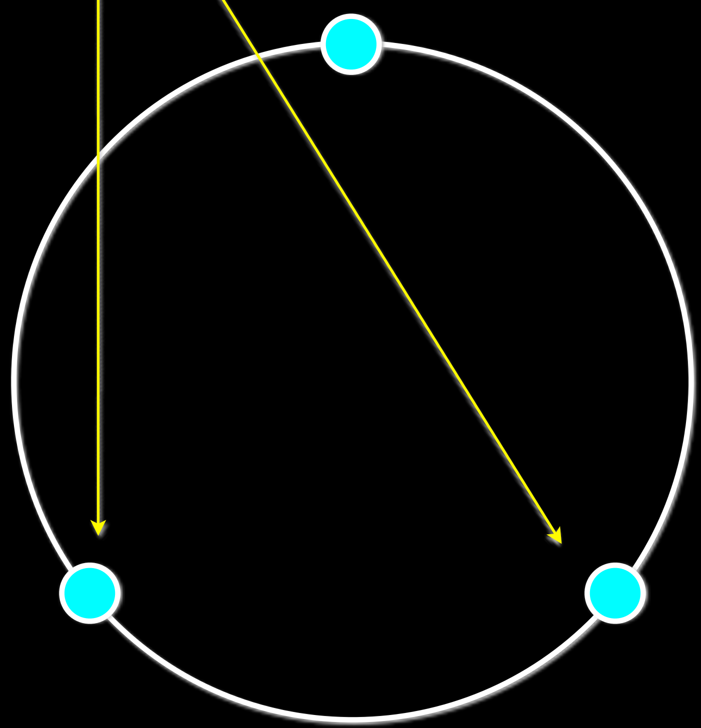

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$$\Psi_R(y) = 0 \xrightarrow{\text{5d Dirac eq.}} (-D_y + M_F)\Psi_L(y) = 0$$

or

$$\Psi_L(y) = 0 \xrightarrow{\text{5d Dirac eq.}} (D_y + M_F)\Psi_R(y) = 0$$

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$\Psi_R(x,0)=0$ $\Psi_R(x,L)=0$

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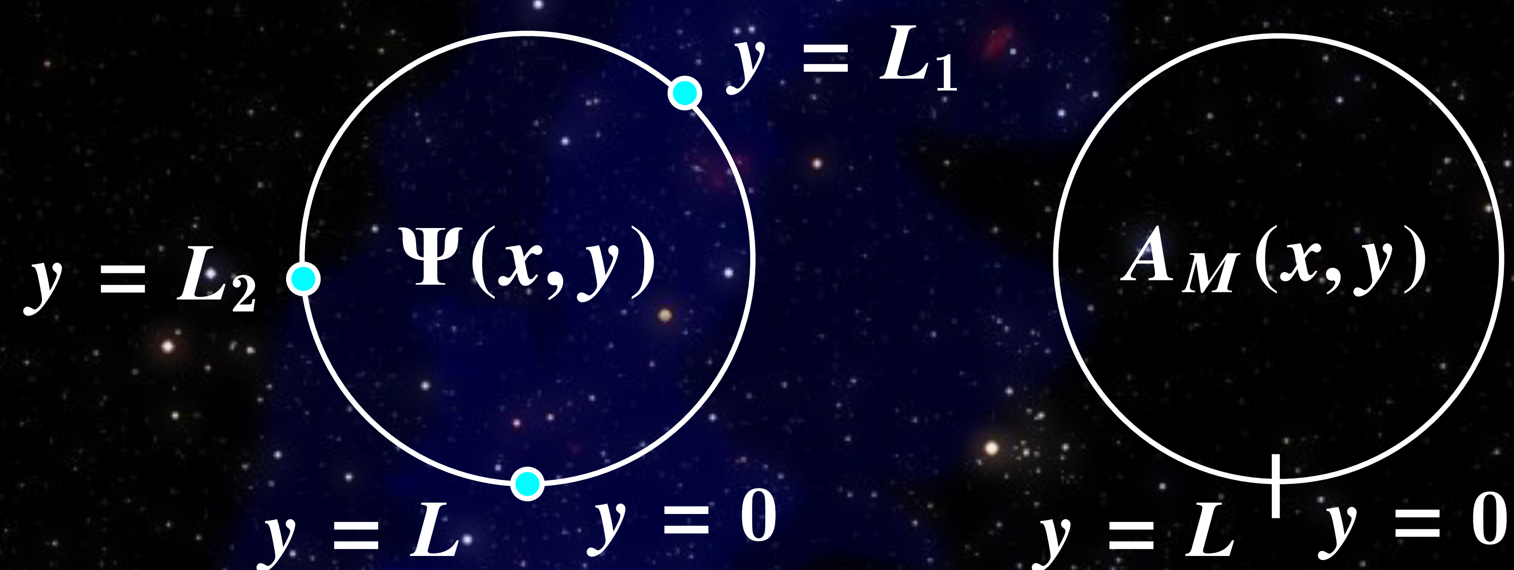

★ **Gauge fields do not feel point interactions to avoid generations.**

★ **The situation is consistent with 5d gauge invariance.**

Generation

- Triply-degenerated chiral zero modes via point interactions

$\Psi_R(x, 0) = 0$ $\quad$ $\Psi_R(x, L_1) = 0$ $\quad$ $\Psi_R(x, L_2) = 0$ $\quad$ $\Psi_R(x, L) = 0$

$$\Psi(x, y) = \sum_{j=1}^{3} \underbrace{\psi_L^{(j)}(x)}_{\text{4-dim. mass eigenstates}} \underbrace{g_0^{(j)}(y)}_{\text{Mode functions}} + (\,\mathrm{KK-modes}\,)$$

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$$\mathcal{D}^\dagger g_0^{(j)}(y)=0 \quad (\mathcal{D}^\dagger\equiv-\partial_y+M_F)$$

Bulk mass

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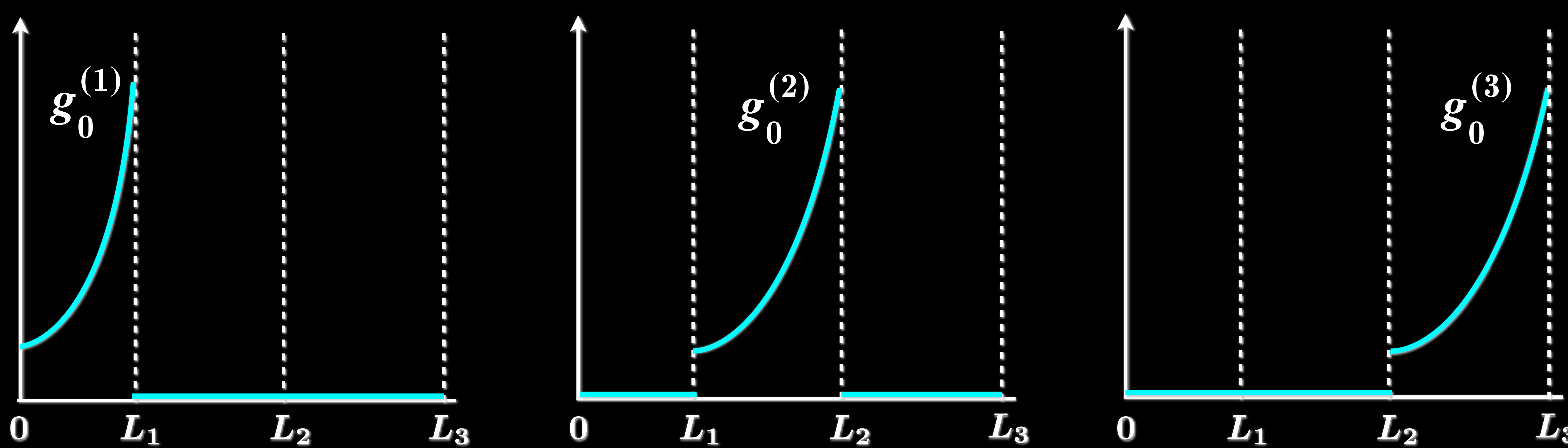

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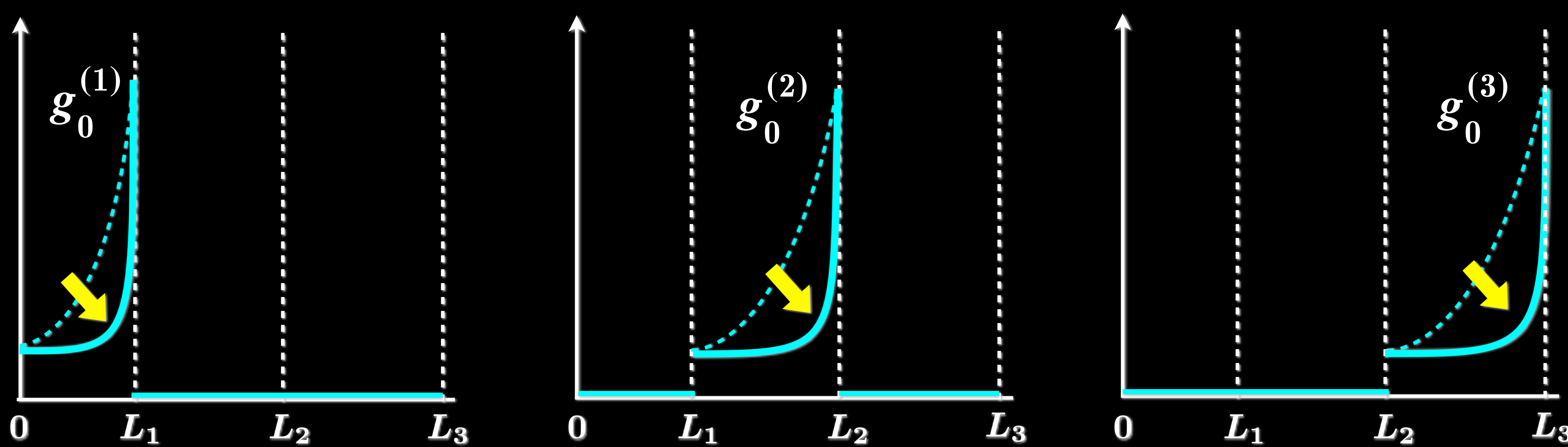
$$\Psi(x,y)=\sum_{j=1}^{3}\psi_L^{(j)}(x)g_0^{(j)}(y)+(\mathrm{KK-modes})$$

$M_F \rightarrow$ large

$$\mathcal{D}^\dagger g_0^{(j)}(y)=0 \quad (\mathcal{D}^\dagger\equiv-\partial_y+M_F)$$



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$$\Psi(x,y)=\sum_{j=1}^{3}\psi_L^{(j)}(x)g_0^{(j)}(y)+(\mathrm{KK-modes})$$

$M_F<0$

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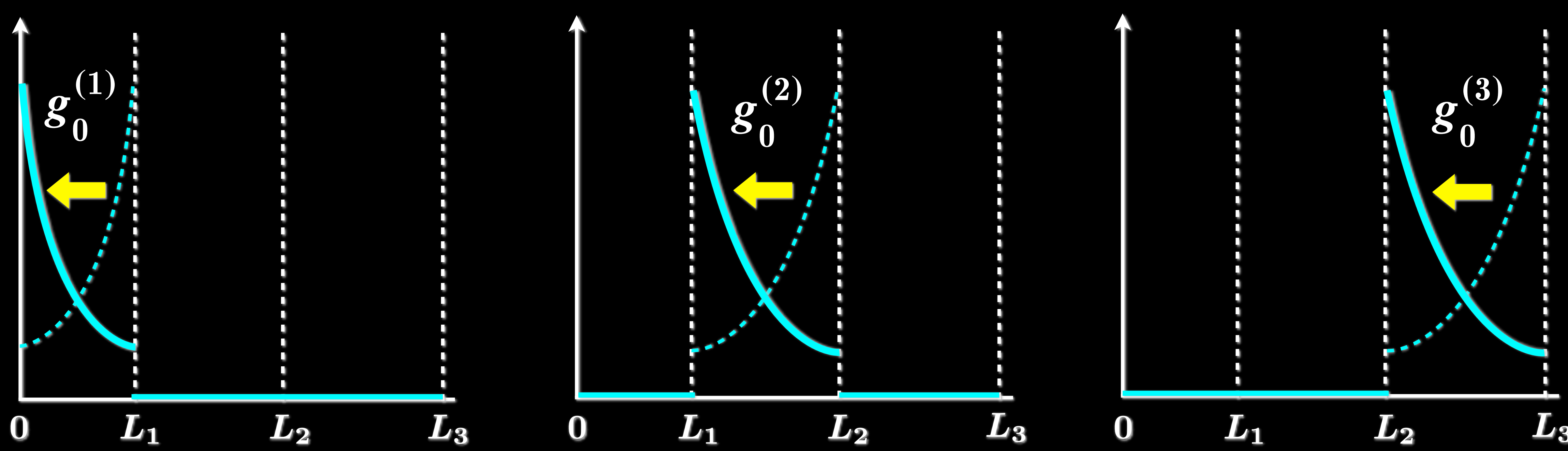

Lepton mass hierarchy

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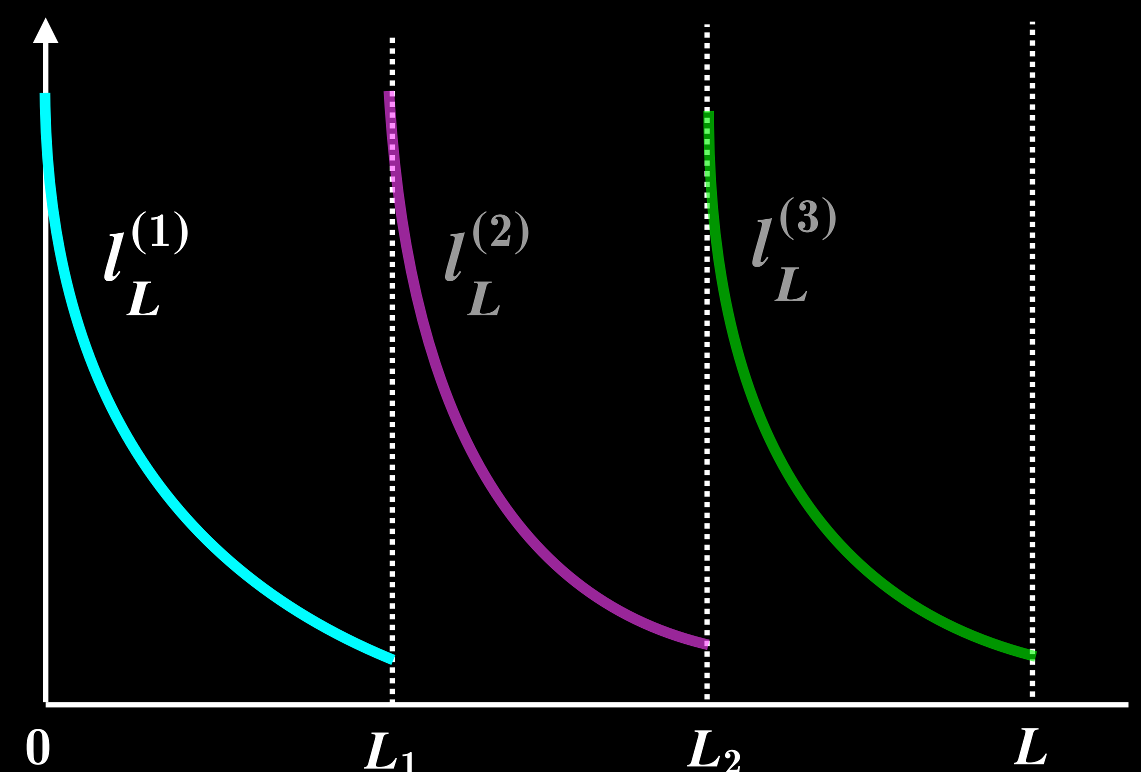
- Three localized SU(2) doublet zero modes



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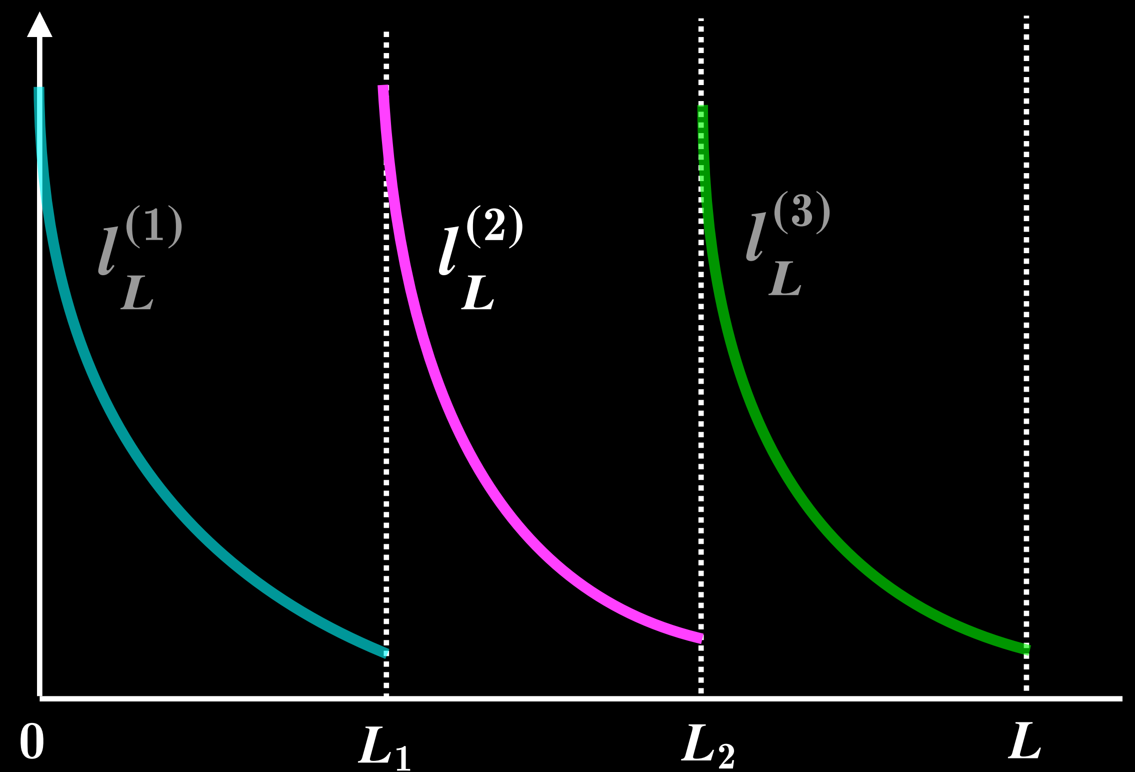

Lepton mass hierarchy

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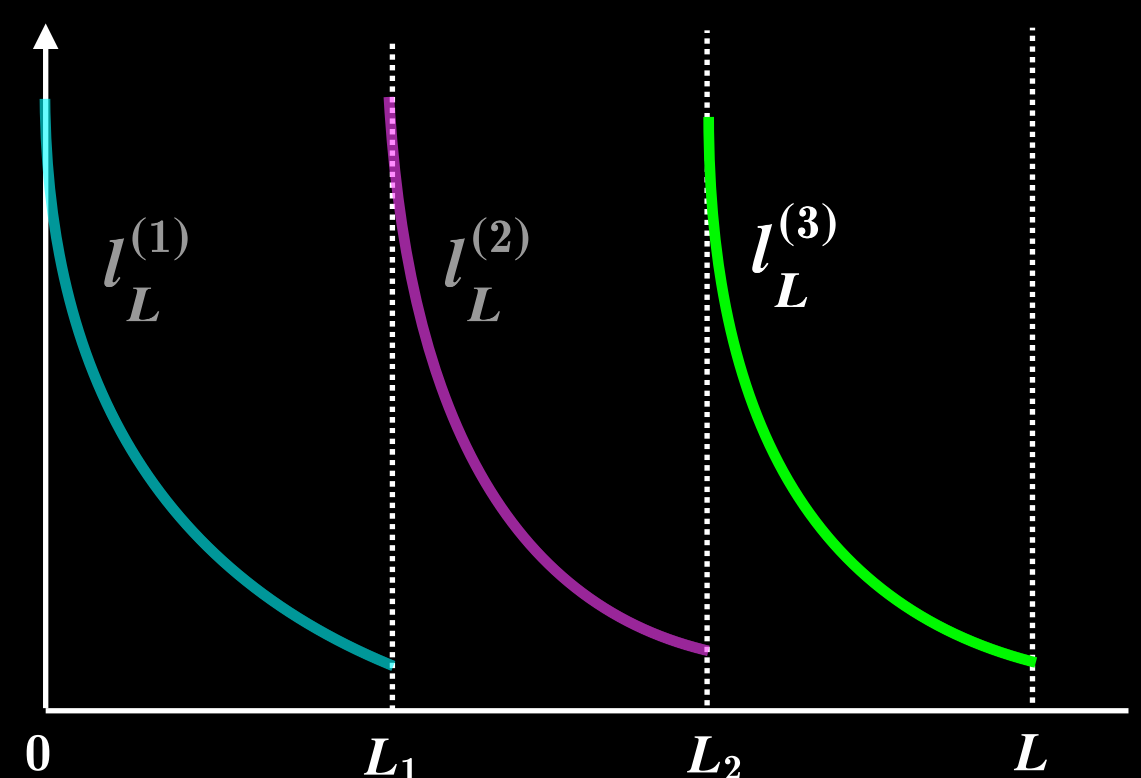

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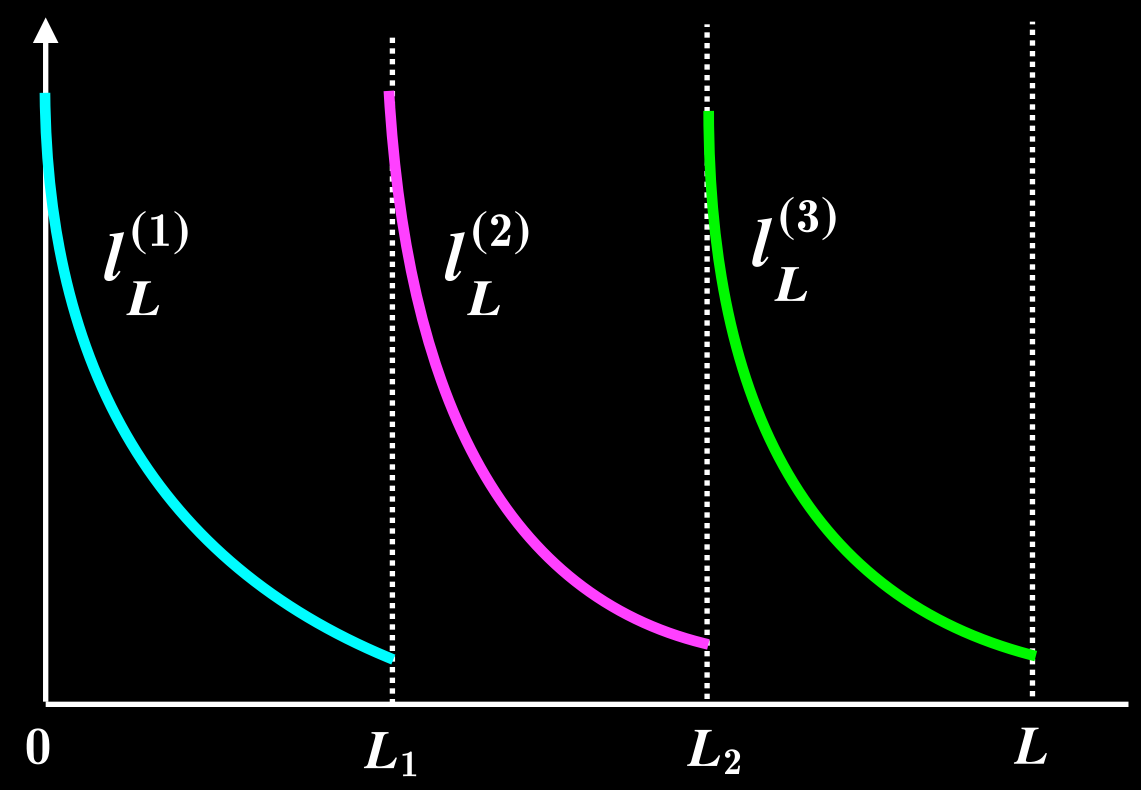
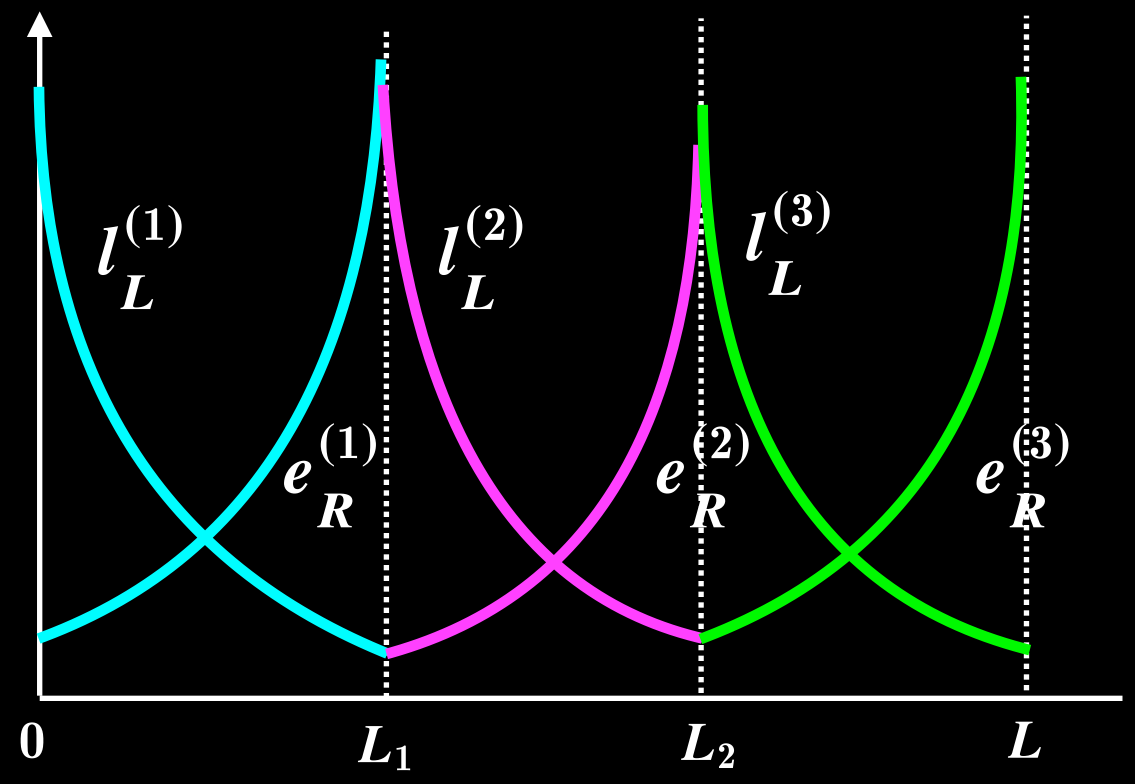


Lepton mass hierarchy

❒ **Three localized SU(2) doublet zero modes**
& Three localized SU(2) singlet zero modes



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- Three localized SU(2) doublet zero modes & Three localized SU(2) singlet zero modes
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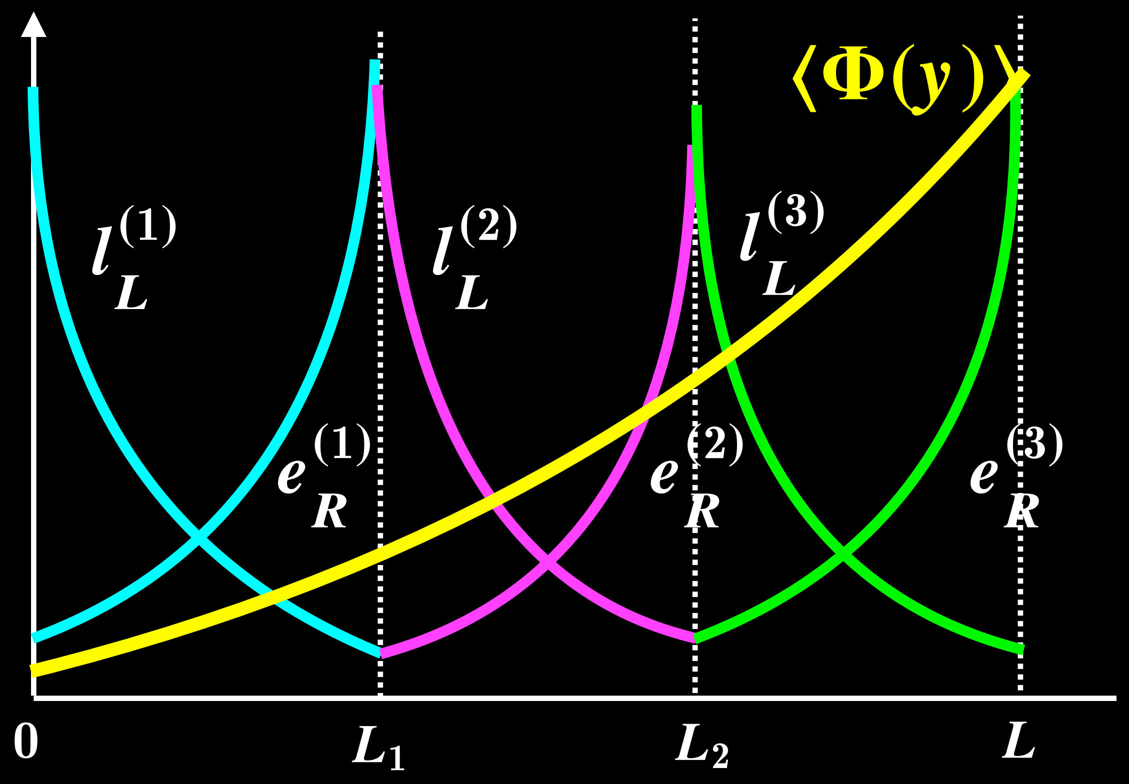
$$\langle \Phi(y) \rangle \sim e^{My}$$



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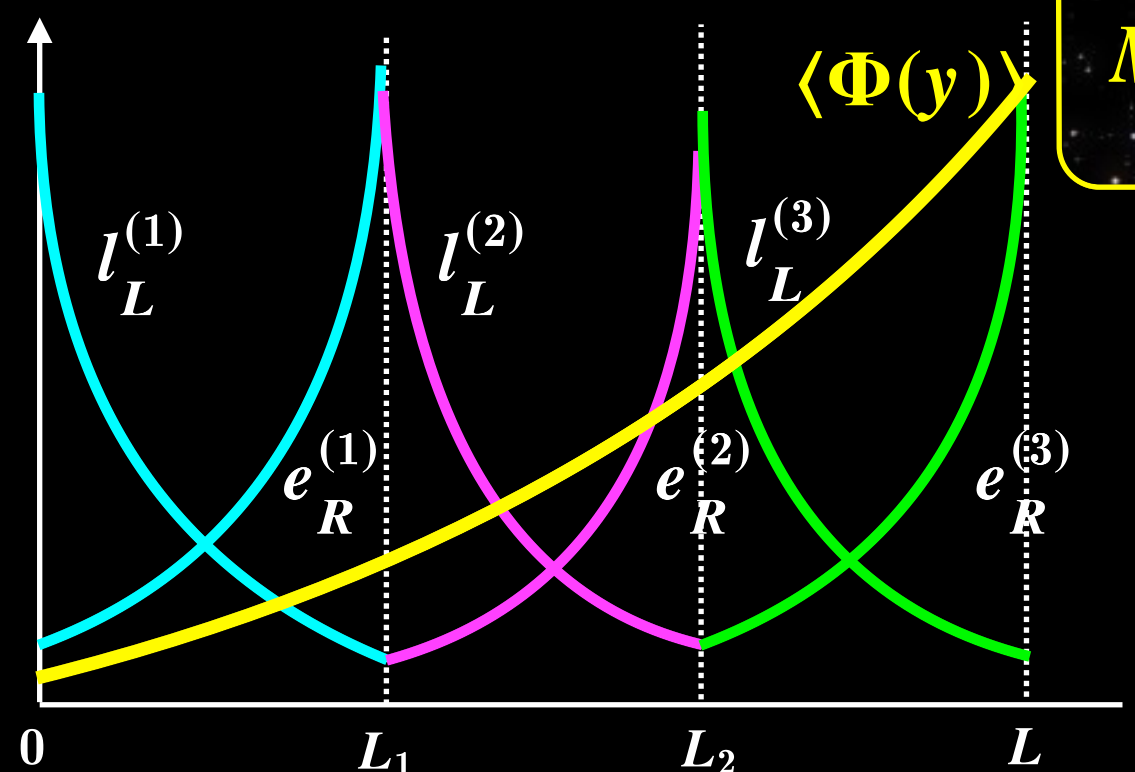

$$M_{ij}^{(e)} = \lambda \int_0^L dy \, \langle \Phi(y) \rangle \langle H(y) \rangle l_L^{(i)}(y) e_R^{(j)}$$

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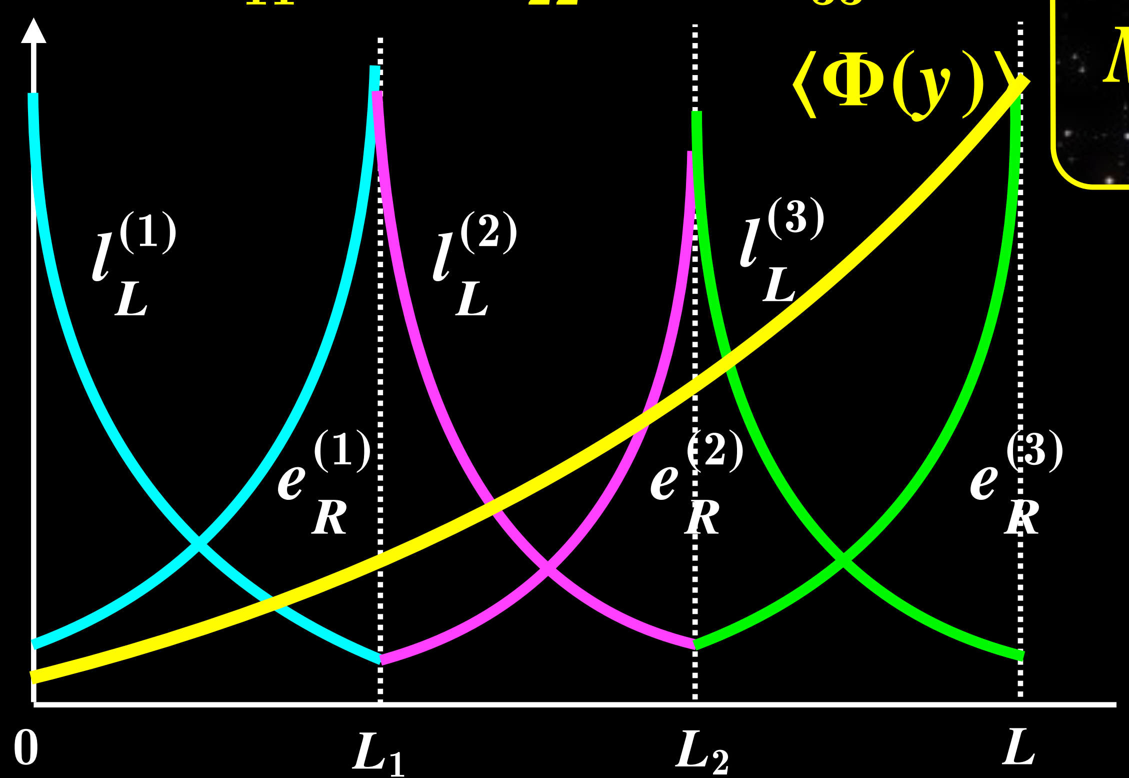

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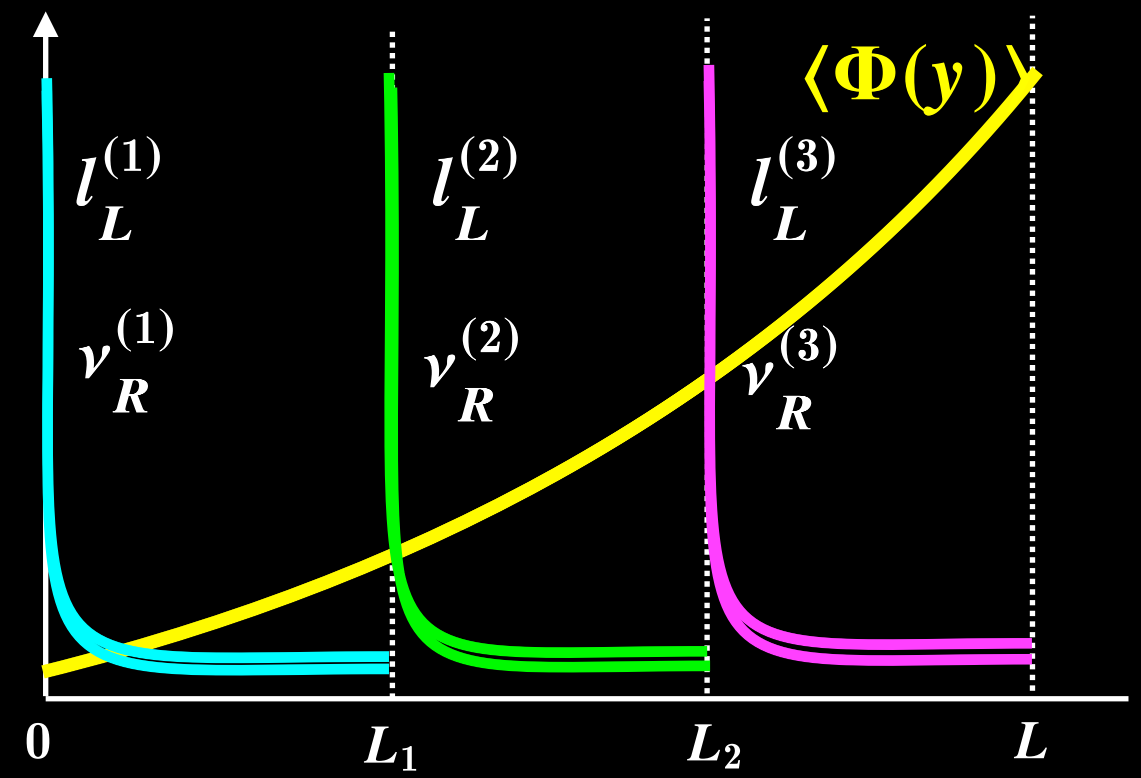
↓

Mass hierarchy !!

Tiny neutrino masses

Tiny neutrino masses

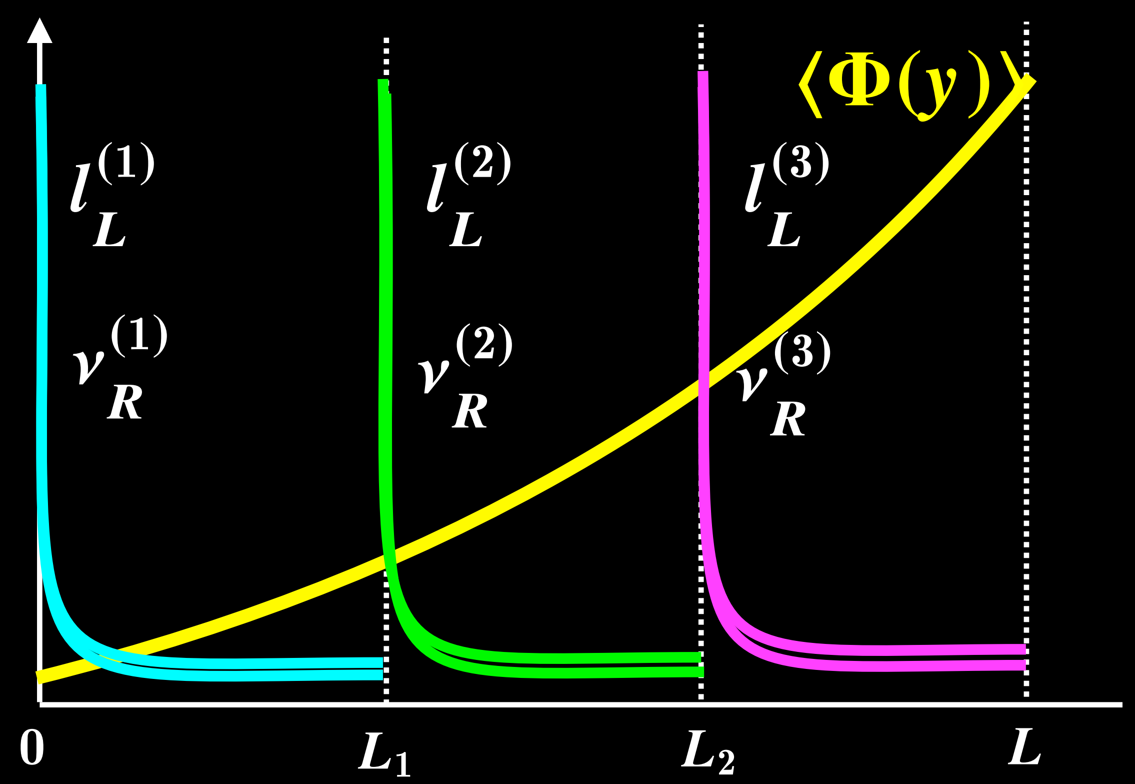
- Three localized SU(2) doublet and singlet with y-dependent VEV



Tiny neutrino masses

- Three localized SU(2) doublet and singlet with y-dependent VEV
- Large bulk mass produce tiny neutrino masses

$$|M_L L| \sim |M_E L| \sim \mathcal{O}(100)$$



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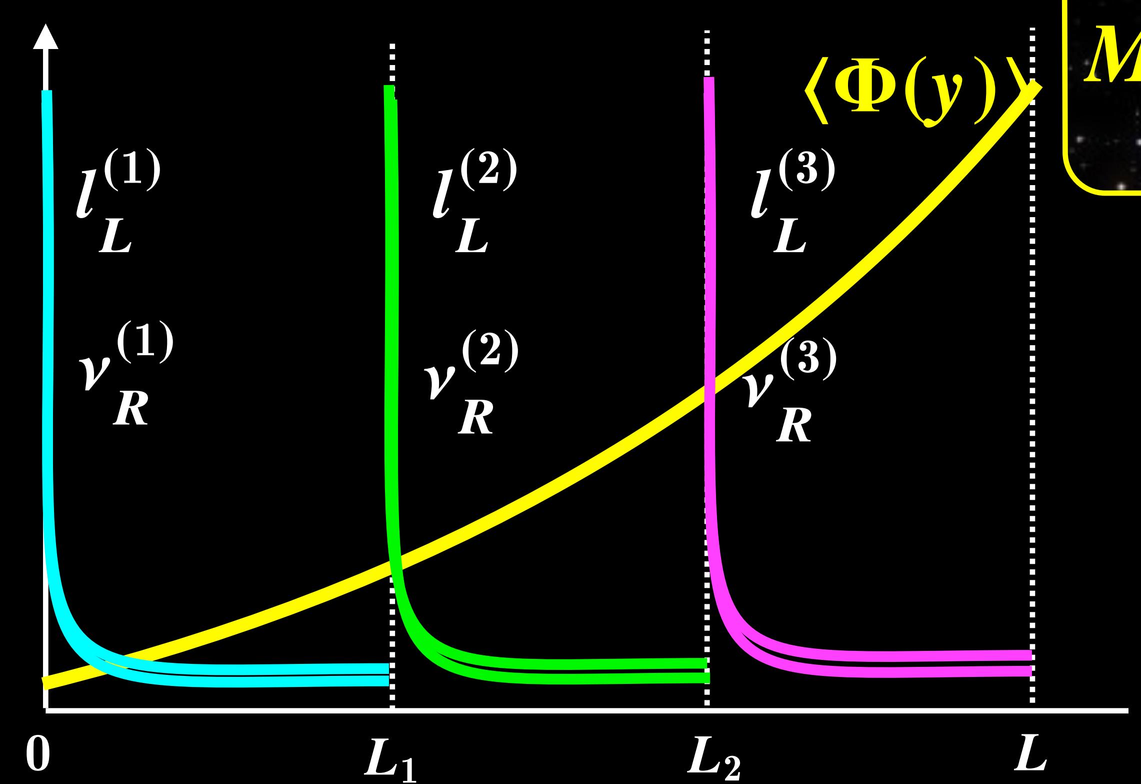

$$M_{ij}^{(\nu)} = \lambda \int_0^L dy \, \langle \Phi(y) \rangle \langle H(y) \rangle^* l_L^{(i)}(y) \nu_R^{(j)}$$

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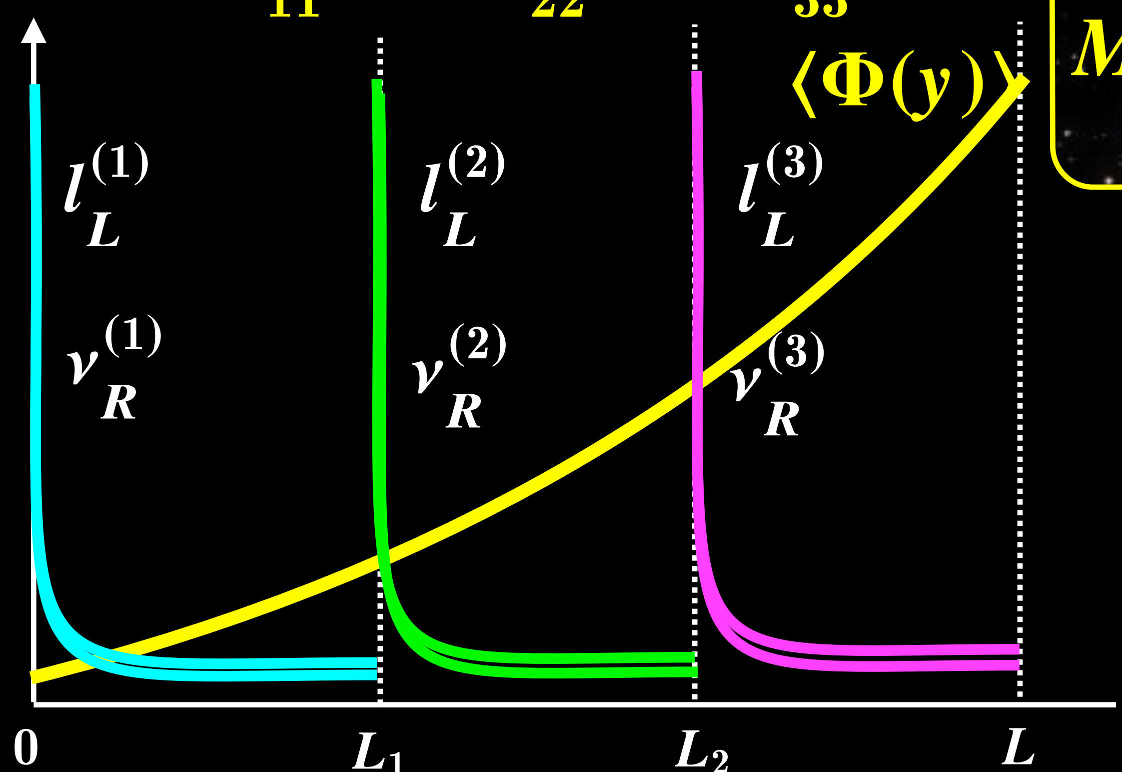

$$M_{ij}^{(\nu)} = \lambda \int_0^L dy\ \langle\Phi(y)\rangle\langle H(y)\rangle^* l_L^{(i)}(y)\nu_R^{(j)}$$

⬇

NH O(0.1)eV masses !!

CP phase

CP phase

- **Twisted BC for the Higgs produce y-dependent phase to the Higgs VEV.**

YF, K.Nishiwaki, M.Sakamoto, PRD 88,115007(2013)

CP phase

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YF, K.Nishiwaki, M.Sakamoto, PRD 88,115007(2013)

Twisted parameter

$$\langle H(y)\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e^{i\frac{\theta}{L}y}$$

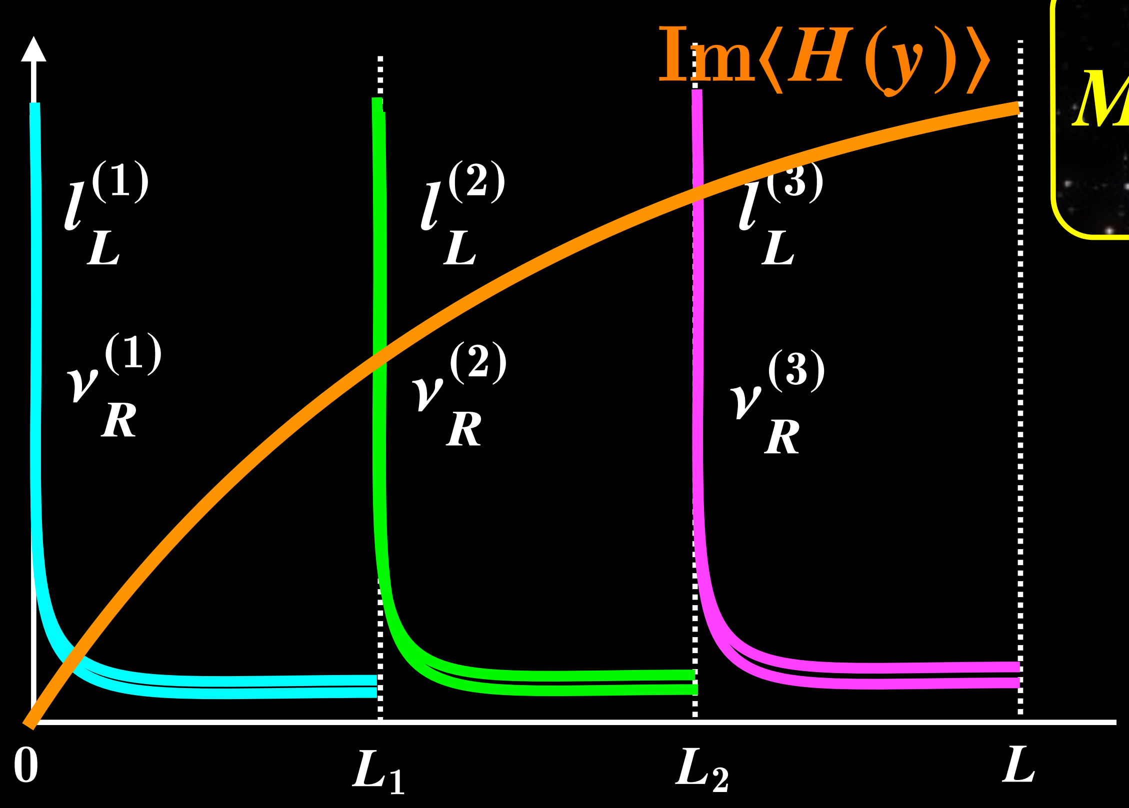
$$\begin{cases} H(L) = e^{i\theta} H(0) \\ \partial_y H(L) = e^{i\theta} \partial_y H(0) \end{cases}$$

CP phase

- Twisted BC for the Higgs produce y-dependent phase to the Higgs VEV.

YF, K.Nishiwaki, M.Sakamoto, PRD 88,115007(2013)

- Each element of mass matrices acquires own phases through the overlap integrals.



$$M_{ij}^{(\nu)} = \lambda \int_0^L dy \, \langle \Phi(y) \rangle \langle H(y) \rangle^* l_L^{(i)}(y) \nu_R^{(j)}$$

↓

CP phase !!

Flavor mixing

Flavor mixing

❒ **The configuration of the point interactions**

Flavor mixing

- The configuration of the point interactions
  - ★ In general, the positions of the point interactions can change with respect to the fermions.

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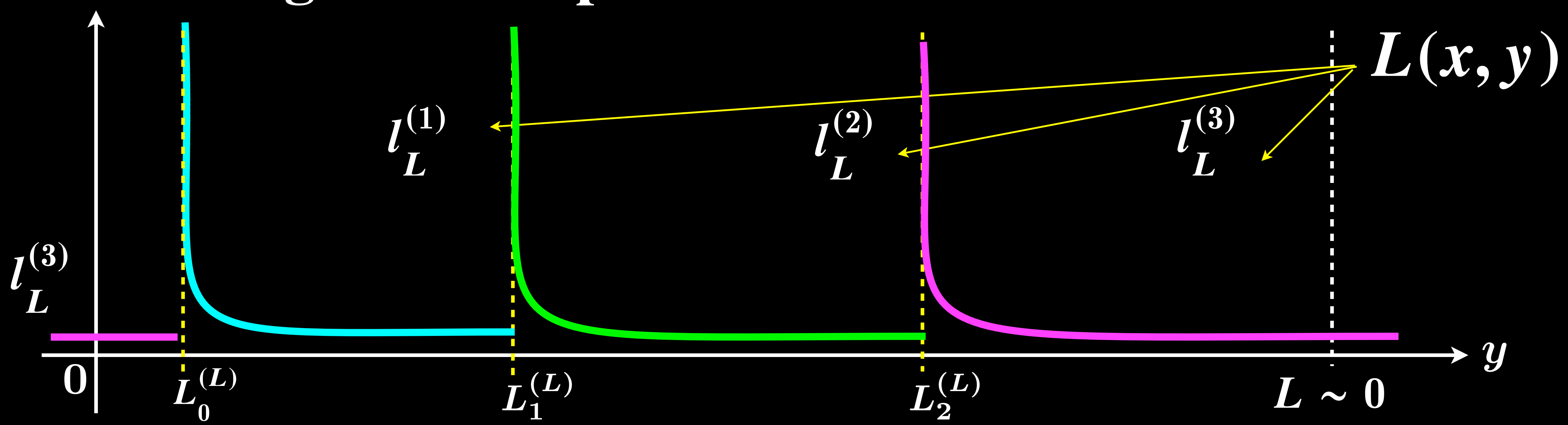

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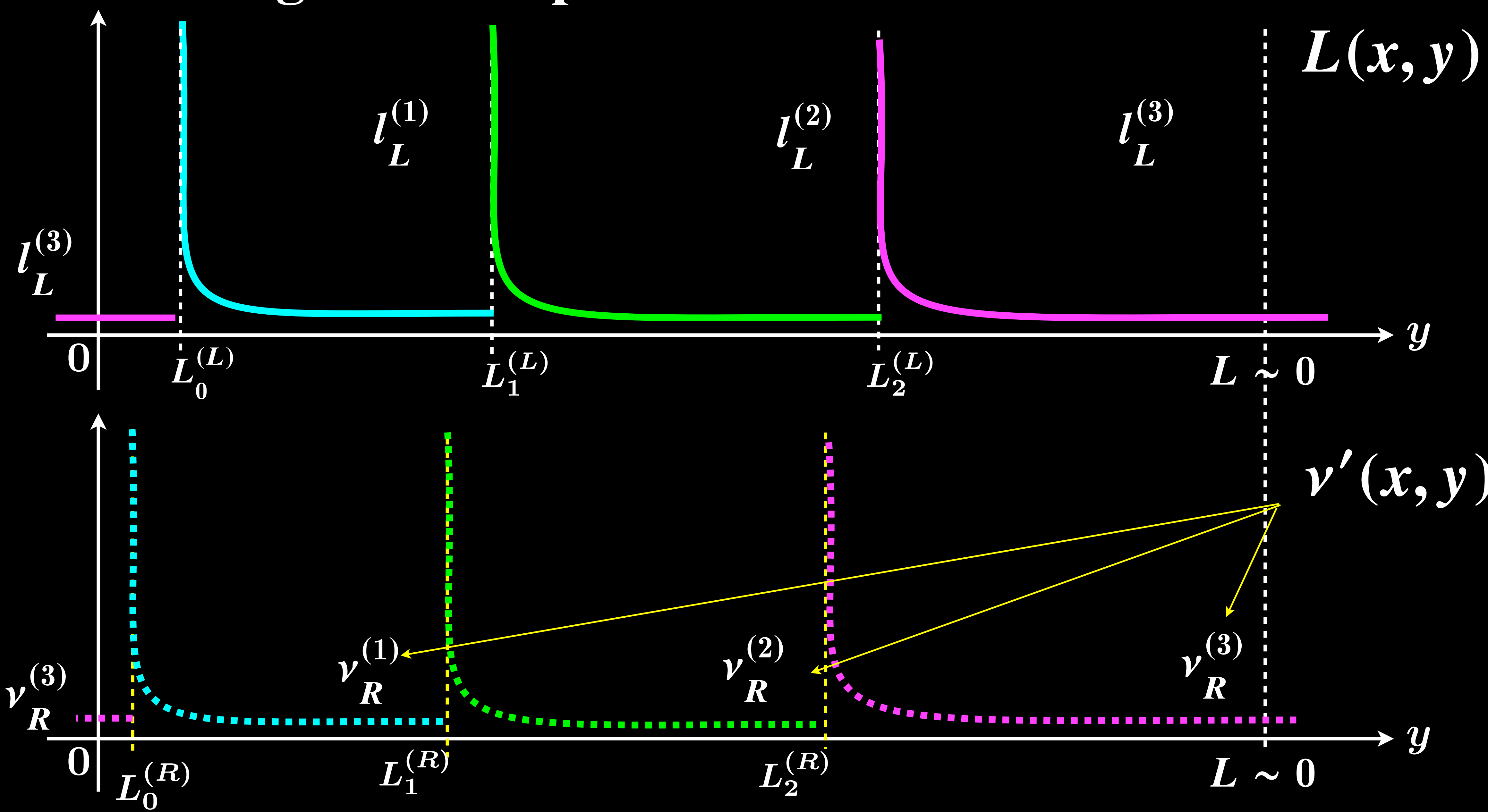

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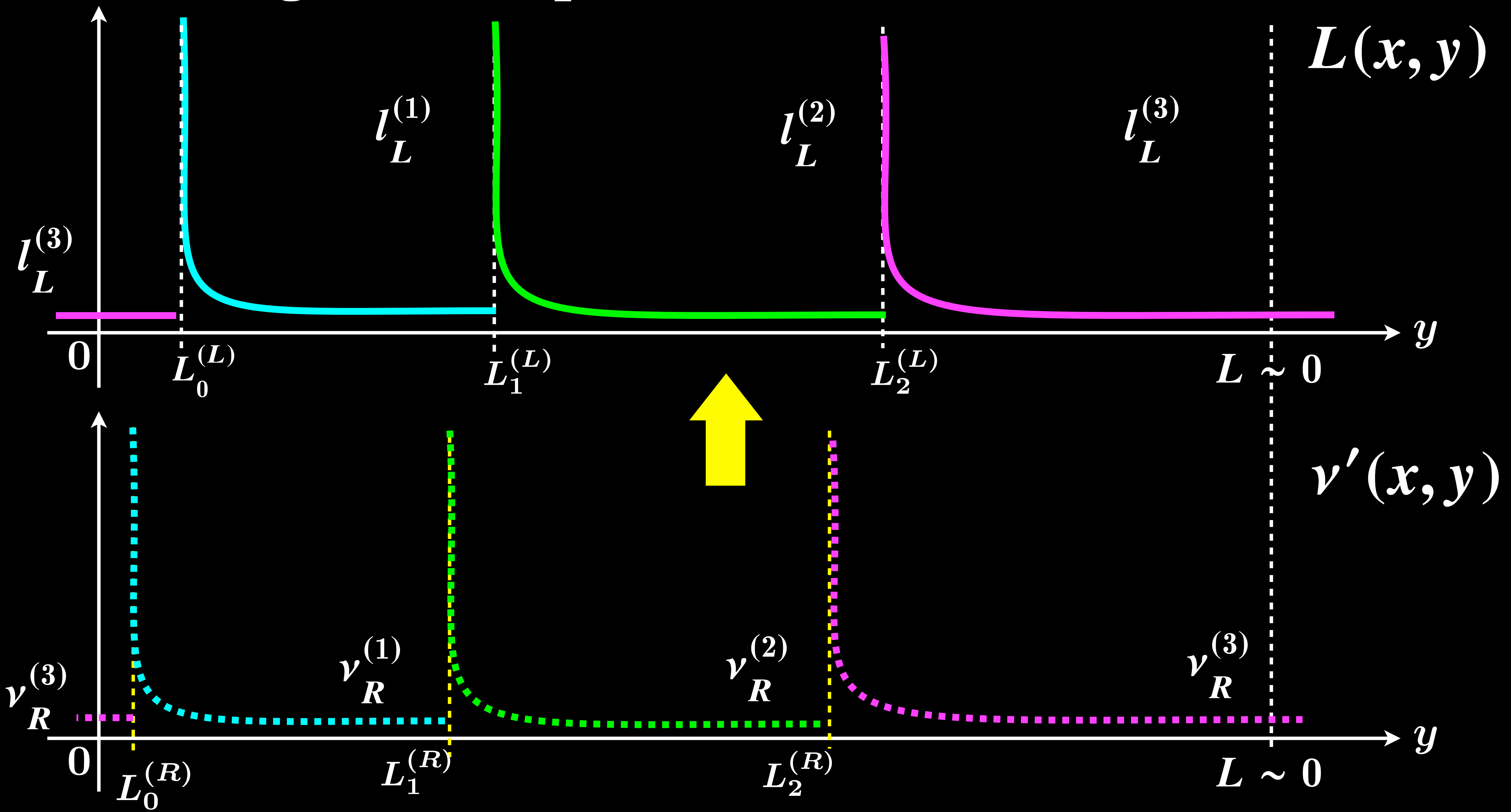

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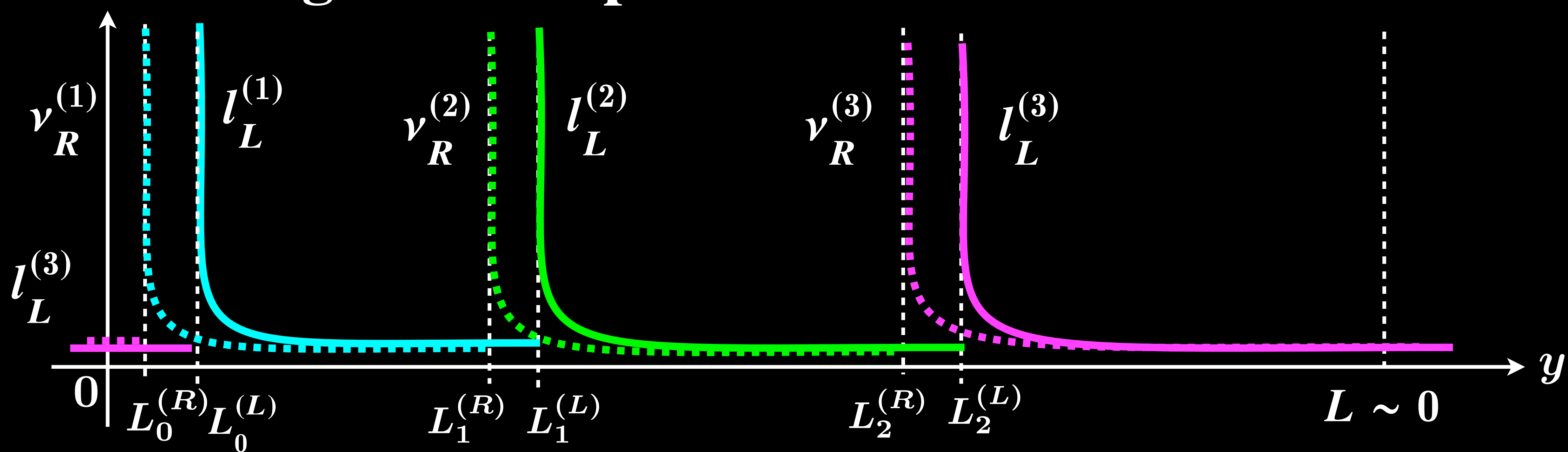

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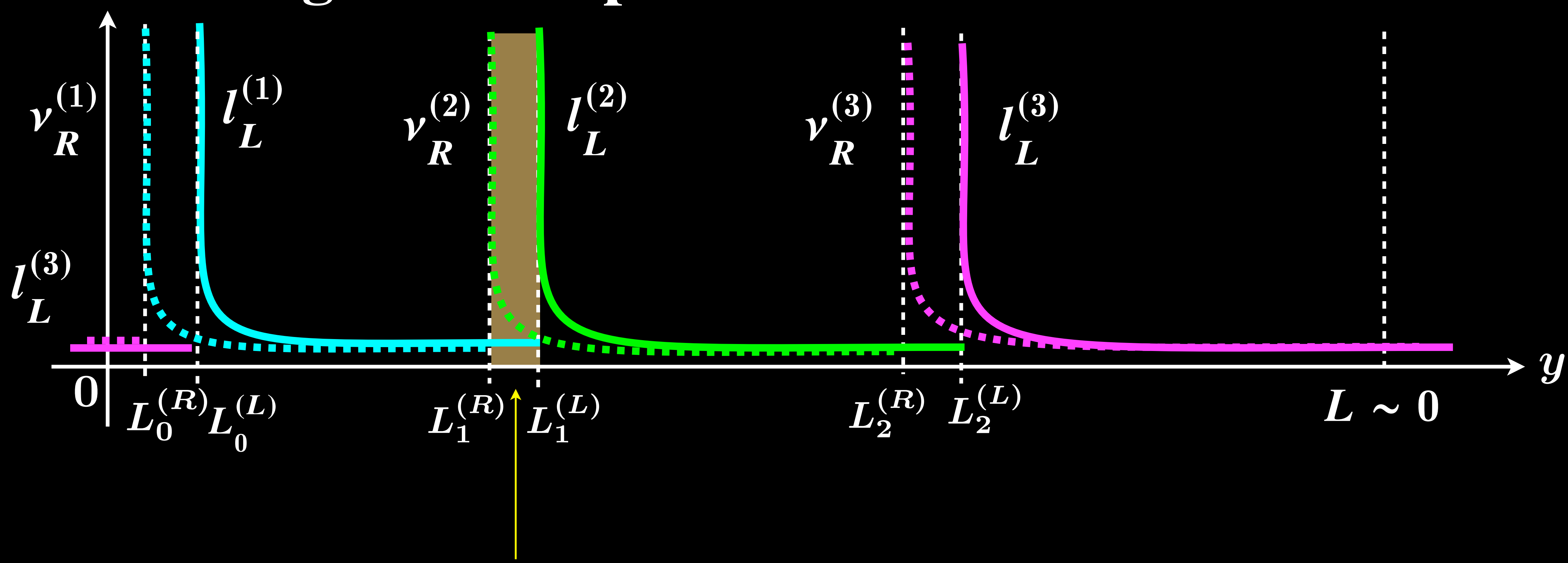

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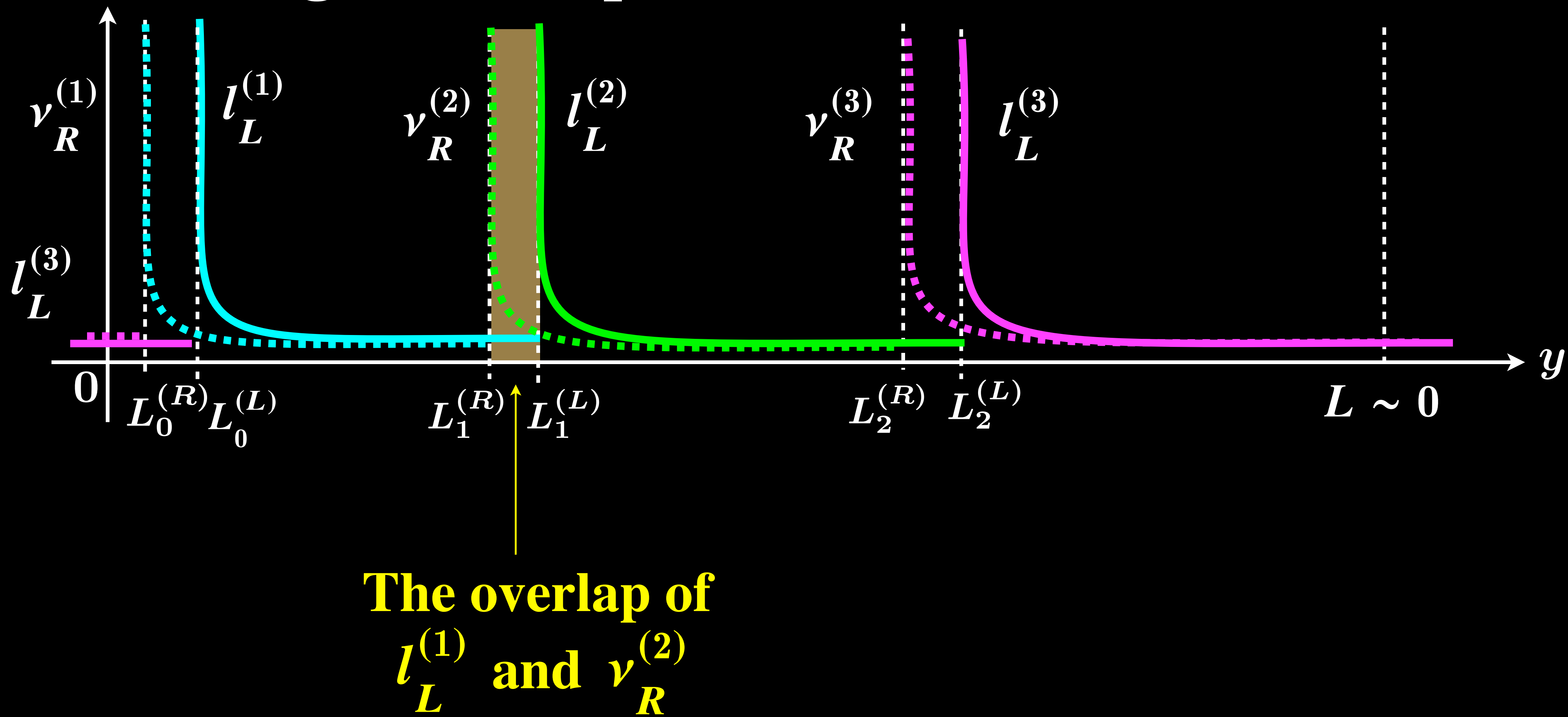

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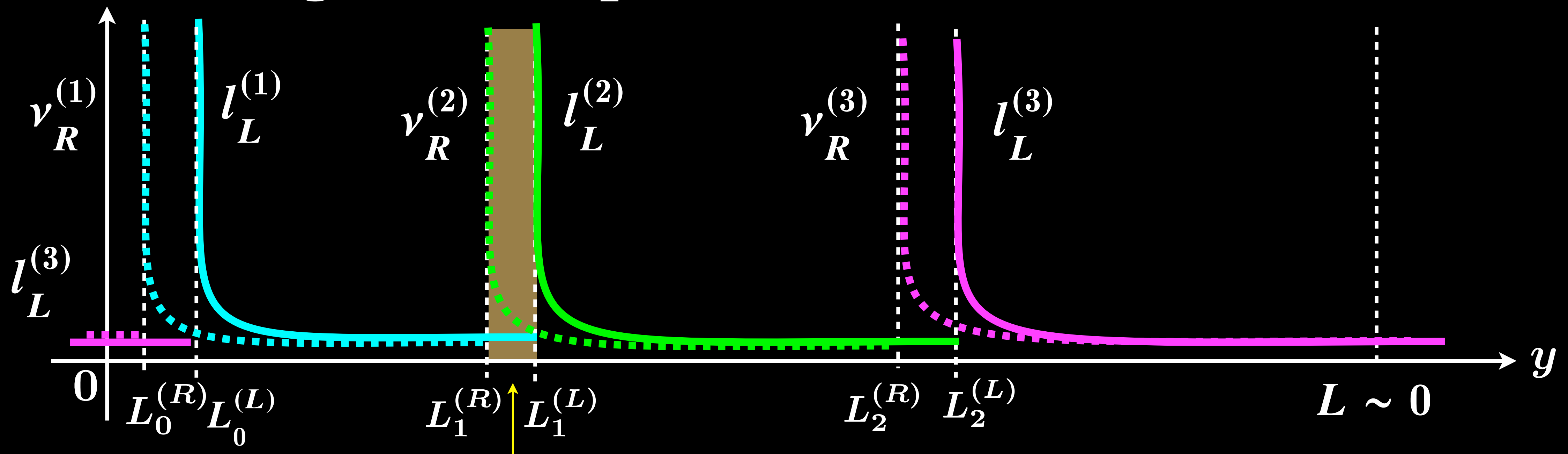

The overlap of $l_L^{(1)}$ and $\nu_R^{(2)}$

$$m_{12}^{(\nu)}$$

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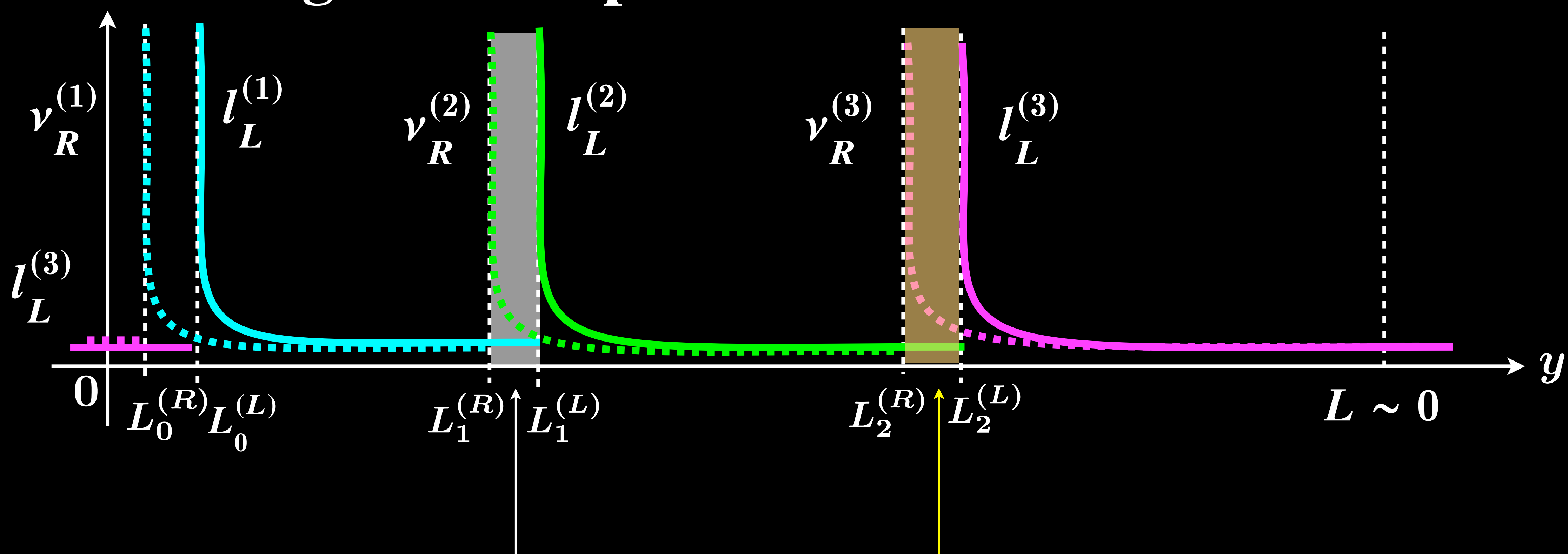

The overlap of $l_L^{(1)}$ and $\nu_R^{(2)}$

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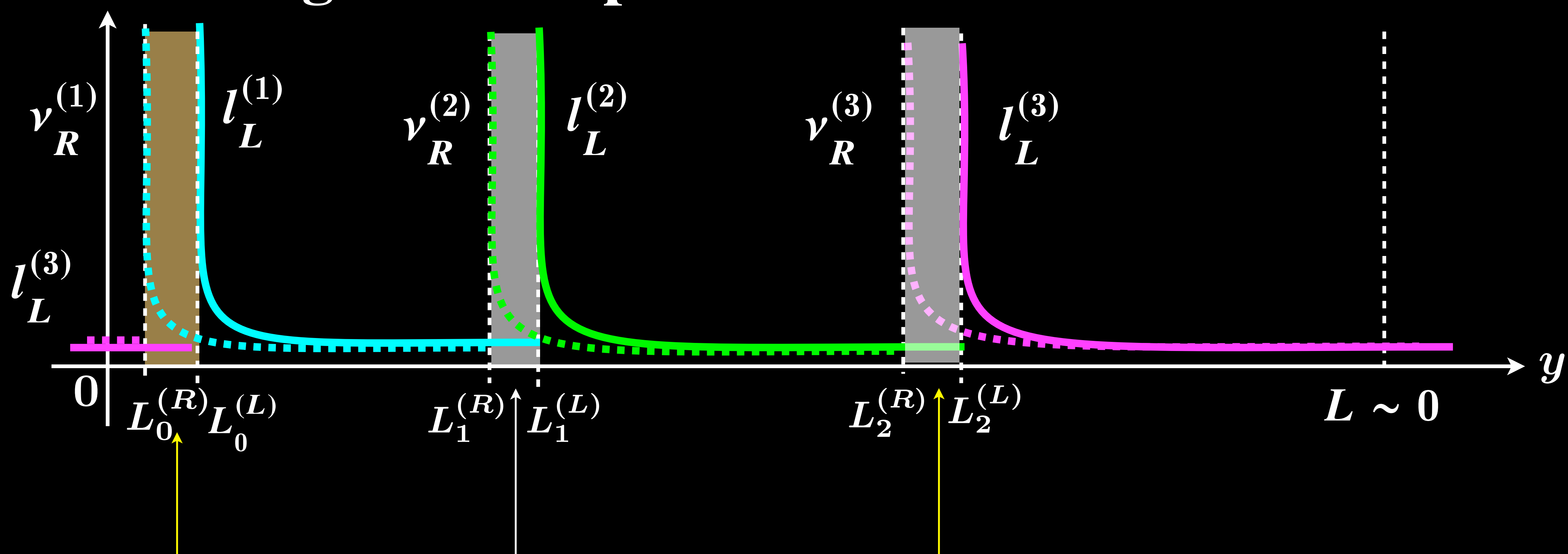

The overlap of $l_L^{(3)}$ and $\nu_R^{(1)}$

$$m_{31}^{(\nu)}$$

The overlap of $l_L^{(1)}$ and $\nu_R^{(2)}$

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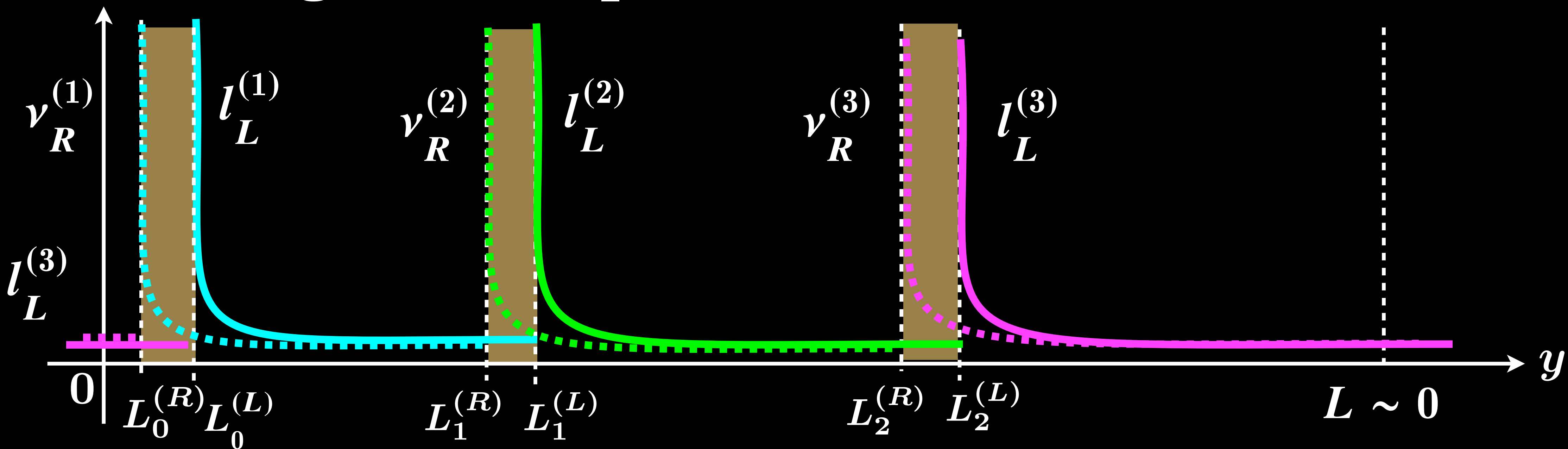

$$M^{(\nu)} = \begin{pmatrix} m_{11}^{(\nu)} & m_{12}^{(\nu)} & 0 \\ 0 & m_{22}^{(\nu)} & m_{23}^{(\nu)} \\ m_{31}^{(\nu)} & 0 & m_{33}^{(\nu)} \end{pmatrix}$$

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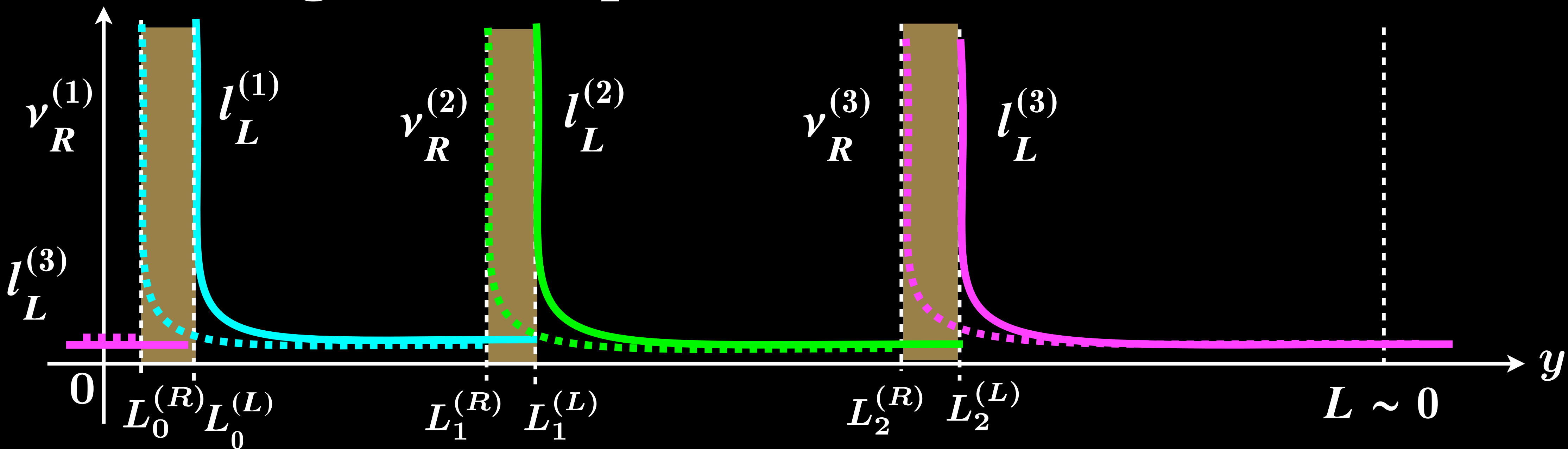

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Source of flavor mixing !!

Flavor mixing

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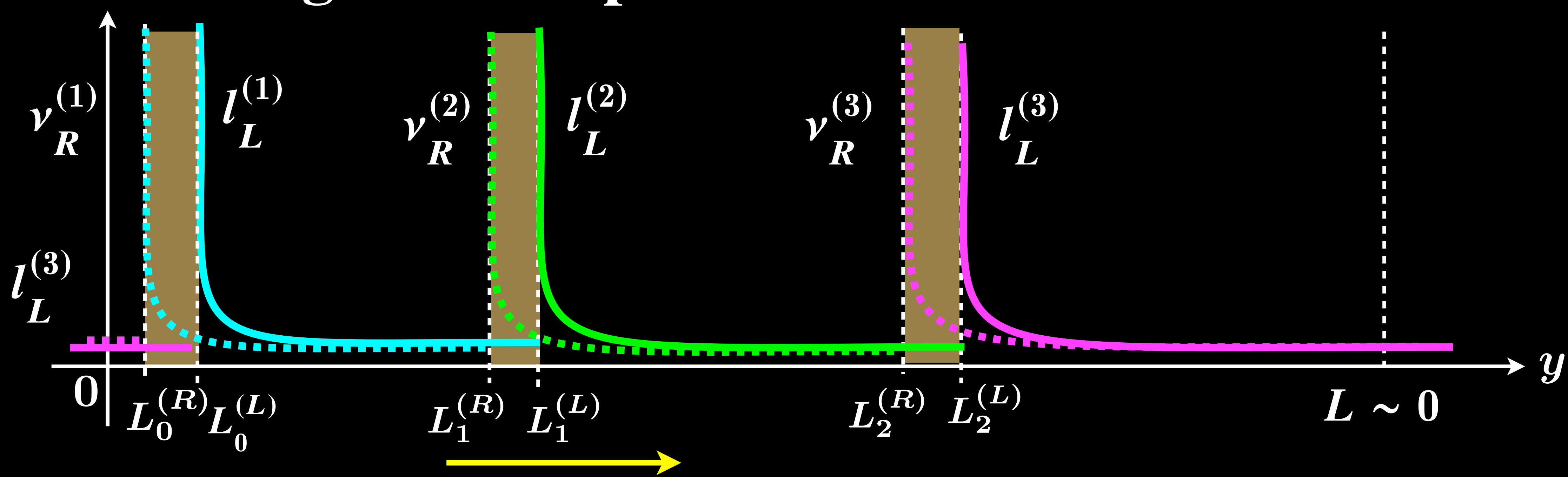

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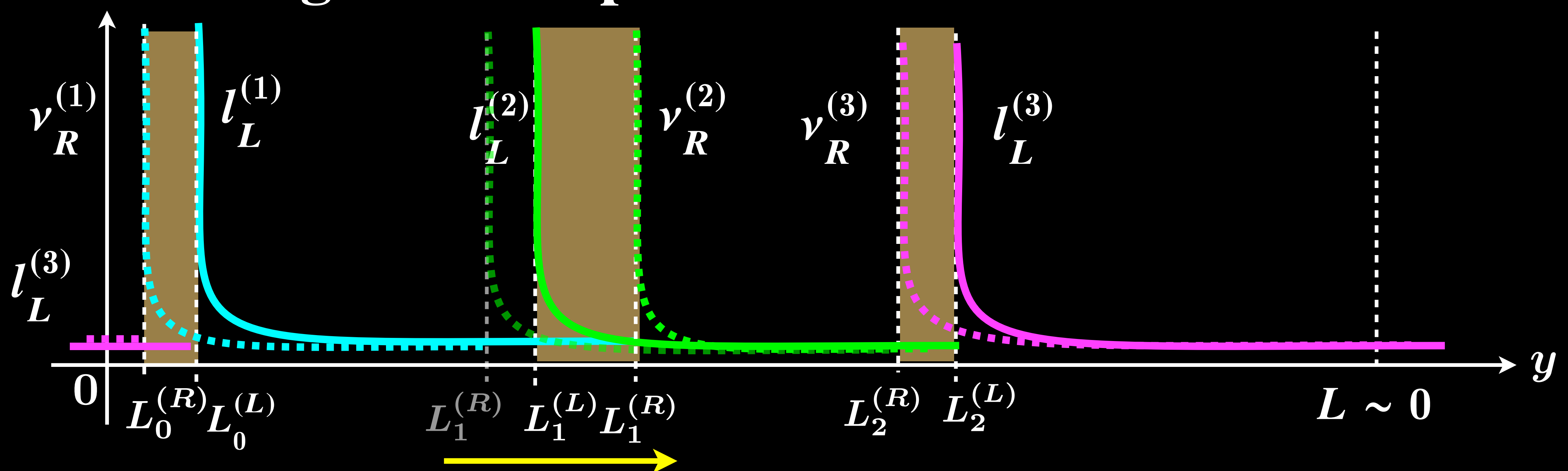

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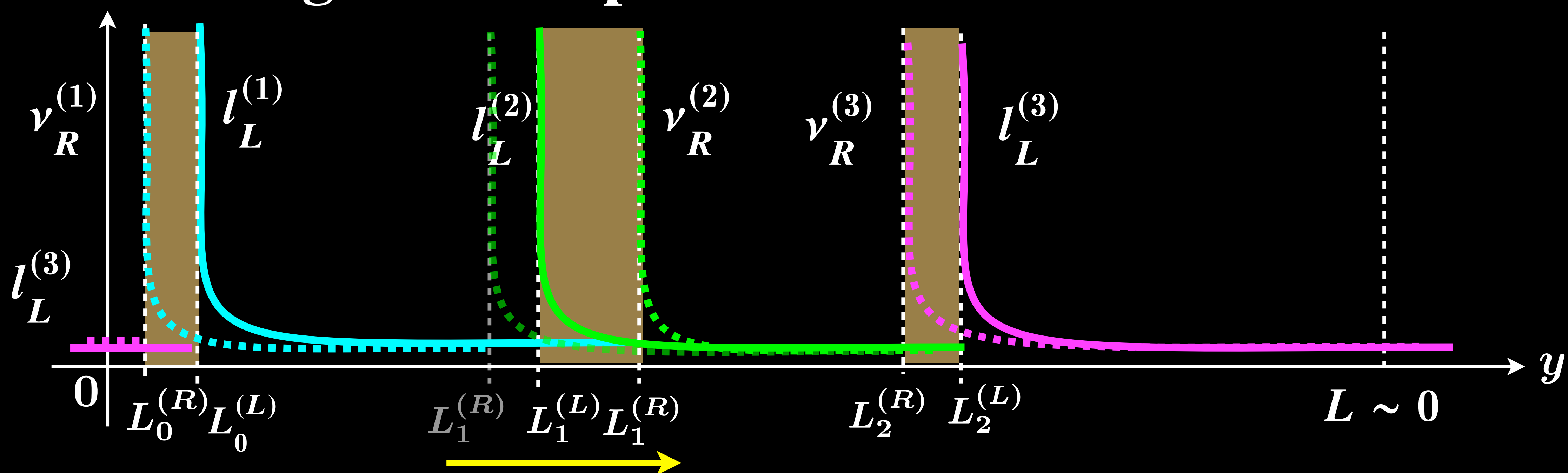

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We can't fill up all the elements !!

of Parameters

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- parameters: 14

$M_L \quad M_\nu \quad M_e$ ⟵ Bulk mass × 3

$L_0^{(l)} \quad L_1^{(l)} \quad L_2^{(l)}$ ⟵ Point interaction for doublet

$L_0^{(\nu)} \quad L_1^{(\nu)} \quad L_2^{(\nu)}$ ⟵ Point interaction for ν-singlet

$L_0^{(e)} \quad L_1^{(e)} \quad L_2^{(e)}$ ⟵ Point interaction for e-singlet

$\lambda^{(\nu)} \quad \lambda^{(e)}$ ⟵ Yukawa coupling × 2

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$$M_L \quad M_\nu \quad M_e$$

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$$L_0^{(\nu)} \quad L_1^{(\nu)} \quad L_2^{(\nu)}$$

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$$\lambda^{(\nu)} \quad \lambda^{(e)}$$

- Physical quantities: 10

$$m_\nu^{(1)} \quad m_\nu^{(2)} \quad m_\nu^{(3)}$$

$$m_e \quad m_\mu \quad m_\tau$$

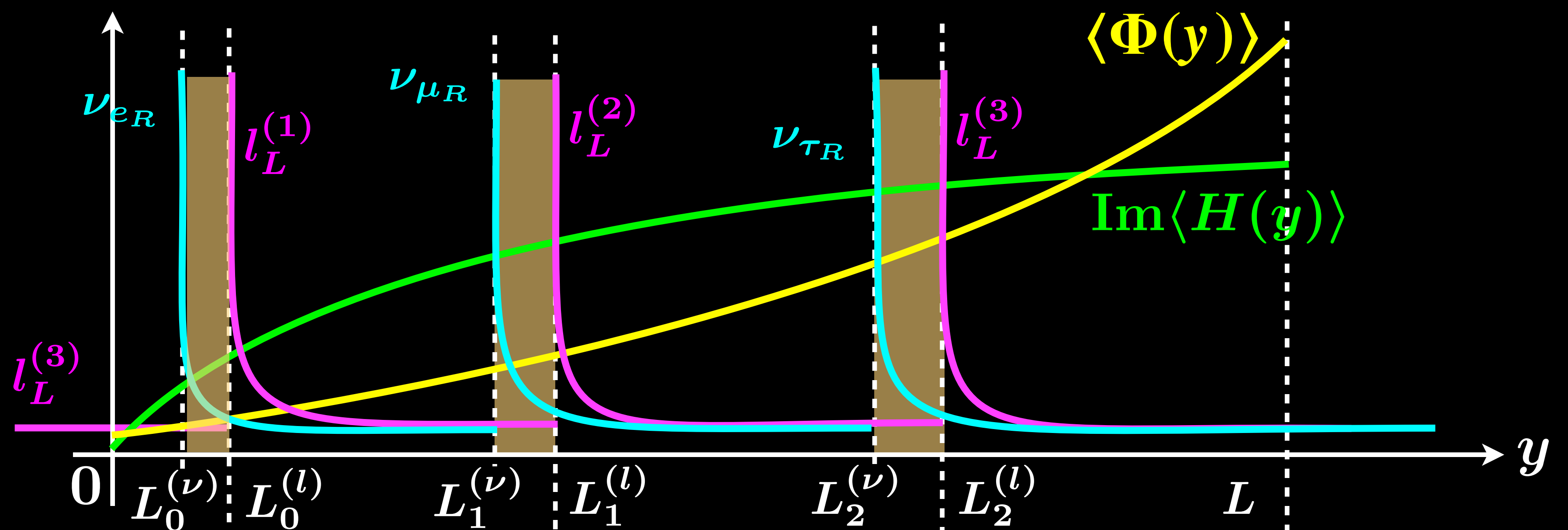
$$\sin\theta_{12} \quad \sin\theta_{23} \quad \sin\theta_{13}$$

$$\delta_{\mathrm{CP}}$$

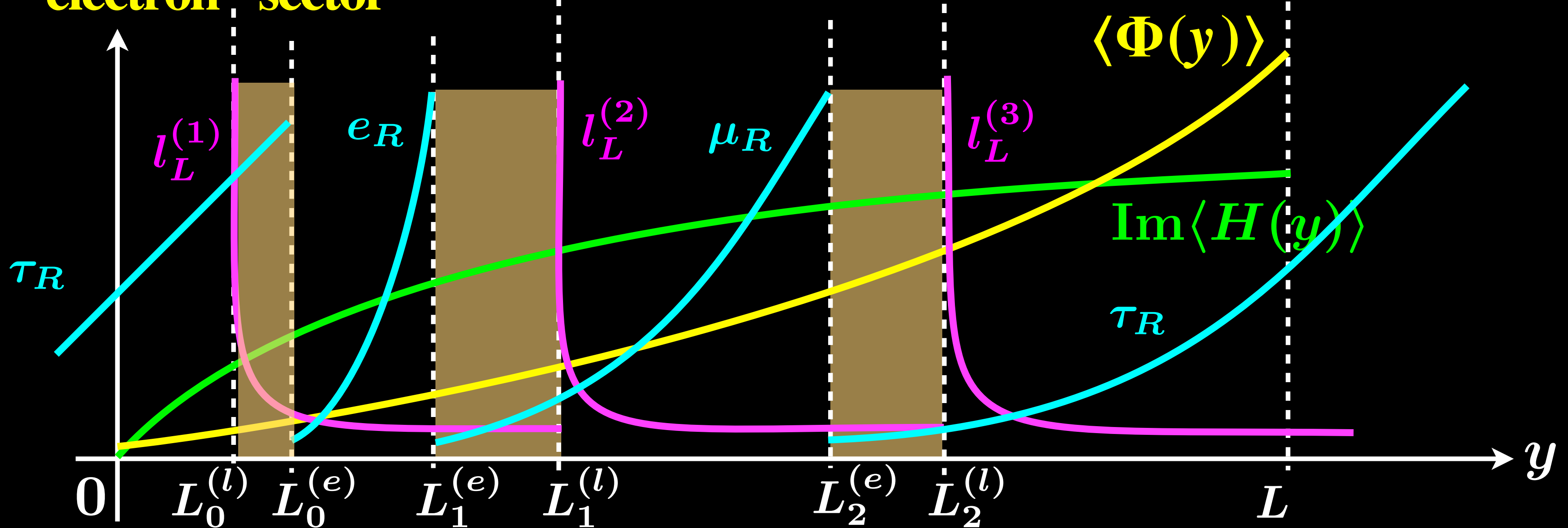
Lepton sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

- neutrino - sector



- electron - sector



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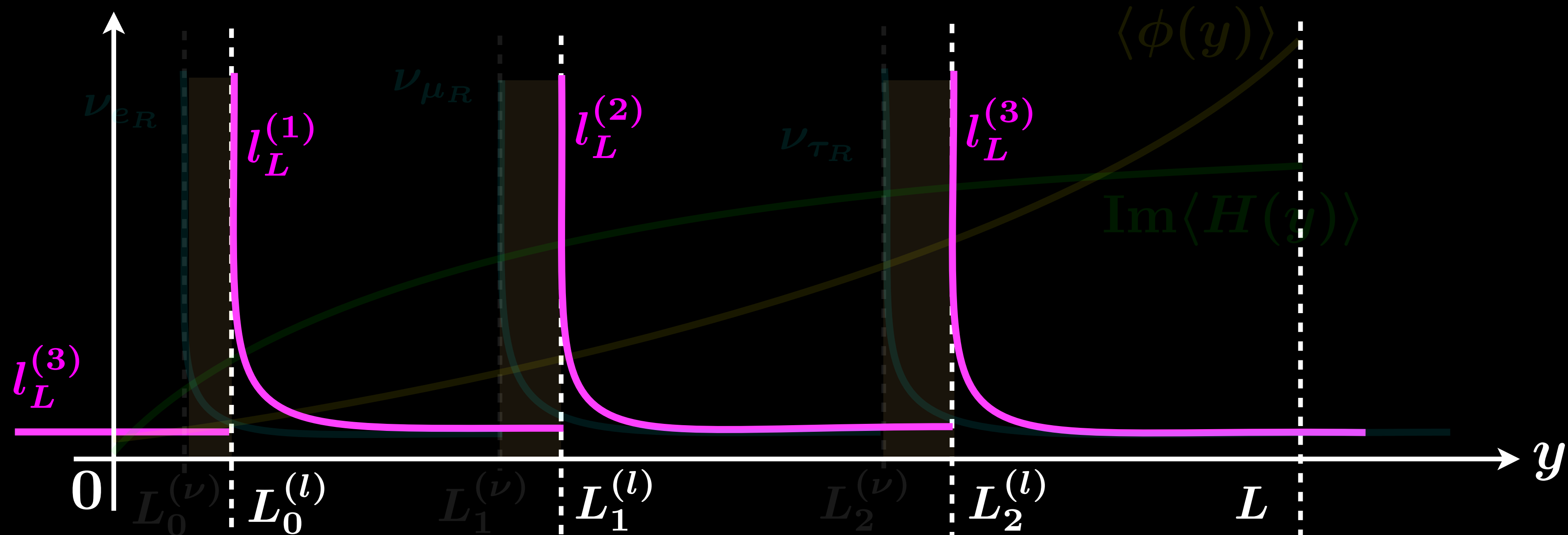

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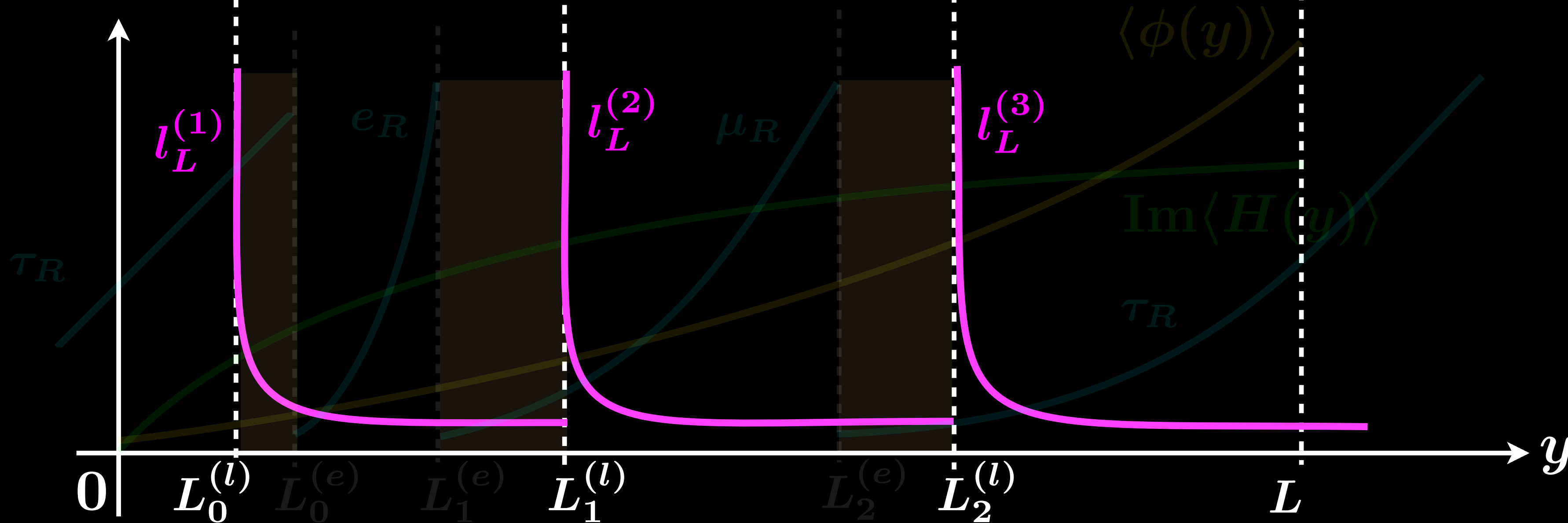

★ Three generations via point interactions.

Lepton sector

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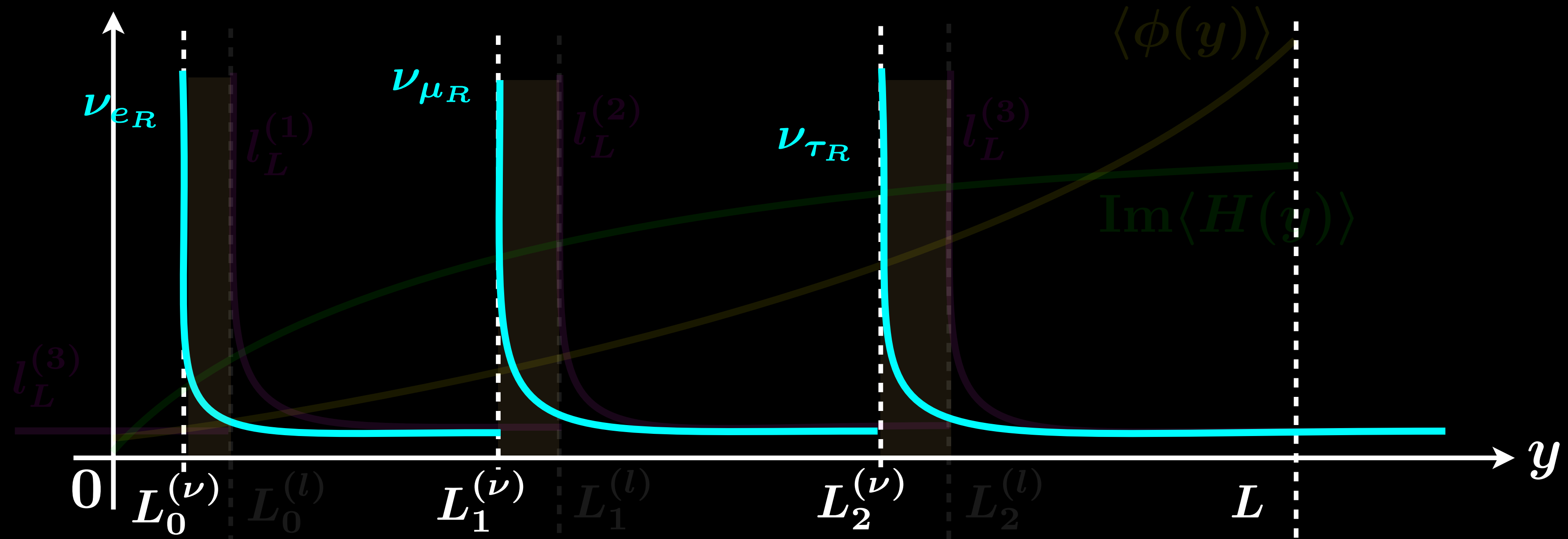

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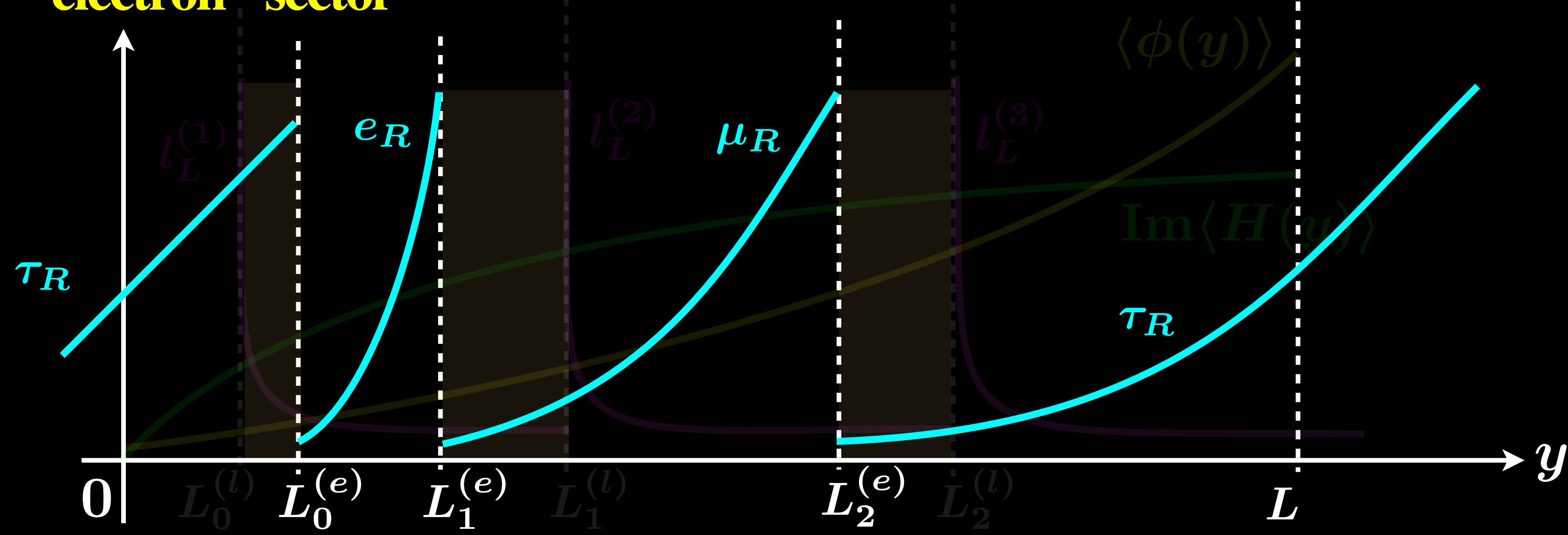

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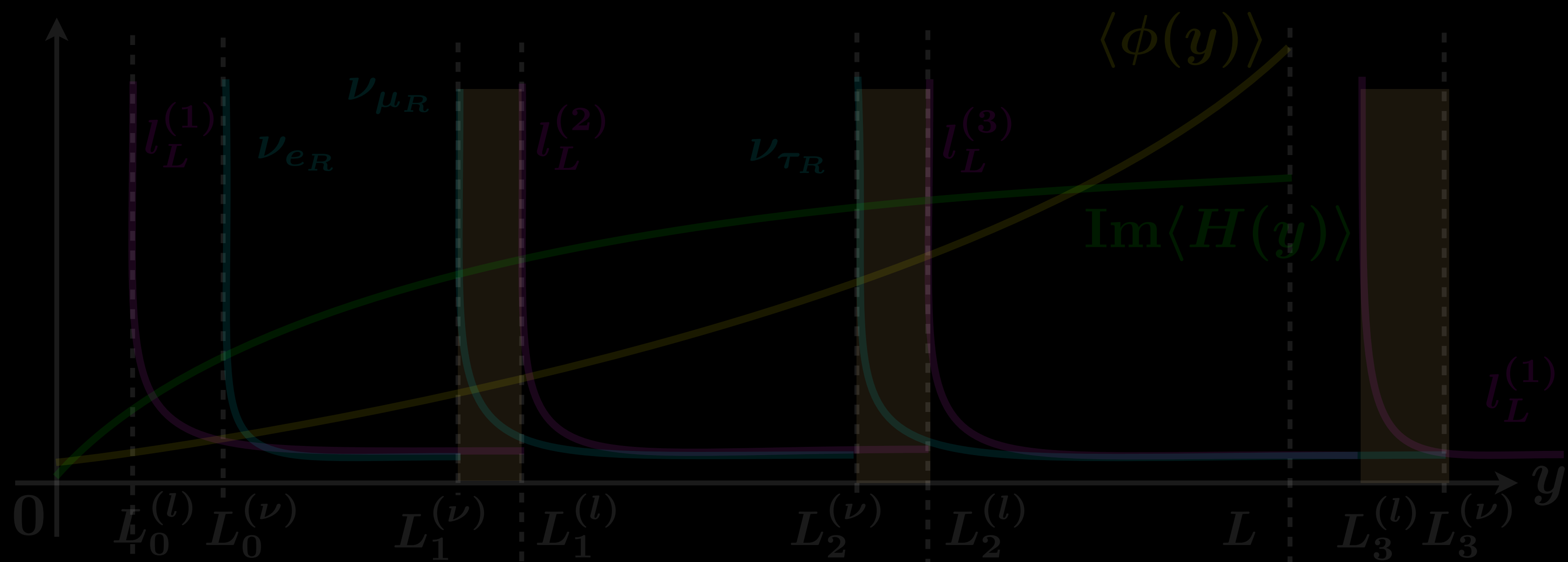

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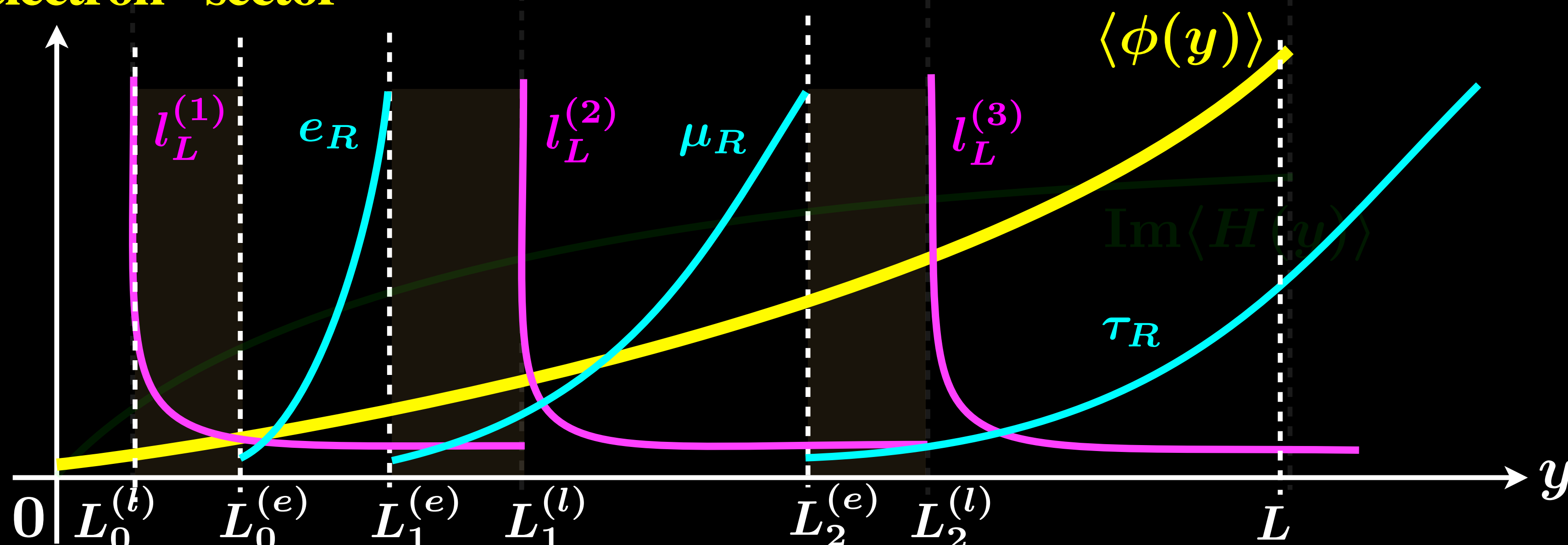

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Lepton sector

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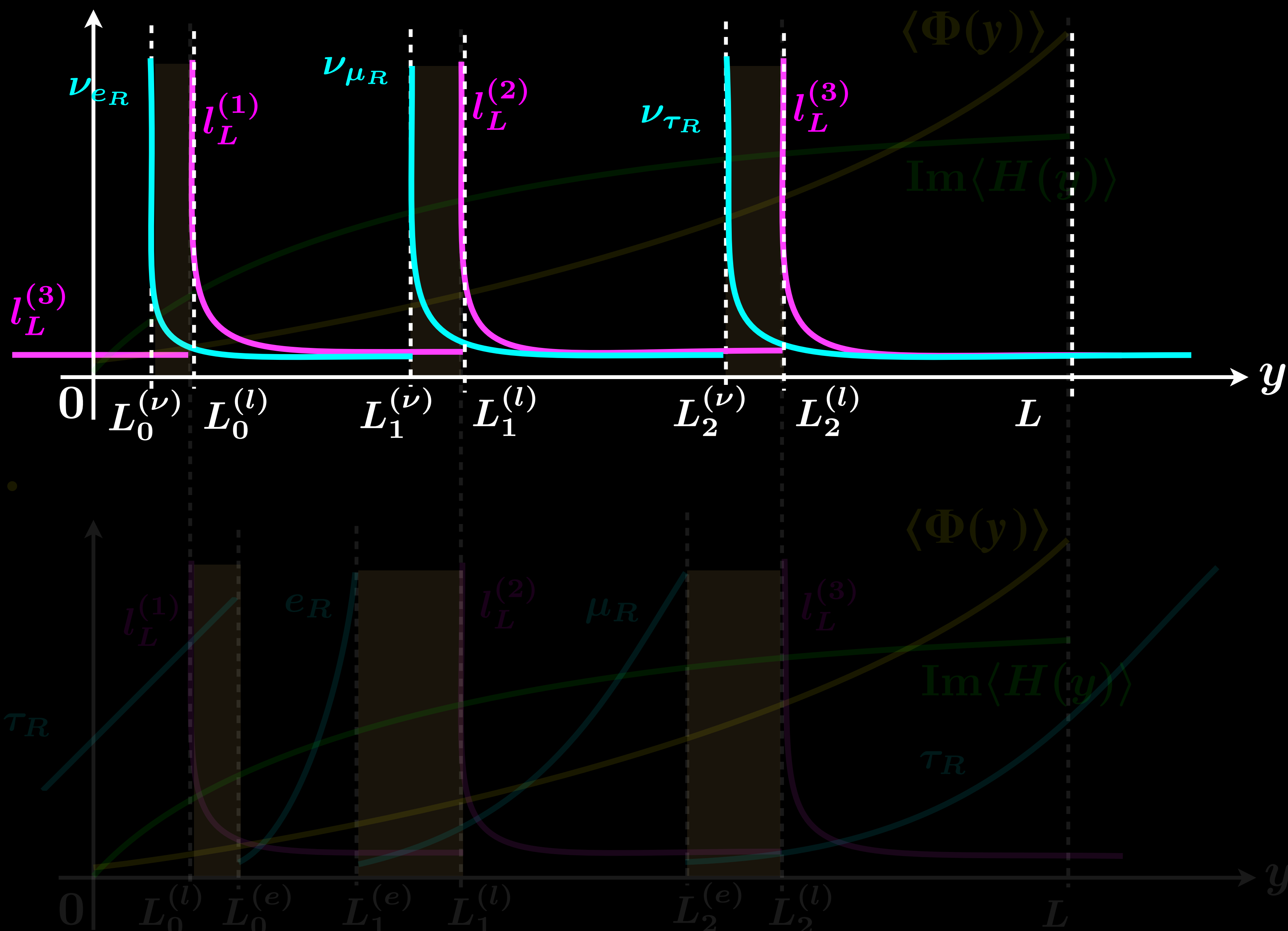

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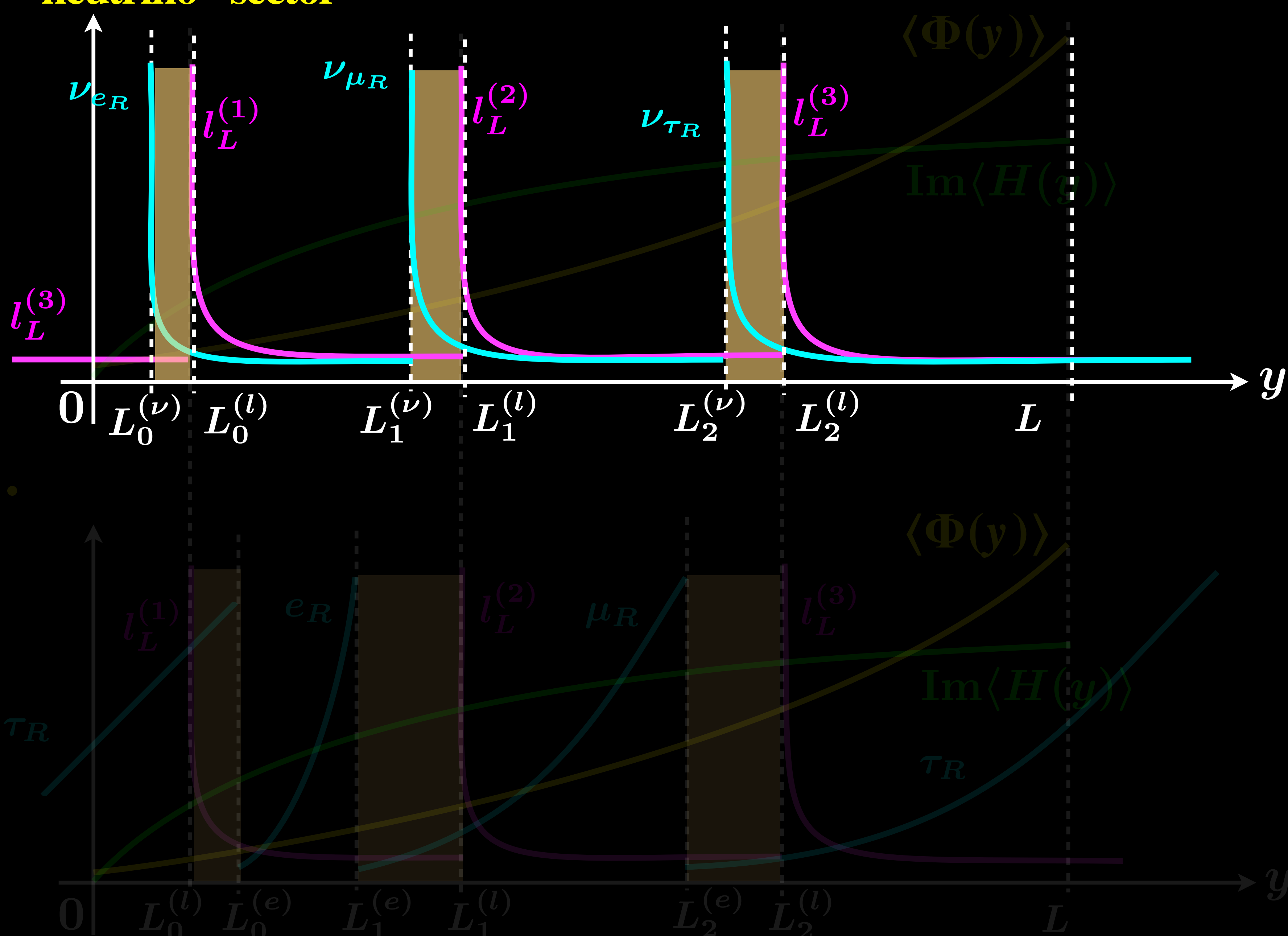

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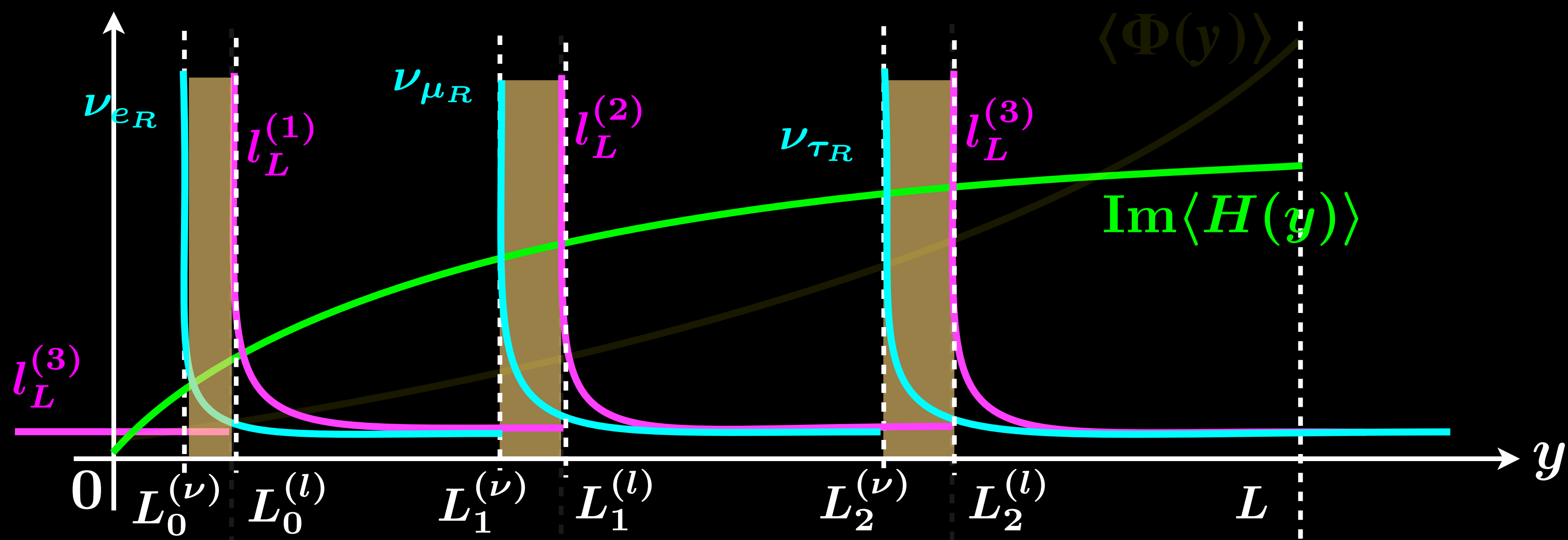

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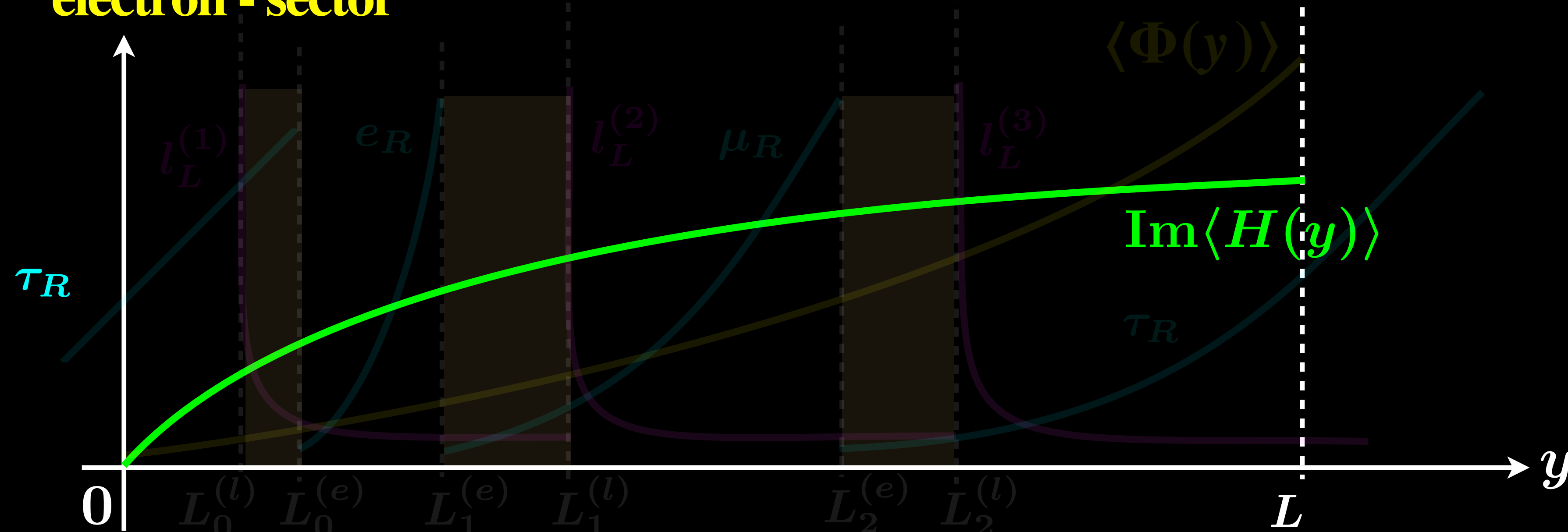

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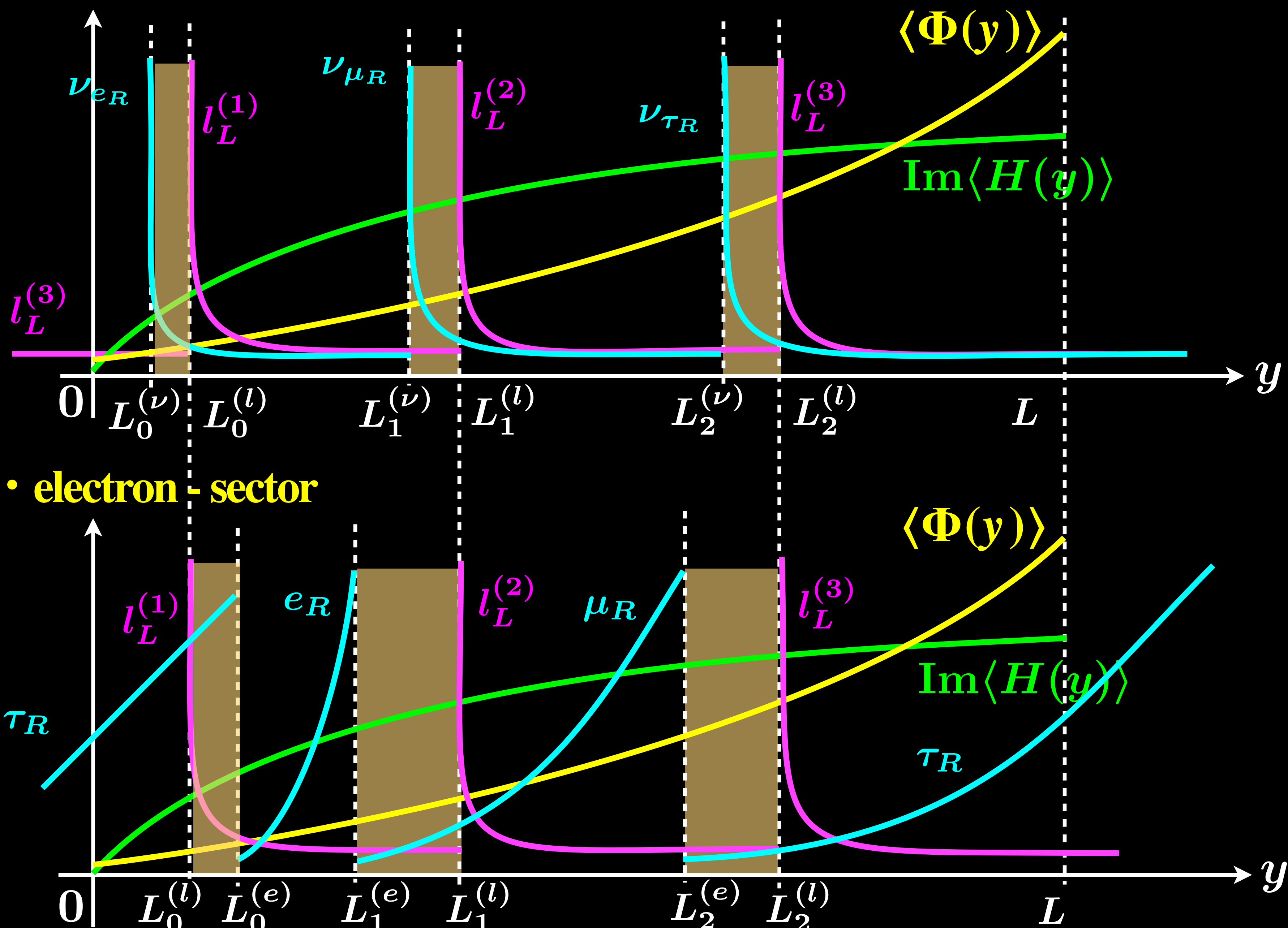

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Lepton sector

❒ Typical example

$$m_1^{(\nu)} = 0.0092\ \mathrm{eV}\ ,\ m_2^{(\nu)} = 0.013\ \mathrm{eV}\ ,\ m_3^{(\nu)} = 0.018\ \mathrm{eV}$$

$$m_e = 0.519\ \mathrm{MeV}\ ,\ m_\mu = 106\ \mathrm{MeV}\ ,\ m_\tau = 1778\ \mathrm{MeV}$$

$$\sin^2\theta_{12} = 0.333\ ,\ \sin^2\theta_{23} = 0.435\ ,\ \sin^2\theta_{13} = 0.0239$$

$$J_{\mathrm{lepton}} = 0.0214\ (\sin\delta = 0.607).$$

Conclusion and Discussion

Conclusion and Discussion

5d gauge theories on a circle
with point interactions

The low energy effective theory

4d gauge theories

Generation
Mass Hierarchy
Tiny neutrino masses
CP phase
Flavor mixing

Conclusion and Discussion

Challenges for the future

- ♣ Point interactions from dynamics…?
- ♣ Application to Gauge-Higgs Unification…?
- ♣ Application to GUT…?
- ♣ Radion stability with point interactions <– Comming soon !?
- ♣ Point interactions on warped metric <– currently underway.
- ♣ FCNC phenomenology

⋮

Back up

Results

$$m_{\nu_1} = 0.0092 \text{ eV}, \qquad m_{\nu_2} = 0.013 \text{ eV}, \qquad m_{\nu_3} = 0.018 \text{ eV},$$

$$m_{\text{electron}} = 0.519 \text{ MeV}, \qquad m_{\text{muon}} = 106 \text{ MeV}, \qquad m_{\text{tau}} = 1.778 \text{ GeV},$$

$$\sin^2\theta_{12} = 0.333, \qquad \sin^2\theta_{23} = 0.435, \qquad \sin^2\theta_{13} = 0.0239,$$

$$J_{\text{lepton}} = 0.0214 \quad (\sin\delta = 0.607).$$

$$\sqrt{\frac{\delta m^2}{\delta m^{2(\text{exp.})}}} = 1.03, \qquad \sqrt{\frac{\Delta m^2}{\Delta m^{2(\text{exp.})}}} = 0.285,$$

$$\frac{m_{\text{electron}}}{m_{\text{electron}}^{(\text{exp.})}} = 1.02, \qquad \frac{m_{\text{muon}}}{m_{\text{muon}}^{(\text{exp.})}} = 0.995, \qquad \frac{m_{\text{tau}}}{m_{\text{tau}}^{(\text{exp.})}} = 1.00,$$

$$\frac{\sin^2\theta_{12}}{\sin^2\theta_{12}^{(\text{exp.})}} = 1.08, \qquad \frac{\sin^2\theta_{23}}{\sin^2\theta_{23}^{(\text{exp.})}} = 1.02, \qquad \frac{\sin^2\theta_{13}}{\sin^2\theta_{13}^{(\text{exp.})}} = 1.02,$$

$$\delta m^2 \equiv m_{\nu_2}^2 - m_{\nu_1}^2,$$

$$\Delta m^2 \equiv m_{\nu_3}^2 - \left(\frac{m_{\nu_1}^2 - m_{\nu_2}^2}{2}\right),$$

Parameters

$$\tilde{L}_0^{(L)} = 0.2565, \qquad \tilde{L}_1^{(L)} = 0.5776, \qquad \tilde{L}_2^{(L)} = 0.9432,$$
$$\tilde{L}_0^{(\mathcal{N})} = 0.08240, \qquad \tilde{L}_1^{(\mathcal{N})} = 0.3909, \qquad \tilde{L}_2^{(\mathcal{N})} = 0.7317,$$
$$\tilde{L}_0^{(E)} = 0.277, \qquad \tilde{L}_1^{(E)} = 0.49, \qquad \tilde{L}_2^{(E)} = 0.79,$$

$$\tilde{M}_L = -136.9, \qquad \tilde{M}_{\mathcal{N}} = 112.1, \qquad \tilde{M}_E = -2.00,$$

$$\tilde{M}_\Phi = 8.67, \qquad \tilde{\lambda}_\Phi = 0.001, \qquad \frac{1}{\tilde{L}_+} = -6.07, \quad \frac{1}{\tilde{L}_-} = 8.69, \qquad \theta = 3,$$

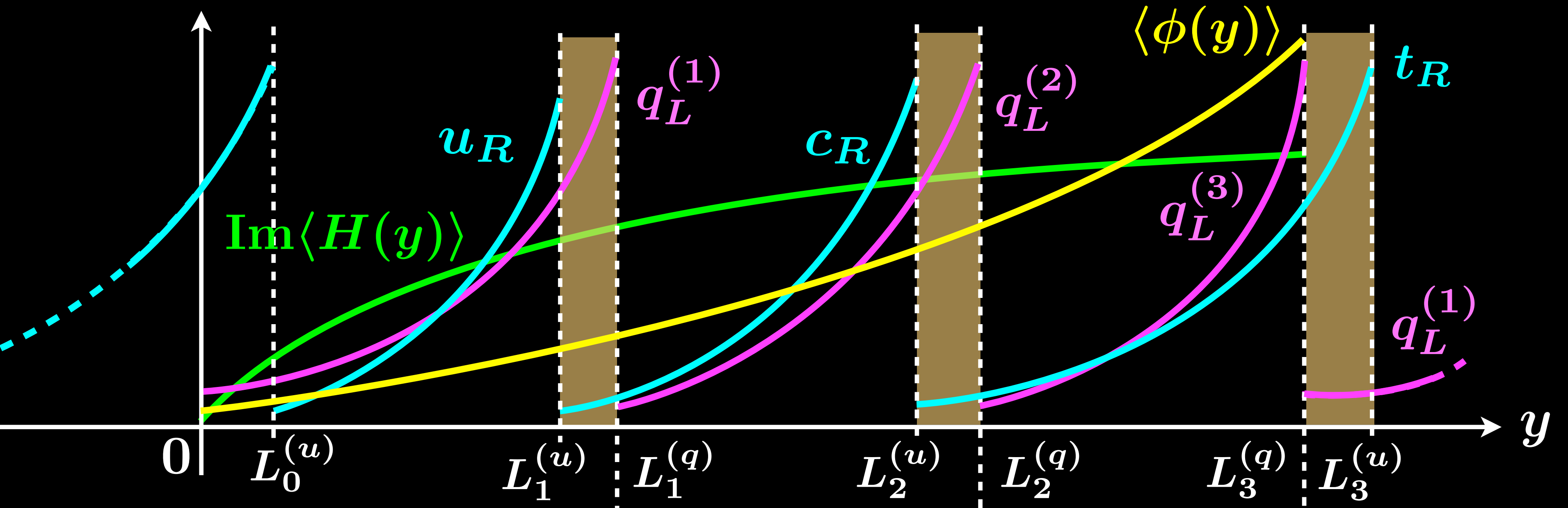
$$\tilde{\mathcal{Y}}^{(\mathcal{N})} = -0.0000309 - 9.15 \times 10^{-6}\, i, \qquad \tilde{\mathcal{Y}}^{(E)} = -0.00309 - 0.000915\, i$$

$$\sqrt{|\mathcal{Y}^{(\mathcal{N})}|} = 0.00568L \text{ and } \sqrt{|\mathcal{Y}^{(E)}|} = 0.0568L.$$

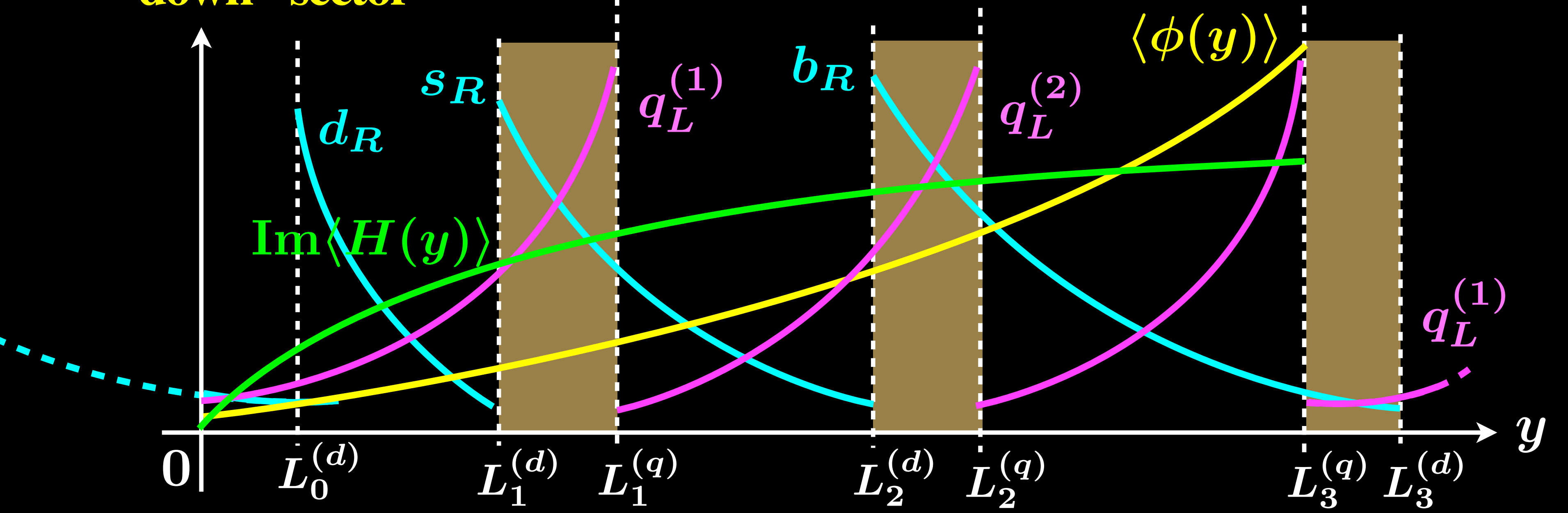
Quark sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

- up - sector



- down - sector

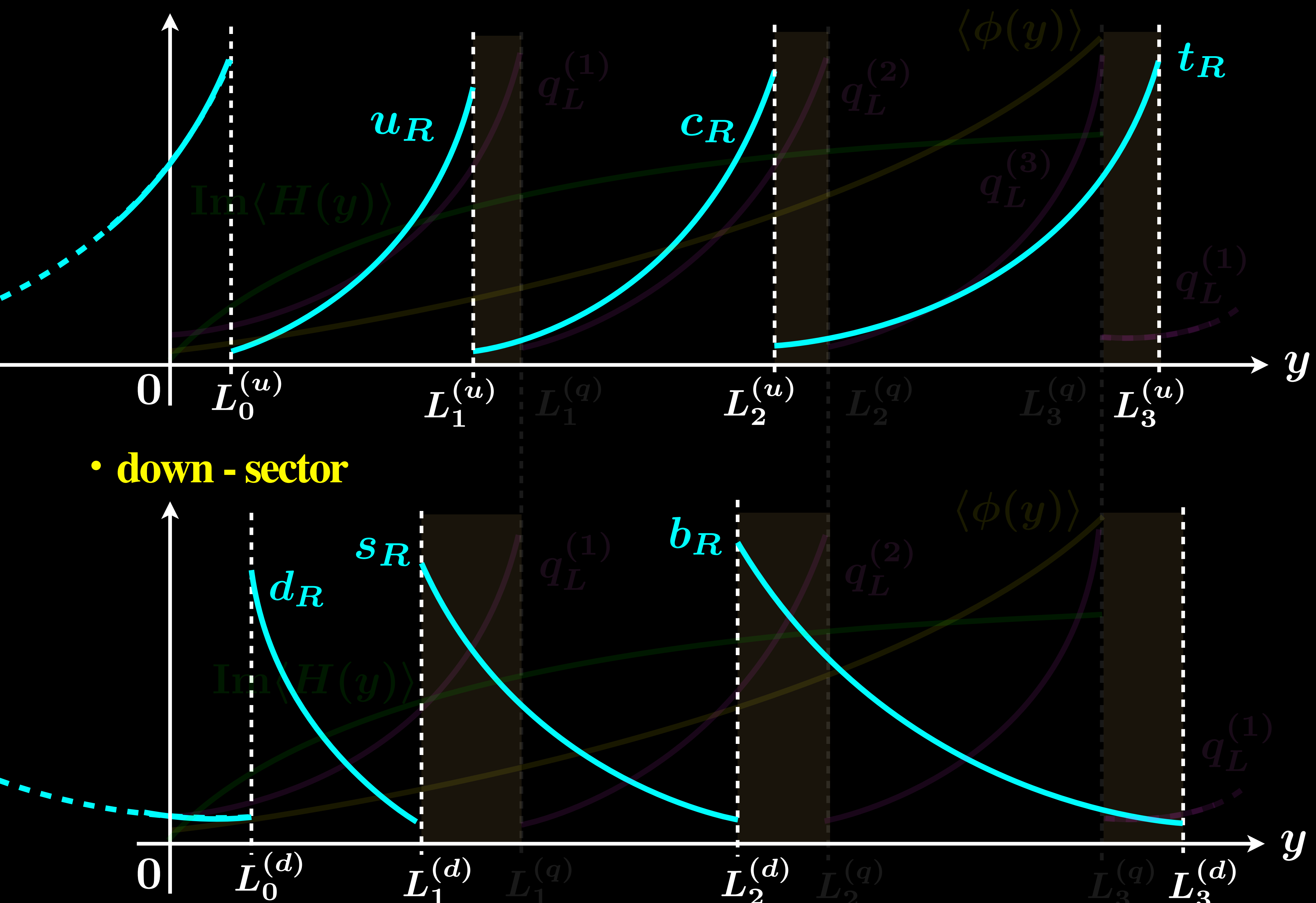


Quark sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

- **up - sector**

- **down - sector**

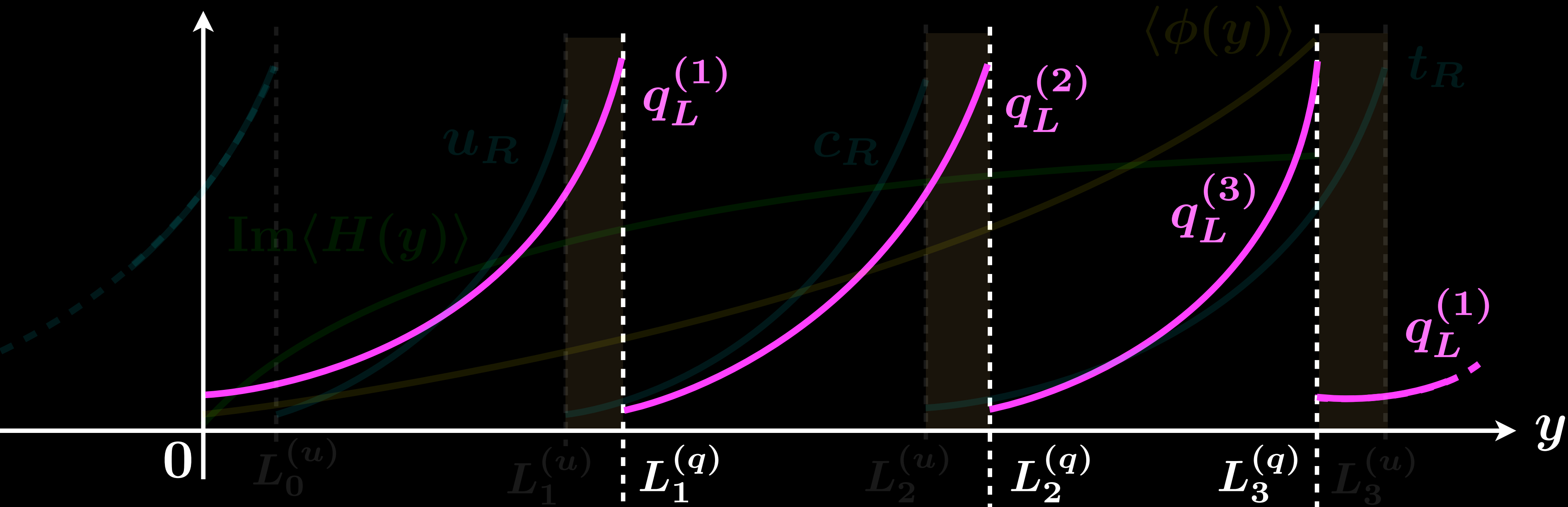


★ **Three generations via point interactions**

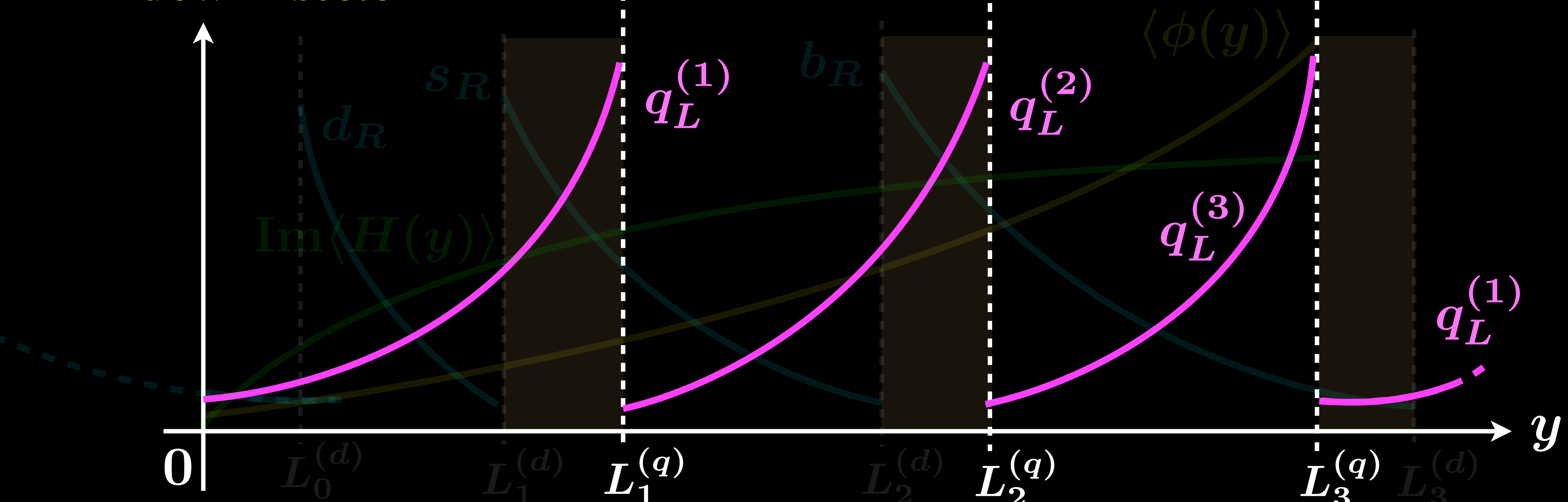
Quark sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

- up - sector



- down - sector

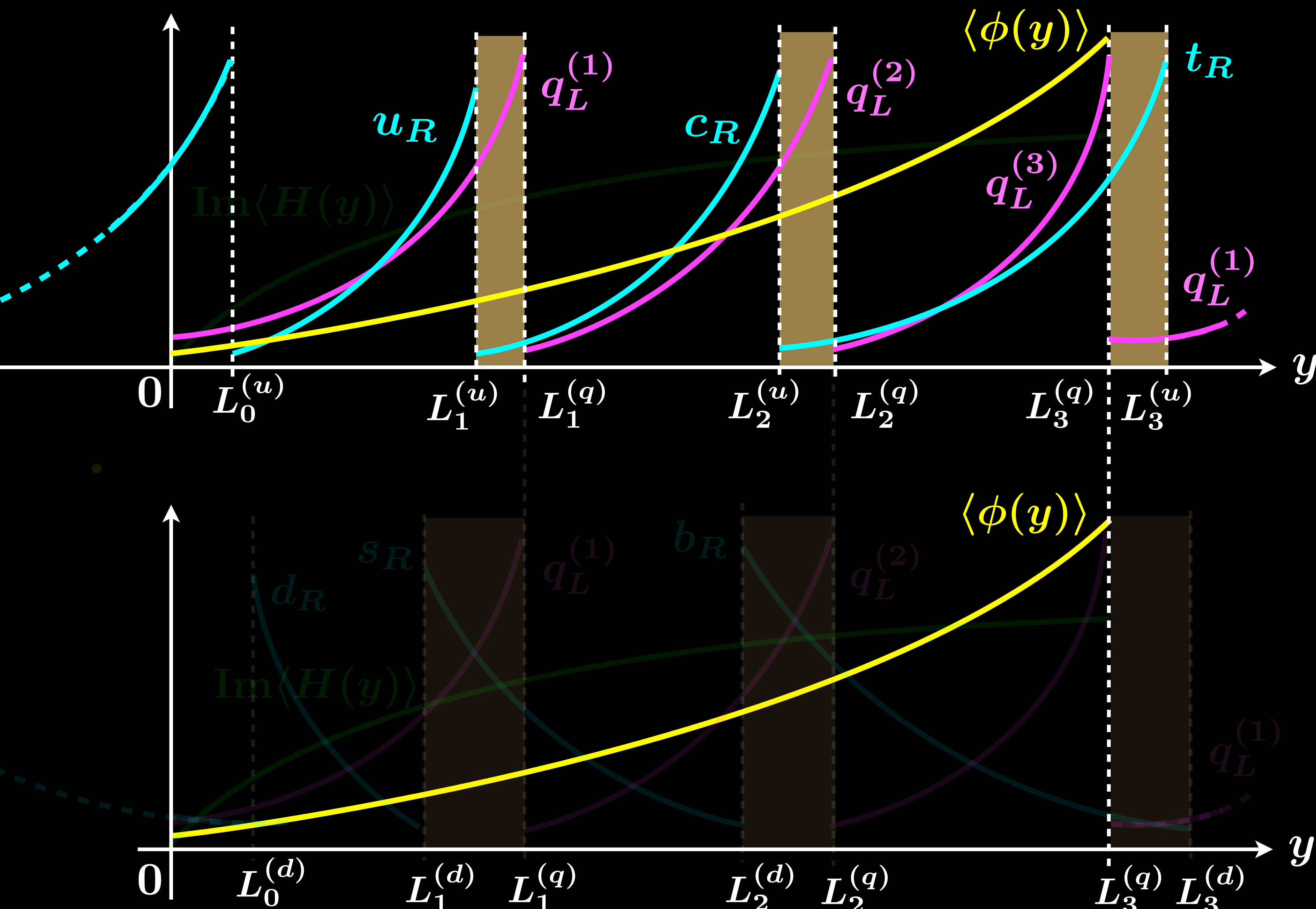


★ Three generations via point interactions

Quark sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

- up - sector



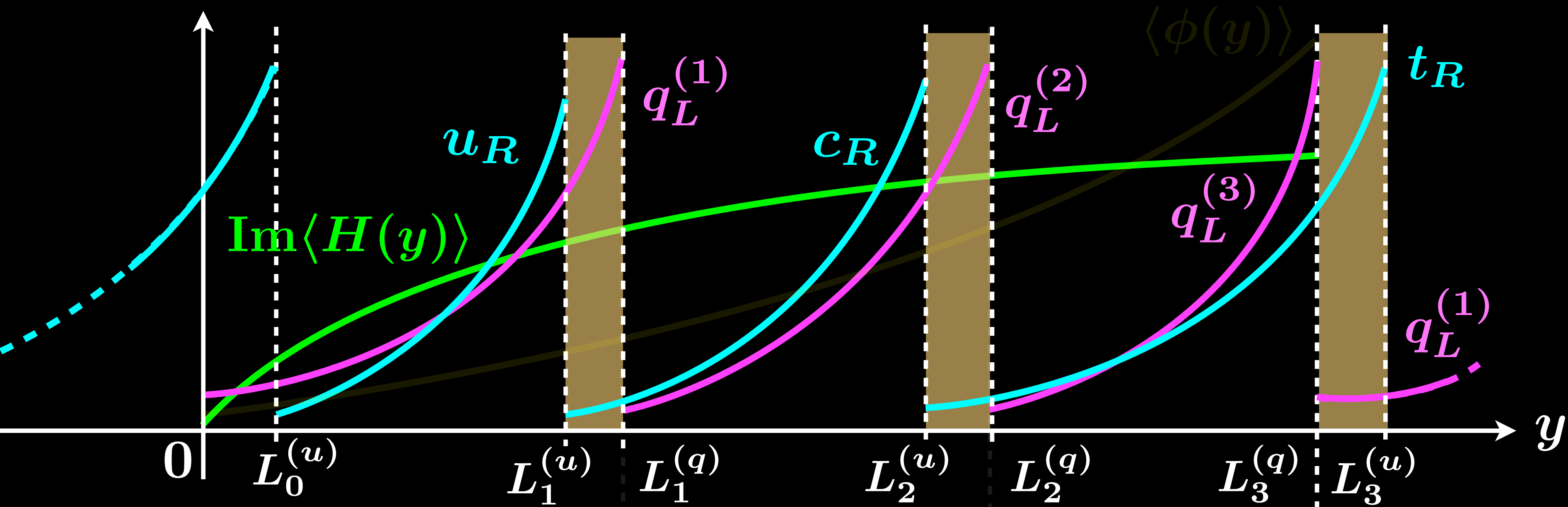
★ Three generations via point interactions

★ Mass hierarchy from $\langle\Phi(y)\rangle$ via the Robin BC

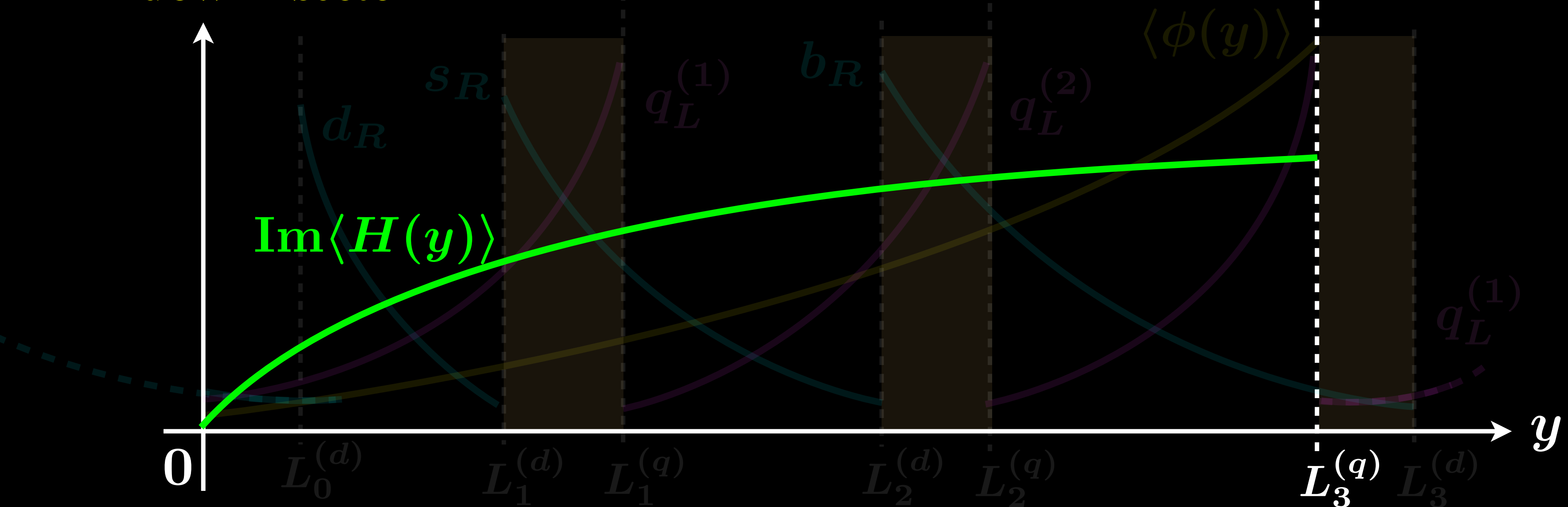
Quark sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

- up - sector



- down - sector



★ Three generations via point interactions

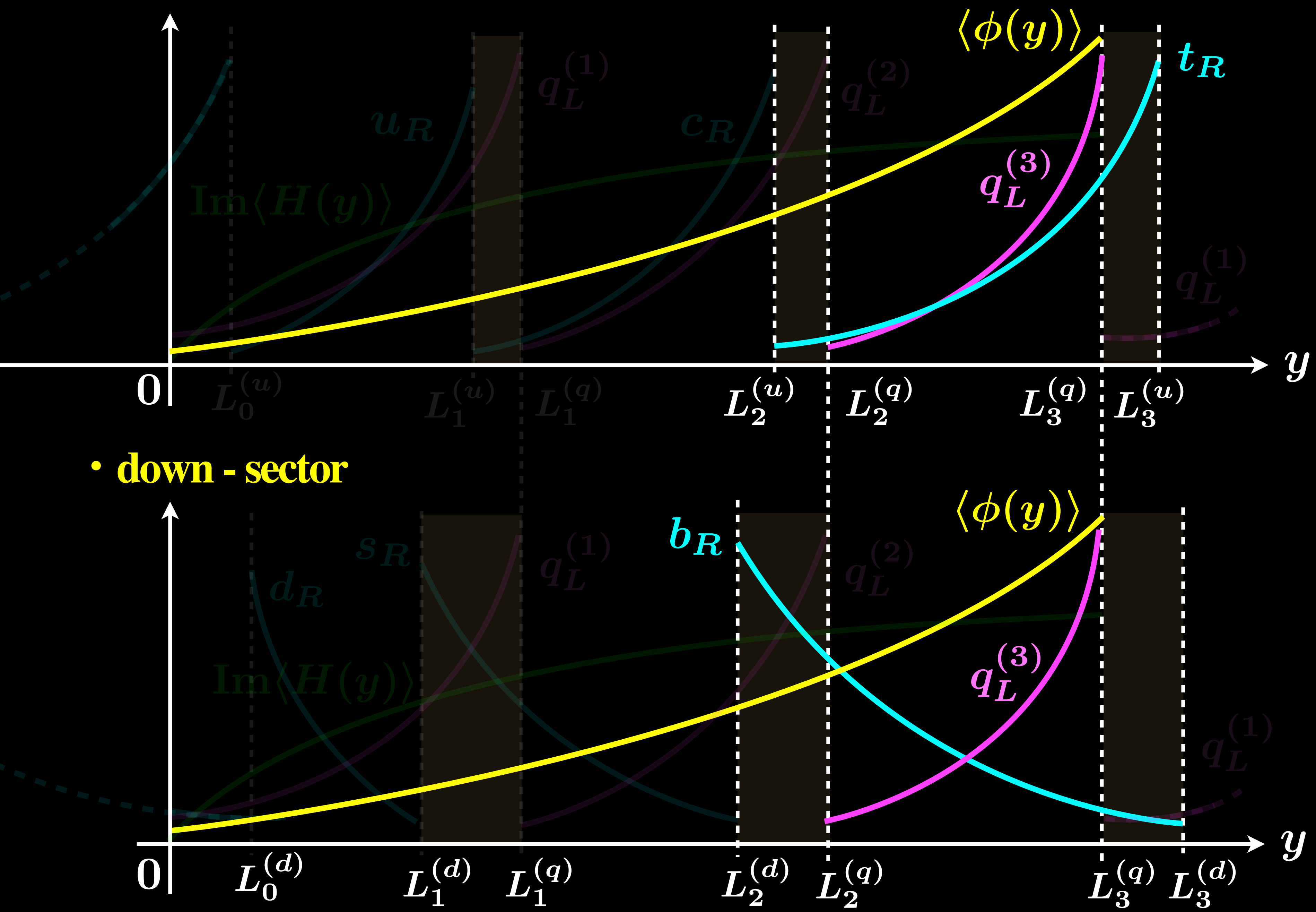
★ Mass hierarchy from $\langle \Phi(y) \rangle$ via the Robin BC

★ CP phase from $\langle H(y) \rangle$ via the twisted BC

Quark sector

$$m_{4d} \sim Y_{5d} \int_0^{L_3} dy \, \langle \Phi(y) \rangle \langle H(y) \rangle \bar{\Psi}' \Psi$$

- up - sector
- down - sector



★ Three generations via point interactions

★ Mass hierarchy from $\langle \Phi(y) \rangle$ via the Robin BC

★ CP phase from $\langle H(y) \rangle$ via the twisted BC

★ $M_t > M_b$ from the configuration

Quark sector

❐ Numerical results

$$\frac{m_{\mathrm{up}}^{(\mathrm{ours})}}{m_{\mathrm{up}}^{(\mathrm{exp.})}} = 0.897 \qquad \frac{m_{\mathrm{charm}}^{(\mathrm{ours})}}{m_{\mathrm{charm}}^{(\mathrm{exp.})}} = 0.978 \qquad \frac{m_{\mathrm{top}}^{(\mathrm{ours})}}{m_{\mathrm{top}}^{(\mathrm{exp.})}} = 1.00$$

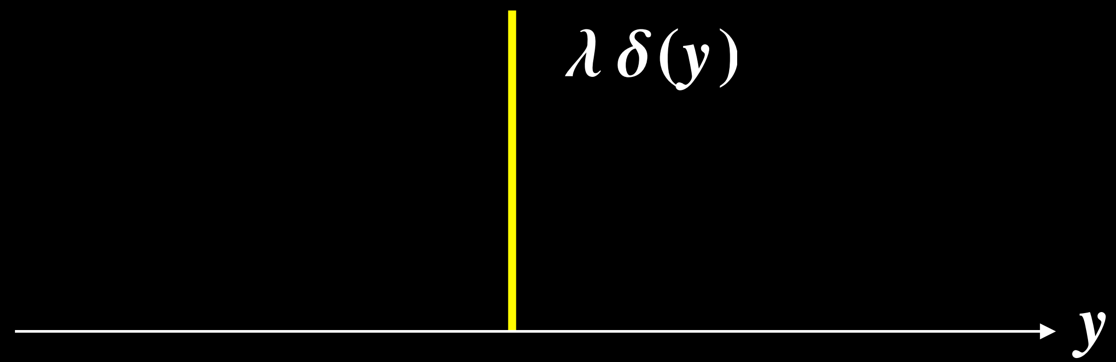
$$\frac{m_{\mathrm{down}}^{(\mathrm{ours})}}{m_{\mathrm{down}}^{(\mathrm{exp.})}} = 1.02 \qquad \frac{m_{\mathrm{strange}}^{(\mathrm{ours})}}{m_{\mathrm{strange}}^{(\mathrm{exp.})}} = 1.07 \qquad \frac{m_{\mathrm{bottom}}^{(\mathrm{ours})}}{m_{\mathrm{bottom}}^{(\mathrm{exp.})}} = 1.00$$

$$\frac{|V_{\mathrm{CKM}}^{(\mathrm{ours})}|}{|V_{\mathrm{CKM}}^{(\mathrm{exp.})}|} = \begin{bmatrix} 0.997 & 1.06 & 0.906 \\ 1.06 & 0.997 & 0.902 \\ 0.957 & 0.900 & 1.00 \end{bmatrix} \qquad \frac{J^{(\mathrm{ours})}}{J^{(\mathrm{exp.})}} = 0.865$$

Point interactions

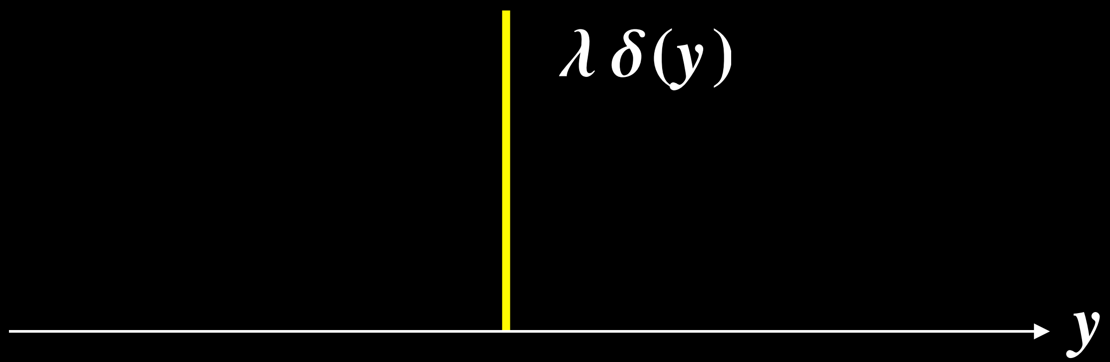
Point interactions

- **Delta function potential**

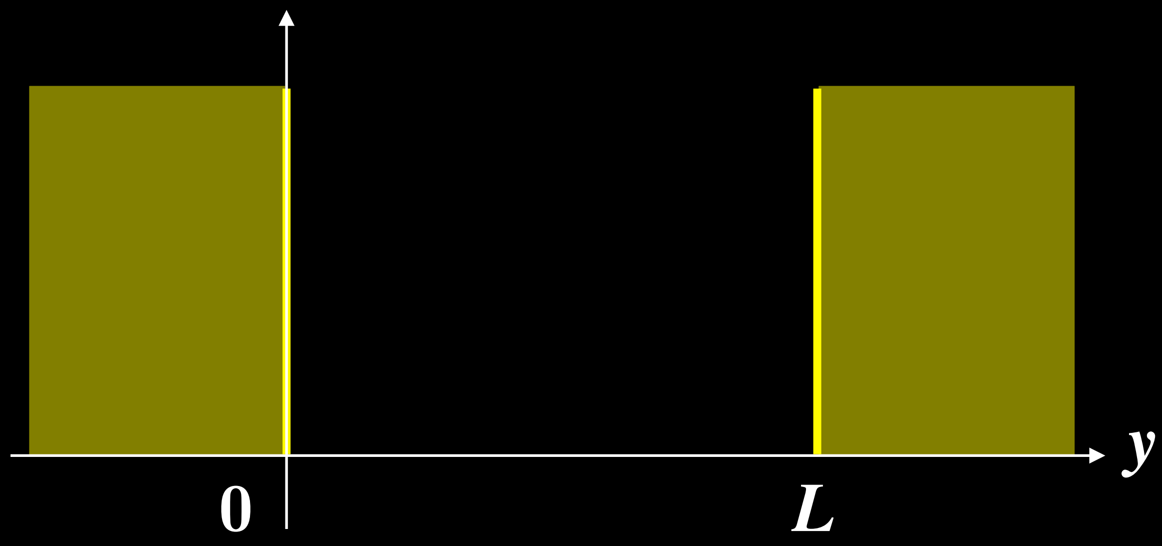


Point interactions

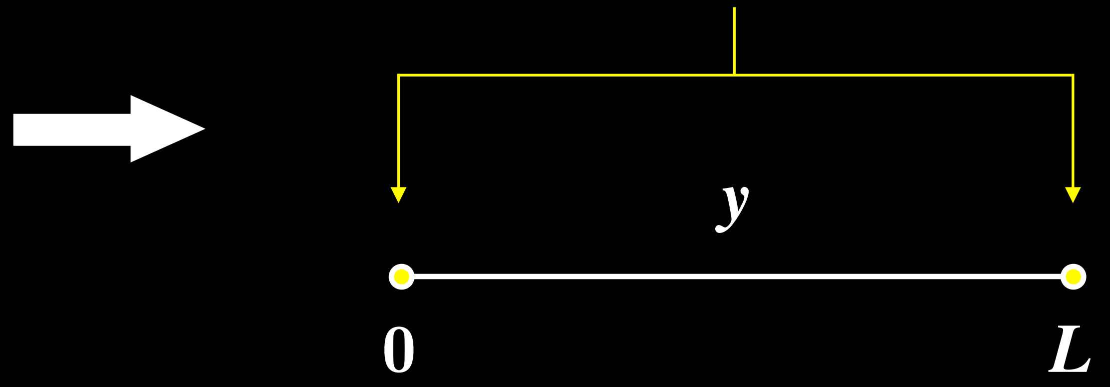
❒ **Delta function potential**



❒ **Infinite square well**

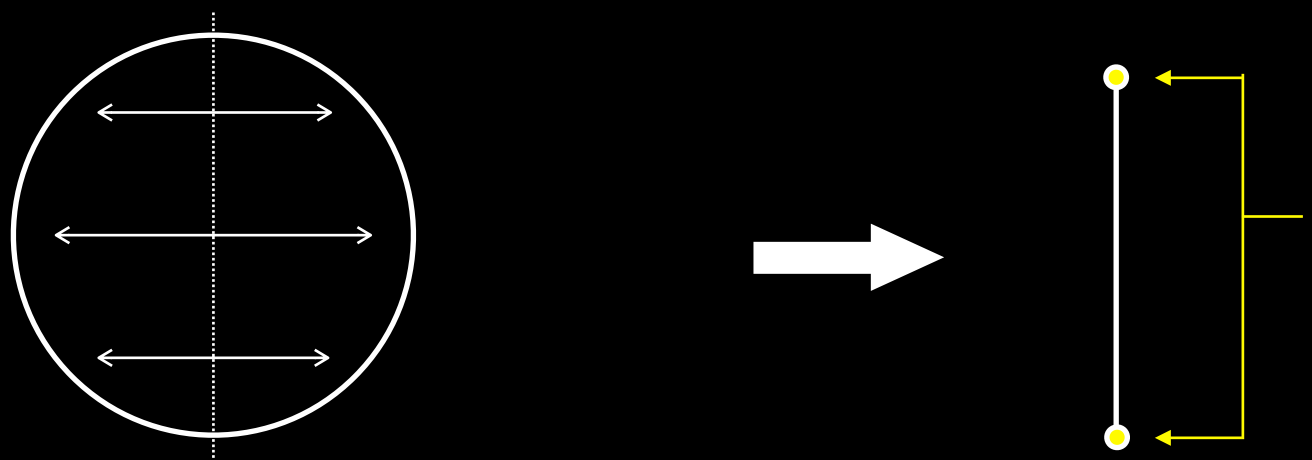


Infinite well can be recognize as point interactions.



Point interactions

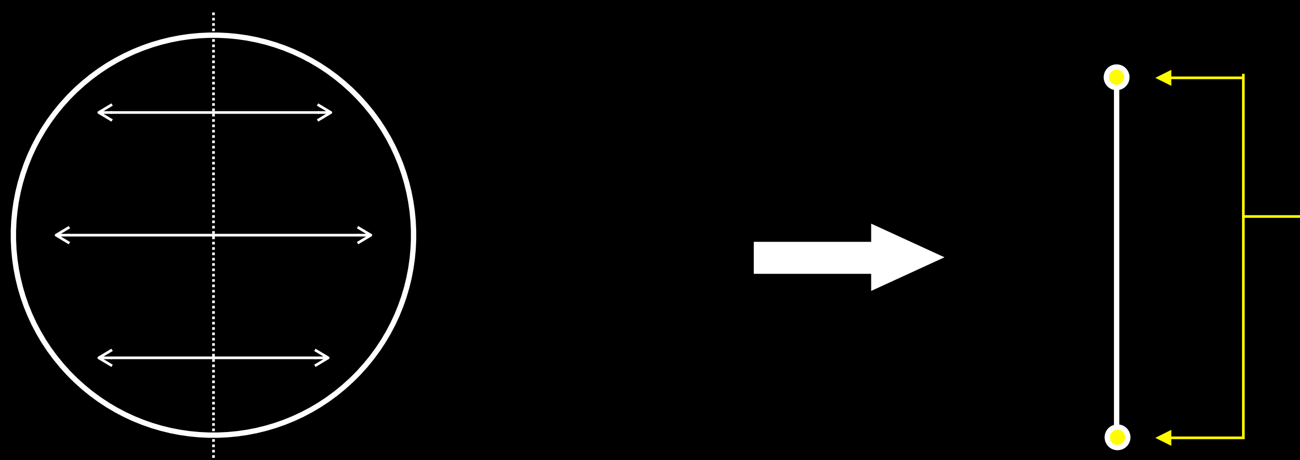
❒ Orbifold fixed point



Fixed points can be recognized as point intearctions.

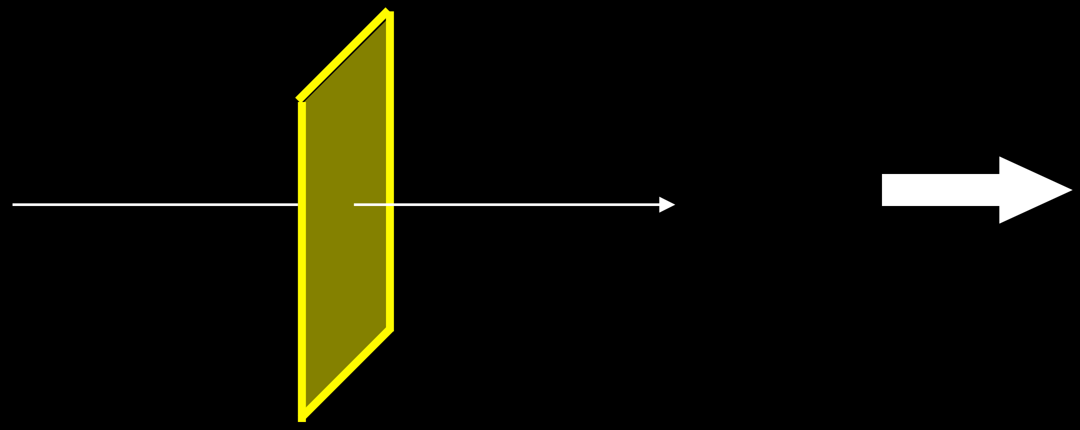
Point interactions

❒ **Orbifold fixed point**



Fixed points can be recognized as point intearctions.

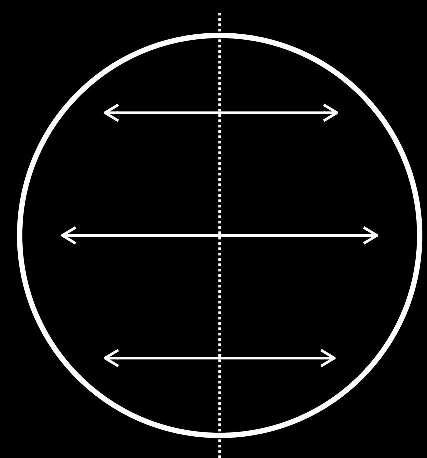
❒ **Zero-thick brane**

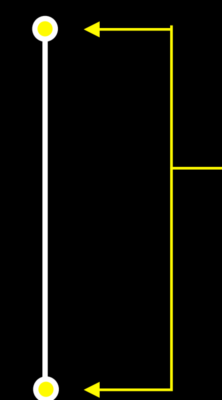


Zero-thick brane can be recognized as point interactions in field theories.

Point interactions

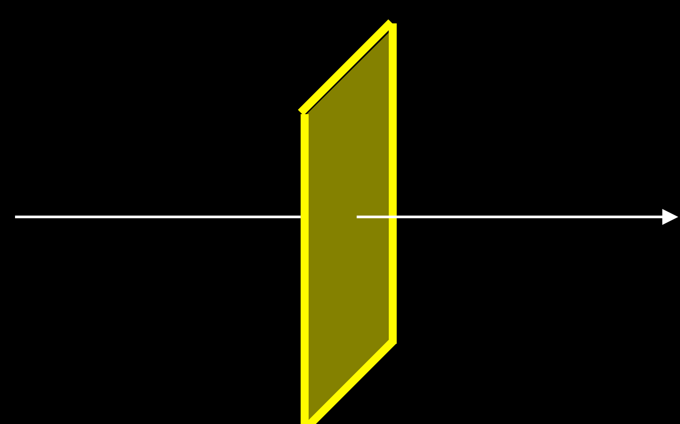
❒ Orbifold fixed point





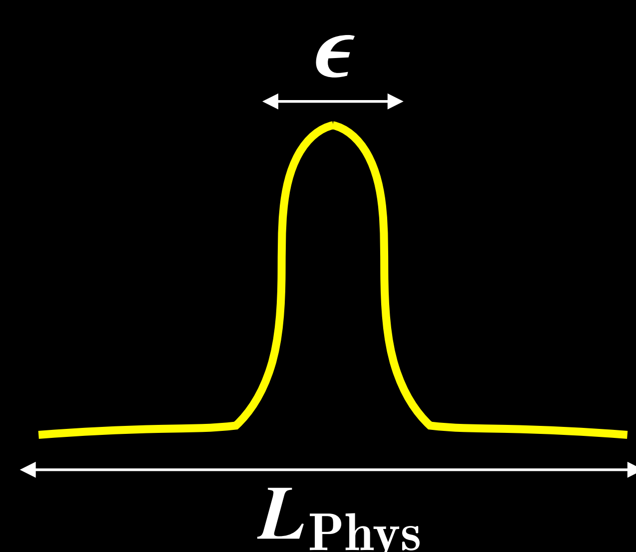
Fixed points can be recognized as point intearctions.

❒ Zero-thick brane

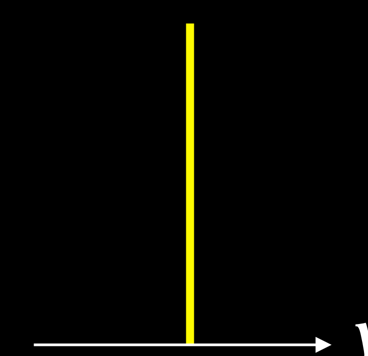


Zero-thick brane can be recognized as point interactions in field theories.

❒ Effective theory



$\epsilon \ll L_{\mathrm{Phys}}$



Point interactions

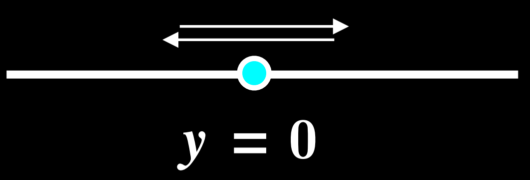
- Point interaction is described by BC's.

Point interactions

❒ Point interaction is described by BC's.

- **Conservation of the probability current**

$$\boxed{j(0+) = j(0-)}$$

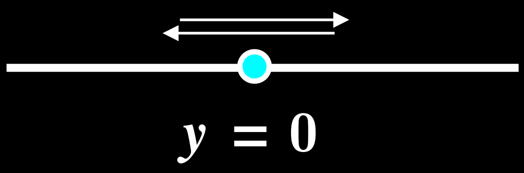


Point interactions

❒ Point interaction is described by BC's.

- Conservation of the probability current

$$\boxed{j(0+) = j(0-)}$$



$$\Updownarrow$$

- General boundary condition for 1d QM

$$\boxed{(U - 1)\begin{pmatrix} \psi(0+) \\ \psi(0-) \end{pmatrix} + iL_0(U + 1)\sigma_3 \begin{pmatrix} \psi'(0+) \\ \psi'(0-) \end{pmatrix} = 0}$$

$$\left(U \in \mathrm{U}(2)\ ,\ \psi' \equiv \frac{d\psi}{dy} \right)$$

[1] M. Reed and B. Simon, Methods of modern mathematical physics II
: Fourier analysis, self-adjointness. 1975.
[2] P. Seva, J. Phys. 36 (1986)667–673.
[3] T. Cheon, T. Fulop, and I. Tsutsui, Annals Phys. 294 (2001) 1–23,

Point interactions

◆ Point interaction

(i) δ-type BC :

Point interactions

◆ Point interaction

(i) δ-type BC : $U = e^{i(\theta+\frac{\pi}{2})} e^{i(\theta-\frac{\pi}{2})\sigma_1}$

Point interactions

◆ Point interaction

(i) δ-type BC ： $U = e^{i(\theta + \frac{\pi}{2})} e^{i(\theta - \frac{\pi}{2})\sigma_1}$

$$(U-1)\begin{pmatrix} \psi(0+) \\ \psi(0-) \end{pmatrix} + iL_0(U+1)\sigma_3 \begin{pmatrix} \psi'(0+) \\ \psi'(0-) \end{pmatrix} = 0$$

Point interactions

◆ Point interaction

(i) δ-type BC ： $U = e^{i(\theta+\frac{\pi}{2})} e^{i(\theta-\frac{\pi}{2})\sigma_1}$

$$(U-1)\begin{pmatrix} \psi(0+) \\ \psi(0-) \end{pmatrix} + iL_0(U+1)\sigma_3 \begin{pmatrix} \psi'(0+) \\ \psi'(0-) \end{pmatrix} = 0$$

$$\Downarrow$$

$$\begin{cases} \psi(0+) = \psi(0-) \\ \psi'(0+) - \psi'(0-) = -\frac{2}{L_0}\tan\theta\ \psi(0+) \end{cases}$$

Point interactions

- **Point interaction**

  (i) δ-type BC

  (ii) Dirichlet BC (Infinite square well) :

Point interactions

- **Point interaction**

  (i) δ-type BC

  (ii) Dirichlet BC (Infinite square well) : $U = -1$

Point interactions

◆ **Point interaction**

(i) δ-type BC

(ii) Dirichlet BC (Infinite square well) : $U=-1$

$$(U-1)\begin{pmatrix}\psi(0+)\\ \psi(0-)\end{pmatrix}+iL_0(U+1)\sigma_3\begin{pmatrix}\psi'(0+)\\ \psi'(0-)\end{pmatrix}=0$$

Point interactions

◆ Point interaction

(i) δ-type BC

(ii) Dirichlet BC (Infinite square well) : $U = -1$

$$(U-1)\begin{pmatrix}\psi(0+)\\ \psi(0-)\end{pmatrix} + iL_0(U+1)\sigma_3\begin{pmatrix}\psi'(0+)\\ \psi'(0-)\end{pmatrix} = 0$$

↓

$$\phi(0+) = 0 = \psi(0-)$$

Point interactions

◆ **Point interaction**

(i) δ-type BC

(ii) Dirichlet BC (Infinite square well)

(iii) Periodic BC ：

Point interactions

◆ Point interaction

(i) δ-type BC

(ii) Dirichlet BC (Infinite square well)

(iii) Periodic BC ： $U = \sigma_1$

Point interactions

◆ Point interaction

(i) δ-type BC

(ii) Dirichlet BC (Infinite square well)

(iii) Periodic BC : $U = \sigma_1$

$$(U-1)\begin{pmatrix} \psi(0+) \\ \psi(0-) \end{pmatrix} + iL_0(U+1)\sigma_3 \begin{pmatrix} \psi'(0+) \\ \psi'(0-) \end{pmatrix} = 0$$

Point interactions

◆ Point interaction

(i) δ-type BC

(ii) Dirichlet BC (Infinite square well)

(iii) Periodic BC : $U = \sigma_1$

$$(U-1)\begin{pmatrix}\psi(0+)\\ \psi(0-)\end{pmatrix} + iL_0(U+1)\sigma_3\begin{pmatrix}\psi'(0+)\\ \psi'(0-)\end{pmatrix} = 0$$

$$\Downarrow$$

$$\begin{cases}\psi(0+) = \psi(0-)\\ \psi'(0+) = \psi'(0-)\end{cases}$$

Point interactions

- **Point interaction**

  (i) δ-type BC

  (ii) Dirichlet BC (Infinite square well)

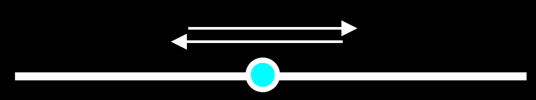
  (iii) Periodic BC

  (vi) Anti-periodic BC

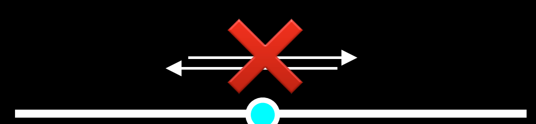
  ⋮

Point interactions

- Point interaction is described by BC's.
- The low energy effective theory (zero mode) is sensitive to the BC's.



4 parameters (scalar)



1 parameters (scalar)

the Robin BC
$$\Phi(y_i) + r_i \partial_y \Phi(y_i) = 0$$

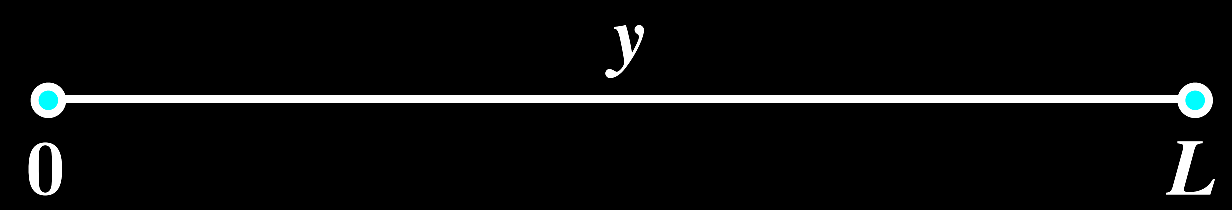
No parameter (spinor, gauge)

$$\mathcal{D} f_n(y_i) = g_n(y_i) = 0$$
or
$$\mathcal{D}^\dagger g_n(y_i) = f_n(y_i) = 0$$

Chiral fermion

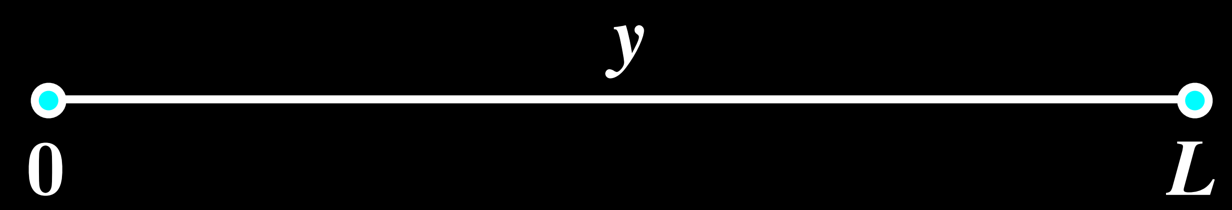
Chiral fermion

- $M^4 \times$ Interval

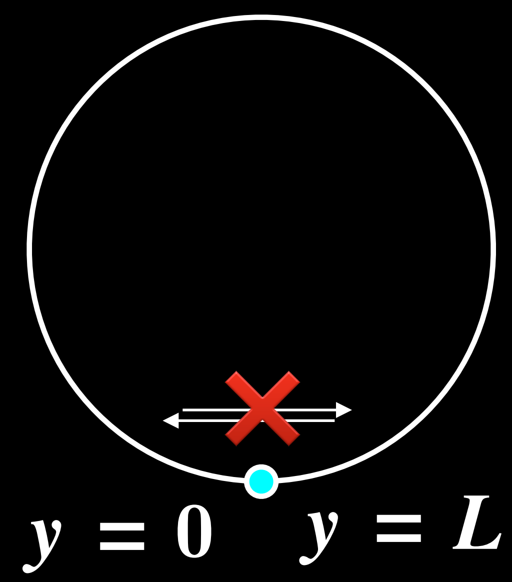


Chiral fermion

- $M^4 \times$ Interval

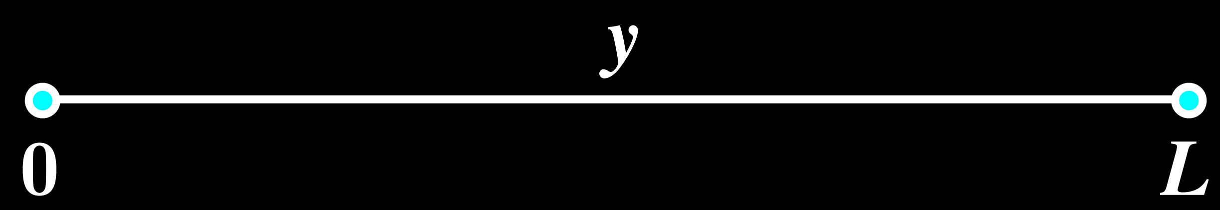


- $M^4 \times S^1$ with point interaction



Chiral fermion

- $M^4 \times$ Interval

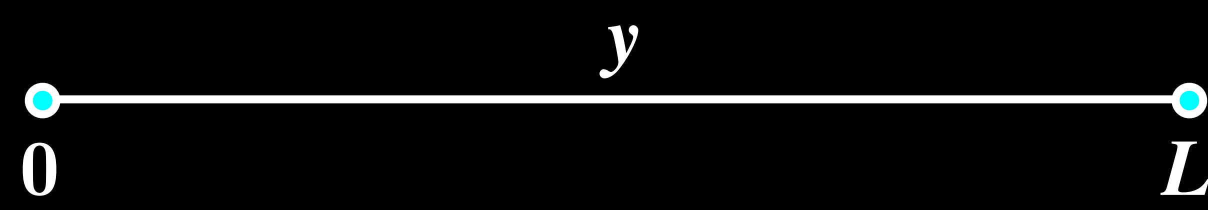


★ **Action**

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

Chiral fermion

❒ $M^4 \times$ Interval



★ **Action**

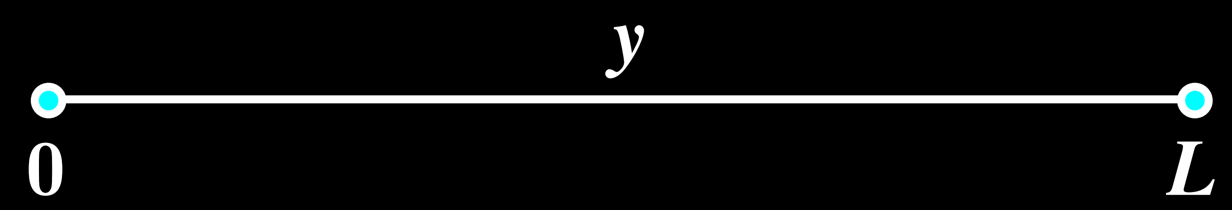
$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

★ **Boundary conditions (BC's)** $(\Psi = \Psi_R + \Psi_L)$

$$\Psi_R(x,y) = 0 \quad \text{or} \quad \Psi_L(x,y) = 0 \qquad @ \quad y = 0, L$$

Chiral fermion

❒ $M^4 \times$ Interval



★ Action

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

★ Boundary conditions (BC's) $(\Psi = \Psi_R + \Psi_L)$

$\Psi_R(x,y) = 0$ or $\Psi_L(x,y) = 0$ @ $y = 0, L$

!

BC's are obtained from the action principle, etc.

$\delta S = 0$ ⟶ (E.O.M) + (Surface term $\bar{\Psi}_R \Psi_L = 0$)

Chiral fermion

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

Chiral fermion

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

$$\Psi(x,y) = \sum_n \Big(\overbrace{\psi_R^{(n)}(x) f_n(y) + \psi_L^{(n)}}^{\text{4d mass eigenstates}}(x) g_n(y) \Big)$$

Mode functions (Complete sets)

Chiral fermion

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

$$\Psi(x,y) = \sum_n \left(\psi_R^{(n)}(x) f_n(y) + \psi_L^{(n)}(x) g_n(y) \right)$$

$$\begin{cases} \mathcal{D}^\dagger \mathcal{D} f_n(y) = m_n^2 f_n(y) \\ \mathcal{D}\mathcal{D}^\dagger g_n(y) = m_n^2 g_n(y) \end{cases}$$

$\oplus$ **boundary conditions**

$$\begin{cases} \mathcal{D} \equiv \partial_y + M_F \\ \mathcal{D}^\dagger \equiv -\partial_y + M_F \end{cases}$$

Chiral fermion

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

$$\Psi(x,y) = \sum_n \left(\psi_R^{(n)}(x) f_n(y) + \psi_L^{(n)}(x) g_n(y) \right)$$

$$\begin{cases} \mathcal{D}^\dagger \mathcal{D} f_n(y) = m_n^2 f_n(y) \\ \mathcal{D}\mathcal{D}^\dagger g_n(y) = m_n^2 g_n(y) \end{cases} \oplus \textbf{ boundary conditions}$$

$$\begin{cases} \mathcal{D} \equiv \partial_y + M_F \\ \mathcal{D}^\dagger \equiv -\partial_y + M_F \end{cases}$$

$$\begin{cases} \mathcal{D}^\dagger g_n(y) = m_n f_n(y) \\ \mathcal{D} f_n(y) = m_n g_n(y) \end{cases}$$

QM SUSY relation

Chiral fermion

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$

$$\Psi(x,y) = \sum_n \left(\psi_R^{(n)}(x) f_n(y) + \psi_L^{(n)}(x) g_n(y) \right)$$

$$\begin{cases} \mathcal{D}^\dagger \mathcal{D} f_n(y) = m_n^2\, f_n(y) \\ \mathcal{D}\mathcal{D}^\dagger g_n(y) = m_n^2\, g_n(y) \end{cases} \oplus \text{ boundary conditions}$$

$$\begin{cases} \mathcal{D} \equiv \partial_y + M_F \\ \mathcal{D}^\dagger \equiv -\partial_y + M_F \end{cases}$$

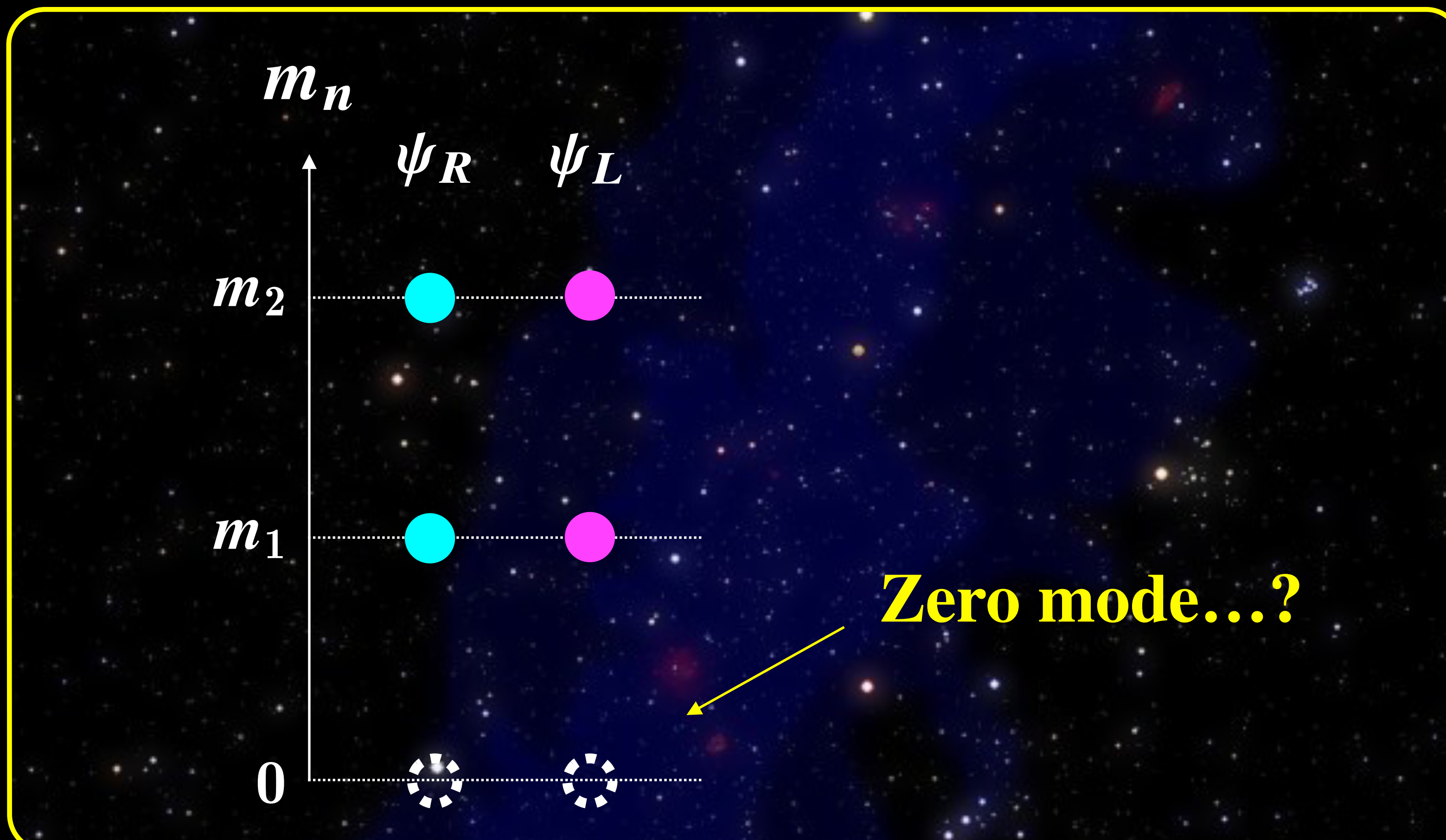
$$\begin{cases} \mathcal{D}^\dagger g_n(y) = m_n\, f_n(y) \\ \mathcal{D} f_n(y) = m_n\, g_n(y) \end{cases}$$

QM SUSY relation

$$= \int d^4x \left((\text{massless zero mode}) + \sum_{n=1}^{\infty} \overline{\psi^{(n)}}(x)(i\gamma^\mu \partial_\mu + m_n)\psi^{(n)}(x) \right)$$

Chiral fermion

$$S = \int d^4x \int_0^L dy\, \overline{\Psi}(x,y)(i\gamma^\mu \partial_\mu + i\gamma^y \partial_y + M_F)\Psi(x,y)$$



$$= \int d^4x \left(\text{(massless zero mode)} + \sum_{n=1}^{\infty} \overline{\psi^{(n)}}(x)(i\gamma^\mu \partial_\mu + m_n)\psi^{(n)}(x) \right)$$

Chiral fermion

❒ Zero mode solution

$$\begin{cases} \mathcal{D}f_0(y) = m_0 g_0(y) = 0 \\ \mathcal{D}^\dagger g_0(y) = m_0 f_0(y) = 0 \end{cases}$$

QM SUSY relation

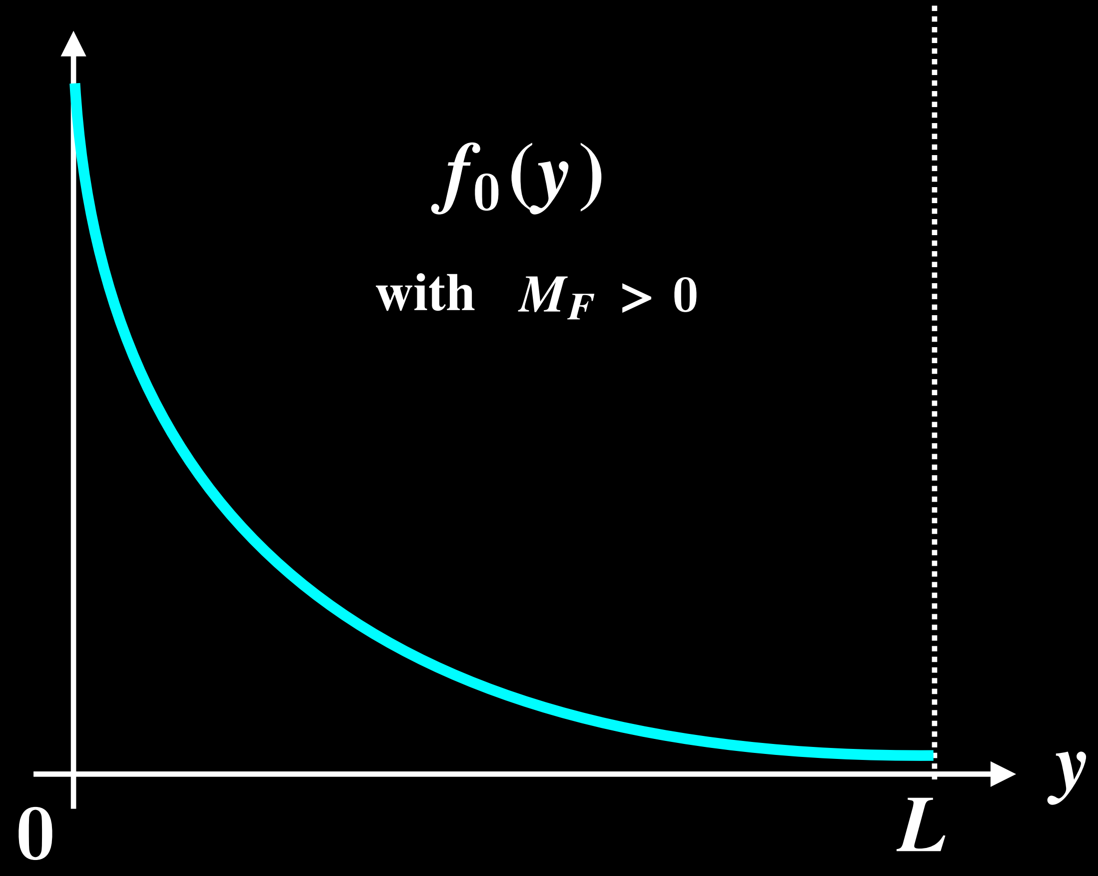
$$\left(\mathcal{D} \equiv \partial_y + M_F \,,\, \mathcal{D}^\dagger \equiv -\partial_y + M_F \right)$$

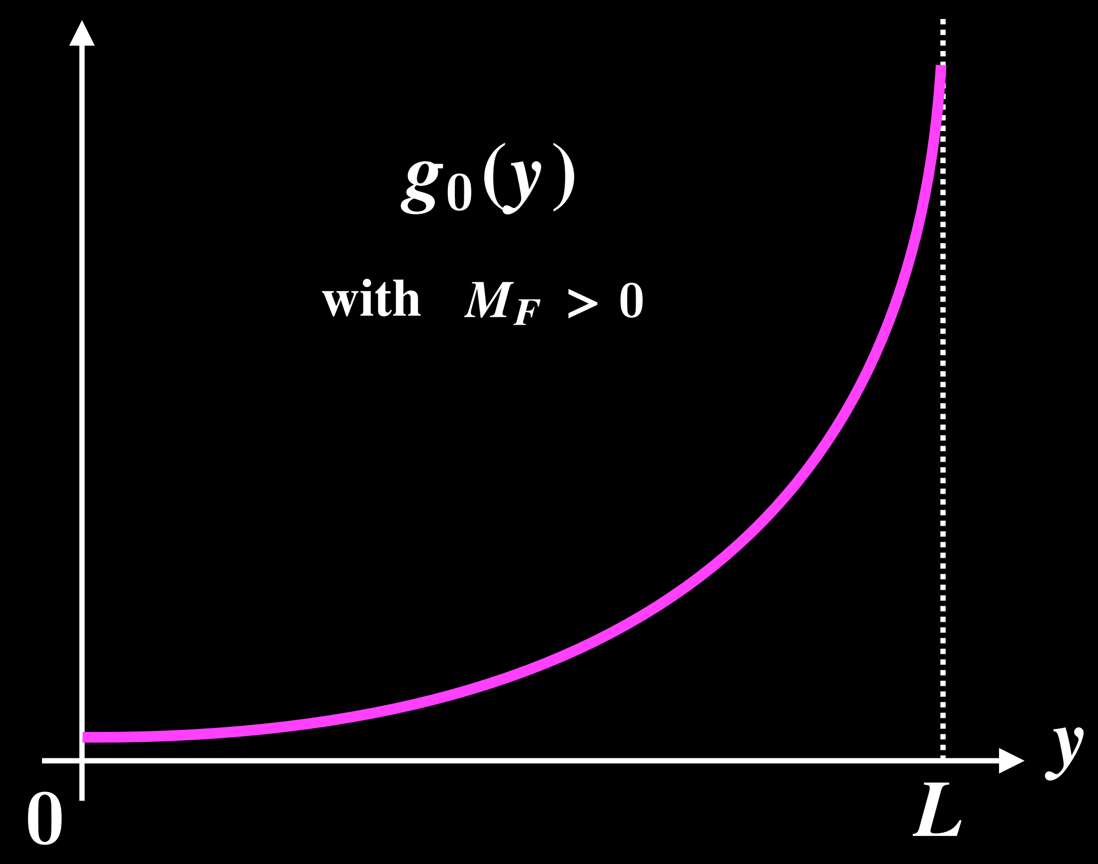
Chiral fermion

❒ Zero mode solution

$$\begin{cases} \mathcal{D} f_0(y) = m_0 g_0(y) = 0 \\ \mathcal{D}^\dagger g_0(y) = m_0 f_0(y) = 0 \end{cases} \Rightarrow \begin{cases} f_0(y) \propto e^{-M_F y} \\ g_0(y) \propto e^{+M_F y} \end{cases}$$

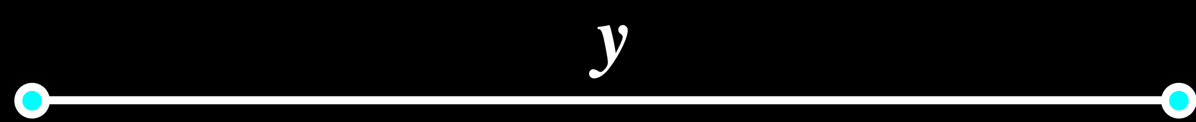
$$\left(\mathcal{D} \equiv \partial_y + M_F \, , \, \mathcal{D}^\dagger \equiv -\partial_y + M_F \right)$$





Chiral fermion

❒ Spectrum

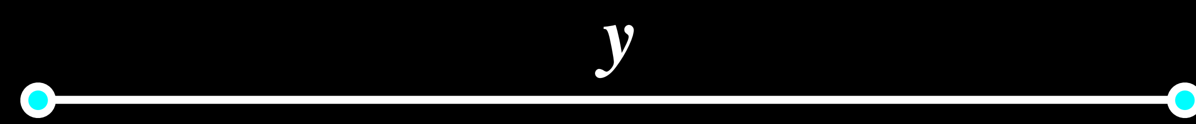


$$\Psi_R(x,0)=0$$
or
$$\Psi_L(x,0)=0$$

$$\Psi_R(x,L)=0$$
or
$$\Psi_L(x,L)=0$$

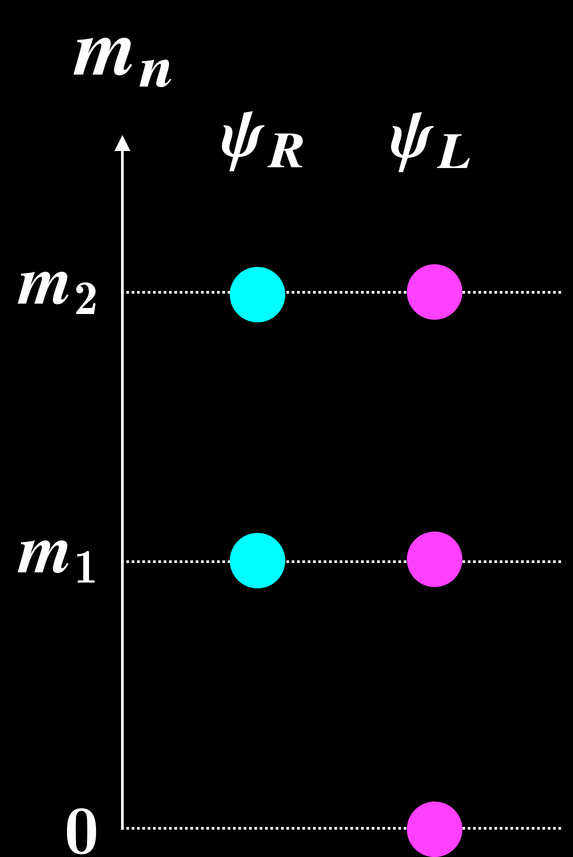
Chiral fermion

❐ Spectrum



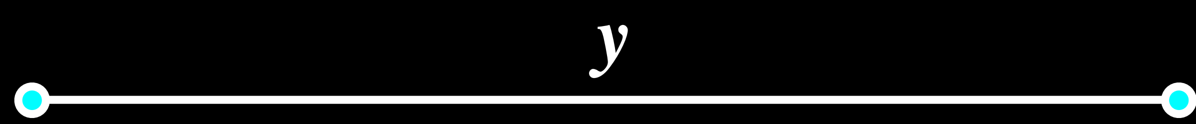
$$\begin{array}{ccc} \Psi_R(x,0)=0 & & \Psi_R(x,L)=0 \\ \text{or} & \xleftrightarrow{\text{(i)}} & \text{or} \\ \Psi_L(x,0)=0 & & \Psi_L(x,L)=0 \end{array}$$

type (i)



Chiral fermion

❒ Spectrum

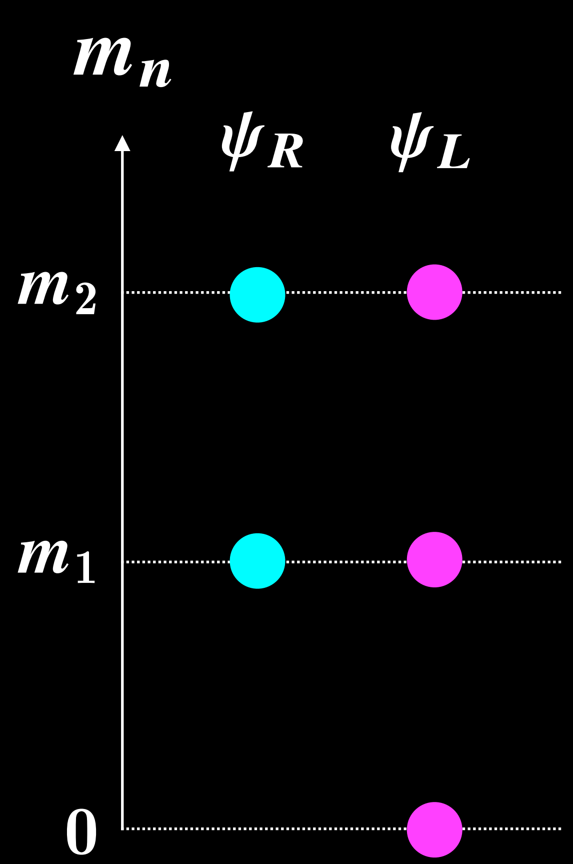


$$\Psi_R(x,0)=0 \quad \xleftrightarrow{\text{(i)}} \quad \Psi_R(x,L)=0$$

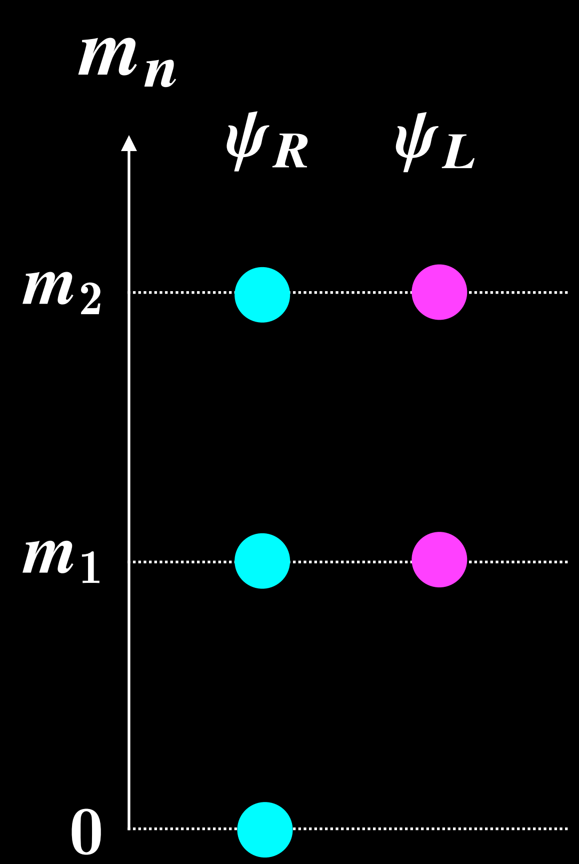
or

$$\Psi_L(x,0)=0 \quad \xleftrightarrow[\text{(ii)}]{} \quad \Psi_L(x,L)=0$$

type (i)



type (ii)

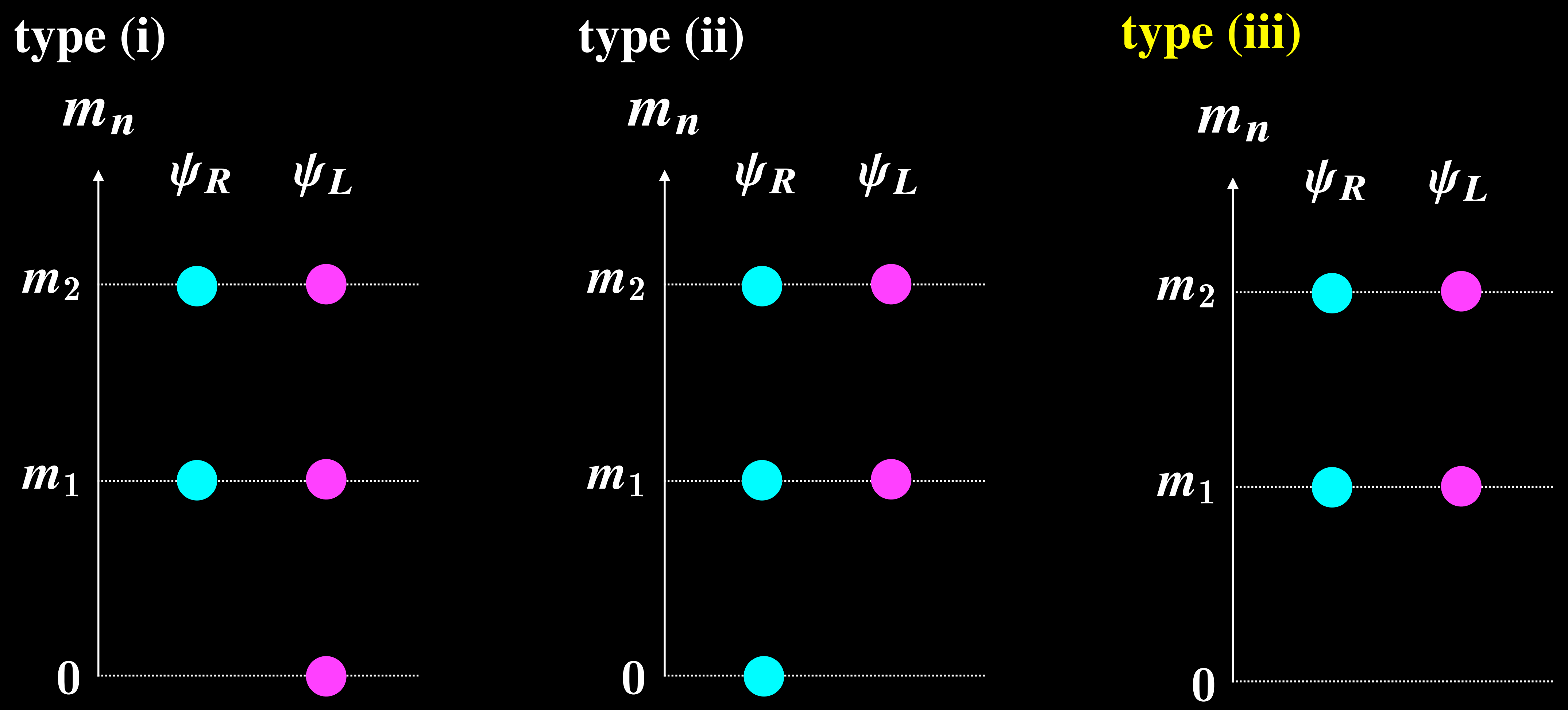


Chiral fermion

❒ Spectrum

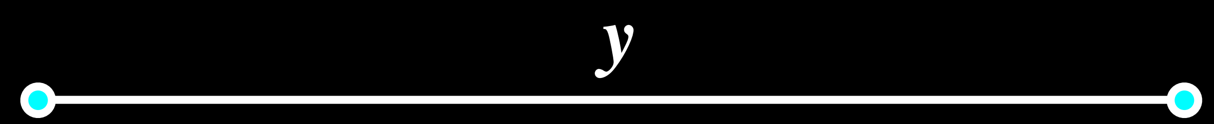
y

$\Psi_R(x,0) = 0$	(i)	$\Psi_R(x,L) = 0$
or	(iii)	or
$\Psi_L(x,0) = 0$	(ii)	$\Psi_L(x,L) = 0$



Chiral fermion

❒ Spectrum

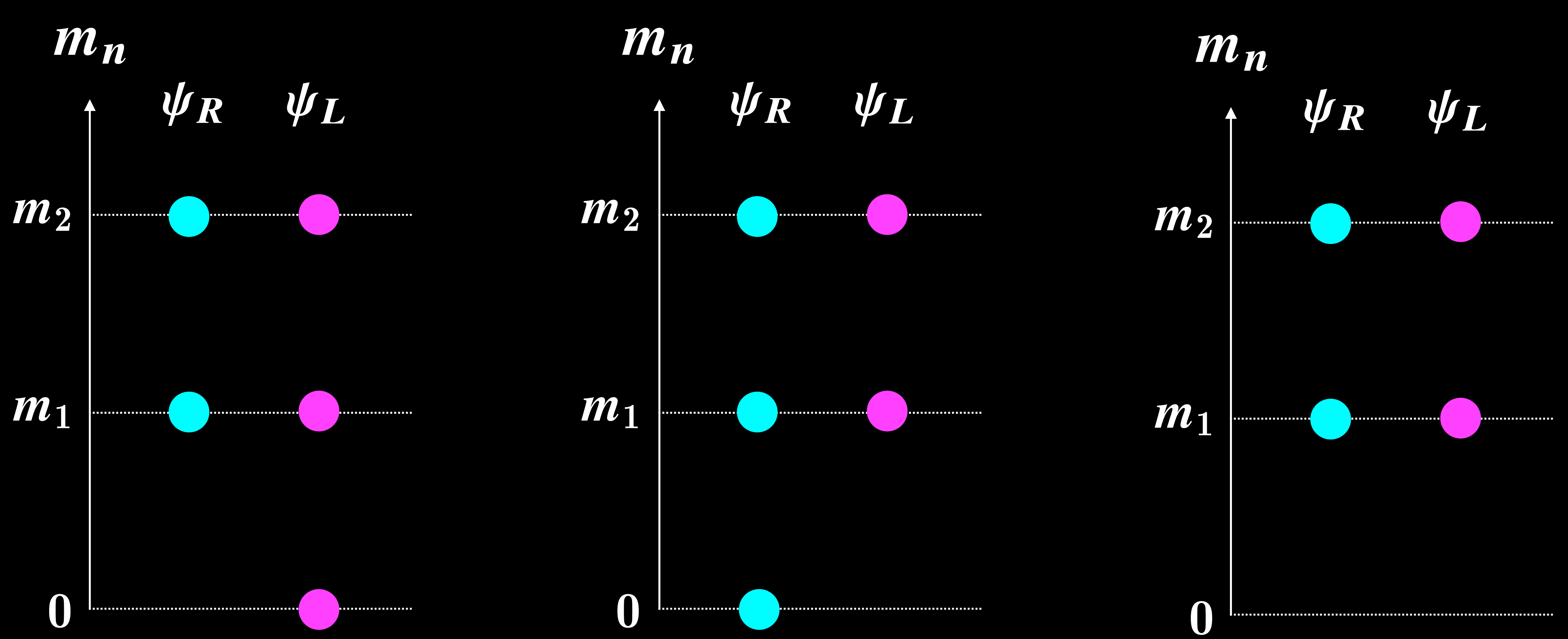


$$\Psi_R(x,0)=0 \quad \text{or} \quad \Psi_L(x,0)=0$$

(i) (iii) (ii)

$$\Psi_R(x,L)=0 \quad \text{or} \quad \Psi_L(x,L)=0$$

type (i) type (ii) type (iii)



Chiral fermion

❒ Spectrum

y

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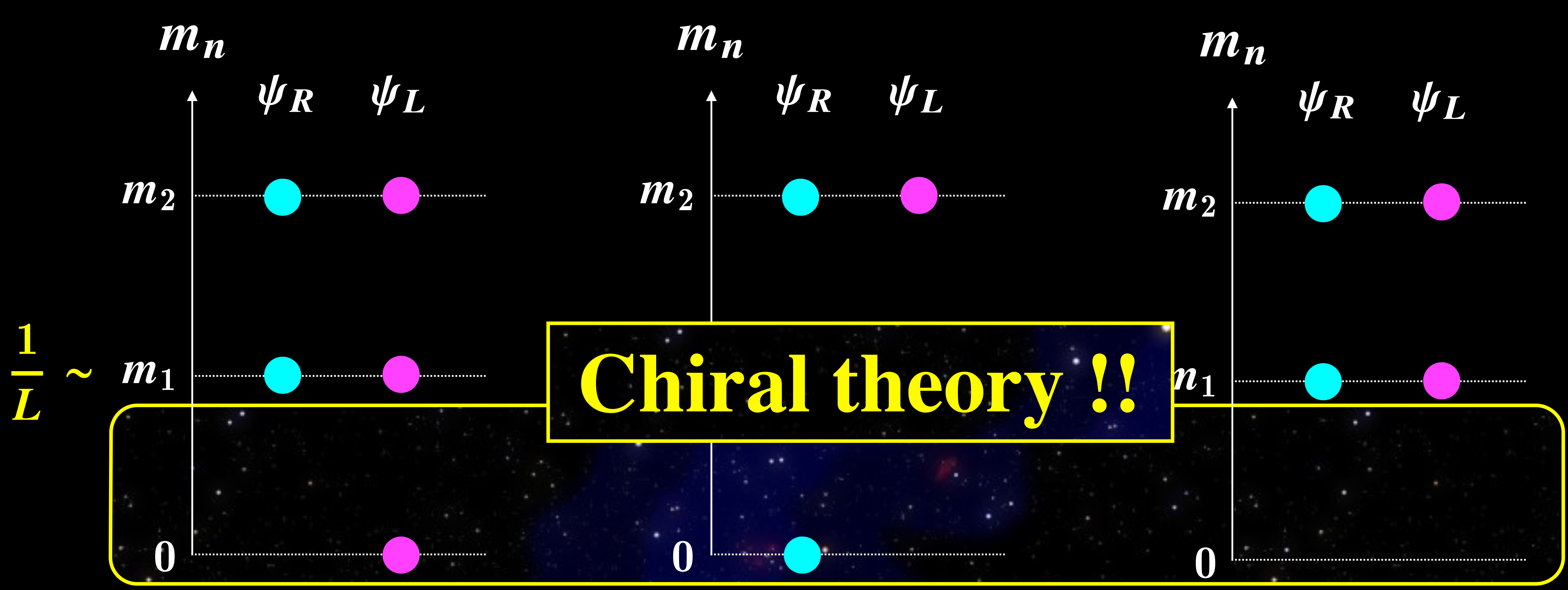
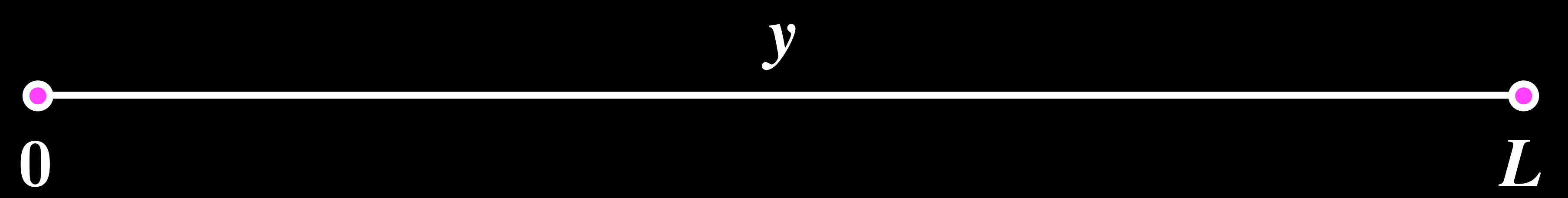
type (i) type (ii) type (iii)



Mass hierarchy

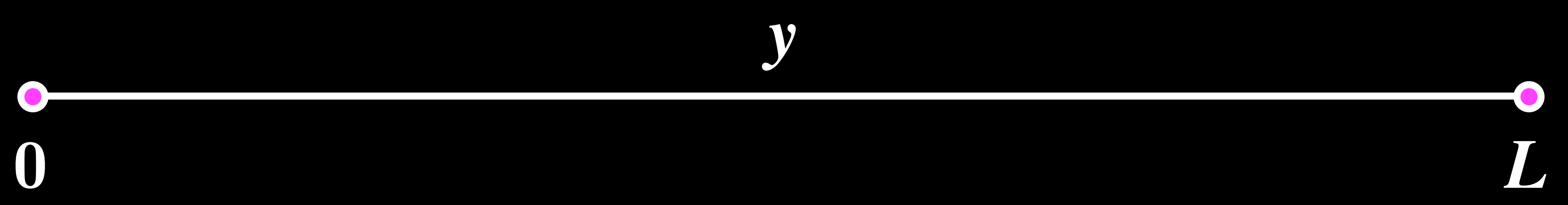
- The Robin BC can produce a y-dependent VEV



$$S = \int d^4x \int_0^L dy\ \Phi^\dagger(\partial^\mu \partial_\mu + \partial_y^2 - M^2)\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2$$

Mass hierarchy

❒ **The Robin BC can produce a y-dependent VEV**



$$S = \int d^4x \int_0^L dy\, \Phi^\dagger(\partial^\mu \partial_\mu + \partial_y^2 - M^2)\Phi - \frac{\lambda}{2}(\Phi^\dagger \Phi)^2$$

+ Robin boundary condition

$$\begin{cases} \Phi(0) + L_+ \partial_y \Phi(0) = 0 \\ \Phi(L) - L_- \partial_y \Phi(L) = 0 \end{cases} \quad (-\infty \leq L_\pm \leq +\infty)$$

Mass hierarchy

- **VEV of the scalar**

$$V_{\rm 4d} = \int_0^L dy \left[\Phi^\dagger(-\partial_y^2 + M^2)\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 \right]$$

Mass hierarchy

- **VEV of the scalar**

$$V_{4\mathrm{d}} = \int_0^L dy \left[\Phi^\dagger (-\partial_y^2 + M^2)\Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 \right]$$

$$\downarrow$$

Find a solution to $\delta V_{4\mathrm{d}} = 0$:

$$(-\partial_y^2 + M^2)\Phi(y) + \lambda |\Phi(y)|^2 \Phi(y) = 0$$

Mass hierarchy

- **VEV of the scalar**

$$V_{4\mathrm{d}} = \int_0^L dy \left[\Phi^\dagger(-\partial_y^2 + M^2)\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 \right]$$

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$$\begin{cases} \Phi(0) + L_+\partial_y\Phi(0) = 0 \\ \Phi(L) - L_-\partial_y\Phi(L) = 0 \end{cases} \qquad (-\infty \leq L_\pm \leq +\infty)$$

Mass hierarchy

❒ VEV of the scalar

(type-i)

$$\phi(y) = \mu_- \frac{\mathrm{sn}(\mu_+\sqrt{\frac{\lambda}{2}}(y-y_0),k)}{\mathrm{cn}(\mu_+\sqrt{\frac{\lambda}{2}}(y-y_0),k)} \quad , \qquad \mu_\pm \equiv \frac{M^2}{\lambda}(1 \pm \sqrt{1-\frac{4\lambda Q}{M^4}}) \qquad k^2 \equiv \frac{\mu_+^2-\mu_-^2}{\mu_+^2}$$

Mass hierarchy

❒ VEV of the scalar

Parameters which are determined by BC's

(type-i)

$$\phi(y) = \mu_- \frac{\mathrm{sn}(\mu_+\sqrt{\frac{\lambda}{2}}(y-y_0),k)}{\mathrm{cn}(\mu_+\sqrt{\frac{\lambda}{2}}(y-y_0),k)} \quad , \qquad \mu_\pm \equiv \frac{M^2}{\lambda}\left(1 \pm \sqrt{1-\frac{4\lambda Q}{M^4}}\right)$$

$$k^2 \equiv \frac{\mu_+^2 - \mu_-^2}{\mu_+^2}$$

Mass hierarchy

❐ VEV of the scalar

Parameters which are determined by BC's

(type-i)

$$\phi(y) = \mu_- \frac{\mathrm{sn}(\mu_+\sqrt{\frac{\lambda}{2}}(y-y_0),k)}{\mathrm{cn}(\mu_+\sqrt{\frac{\lambda}{2}}(y-y_0),k)} \quad , \qquad \mu_\pm \equiv \frac{M^2}{\lambda}(1 \pm \sqrt{1-\frac{4\lambda Q}{M^4}}), \quad k^2 \equiv \frac{\mu_+^2-\mu_-^2}{\mu_+^2}$$

(type-ii)

$$\phi(y) = \frac{\nu}{\mathrm{cn}(\sqrt{\frac{\lambda}{2}}\frac{\mu}{k}(y-y_0),k)} \quad , \qquad \mu \equiv \frac{M^2}{\lambda}(1+\sqrt{1+\frac{4\lambda|Q|}{M^4}}), \quad \nu \equiv \frac{M^2}{\lambda}(\sqrt{1+\frac{4\lambda|Q|}{M^4}}-1), \quad k^2 \equiv \frac{\mu^2}{\mu^2+\nu^2}$$

CP phase

CP phase

❒ VEV of the Higgs

★ **Twisted BC**

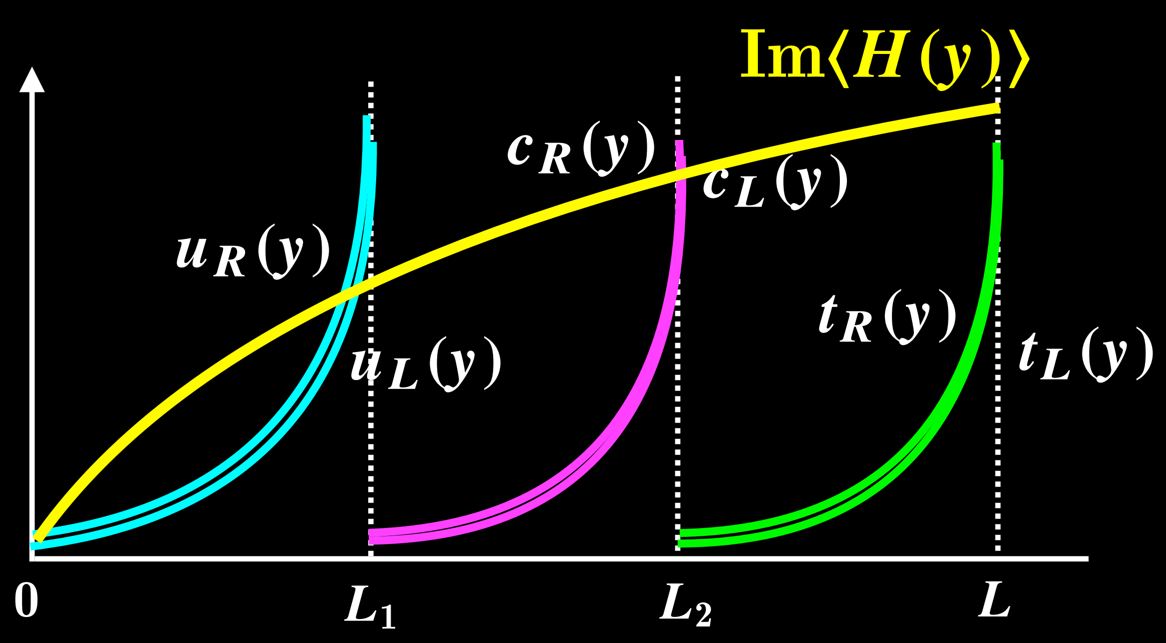
$$\langle H(y)\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} e^{i\frac{\pi}{L}y}$$

CP phase

❒ VEV of the Higgs

★ Twisted BC

$$\langle H(y) \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} e^{i\frac{\pi}{L}y}$$

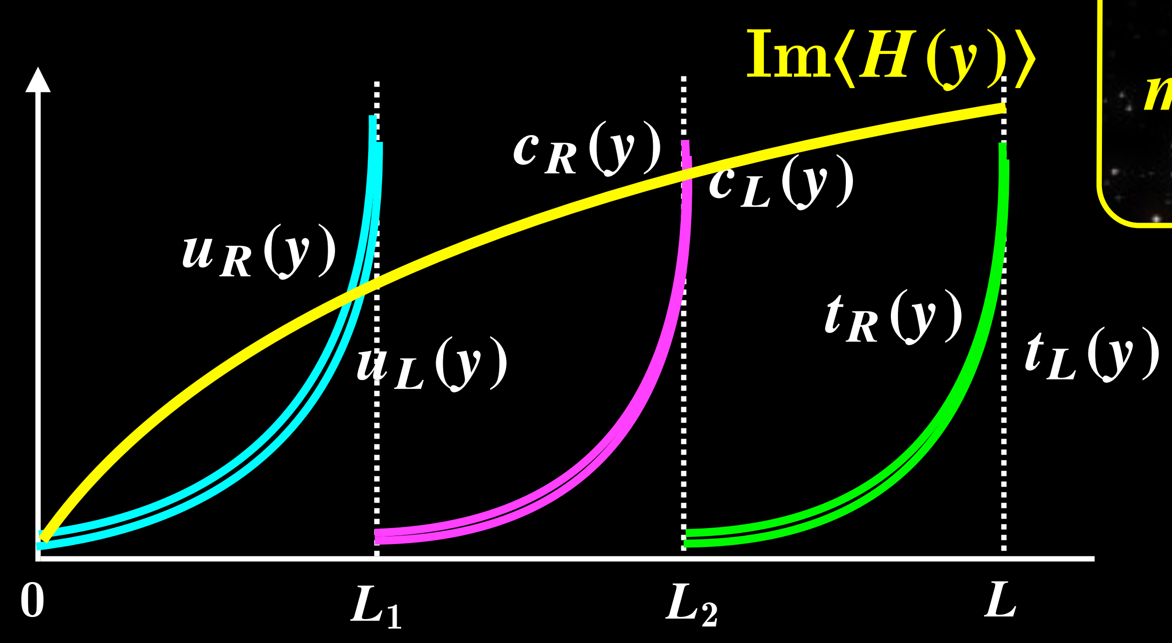


CP phase

❒ VEV of the Higgs

★ Twisted BC

$$\langle H(y)\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} e^{i\frac{\pi}{L}y}$$

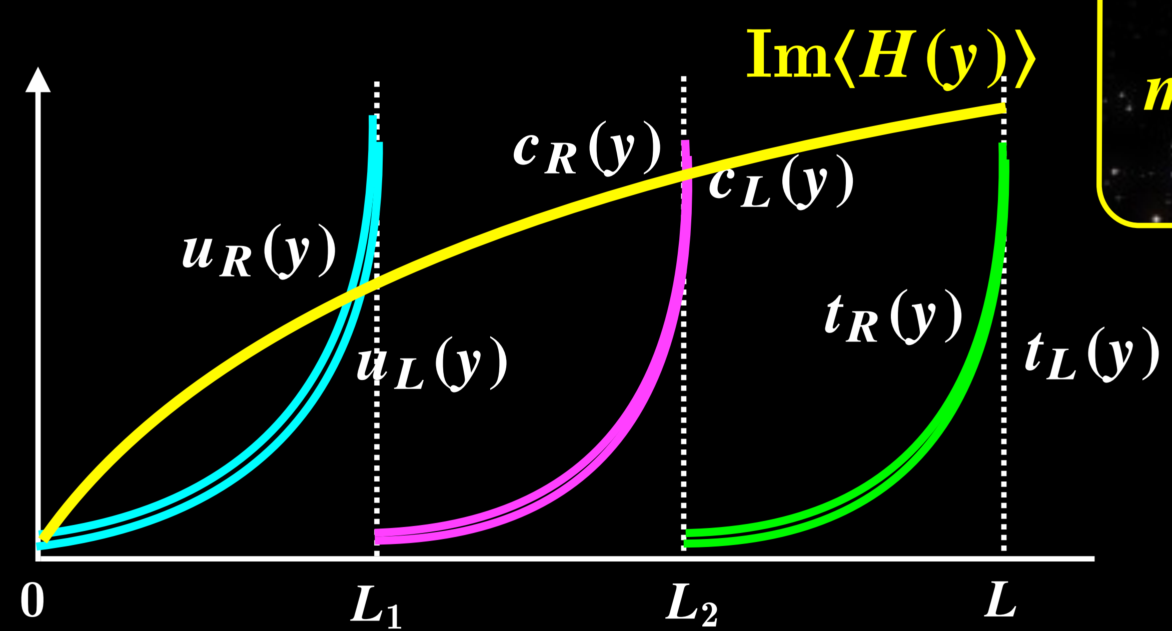


$$m_{ij}^{(u)} = \lambda \int_0^L dy\, \langle\Phi(y)\rangle\langle H(y)\rangle q_L^{(i)}(y) q_R^{(j)}(y)$$

CP phase

- **VEV of the Higgs**
  - ★ Twisted BC

$$\langle H(y)\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} e^{i\frac{\pi}{L}y}$$



$$\boxed{m_{ij}^{(u)} = \lambda \int_0^L dy\, \langle \Phi(y)\rangle \langle H(y)\rangle q_L^{(i)}(y) q_R^{(j)}(y)}$$

↓

CP phase !!