ASTROPARTICLES
Astroparticles and High Energy Physics Group

The Flavour Symmetry Q_6 in a $SU(5)$ GUT Model

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IFIC
CSIC-Unv. de Valencia

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This work is in collaboration with:

J. C. Gómez-Izquierdo

M. Mondragón

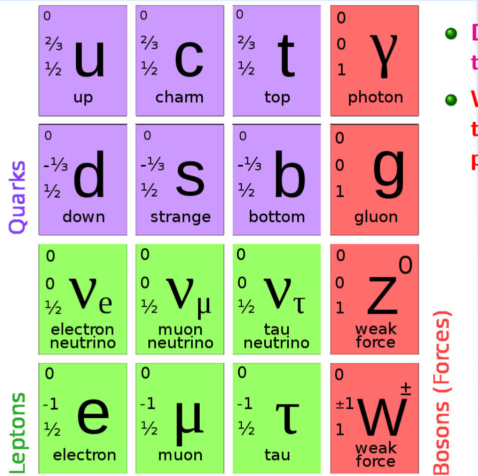
Instituto de Física
UNAM

outline

1. Motivation
2. SUSY $SU(5) \otimes U(1) \otimes Q_6$ model
   - The symmetry $\mathbf{Q_6}$
   - Matter content
   - Mass matrices
   - The mixing matrices
3. Conclusions

Why do we have to go beyond the standard model?

Periodic table of Particle Physics



- **Does not exist a theoretical principle that fix the number of particles.**
- **We need three families for reproduce the experimental data: replication problem.**

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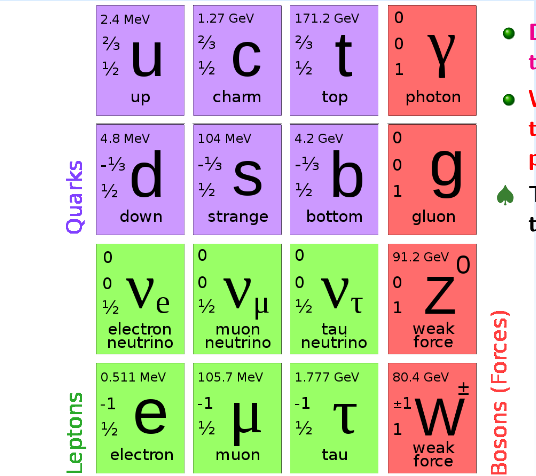

- **Does not exist a theoretical principle that fix the number of particles.**
- **We need three families for reproduce the experimental data: replication problem.**
- **The experimental evidence points to:**
  - Oscillation of the neutrinos between their flavour states.
    [Phys. Rept. **460**:1-129.]
    - ♯ The neutrinos are not massless particles.
    - ♯ Flavour mixing in leptonic sector.
    - ⋮

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Theoretical evidence

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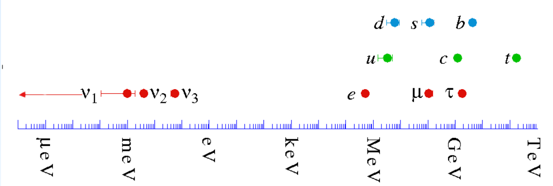

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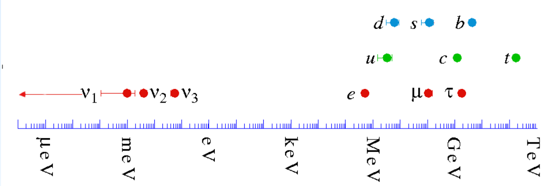
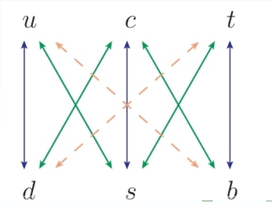
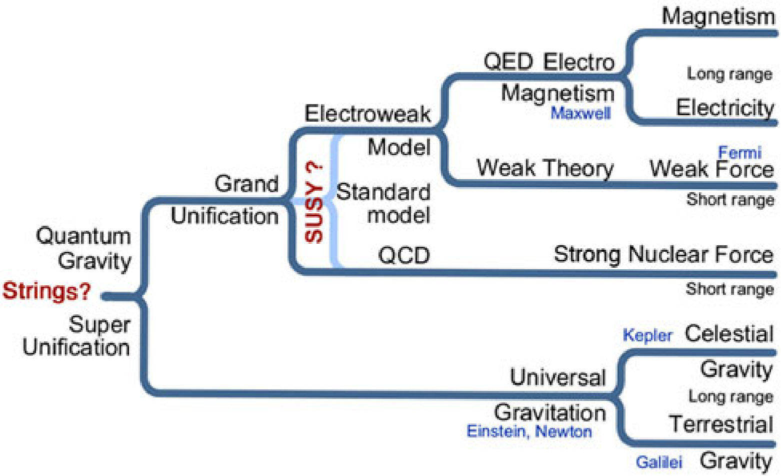


♢ Flavour-changing neutral currents (FCNC) problem: Why do we not observe "horizontal" inter-generation transitions?



But... How far beyond...?



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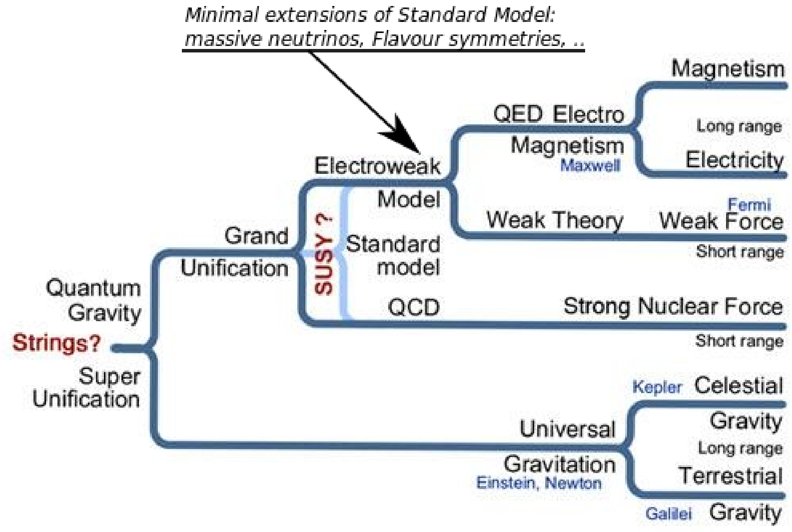

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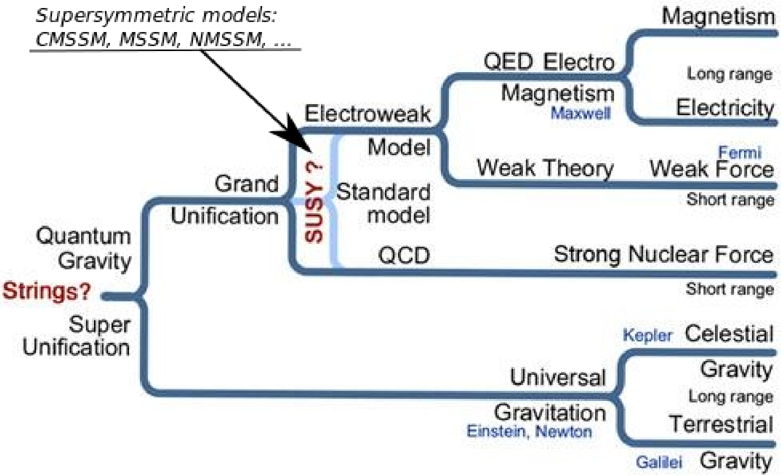
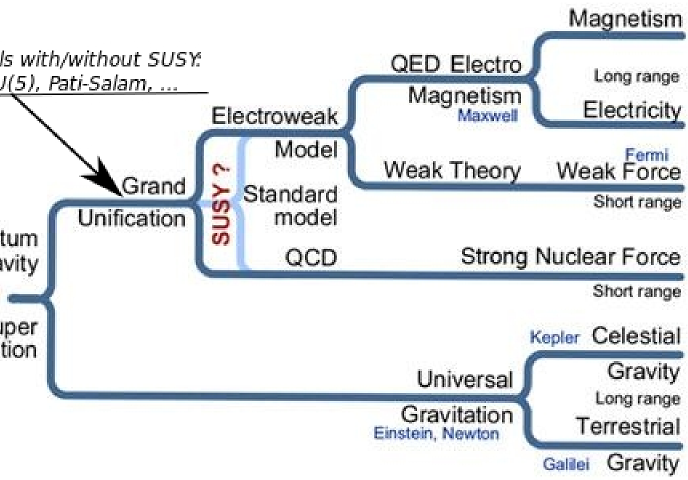
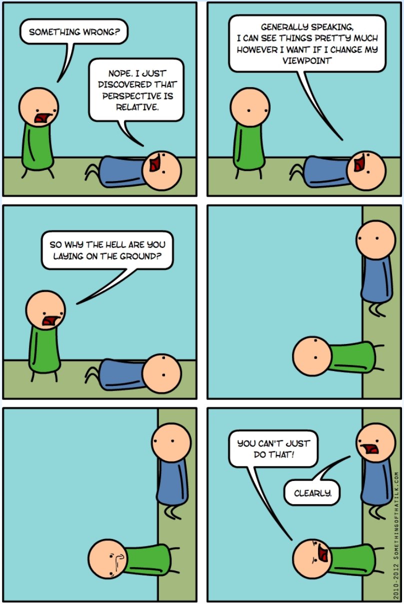


But... How far beyond...?

GUT models with/without SUSY:
SO(10), SU(5), Pati-Salam, ...



"The answer"



- Taking into account the results obtained in the SM$\otimes S_3$ model.

  [See Myriam's talk]

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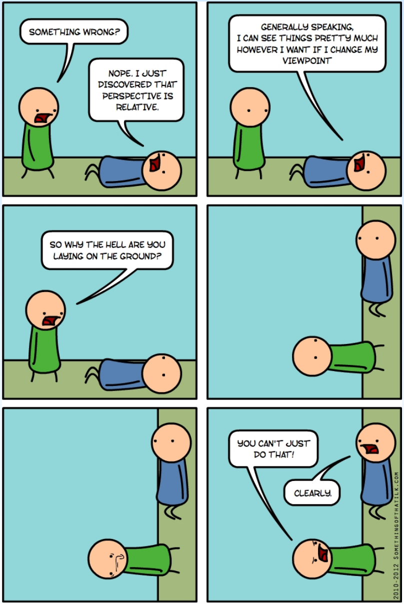

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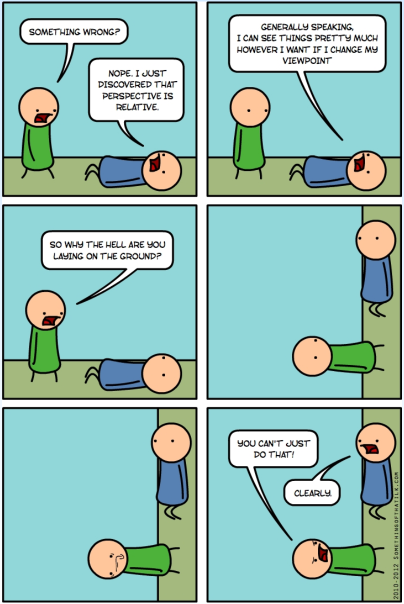

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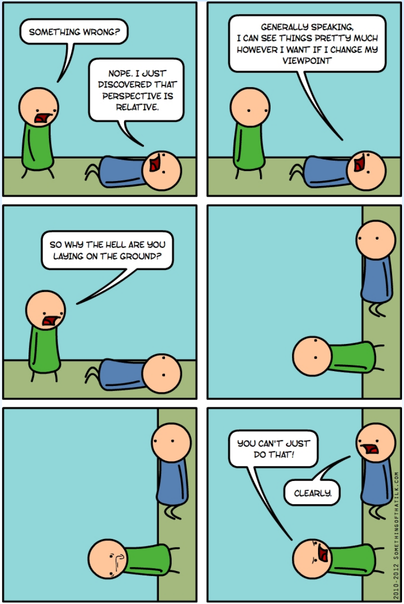

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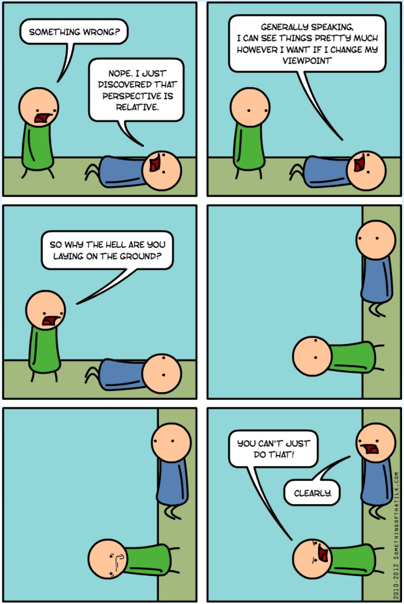

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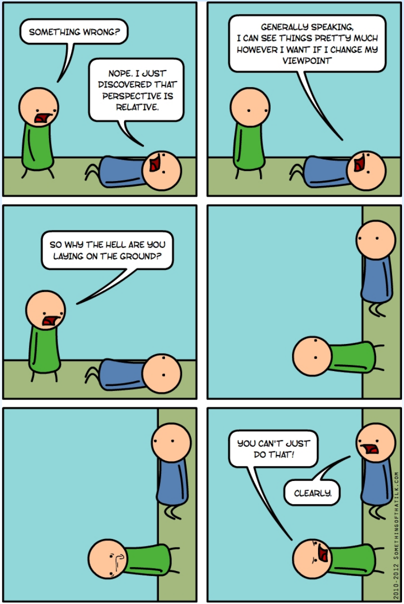

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  - The R-parity is conserved.
  - The Higgs sector is extended.
  - The Q_6 Group is the flavour symmetry.

$SU(5) \otimes U(1) \otimes Q_6$ model

- Why a $SU(5) \otimes U(1)$ gauge group?

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  [Carolina Arbeláez et al. Phys. Rev. D 89 (2014) 055003]

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  [K. S. Babu and Jisuke Kubo. Phys. Rev., D71:056006, 2005; Yuji Kajiyama, et, al. Nucl. Phys., B743:74-103;

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In this theoretical framework, we are mainly interested in showing that masses and flavour mixings of quarks and leptons can be reproduced in good agreement with experimental data.

[J.C. Gómez-Izquierdo, et, al. arXiv:1312.7385]

The binary dihedral group Q_6 has 12 elements:

$$\mathbf{Q}_6 = \{\mathbf{1}, \mathbf{A}, \mathbf{A}^2, \mathbf{A}^3, \mathbf{A}^4, \mathbf{A}^5, \mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \mathbf{A}^3\mathbf{B}, \mathbf{A}^4\mathbf{B}, \mathbf{A}^5\mathbf{B}\},$$

$$\mathbf{A} = \begin{pmatrix} \cos(\pi/3) & \sin(\pi/3) \\ -\sin(\pi/3) & \cos(\pi/3) \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

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The irreducible representations are:

- Four singles $\mathbf{1}_{+,0}$, $\mathbf{1}_{+,2}$, $\mathbf{1}_{-,1}$, $\mathbf{1}_{-,3}$.
- Two doublets $\mathbf{2}_1$, $\mathbf{2}_2$.

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Multiplication rules

$\mathbf{1}_{+,2} \otimes \mathbf{1}_{+,2} = \mathbf{1}_{+,0}$, $\mathbf{1}_{-,3} \otimes \mathbf{1}_{-,3} = \mathbf{1}_{+,2}$, $\mathbf{1}_{-,1} \otimes \mathbf{1}_{-,1} = \mathbf{1}_{+,2}$, $\mathbf{1}_{-,1} \otimes \mathbf{1}_{-,3} = \mathbf{1}_{+,0}$, $\mathbf{1}_{+,2} \otimes \mathbf{1}_{-,1} = \mathbf{1}_{-,3}$, $\mathbf{1}_{+,2} \otimes \mathbf{1}_{-,3} = \mathbf{1}_{-,1}$, $\mathbf{2}_1 \otimes \mathbf{1}_{+,2} = \mathbf{2}_1$, $\mathbf{2}_1 \otimes \mathbf{1}_{-,3} = \mathbf{2}_2$, $\mathbf{2}_1 \otimes \mathbf{1}_{-,1} = \mathbf{2}_2$, $\mathbf{2}_2 \otimes \mathbf{1}_{+,2} = \mathbf{2}_2$, $\mathbf{2}_2 \otimes \mathbf{1}_{-,3} = \mathbf{2}_1$, $\mathbf{2}_2 \otimes \mathbf{1}_{-,1} = \mathbf{2}_1$;

$$\overbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}^{\mathbf{2}_1} \otimes \overbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}^{\mathbf{2}_1} = \overbrace{(x_1 y_2 - x_2 y_1)}^{\mathbf{1}_{+,0}} + \overbrace{(x_1 y_1 + x_2 y_2)}^{\mathbf{1}_{+,2}} + \overbrace{\begin{pmatrix} -x_1 y_2 - x_2 y_1 \\ x_1 y_1 - x_2 y_2 \end{pmatrix}}^{\mathbf{2}_2}$$

$$\overbrace{\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}}^{\mathbf{2}_2} \otimes \overbrace{\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}}^{\mathbf{2}_2} = \overbrace{(a_1 b_1 + a_2 b_2)}^{\mathbf{1}_{+,0}} + \overbrace{(a_1 b_2 - a_2 b_1)}^{\mathbf{1}_{+,2}} + \overbrace{\begin{pmatrix} -a_1 b_1 + a_2 b_2 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}}^{\mathbf{2}_2}$$

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Matter content

	$SU(5)$	Q_6	$U(1)$
H_I^d	$\bar{\mathbf{5}}$	$\mathbf{2}_1$	$-x$
H_3^d	$\bar{\mathbf{5}}$	$\mathbf{1}_{+,2}$	$-x$
H_I^u	$\mathbf{5}$	$\mathbf{2}_1$	x
H_3^u	$\mathbf{5}$	$\mathbf{1}_{+,2}$	x
F_I	$\bar{\mathbf{5}}$	$\mathbf{2}_2$	$\frac{3x}{2}$
F_3	$\bar{\mathbf{5}}$	$\mathbf{1}_{-,3}$	$\frac{3x}{2}$
T_I	$\mathbf{10}$	$\mathbf{2}_2$	$-\frac{x}{2}$
T_3	$\mathbf{10}$	$\mathbf{1}_{-,3}$	$-\frac{x}{2}$
N_I^c	$\mathbf{1}$	$\mathbf{2}_2$	$-\frac{5x}{2}$
N_3^c	$\mathbf{1}$	$\mathbf{1}_{-,1}$	$-\frac{5x}{2}$
ϕ	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$5x$
$\bar{\phi}$	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$-5x$
$H_{\bar{45}}$	$\bar{\mathbf{45}}$	$\mathbf{1}_{+,2}$	$-x$
H_{45}	$\mathbf{45}$	$\mathbf{1}_{+,2}$	x
Φ	$\mathbf{24}$	$\mathbf{1}_{+,0}$	0

J. C. Gómez, at, al. J.Phys.Conf.Ser. 485 (2014) 012057

J.C. Gómez-Izquierdo, et, al. arXiv:1312.7385

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- ϕ and $\bar{\phi}$ are scalars.

Matter content

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H^d_3	$\bar{\mathbf{5}}$	$\mathbf{1}_{+,2}$	$-x$
H^u_I	$\mathbf{5}$	$\mathbf{2}_1$	x
H^u_3	$\mathbf{5}$	$\mathbf{1}_{+,2}$	x
F_I	$\bar{\mathbf{5}}$	$\mathbf{2}_2$	$\frac{3x}{2}$
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N^c_I	$\mathbf{1}$	$\mathbf{2}_2$	$-\frac{5x}{2}$
N^c_3	$\mathbf{1}$	$\mathbf{1}_{-,1}$	$-\frac{5x}{2}$
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- ϕ and $\bar{\phi}$ are scalars. ϕ gives mass to the RH neutrinos.

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- ϕ and $\bar{\phi}$ are scalars. ϕ gives mass to the RH neutrinos. $\bar{\phi}$ cancel anomalies in the $U(1)$.
- Three families of Higgs type H_i^u and H_j^d.

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- Three families of Higgs type H_i^u and H_j^d.
- The N_i denotes the right-handed neutrino.

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T_3	$\mathbf{10}$	$\mathbf{1}_{-,3}$	$-\frac{x}{2}$
N^c_I	$\mathbf{1}$	$\mathbf{2}_2$	$-\frac{5x}{2}$
N^c_3	$\mathbf{1}$	$\mathbf{1}_{-,1}$	$-\frac{5x}{2}$
ϕ	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$5x$
$\bar{\phi}$	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$-5x$
$H_{\overline{45}}$	$\overline{\mathbf{45}}$	$\mathbf{1}_{+,2}$	$-x$
H_{45}	$\mathbf{45}$	$\mathbf{1}_{+,2}$	x
Φ	$\mathbf{24}$	$\mathbf{1}_{+,0}$	0

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- ϕ and $\bar{\phi}$ are scalars. ϕ gives mass to the RH neutrinos. $\bar{\phi}$ cancel anomalies in the $U(1)$.
- Three families of Higgs type H^u_i and H^d_j.
- The N_i denotes the right-handed neutrino.

Chain symmetry breaking.

$$SU(5) \otimes U(1) \otimes Q_6 \underset{\phi}{\longrightarrow} SU(5) \otimes Q_6$$

Matter content

	$SU(5)$	Q_6	$U(1)$
H_I^d	$\bar{\mathbf{5}}$	$\mathbf{2}_1$	$-x$
H_3^d	$\bar{\mathbf{5}}$	$\mathbf{1}_{+,2}$	$-x$
H_I^u	$\mathbf{5}$	$\mathbf{2}_1$	x
H_3^u	$\mathbf{5}$	$\mathbf{1}_{+,2}$	x
F_I	$\bar{\mathbf{5}}$	$\mathbf{2}_2$	$\frac{3x}{2}$
F_3	$\bar{\mathbf{5}}$	$\mathbf{1}_{-,3}$	$\frac{3x}{2}$
T_I	$\mathbf{10}$	$\mathbf{2}_2$	$-\frac{x}{2}$
T_3	$\mathbf{10}$	$\mathbf{1}_{-,3}$	$-\frac{x}{2}$
N_I^c	$\mathbf{1}$	$\mathbf{2}_2$	$-\frac{5x}{2}$
N_3^c	$\mathbf{1}$	$\mathbf{1}_{-,1}$	$-\frac{5x}{2}$
ϕ	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$5x$
$\bar{\phi}$	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$-5x$
$H_{\bar{45}}$	$\bar{\mathbf{45}}$	$\mathbf{1}_{+,2}$	$-x$
H_{45}	$\mathbf{45}$	$\mathbf{1}_{+,2}$	x
Φ	$\mathbf{24}$	$\mathbf{1}_{+,0}$	0

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$$SU(5)\otimes U(1)\otimes Q_6 \underset{\phi}{\longrightarrow} SU(5)\otimes Q_6$$

$$SU(5)\otimes Q_6 \underset{\Phi}{\longrightarrow} \text{MSSM}$$

Matter content

	$SU(5)$	Q_6	$U(1)$
H^d_I	$\bar{\mathbf{5}}$	$\mathbf{2}_1$	$-x$
H^d_3	$\bar{\mathbf{5}}$	$\mathbf{1}_{+,2}$	$-x$
H^u_I	$\mathbf{5}$	$\mathbf{2}_1$	x
H^u_3	$\mathbf{5}$	$\mathbf{1}_{+,2}$	x
F_I	$\bar{\mathbf{5}}$	$\mathbf{2}_2$	$\frac{3x}{2}$
F_3	$\bar{\mathbf{5}}$	$\mathbf{1}_{-,3}$	$\frac{3x}{2}$
T_I	$\mathbf{10}$	$\mathbf{2}_2$	$-\frac{x}{2}$
T_3	$\mathbf{10}$	$\mathbf{1}_{-,3}$	$-\frac{x}{2}$
N^c_I	$\mathbf{1}$	$\mathbf{2}_2$	$-\frac{5x}{2}$
N^c_3	$\mathbf{1}$	$\mathbf{1}_{-,1}$	$-\frac{5x}{2}$
ϕ	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$5x$
$\bar{\phi}$	$\mathbf{1}$	$\mathbf{1}_{+,2}$	$-5x$
$H_{\overline{45}}$	$\overline{\mathbf{45}}$	$\mathbf{1}_{+,2}$	$-x$
H_{45}	$\mathbf{45}$	$\mathbf{1}_{+,2}$	x
Σ	$\mathbf{24}$	$\mathbf{1}_{+,0}$	0

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Superpotential

$$
\begin{aligned}
\mathbf{w}_Y = {} & \sqrt{2} y_1^d \left(F_1 T_2 - F_2 T_1\right) H_3^d + \sqrt{2} y_2^d \left(F_1 T_3 H_2^d - F_2 T_3 H_1^d\right) \\
& + \sqrt{2} y_3^d F_3 \left(T_1 H_2^d - T_2 H_1^d\right) + \sqrt{2} y_4^d F_3 T_3 H_3^d \\
& + \frac{y_1^u}{4} \left(T_1 T_2 - T_2 T_1\right) H_3^u + \frac{y_2^u}{4} \left(T_1 T_3 H_2^u - T_2 T_3 H_1^u\right) \\
& + \frac{y_3^u}{4} T_3 \left(T_1 H_2^u - T_2 H_1^u\right) + \frac{y_4^u}{4} T_3 T_3 H_3^u \\
& + \sqrt{2} Y_1 \left(F_1 T_2 - F_2 T_1\right) H_{\overline{45}} + \sqrt{2} Y_2 F_3 T_3 H_{\overline{45}} \\
& + \frac{\tilde{Y}_1}{4} \left(T_1 T_2 - T_2 T_1\right) H_{45} + \frac{\tilde{Y}_2}{4} T_3 T_3 H_{45} \\
& + y_1^n \left(N_1^c F_2 - N_2^c F_1\right) H_3^u + y_2^n \left(N_1^c F_3 H_2^u - N_2^c F_3 H_1^u\right) \\
& + y_3^n N_3^c \left(F_1 H_2^u + F_2 H_1^u\right) + y_1^m \left(N_1^c \phi N_1^c + N_2^c \phi N_2^c\right) + y_2^m N_3^c \phi N_3^c
\end{aligned}
$$

Scalars

$$
\langle \mathbf{H}^u \rangle = \begin{pmatrix} 0 \\ h^{0u} \end{pmatrix}, \qquad \langle \mathbf{H}^d \rangle = \begin{pmatrix} h^{0d} \\ 0 \end{pmatrix}, \qquad \langle H_{45} \rangle_{\alpha}^{\alpha 5} = v_{45},
$$

$$
\langle H_{45} \rangle_4^{45} = -3 v_{45}, \quad \left\langle H_{\overline{45}} \right\rangle_{\alpha 5}^{\alpha} = v_{\overline{45}}, \qquad \left\langle H_{\overline{45}} \right\rangle_{45}^{4} = -3 v_{\overline{45}},
$$

$$
\langle \phi \rangle = v_s, \quad \alpha, \beta = 1, 2, 3.
$$

The mass matrices for Dirac fermions

$$\mathbf{M}_u = \begin{pmatrix} 0 & -2\tilde{Y}_1 v_{45} & \bar{y}^u h_2^{0u} \\ 2\tilde{Y}_1 v_{45} & 0 & -\bar{y}^u h_1^{0u} \\ \bar{y}^u h_2^{0u} & -\bar{y}^u h_1^{0u} & y_4^u h_3^{0u} \end{pmatrix}, \quad \mathbf{M}_{(\ell,d)} = \begin{pmatrix} 0 & y_1^d h_3^{0d} + 2Y_1 v_{\bar{45}} & y_2^d h_2^{0d} \\ -y_1^d h_3^{0d} - 2Y_1 v_{\bar{45}} & 0 & -y_2^d h_1^{0d} \\ y_3^d h_2^{0d} & -y_3^d h_1^{0d} & y_4^d h_3^{0d} + 2Y_2 v_{\bar{45}} \end{pmatrix},$$

$$\mathbf{M}_{(\ell)} = \begin{pmatrix} 0 & -\left(y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}}\right) & y_3^d h_2^{0d} \\ y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}} & 0 & -y_3^d h_1^{0d} \\ y_2^d h_2^{0d} & -y_2^d h_1^{0d} & y_4^d h_3^{0d} - 6Y_2 v_{\bar{45}} \end{pmatrix}, \quad \mathbf{M}_D = \begin{pmatrix} 0 & y_1^n h_3^{0u} & y_2^n h_2^{0u} \\ -y_1^n h_3^{0u} & 0 & -y_2^n h_1^{0u} \\ y_3^n h_2^{0u} & y_3^n h_1^{0u} & 0 \end{pmatrix}.$$

The mass matrices for Dirac fermions

$$\mathbf{M}_u = \begin{pmatrix} 0 & -2\tilde{Y}_1 v_{45} & \bar{y}^u h_2^{0u} \\ 2\tilde{Y}_1 v_{45} & 0 & -\bar{y}^u h_1^{0u} \\ \bar{y}^u h_2^{0u} & -\bar{y}^u h_1^{0u} & y_4^u h_3^{0u} \end{pmatrix}, \quad \mathbf{M}_{(\ell,d)} = \begin{pmatrix} 0 & y_1^d h_3^{0d} + 2Y_1 v_{\bar{45}} & y_2^d h_2^{0d} \\ -y_1^d h_3^{0d} - 2Y_1 v_{\bar{45}} & 0 & -y_2^d h_1^{0d} \\ y_3^d h_2^{0d} & -y_3^d h_1^{0d} & y_4^d h_3^{0d} + 2Y_2 v_{\bar{45}} \end{pmatrix},$$

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With $h_2^{0u} = h_1^{0u} \equiv h^{0u}$ and $h_2^{0d} = h_1^{0d} \equiv h^{0d}$

NNI Textures

$$\mathbf{m}_k = \mathbf{U}_{\pi/4}^T \hat{\mathbf{M}}_k \mathbf{U}_{\pi/4} = \begin{pmatrix} 0 & \pm\tilde{A}_k & 0 \\ \mp\tilde{A}_k & 0 & -\sqrt{2}\tilde{B}_k \\ 0 & -\sqrt{2}\tilde{C}_k & \tilde{D}_k \end{pmatrix}, \quad \mathbf{U}_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}; k = u, d, \ell$$

The mass matrices for Dirac fermions

$$\mathbf{M}_u = \begin{pmatrix} 0 & -2\tilde{Y}_1 v_{45} & \bar{y}^u h_2^{0u} \\ 2\tilde{Y}_1 v_{45} & 0 & -\bar{y}^u h_1^{0u} \\ \bar{y}^u h_2^{0u} & -\bar{y}^u h_1^{0u} & y_4^u h_3^{0u} \end{pmatrix}, \quad \mathbf{M}_{(\ell,d)} = \begin{pmatrix} 0 & y_1^d h_3^{0d} + 2Y_1 v_{\bar{45}} & y_2^d h_2^{0d} \\ -y_1^d h_3^{0d} - 2Y_1 v_{\bar{45}} & 0 & -y_2^d h_1^{0d} \\ y_3^d h_2^{0d} & -y_3^d h_1^{0d} & y_4^d h_3^{0d} + 2Y_2 v_{\bar{45}} \end{pmatrix},$$

$$\mathbf{M}_{(\ell)} = \begin{pmatrix} 0 & -\left(y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}}\right) & y_3^d h_2^{0d} \\ y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}} & 0 & -y_3^d h_1^{0d} \\ y_2^d h_2^{0d} & -y_2^d h_1^{0d} & y_4^d h_3^{0d} - 6Y_2 v_{\bar{45}} \end{pmatrix}, \quad \mathbf{M}_D = \begin{pmatrix} 0 & y_1^n h_3^{0u} & y_2^n h_2^{0u} \\ -y_1^n h_3^{0u} & 0 & -y_2^n h_1^{0u} \\ y_3^n h_2^{0u} & y_3^n h_1^{0u} & 0 \end{pmatrix}.$$

With $h_2^{0u} = h_1^{0u} \equiv h^{0u}$ and $h_2^{0d} = h_1^{0d} \equiv h^{0d}$

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This matrix are similar to obtained in $S_3 \otimes Z_2$ model.

The mass matrices for Dirac fermions

$$\mathbf{M}_u = \begin{pmatrix} 0 & -2\tilde{Y}_1 v_{45} & \bar{y}^u h_2^{0u} \\ 2\tilde{Y}_1 v_{45} & 0 & -\bar{y}^u h_1^{0u} \\ \bar{y}^u h_2^{0u} & -\bar{y}^u h_1^{0u} & y_4^u h_3^{0u} \end{pmatrix}, \quad \mathbf{M}_{(\ell,d)} = \begin{pmatrix} 0 & y_1^d h_3^{0d} + 2Y_1 v_{\bar{45}} & y_2^d h_2^{0d} \\ -y_1^d h_3^{0d} - 2Y_1 v_{\bar{45}} & 0 & -y_2^d h_1^{0d} \\ y_3^d h_2^{0d} & -y_3^d h_1^{0d} & y_4^d h_3^{0d} + 2Y_2 v_{\bar{45}} \end{pmatrix},$$

$$\mathbf{M}_{(\ell)} = \begin{pmatrix} 0 & -\left(y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}}\right) & y_3^d h_2^{0d} \\ y_1^d h_3^{0d} - 6Y_1 v_{\bar{45}} & 0 & -y_3^d h_1^{0d} \\ y_2^d h_2^{0d} & -y_2^d h_1^{0d} & y_4^d h_3^{0d} - 6Y_2 v_{\bar{45}} \end{pmatrix}, \quad \mathbf{M}_D = \begin{pmatrix} 0 & y_1^n h_3^{0u} & y_2^n h_2^{0u} \\ -y_1^n h_3^{0u} & 0 & -y_2^n h_1^{0u} \\ y_3^n h_2^{0u} & y_3^n h_1^{0u} & 0 \end{pmatrix}.$$

With $h_2^{0u} = h_1^{0u} \equiv h^{0u}$ and $h_2^{0d} = h_1^{0d} \equiv h^{0d}$

NNI Textures

$$\mathbf{m}_k = \mathbf{U}_{\pi/4}^T \hat{\mathbf{M}}_k \mathbf{U}_{\pi/4} = \begin{pmatrix} 0 & \pm\tilde{A}_k & 0 \\ \mp\tilde{A}_k & 0 & -\sqrt{2}\tilde{B}_k \\ 0 & -\sqrt{2}\tilde{C}_k & \tilde{D}_k \end{pmatrix}, \quad \mathbf{U}_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}; \; k = u, d, \ell$$

This matrix are similar to obtained in $S_3 \otimes Z_2$ model.

For the quarks up: $\tilde{B}_u = \tilde{C}_u$.

The neutrino mass matrix

$$\mathbf{M}_\nu = \mathbf{M}_D^T \mathbf{M}_R^{-1} \mathbf{M}_D, \qquad \mathbf{M}_R = \mathrm{diag}\left\{M_{R_1}, M_{R_2}, M_{R_3}\right\}$$

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$$\mathbf{M}_\nu = \begin{pmatrix} \frac{\left(y_1^n h_3^{0u}\right)^2}{M_{R_2}} + \frac{\left(y_3^n h_2^{0u}\right)^2}{M_{R_3}} & \frac{\left(y_3^n\right)^2 h_2^{0u} h_1^{0u}}{M_{R_3}} & \frac{y_1^n h_3^{0u} y_2^n h_1^{0u}}{M_{R_2}} \\ \frac{\left(y_3^n\right)^2 h_2^{0u} h_1^{0u}}{M_{R_3}} & \frac{\left(y_1^n h_3^{0u}\right)^2}{M_{R_1}} + \frac{\left(y_3^n h_1^{0u}\right)^2}{M_{R_3}} & \frac{y_1^n h_3^{0u} y_2^n h_2^{0u}}{M_{R_1}} \\ \frac{y_1^n h_3^{0u} y_2^n h_1^{0u}}{M_{R_2}} & \frac{y_1^n h_3^{0u} y_2^n h_2^{0u}}{M_{R_1}} & \frac{\left(y_2^n h_2^{0u}\right)^2}{M_{R_1}} + \frac{\left(y_2^n h_1^{0u}\right)^2}{M_{R_2}} \end{pmatrix}.$$

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$$\mathbf{M}_\nu = \mathbf{M}_D^T \mathbf{M}_R^{-1} \mathbf{M}_D , \qquad \mathbf{M}_R = \mathrm{diag}\left\{ M_{R_1}, M_{R_2}, M_{R_3} \right\}$$

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This matrix can be reduced:

$$\mathbf{m}_\nu = u_\theta^T \mathbf{M}_\nu u_\theta = \begin{pmatrix} b_\nu & a_\nu & c_\nu \\ a_\nu & \mu_0 & 0 \\ c_\nu & 0 & d_\nu \end{pmatrix} \quad \text{and} \quad u_\theta = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\theta & 0 & \cos\theta \\ 0 & 1 & 0 \end{pmatrix},$$

$$\tan\theta = \frac{M_{R_2}}{M_{R_1}}$$

Orthogonal matrix for Dirac Fermions.

$$\mathbf{u}_{fL} = \mathbf{Q}_f \mathbf{O}_{fL}, \quad \mathbf{O}_{fL} = (|\, f_1\rangle, |\, f_2\rangle, |\, f_3\rangle), \qquad \mathbf{Q}_f = \mathrm{Diag}\left(1, \exp(-i\eta_{f_2}), \exp(-i\eta_{f_3})\right)$$

$$|\, f_i\rangle = N_{f_i} \begin{pmatrix} (\tilde{m}^2_{f_i} - |\tilde{A}_f|^2 - |\tilde{C}_f|^2)|\tilde{A}_f||\tilde{B}_f| \\ (\tilde{m}^2_{f_i} - |\tilde{A}_f|^2)|\tilde{C}_f||\tilde{D}_f| \\ (\tilde{m}^2_{f_i} - |\tilde{A}_f|^2)(\tilde{m}^2_{f_i} - |\tilde{A}_f|^2 - |\tilde{C}_f|^2) \end{pmatrix}, \quad |\, f_3\rangle = N_{f_3} \begin{pmatrix} (1 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2)|\tilde{A}_f||\tilde{B}_f| \\ (1 - |\tilde{A}_f|^2)|\tilde{C}_f||\tilde{D}_f| \\ (1 - |\tilde{A}_f|^2)(1 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2) \end{pmatrix}.$$

$$|\tilde{A}_f| = \frac{q_f}{y_f}, \; (|\tilde{B}_f|, |\tilde{C}_f|) = \sqrt{\frac{1 + P_f - y_f^4 \mp R_f}{2} - |\tilde{A}_f|^2}, \; |\tilde{D}_f| \equiv y_f$$

$$P_f = \tilde{m}^2_{1f} + \tilde{m}^2_{2f}, \; q_f = \sqrt[4]{\tilde{m}^2_{1f}\tilde{m}^2_{2f}},$$

$$R_f = \sqrt{\left(1 + P_f - y_f^4\right)^2 - 4\left(P_f + q_f^4\right) + 8 q_f^2 y_f^2}.$$

Orthogonal matrix for Dirac Fermions.

$$\mathbf{u}_{fL} = \mathbf{Q}_f \mathbf{O}_{fL}, \quad \mathbf{O}_{fL} = (|\, f_1\rangle, |\, f_2\rangle, |\, f_3\rangle)\,, \qquad \mathbf{Q}_f = \mathrm{Diag}\left(1, \exp(-i\eta_{f_2}), \exp(-i\eta_{f_3})\right)$$

$$|\, f_i\rangle = N_{f_i}\begin{pmatrix} (\tilde{m}^2_{f_i} - |\tilde{A}_f|^2 - |\tilde{C}_f|^2)|\tilde{A}_f||\tilde{B}_f| \\ (\tilde{m}^2_{f_i} - |\tilde{A}_f|^2)|\tilde{C}_f||\tilde{D}_f| \\ (\tilde{m}^2_{f_i} - |\tilde{A}_f|^2)(\tilde{m}^2_{f_i} - |\tilde{A}_f|^2 - |\tilde{C}_f|^2) \end{pmatrix}, \qquad |\, f_3\rangle = N_{f_3}\begin{pmatrix} (1 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2)|\tilde{A}_f||\tilde{B}_f| \\ (1 - |\tilde{A}_f|^2)|\tilde{C}_f||\tilde{D}_f| \\ (1 - |\tilde{A}_f|^2)(1 - |\tilde{A}_f|^2 - |\tilde{C}_f|^2) \end{pmatrix}.$$

$$|\tilde{A}_f| = \frac{q_f}{y_f}, \; (|\tilde{B}_f|, |\tilde{C}_f|) = \sqrt{\frac{1 + P_f - y_f^4 \mp R_f}{2} - |\tilde{A}_f|^2}, \; |\tilde{D}_f| \equiv y_f$$

$$P_f = \tilde{m}^2_{1f} + \tilde{m}^2_{2f}, \; q_f = \sqrt[4]{\tilde{m}^2_{1f}\tilde{m}^2_{2f}},$$

$$R_f = \sqrt{\left(1 + P_f - y_f^4\right)^2 - 4\left(P_f + q_f^4\right) + 8q_f^2 y_f^2}.$$

The CKM matrix

$$V_{CKM} = \mathbf{O}_u^\top \mathbf{Q}_q \mathbf{O}_d, \qquad \mathbf{Q}_q = \mathbf{Q}_u^\dagger \mathbf{Q}_d = \mathrm{diag}\left(1, \exp i\alpha, \exp i\beta\right)$$

CKM matrix

$$V_{CKM} = V_{CKM}(y_d, \alpha, \beta).$$

$$\chi^2 : |V_{ud}^{ex}| = 0.97427 \pm 0.00015, |V_{us}^{ex}| = 0.2253 \pm 0.007,$$

$$|V_{ub}^{ex}| = 0.00351 \pm 0.00015, \mathcal{J}_q^{ex} = (2.96 \pm 0.18) \times 10^{-5}$$

Free parameters values

$$y_d = 0.981977^{+0.002843}_{-0.002117}, \alpha = (168.78^{+2.10}_{-1.11})^\circ, \beta = (132^{+105}_{-25})^\circ.$$

$$\text{a } 70\,\%\, C.L \text{ con } \chi^2 = 0.0515.$$

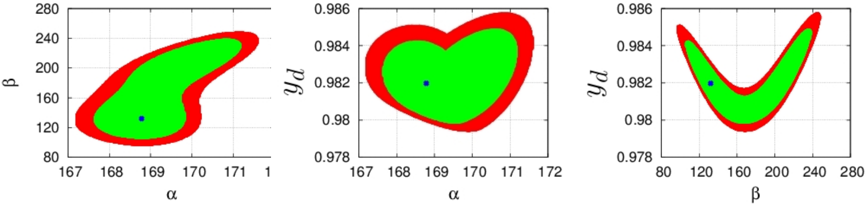


Figure: Allowed region for the three free parameters in the quark sector at 70% (green) and 90% (red) confidence level.

Neutrino Sector

Masses and sum rule

Neutrino Sector

Masses and sum rule

With $M_{R_1} = M_{R_2}$
$\rightarrow \tan\theta = 1$

Neutrino Sector

Masses and sum rule

With $M_{R_1} = M_{R_2}$
$\rightarrow \tan\theta = 1$

$$\tilde{\mathbf{M}}_\nu = \mathrm{Diag}\left(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\right) = \mathbf{U}_\nu^T \mathbf{M}_\nu \mathbf{U}_\nu,$$

$$\mathbf{U}_\nu = u_{\pi/4}\mathbf{u}_\nu, \quad u_{\pi/4} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}.$$

$$\mathbf{m}_\nu = \begin{pmatrix} A_\nu^2 + 2B_\nu^2 & \sqrt{2}A_\nu C_\nu & 0 \\ \sqrt{2}A_\nu C_\nu & 2C_\nu^2 & 0 \\ 0 & 0 & A_\nu^2 \end{pmatrix} = \mathbf{P}_\nu \hat{\mathbf{m}}_\nu \mathbf{P}_\nu,$$

$$\mathbf{P}_\nu = \mathrm{Diag.}(\exp i\eta_{\nu_1}, \exp i\eta_{\nu_2}, \exp i\eta_{\nu_1}), \quad \mathbf{u}_\nu = \mathbf{P}_\nu^\dagger \mathbf{O}_\nu.$$

$$|A_\nu|^2 = m_{\nu_3}, \; |B_\nu|^2_\mp = \frac{1}{4}\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3} \mp R_\nu\right),$$

$$|C_\nu|^2_\pm = \frac{1}{4}\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3} \pm R_\nu\right),$$

$$R_\nu \equiv \sqrt{\left(m_{\nu_2} + m_{\nu_1} - m_{\nu_3}\right)^2 - 4m_{\nu_2}m_{\nu_1}}.$$

$|B_\nu|^2_-$ *and* $|C_\nu|^2_+$ must be real $\Leftrightarrow$

$$m_{\nu_3} \leq (\sqrt{m_{\nu_2}} - \sqrt{m_{\nu_1}})^2$$

$$\mathbf{O}_\nu = \begin{pmatrix} \cos\theta_\nu & \sin\theta_\nu & 0 \\ -\sin\theta_\nu & \cos\theta_\nu & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sin\theta_\nu = \sqrt{\frac{m_{\nu_2} - m_{\nu_1} + m_{\nu_3} - R_\nu}{2\left(m_{\nu_2} - m_{\nu_1}\right)}}$$

$$\cos\theta_\nu = \sqrt{\frac{m_{\nu_2} - m_{\nu_1} - m_{3\nu} + R_\nu}{2\left(m_{\nu_2} - m_{\nu_1}\right)}}$$

Por lo tanto

$$\mathbf{V}_{PMNS} = \mathbf{U}_\ell^\dagger \mathbf{U}_\nu = \mathbf{O}_{\ell L}^T \mathbf{Q}_\ell^\dagger \mathbf{S}_{23} \mathbf{P}_\nu^\dagger \mathbf{O}_{\nu L}$$

$$|\sin\theta_{13}| = |O_{21\ell}|, \; |\sin\theta_{23}| = \frac{|O_{22\ell}|}{\sqrt{1 - |O_{21\ell}|^2}},$$

$$|\tan\theta_{12}|^2 = \frac{|O_{11\ell}O_{12\nu} + O_{31\ell}O_{22\nu}\exp\left(i\bar{\eta}_{3e}\right)|^2}{|O_{11\ell}O_{11\nu} + O_{31\ell}O_{21\nu}\exp\left(i\bar{\eta}_{3e}\right)|^2}$$

Lepton Sector

$$V_{PMNS} = V_{PMNS}(y_\ell, m_{\nu_3}, \exp(i\bar{\eta}_{3e}), \exp(i\eta_{2e}))$$

$$\chi^2 : \sin^2\theta^\ell_{23} = 0.52 \pm 0.0, \sin^2 2\theta^\ell_{13} = 0.076 \pm 0.068.$$

$$y_e = 0.8478^{+0,0045}_{-0.0046}, \quad \text{a } 1\sigma \text{ y } \chi^2 = 0.85,$$

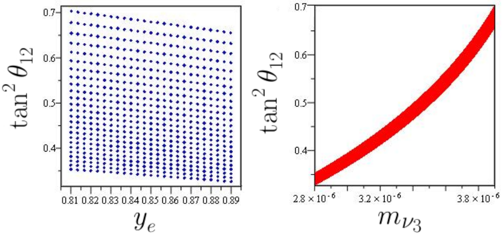
$$\theta^{\ell th}_{23} = 46.18^{+0.66}_{-0.65} \quad \text{and} \quad \theta^{\ell th}_{13} = 3.38^{+0.03}_{-0.02}.$$

Sum Rule,

$$m_{\nu_3} \leq \left(\sqrt[4]{m^2_{\nu_3} + \Delta m^2_\odot + \Delta m^2_{ATM}} - \sqrt[4]{m^2_{\nu_3} + \Delta m^2_{ATM}}\right)^2$$

$$0 \leq m_{\nu_3} \leq 4 \times 10^{-6}\ eV$$

$$\theta^{\ell th}_{12} = 36.62 \pm 4.06, \text{ a } 90\%\ C.L.$$



Neutrino Sector

Inverted Hierarchy

With $M_{R_1} \neq M_{R_2} \rightarrow \tan\theta = \frac{M_{R_2}}{M_{R_1}}$

Neutrino Sector

Inverted Hierarchy

With $M_{R_1} \neq M_{R_2} \to \tan\theta = \frac{M_{R_2}}{M_{R_1}}$

$$\mathbf{O}_\nu = \begin{pmatrix} \sqrt{\frac{\widetilde{\sigma}_{1[3]}(1-\delta_\nu)f_{\nu 1[3]}}{D_{\nu 1[3]}}} & \sqrt{\frac{\widetilde{\sigma}_{2[1]}(1-\delta_\nu)f_{\nu 2[1]}}{D_{\nu 2[1]}}} & \sqrt{\frac{\delta_\nu(1-\delta_\nu)}{D_{\nu 3[2]}}} \\ \sqrt{\frac{\widetilde{\sigma}_{2[1]}f_{\nu 1[3]}}{D_{\nu 1[3]}}} & -\sqrt{\frac{\widetilde{\sigma}_{1[3]}f_{\nu 2[1]}}{D_{\nu 2[1]}}} & \sqrt{\frac{\widetilde{\sigma}_{1[3]}\widetilde{\sigma}_{2[1]}\delta_\nu}{D_{\nu 3[2]}}} \\ -\sqrt{\frac{\widetilde{\sigma}_{1[3]}\delta_\nu f_{\nu 2[1]}}{D_{\nu 1[3]}}} & -\sqrt{\frac{\widetilde{\sigma}_{2[1]}\delta_\nu f_{\nu 1[3]}}{D_{\nu 2[1]}}} & \sqrt{\frac{f_{\nu 1[3]}f_{\nu 2[1]}}{D_{\nu 3[2]}}} \end{pmatrix},$$

$$D_{\nu 1[3]} = (1-\delta_\nu)\left(\widetilde{\sigma}_{1[3]} + \widetilde{\sigma}_{2[1]}\right)\left(1-\widetilde{\sigma}_{1[3]}\right),$$
$$D_{\nu 2} = (1-\delta_\nu)\left(\widetilde{\sigma}_{1[3]} + \widetilde{\sigma}_{2[1]}\right)\left(1+\widetilde{\sigma}_{2[1]}\right),$$
$$D_{\nu 3[2]} = (1-\delta_\nu)\left(1-\widetilde{\sigma}_{1[3]}\right)\left(1+\widetilde{\sigma}_{2[1]}\right).$$
$$f_{\nu 1[3]} = \left(1-\widetilde{\sigma}_{1[3]}-\delta_\nu\right),$$
$$f_{\nu 2[1]} = \left(1+\widetilde{\sigma}_{2[1]}-\delta_\nu\right),$$

Then, one obtains that leptonic mixing matrix takes the form

$$\mathbf{V}_{PMNS} = \mathbf{O}_{\ell L}^T \mathbf{Q}_\ell^\dagger \mathbf{S}_{23} \mathbf{P}_\nu^\dagger \mathbf{O}_{\nu L}. \tag{1}$$

Now $\mathbf{S}_{23} = \mathbf{U}_{\pi/4}^T u_{\theta=\pi/4}$ is not the permutation matrix show in the previous section. The explicit form of $\mathbf{S}_{23}$ is

$$\mathbf{S}_{23} = \begin{pmatrix} S_{\theta+\frac{\pi}{4}} & 0 & C_{\theta+\frac{\pi}{4}} \\ C_{\theta+\frac{\pi}{4}} & 0 & S_{\theta+\frac{\pi}{4}} \\ 0 & 1 & 0 \end{pmatrix}, \tag{2}$$

where $S_{\theta+\frac{\pi}{4}} = \sin\left(\theta+\frac{\pi}{4}\right)$ and $C_{\theta+\frac{\pi}{4}} = \cos\left(\theta+\frac{\pi}{4}\right)$

$$\widetilde{\sigma}_{1[3]} = \frac{\widetilde{m}_{\nu_{1[3]}} - \widetilde{\mu}}{1-\widetilde{\mu}}, \quad \widetilde{\sigma}_{2[1]} = \frac{\left|\widetilde{m}_{\nu_{2[1]}} - \widetilde{\mu}\right|}{1-\widetilde{\mu}}, \quad \widetilde{m}_{\nu_{1[3]}} = \frac{m_{\nu_{1[3]}}}{m_{\nu_{3[2]}}}, \quad \widetilde{m}_{\nu_{2[1]}} = \frac{m_{\nu_{2[1]}}}{m_{\nu_{3[2]}}}, \quad \widetilde{\mu} = \frac{|\mu_0|}{m_{\nu_{3[2]}}}.$$

Lepton Sector

$$V_{PMNS} = V_{PMNS}(y_\ell, m_{\nu_3}, \theta, \exp(i\bar{\eta}_{3e}), \exp(i\eta_{2e}))$$

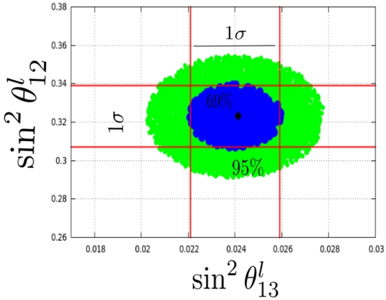
$$\text{All phases} \in [0, 2\pi], \quad y_e = 0.8478^{+0,0045}_{-0.0046},$$

$$\chi^2 = 0.2$$

$$m_{\nu_2} = \left(6.24^{+3.74}_{-1.46}\right) \times 10^{-2}, m_{\nu_1} = \left(6.18^{+3.77}_{-1.48}\right) \times 10^{-2},$$

$$m_{\nu_3} = \left(3.65^{+5.12}_{-3.45}\right) \times 10^{-2}.$$

$$\theta_{12}^{\ell th} = \left(34.64^{+1.89}_{-1.98}\right)^\circ, \quad \theta_{23}^{\ell th} = \left(48.62^{+3.91}_{-3.84}\right)^\circ \quad \text{and} \quad \theta_{13}^{\ell th} = 8.93^{+0.65}_{-0.75}.$$



Conclusions

we present a renormalizable SUSY $SU(5) \otimes U(1) \otimes Q_6$ model, with unbroken R-parity and the discrete group Q(6) as flavour symmetry. We add three right-handed neutrinos to implement the see-saw mechanism, and we study the general case where the three masses are non-degenerate. Within this theoretical framework, we can determine the masses and mixing of both quarks and leptons in good agreement with the experimental data. On the other hand, in the limit when the first and second masses of the right handed neutrinos are degenerate, the model predicts a strong inverted hierarchy and a sum rule among the neutrino masses, which leads to a value for the reactor mixing angle different from zero but very small.