

Towards predictive flavour models in SUSY SU(5) GUTs with doublet-triplet splitting

Vinzenz Maurer



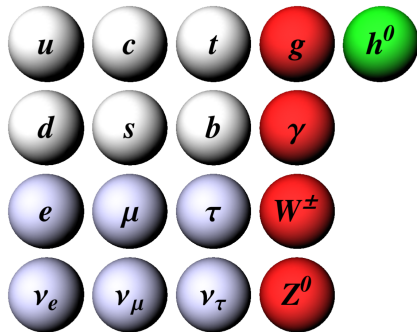
18th June 2014

FLASY 2014

Based on Antusch, de Medeiros Varzielas, V.M., Sluka & Spinrath arXiv:1405.6962

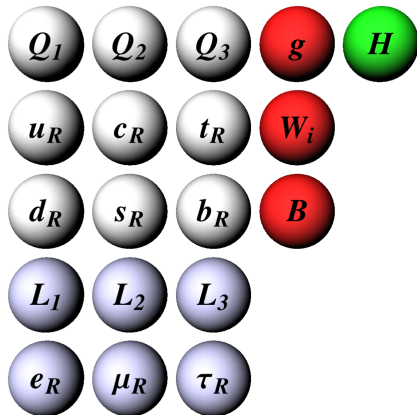
Motivation: Playing Particle Zoo Keeper

1 particles



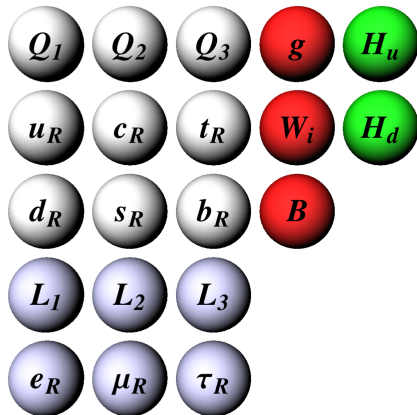
Motivation: Playing Particle Zoo Keeper

- 1 particles
- 2 SM multiplets



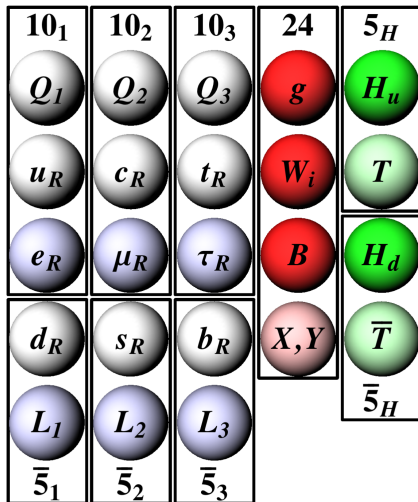
Motivation: Playing Particle Zoo Keeper

- 1 particles
- 2 SM multiplets
- 3 MSSM multiplets



Motivation: Playing Particle Zoo Keeper

- 1 particles
- 2 SM multiplets
- 3 MSSM multiplets
- 4 GUT multiplets



GUT Symmetry Breaking

Make new states heavy enough to suppress proton decay

- $X, Y \Rightarrow$ GUT scale $\gtrsim 10^{16}$ GeV
- $T, \bar{T} \rightarrow$ Doublet-Triplet splitting problem

Solution(s) in 4D SU(5): (Double) Missing Partner Mechanism

[Masiero, Nanopoulos, Tamvakis, Yanagida '82; Grinstein '82; Hisano, Moroi, Tobe, Yanagida '95]

Correct Relations for Flavour Observables

- Yukawa couplings and mixing angles
- Georgi-Jarlskog relations $y_\mu/y_s = 3 = y_d/y_e$ disfavoured

Solved with effective Yukawa couplings from higher dim. operators

$$\Rightarrow y_\mu = 6 y_s, y_e = \frac{1}{2} y_d \text{ and } y_\tau = \frac{3}{2} y_b$$

[Antusch, Spinrath '09]

Strategy

The Missing Partner Mechanism

Observation in GUT \supset SM embedding

$$\mathbf{5} \rightarrow (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} + (\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$$
$$\mathbf{50} \rightarrow (\mathbf{1}, \mathbf{1})_2 + (\mathbf{3}, \mathbf{1})_{\frac{1}{3}} + (\bar{\mathbf{3}}, \mathbf{2})_{\frac{7}{6}} + (\mathbf{6}, \mathbf{1})_{-\frac{4}{3}} + (\bar{\mathbf{6}}, \mathbf{3})_{\frac{1}{3}} + (\mathbf{8}, \mathbf{2})_{-\frac{1}{2}}$$

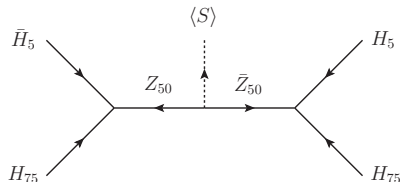
[Masiero, Nanopoulos, Tamvakis, Yanagida '82; Grinstein '82]

$$W_{\text{MPM}} = \bar{H}_5 H_{75} Z_{50} + \bar{Z}_{50} H_{75} H_5$$
$$+ \langle S \rangle Z_{50} \bar{Z}_{50}$$

\Rightarrow

$$m_D = (0)$$

$$m_T = \begin{pmatrix} 0 & V \\ V & \langle S \rangle \end{pmatrix}$$



- $\langle H_{75} \rangle$ breaks $SU(5) \rightarrow SM$
- Non-trivial charge of S , otherwise $W \supset H_5 H_{75}^2 \bar{H}_5 / M_{\text{Pl}}$

The Missing Partner Mechanism

Observation in GUT \supset SM embedding

$$\mathbf{5} \rightarrow (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} + (\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$$

$$\mathbf{50} \rightarrow (\mathbf{1}, \mathbf{1})_2 + (\mathbf{3}, \mathbf{1})_{\frac{1}{3}} + (\bar{\mathbf{3}}, \mathbf{2})_{\frac{7}{6}} + (\mathbf{6}, \mathbf{1})_{-\frac{4}{3}} + (\bar{\mathbf{6}}, \mathbf{3})_{\frac{1}{3}} + (\mathbf{8}, \mathbf{2})_{-\frac{1}{2}}$$

[Masiero, Nanopoulos, Tamvakis, Yanagida '82; Grinstein '82]

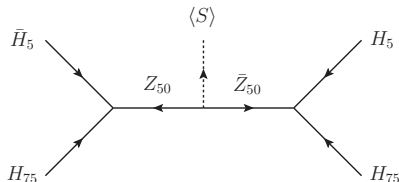
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- Non-trivial charge of S , otherwise $W \supset H_5 H_{75}^2 \bar{H}_5 / M_{\text{Pl}}$

The Missing Partner Mechanism

$$W_{\text{Yukawa}} = \mathcal{T}_i \mathcal{F}_j \bar{H}_5 + \mathcal{T}_i \mathcal{T}_j H_5$$

Integrate out all colour triplets ($T_5, \bar{T}_5, T_{50}, \bar{T}_{50}$) \Rightarrow

$$W_{\text{eff}} \supset \frac{1}{M_T^{\text{eff}}} \overbrace{(Q_i Q_j + \bar{U}_i \bar{E}_j)}^{\mathcal{T}_i \mathcal{T}_j} \overbrace{(Q_m L_n + \bar{U}_m \bar{D}_n)}^{\mathcal{T}_m \mathcal{F}_n}$$

with

$$M_T^{\text{eff}} = (m_T^{-1})_{11}^{-1} = \frac{V^2}{\langle S \rangle}$$

Some numbers: $M_T^{\text{eff}} \gtrsim 10^{17}$ GeV, $V \approx 10^{16}$ GeV $\Rightarrow \langle S \rangle \lesssim 10^{15}$ GeV

The Missing Partner Mechanism

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The Missing Partner Mechanism

$$W_{\text{Yukawa}} = T_i \mathcal{F}_j \bar{H}_5 + T_i T_j H_5$$

Integrate out all colour triplets ($T_5, \bar{T}_5, T_{50}, \bar{T}_{50}$) \Rightarrow

$$W_{\text{eff}} \supset \frac{1}{M_T^{\text{eff}}} \overbrace{(Q_i Q_j + \bar{U}_i \bar{E}_j)}^{T_i T_j} \overbrace{(Q_m L_n + \bar{U}_m \bar{D}_n)}^{T_m \mathcal{F}_n}$$

with

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Some numbers: $M_T^{\text{eff}} \gtrsim 10^{17}$ GeV, $V \approx 10^{16}$ GeV $\Rightarrow \langle S \rangle \lesssim 10^{15}$ GeV

Left-overs from **50** and $\bar{\mathbf{50}}$ in RGEs at 10^{15} GeV

Theory becomes non-perturbative much below M_{Pl} ☹️

The Double Missing Partner Mechanism

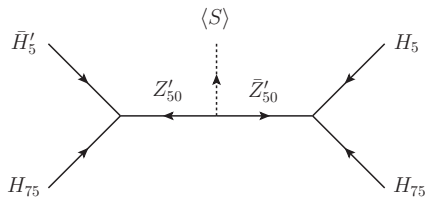
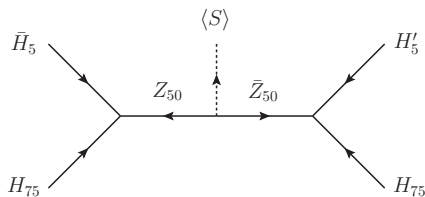
Double the **5**'s and **50**'s with $H'_5, \bar{H}'_5, Z'_{50}, \bar{Z}'_{50}$!

[Hisano, Moroi, Tobe, Yanagida '95]

$$\begin{aligned}
 W_{\text{DMPM}} = & \bar{H}_5 H_{75} Z_{50} + \bar{Z}_{50} H_{75} H'_5 \\
 & + \bar{H}'_5 H_{75} Z'_{50} + \bar{Z}'_{50} H_{75} H_5 \\
 & + \langle S \rangle Z_{50} \bar{Z}_{50} + \langle S \rangle Z'_{50} \bar{Z}'_{50} \\
 & + \langle S_{\mu'} \rangle H'_5 \bar{H}'_5 \\
 \Rightarrow &
 \end{aligned}$$

$$m_D = \begin{pmatrix} 0 & 0 \\ 0 & \langle S_{\mu'} \rangle \end{pmatrix}$$

$$m_T = \begin{pmatrix} 0 & 0 & 0 & V \\ 0 & \langle S_{\mu'} \rangle & V & 0 \\ V & 0 & \langle S \rangle & 0 \\ 0 & V & 0 & \langle S \rangle \end{pmatrix}$$



Again only H_5, \bar{H}_5 couple to SM matter

The Double Missing Partner Mechanism

Analogous to MPM

$$M_T^{\text{eff}} = (m_T^{-1})_{11}^{-1} = \frac{V^4}{\langle S \rangle^2 \langle S_{\mu'} \rangle}$$

Some numbers:

$$M_T^{\text{eff}} \sim 10^{17} \text{ GeV}, V \approx 10^{16} \text{ GeV}, \langle S_{\mu'} \rangle \approx 10^{11} \text{ GeV}$$

$$\Rightarrow \langle S \rangle \approx 10^{18} \text{ GeV}$$

50 and $\overline{50}$ in RGEs at 10^{18} GeV

Theory perturbative up to (almost) M_{Pl} 😊

The Double Missing Partner Mechanism

Analogous to MPM

$$M_T^{\text{eff}} = (m_T^{-1})_{11}^{-1} = \frac{V^4}{\langle S \rangle^2 \langle S_{\mu'} \rangle} \approx \frac{M_T^2}{\mu'}$$

Some numbers:

$$M_T^{\text{eff}} \sim 10^{17} \text{ GeV}, V \approx 10^{16} \text{ GeV}, \langle S_{\mu'} \rangle \approx 10^{11} \text{ GeV}$$

$$\Rightarrow \langle S \rangle \approx 10^{18} \text{ GeV}$$

50 and $\overline{50}$ in RGEs at 10^{18} GeV

Theory perturbative up to (almost) M_{Pl} 😊

Yukawa Coupling Ratios

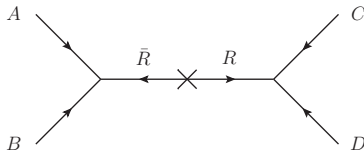
- Higher dim. effective operators for Yukawa couplings with $\langle H_{24} \rangle$
 \Rightarrow GUT breaking effects proportional to hypercharge
- Only predictive if index contraction is fixed:

$$W \supset (AB)_R (CD)_{\bar{R}}$$

using messenger fields in SU(5) irrep \mathbf{R}

- Used ratios:

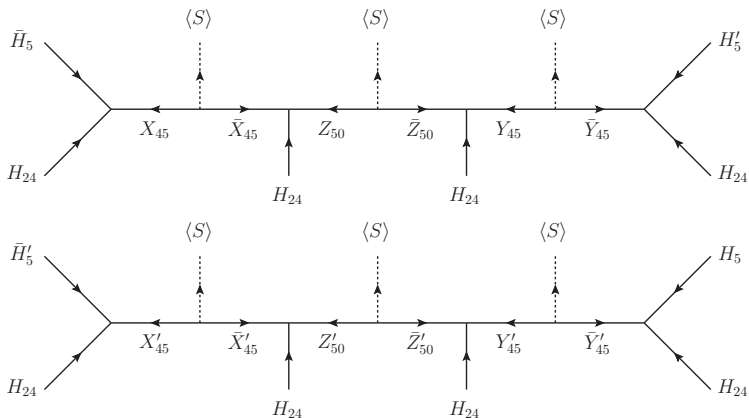
AB	CD	\mathbf{R}	$(Y_e)_{ji}/(Y_d)_{ij}$
$H_{24} \mathcal{F}$	$\mathcal{T} \bar{H}_{45}$	$\bar{\mathbf{45}}$	$-\frac{1}{2}$
$H_{24} \mathcal{F}$	$\mathcal{T} \bar{H}_5$	$\bar{\mathbf{5}}$	$-\frac{3}{2}$
$H_{24} \mathcal{T}$	$\mathcal{F} \bar{H}_5$	$\mathbf{10}$	6



[Antusch, Spinrath '09]

Double Missing Partner with a 24

Replace $H_{75} \rightarrow H_{24}^2/\Lambda$



Additional messenger sector: $X_{45}, \bar{X}_{45}, Y_{45}, \bar{Y}_{45}, X'_{45}, \bar{X}'_{45}, Y'_{45}, \bar{Y}'_{45}$

[Zhang, Zheng '12]

Grand Unification and its Breaking

The Need for two **24**'s

Minimal SU(5):

- Single **24** contains one SU(2) triplet and one SU(3) octet that obtain equal masses
- Gauge coupling unification needs $M_T \sim 10^{15}$ GeV ☹️
- Non-renormalizable terms for **24** \Rightarrow split triplet and octet \Rightarrow higher M_T possible

[Murayama, Pierce '02]

[Bajc, Perez, Senjanovic '02]

\rightarrow

generate higher dim. operator with second **24**

The Need for two **24**'s

Not taking the limit where one **24** is heavy and the other isn't:

- Additional SU(2) triplet and SU(3) octet (and leptoquark superfield) gives more freedom in GUT threshold corrections
- More possible choices for superpotentials that do not reduce to non-renormalizable single **24** versions

Intended Yukawa coupling ratios require **24** with non-trivial charge under some “shaping symmetry”

- Single **24** renormalizable superpotential that breaks SU(5) has no shaping symmetry
- Adding another **24** allows for shaping symmetry and VEV

Introduce H'_{24} in addition to H_{24}

[Antusch, de Medeiros Varzielas, VM, Sluka, Spinrath arXiv:1405.6962]

GUT Breaking Superpotentials with two **24**'s

Requirements:

renormalizable, VEVs for both **24**'s, no massless states in vacuum

(a) $W = M_{24} \text{tr} H_{24}^2 + M'_{24} \text{tr} H'_{24}{}^2 + \kappa' \text{tr} H_{24} H'_{24}{}^2 + \lambda \text{tr} H_{24}^3,$
 \mathbb{Z}_2 with $Q(H_{24}) = 0$ and $Q(H'_{24}) = 1$.

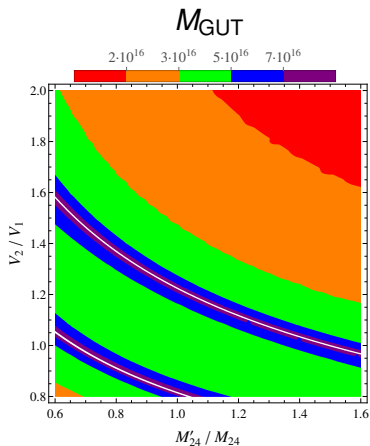
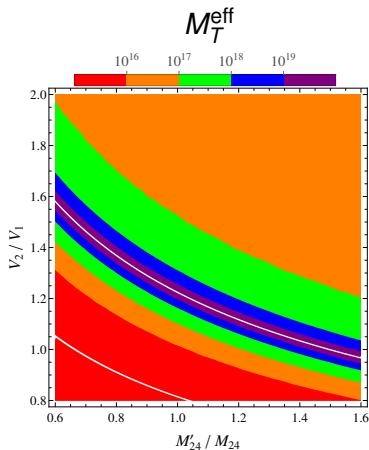
(b) $W = \tilde{M}_{24} \text{tr} H_{24} H'_{24} + \lambda \text{tr} H_{24}^3 + \lambda' \text{tr} H'_{24}{}^3,$
 \mathbb{Z}_3 with $Q(H_{24}) = 2$ and $Q(H'_{24}) = 1$.

(c) $W = \tilde{M}_{24} \text{tr} H_{24} H'_{24} + \lambda \text{tr} H_{24}^3 + \kappa' \text{tr} H_{24} H'_{24}{}^2,$
 \mathbb{Z}_4^R with $Q(H_{24}) = 2$ and $Q(H'_{24}) = 0$ (and $Q(\theta) = 1$).

(d) The trivial case with no additional shaping symmetry.

Grand Unification: Two Loop Analysis

(a) $W = M_{24} \text{tr} H_{24}^2 + M'_{24} \text{tr} H'_{24}{}^2 + \kappa' \text{tr} H_{24} H'_{24}{}^2 + \lambda \text{tr} H_{24}^3$



$M_{\text{SUSY}} = 1 \text{ TeV}$, $M = \sqrt{|M_{24}|^2 + |M'_{24}|^2} = 10^{15} \text{ GeV}$ and equal phases for V_1 , V_2 and M_{24} , M'_{24}
 White stripes: $M_{T(24)}$ or $M_{O(24)} < 10^{13} \text{ GeV}$

Grand Unification: Two Loop Analysis

$$(b) \quad W = \tilde{M}_{24} \operatorname{tr} H_{24} H'_{24} + \lambda \operatorname{tr} H_{24}^3 + \lambda' \operatorname{tr} H'_{24}{}^3$$

$$(c) \quad W = \tilde{M}_{24} \operatorname{tr} H_{24} H'_{24} + \lambda \operatorname{tr} H_{24}^3 + \kappa' \operatorname{tr} H_{24} H'_{24}{}^2$$

$$M_T^{\text{eff, 2-loop}} = \left(\frac{M_{\text{SUSY}}}{1 \text{ TeV}} \right)^{0.74} \cdot \begin{cases} 5.4 \cdot 10^{16} \text{ GeV} \left(\frac{M}{10^{15} \text{ GeV}} \right)^{-0.15} & (b) \\ 7.1 \cdot 10^{15} \text{ GeV} \left(\frac{M}{10^{15} \text{ GeV}} \right)^{-0.19} & (c) \end{cases}$$

$$M_{\text{GUT}}^{2\text{-loop}} = |\sin 2\beta_V|^{-0.48} \left(\frac{M_{\text{SUSY}}}{1 \text{ TeV}} \right)^{-0.41} \cdot \begin{cases} 2.95 \cdot 10^{16} \text{ GeV} \left(\frac{M}{10^{15} \text{ GeV}} \right)^{-0.62} & (b) \\ 4.89 \cdot 10^{16} \text{ GeV} \left(\frac{M}{10^{15} \text{ GeV}} \right)^{-0.64} & (c) \end{cases}$$

$$\text{with } \tan \beta_V \equiv \frac{V_1}{V_2}.$$

Some numbers: $M_T^{\text{eff}} \gtrsim 10^{17} \text{ GeV}$

$$\Rightarrow M_{\text{SUSY}} \gtrsim 2.3 \text{ TeV (b), } 35 \text{ TeV (c)}$$

Flavour GUT Models with DMPM

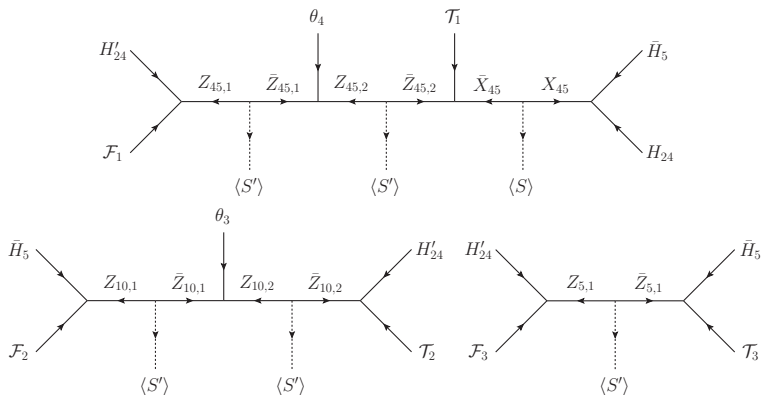
- Yukawa matrix structure:

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_e = \begin{pmatrix} -\frac{1}{2}y_d & 0 & 0 \\ 0 & 6y_s & 0 \\ 0 & 0 & -\frac{3}{2}y_b \end{pmatrix}$$

$$Y_u = \begin{pmatrix} \epsilon_1^2 \epsilon_2^2 & \epsilon_1^2 \epsilon_2 & \epsilon_1 \epsilon_2 \\ \epsilon_1^2 \epsilon_2 & \epsilon_1^2 & \epsilon_1 \\ \epsilon_1 \epsilon_2 & \epsilon_1 & y_{33} \end{pmatrix}$$

- Hierarchy via $\epsilon_i = \frac{\theta_i}{\Lambda}$ with some messenger scale Λ
- For explicit W_u, W_d , see arXiv:1405.6962

Model: Diagonal Y_d and Y_e Yukawa Matrices



- Messenger masses via $\langle S \rangle$, $\langle S' \rangle \Rightarrow$ no corrections to ratios
- Superpotential (a) $\rightarrow \mathbb{Z}_2$ charged H'_{24} , more free M_T^{eff}

Model: Diagonal Y_d and Y_e Yukawa Matrices

Effective/Planck-scale suppressed operators generate μ and μ' with:

$$\mu = \frac{\langle\theta_3\rangle\langle\theta_4\rangle^4}{M_{\text{Pl}}^4}, \quad \mu' = \frac{\langle\theta_3\rangle^4}{M_{\text{Pl}}^4},$$

Number example:

$$\langle H_{24} \rangle \sim 2 \cdot 10^{16} \text{ GeV}$$

$$\langle S \rangle \sim 10^{18} \text{ GeV}$$

$$y_b \sim \frac{\langle H'_{24} \rangle}{\langle S' \rangle} \approx 0.18$$

$$y_s \sim \frac{\langle H'_{24} \rangle \langle \theta_3 \rangle}{\langle S' \rangle^2} \approx 3 \cdot 10^{-3}$$

$$y_d \sim \frac{\langle H_{24} \rangle \langle H'_{24} \rangle \langle \theta_4 \rangle}{\langle S' \rangle^2 \langle S \rangle} \approx 1.6 \cdot 10^{-4}$$

$$M_T^{\text{eff}} \approx 2 \cdot 10^{18} \text{ GeV}$$

$$\mu \approx \mathcal{O}(100) \text{ GeV}$$

$$\mu' \approx \mathcal{O}(10) \text{ TeV}$$

$$M_T \approx 2 \cdot 10^{11} \text{ GeV}$$

[Numbers from Antusch, VM '13 for $\tan \beta = 30$]

Model: Cabibbo Angle θ_C from Y_d (and Y_e)

- Yukawa matrix structure:

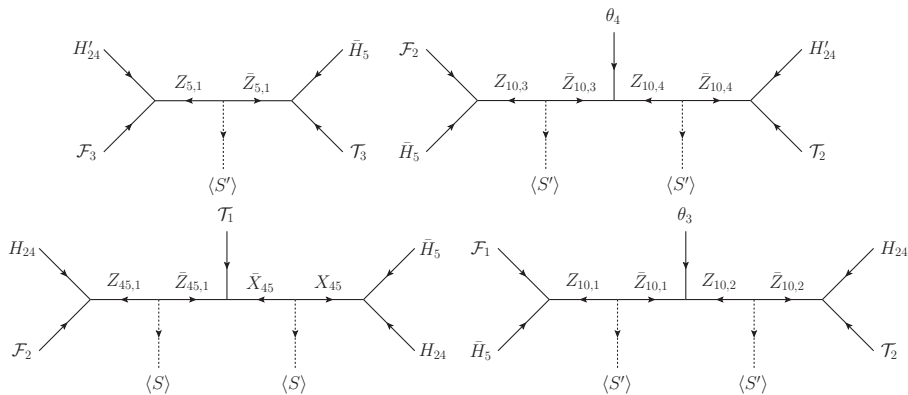
$$Y_d = \begin{pmatrix} 0 & y_{d,12} & 0 \\ y_{d,21} & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 6y_{d,21} & 0 \\ -\frac{1}{2}y_{d,12} & 6y_s & 0 \\ 0 & 0 & -\frac{3}{2}y_b \end{pmatrix}$$

$$Y_u = \begin{pmatrix} y_{11} & y_{12} & 0 \\ y_{12} & y_{22} & y_{23} \\ 0 & y_{23} & y_{33} \end{pmatrix}$$

- \mathcal{T}_2 couples to \mathcal{F}_1 and $\mathcal{F}_2 \Rightarrow$ superpotential (b) to forbid $(Y_d)_{11}$

[Motivated by Antusch, King, Malinsky, Spinrath '09; Antusch, Gross, VM, Sluka '13]

Model: Cabibbo Angle θ_C from Y_d (and Y_e)



Number example:

$$M_T^{\text{eff}} \approx 2 \cdot 10^{18} \text{ GeV},$$

$$\mu \approx \mathcal{O}(300) \text{ GeV},$$

$$M_T \approx 2 \cdot 10^{11} \text{ GeV}$$

$$\mu' \approx \mathcal{O}(10) \text{ TeV}$$

Proton Decay

Proton Decay via Dimension Five Operators

Colour triplets in H_5, \bar{H}_5 couple to matter:

$$W_T = \epsilon_{\alpha\beta} \left(-\frac{1}{2} (Y_{qq})_{ij} \epsilon_{abc} T^a Q_i^{\alpha b} Q_j^{\beta c} + (Y_{ql})_{ij} \bar{T}^a Q_i^{\alpha a} L_j^{\beta} \right) \\ + (Y_{ue})_{ij} T^a \bar{U}_i^a \bar{E}_j - (Y_{ud})_{ij} \epsilon_{abc} \bar{T}^a \bar{U}_i^b \bar{D}_j^c + M_T T^a \bar{T}^a .$$

Messenger sector also generates specific ratios between

$$Y_{ql}, Y_{ud} \leftrightarrow Y_d \text{ and } Y_{qq}, Y_{ue} \leftrightarrow Y_u$$

$Y_{ql}, Y_{ud}, Y_{qq}, Y_{ue}$ predicted from fit of Y_d, Y_e, Y_u

Proton Decay via Dimension Five Operators

Model with diagonal Y_d and Y_e :

$$Y_{ql} = \text{diag} \left(y_d, y_s, -\frac{3}{2}y_b \right) \quad Y_{ud} = \text{diag} \left(\frac{2}{3}y_d, -4y_s, y_b \right)$$

$$Y_{qq} = Y_u$$

$$Y_{ue} = Y_u$$

Model with Cabibbo angle from Y_d, Y_e :

$$Y_{ql} = \begin{pmatrix} 0 & y_{d,12} & 0 \\ y_{d,21} & y_s & 0 \\ 0 & 0 & -\frac{3}{2}y_b \end{pmatrix} \quad Y_{ud} = \begin{pmatrix} 0 & \frac{2}{3}y_{d,12} & 0 \\ -4y_{d,21} & -4y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

$$Y_{qq} = Y_u$$

$$Y_{ue} = Y_u$$

Extensive table of ratios to triplet Yukawa couplings \rightarrow arXiv:1405.6962

Proton Decay via Dimension Five Operators

$$W_{\beta} = \frac{1}{M_T^{\text{eff}}} \left[\frac{1}{2} Y_{qq}^{ij} Y_{ql}^{mn} Q_i Q_j Q_m L_n + Y_{ue}^{ij} Y_{ud}^{mn} \bar{U}_i \bar{E}_j \bar{U}_m \bar{D}_n \right]$$

- $M_T^{\text{eff}} \gtrsim 10^{17}$ GeV due to two **24**'s
- Full control over all 3+4 Yukawa matrices
- Diagonal Y_{ql} and $Y_{ud} \rightarrow$ suppressed flavour transitions

RGE running/matching and dressing with superpartner fields:

$$A_{LLLL} \sim \tan \beta \text{ and } A_{RRRR} \sim (\tan \beta)^2$$

$$\rightarrow \boxed{\frac{y_{\tau}}{y_b} = -\frac{3}{2} \text{ needs } \tan \beta \approx 25} \text{ vs. } \boxed{y_{\tau} = y_b = y_t \text{ needs } \tan \beta \approx 50}$$

Summary and Conclusions

Summary and Conclusions

- DMPM can be successfully combined with predictive Yukawa coupling ratios using novel CG factors
- Adding another **24** allows for high enough M_T^{eff} to avoid problems with proton decay
- Adjoint field charged under shaping symmetry allows predictive Yukawa coupling ratios
- No danger from M_{Pl} suppressed operators due to charge assignment and messenger UV completion

Thank you for your attention!