Non-resonant $B \to K\pi ll$

based on works with Diganta Das, GH, Martin Jung, Alex Shires,
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Flavor physics program: Test the SM, explore its borders and the physics beyond!

FCNCs are ideally suited to do so.

3 fb^{-1} at LHCb: about 3000 events for $B \to K^* \mu\mu$.

Allows precision physics, and angular analysis via $B \to K^*(\to K\pi)\mu\mu$ and $B_s \to \varphi(\to KK)\mu\mu$ alike.

Unprecendented event rates give new opportunities and new background e.g. other sources of $B \to K\pi\mu\mu$.

Other sources of $B \to K\pi\mu\mu$:

– S-wave resonances: Becirevic et al, Egede et al, Descotes-Genon et al, this work

– P,D ... resonances this work

	J^P	mass [MeV]	width [MeV]	branching ratio to $\bar{K}\pi$
κ	0^+	658	557	~ 100 % Descotes-Genon et al
$K^{*0}(892)$	1^-	895.8	47.4	~ 100 %
$K^*(1410)$	1^-	1414	232	~ 7 %
$K_0^*(1430)$	0^+	1425	270	~ 100 %
$K_2^{*0}(1430)$	2^+	1432	109	~ 50 %
$K^*(1680)$	1^-	1717	322	~ 39 %
$K_3^*(1780)$	3^-	1776	159	~ 19 %

– non-resonant: infinite tower of ℓ-waves this work

– known as contribution to inclusive $B \to X s l l$ decays towards end point Buchalla, Isidori ’98

– can be treated within OPE at low recoil due to hard momentum exchange Buchalla, Isidori ’98

– At $1/m_b$ a la Grinstein, Pirjol ’04 employ Isgur-Wise relations this work, transversity amp’s factorize at LO in $1/m_b$, corrections suppressed

$$H_{0,\|}^{L/R} = C_-^{L/R}(q^2)\, F_{0,\|}(q^2, p^2, \cos\vartheta_K) \quad (1)$$

$$H_{\perp}^{L/R} = C_+^{L/R}(q^2)\, F_{\perp}(q^2, p^2, \cos\vartheta_K) \quad (2)$$

$C_\pm^{L/R}(q^2)$: short-distance, universal, as in $B \to K^{(*)} ll$ Bobeth,GH,van Dyk

$F_i = F_i(q^2, p^2, \cos\vartheta_K)$ form factors

Non-resonant $B \to K\pi ll$ short-distance structure

Short-distance coefficients:

$$C^L_\pm(q^2) = C_9^{\rm eff}(q^2) \pm C_9' - (C_{10} \pm C_{10}') + \kappa \frac{2m_b m_B}{q^2}(C_7^{\rm eff} \pm C_7')\,, \tag{3}$$

$$C^R_\pm(q^2) = C_9^{\rm eff}(q^2) \pm C_9' + C_{10} \pm C_{10}' + \kappa \frac{2m_b m_B}{q^2}(C_7^{\rm eff} \pm C_7')\,, \tag{4}$$

Combinations which can be probed in non-resonant decays:

$$\rho_1^\pm = \frac{1}{2}(|C^R_\pm|^2 + |C^L_\pm|^2)\,, \quad \delta\rho = \frac{1}{4}(|C^R_-|^2 - |C^L_-|^2)\,, \quad \rho_2^\pm = \frac{1}{4}(C^R_+ C^{R*}_- \mp C^L_- C^{L*}_+)\,, \tag{5}$$

NEW to non-resonant decays (beyond $B \to K^{(*)}ll$): $\delta\rho, \rho_2^-$ this work

$$\delta\rho = \mathrm{Re}\left[\left(C_9^{\rm eff} - C_9' + \kappa\frac{2m_b m_B}{q^2}(C_7^{\rm eff} - C_7')\right)\left(C_{10} - C_{10}'\right)^*\right]\,, \tag{6}$$

$$\mathrm{Re}\rho_2^- = \frac{1}{2}\left[|C_{10}|^2 - |C_{10}'|^2 + \left|C_9^{\rm eff} + \kappa\frac{2m_b m_B}{q^2}C_7^{\rm eff}\right|^2 - \left|C_9' + \kappa\frac{2m_b m_B}{q^2}C_7'\right|^2\right]\,, \tag{7}$$

$$\mathrm{Im}\rho_2^- = \mathrm{Im}\left[C_{10}'\left(C_9^{\rm eff} + \kappa\frac{2m_b m_B}{q^2}C_7^{\rm eff}\right)^* - C_{10}\left(C_9' + \kappa\frac{2m_b m_B}{q^2}C_7'\right)^*\right]\,. \tag{8}$$

Non-resonant $B \to K\pi ll$ form factors

$$F_0 = \frac{\mathcal{N}_{nr}}{2}\left[\lambda^{1/2} w_+(q^2, p^2, \cos\vartheta_K) + \frac{1}{p^2}\{(m_K^2 - m_\pi^2)\lambda^{1/2} - (m_B^2 - q^2 - p^2)\lambda_p^{1/2}\cos\vartheta_K\} w_-(q^2, p^2, \cos\vartheta_K)\right],$$

$$F_\| = \mathcal{N}_{nr}\sqrt{\lambda_p \frac{q^2}{p^2}}\, w_-(q^2, p^2, \cos\vartheta_K), \qquad F_\perp = \frac{\mathcal{N}_{nr}}{2}\sqrt{\lambda\lambda_p \frac{q^2}{p^2}}\, h(q^2, p^2, \cos\vartheta_K). \tag{9}$$

Need $w_\pm, h$; Dipole form factors $w'_\pm, h'$ have been eliminated by EOM at $O(1/m_b)$.

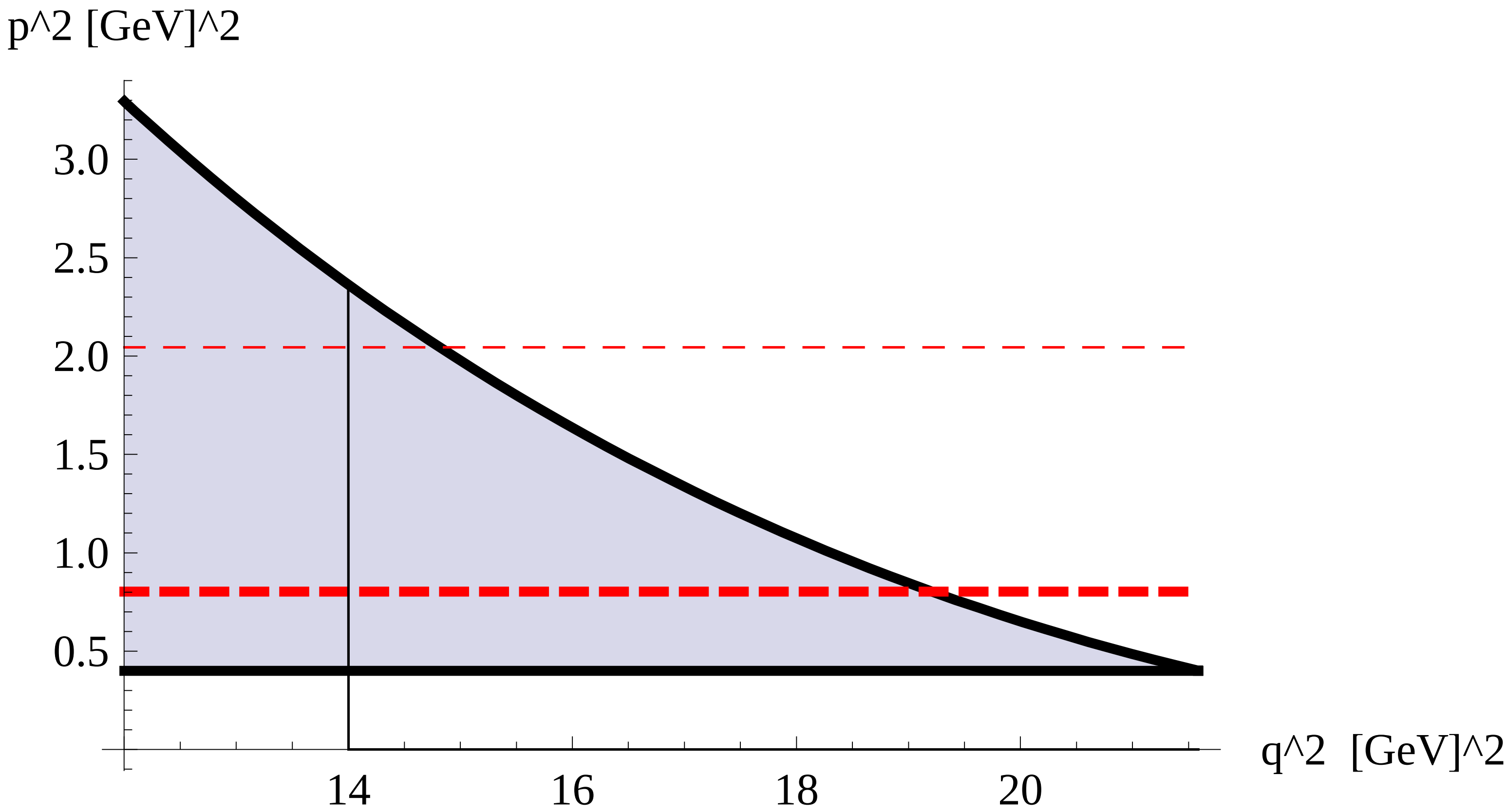
$$\langle \bar{K}^i(p_K)\pi^j(p_\pi)|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}(p_B)\rangle = ic_{ij}\left[w_+ p_\mu + w_- P_\mu + r q_\mu + ih\epsilon_{\mu\alpha\beta\gamma} p_B^\alpha p^\beta P^\gamma\right], \tag{10}$$

$$\langle \bar{K}^i(p_K)\pi^j(p_\pi)|\bar{s}iq^\nu\sigma_{\mu\nu}(1+\gamma_5)b|\bar{B}(p_B)\rangle = -ic_{ij}m_B\left[w'_+ p_\mu + w'_- P_\mu + r' q_\mu + ih'\varepsilon_{\mu\alpha\beta\gamma} p_B^\alpha p^\beta P^\gamma\right], \tag{11}$$

$w_\pm, h$ available from HHχ PT Lee, Liu, Wise, Burdman, Donoghue and later works, valid for soft K, π momenta in B cms. Form factors could be included by other means; desireable for control outside of $p^2 \lesssim 1$ GeV^2.

Ready for phenomenology!

$B \to K\pi ll$ phase space at low recoil $\mathcal{O}(m_b^2) \sim q^2 \gtrsim 14 - 15$ GeV^2



in low recoil region $\leftrightarrow$ small $p^2 \sim \mathcal{O}(1\,\text{GeV}^2)$

Angular expansion and zero recoil

The transversity amplitudes can be wave-expanded. In the OPE, this is equivalent for form factors:

$$F_0 = \sum_{\ell=0} a_0^\ell(q^2,p^2)\, P_\ell^{m=0}(\cos\vartheta_K)\,, \qquad F_{\parallel,\perp} = \sum_{\ell=1} a_{\parallel,\perp}^\ell(q^2,p^2)\, \frac{P_\ell^{m=1}(\cos\vartheta_K)}{\sin\vartheta_K}\,,$$

Useful for phenomenology including resonances and to understand endpoint relations. this work

Transversity amplitudes of weak decays $B \to K_J ll$ at endpoint are subjected to kinematic constraints (absence of direction) GH, Zwicky'13

$$\begin{aligned}
&H_0^{L/R} = 0 + \mathcal{O}\left(\sqrt{\lambda}\right)\,, && (J=0)\\
&H_\perp^{L/R} = 0 + \mathcal{O}\left(\sqrt{\lambda}\right)\,, \qquad H_\parallel^{L/R} = -\sqrt{2}H_0^{L/R} + \mathcal{O}(\lambda)\,, && (J=1)\\
&H_{0,\parallel,\perp}^{L/R} = 0 + \mathcal{O}\left(\lambda^{(J-1)/2}\right)\,. && (J\geq 2)
\end{aligned} \tag{12}$$

Endpoint of non-resonant decays at $\lambda_*(m_B^2, q^2, p^2) = 0$ is dominated by $J = 1$ amplitudes $F_{\perp,\|}$ (only ones non-vanishing in SM+SM′ basis), which are related. Explicitly

$$\hat{a}_0^1 = \hat{a}_{\|}^1 = -\sqrt{\frac{q^2}{p^2}\lambda_p}\; w_-\Big|_{\lambda=\lambda_*}$$

Non-resonant decays feature endpoint structure as $J = 1$ modes, however, in general at different q^2.

Example: $F_L = 1/3$ etc. or isotropicity in lepton and kaon-angle distributions, but not the one in the angle between the two decay planes. this work

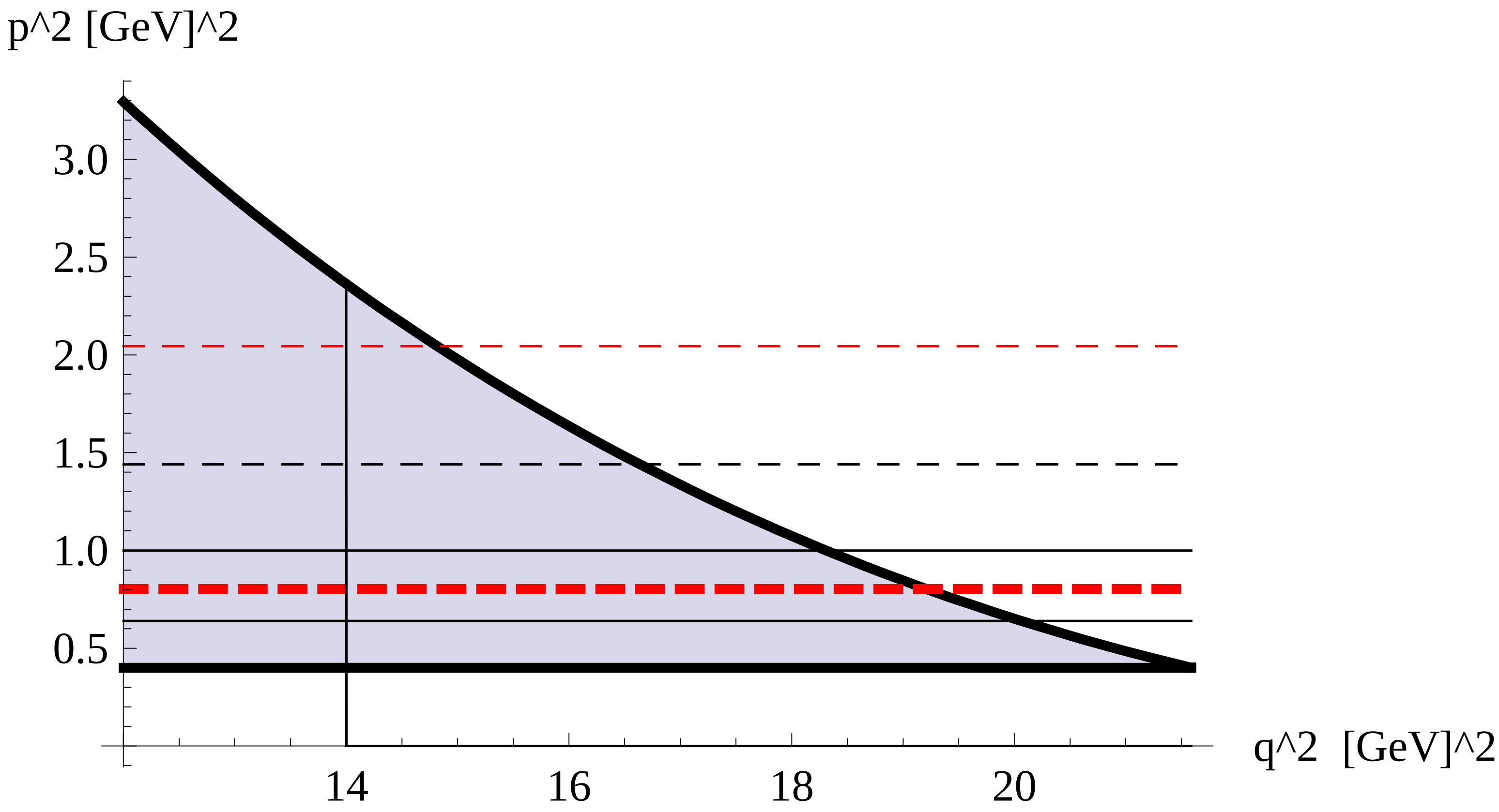
$d\Gamma^5$ phase space can be deduced from K_{l4}.

$$d^5\Gamma = \frac{1}{2\pi}\left[\sum c_i(\vartheta_\ell,\varphi)I_i(q^2,p^2,\cos\vartheta_K)\right] dq^2 dp^2 d\cos\vartheta_K d\cos\vartheta_\ell d\varphi\,, \tag{13}$$

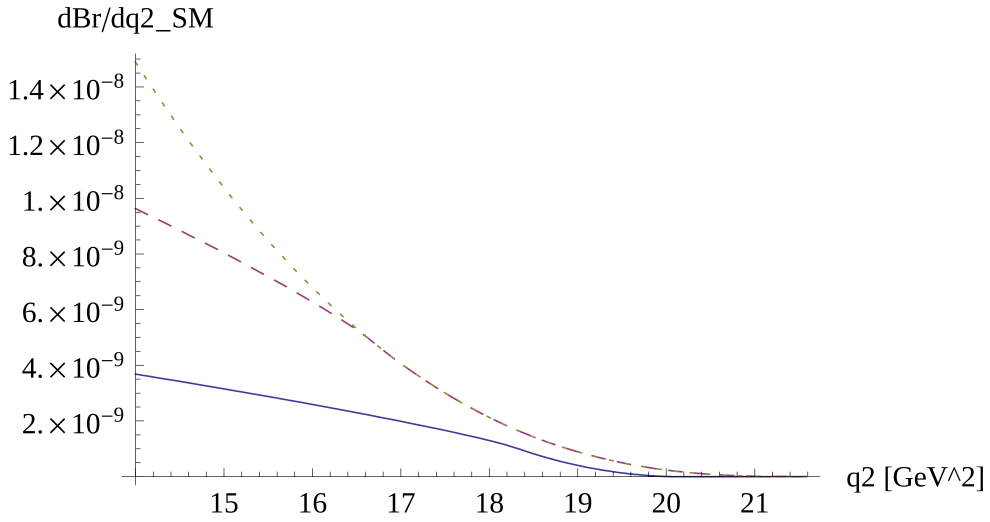
$$\begin{aligned}
I_1 &= \frac{1}{8}\left[|\mathcal{F}_0|^2\rho_1^- + \frac{3}{2}\sin^2\vartheta_K\{|\mathcal{F}_\parallel|^2\rho_1^- + |\mathcal{F}_\perp|^2\rho_1^+\}\right],\\
I_2 &= -\frac{1}{8}\left[|\mathcal{F}_0|^2\rho_1^- - \frac{1}{2}\sin^2\vartheta_K\{|\mathcal{F}_\parallel|^2\rho_1^- + |\mathcal{F}_\perp|^2\rho_1^+\}\right],\\
I_3 &= \frac{1}{8}\left[|\mathcal{F}_\perp|^2\rho_1^+ - |\mathcal{F}_\parallel|^2\rho_1^-\right]\sin^2\vartheta_K\,,\\
&\ldots\\
I_6 &= -\left[Re(\mathcal{F}_\parallel\mathcal{F}_\perp^*)Re\rho_2^+ + Im(\mathcal{F}_\parallel\mathcal{F}_\perp^*)Im\rho_2^-\right]\sin^2\vartheta_K\,,\\
I_7 &= \delta\rho Im(\mathcal{F}_0\mathcal{F}_\parallel^*)\sin\vartheta_K\,,\\
I_8 &= \frac{1}{2}\left[Re(\mathcal{F}_0\mathcal{F}_\perp^*)Im\rho_2^+ - Im(\mathcal{F}_0\mathcal{F}_\perp^*)Re\rho_2^-\right]\sin\vartheta_K\,,\\
I_9 &= \frac{1}{2}\left[Re(\mathcal{F}_\perp\mathcal{F}_\parallel^*)Im\rho_2^+ + Im(\mathcal{F}_\perp\mathcal{F}_\parallel^*)Re\rho_2^-\right]\sin^2\vartheta_K\,,
\end{aligned} \tag{14}$$

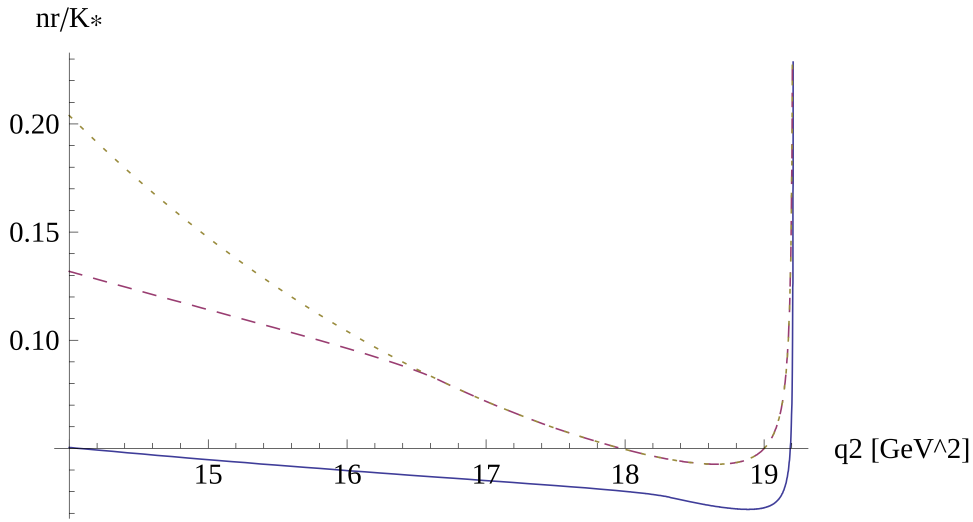
Framework allows to incorporate resonances via angular expansion!

$B \to K^* \mu\mu$ cuts: solid: K^*-signal cuts; dashed $p^2_{min} < p^2 < 1.44$: "total S+P" window



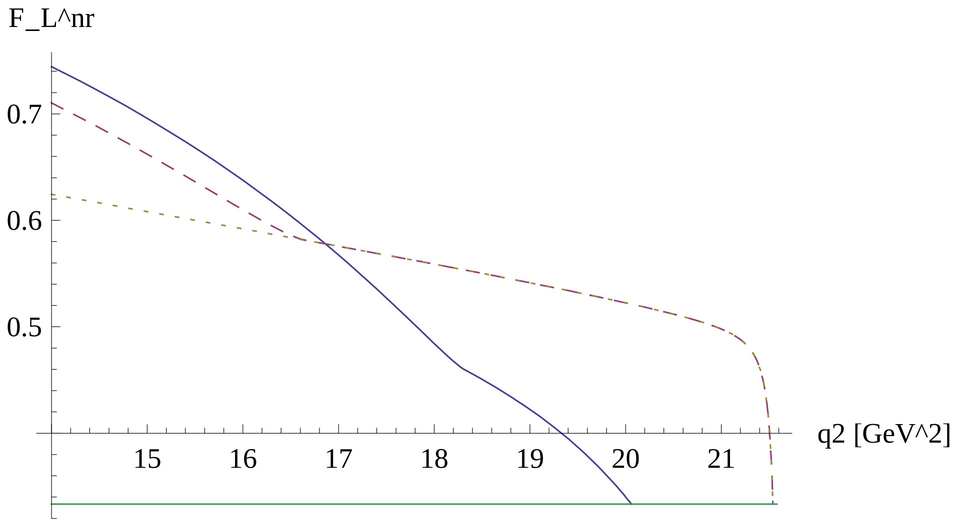
SM branching ratios w/o cuts





Brs few $\times 10^{-8}$; with signal cuts few percent BDG to $B \to K^* \mu\mu$

blue solid: $B \to K^* \mu\mu$ signal cut



blue solid: $B \to K^* \mu\mu$ signal cut; green horizontal line: $1/3$

Endpoint $B \to K^* \mu\mu$: $q^2 = 19.2\,$GeV^2,
non-resonant endpoint: $q^2 = (m_B - p_{min}^2)^2$ (cut-dependent).

– Universal structure of OPE allows via angular analysis to obtain short-distance and form factor-free observables as in $B \to K^{(*)} ll$.
– Framework allows to include non-resonant and resonant decays simultaneously

$$\mathcal{F}_0 \equiv \mathcal{F}_0(q^2, p^2, \cos\vartheta_K) = F_0(q^2, p^2, \cos\vartheta_K) + \sum_R P^0_{J_R}(\cos\vartheta_K) \cdot F_{0\,J_R}(q^2, p^2)\,, \tag{15}$$

$$\mathcal{F}_i \equiv \mathcal{F}_i(q^2, p^2, \cos\vartheta_K) = F_i(q^2, p^2, \cos\vartheta_K) + \sum_R \frac{P^1_{J_R}(\cos\vartheta_K)}{\sin\vartheta_K} \cdot F_{i\,J_R}(q^2, p^2)\,, \quad i = \parallel, \perp\,.$$

- We can probe interference effects (relative strong phases), e;g. in ratios

$$\frac{J_{7c}}{J_6} = \frac{\mathrm{Im}(F_{0P} F^*_{\parallel P})}{\mathrm{Re}(F_{\parallel P} F^*_{\perp P})}\,, \tag{16}$$

$$\frac{J_{8c}}{J_3} = 2\,\frac{\mathrm{Im}(F_{0P} F^*_{\perp P})}{|F_{\perp P}|^2 - |F_{\parallel P}|^2}\,, \tag{17}$$

$$\frac{J_9}{J_3} = 2\,\frac{\mathrm{Im}(F_{\perp P} F^*_{\parallel P})}{|F_{\perp P}|^2 - |F_{\parallel P}|^2}\,. \tag{18}$$

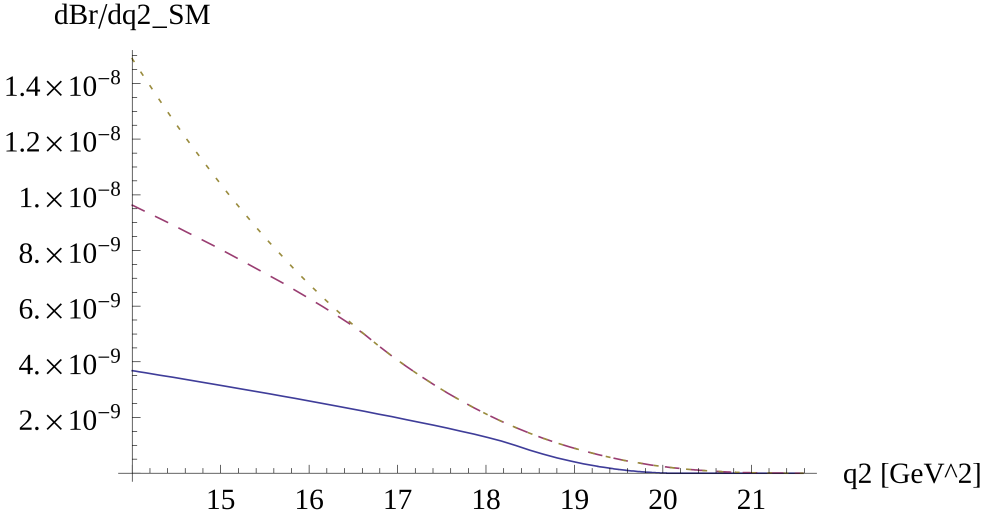
which can be at 0.1 level for maximal phase between non-resonant and K^*.

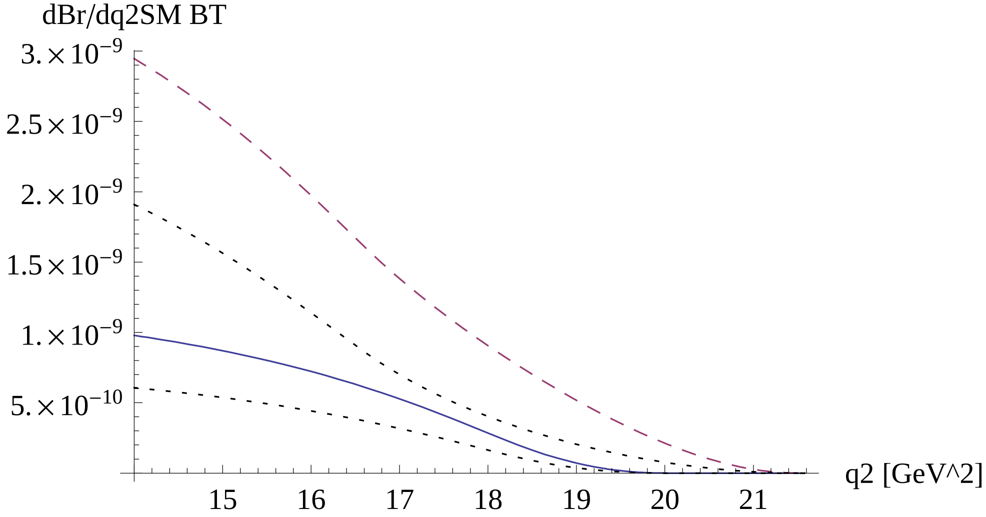
– With non-zero phases there are contributions to $B \to K^* ll$ null tests, e.g. J_7

$$\tilde{J}_{7c}^{\rm SM}(q^2) = \int dp^2 J_{7c}^{\rm SM} = -\rho_2^{\rm SM} \int dp^2 Im(F_{0P} F^*_{\|P}) \quad (19)$$

which can be at the 5 percent level compared to the $B \to K^* ll$ rate.

Non-resonant vs resonant S-wave





blue solid: $B \to K^* \mu\mu$ signal cut

rirght plot: S-wave from Becirevic Tayduganov line shape

$$BW_S(p^2) = \mathcal{N}_S \left[\frac{-g_\kappa}{(m_\kappa - i\Gamma_\kappa/2)^2 - p^2} + \frac{1}{(m_{K_0^*} - i\Gamma_{K_0^*}/2)^2 - p^2} \right] ,$$

with $|g_\kappa|$ within 0 and 0.2 and $arg g_\kappa$ within $\pi/2$ (largerst effect) and π.

–Non-resonant decays provide a background to important signal modes $B \to K^*(\to K\pi)\mu\mu$ and $B_s \to \varphi(\to KK)\mu\mu$. We present distributions for low recoil.

– SM branching ratios are at few 10^{-8}; contributions can be much suppressed by cuts around the K^* and notably, the narrow φ.

– It would be useful to have $B \to K\pi$ form factors available by other means (lattice QCD).

– We find that the dominant background is non-resonant. Nonresonant distributions helpful in modelling background.

– Further applications on extracting strong phase, pollution to the angular coefficients in the $B \to K^*ll$ analysis.

– More opportunitiies awaiting $B \to K\pi\mu\mu$ –STAY TUNED

$$\frac{d^5\Gamma(S+P+D)}{dq^2 dp^2 d\cos\vartheta_K d\cos\vartheta_\ell d\varphi} = \frac{1}{2\pi}\Bigg[\sum_{i=1,2} c_i \left(J_{icc}\cos^2\vartheta_K + J_{iss}\sin^2\vartheta_K + J_{ic}\cos\vartheta_K\right.$$
$$\left. + J_{issc}\sin^2\vartheta_K\cos\vartheta_K + J_{isscc}\sin^2\vartheta_K\cos^2\vartheta_K\right)$$
$$+ \sum_{i=3,6,9} c_i \left(J_{icc}\cos^2\vartheta_K + J_i + J_{ic}\cos\vartheta_K\right)\sin^2\vartheta_K$$
$$+ \sum_{i=4,5,7,8} c_i \left(J_{icc}\cos^2\vartheta_K + J_{iss}\sin^2\vartheta_K + J_{ic}\cos\vartheta_K + J_{issc}\sin^2\vartheta_K\cos\vartheta_K\right)\sin\vartheta_K\Bigg]. \qquad (20)$$

For pure P- wave, only the coefficients $J_{iss,icc}, i = 1,2$ and $J_i, i = 3,6,9$ and J_{ic} for $i = 4,5,7,8$ remain.

E.G. $J_{7c} = -\delta\rho Im(3F_{0D}F^*_{\parallel D} + 3F_{0S}F^*_{\parallel D} + F_{0P}F^*_{\parallel P})$