Lepton mixing from generalized CP and Delta(96) family symmetry

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In collaboration with Stephen F. King

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Outline

1. General approach
2. $\Delta(96)$ and generalized CP
3. Predictions for lepton flavor mixing in $\Delta(96)$ and generalized CP
4. Summary

Neutrino mixing: what we know and don't know

(Taken from NuFIT, JHEP 1212:123,2012.)

	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.302^{+0.013}_{-0.012}$	$0.267 \to 0.344$	$0.311^{+0.013}_{-0.013}$	$0.273 \to 0.354$
$\theta_{12}/^\circ$	$33.36^{+0.81}_{-0.78}$	$31.09 \to 35.89$	$33.87^{+0.82}_{-0.80}$	$31.52 \to 36.49$
$\sin^2\theta_{23}$	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$	$0.342 \to 0.667$	$0.416^{+0.036}_{-0.029} \oplus 0.600^{+0.019}_{-0.026}$	$0.341 \to 0.670$
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.3}_{-1.3}$	$35.8 \to 54.8$	$40.1^{+2.1}_{-1.6} \oplus 50.7^{+1.2}_{-1.5}$	$35.7 \to 55.0$
$\sin^2\theta_{13}$	$0.0227^{+0.0023}_{-0.0024}$	$0.0156 \to 0.0299$	$0.0255^{+0.0024}_{-0.0024}$	$0.0181 \to 0.0327$
$\theta_{13}/^\circ$	$8.66^{+0.44}_{-0.46}$	$7.19 \to 9.96$	$9.20^{+0.41}_{-0.45}$	$7.73 \to 10.42$
$\delta_{CP}/^\circ$	300^{+66}_{-138}	$0 \to 360$	298^{+59}_{-145}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.18}_{-0.19}$	$7.00 \to 8.09$	$7.51^{+0.21}_{-0.15}$	$7.04 \to 8.12$
$\frac{\Delta m^2_{31}}{10^{-3}\ \text{eV}^2}$ (N)	$+2.473^{+0.070}_{-0.067}$	$+2.276 \to +2.695$	$+2.489^{+0.055}_{-0.051}$	$+2.294 \to +2.715$
$\frac{\Delta m^2_{32}}{10^{-3}\ \text{eV}^2}$ (I)	$-2.427^{+0.042}_{-0.065}$	$-2.649 \to -2.242$	$-2.468^{+0.073}_{-0.065}$	$-2.678 \to -2.252$

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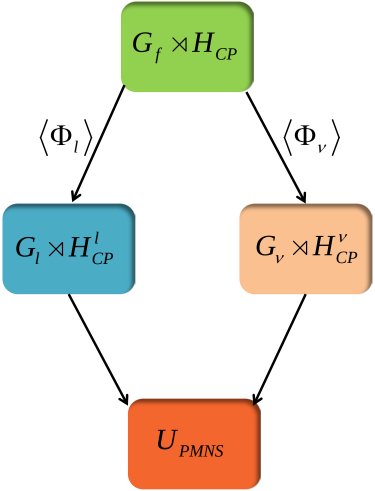
Unknown quantities:① octant of θ_{23} ②Majorana CP phases: α_{21} and α_{31} ③ absolute neutrino masses and their order.

The philosophy of Family symmetry and Generalized CP symmetry



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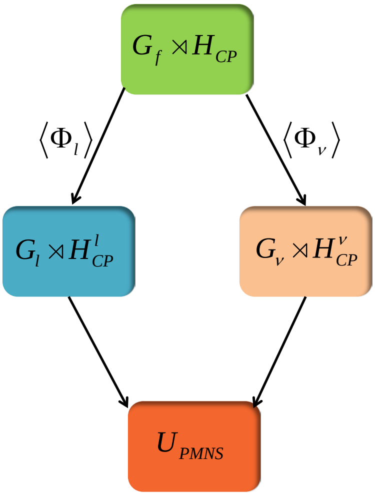

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- Dirac CP would be generally conserved and Majorana CP phases are not constrained if only family symmetry is imposed.

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with $x' = (t, -\mathbf{x})$, X is a unitary matrix.

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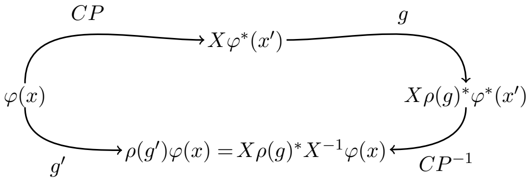

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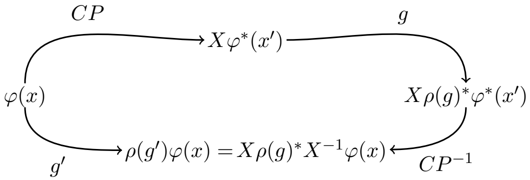
Consistency equation:

$$X\rho^*(g)X^{-1} = \rho(g'), \quad g, g' \in G_f\,.$$



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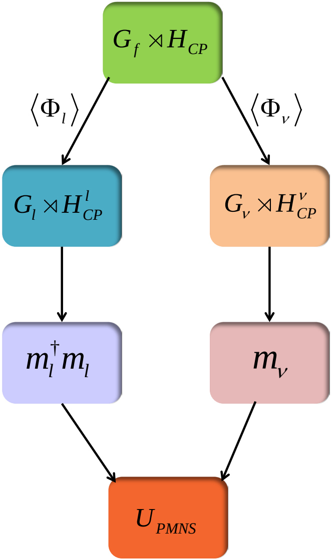
Lepton mixing from remnant symmetries

Neutrino mass term: $\nu_i^T C^{-1} (m_\nu)_{ij} \nu_j$

Charged lepton mass term: $\bar{l}_{R,i} (m_l)_{ij} l_{L,j} + h.c.$



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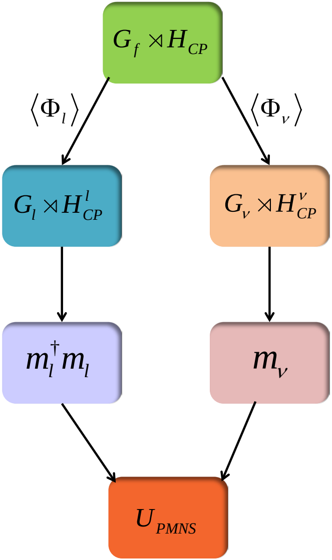
- Invariant under remnant flavor symmetry

$$\rho_{\mathbf{3}}^T(g_{\nu_i}) m_\nu \rho_{\mathbf{3}}(g_{\nu_i}) = m_\nu, \quad g_{\nu_i} \in G_\nu,$$
$$\rho_{\mathbf{3}}^\dagger(g_{l_i}) m_l^\dagger m_l \rho_{\mathbf{3}}(g_{l_i}) = m_l^\dagger m_l, \quad g_{l_i} \in G_l\,.$$



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$$X_{\nu\mathbf{3}}^T m_\nu X_{\nu\mathbf{3}} = m_\nu^*, \qquad X_{\nu\mathbf{3}} \in H_{CP}^\nu,$$
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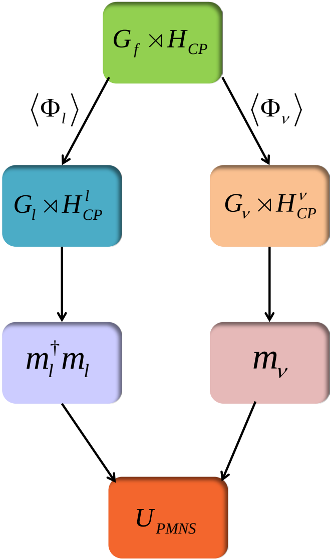

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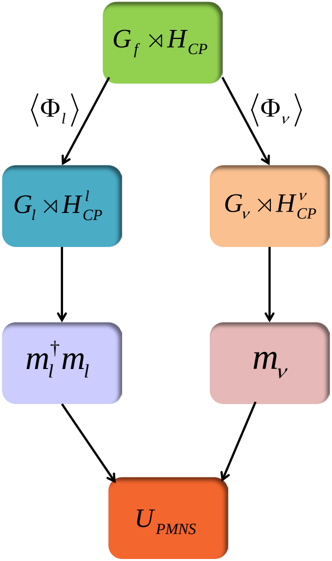
- For $G'_\nu = h G_\nu h^{-1}$, $G'_l = h G_l h^{-1}$

$$H_{CP}^{\nu'} = \rho(h) H_{CP}^\nu \rho^T(h), \; H_{CP}^{l'} = \rho(h) H_{CP}^l \rho^T(h),$$
$$m'_\nu = \rho_{\mathbf{3}}^*(h) m_\nu \rho_{\mathbf{3}}^\dagger(h), \qquad m_l'^\dagger m'_l = \rho_{\mathbf{3}}(h) m_l^\dagger m_l \rho_{\mathbf{3}}^\dagger(h)$$

The same predictions for U_{PMNS}!



Group theory of $\Delta(96)$

- $\Delta(96)$ is a non-abelian finite subgroup of SU(3) of order 96, it is isomorphic to $(Z_4 \times Z_4) \rtimes S_3$:

$$S^2 = T^3 = U^2 = (ST)^3 = 1, \quad SU = US$$
$$(TU)^8 = 1, \quad (TUT^2U)^3 = 1, \quad (UTSUT^2UT)^2 = 1\,.$$

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- Working basis $\hookrightarrow$ “CP" basis
  Generators in triplet representation $\mathbf{3}$: $\omega = e^{2\pi i/3}$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$U = \frac{1}{3}\begin{pmatrix} -1-\sqrt{3} & -1 & -1+\sqrt{3} \\ -1 & -1+\sqrt{3} & -1-\sqrt{3} \\ -1+\sqrt{3} & -1-\sqrt{3} & -1 \end{pmatrix}.$$

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- Generalised CP consistent with $\Delta(96) \Leftarrow$ consistency equation

$$\left.\begin{array}{l} X(\mathfrak{u})\rho^*(S)X^{-1}(\mathfrak{u}) = \rho(\mathfrak{u}(S)) = \rho(S), \\ X(\mathfrak{u})\rho^*(T)X^{-1}(\mathfrak{u}) = \rho(\mathfrak{u}(T)) = \rho(T^2), \\ X(\mathfrak{u})\rho^*(U)X^{-1}(\mathfrak{u}) = \rho(\mathfrak{u}(U)) = \rho(U)\,. \end{array}\right\}$$

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- Including inner automorphism, the generalized CP transformation is

$$X = \rho(h), \quad h \in \Delta(96)\,,$$

which is of the same form as flavor symmetry transformation.

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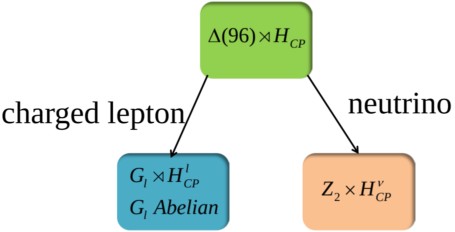

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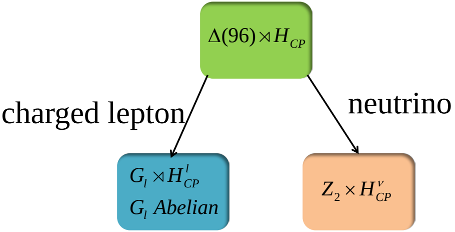

①U_{PMNS} is determined up to permutations of rows and columns.
② Mixing angles and CP phases are predicted in terms of one parameter θ.

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Mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{3}\left[1-\cos\left(\frac{\pi}{6}-2\theta\right)\right], \quad \sin^2\theta_{12} = \frac{1}{2+\cos\left(\frac{\pi}{6}-2\theta\right)}\,,$$

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CP is conserved!

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Mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{3}\left[1-\cos\left(\frac{\pi}{6}-2\theta\right)\right], \quad \sin^2\theta_{12} = \frac{1}{2+\cos\left(\frac{\pi}{6}-2\theta\right)}\,,$$

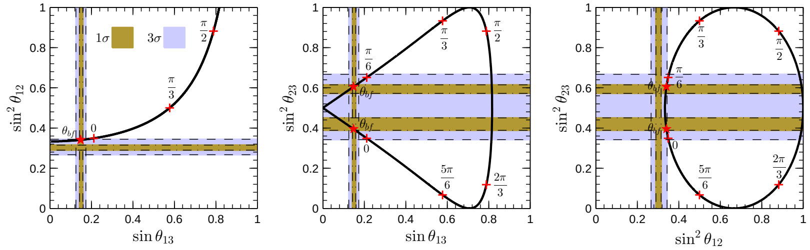
$$\sin^2\theta_{23} = \frac{1+\sin 2\theta}{2+\cos\left(\frac{\pi}{6}-2\theta\right)}, \quad \tan\delta_{CP} = \tan\alpha_{21} = \tan\alpha_{31} = 0\,.$$

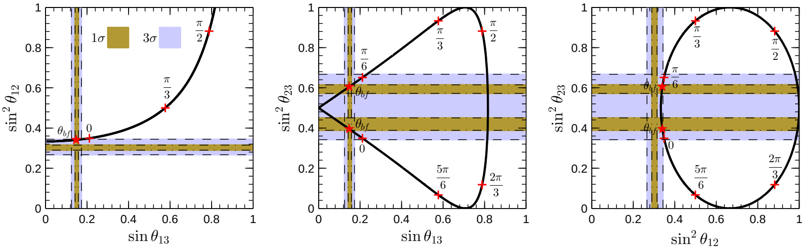
CP is conserved!

Best fitting:

$$\theta_{\rm bf} \simeq 0.0798, \quad \sin^2\theta_{13}(\theta_{\rm bf}) \simeq 0.0218, \quad \sin^2\theta_{12}(\theta_{\rm bf}) \simeq 0.341,$$

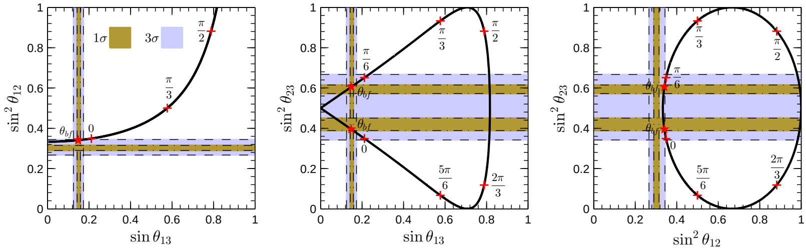
$$\sin^2\theta_{23}(\theta_{\rm bf}) \simeq 0.395 \text{ or } 0.605, \quad \chi^2_{\rm min} \simeq 9.548(9.303) \text{ for } \theta_{23} < (>)\pi/4\,.$$


1σ
3σ
$\sin^2\theta_{12}$
$\sin\theta_{13}$
$\sin^2\theta_{23}$
$\sin\theta_{13}$
$\sin^2\theta_{23}$
$\sin^2\theta_{12}$
θ_{bf}
0
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{5\pi}{6}$



■ Scenario 1.2: $G_l = Z_3^T,\ G_\nu = Z_2^S,\ H_{CP}^\nu = \{\rho(UTSUT^2UT), \rho(UTUT^2UST)\}$

$$U_{PMNS} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2i\cos\theta & \sqrt{2} & -2i\sin\theta \\ -\sqrt{3}\sin\theta - i\cos\theta & \sqrt{2} & -\sqrt{3}\cos\theta + i\sin\theta \\ \sqrt{3}\sin\theta - i\cos\theta & \sqrt{2} & \sqrt{3}\cos\theta + i\sin\theta \end{pmatrix}.$$



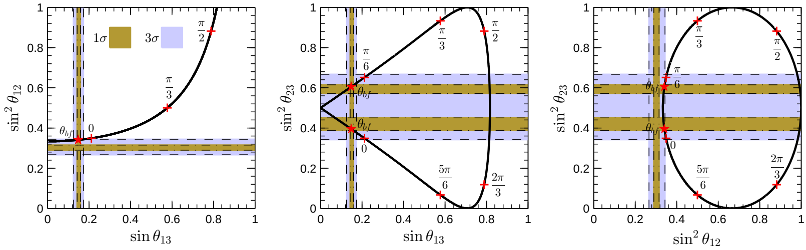
■ Scenario 1.2: $G_l = Z_3^T,\, G_\nu = Z_2^S,\, H^\nu_{CP} = \{\rho(UTSUT^2UT), \rho(UTUT^2UST)\}$

$$U_{PMNS} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2i\cos\theta & \sqrt{2} & -2i\sin\theta \\ -\sqrt{3}\sin\theta - i\cos\theta & \sqrt{2} & -\sqrt{3}\cos\theta + i\sin\theta \\ \sqrt{3}\sin\theta - i\cos\theta & \sqrt{2} & \sqrt{3}\cos\theta + i\sin\theta \end{pmatrix}.$$

Mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{3}\left(1-\cos 2\theta\right), \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta}, \quad \sin^2\theta_{23} = \frac{1}{2},$$

$$|J_{CP}| = \frac{1}{6\sqrt{3}}\left|\sin 2\theta\right|, \quad \cot\delta_{CP} = \tan\alpha_{21} = \tan\alpha_{31} = 0.$$



■ Scenario 1.2: $G_l = Z_3^T,\ G_\nu = Z_2^S,\ H_{CP}^\nu = \{\rho(UTSUT^2UT), \rho(UTUT^2UST)\}$

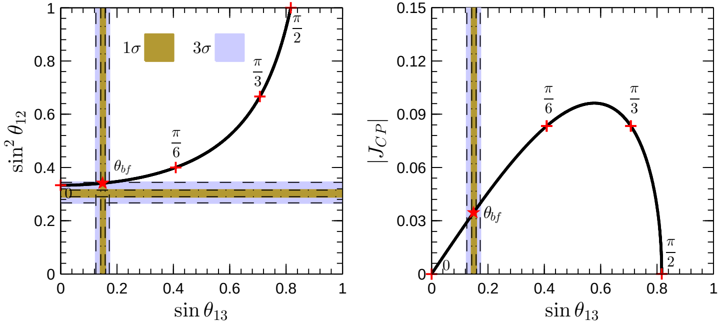
$$U_{PMNS} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2i\cos\theta & \sqrt{2} & -2i\sin\theta \\ -\sqrt{3}\sin\theta - i\cos\theta & \sqrt{2} & -\sqrt{3}\cos\theta + i\sin\theta \\ \sqrt{3}\sin\theta - i\cos\theta & \sqrt{2} & \sqrt{3}\cos\theta + i\sin\theta \end{pmatrix}.$$

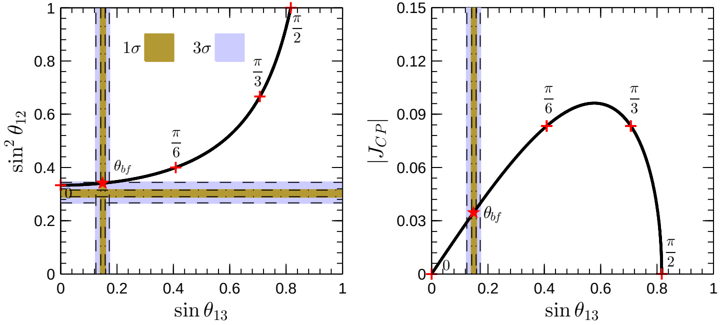
Mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{3}(1-\cos 2\theta), \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta}, \quad \sin^2\theta_{23} = \frac{1}{2},$$

$$|J_{CP}| = \frac{1}{6\sqrt{3}}|\sin 2\theta|, \quad \cot\delta_{CP} = \tan\alpha_{21} = \tan\alpha_{31} = 0.$$

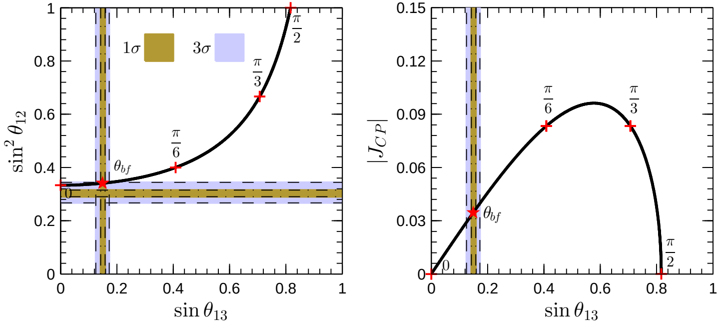
Maximal Dirac CP violation and maximal atmospheric mixing!


1σ
3σ
$\sin^2\theta_{12}$
$\sin\theta_{13}$
θ_{bf}
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
0
$|J_{CP}|$
$\sin\theta_{13}$
θ_{bf}
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
0



Scenario 1.3: $G_l = Z_3^T,\ G_\nu = Z_2^S,\ H_{CP}^\nu = \left\{\rho(T^2UT^2), \rho(ST^2UT^2)\right\}$

$$U_{PMNS} = \frac{1}{\sqrt{3}}\begin{pmatrix} e^{-\frac{i\pi}{8}}\cos\left(\frac{\pi}{24}-\theta\right)-e^{\frac{3i\pi}{8}}\cos\left(\frac{\pi}{24}+\theta\right) & 1 & -e^{-\frac{i\pi}{8}}\sin\left(\frac{\pi}{24}-\theta\right)-e^{\frac{3i\pi}{8}}\sin\left(\frac{\pi}{24}+\theta\right) \\ -e^{-\frac{i\pi}{8}}\sin\left(\frac{5\pi}{24}-\theta\right)+e^{\frac{3i\pi}{8}}\sin\left(\frac{5\pi}{24}+\theta\right) & 1 & -e^{-\frac{i\pi}{8}}\cos\left(\frac{5\pi}{24}-\theta\right)-e^{\frac{3i\pi}{8}}\cos\left(\frac{5\pi}{24}+\theta\right) \\ -e^{-\frac{i\pi}{8}}\sin\left(\frac{\pi}{8}+\theta\right)+e^{\frac{3i\pi}{8}}\sin\left(\frac{\pi}{8}-\theta\right) & 1 & e^{-\frac{i\pi}{8}}\cos\left(\frac{\pi}{8}+\theta\right)+e^{\frac{3i\pi}{8}}\cos\left(\frac{\pi}{8}-\theta\right) \end{pmatrix}.$$



Scenario 1.3: $G_l = Z_3^T,\ G_\nu = Z_2^S,\ H_{CP}^\nu = \left\{\rho(T^2UT^2), \rho(ST^2UT^2)\right\}$

$$U_{PMNS} = \frac{1}{\sqrt{3}}\begin{pmatrix} e^{-\frac{i\pi}{8}}\cos\left(\frac{\pi}{24}-\theta\right) - e^{\frac{3i\pi}{8}}\cos\left(\frac{\pi}{24}+\theta\right) & 1 & -e^{-\frac{i\pi}{8}}\sin\left(\frac{\pi}{24}-\theta\right) - e^{\frac{3i\pi}{8}}\sin\left(\frac{\pi}{24}+\theta\right) \\ -e^{-\frac{i\pi}{8}}\sin\left(\frac{5\pi}{24}-\theta\right) + e^{\frac{3i\pi}{8}}\sin\left(\frac{5\pi}{24}+\theta\right) & 1 & -e^{-\frac{i\pi}{8}}\cos\left(\frac{5\pi}{24}-\theta\right) - e^{\frac{3i\pi}{8}}\cos\left(\frac{5\pi}{24}+\theta\right) \\ -e^{-\frac{i\pi}{8}}\sin\left(\frac{\pi}{8}+\theta\right) + e^{\frac{3i\pi}{8}}\sin\left(\frac{\pi}{8}-\theta\right) & 1 & e^{-\frac{i\pi}{8}}\cos\left(\frac{\pi}{8}+\theta\right) + e^{\frac{3i\pi}{8}}\cos\left(\frac{\pi}{8}-\theta\right) \end{pmatrix}.$$

Lepton mixing angles and CP phases :

$$\sin^2\theta_{13}=\frac{1}{3}-\frac{\sqrt{6}+\sqrt{2}}{12}\cos 2\theta,\quad \sin^2\theta_{12}=\frac{4}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta},$$

$$\sin^2\theta_{23}=\frac{4+\left(\sqrt{6}-\sqrt{2}\right)\cos 2\theta}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta},\quad |J_{CP}|=\frac{1}{6\sqrt{3}}\left|\sin 2\theta\right|,$$

$$\sin^2\theta_{13} = \frac{1}{3} - \frac{\sqrt{6}+\sqrt{2}}{12}\cos 2\theta, \quad \sin^2\theta_{12} = \frac{4}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta},$$

$$\sin^2\theta_{23} = \frac{4+\left(\sqrt{6}-\sqrt{2}\right)\cos 2\theta}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta}, \quad |J_{CP}| = \frac{1}{6\sqrt{3}}\left|\sin 2\theta\right|,$$

$$|\tan\delta_{CP}| = \left|\frac{4\sqrt{2}+\left(1+\sqrt{3}\right)\cos 2\theta}{1-\sqrt{3}-\sqrt{2}\cos 2\theta}\tan 2\theta\right|,$$

$$|\tan\alpha_{21}| = \left|\frac{\sqrt{6}+\sqrt{2}+4\cos 2\theta+\left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}{\sqrt{6}+\sqrt{2}+4\cos 2\theta-\left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}\right|,$$

$$\left|\tan\alpha'_{31}\right| = \left|\frac{4\sin 2\theta}{2-3\sqrt{3}+\left(2+\sqrt{3}\right)\cos 4\theta}\right|.$$

$$\sin^2\theta_{13}=\frac{1}{3}-\frac{\sqrt{6}+\sqrt{2}}{12}\cos2\theta,\quad \sin^2\theta_{12}=\frac{4}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos2\theta},$$

$$\sin^2\theta_{23}=\frac{4+\left(\sqrt{6}-\sqrt{2}\right)\cos2\theta}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos2\theta},\quad |J_{CP}|=\frac{1}{6\sqrt{3}}\left|\sin2\theta\right|,$$

$$|\tan\delta_{CP}|=\left|\frac{4\sqrt{2}+\left(1+\sqrt{3}\right)\cos2\theta}{1-\sqrt{3}-\sqrt{2}\cos2\theta}\tan2\theta\right|,$$

$$|\tan\alpha_{21}|=\left|\frac{\sqrt{6}+\sqrt{2}+4\cos2\theta+\left(\sqrt{6}-\sqrt{2}\right)\sin2\theta}{\sqrt{6}+\sqrt{2}+4\cos2\theta-\left(\sqrt{6}-\sqrt{2}\right)\sin2\theta}\right|,$$

$$|\tan\alpha'_{31}|=\left|\frac{4\sin2\theta}{2-3\sqrt{3}+\left(2+\sqrt{3}\right)\cos4\theta}\right|.$$

- Both mixing angles and CP phases depend on θ.

$$\sin^2\theta_{13}=\frac{1}{3}-\frac{\sqrt{6}+\sqrt{2}}{12}\cos 2\theta,\quad \sin^2\theta_{12}=\frac{4}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta},$$

$$\sin^2\theta_{23}=\frac{4+\left(\sqrt{6}-\sqrt{2}\right)\cos 2\theta}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta},\quad |J_{CP}|=\frac{1}{6\sqrt{3}}\left|\sin 2\theta\right|,$$

$$|\tan\delta_{CP}|=\left|\frac{4\sqrt{2}+\left(1+\sqrt{3}\right)\cos 2\theta}{1-\sqrt{3}-\sqrt{2}\cos 2\theta}\tan 2\theta\right|,$$

$$|\tan\alpha_{21}|=\left|\frac{\sqrt{6}+\sqrt{2}+4\cos 2\theta+\left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}{\sqrt{6}+\sqrt{2}+4\cos 2\theta-\left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}\right|,$$

$$|\tan\alpha'_{31}|=\left|\frac{4\sin 2\theta}{2-3\sqrt{3}+\left(2+\sqrt{3}\right)\cos 4\theta}\right|.$$

- Both mixing angles and CP phases depend on θ.
- This pattern can also be derived from $\Delta(48)\hookrightarrow$ G.-J. Ding and Y.-L. Zhou, arXiv:1404.0592,JHEP 1406 (2014) 023.

$$\sin^2\theta_{13}=\frac{1}{3}-\frac{\sqrt{6}+\sqrt{2}}{12}\cos 2\theta,\quad \sin^2\theta_{12}=\frac{4}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta},$$

$$\sin^2\theta_{23}=\frac{4+\left(\sqrt{6}-\sqrt{2}\right)\cos 2\theta}{8+\left(\sqrt{6}+\sqrt{2}\right)\cos 2\theta},\quad |J_{CP}|=\frac{1}{6\sqrt{3}}\left|\sin 2\theta\right|,$$

$$|\tan\delta_{CP}|=\left|\frac{4\sqrt{2}+\left(1+\sqrt{3}\right)\cos 2\theta}{1-\sqrt{3}-\sqrt{2}\cos 2\theta}\tan 2\theta\right|,$$

$$|\tan\alpha_{21}|=\left|\frac{\sqrt{6}+\sqrt{2}+4\cos 2\theta+\left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}{\sqrt{6}+\sqrt{2}+4\cos 2\theta-\left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}\right|,$$

$$|\tan\alpha'_{31}|=\left|\frac{4\sin 2\theta}{2-3\sqrt{3}+\left(2+\sqrt{3}\right)\cos 4\theta}\right|.$$

- Both mixing angles and CP phases depend on θ.
- This pattern can also be derived from $\Delta(48)\hookrightarrow$ G.-J. Ding and Y.-L. Zhou, arXiv:1404.0592,JHEP 1406 (2014) 023.

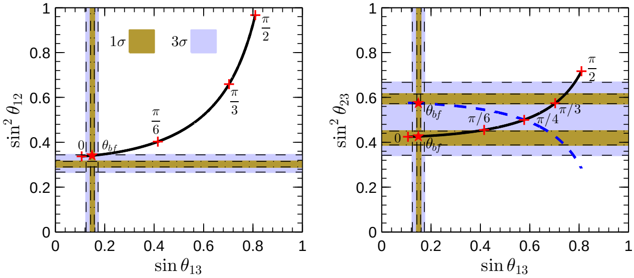
Best fitting:

$$\theta_{\rm bf}=\pm 0.130,\quad \chi^2_{\rm min}=9.124(9.838),$$

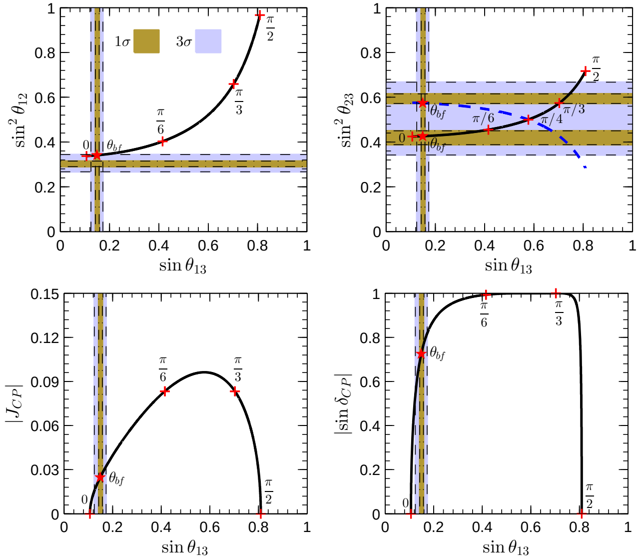
$$\sin^2\theta_{13}(\theta_{\rm bf})=0.0222,\quad \sin^2\theta_{12}(\theta_{\rm bf})=0.341,\quad \sin^2\theta_{23}(\theta_{\rm bf})=0.426(0.574),$$

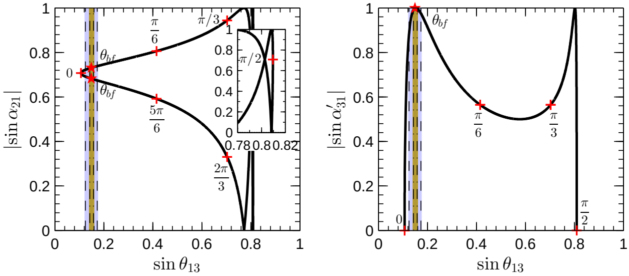
$$|\sin\delta_{\rm CP}(\theta_{\rm bf})|=0.725,\quad |\sin\alpha_{21}(\theta_{\rm bf})|=0.682 \text{ or } 0.731,\quad \left|\sin\alpha'_{31}(\theta_{\rm bf})\right|=0.999.$$

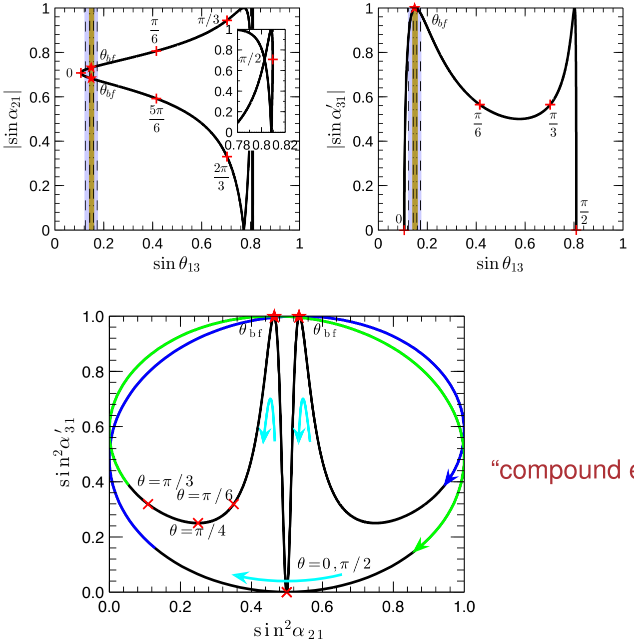
Correlations between different observables



Correlations between different observables




$|\sin\alpha_{21}|$
$\sin\theta_{13}$
θ_{bf}
$\frac{\pi}{6}$
$\pi/3$
$\pi/2$
$\frac{5\pi}{6}$
$\frac{2\pi}{3}$
$|\sin\alpha'_{31}|$
$\frac{\pi}{3}$
$\frac{\pi}{2}$



"compound eye" diagram

Scenario 2: $G_l = Z_3^T$, $G_\nu = Z_2^U$, $H_{CP}^\nu = \{\rho(T^2UTUT^2), \rho(UT^2UTUT^2)\}$

$$U_{PMNS} = \frac{1}{2\sqrt{3}}\begin{pmatrix} -1-\sqrt{3} & \left(\sqrt{3}-1\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & \left(\sqrt{3}-1\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ 2 & 2\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & 2\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ \sqrt{3}-1 & -\left(1+\sqrt{3}\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & -\left(1+\sqrt{3}\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \end{pmatrix}.$$

Scenario 2: $G_l = Z_3^T$, $G_\nu = Z_2^U$, $H_{CP}^\nu = \left\{\rho(T^2UTUT^2), \rho(UT^2UTUT^2)\right\}$

$$U_{PMNS} = \frac{1}{2\sqrt{3}}\begin{pmatrix} -1-\sqrt{3} & \left(\sqrt{3}-1\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & \left(\sqrt{3}-1\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ 2 & 2\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & 2\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ \sqrt{3}-1 & -\left(1+\sqrt{3}\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & -\left(1+\sqrt{3}\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \end{pmatrix}.$$

Mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{12}\left[4-\sqrt{3}-\sqrt{3}\cos 2\theta - \left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta\right], \quad \sin^2\theta_{12} = 1 - \frac{2+\sqrt{3}}{6\cos^2\theta_{13}},$$

$$\sin^2\theta_{23} = \frac{4-2\sqrt{2}\sin 2\theta}{8+\sqrt{3}+\sqrt{3}\cos 2\theta + \left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}, \qquad |J_{CP}| = \frac{1}{12\sqrt{6}}\left|\sin 2\theta\right|,$$

Scenario 2: $G_l = Z_3^T,\ G_\nu = Z_2^U,\ H_{CP}^\nu = \{\rho(T^2UTUT^2), \rho(UT^2UTUT^2)\}$

$$U_{PMNS} = \frac{1}{2\sqrt{3}} \begin{pmatrix} -1-\sqrt{3} & \left(\sqrt{3}-1\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & \left(\sqrt{3}-1\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ 2 & 2\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & 2\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ \sqrt{3}-1 & -\left(1+\sqrt{3}\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & -\left(1+\sqrt{3}\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \end{pmatrix}.$$

Mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{12}\left[4-\sqrt{3}-\sqrt{3}\cos 2\theta - \left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta\right], \quad \sin^2\theta_{12} = 1 - \frac{2+\sqrt{3}}{6\cos^2\theta_{13}},$$

$$\sin^2\theta_{23} = \frac{4-2\sqrt{2}\sin 2\theta}{8+\sqrt{3}+\sqrt{3}\cos 2\theta + \left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}, \qquad |J_{CP}| = \frac{1}{12\sqrt{6}}\left|\sin 2\theta\right|,$$

$$|\tan\delta_{CP}| = \left|\frac{\left(6-2\sqrt{3}\right)(1-\cos 4\theta) + \left(6\sqrt{2}+16\sqrt{6}\right)\sin 2\theta + 3\sqrt{2}\sin 4\theta}{24+18\sqrt{3}+\left(24-8\sqrt{3}\right)\cos 2\theta - 6\sqrt{2}\sin 2\theta + 6\sqrt{3}\cos 4\theta - \left(15\sqrt{2}+4\sqrt{6}\right)\sin 4\theta}\right|,$$

$$|\tan\alpha_{21}| = \left|\frac{2\left(2+\sqrt{3}\right)(1+\cos 2\theta) + \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}{1-\cos 2\theta + \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}\right|,$$

$$|\tan\alpha'_{31}| = \left|\frac{2\left(2+\sqrt{3}\right)(1-\cos 2\theta) - \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}{1+\cos 2\theta - \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}\right|.$$

Scenario 2: $G_l = Z_3^T$, $G_\nu = Z_2^U$, $H_{CP}^\nu = \{\rho(T^2UTUT^2), \rho(UT^2UTUT^2)\}$

$$U_{PMNS} = \frac{1}{2\sqrt{3}} \begin{pmatrix} -1-\sqrt{3} & \left(\sqrt{3}-1\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & \left(\sqrt{3}-1\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ 2 & 2\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & 2\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \\ \sqrt{3}-1 & -\left(1+\sqrt{3}\right)\sin\theta + 2e^{-\frac{i\pi}{4}}\cos\theta & -\left(1+\sqrt{3}\right)\cos\theta - 2e^{-\frac{i\pi}{4}}\sin\theta \end{pmatrix}.$$

Mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{12}\left[4-\sqrt{3}-\sqrt{3}\cos 2\theta - \left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta\right], \quad \sin^2\theta_{12} = 1 - \frac{2+\sqrt{3}}{6\cos^2\theta_{13}},$$

$$\sin^2\theta_{23} = \frac{4-2\sqrt{2}\sin 2\theta}{8+\sqrt{3}+\sqrt{3}\cos 2\theta + \left(\sqrt{6}-\sqrt{2}\right)\sin 2\theta}, \qquad |J_{CP}| = \frac{1}{12\sqrt{6}}\left|\sin 2\theta\right|,$$

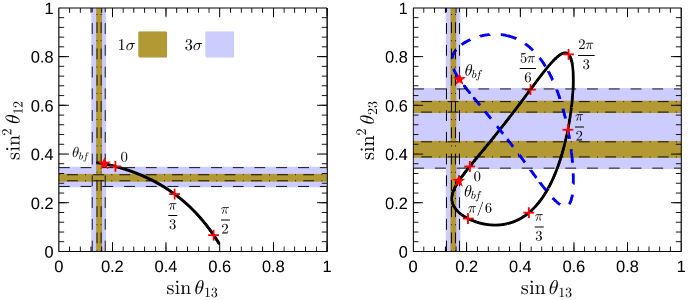
$$|\tan\delta_{CP}| = \left|\frac{\left(6-2\sqrt{3}\right)(1-\cos 4\theta) + \left(6\sqrt{2}+16\sqrt{6}\right)\sin 2\theta + 3\sqrt{2}\sin 4\theta}{24+18\sqrt{3}+\left(24-8\sqrt{3}\right)\cos 2\theta - 6\sqrt{2}\sin 2\theta + 6\sqrt{3}\cos 4\theta - \left(15\sqrt{2}+4\sqrt{6}\right)\sin 4\theta}\right|,$$

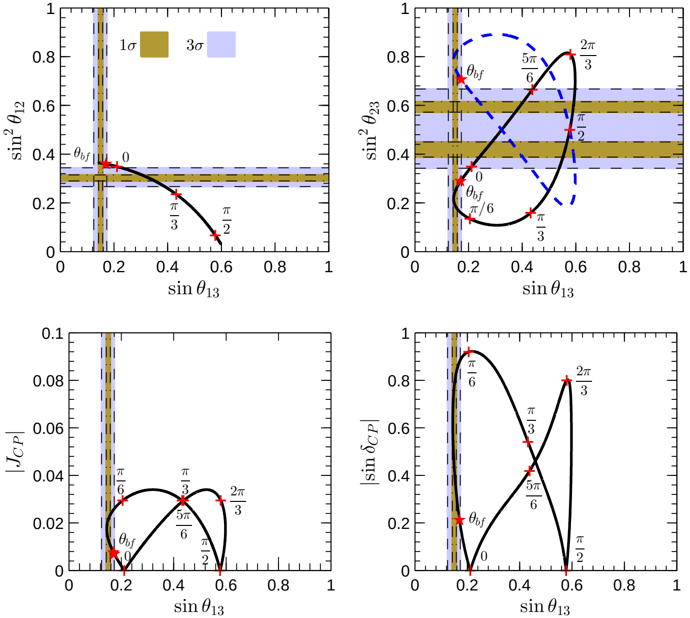
$$|\tan\alpha_{21}| = \left|\frac{2\left(2+\sqrt{3}\right)(1+\cos 2\theta) + \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}{1-\cos 2\theta + \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}\right|,$$

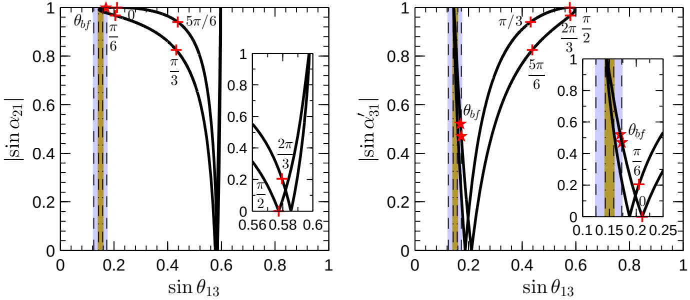
$$|\tan\alpha'_{31}| = \left|\frac{2\left(2+\sqrt{3}\right)(1-\cos 2\theta) - \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}{1+\cos 2\theta - \left(\sqrt{6}+\sqrt{2}\right)\sin 2\theta}\right|.$$

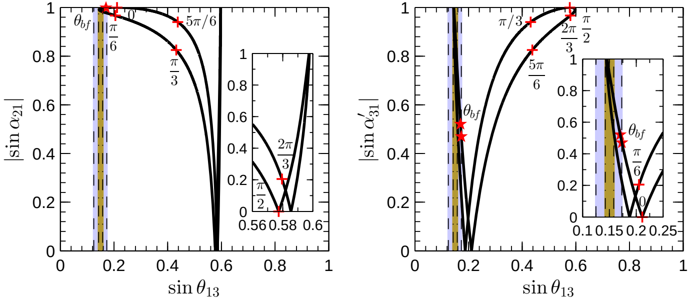
All mixing parameters depend on θ!

$\chi^2_{\mathbf{min}} = 51.645(57.745)$ for $\theta_{23} < 45^\circ$ ($\theta_{23} > 45^\circ$).


1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
$\sin^2\theta_{12}$
$\sin\theta_{13}$
1σ
3σ
θ_{bf}
0
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\sin^2\theta_{23}$
$\frac{2\pi}{3}$
$\frac{5\pi}{6}$
$\frac{\pi}{2}$
$\pi/6$
$\frac{\pi}{3}$

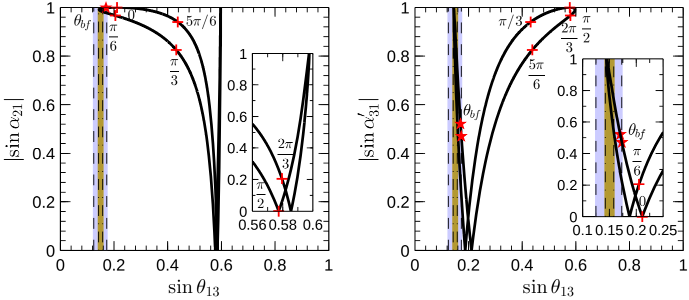

1σ
3σ
$\sin^2\theta_{12}$
$\sin\theta_{13}$
θ_{bf}
0
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\sin^2\theta_{23}$
$\frac{5\pi}{6}$
$\frac{2\pi}{3}$
$\pi/6$
$|J_{CP}|$
$\frac{\pi}{6}$
$|\sin\delta_{CP}|$


$|\sin\alpha_{21}|$
$\sin\theta_{13}$
θ_{bf}
0
$\frac{\pi}{6}$
$5\pi/6$
$\frac{\pi}{3}$
$\frac{2\pi}{3}$
$\frac{\pi}{2}$
$|\sin\alpha'_{31}|$
$\sin\theta_{13}$
$\pi/3$
$\frac{2\pi}{3}$
$\frac{\pi}{2}$
$\frac{5\pi}{6}$
θ_{bf}
θ_{bf}
$\frac{\pi}{6}$
0



■ Scenario 3.1: $G_l = Z_3^T$, $G_\nu = Z_2^{UTUT^2UST}$, $H_{CP}^\nu = \{\rho(S), \rho(UTSUT^2UT)\}$

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2}\cos\theta & \sqrt{2}\sin\theta \\ -1 & \sqrt{2}\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2}\sin\theta - i\sqrt{3}\cos\theta \\ -1 & \sqrt{2}\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2}\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix}.$$



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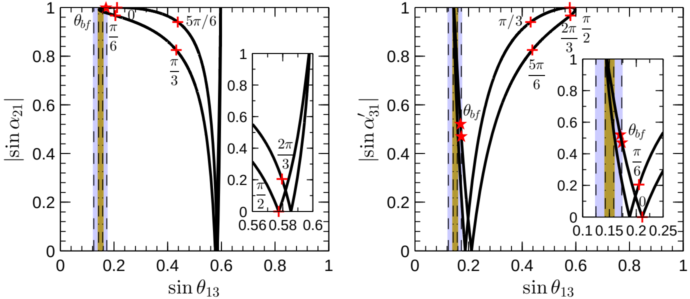

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θ_{23} and δ_{CP} are maximal, and excellent agreement with experimental data can be achieved. $\hookrightarrow$ C.-C.Li and G.-J.Ding, Nucl. Phys. B881, 206 (2014).

Scenario 3.2: $G_l = Z_3^T$, $G_\nu = Z_2^{UTUT^2UST}$, $H_{CP}^\nu = \{\rho(T^2UTUT^2), \rho(UT^2ST)\}$

$$U_{PMNS} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \sqrt{2}\,e^{-\frac{i\pi}{4}}\cos\theta & \sqrt{2}\,e^{-\frac{i\pi}{4}}\sin\theta \\ -1 & \sqrt{3}\,\sin\theta+\sqrt{2}\,e^{-\frac{i\pi}{4}}\cos\theta & -\sqrt{3}\,\cos\theta+\sqrt{2}\,e^{-\frac{i\pi}{4}}\sin\theta \\ -1 & -\sqrt{3}\,\sin\theta+\sqrt{2}\,e^{-\frac{i\pi}{4}}\cos\theta & \sqrt{3}\,\cos\theta+\sqrt{2}\,e^{-\frac{i\pi}{4}}\sin\theta \end{pmatrix}.$$

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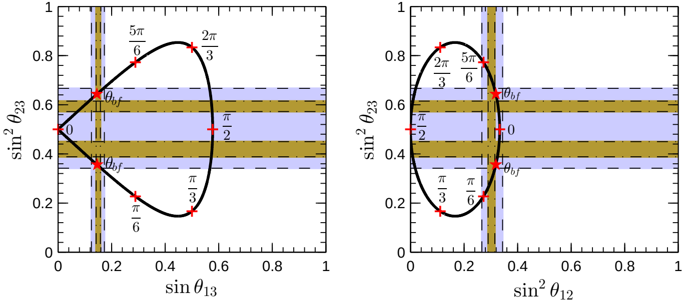
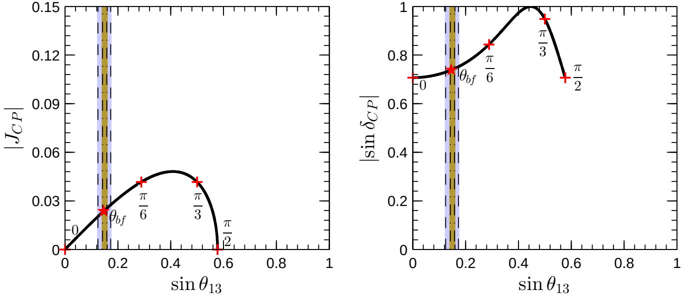
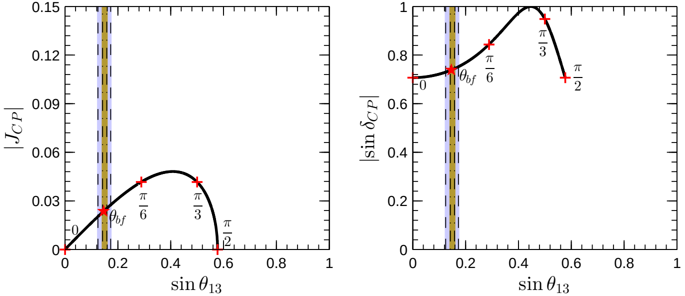
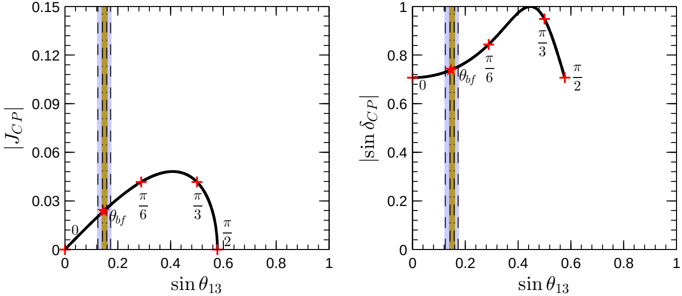



0.15
0.12
0.09
0.06
0.03
0
$|J_{CP}|$
0
0.2
0.4
0.6
0.8
1
$\sin\theta_{13}$
0
θ_{bf}
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
1
0.8
0.6
0.4
0.2
0
$|\sin\delta_{CP}|$
$\sin\theta_{13}$



- Scenario 4: $G_l = Z_8^{UTS}$, $G_\nu = Z_2^U$, $H_{CP}^\nu = \{\rho(S), \rho(SU)\}$

$$U_{PMNS} = \frac{1}{2}\begin{pmatrix} -i\left(\sqrt{2}\cos\theta + \sin\theta\right) & 1 & -i\left(\cos\theta - \sqrt{2}\sin\theta\right) \\ i\sqrt{2}\sin\theta & \sqrt{2} & i\sqrt{2}\cos\theta \\ i\left(\sqrt{2}\cos\theta - \sin\theta\right) & 1 & -i\left(\cos\theta + \sqrt{2}\sin\theta\right) \end{pmatrix}.$$



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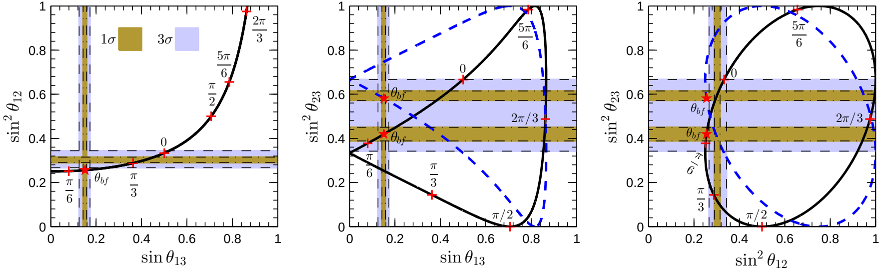
Lepton mixing parameters:

$$\sin^2\theta_{13} = \frac{1}{8}\left(3 - \cos 2\theta - 2\sqrt{2}\sin 2\theta\right), \quad \sin^2\theta_{12} = \frac{2}{5 + \cos 2\theta + 2\sqrt{2}\sin 2\theta},$$

$$\sin^2\theta_{23} = \frac{2 + 2\cos 2\theta}{5 + \cos 2\theta + 2\sqrt{2}\sin 2\theta}, \quad \tan\delta_{CP} = \tan\alpha_{21} = \tan\alpha_{31} = 0.$$


1σ
3σ
θbf
sin θ13
sin² θ12
sin² θ23
0
π/6
π/3
π/2
5π/6
2π/3

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Thank you for your attention!