

Theory Perspective on Flavour

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and Consequences in Accelerators and Cosmology

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Flavour problem

flavour sector of the SM
[minimally extended to include
massive Majorana neutrinos]

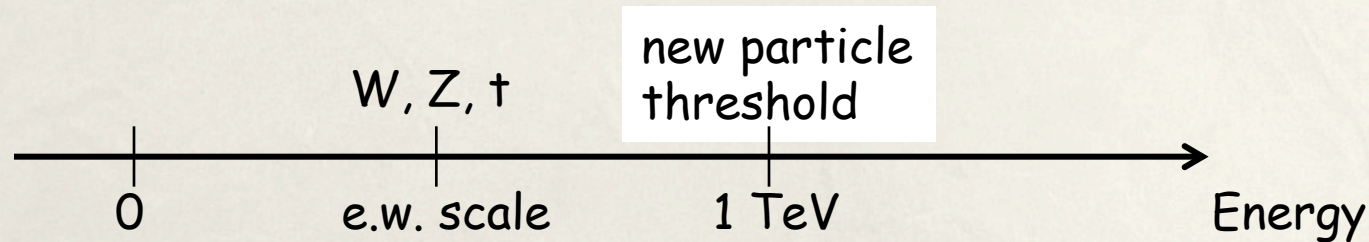


22 parameters: γ
18 measured
(+ 1 mass + 3 phases)

two aspects

1 origin of γ

2 new particle threshold



how to avoid large FCNC and CPV induced by the new particles?

less urgent? no evidence for a new threshold so far

this talk: main focus on 1, but also comments on 2

not a systematic review of models, rather a reappraisal of few well-known ideas
as an introduction to the presentations of this Workshop

approaches

1 \mathcal{Y} should be deduced from first principles

most striking fact: nothing approaching a standard theory of \mathcal{Y} , despite decades of experimental progress and theoretical efforts

2 \mathcal{Y} are due to chance

many variants

bottom-up: anarchy, FN models, fermions in ED, partial compositeness
top-down: fundamental theory with a landscape of ground states

observed \mathcal{Y} are environmental and cannot be fully predicted



relative sizes of solar planetary orbits

assumptions

knowledge of statistical distribution of \mathcal{Y} in the fundamental theory

the observed \mathcal{Y} are typical

[any anthropic selection?]

relevant questions

how typical are the \mathcal{Y} we observe?

which is the statistical distribution of \mathcal{Y} in the fundamental theory?

fundamental theory



[symmetry and/or dynamical principle]



\mathcal{Y}

the quark sector

any empirical evidence for G_f from the quark sector?

$$G_f = U(1)_{FN}$$

[Froggatt, Nielsen 1979]

mass ratios and mixing angles are small, hierarchical parameters

$$\frac{m_u}{m_t} \ll \frac{m_c}{m_t} \ll 1 \quad \frac{m_d}{m_b} \ll \frac{m_s}{m_b} \ll 1 \quad |V_{ub}| \ll |V_{cb}| \ll |V_{us}| \equiv \lambda < 1$$

easily reproduced by $G_f = U(1)_{FN}$

mass ratios and mixing angles are powers of a small SB parameter λ

$U(1)_{FN}$ broken by

$$\lambda = \frac{\langle \varphi \rangle}{\Lambda_f} \approx 0.2$$

<i>flavon</i>	Q_{FN}
φ	-1

$$y_u = F_{U^c} Y_u F_Q$$

$$y_d = F_{D^c} Y_d F_Q$$

call this map
"hierarchy"

$$F_X = \begin{pmatrix} \lambda^{FN(X_1)} & 0 & 0 \\ 0 & \lambda^{FN(X_2)} & 0 \\ 0 & 0 & \lambda^{FN(X_3)} \end{pmatrix}$$

$$Y_{u,d} \approx O(1)$$

undetermined by $U(1)_{FN}$

$FN(X_i)$ are $U(1)_{FN}$ charges

($X = Q, U^c, D^c$)

not a mere book-keeping

take $\text{FN}(Q_1) > \text{FN}(Q_2) > \text{FN}(Q_3) \geq 0$

$$\left(V_{u,d}\right)_{ij} \approx \frac{F_{Q_i}}{F_{Q_j}} < 1 \quad (i < j) \quad V_{CKM} = V_u^+ V_d$$

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx O(1)$$

$$V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

[O.K. within a factor of 2]

independently from the specific charge choice

correct orders of magnitude of V_{ij}
reproduced by e.g.

$$\text{FN}(Q) = (3, 2, 0)$$

correct orders of magnitude of
quark/charged lepton mass ratios
[up to a couple of moderate tunings]
reproduced by e.g.

$$\text{FN}(U^c) = \text{FN}(E^c) = \text{FN}(Q) = (3, 2, 0)$$

$$\text{FN}(D^c) = \text{FN}(L) = (2, 0, 0)$$

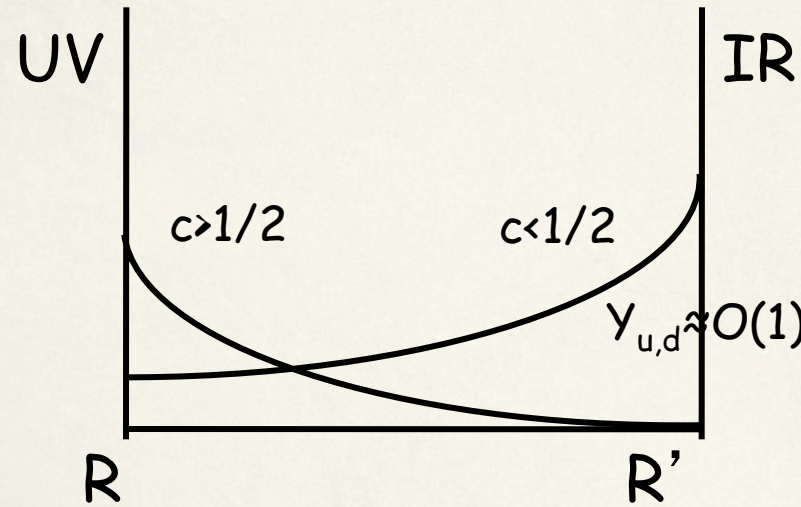
charge assignment compatible with $SU(5)$ gauge unification

we have recently tested this scenario in the context of an $SO(10)$ model
all FN charges \leftrightarrow 4 parameters [F, Patel, Vicino to appear]

is a symmetry really needed ?

$$y_u = F_{U^c} Y_u F_Q$$

$$y_d = F_{D^c} Y_d F_Q$$



split fermions in an Extra Dimension

$$F_{X_i} = \sqrt{\frac{2\mu_i}{1 - e^{-2\mu_i r}}}$$

ED	μ_i	r
Flat $[0, \pi R]$	M_i / Λ	$\Lambda \pi R$
Warped $[R, R']$	$1/2 - M_i R$	$\log R'/R$

no symmetry:
hierarchy produced by geometry

M_i = bulk mass of fermion X_i
 $Y_{u,d} = O(1)$ Yukawa couplings between bulk fermions and a Higgs localized at one brane

partial compositeness

$$F_{X_i} = \Delta_i M_i^{-1}$$

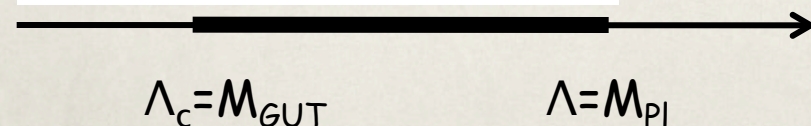
M_i = masses of composite fermions
 Δ_i = elementary-composite mixing
 $Y_{u,d} = O(1)$ Yukawa couplings in composite sector

chiral multiplets X_i of the MSSM coupled to a superconformal sector

[Nelson-Strassler 0006251]

$$F_{X_i} = \left(\frac{\Lambda_c}{\Lambda} \right)^{\frac{\gamma_i}{2}} < 1$$

γ_i anomalous dimension of X_i



dangerous FCNC

the "hierarchy" map can support a Maximal Flavour Symmetry similar to G_{MFV}

flavour group felt by quarks can be as large as G_{MFV} , but there are more spurions

$$F_Q, F_U^c, F_D^c, Y_u, Y_d$$

true flavour symmetry can be weaker, dep. on the way "hierarchy" is realized, as e.g. in FN models [Dudas, von Gersdorff, Parmentier, Pokorski 1007.5208]

maximal symmetry applies to RS models [RS-GIM Agashe, Perez, Soni 0408134]

one concrete example $O_K^4 = (\bar{s}_L d_R)(\bar{s}_R d_L)$ contributions to ϵ_K are both chiral and RG enhanced

arises from $\frac{1}{\Lambda_{NP}^2} (\bar{Q} F_Q^+ \gamma_\mu F_Q Q) (\bar{D}^c F_{D^c}^+ \gamma^\mu F_{D^c} D^c) \rightarrow C_K^4 \approx \frac{1}{\Lambda_{NP}^2} \frac{1}{\langle Y_d \rangle^2} \frac{2 m_d m_s}{v^2}$

$$\text{Im}(C_K^4) < (160 \times 10^3 \text{ TeV})^{-2}$$

$$\text{Im}(C_K^4) \approx \text{Re}(C_K^4)$$



$$\langle Y_d \rangle \Lambda_{NP} > 20 \text{ TeV}$$

confirmed by explicit computation in RS O_K^4 from tree-level KK gluon exchange
[also neutron EDM $\rightarrow M_{KK} > O(10) \text{ TeV}$]

$$M_{KK} > (22 \pm 6) \text{ TeV}$$

[Csaki, Falkowski, Weiler 0804.1954
Von Gersdorff 1311.2078]

FCNC and/or CPV not sufficiently suppressed if there is New Physics at the TeV scale

some lessons from the quark sector

Pattern of quark masses and mixing angles well-explained by a hierarchy map: underlying $Y_{u,d}$ are $O(1)$
hierarchy realized in several different frameworks: FN, RS, NS,....
symmetry is not a necessary ingredient

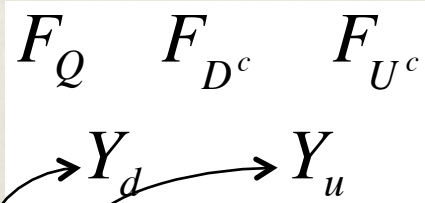
correct order-of-magnitude predictions

compatible with $SU(5)/SO(10)$ GUTs

compatible with/incorporated in known solutions to the hierarchy problem

additional ingredients needed to control the new sources of FC/CPV arising from New Physics at the TeV scale

alignment
universality
...



some symmetry ?

large number of independent $O(1)$ parameters:
test of statistical distributions

additional constraints?

present precision in quark mass/mixing parameters

testable predictions beyond order-of-magnitude accuracy ?

the lepton sector

small parameters

	$\sin^2 \vartheta_{23} - 1/2$	$\sin^2 \vartheta_{13}$	$\Delta m_{21}^2 / \Delta m_{31}^2 $	$\Delta m_{21} / m_1$
NH	$0.067^{+0.032}_{-0.128}$	0.0234 ± 0.0020	0.0306 ± 0.0011	–
IH	$0.073^{+0.025}_{-0.043}$	0.0240 ± 0.0019	0.0319 ± 0.0009	≤ 0.016

1 accidental origin



$$F_{E_1^c} \gg F_{E_2^c} \gg F_{E_3^c}$$

$$F_{L_1} = F_{L_2} = F_{L_3}$$

[viable both for Majorana or Dirac neutrinos, here focus on Majorana]

Anarchy

[Hall, Murayama, Weiner 1999
De Gouvea, Murayama 1204.1249]

$$m_\nu \propto \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

mixing angles
and mass ratios
from random $O(1)$
quantities

$$|U_{PMNS}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix}$$

consistent with data

$\vartheta_{13} \approx 0.15$ rad and the hint for non maximal ϑ_{23}
have strengthened the case for anarchy

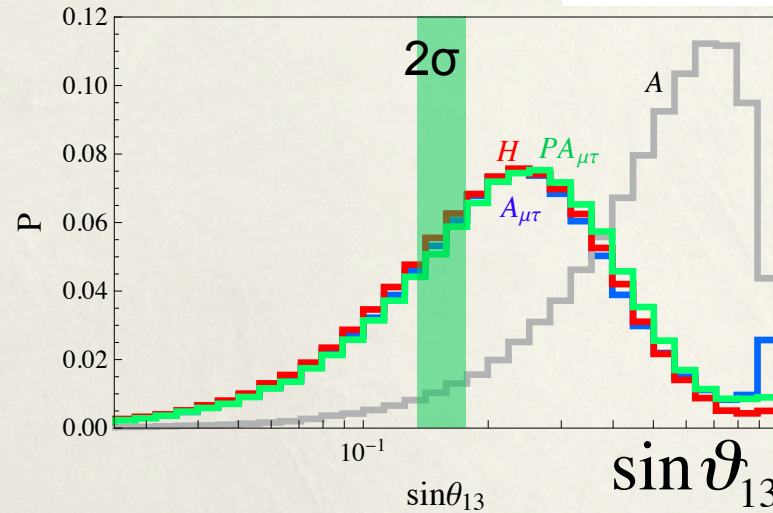
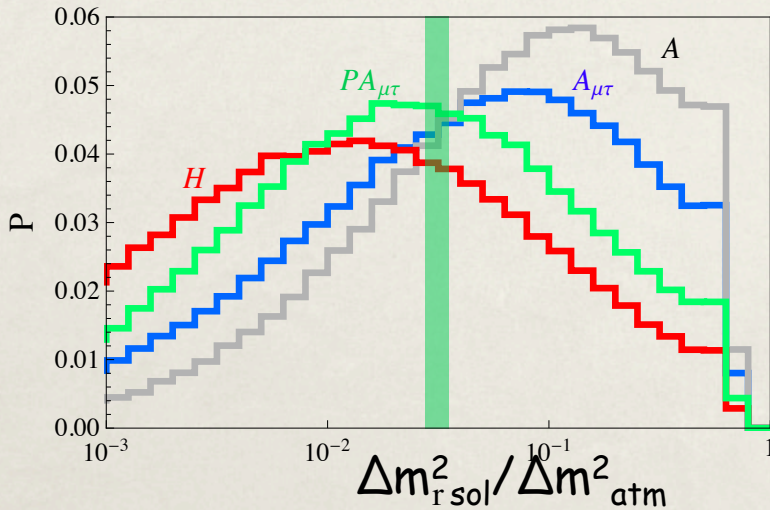
2 not entirely accidental

- variants of Anarchy e.g. in $U(1)_{FN}$ models, quarks and leptons treated on equal foot
- compatible with $SU(5)$ unification

[Buchmuller, Domcke, Schmitz, 1111.387;
Altarelli, F, Masina, Merlo 1207.0587;
Bergstrom, Meloni, Merlo, 1403.4528]

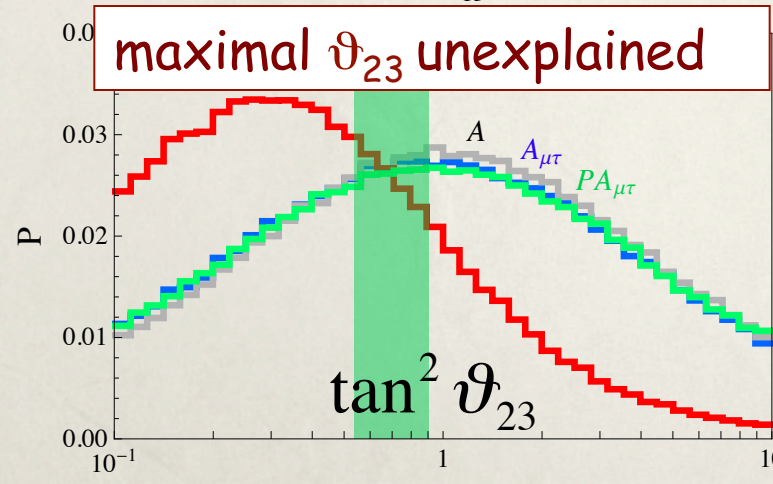
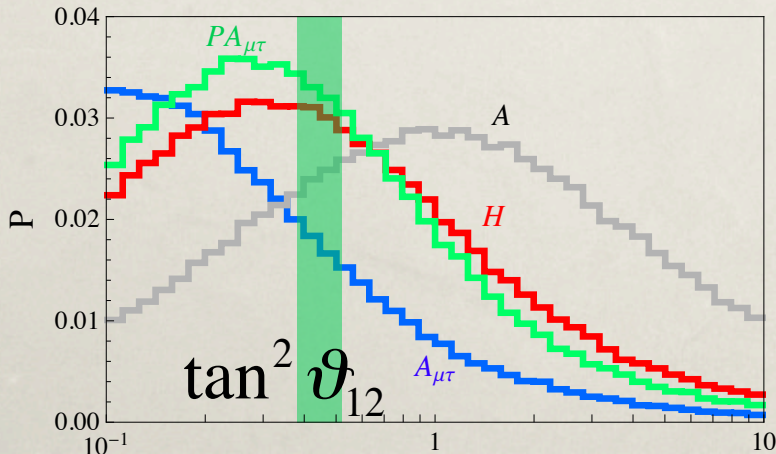
$$F(L_i) = \lambda^{FN(L_i)}$$

	$FN(L)$	λ
A	(0,0,0)	
$A_{\mu\tau}$	(1,0,0)	0.25
$PA_{\mu\tau}$	(2,0,0)	0.35
H	(2,1,0)	0.45



$$\sin^2 \vartheta_{13} \approx \frac{\Delta m_{12}^2}{\Delta m_{13}^2}$$

NH favoured



difficult to go beyond order-of-magnitude predictions

constraints from lepton flavour violation

take the limit $m_\nu = 0$

if MFV applied, we would expect no LFV [y_e diagonal]



in our setup, in general

F_{E^c}, F_L, Y_e do not commute
[not even when F_L is universal]
LFV expected at some level

dominant LFV
dipole operator

$$L_{dip} = \frac{e}{\Lambda_{NP}^2} E^c (\sigma_{\mu\nu} F^{\mu\nu}) \underbrace{(F_{E^c} Y_e Y_e^+ Y_e F_L)}_{\text{not diagonal}} (H^+ L)$$

when $y_e = F_{E^c} Y_e F_L$ diagonal

Explicit computation in RS

[Agashe, Blechman, Petriello 0606021
Csaki, Grossman, Tanedo, Tsai 1004.2037]

comparable bounds from e EDM

[Keren-Zur, Lodone, Nardecchia,
Pappadopulo, Rattazzi, Vecchi, 1205.5803]

$$BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}$$

$$M_{KK} > O(10) \text{ TeV}$$



F_L universality is not enough

a sufficient condition for
the absence of LFV:

$$F_{E^c}, Y_e, F_L$$

diagonal in the same basis

for instance:

$$F_L \propto 1$$

$$F_{E^c} \propto Y_e Y_e^+$$

[M.C. Chen and Yu, 08042503
Perez, Randall 0805.4652]

anything special from data, requiring a symmetry?

- 1 ϑ_{23} maximal ?
- 2 $\delta_{CP} = -\pi/2$?
- 3 U_{PMNS} close to TB (BM,...) ?

3 examples from a longer list...

1 today most precise single determination of ϑ_{23} is from T2K ($P_{\mu\mu}$)

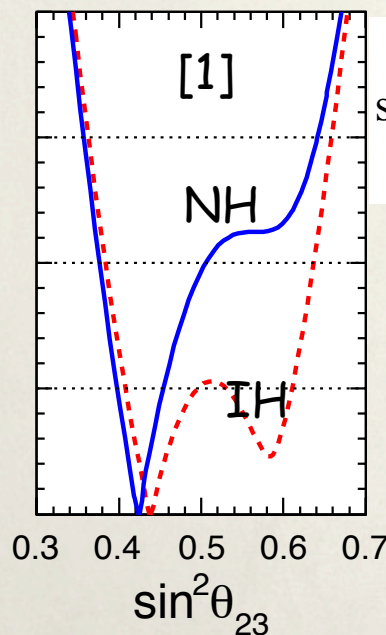
$$[1403.1532] \quad \sin^2 \vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & \text{(NH)} \\ 0.511^{+0.055}_{-0.055} & \text{(IH)} \end{cases}$$

well compatible with ϑ_{23} maximal

global fits hint at ϑ_{23} non-maximal
main effect: interplay between
SBL reactor experiments (P_{ee}) and
LBL experiments searching ($P_{\mu e}$)

$$P_{ee} = 1 - \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m_{32}^2 L}{4E} + \dots$$

$$P_{\mu e} = \sin^2 \vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m_{32}^2 L}{4E} + \dots$$



$$[2] \quad \sin^2 \vartheta_{23} = \begin{cases} 0.567^{+0.032}_{-0.128} & \text{(NH)} \\ 0.573^{+0.025}_{-0.043} & \text{(IH)} \end{cases}$$

global fit:
[1] Capozzi, Fogli, Lisi, Marrone,
Montanino, Palazzo 1312.2878
[2] Forero, Tortola, Valle
1405.7540

a small change of P_{ee} and/or $P_{\mu e}$ within about 1σ can bring back ϑ_{23} to maximal

difficult to improve ϑ_{23} from $P_{\mu\mu}$

$$\delta\vartheta_{23} \approx \sqrt{\delta P_{\mu\mu}} / 2$$

$$\delta P_{\mu\mu} \approx 0.01$$



$$\delta\vartheta_{23} \approx 0.05 \text{ rad } (2.9^\circ)$$

ϑ_{23} nearly maximal would be a crucial piece of information

ϑ_{23} cannot be made maximal by RGE evolution
[barring tuning of b.c. and/or ad hoc threshold corrections]

no maximal ϑ_{23} from an **exact** symmetry

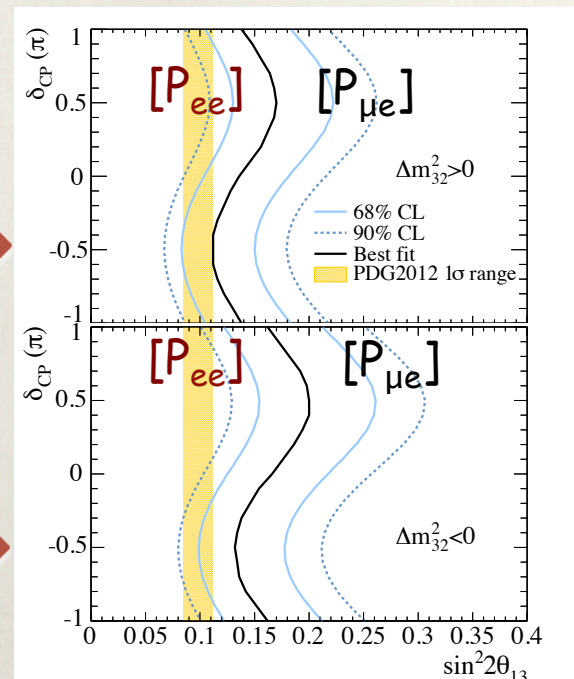
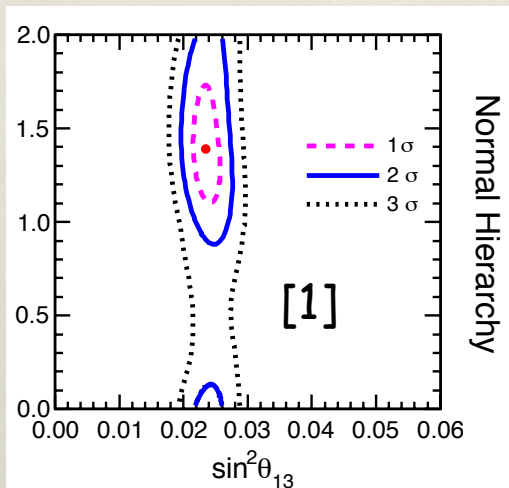
broken **abelian symmetries** do not work
[not a theorem but no counterexamples]



we are left with broken **non-abelian symmetries**

2

$$\delta_{CP} = -\pi/2 ?$$



[T2K: 1311.4750 and 1311.4114]

3

 U_{PMNS} close to TB (BM,...) ?

discrete flavor symmetries showed
very efficient to reproduce U_{TB} , U_{BM}, \dots

indirect: symmetries of m_ν and $(m_e + m_e)$
have no direct relation to G_f

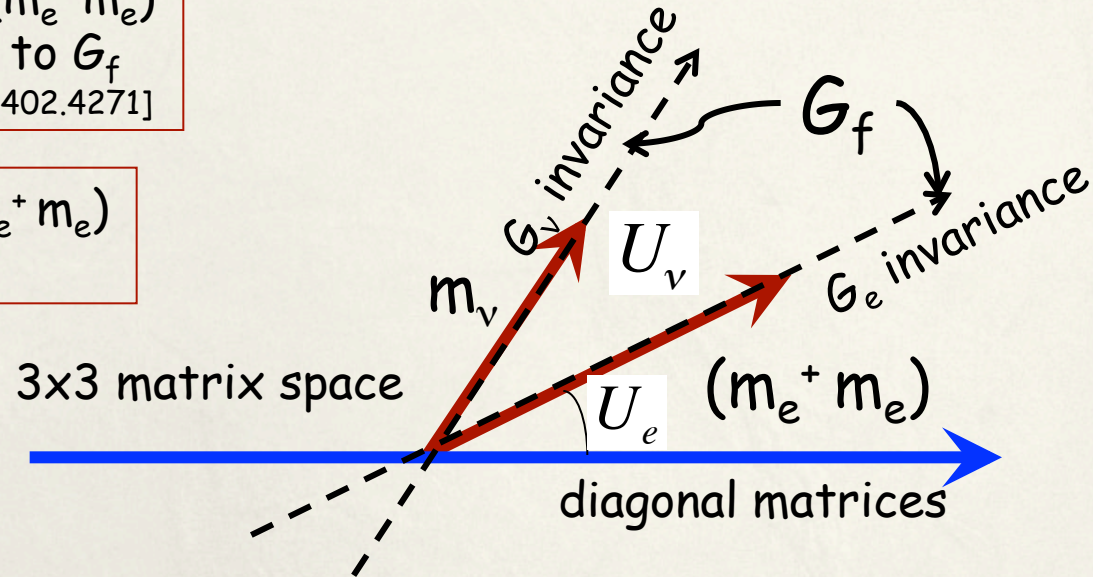
[see King, Merle, Morisi, Shimizu and Tanimoto 1402.4271]

direct: symmetries of m_ν and $(m_e + m_e)$
are subgroups of G_f

4 predictions

$$\vartheta_{12}^0 \quad \vartheta_{23}^0 \quad \vartheta_{13}^0 \quad \delta^0 \pmod{\pi}$$

$$U_{PMNS} = U_{PMNS}^0 + \text{corrections}$$



Majorana neutrinos
imply $G_\nu \leq Z_2 \times Z_2$

smallest group leading to TB:
 $S_4 \approx (A_4 + \text{accidental symmetry})$

neutrino masses fitted,
not predicted.

expectation for $U_{PMNS}^0 = U_{TB}$

$$\begin{cases} \vartheta_{13}^0 = 0 \\ \vartheta_{23}^0 = \frac{\pi}{4} \end{cases}$$



$$\begin{cases} \vartheta_{13} = O(\text{few degrees}) \\ \vartheta_{23} = \text{close to } \frac{\pi}{4} \end{cases}$$

not to spoil the
agreement with ϑ_{12}

wrong!

1 add large corrections $O(\vartheta_{13}) \approx 0.2$

- predictability is lost since in general correction terms are many
- new dangerous sources of FC/CPV if NP is at the TeV scale

2 relax symmetry requirements

[Hernandez, Smirnov 1204.0445]

G_e as before

$$G_\nu = Z_2$$

2 predictions:

2 combinations of

$$\vartheta_{12}^0 \quad \vartheta_{23}^0 \quad \vartheta_{13}^0 \quad \delta_{CP}$$

two deformations of TB, called Trimaximal [TM] mixing

TM₁

$$U^0 = U_{TB} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & e^{i\delta} \sin \alpha \\ 0 & -e^{-i\delta} \sin \alpha & \cos \alpha \end{pmatrix}$$

TM₂

$$U^0 = U_{TB} \times \begin{pmatrix} \cos \alpha & 0 & e^{i\delta} \sin \alpha \\ 0 & 1 & 0 \\ -e^{-i\delta} \sin \alpha & 0 & \cos \alpha \end{pmatrix}$$

leads to testable sum rules

$$\sin^2 \vartheta_{12} = \frac{1}{3} - \frac{2}{3} \sin^2 \vartheta_{13} + O(\sin^4 \vartheta_{13})$$

$$\sin^2 \vartheta_{12} = \frac{1}{3} + \frac{1}{3} \sin^2 \vartheta_{13} + O(\sin^4 \vartheta_{13})$$

$$\sin^2 \vartheta_{23} = \frac{1}{2} - \sqrt{2} \sin \vartheta_{13} \cos \delta_{CP} + O(\sin^2 \vartheta_{13})$$

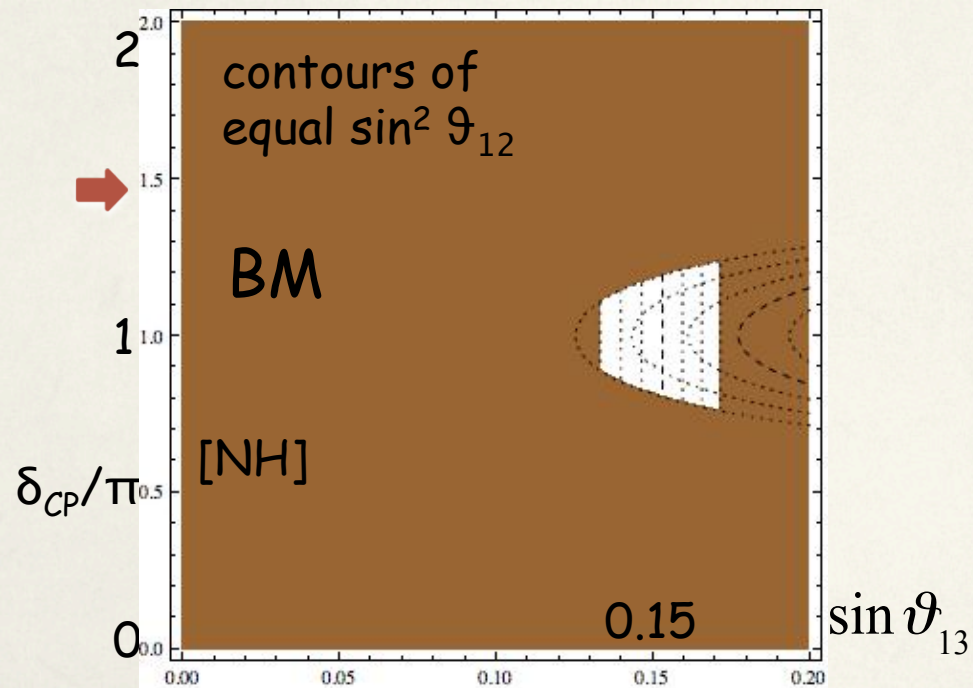
$$\sin^2 \vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}} \sin \vartheta_{13} \cos \delta_{CP} + O(\sin^2 \vartheta_{13})$$

4 change LO pattern

$$U_{PMNS}^0 = U_{BM}$$

corrected by $U_{e_{12}}$

$$\sin^2 \vartheta_{12} = \frac{1}{2} + \sin \vartheta_{13} \cos \delta_{CP} + O(\sin^2 \vartheta_{13})$$



5 include CP in the SB pattern

$$G_{CP} = G_f \rtimes CP$$

[F. F. C. Hagedorn and R. Ziegler 1211.5560, 1303.7178
Ding, King, Luhn, Stuart 1303.6180
Ding, King, Stuart 1307.4212]

$$G_e$$

$$G_\nu = Z_2 \times CP$$

mixing angles and CP violating phases

$$(\vartheta_{12}^0, \vartheta_{23}^0, \vartheta_{13}^0, \delta^0, \alpha^0, \beta^0)$$

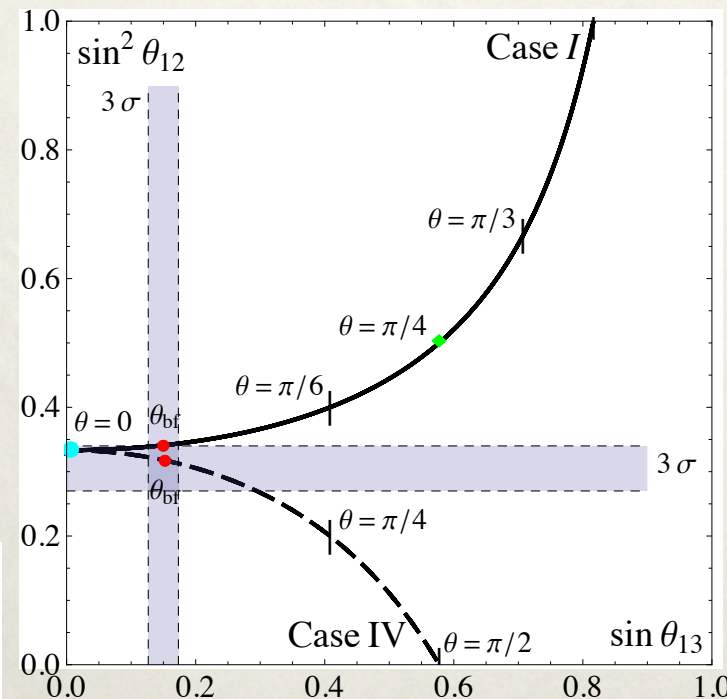
predicted in terms of a single real parameter $0 \leq \vartheta \leq \pi$

2 examples with $G_f = S_4$ $G_e = Z_3$

$$\sin^2 \vartheta_{23}^0 = \frac{1}{2} \quad |\sin \delta^0| = 1$$

$$\sin \alpha^0 = 0$$

$$\sin \beta^0 = 0$$



no conclusions...



back up slides

Conclusions

flavour symmetries are a useful tool in our quest of the origin of \mathcal{Y} but no compelling and unique picture have emerged so far.
Present data can be described within widely different frameworks [despite the constant, impressive progress on the experimental side]

simple schemes with a minimal amount of structure can well reproduce the main features of \mathcal{Y} in both quark and lepton sectors also in a GUT framework

main drawbacks: -- no precise questions/no precision tests allowed
[e.g. maximal ϑ_{23} unexplained]

-- more structure needed to suppress FCNC and CPV if there is new physics at the TeV scale

some special features [ϑ_{23} maximal, $\delta_{CP} = -\pi/2$, $U_{PMNS} \approx TB, BM, \dots$] can survive experimental refinements and guide us in the search of a unifying principle for the flavour sector.

θ_{23} maximal from some flavour symmetries ?

$\vartheta_{23} = \pi/4$ can never arise in the limit of an **exact realistic symmetry**

a no-go theorem

[F. 2004]

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

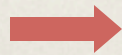
symmetric limit

symmetry breaking effects:
vanishing when flavour symmetry F is **exact**

realistic symmetry:

(1) $|\delta m_l^0| < |m_l^0|$

(2) m_l^0 has rank ≤ 1



$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan \vartheta_{23}^0 = \tan \vartheta_{23}^\nu \cos \vartheta_{12}^e + \left(\frac{\tan \vartheta_{13}^\nu}{\cos \vartheta_{23}^\nu} \right) \sin \vartheta_{12}^e$$

undetermined

$$\vartheta_{23} = \frac{\pi}{4}$$

determined entirely by breaking effects
(different, in general, for ν and e sectors)

$G_f = \text{discrete flavor symmetry}$

$$U_{PMNS} = U_{PMNS}^0 + \text{corrections} \quad \leftarrow$$

some simple pattern, exactly reproduced by a flavor symmetry

well motivated before 2012

$$U_{PMNS}^0 = U_{TB} \equiv \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \quad \text{Tribimaximal Mixing}$$

discrete flavor symmetries showed very efficient to reproduce U_{PMNS}^0

still justified today?

$$U_{TB} \approx \begin{pmatrix} 0.82 & 0.58 & 0 \\ -0.41 & 0.58 & -0.71 \\ -0.41 & 0.58 & 0.71 \end{pmatrix}$$

$$|U_{PMNS}| = \begin{pmatrix} 0.80 \div 0.85 & 0.51 \div 0.59 & 0.13 \div 0.18 \\ 0.21 \div 0.54 & 0.42 \div 0.73 & 0.58 \div 0.81 \\ 0.22 \div 0.55 & 0.41 \div 0.73 & 0.57 \div 0.80 \end{pmatrix}$$

[3σ ranges from Gonzalez-Garcia, Maltoni, Salvado, Schwetz 1209.3023]

2011/2012 breakthrough

- LBL experiments searching for $\nu_\mu \rightarrow \nu_e$ conversion
- SBL reactor experiments searching for anti- ν_e disappearance

[see Fogli's talk]

	Lisi [NeuTel 2013]	[1209.3023] [G-Garcia, Maltoni, Salvado, Schwetz]
$\sin^2 \vartheta_{13}$	$0.0241^{+0.0025}_{-0.0025}$ (NO) $0.0244^{+0.0023}_{-0.0025}$ (IO)	$0.0227^{+0.0023}_{-0.0024}$
$\sin^2 \vartheta_{23}$	$0.386^{+0.024}_{-0.021}$ (NO) $0.392^{+0.039}_{-0.022}$ (IO)	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$



10 σ away from 0

impact on flavor symmetry (part 3)

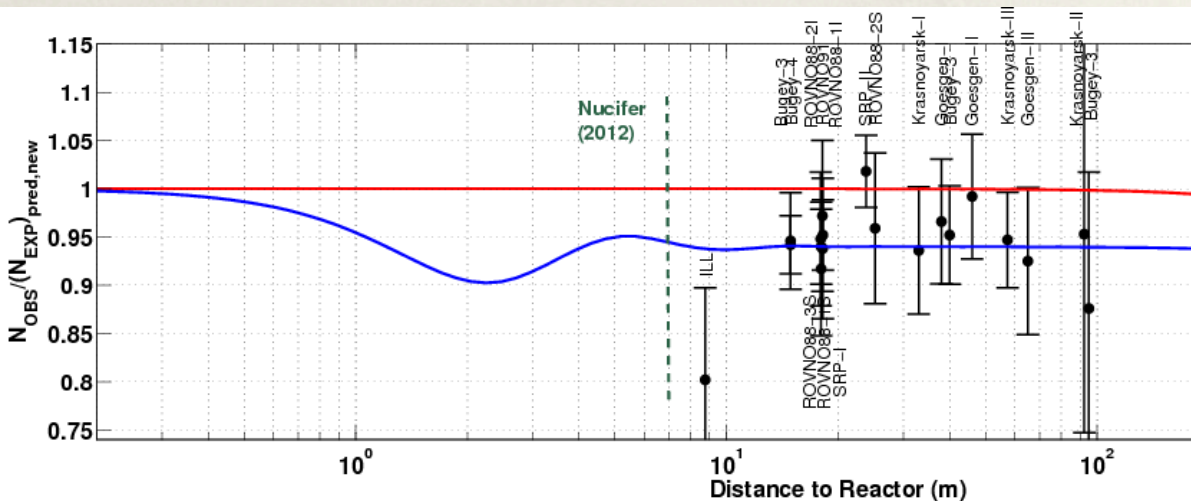


hint for non maximal ϑ_{23}

sterile neutrinos coming back

1 reactor anomaly (anti- ν_e disappearance)

re-evaluation of reactor anti- ν_e flux: new estimate 3.5% higher than old one



$$(\Phi_{\text{exp}} - \Phi_{\text{th}}) / \Phi_{\text{th}} \approx -6\%$$

[th. uncertainty?]

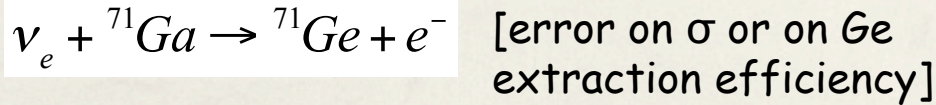
very SBL $L \leq 100$ m

$$\vartheta_{es} \approx 0.2$$

$$\Delta m^2 \approx m_s^2 \geq 1 \text{ eV}^2$$

supported by the **Gallium anomaly**

ν_e flux measured from high intensity radioactive sources in Gallex, Sage exp



most recent cosmological limits

[depending on assumed cosmological model, data set included,...]

relativistic degrees of freedom at recombination epoch

$$N_{\text{eff}} = 3.30 \pm 0.27$$

[Planck, WMAP, BAO, high multiple CMB data]

fully thermalized non relativistic ν

$$N_{\text{eff}} < 3.80 \quad (95\% \text{ CL})$$

$$m_s < 0.42 \text{ eV} \quad (95\% \text{ CL})$$

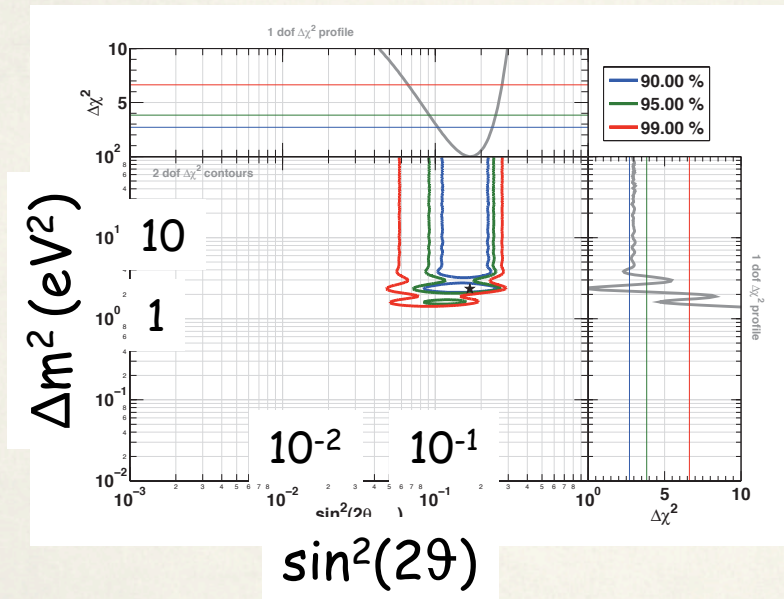
2 long-standing claim

evidence for $\nu_\mu \rightarrow \nu_e$ appearance in accelerator experiments

exp		$E(\text{MeV})$	$L(\text{m})$
<i>LSND</i>	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$10 \div 50$	30
<i>MiniBoone</i>	$\nu_\mu \rightarrow \nu_e$	$300 \div 3000$	541
	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$		

3.8σ

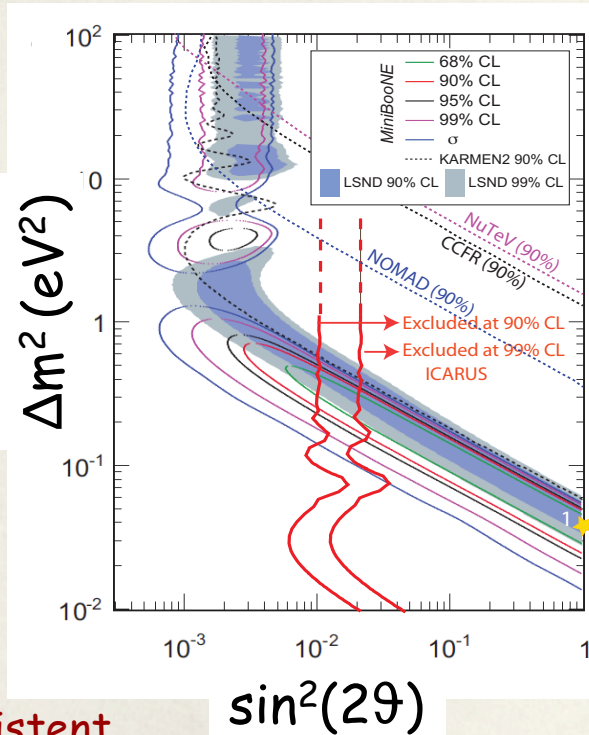
3.8σ [signal from low-energy region]



parameter space limited by negative results from Karmen and ICARUS

$$\vartheta_{e\mu} \approx 0.035$$

$$\Delta m^2 \approx 0.5 \text{ eV}^2$$



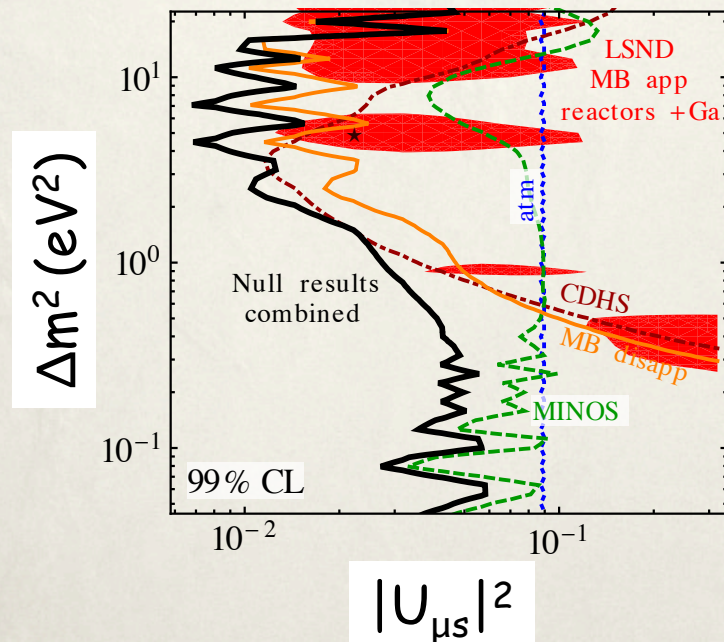
3

interpretation in 3+1 scheme: **inconsistent** (more than 1s disfavored by cosmology)

$$\underbrace{\vartheta_{e\mu}}_{0.035} \approx \underbrace{\vartheta_{es}}_{0.2} \times \vartheta_{\mu s} \quad \rightarrow \quad \vartheta_{\mu s} \approx 0.2$$

predicted suppression in ν_μ disappearance experiments: **undetected**

by ignoring LSND/Miniboone data the reactor anomaly can be accommodated by $m_s \geq 1 \text{ eV}$ and $\vartheta_{es} \approx 0.2$
[not suitable for WDM, more on this later]



predictions based on $G_f = A_4 \times Z_3 \times U(1)_{FN}$ [+ SEE-SAW] [Altarelli, F 2005]

lepton mixing is TB, by construction, plus NLO corrections of order $0.005 < u < 0.05$
 at the LO neutrino mass spectrum depends on two complex parameters
 there is a sum rule among (complex) mass eigenvalues $m_{1,2,3}$

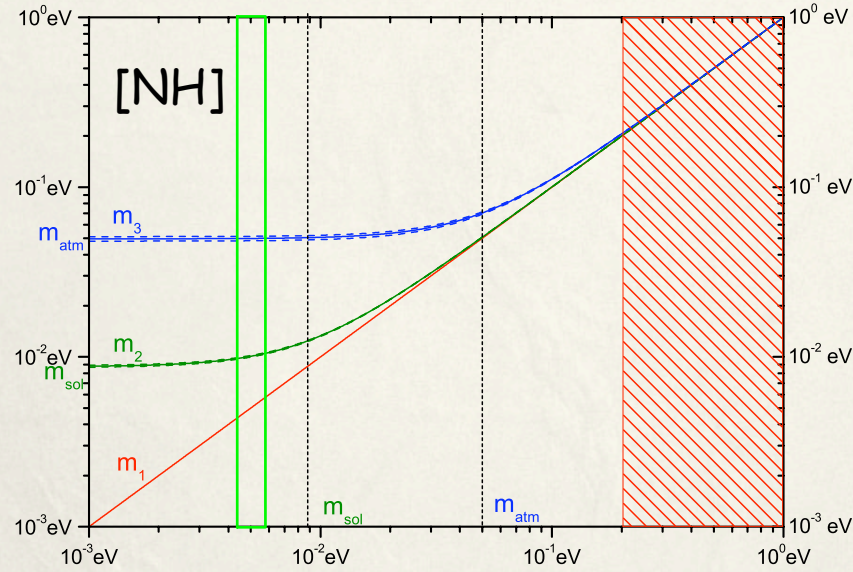
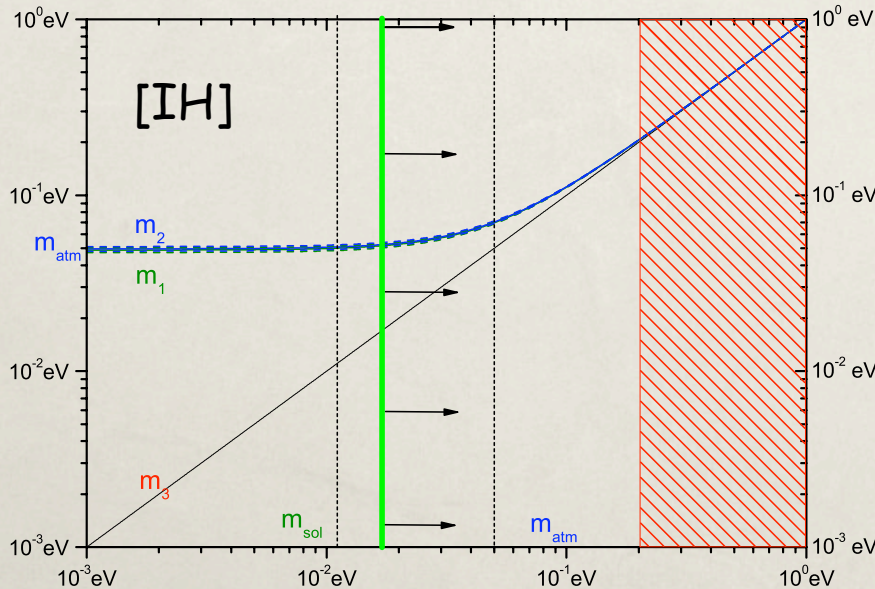
$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

both normal [NH] and inverted [IH] hierarchy are allowed

in the NH case the sum rule completely determines the spectrum

$$m_1 \approx 0.005 \text{ eV} \quad m_2 \approx 0.01 \text{ eV} \quad m_3 \approx 0.05 \text{ eV}$$

$$|m_{ee}| \approx 0.007 \text{ eV}$$



in the IH case the sum rule provides a lower bound on m_3

$$m_3 \geq 0.017 \text{ eV}$$

$$|m_{ee}| \geq 0.017 \text{ eV}$$

NLO corrections are negligible for NH and for IH close to the lower bound